The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

SMR 1760 - 20

COLLEGE ON

PHYSICS OF NANO-DEVICES

10 - 21 July 2006

Coherent excitonic matter

Presented by:

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Coherent excitonic matter

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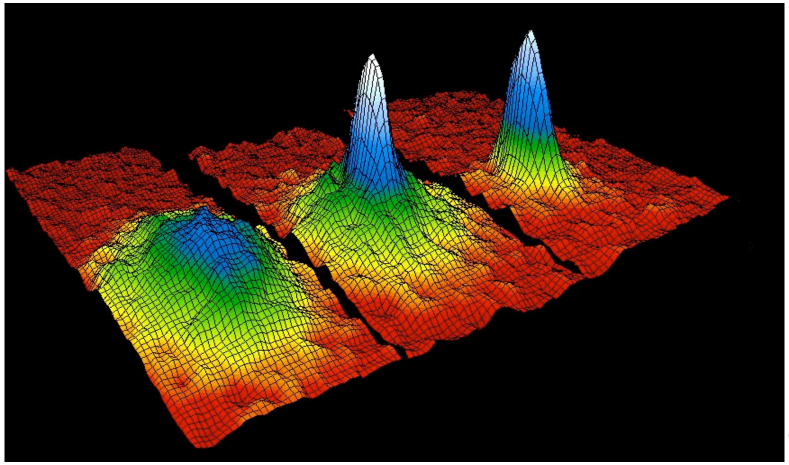
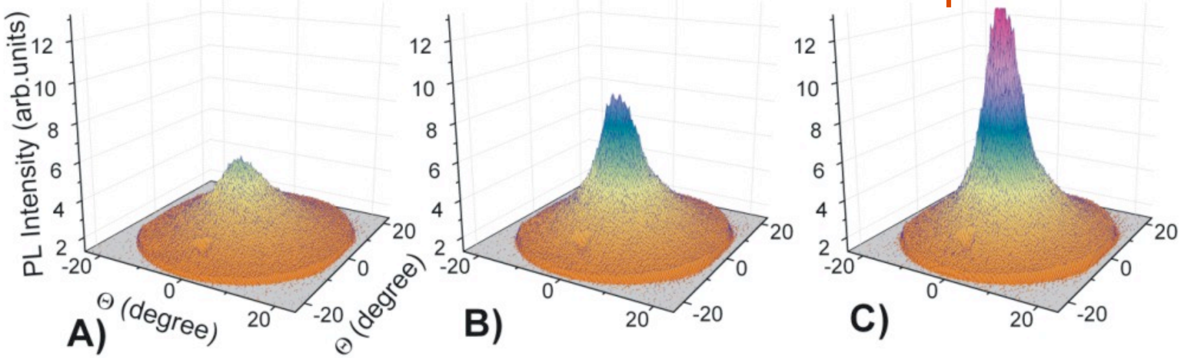


Rb atom condensate, JILA, Colorado

Momentum distribution of cold atoms

Momentum distribution of cold exciton-polaritons



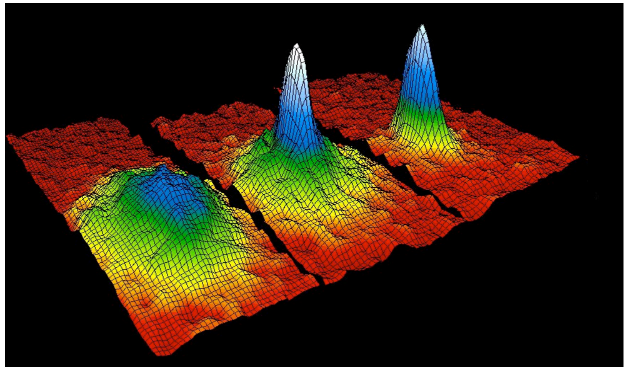
Exciton condensate ?, Kasprzak et al 2006

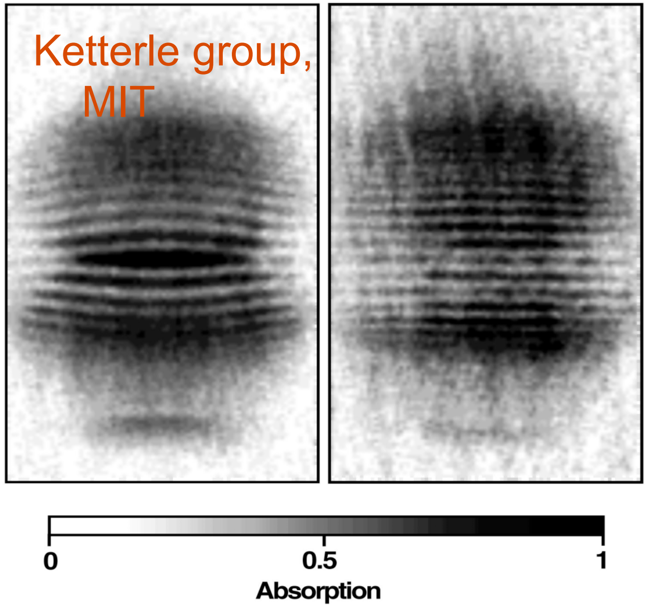
Bose-Einstein Condensation

- Macroscopic occupation of the ground state
  - Originally seen as a consequence of statistical physics of weakly interacting bosons
- Macroscopic quantum coherence
  - Interactions (exchange) give rise to macroscopic synchronisation

    $\psi \;\text{->}\; \psi\, e^{i\phi}$

  Genuine symmetry breaking, distinct from BEC
- Superfluidity
  - Rigidity of wavefunction – stiffness of the phase – gives rise to collective modes
- An array of two-level systems may have precisely the same character





Christiaan Huygens 1629-95

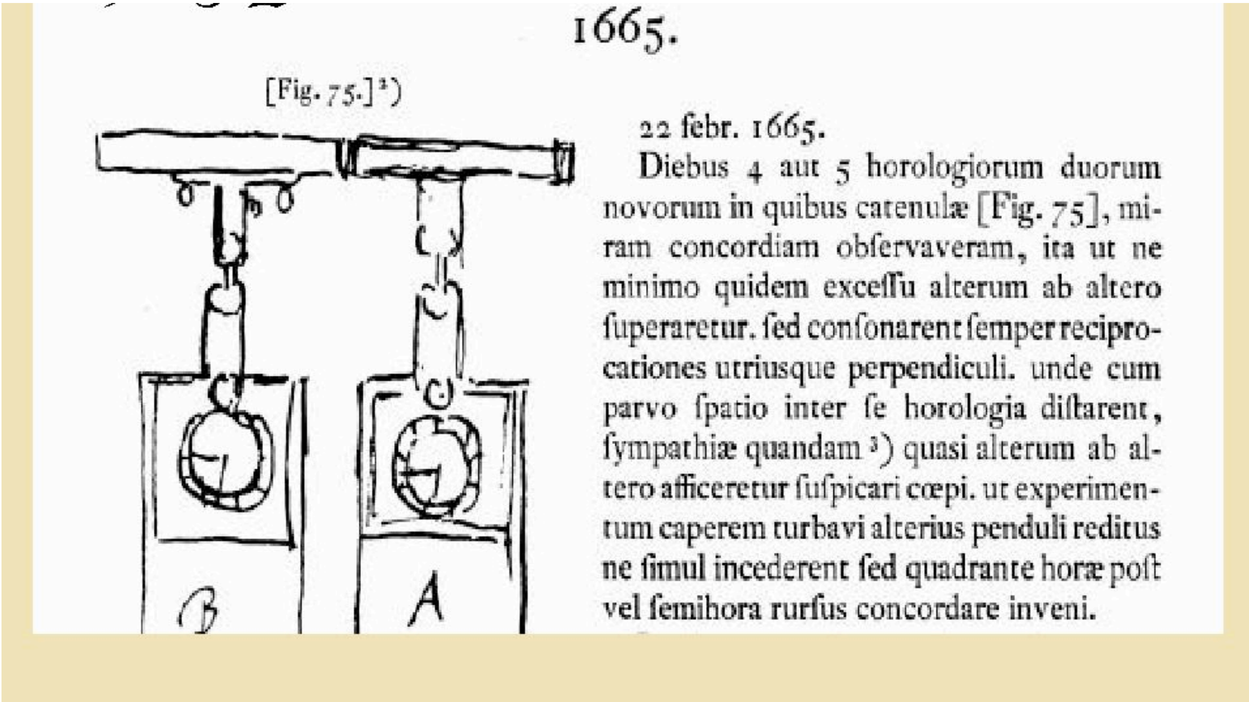
1656 – Patented the pendulum clock

1663 – Elected to Royal Society

1662-5 With Alexander Bruce, and sponsored by the Royal Society, constructed maritime pendulum clocks – periodically communicating by letter

Huygens Clocks

In early 1665, Huygens discovered ``..an odd kind of sympathy perceived by him in these watches [two pendulum clocks] suspended by the side of each other."



1665.

22 febr. 1665.

Diebus 4 aut 5 horologiorum duorum novorum in quibus catenulæ [Fig. 75], miram concordiam obſervaveram, ita ut ne minimo quidem exceſſu alterum ab altero ſuperaretur. ſed conſonarent ſemper reciprocationes utriusque perpendiculi. unde cum parvo ſpatio inter ſe horologia diſtarent, ſympathiæ quandam [3]) quasi alterum ab altero afficeretur ſuſpicari cœpi. ut experimentum caperem turbavi alterius penduli reditus ne ſimul incederent ſed quadrante horæ poſt vel ſemihora rurſus concordare inveni.

He deduced that effect came from "imperceptible movements" of the common frame supporting the clocks

"Huygens' Clocks", M. Bennett et al, Proc. Roy. Soc. A **458**, 563 (2002)

Spontaneous synchronization

- Spontaneous synchronisation is a general property of coupled oscillators, when there is
  - feedback from neighbours
  - non-linearity
  - not “too much” random noise from the environment
- Many examples in biology
  - synchronized insect emergence: 13 year and 17 year locusts (cicadas)
  - synchonisation in heart muscle
  - epilepsy
- and in physics
  - lasers
  - superconductors
  - Bose-Einstein condensation
- and in both ….
  - Magnetic Resonance Imaging (MRI)

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Issues for these lectures

- Characteristics of a Bose condensate
- Excitons, and why they might be candidates for BEC

How do you make a BEC wavefunction based on pairs of fermions?

- BCS (interaction-driven high density limit) to Bose (low density limit) crossover

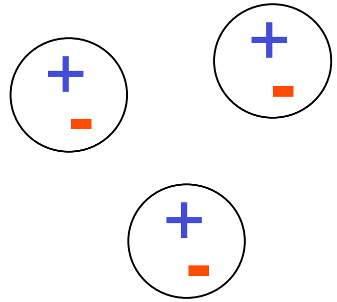
- Excitons may decay directly into photons

What happens to the photons if the "matter" field is coherent?

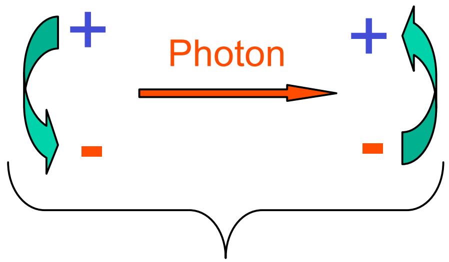
- Two level systems interacting via photons

How do you couple to the environment ?

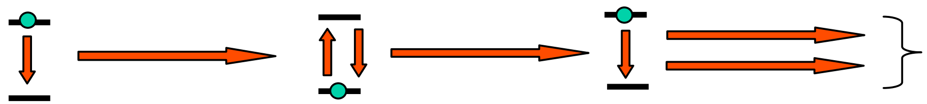
- Decoherence phenomena and the relationship to lasers



Excitons are the solid state analogue of positronium



Combined excitation is called a **polariton**



Emission

Absorption and re-emission

Stimulated emission

These photons have a fixed phase relationship

Stimulated Emission > Absorption

⇒ Laser

Outline

- General review
- Exciton condensation
  - mean field theory of Keldysh – BCS analogy
  - BCS-BEC crossover
  - broken symmetries, tunnelling, and (absence of) superfluidity
- Polaritons (coherent mixture of exciton and photon)
  - mean field theory
  - BCS-BEC crossover (again) and 2D physics
  - signatures of condensation
  - disorder
  - pairbreaking
  - phase-breaking and decoherence
- Review of Experiment intermingled
- Other systems (if there is time)
  - quantum Hall bilayers
  - “triplons” in quantum spin systems
  - ultracold fermions and the Feshbach resonance

Background material and details for the lectures

I will not give detailed derivations in lectures, but they can all be found in these papers

Reviews

Bose-Einstein Condensation, ed Griffin, Snoke, and Stringari, CUP, (1995)

PB Littlewood and XJ Zhu, Physica Scripta T68, 56 (1996)

P. B. Littlewood, P. R. Eastham, J. M. J. Keeling, F. M. Marchetti, B. D. Simons, M. H. Szymanska. J. Phys.: Condens. Matter 16 (2004) S3597-S3620. cond-mat/0407058

Basic equilibrium models:

Mean field theory (excitons): C. Comte and P. Nozieres, J. Phys. (Paris),43, 1069 (1982); P. Nozieres and C. Comte, ibid., 1083 (1982); P. Nozieres, Physica 117B/118B, 16 (1983).

Mean field theory (polaritons): P. R. Eastham, P. B. Littlewood , Phys. Rev. B 64, 235101 (2001) cond-mat/0102009

BCS-BEC crossover (polaritons): Jonathan Keeling, P. R. Eastham, M. H. Szymanska, P. B. Littlewood, Phys. Rev. Lett. 93, 226403 (2004) cond-mat/0407076; Phys. Rev. B 72, 115320 (2005)

Effects of disorder

F. M. Marchetti, B. D. Simons, P. B. Littlewood, Phys. Rev. B 70, 155327 (2004) cond-mat/0405259

Decoherence and non-equilibrium

M. H. Szymanska, P. B. Littlewood, B. D. Simons, Phys. Rev. A 68, 013818 (2003) cond-mat/0303392

M. H. Szymanska, J. Keeling, P. B. Littlewood Phys. Rev. Lett. 96 230602 (2006); cond-mat/0603447

F. M. Marchetti, J. Keeling, M. H. Szymanska, P. B. Littlewood, Phys. Rev. Lett. 96, 066405 (2006) cond-mat/0509438

Physical signatures

Alexander V. Balatsky, Yogesh N. Joglekar, Peter B. Littlewood, Phys. Rev. Lett. 93, 266801 (2004). cond-mat/0404033

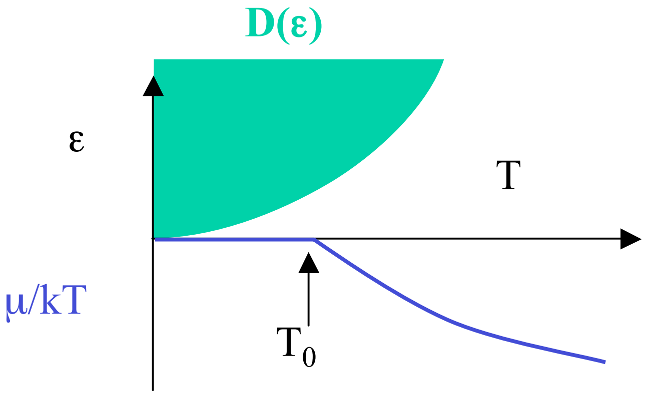
Jonathan Keeling, L. S. Levitov, P. B. Littlewood, Phys. Rev. Lett. 92, 176402 (2004) cond-mat/0311032

P. R. Eastham, P. B. Littlewood cond-mat/0511702

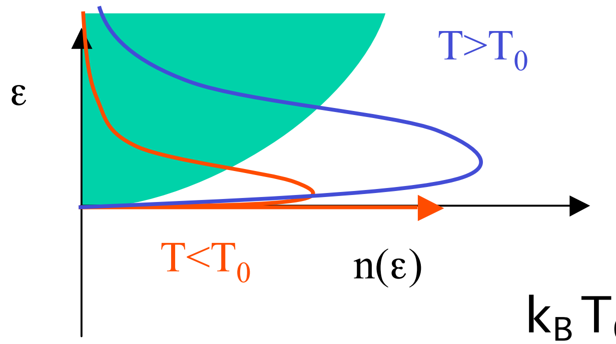
Bose-Einstein condensation

- Macroscopic ground state occupation $n = \int d\epsilon \frac{D(\epsilon)}{e^{\beta(\epsilon-\mu)} - 1}$ finite as $\mu \to 0$

Density of states



Thermal occupation



$k_B T_0 = \frac{1.3}{r_s^2} Ryd. \propto \frac{n^{2/3}}{m}$

- Macroscopic phase coherence

Condensate described by macroscopic wave function $\psi\, e^{i\phi}$ which arises from interactions between particles

$\psi \;\to\; \psi\, e^{i\phi}$

Genuine symmetry breaking, distinct from BEC

Couple to internal degrees of freedom - e.g. dipoles, spins

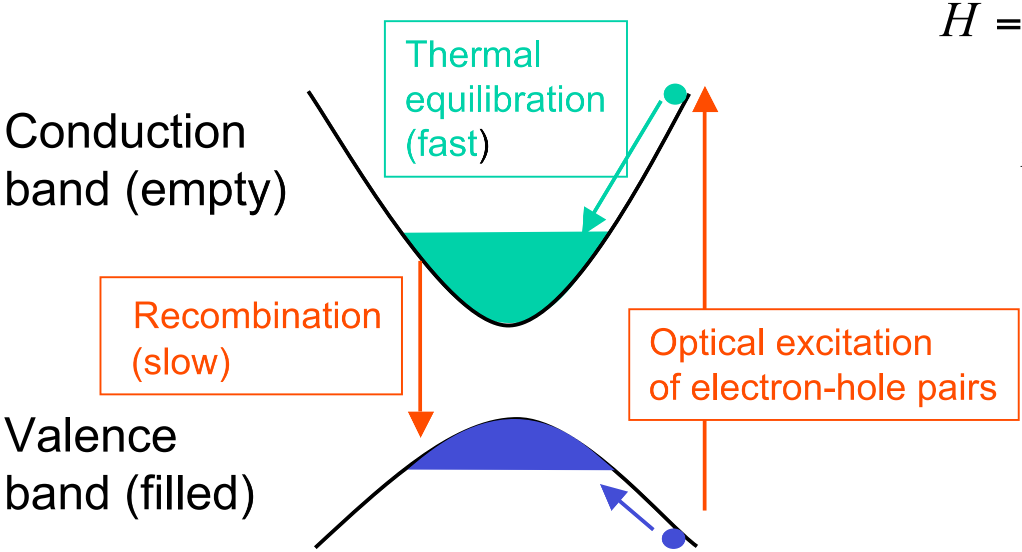
- Superfluidity

Implies linear Goldstone mode in an infinite system with dispersion $\omega = v_s k$ and hence a superfluid stiffness $\alpha\, v_s$

BEC myths

- BEC requires delocalised “free” particles -> condensation in momentum space
  - disorder is not necessarily bad (especially for polaritons)
  - can construct a model with BEC ground state that is completely disordered and has no spatial coordinates
- BEC requires the particles to be good bosons (i.e. separation >> radius)
  - crossover to dense limit analogous to BCS, condensate of pairs of strongly overlapping fermions
  - exciton-exciton scattering is only bad if the system is out of equilibrium
- BEC is a phenomenon of statistical physics of weakly interacting bosons
  - in most likely situations for observation of excitonic BEC, interactions will dominate
  - it is a quantum phase transition --- focus on phase coherence, order parameter
- BEC coherence is distinct from the coherence in a laser
  - distinction between phase-locking of classical oscillators, lasers, and BEC is subtle
  - however, a polariton laser is not necessarily BEC of polaritons
  - decoherence and non-equilibrium effects are the principal enemy of BEC

Excitons in semiconductors



$$H = \sum_i [T_i^e + T_i^h] + \sum_{i,j} [V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}]$$

$$T_i^\alpha = \frac{p_{i\alpha}^2}{2m_\alpha} \qquad V_{ij}^{\alpha\beta} = \frac{e^2}{\varepsilon |r_{i\alpha} - r_{j\beta}|}$$

At high density - an electron-hole plasma

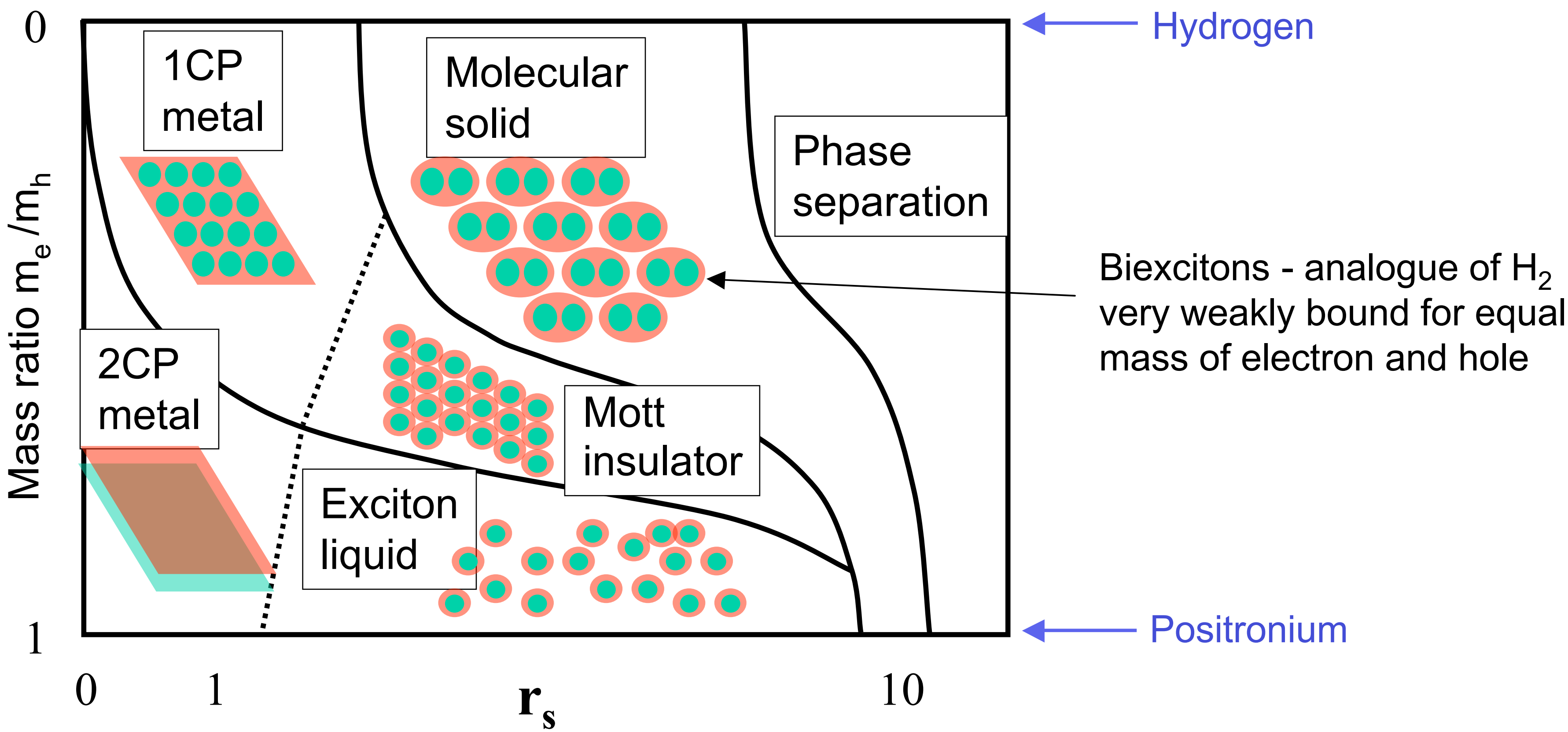
At low density - excitons

Exciton - bound electron-hole pair (analogue of hydrogen, positronium)
In GaAs, $m^* \sim 0.1\ m_e$, $\varepsilon = 13$

- ✍ Rydberg = 5 meV (13.6 eV for Hydrogen)
- Bohr radius = 7 nm (0.05 nm for Hydrogen)

Measure density in terms of a dimensionless parameter r_s - average spacing between excitons in units of a_{Bohr} $\quad 1/n = \frac{4\pi}{3} a_{Bohr}^3 r_s^3$

Speculative phase diagram of electron-hole system (T=0)



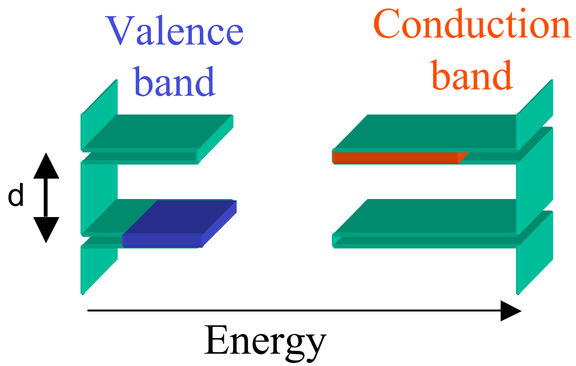
Radius containing one electron and one hole in units of Bohr radius

PBLittlewood and XJZhu Physica Scripta T68, 56 (1996)

Interacting electrons and holes in double quantum well

Two parabolic bands, direct gap, equal masses

Layers of electrons and holes in quantum wells spaced a distance d apart



$$H = \sum_i [T_i^e + T_i^h] + \sum_{i,j} [V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}] \qquad T_i^\alpha = \frac{p_{i\alpha}^2}{2m_\alpha} \qquad V_{ij}^{\alpha\beta} = \frac{e^2}{\varepsilon |r_{i\alpha} - r_{j\beta}|}$$

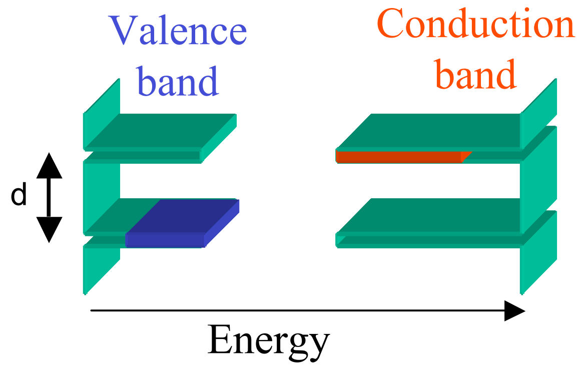
Units: density- $n = 1/\pi (r_s a_B)^2$ $\qquad a_B = \varepsilon \hbar^2 / me^2$

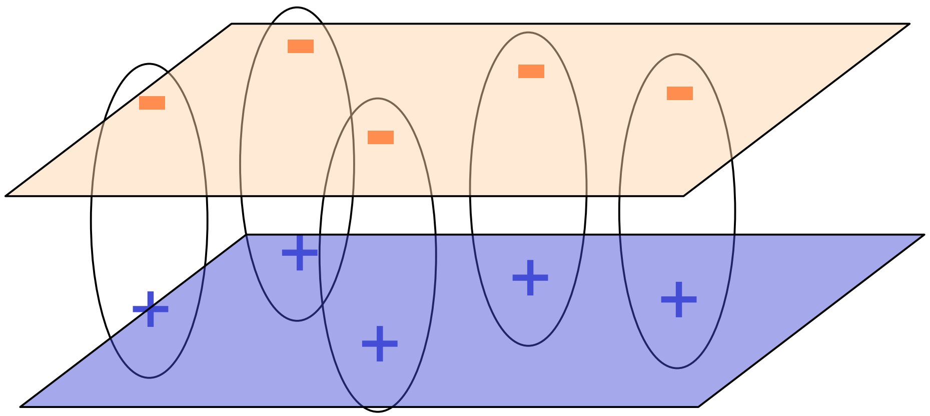
energy- Rydberg $e^2 / 2\varepsilon a_B$

Ignore interband exchange - spinless problem

Ignore biexcitons - disfavoured by dipole-dipole repulsion

Coupled Quantum Wells





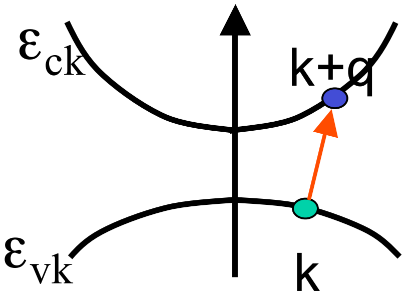
Neutral bosons with repulsive dipolar interaction in 2D

Binding energy few meV in GaAs
Bohr radius ~ 10 nm

Long lifetime up to 100 nsec – recombination by tunnelling through barrier

Excitonic insulator

A dilute Bose gas should condense - generalisation to dense electron-hole system is usually called an excitonic insulator



Single exciton wavefunction (ϕ_k is Fourier transform of hydrogenic wavefunction)

$$\Phi(q=0) = \sum_k \phi_k a_{ck}^{+} a_{vk} \left|0\right\rangle$$

This is not a boson

$e^{\tilde{o}'}\ {}_k ?_k a^y_{ck} a_{vk} j0 >$? $''P\ {}_k ?_k a^y_{ck} a_{vk}\ '^N j0 >$?

Coherent wavefunction for condensate in analogy to BCS theory of superconductivity

[Keldysh and Kopaev 1964]

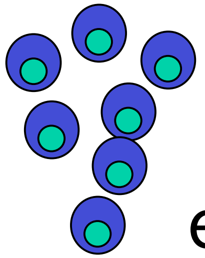
$$\Phi_{BCS} = \prod_k \left[u_k + v_k a_{ck}^{+} a_{vk}\right] \left|0\right\rangle ; \quad \left|u_k\right|^2 + \left|v_k\right|^2 = 1$$

u_k ; v_k variational solutions of H = K.E. + Coulomb interaction

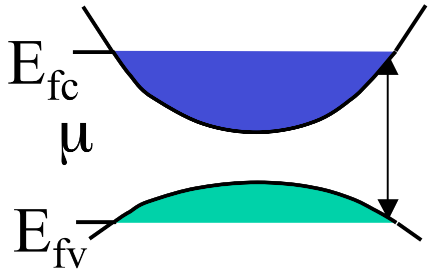
Same wavefunction can describe a Bose condensate of excitons at low density, as well as two overlapping Fermi liquids of electrons and holes at high density

Mean field theory of excitonic insulator

$$\Phi_{BCS} = \prod_k \left[u_k + v_k a^+_{ck} a_{vk} \right] |0\rangle$$



$$e^{\sum_k \phi_k a^\dagger_{ck} a_{vk}} = \prod_k \left[1 + \phi_k a^\dagger_{ck} a_{vk} \right]$$

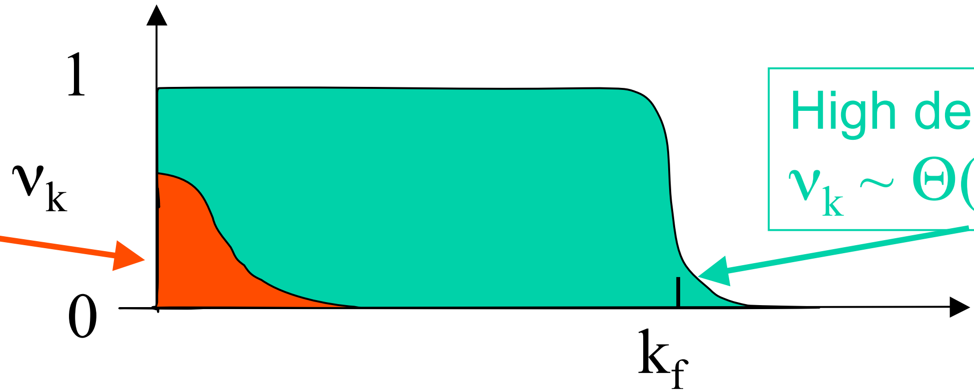


Bose condensation of excitons

BCS-like instability of Fermi surfaces

Low density

$\nu_k \sim n^{1/2} \phi_k$



High density

$\nu_k \sim \Theta(k-k_f)$

Special features: order parameter; gap

$$\langle a^+_{ck} a_{vk} \rangle = u_k v_k = (\Delta_k / 2E_k); \quad E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$$

Excitation spectra

+(-)E_k is energy to add (remove) particle-hole pair from condensate (total momentum zero)

$$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$$

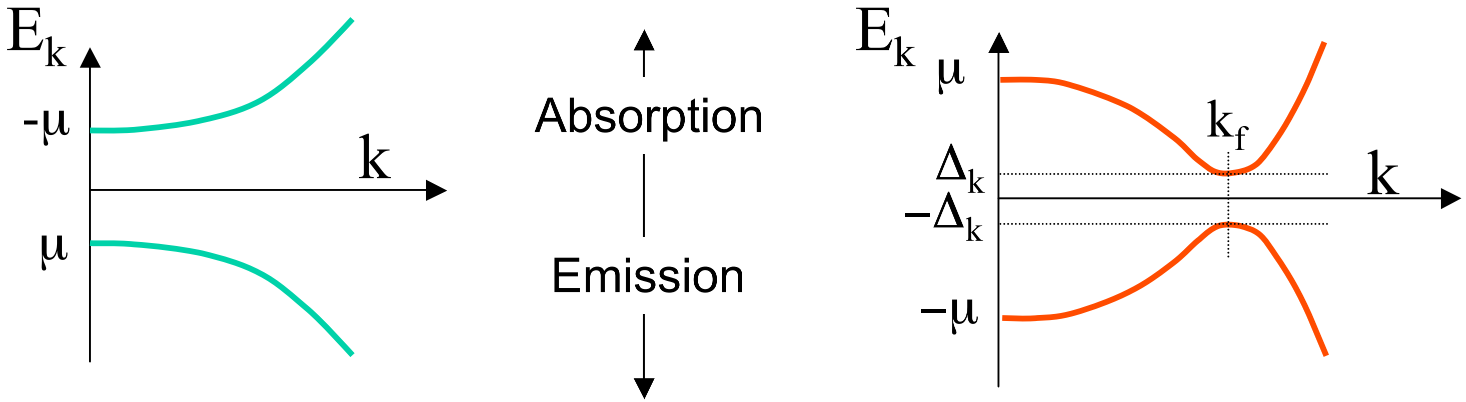
Band energy

Chemical potential (<0 for bound exciton)

Correlation energy

Low density $\mu<0$
Chemical potential
below band edge

High density $\mu>0$
No bound exciton
below band edge



Mean field solution

$$H_{eh} = \sum_k \left(\epsilon_{ck} a^\dagger_{ck} a_{ck} + \epsilon_{vk} a^\dagger_{vk} a_{vk}\right) + \frac{1}{2}\sum_q \left(V^{ee}_q \rho^e_q \rho^e_{-q} + V^{hh}_q \rho^h_q \rho^h_{-q} - 2V^{eh}_q \rho^e_q \rho^h_{-q}\right)$$

$$V^{ee}_q = V^{hh}_q = 2\pi/q \;;\; V^{eh}_q = 2\pi e^{-qd}/q \quad \rho_q = \sum_k a^\dagger_{k+q} a_k$$ 2D coulomb; layer separation *d*

$$\epsilon_{vk} = -E_{gap} - \epsilon_{ck}$$ Particle hole symmetry (a simplification)*

$$|\Psi\rangle = \prod_k \left(u_k + v_k a^\dagger_{ck} a_{vk}\right)|vac\rangle \;; \quad |u_k|^2 + |v_k|^2 = 1$$

$$f = \langle H_{eh}\rangle - \mu\langle n\rangle$$

- Variational (BCS) wavefunction
- Introduce chemical potential
- Minimise free energy per particle

$$\xi_k = \epsilon_k - \mu - 2\sum_{k'} V^{ee}_{k-k'} n_{k'}$$ Renormalised single particle energy

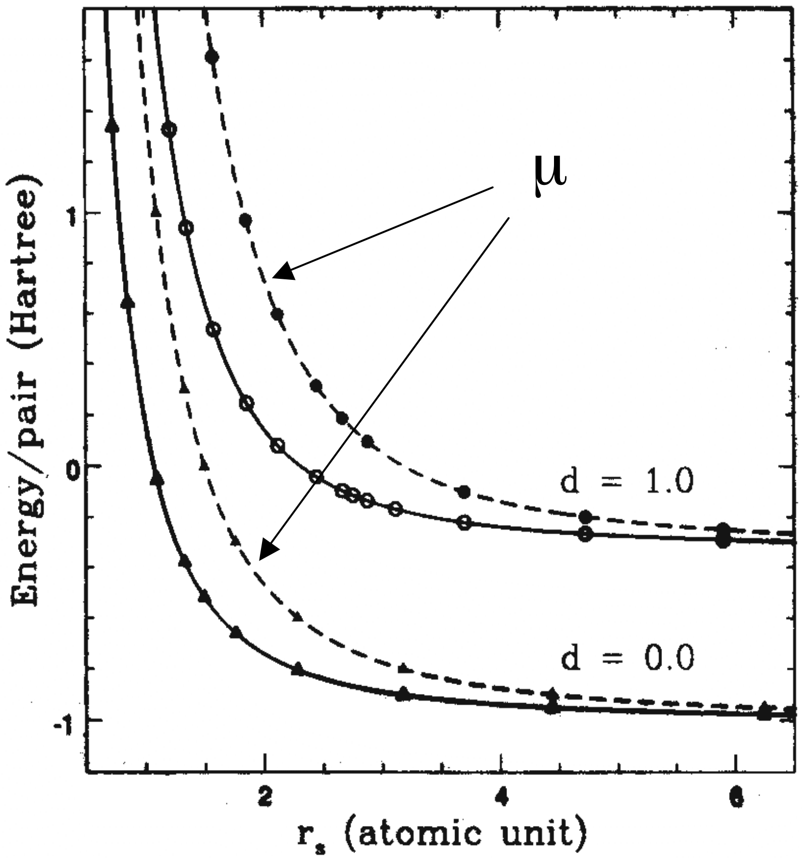
$$\Delta_k = 2\sum_{k'} V^{eh}_{k-k'} \left\langle a^\dagger_{ck} a_{hk}\right\rangle = \sum_{k'} V^{eh}_{k-k'} \Delta_{k'}/E_{k'}$$ Gap equation

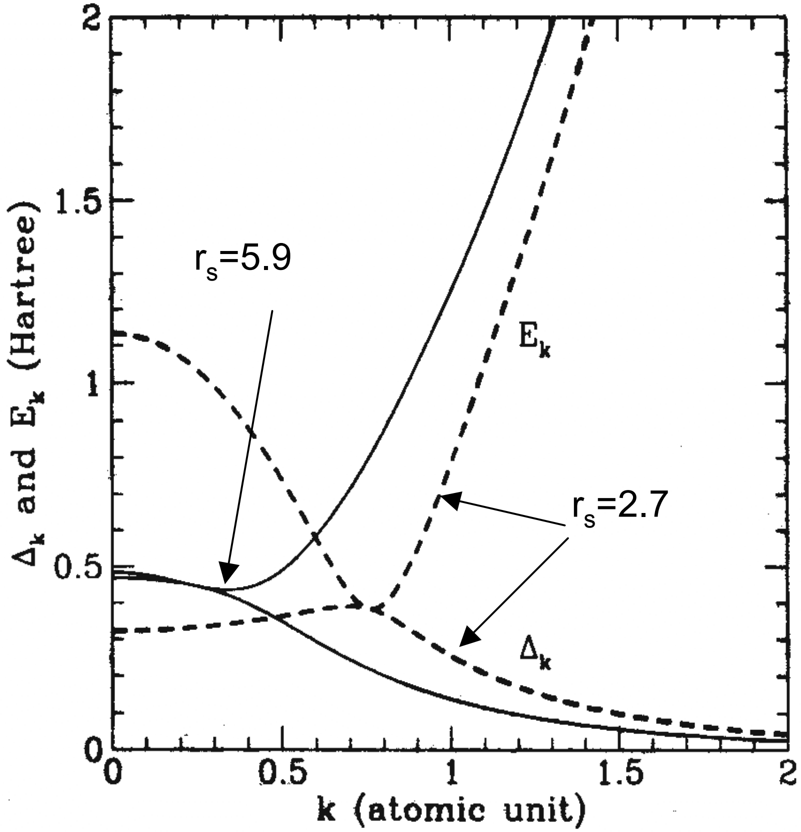
$$E^2_k = \xi^2_k + \Delta^2_k$$ New spectrum of quasiparticles with gap

Comte and Nozieres, J.Phys. (Paris) 43, 1069 (1992)
Zhu et al PRL 74, 1633 (1995)

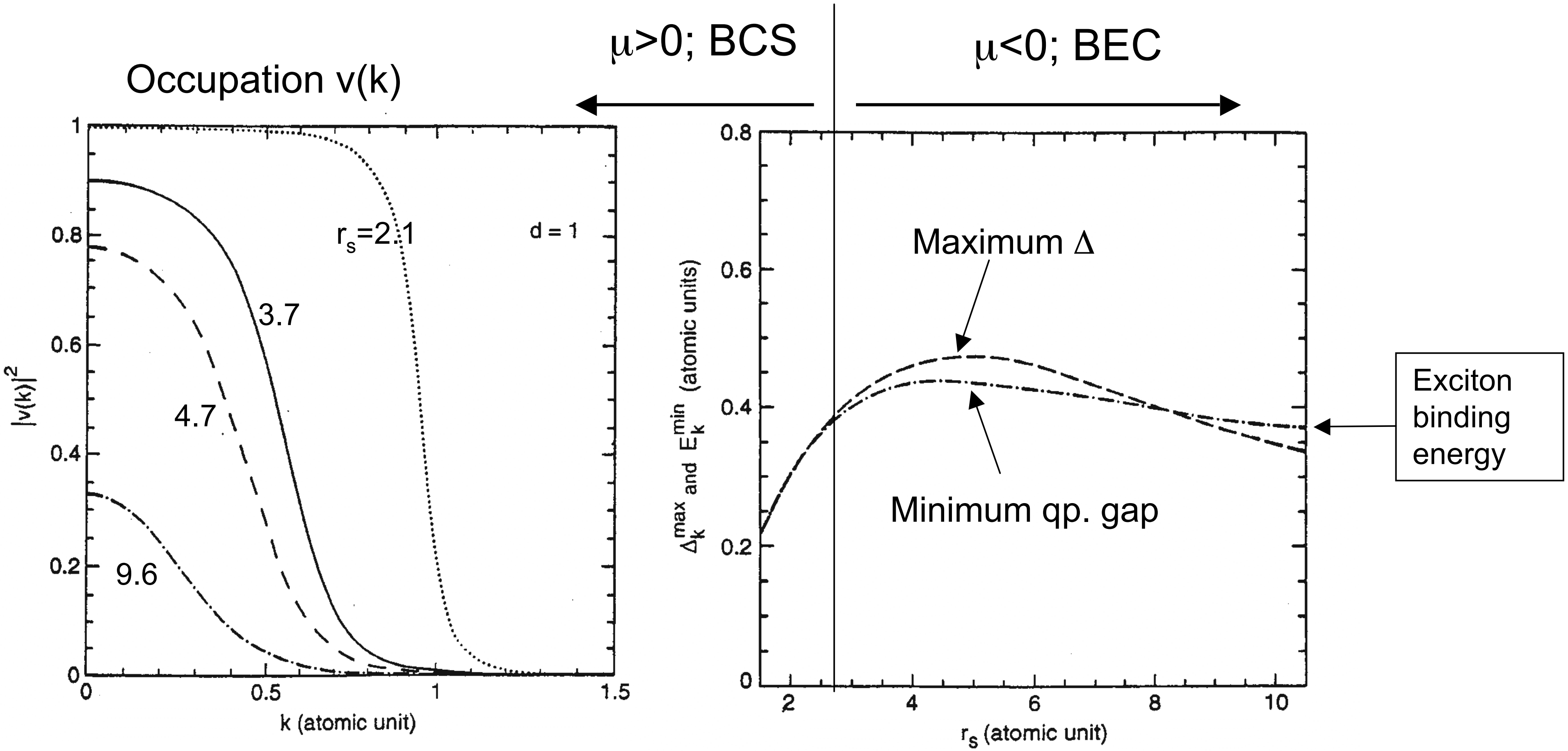
* Parabolic dispersion means that plasma is always weakly unstable even as $r_s \to 0$

2D exciton condensate: Mean field solution





Crossover from BCS to BEC



Smooth crossover between BCS-like fermi surface instability and exciton BEC

Model: 2D quantum wells separated by distance = 1 Bohr radius Zhu et al PRL 74, 1633 (1995)

Improved solution: Variational Monte Carlo

$$|\Psi\rangle = e^{-\sum_{i\neq j} u^{ee}(r_{i;e}-r_{j;e})} \times e^{-\sum_{i\neq j} u^{hh}(r_{i;h}-r_{j;h})} \times e^{-\sum_{ij} u^{eh}(r_{i;e}-r_{j;h})} |\Psi_{\text{mean field}}\rangle$$

Jastrow factors - smooth functions with variational parameters to add extra correlations to wavefunction

Either 2 component plasma (Slater determinant of plane waves) or BCS (Slater determinant of e-h pairs)

Zhu et al, PRB 54, 13575 (1996)

Search through variational parameter space by Monte Carlo
Output: Better energies, also pair correlation functions $g(|r_1 - r_2|) = \langle \psi(r_1)\psi(r_2)\rangle$

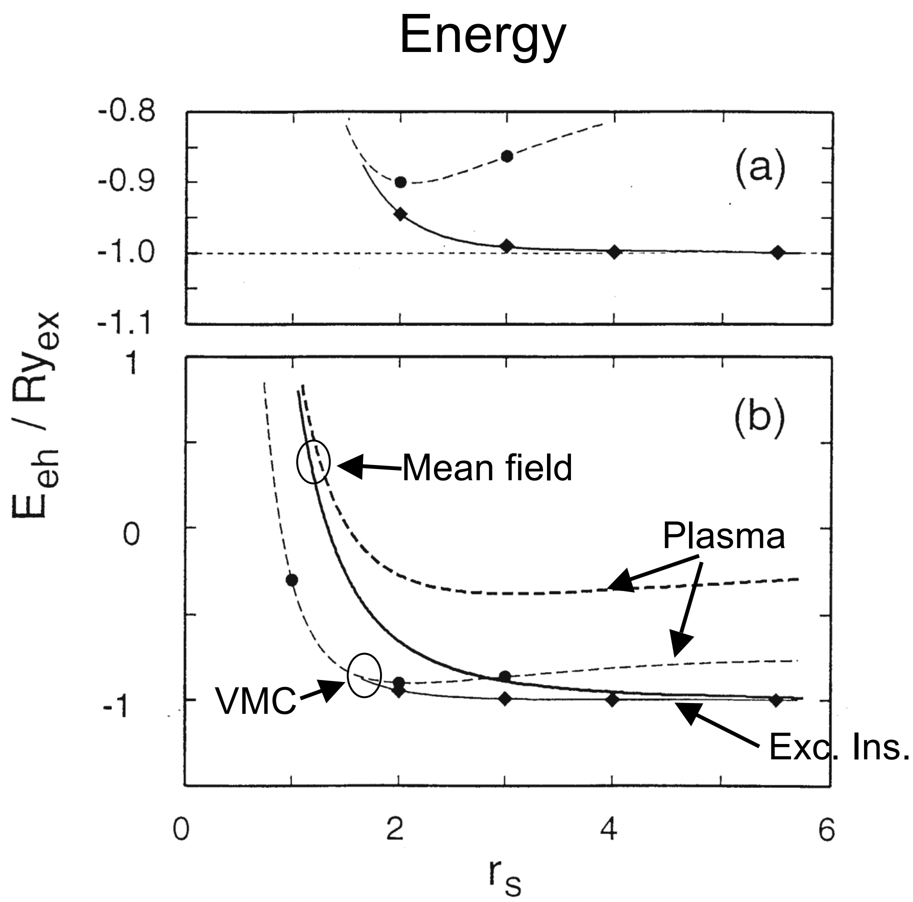
Further improvements possible :

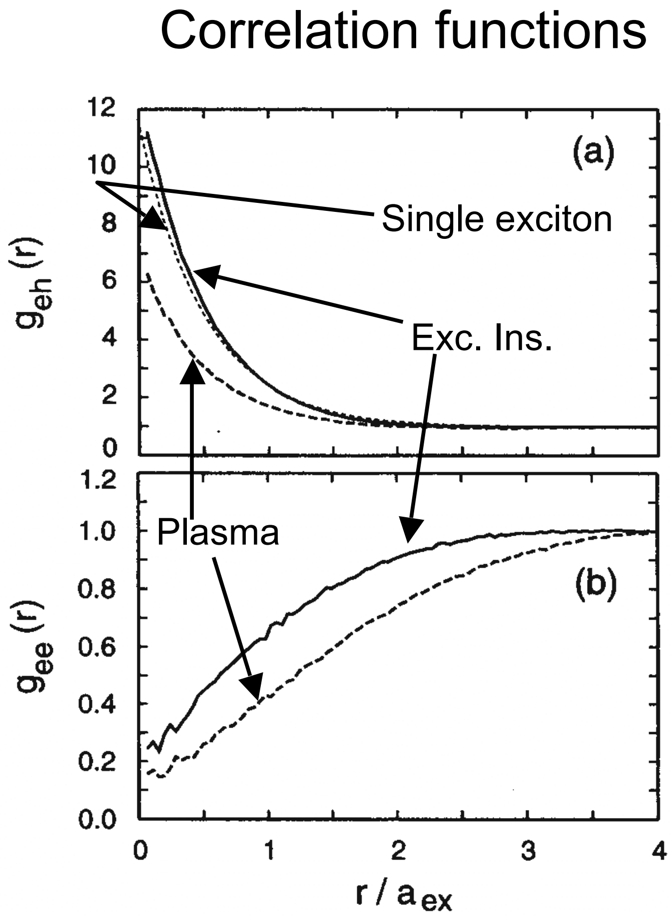
Diffusion Monte Carlo (fixed node) [de Palo et al, cond-mat/0201414]

Path Integral Monte Carlo (finite T) [Shumway and Ceperley, cond-mat/9909434]

Include biexcitons, Wigner crystal phases etc...

3D exciton condensate - mean field vs VMC





Zhu et al PRB 54, 13575 (1996)

Conclusions from numerics

- Condensation is a robust process
  - energy scale is fraction of exciton Rydberg (few meV in GaAs)
- No evidence for droplet formation
  - positive compressibility
  - bi-excitons ignored here, but X-X interaction repulsive in bilayers
  - contrast to multivalley bulk semiconductors like Ge, Si
- BCS-like wavefunction captures smoothly the crossover from high to low densities
- Solid phases also competitive in energy (but higher for moderate r_s)
- So it should be easy to make experimentally …..

2D BEC - no confining potential

Interpolation – by hand – between two limits

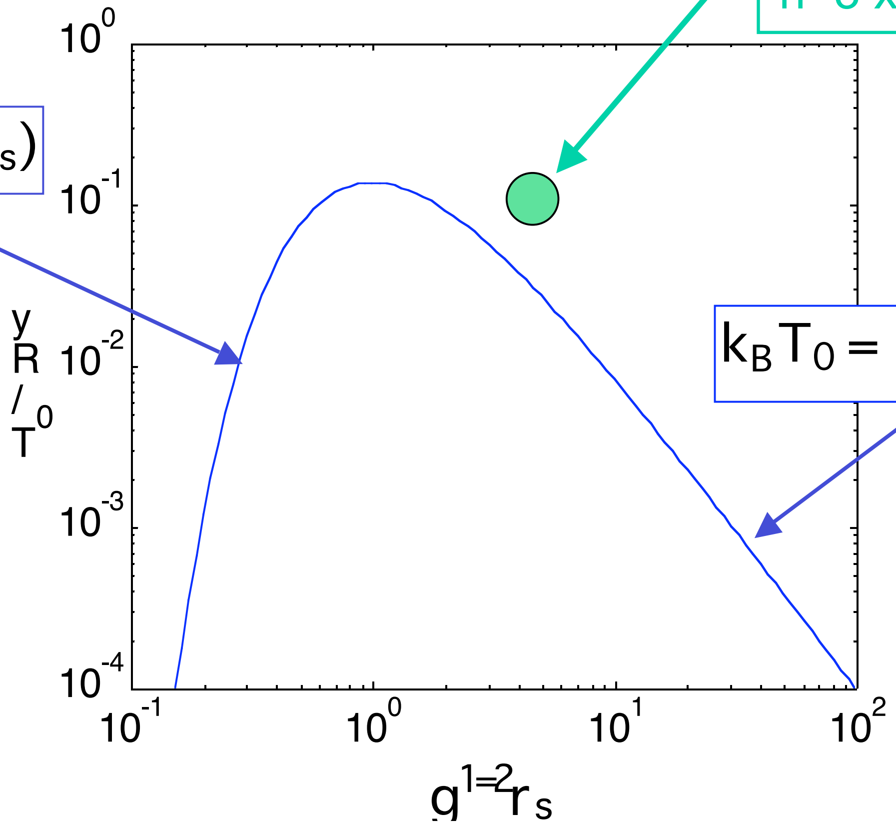
GaAs CQW
T = 4 K
n=3 x 10^{11} cm^{-2}

$k_B T_0 \propto Ry^* \exp(-2/r_s)$

$$Ry^* = \frac{\mu}{m\varepsilon^2}\, 13.6\, \mathrm{eV}$$

$$a_o^* = \frac{\varepsilon m}{\mu} a_{Bohr}$$

$$r_s a_0^* = (\pi n)^{-1/2}$$



$$k_B T_0 = \frac{\pi \hbar^2 n}{2Mg} = \frac{\mu}{M}\frac{1}{g r_s^2} Ry^*$$

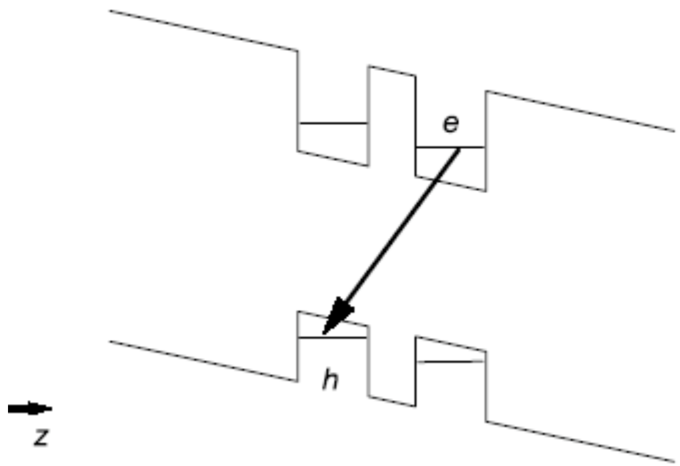
Mean field - should be K-T transition, but OK to estimate energy scales

Some experimental systems

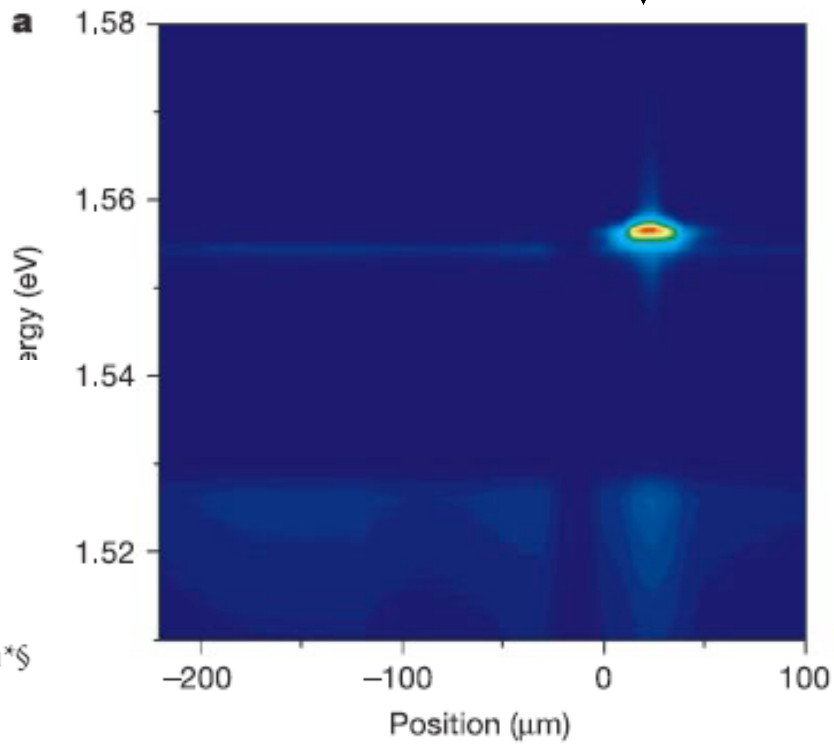
- Cu_2O - long-lived optically excited excitons (dipole-forbidden)
  - anomalous transport [Fortin et al PRL 70, 2951 (1993)] & luminescence [Lin and Wolfe, PRL 71, 1222 (1993)]
  - dominated by Auger recombination [O'Hara and Wolfe PRB 62 12909 (2000)]
- Biexcitons in CuCl - analogue of H_2
  - coherently driven, not thermalised [Chase et al, PRL 42, 1231 (1979); Hasuo et al PRL 70, 1303 (1992); Kuwata-Gonokami et al, JPhysSocJpn 71, 1257 (2002)]
- Double quantum well - keep electrons and holes physically apart
  - Optical excitation in double wells [Fukuzawa et al, PRL 64, 3066 (1990); Kash et al, PRL 66, 2247 (1990) , Butov et al PRL (2001), 2*Nature (2002), Snoke et al, Nature (2002)]
  - Indirect Γ-X exciton at GaAs/AlAs interface [Butov et al, PRL 73, 301 (1994)]
  - Separately gated electron and hole layers [Sivan et al, PRL 68, 1196 (1992)]
  - Type II quantum wells (artificial 2D semimetal) [Lakrimi et al, PRL 79, 3034 (1997)]

- Optical microcavities
  - stimulated emission observed, also coherent driving [Pau et al, PRA 54, 1789 (1996); Senellart and Bloch, PRL 82, 1233 (1999); Le Si Dang et al. PRL 81, 3920 (1998); Stevenson et al., PRL 2000, Deng et al, Science (2002)]
- Josephson Junction array in microwave cavity
  - quantum coherence, or coupled oscillators ? [Barbara et al, PRL 82, 1963 (1999)]
- Quantum Hall bilayers
  - Zero-bias anomaly [Spielman et al, PRL 84, 5808 (2001); 87, 36803 (2002)]
  - Zero Hall effect in counterflow [Kellogg et al, Tutuc et al, 2004]
- "Tripleton" BEC in quantum spin systems
  - Tl $CuCl_3$ [Ruegg et al. 2003]; BaCuSiO [Jaime et al 2002]

Excitons in coupled quantum wells



Sharp recombination emission from “trap”

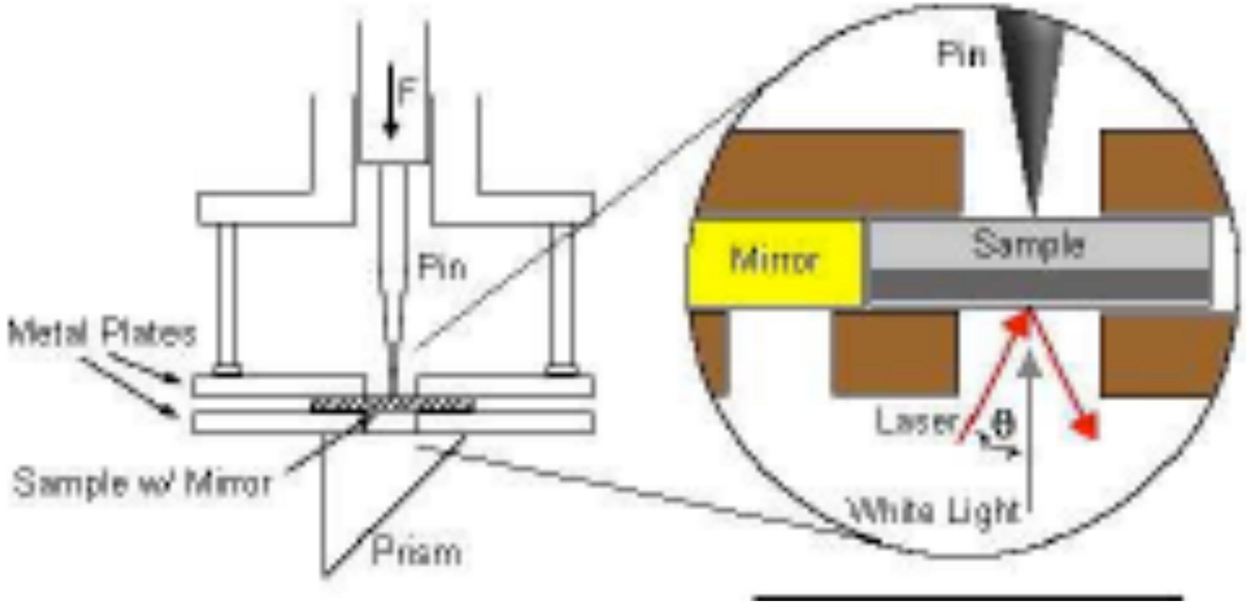


Towards Bose–Einstein condensation of excitons in potential traps

L. V. Butov*, C. W. Lai*, A. L. Ivanov†, A. C. Gossard‡ & D. S. Chemla*§

Nature 2002

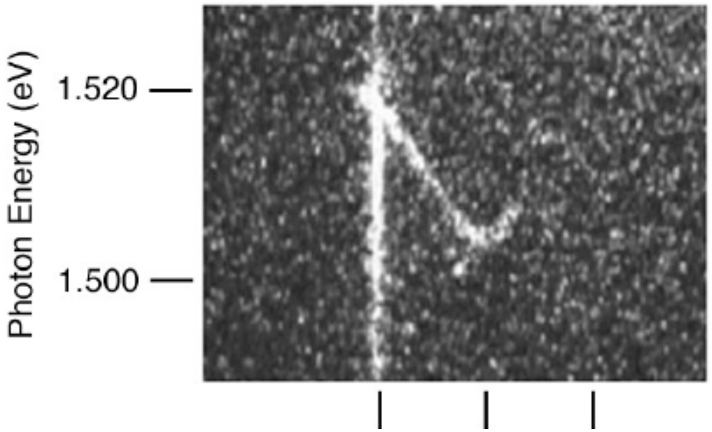
Stress trap



Snoke 2004

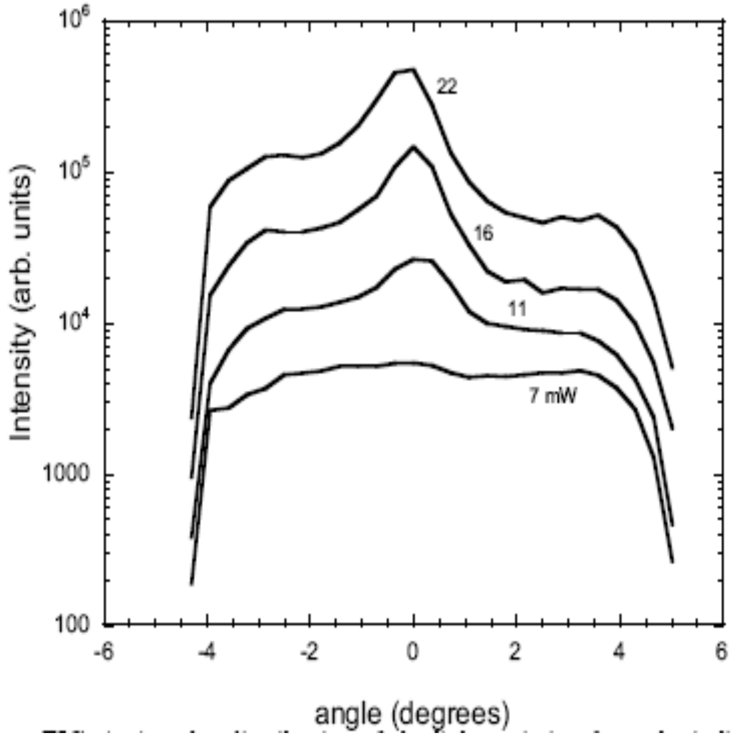
Excitons confined in stress-induced harmonic traps

a)

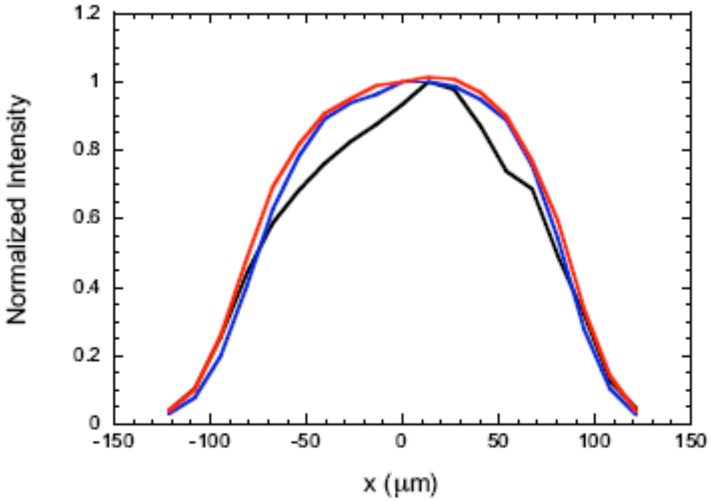


Snoke, Liu, Voros, Pfeiffer and West 2004

Angular pattern of emission



c) Spatial pattern of emission



Not yet convincing for BEC

- – unexpected scaling of peak with density
- – no change in width/shape

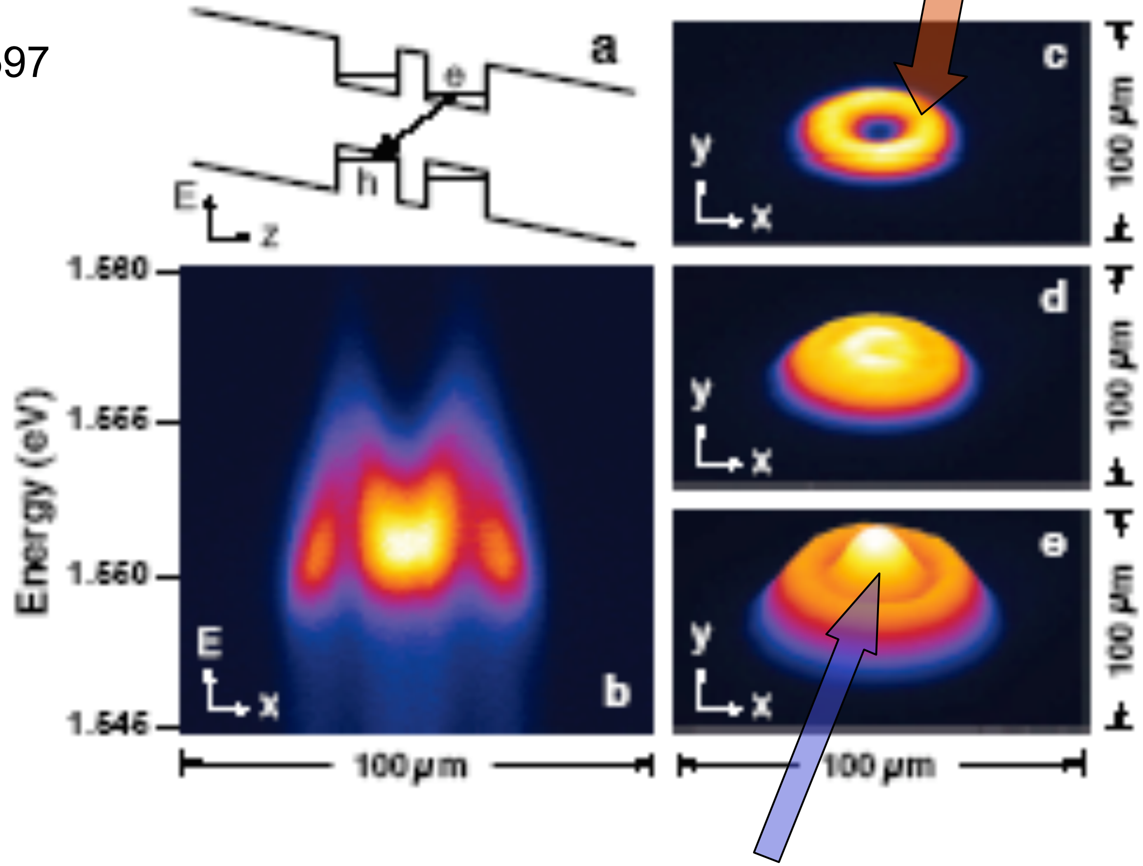
Optical trap

Trapping of Cold Excitons with Laser Light

A.T. Hammack,[1] M. Griswold,[1] L.V. Butov,[1] L.E. Smallwood,[2] A.L. Ivanov,[2] and A.C. Gossard[3]

cond-mat/0603597

Optical excitation of hot excitons in a ring

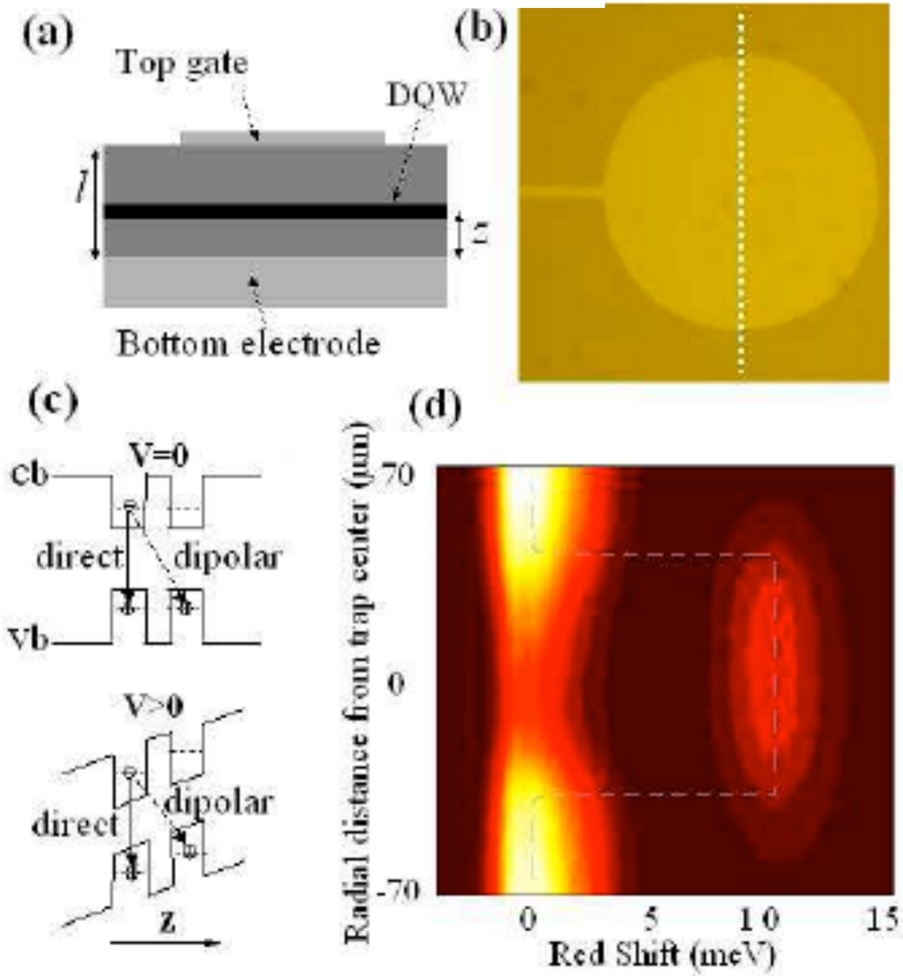


Dipole repulsion traps cold excitons in center

Artificial trapping of a stable high-density dipolar exciton fluid

Gang Chen, Ronen Rapaport, L. N. Pffeifer, K. West, P. M. Platzman, Steven Simon[1] and Z. Vörös, and D. Snoke[2]

cond-mat/0601719



7/19/06

Experimental signatures

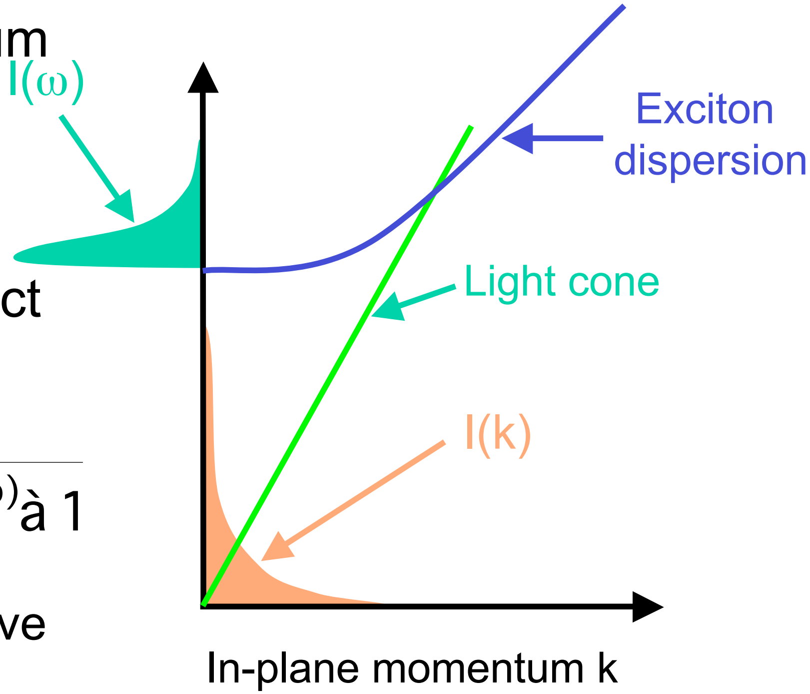
- Phase-coherent luminescence - order parameter is a macroscopic dipole

$$\text{Polarisation } P \propto \sum_k \langle a^{\dagger}_{ck} a_{vk} \rangle \propto \ldots e^{i\omega t}$$

  - Should couple photons and excitons right from the start - polaritons
- Gap in absorption/luminescence spectrum
  - small and low intensity in BEC regime
- Momentum and energy-dependence of luminescence spectrum $I(k,\omega)$ gives direct measure of occupancy

$$n_k = \frac{1}{e^{\beta(E_k - \mu)} - 1}$$

  - 2D Kosterlitz-Thouless transition
  - confined in unknown trap potential
  - only excitons within light cone are radiative



Angular profile of light emission

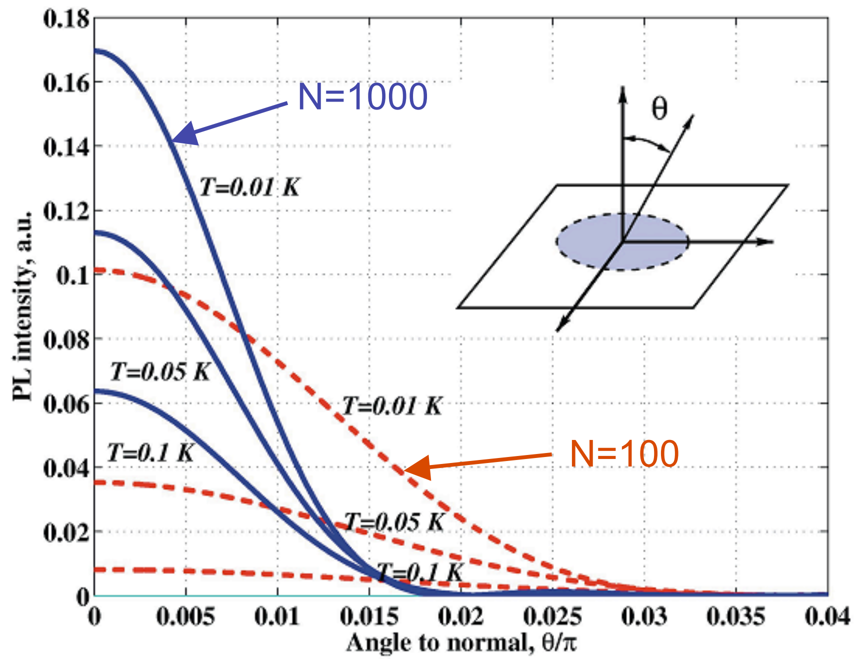
Keeling et al, cond-mat/0311032

- Emitted photon carries momentum of electron-hole pair
- Condensation (to $k_{//} \sim 0$) then has signature in sharp peak for emission perpendicular to 2D trap.
- In 2D the phase transition is of Kosterlitz-Thouless type – no long range order below T_c
- Peak suppressed once thermally excited phase fluctuations reach size of droplet

$$R \sim \xi_T = \left(\frac{\hbar^2}{4m}\frac{1}{kT}\right)^{1/2}$$

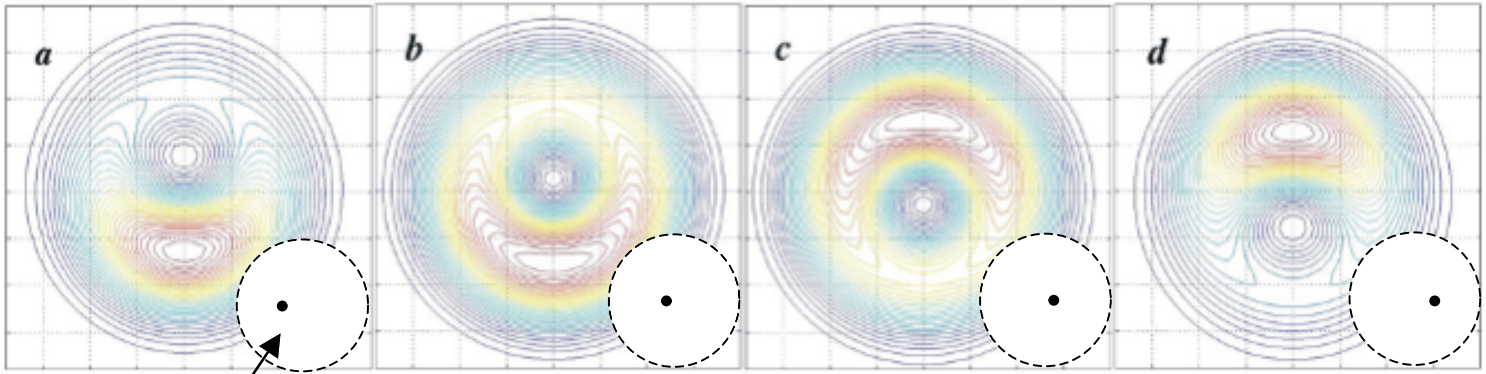
$$T < T_{BEC}/\ln(R/\xi_T)$$

Parameters estimated for coupled quantum wells of separation ~ 5 nm; trap size ~ 10 μm; T_{BEC} ~ 1K

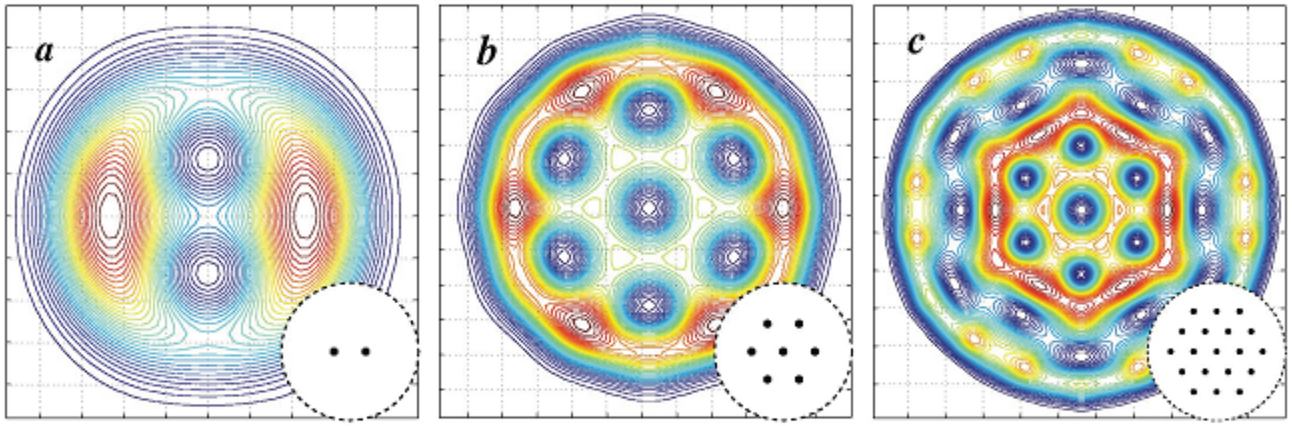


Vortices

Angular emission into θ_x , θ_y

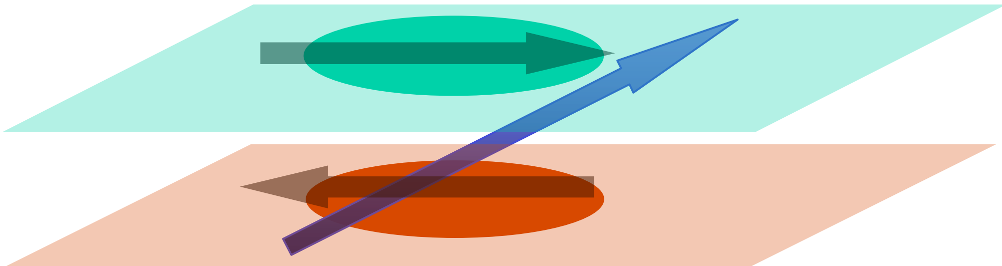


Vortex position (x,y) inside droplet



Dipolar superfluid

- What could be the superfluid response?
  - exciton transport carries no charge or mass
  - in a bilayer have a static dipole

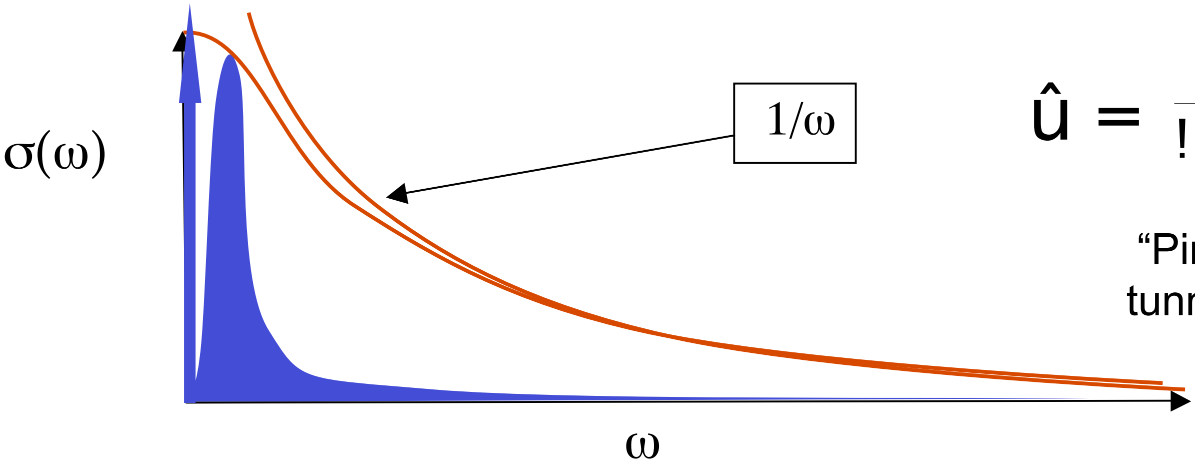


$B(t) = B_o e^{i!t}\hat{z}$

$\acute{E}E = i!B_o d e^{i!t}\hat{x}$

$F = i!B_o e d e^{i!t}\hat{x}$

$j_{dipole} = \hat{u}(!)F$



$\hat{u} = \frac{i\acute{u}_s}{! + i\hat{\imath}} = \grave{u}\acute{u}_s\hat{\imath}(!) + i\frac{\acute{u}_s}{!}$

"Pinning" of the phase by interlayer tunnelling shifts response to nonzero frequency

Joglekar, Balatsky, PBL, 2004

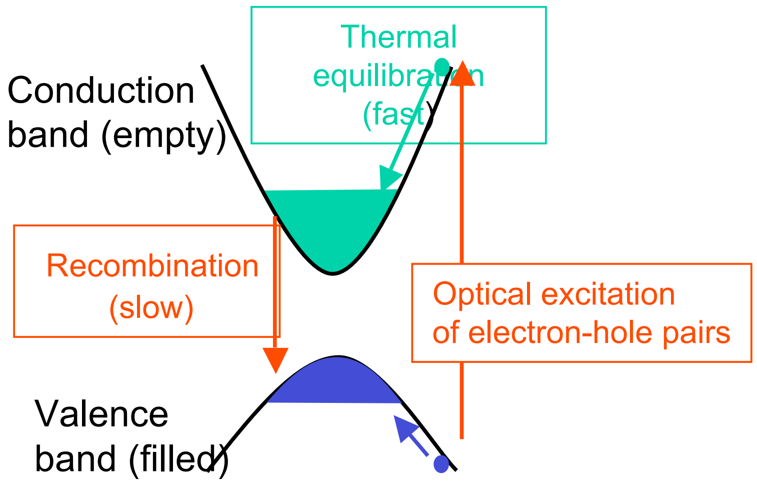
Coupled quantum wells of electrons and holes

- Considerable effort being expended on this at the moment
- High densities have been reliably reached
- Several different kinds of traps have been demonstrated
- Not yet a reliable and convincing demonstration of BEC ….

- Except for electron bilayers in quantum Hall regime at ½ filling.

Recap

Exciton liquid in semiconductors



Interacting electrons and holes
Characteristic energy scale is the exciton Rydberg

$$H_{eh} = \sum_k \left(\varepsilon_{ck} a^\dagger_{ck} a_{ck} + \varepsilon_{vk} a^\dagger_{vk} a_{vk} \right) + \frac{1}{2} \sum_q \left[V^{ee}_q \rho^e_q \rho^e_{-q} + V^{hh}_q \rho^h_q \rho^h_{-q} - 2V^{eh}_q \rho^e_q \rho^h_{-q} \right]$$

A very good wavefunction to capture the crossover from low to high density is BCS

$$|\Psi_0\rangle = \prod_k \left(u_k + v_k a^\dagger_{ck} a_{vk} \right) |vac\rangle \; ; \quad |u_k|^2 + |v_k|^2 = 1$$

Just like a BCS superconductor, this has an order parameter, and a gap

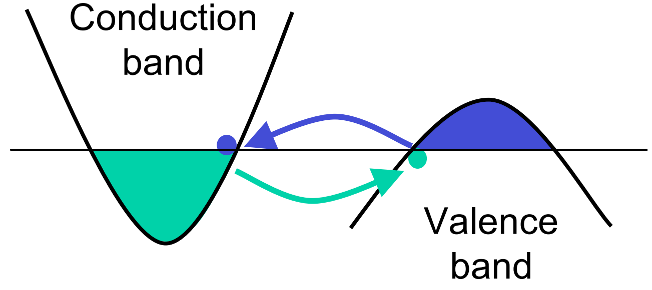
$$\left\langle a^+_{ck} a_{vk} \right\rangle = u_k v_k = (\Delta_k / 2E_k); \qquad E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$$

The order parameter has an undetermined phase $\Rightarrow$ superfluid.

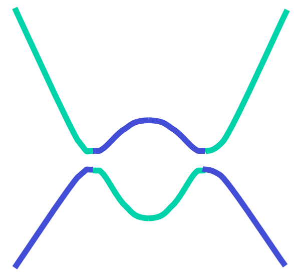
Unfortunately, there are some terms in H that have been left out

Digression: tunnelling and recombination

- Our Hamiltonian has only included interaction between electron and hole densities, and no e-h recombination
- In a semimetal tunnelling between electron and hole pockets is allowed



If pockets related by symmetry, generates single particle terms $ta^{y}_{ck}a_{vk}$



Rediagonalise $(ë_k; ì_k) =$ linear combinations of $(a_{vk} a_{ck})$

Introduces single particle gap

New Coulomb coupling terms $V_1 t\, ë^{y} ë^{y} ë ì \;\; ; \;\; V_2 t^2\, ë^{y} ë^{y} ì ì \;\; :$

If pockets are unrelated by symmetry, still the eigenstates are Bloch states

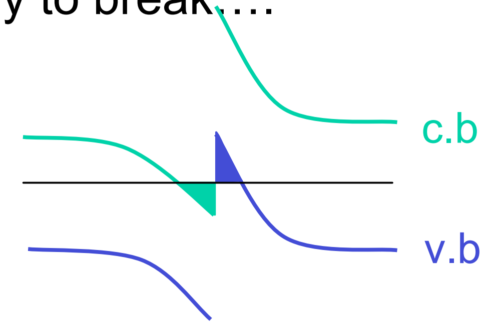
$$\hat{V} = \sum_{n_1;:::;n_4} \sum_{kk^0q} hn_1k; n_2k^0 jVjn_3k^0 + q; n_4k à q i \; â \; a^{y}_{n_1k} a^{y}_{n_2k^0} a_{n_3k^0+q} a_{n_4kàq}$$

In general, terms of the form $V_1\, ë^{y} ë^{y} ë ì \;\; ; \;\; V_2\, ë^{y} ë^{y} ì ì \;\; :$

Most general Hamiltonian does not separately conserve particles and holes

Tunnelling and recombination - 2

- Single particle gap - trivial physics, no extra symmetry to break….



E.g. Artificial 2D semimetal - GaSb/InAs interface
electron-hole mixing introduces gap [Lakrimi et al 1997]
In QH bilayers: tunnelling between layers -> S/AS splitting

- Consider the effect of general Coulomb matrix elements at zeroth order

$\langle \psi^\dagger \psi \rangle \propto |\Delta| e^{i\phi}$ Mean field approximation

$\langle V_2 \psi^\dagger \psi^\dagger \psi \psi \rangle \propto V_2 |\Delta|^2 \cos(2\phi) \longrightarrow$ Josephson-like term; fixes phase; gapped Goldstone mode

$\langle V_1 \psi^\dagger \psi^\dagger \psi \psi \rangle \propto V_1 n_{\psi} |\Delta| \cos(\phi - \phi_0) \longrightarrow$ Symmetry broken at all T; just like band-structure gap

- No properties to distinguish this phase from a normal dielectric, except in that these symmetry breaking effects may be small
- In that case, better referred to as a commensurate charge density wave

Not unfamiliar or exotic at all (but not a superfluid either)

Tunnelling and recombination - 3

- If electron and hole not degenerate, recombination accompanied by emission of a photon

$$H_{dipole} = g \sum_{q} a^{\dagger}_{ck+q} a_{vk} \psi_q + h.c. + \omega_q \psi^{\dagger}_q \psi_q$$

- Evaluate at zeroth order

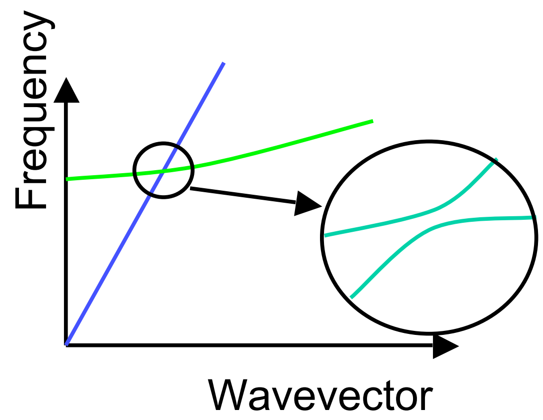
$$\langle H_{dipole} \rangle_{m.f.} = g \langle \psi_q \rangle |\Delta| e^{i\phi} e^{i(\omega_q - \mu)t} + c.c.$$

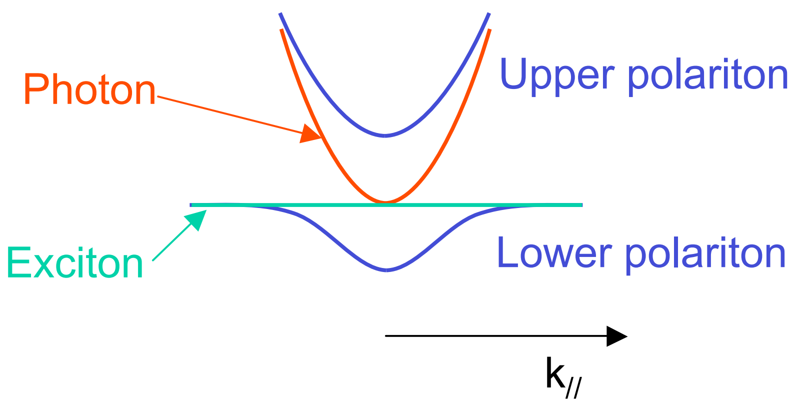
- Phase of order parameter couples to phase of electric field
- Resonant radiation emitted/absorbed at frequency = chemical potential
- Behaves just like an antenna (coherent emission, not incoherent luminescence)

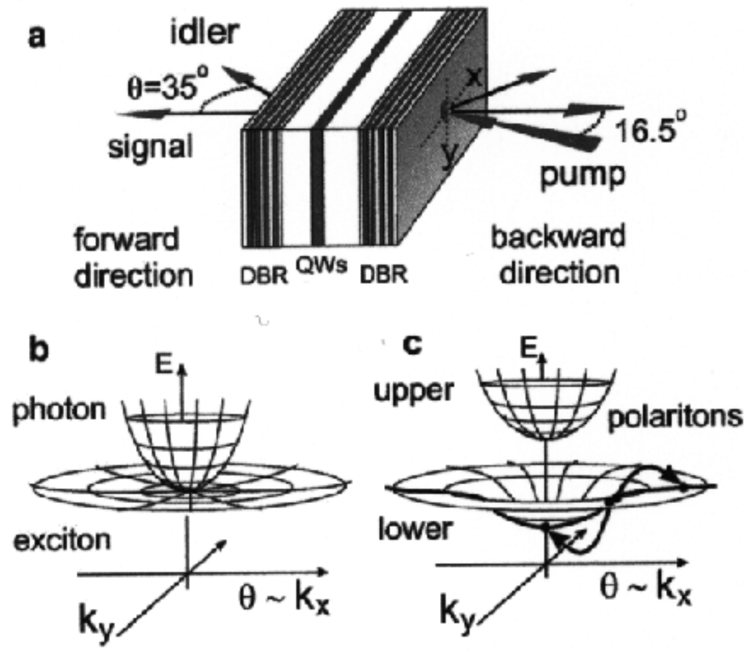
Must include light and matter on an equal footing from the start - POLARITONS

Optical microcavities and polaritons

- Correct *linear* excitations about the ground state are mixed modes of excitonic polarisation and light - polaritons
- Optical microcavities allow one to confine the optical modes and control the interactions with the electronic polarisation
  - small spheres of e.g. glass
  - planar microcavities in semiconductors
  - excitons may be localised - e.g. as 2-level systems in rare earth ions in glass
  - RF coupled Josephson junctions in a microwave cavity

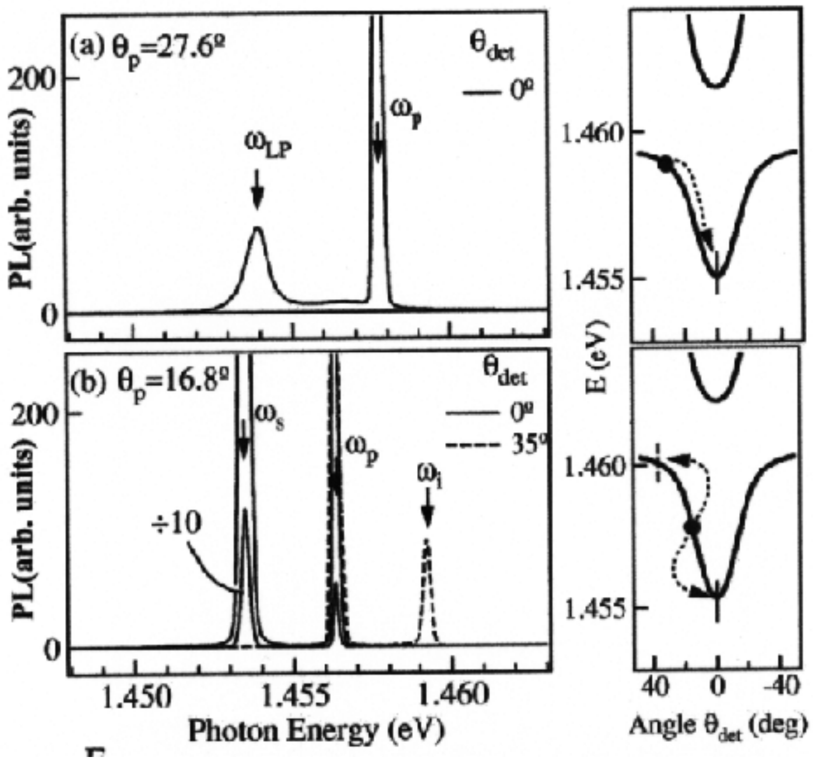


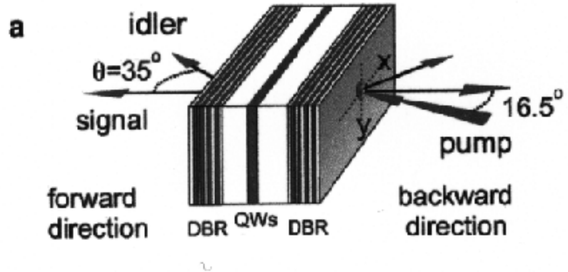


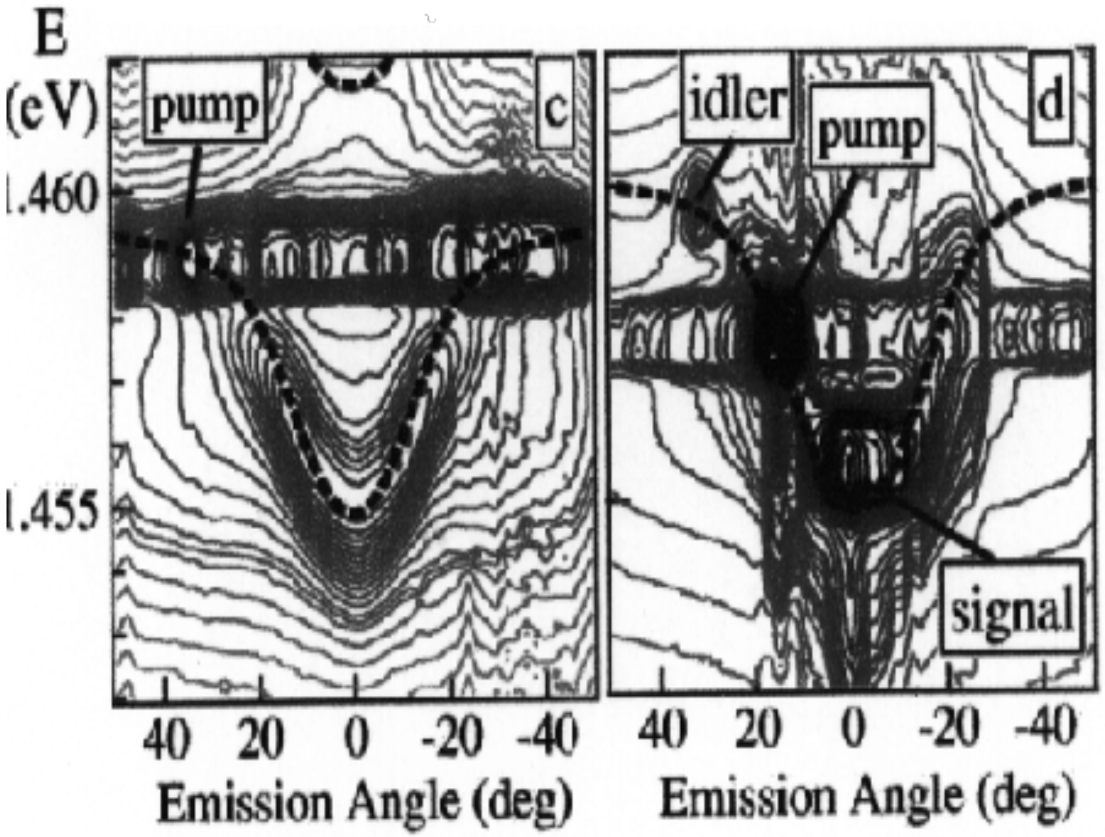


Resonantly pumped microcavity

Address in plane momentum by measurement or excitation as function of angle



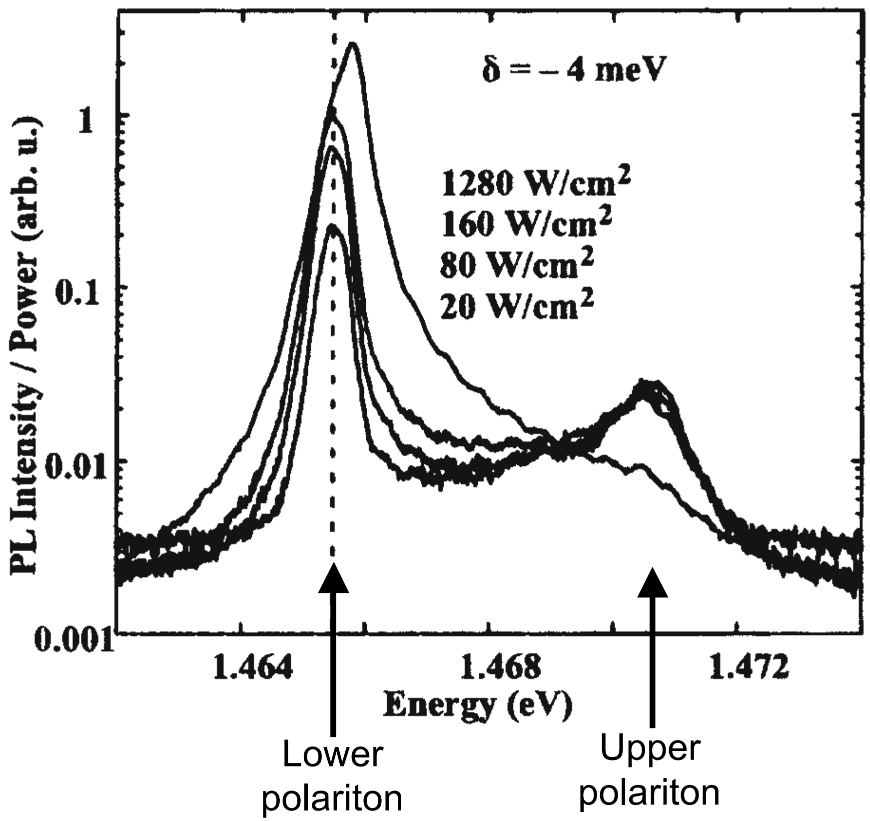


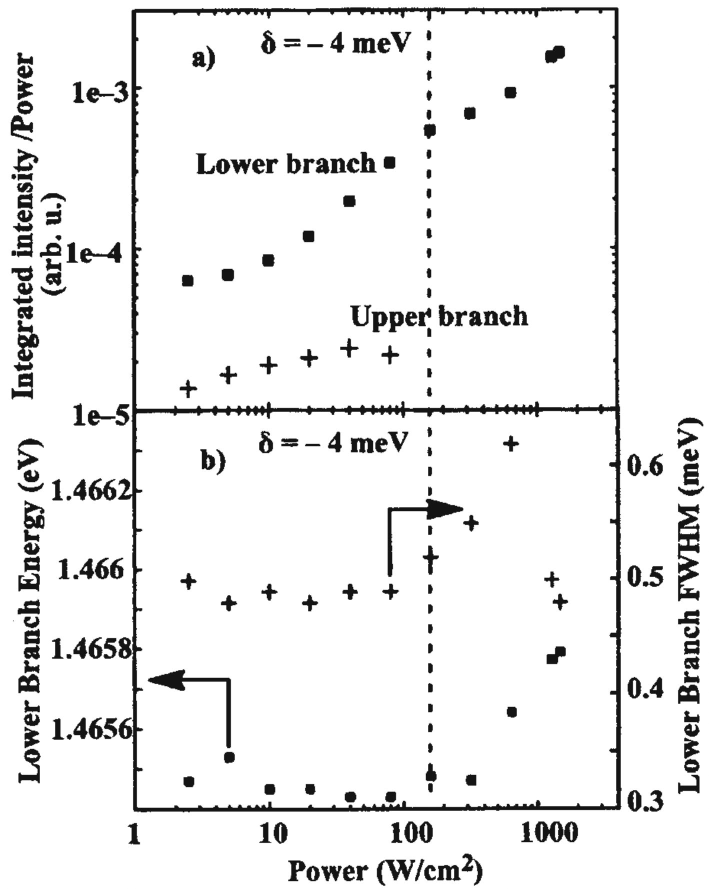


Baumberg et al Phys Rev B 62, 16247 (2000)

Photoluminescence from non-resonantly pumped microcavity

PL normalised to pump intensity



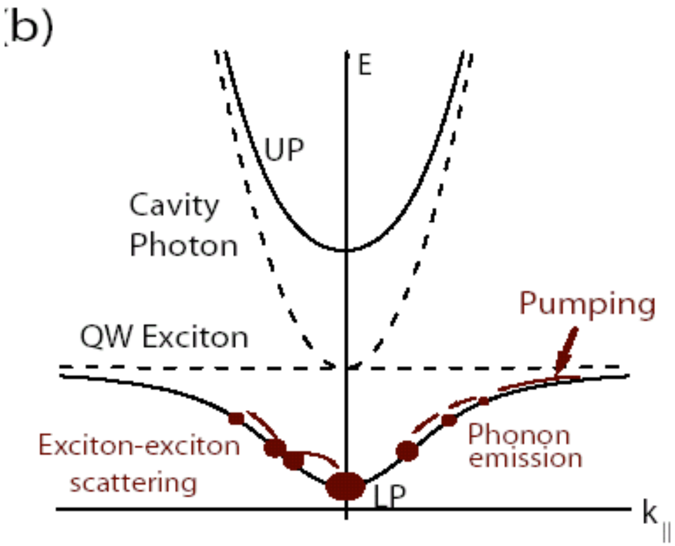


Excitation at ~ 1.7 eV

Senellart & Bloch, PRL 82, 1233 (1999)

Non-resonant(?) pumping in Lower Polariton Branch

Deng et al 2002



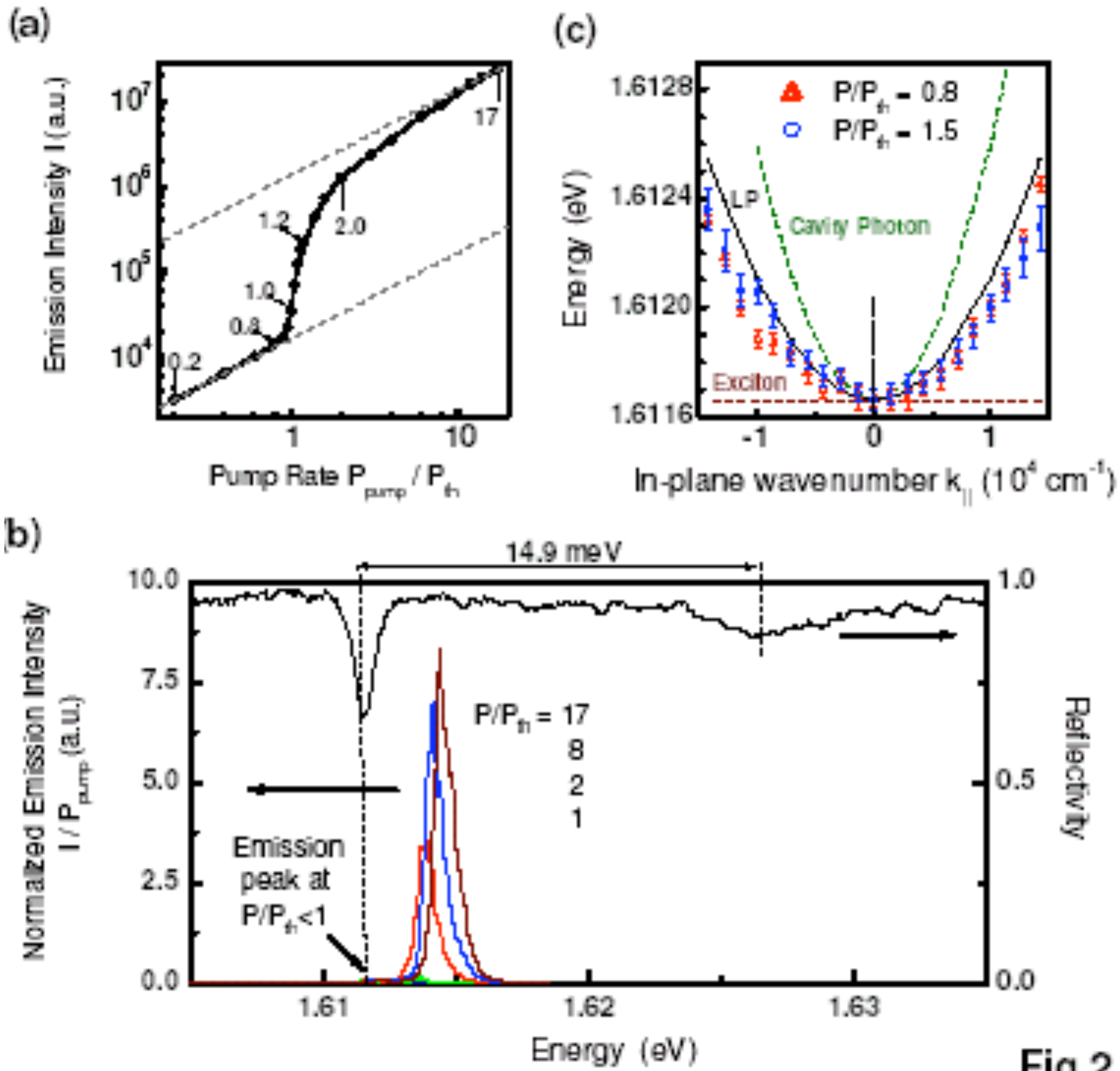


Fig.2

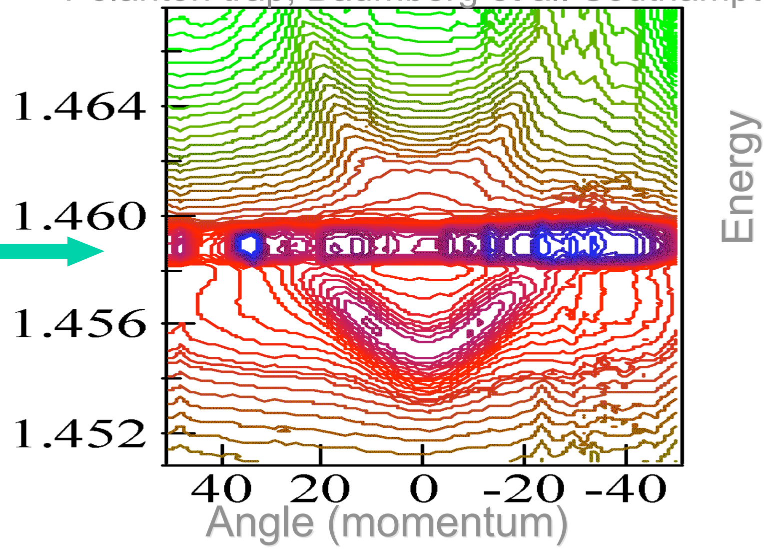
Substantial blue shift appears at threshold
Polariton dispersion seen above threshold

Polaritons

Angular pattern of emission

Pumping

Polariton trap, Baumberg et al. Southampton



Spatial pattern of emission

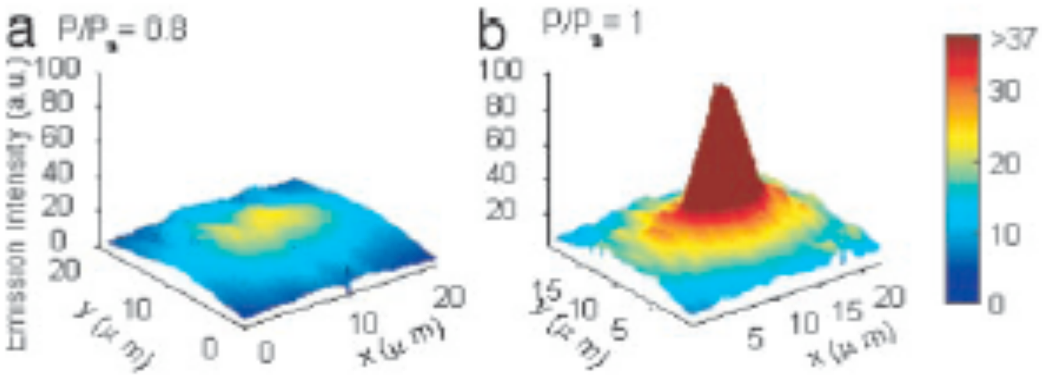
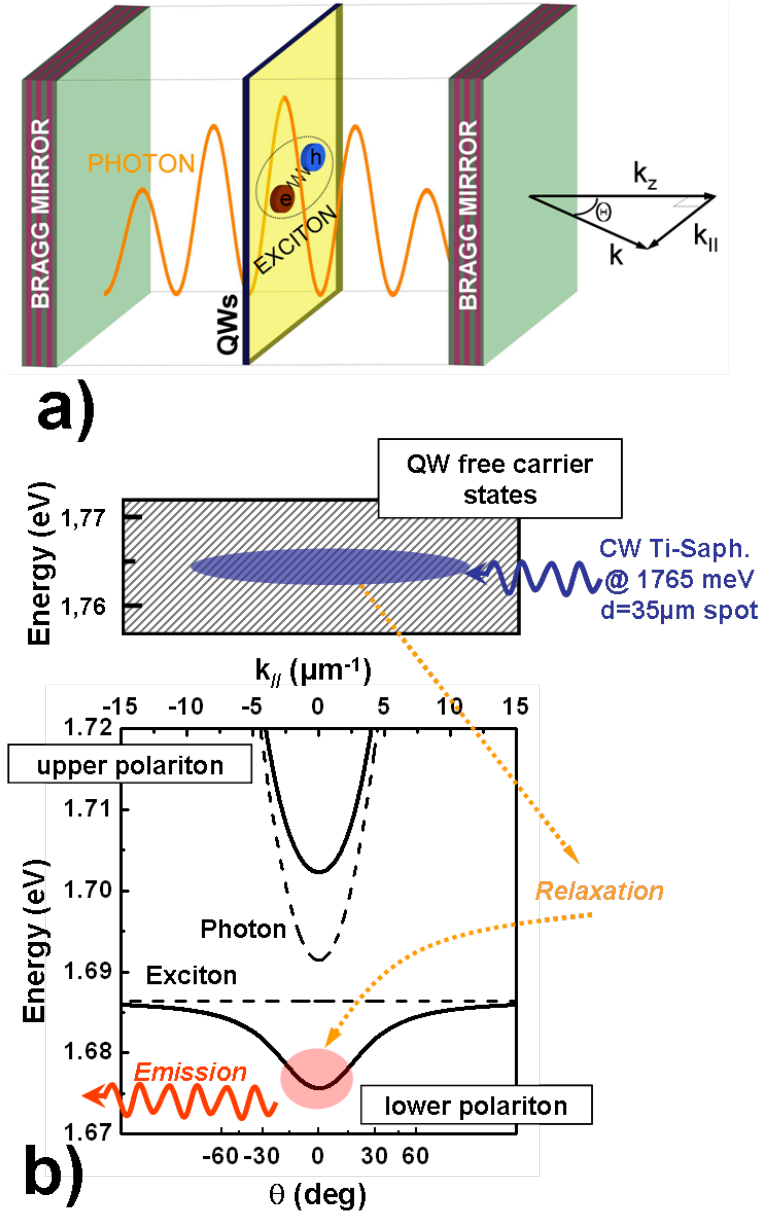


Fig. 5. Spatial profiles of LPs at $P/P_{th} = 0.8$ (*a*) and $P/P_{th} = 1$ (*b*).

Deng et al. PNAS 100, 15318 (2003)

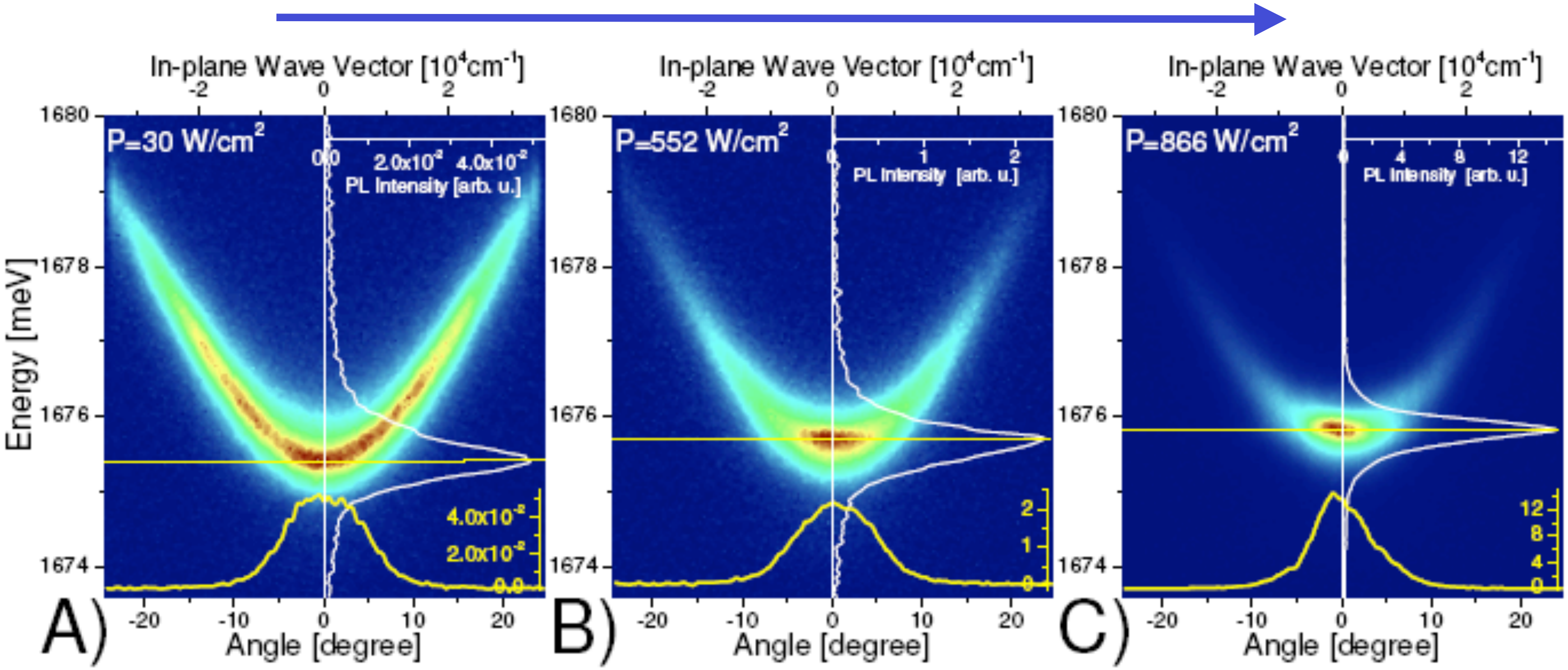


Microcavity polaritons

Experiments:
Kasprzak et al 2006
CdTe microcavities

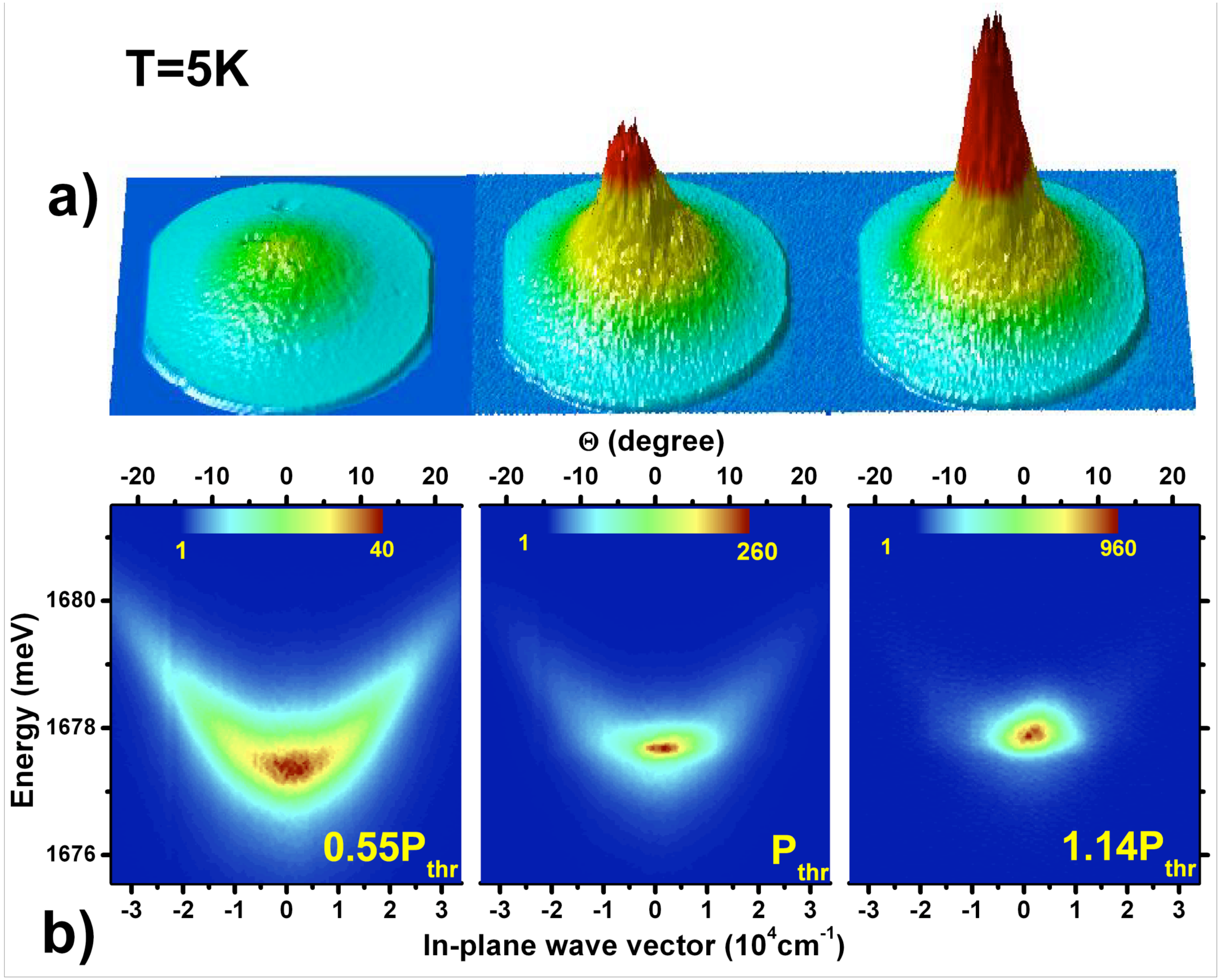
II-VI quantum well microcavities

Increasing pumping

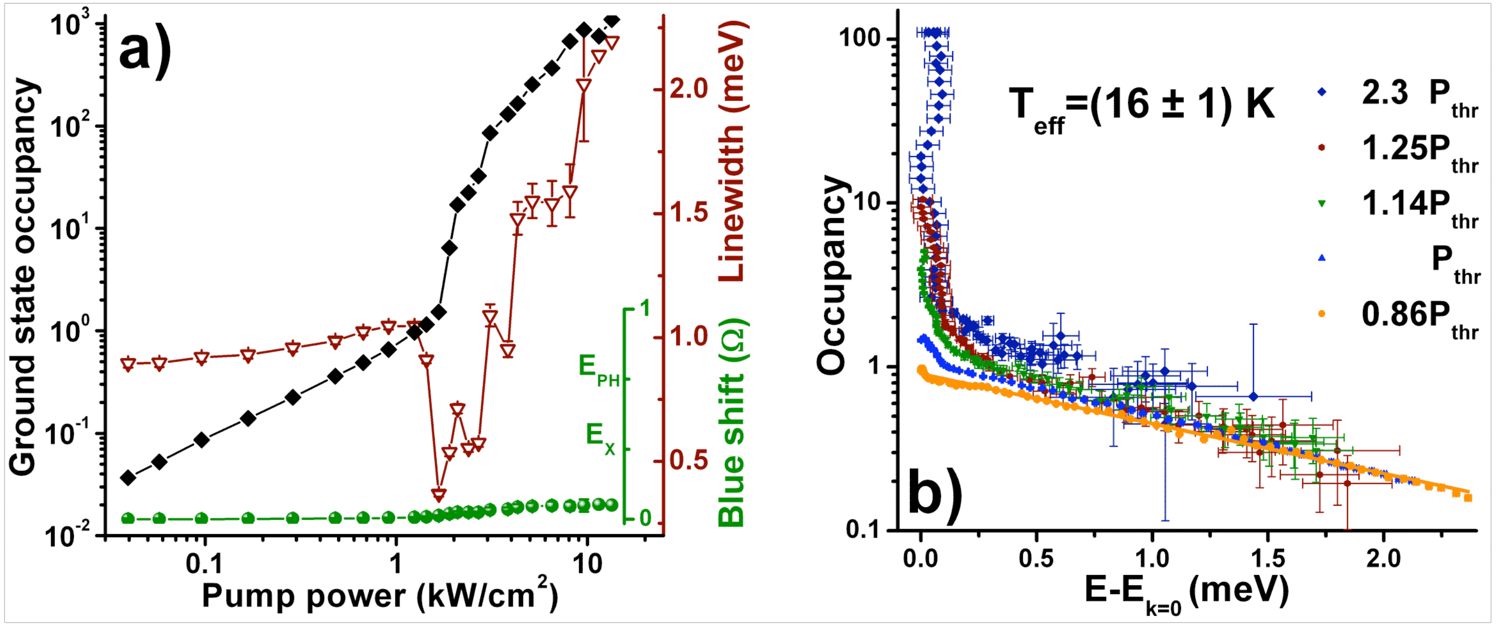


Kasprzak, Dang, unpublished

Occupancy as a function of power



Distribution at varying density

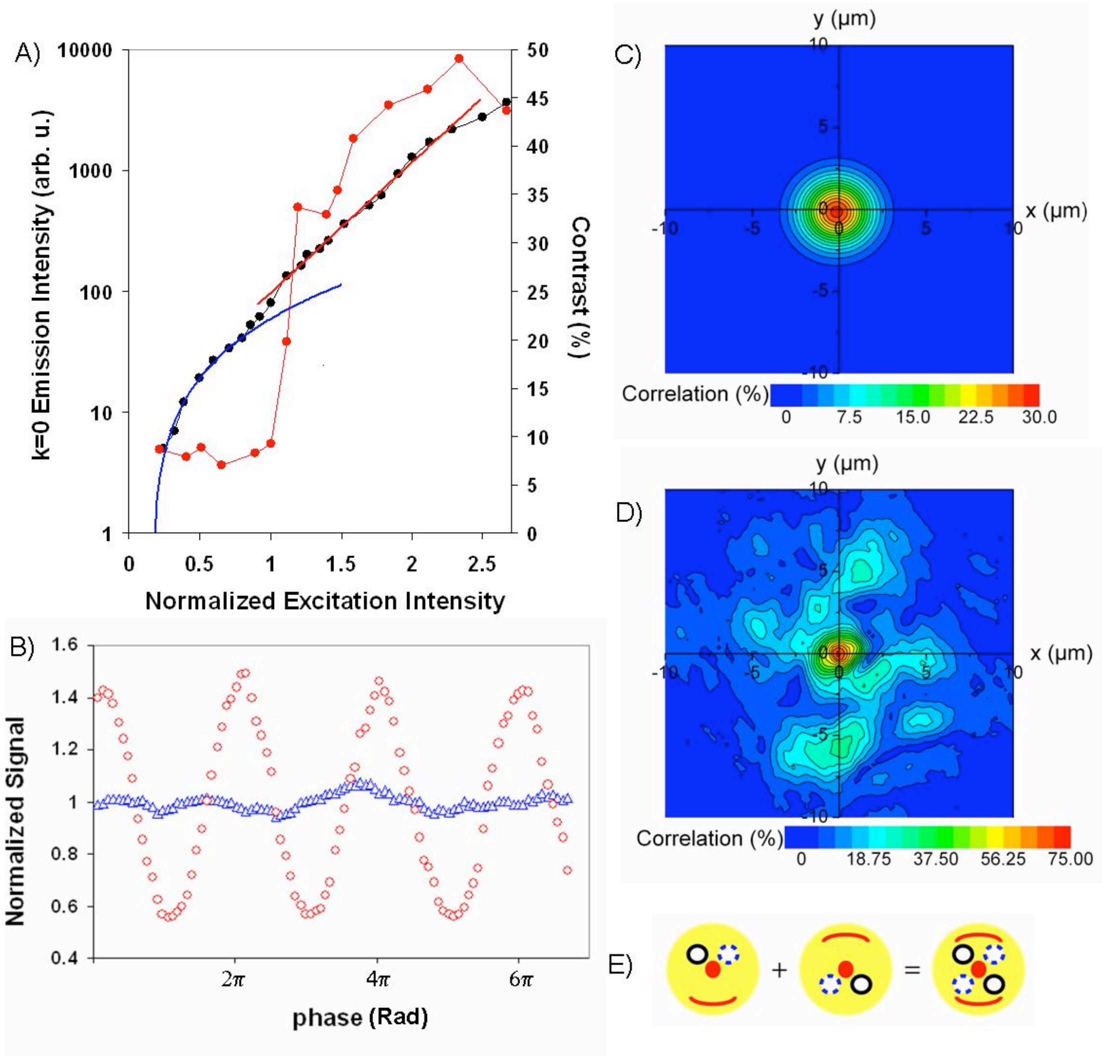


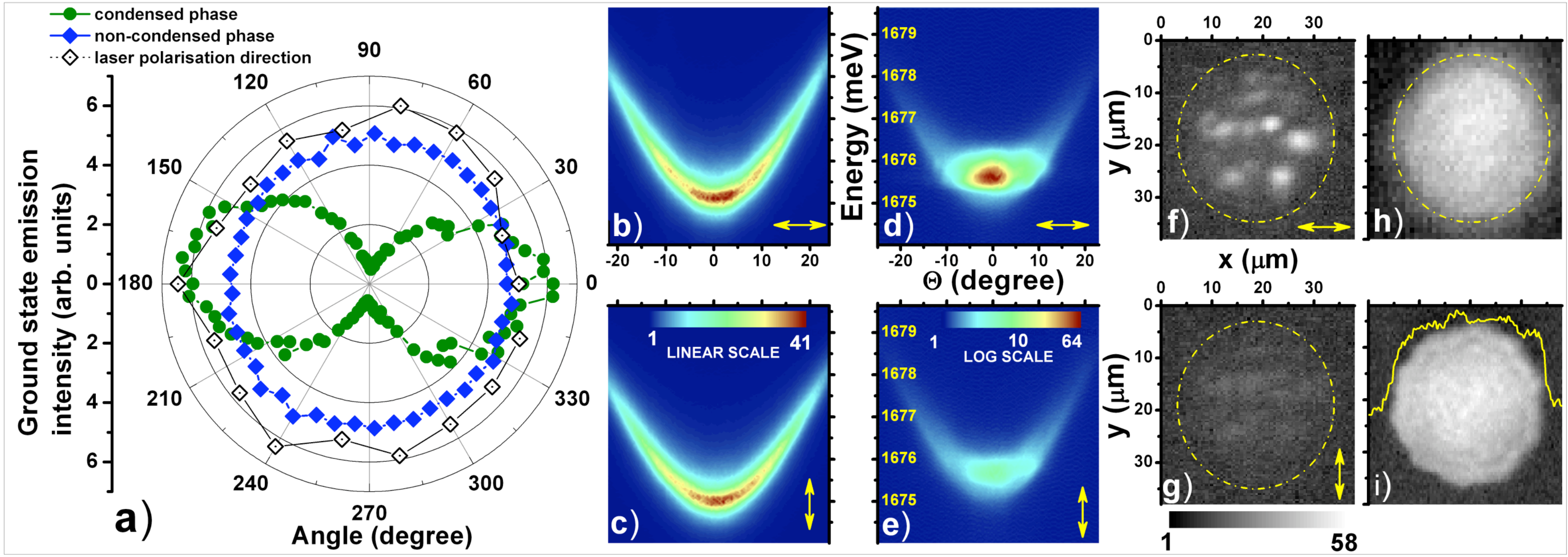
Blue shift used to estimate density
High energy tail of distribution used to fix temperature
Onset of non-linearity gives estimate of critical density
Linewidth well above transition is *inhomogeneous*

Measurement of first order coherence

Temperature and density estimates predict a phase coherence length ~ 5 μm

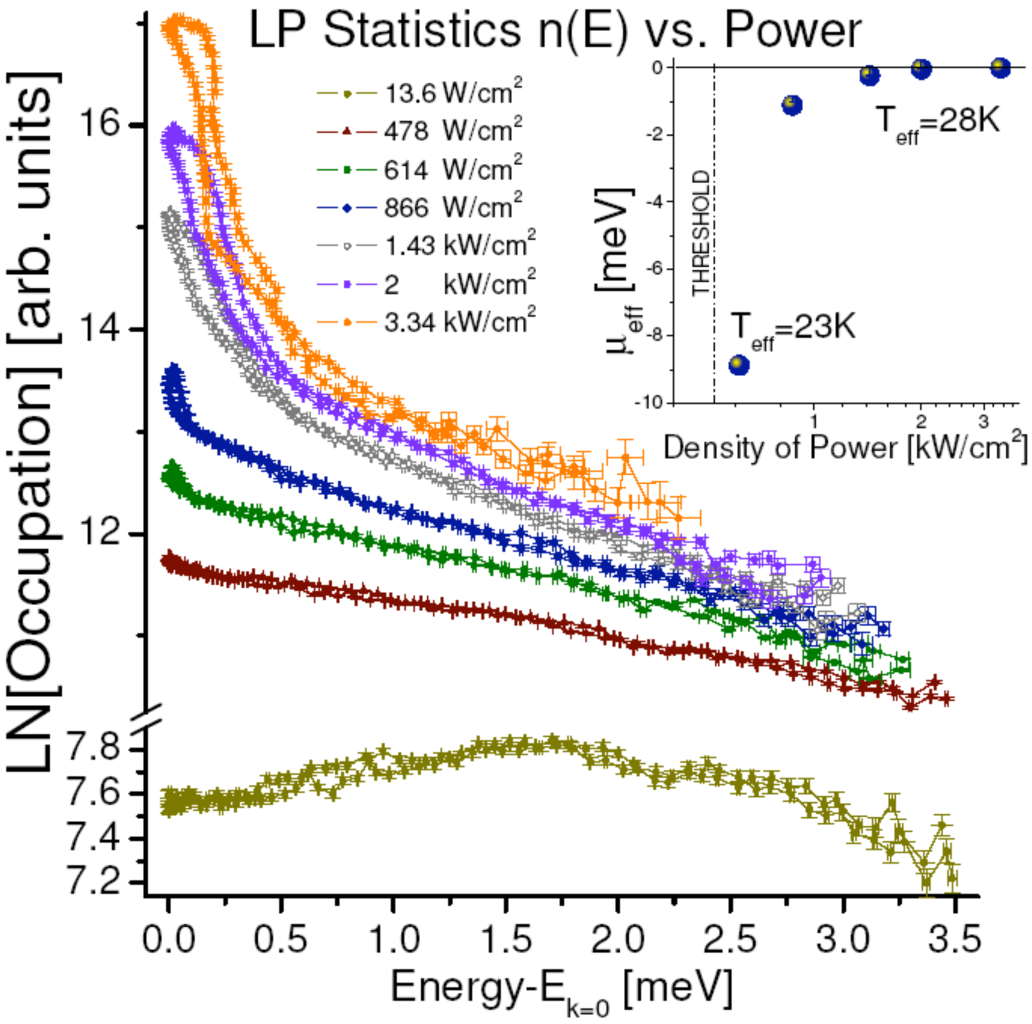
Experiment also shows broken polarisation symmetry





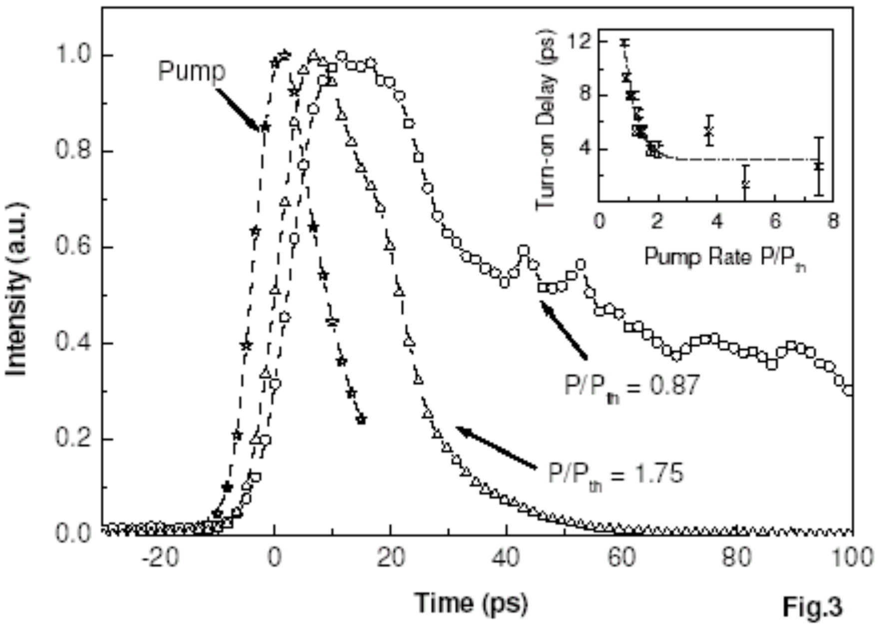
condensed phase
non-condensed phase
laser polarisation direction
Ground state emission intensity (arb. units)
Angle (degree)
a)
b)
c)
d)
e)
f)
g)
h)
i)
Energy (meV)
Θ (degree)
LINEAR SCALE
LOG SCALE
x (μm)
y (μm)

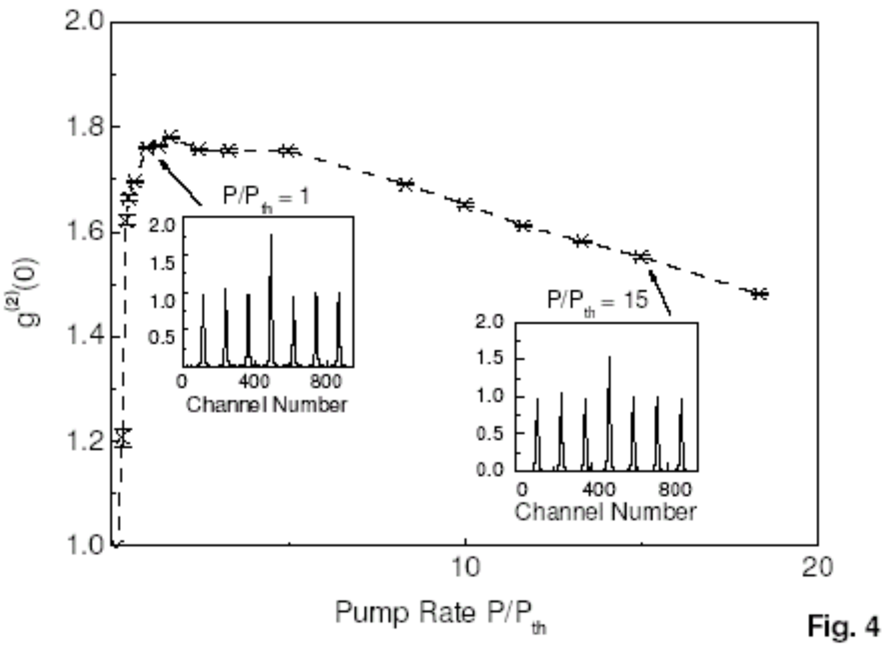
Polariton distribution n(E)



Coherence?

Second order correlator





Not coincident with pump
Hence not coherent FWM

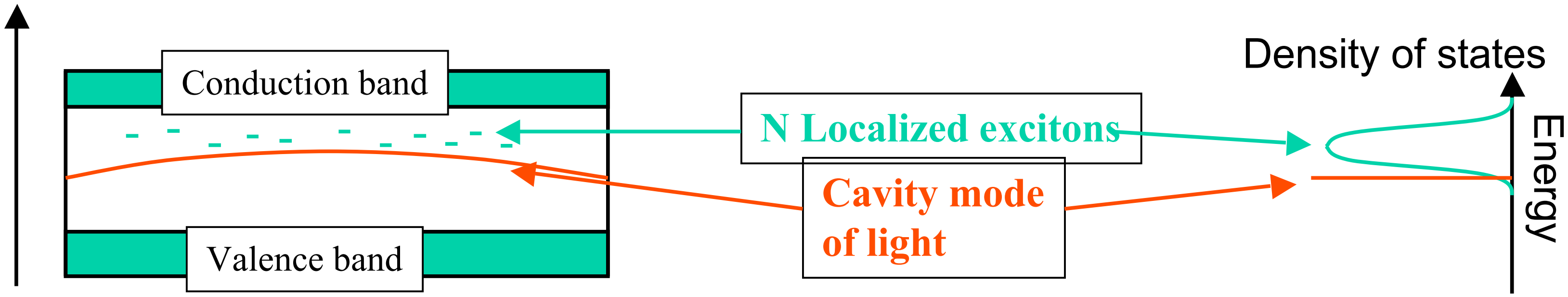
Deng et al 2002

Polariton condensates ?

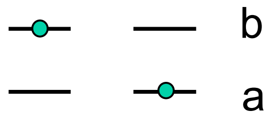
- Composite particle – mixture of electron-hole pair and photon
  - How does this affect the ground state
- Extremely light mass (~ 10^{-5} m_e) means that polaritons are large, and overlap strongly at low-density
  - BEC – "BCS" crossover
- Two-dimensional physics
  - BKT
- Polariton lifetime is short
  - Non-equilibrium, pumped dynamics
  - Decoherence ?

Microcavity polaritons

A simplified model - the excitons are localised and replaced by 2-level systems and coupled to a single optical mode in the microcavity



$$H = \sum_i \varepsilon_i \left(b_i^+ b_i - a_i^+ a_i \right)$$ 2-level system



$a_i^\dagger a_i + b_i^\dagger b_i = 1$

$$+ \omega \psi^+ \psi$$ photon

$$+ \frac{g}{\sqrt{N}} \sum_{i=1\ldots N} (b_i^+ a_i \psi + \psi^+ a_i^+ b_i)$$ Dipole coupling

Fermionic representation

- a_i creates valence hole, b^+_i creates conduction electron on site i

Photon mode couples equally to large number N of excitons since $\lambda \gg a_{Bohr}$

R.H. Dicke, Phys.Rev.**93**,99 (1954)
K.Hepp and E.Lieb, Ann.Phys.(NY) **76**, 360 (1973)

Localized excitons in a microcavity - the Dicke model

- Simplifications
  - Single cavity mode
  - Equilibrium enforced by not allowing excitations to escape
  - Thermal equilibrium assumed (at finite excitation)
  - No exciton collisions or ionisation (OK for dilute, disordered systems)

    Work in k-space, with Coulomb added - then solution is extension of Keldysh mean field theory (used by Schmitt-Rink and Chemla for driven systems)

    Important issues are not to do with localisation/delocalisation or binding/unbinding of e-h pairs but with decoherence
- Important physics
  - Fermionic structure for excitons (saturation; phase-space filling)
  - Strong coupling limit of excitons with light
- To be added later
  - Decoherence (phase-breaking, pairbreaking) processes
  - Non-equilibrium (pumping and decay)

Localized excitons in a microcavity - the Dicke model

$$H = \sum_i \varepsilon_i \left(b_i^+ b_i - a_i^+ a_i\right) + \omega\psi^+\psi + \frac{g}{\sqrt{N}}\sum_i (b_i^+ a_i \psi + \psi^+ a_i^+ b_i)$$

Excitation number (excitons + photons) conserved

$$L = \psi^+\psi + \tfrac{1}{2}\sum_i \left(b_i^+ b_i - a_i^+ a_i\right)$$

Variational wavefunction (BCS-like) is exact in the limit N → ∞ , L/N ~ const.
(easiest to show with coherent state path integral and 1/N expansion)

$$\left|\lambda, u, v\right\rangle = e^{\lambda\psi^+}\prod_i \left[v_i b_i^+ + u_i a_i^+\right]\left|0\right\rangle \qquad u_i^2 + v_i^2 = 1$$

Two coupled order parameters { Coherent photon field $\langle\psi\rangle$, Exciton condensate $\langle a_i^\dagger b_i\rangle$

Excitation spectrum has a gap

PR Eastham & PBL, Solid State Commun. 116, 357 (2000); Phys. Rev. B **64**, 235101 (2001)

Phase coherence

Hamiltonian as a spin model

$$H = \omega\psi^{\dagger}\psi + \sum_i \epsilon_i S_i^z + \frac{g}{\sqrt{N}}\sum_i \left(\psi S_i^+ + S_i^- \psi^{\dagger}\right)$$

Another way to write the wavefunction - a ferromagnet

$$|\lambda; w_i\rangle = \exp[\lambda\psi^{\dagger} + \sum_i w_i e^{i\theta_i} S_i^+]|0\rangle$$

Coherent ground state is phase locked - θ_i identical, self-consistent solution for λ, ω_i

$$(\omega - \mu)\lambda = \frac{2g^2\lambda}{N}\sum \frac{1}{\sqrt{(\epsilon_i - \mu)^2 + 4g^2\lambda^2}}$$

From Heisenberg equations of motion get the same solution by treating spins as classical objects precessing around self-consistently determined field

$$i\frac{d\psi}{dt} = (\omega - \mu)\psi + \frac{g}{\sqrt{N}}\sum_i S_i^-$$

$$i\frac{d}{dt}S_i^- = (\epsilon_i - \mu)S_i^- - \frac{2g}{\sqrt{N}}\psi\sum_i S_i^z$$

- coherent motion in classical electric field E(t) [Galitskii et al., JETP **30**,117 (1970)]

Generalisation from S=1/2 to large S will describe coupled macroscopic oscillators, e.g. Josephson junctions in a microwave cavity

Dictionary of broken symmetries

- Connection to excitonic insulator generalises the BEC concept – different guises

$$e^{\lambda \sum_k \phi_k a^\dagger_{ck} a_{vk}} = \prod_k \left(1 + \lambda \phi_k a^\dagger_{ck} a_{vk}\right)$$

- Rewrite as spin model

$$S_i^+ = a^\dagger_{ci} a_{vi} \quad ; \quad S_i^z = a^\dagger_{ci} a_{ci} - a^\dagger_{vi} a_{vi}$$

- XY Ferromagnet / Quantum Hall bilayer

$$|\psi\rangle = \exp\left[\sum_i \psi_i e^{i\phi_i} S_i^+\right]|0\rangle$$

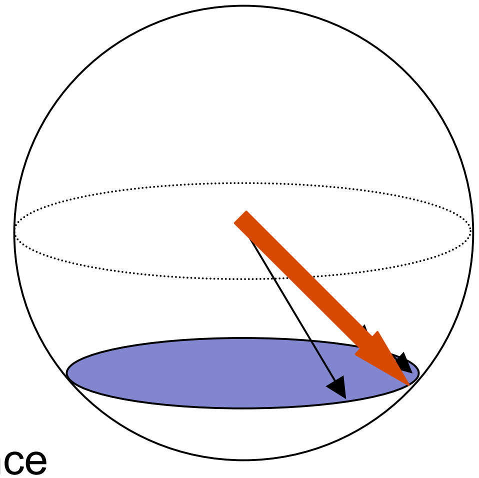
- Couple to an additional Boson mode:
  photons -> polaritons;
  molecules -> cold fermionic atoms near Feshbach resonance

$$|\lambda;\psi\rangle = \exp\left[\lambda \psi^\dagger + \sum_i \psi_i e^{i\phi_i} S_i^+\right]|0\rangle$$

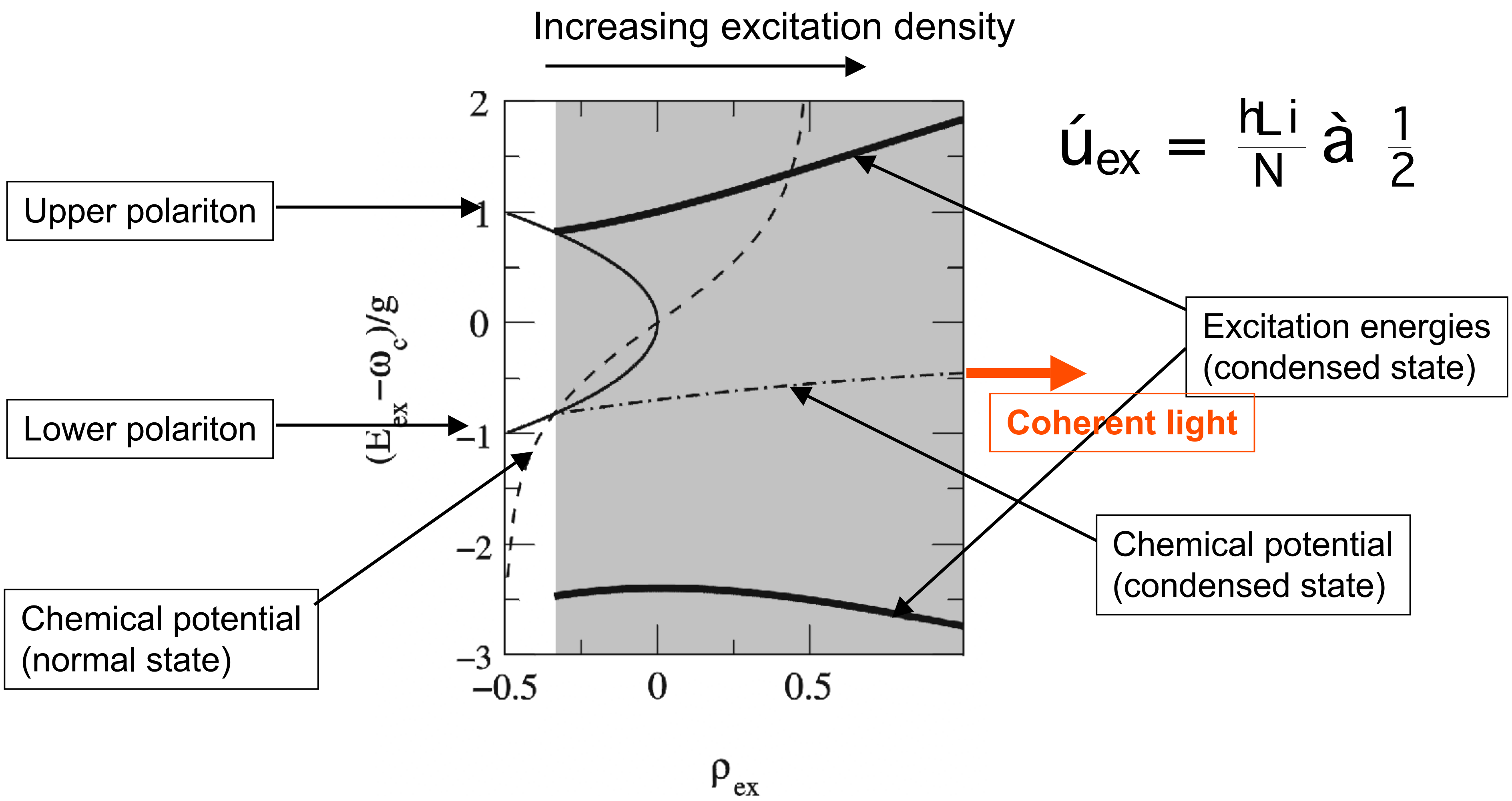
- Charge or spin density wave

$$\sum_k \left\langle a^\dagger_{mk+q} a_{nk} \right\rangle = \rho_{mn}(q)$$

Dynamics – precession in self-consistent field

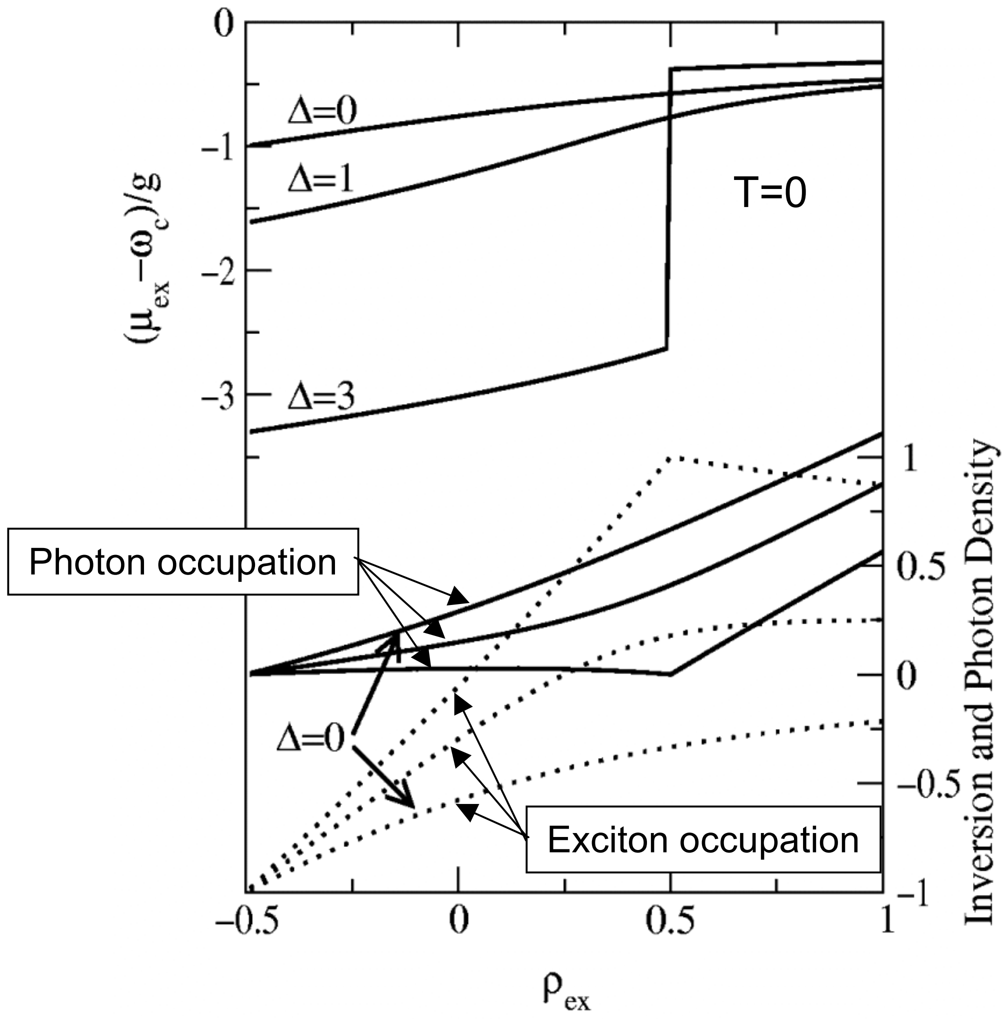


Condensation in the Dicke model (g/T = 2)

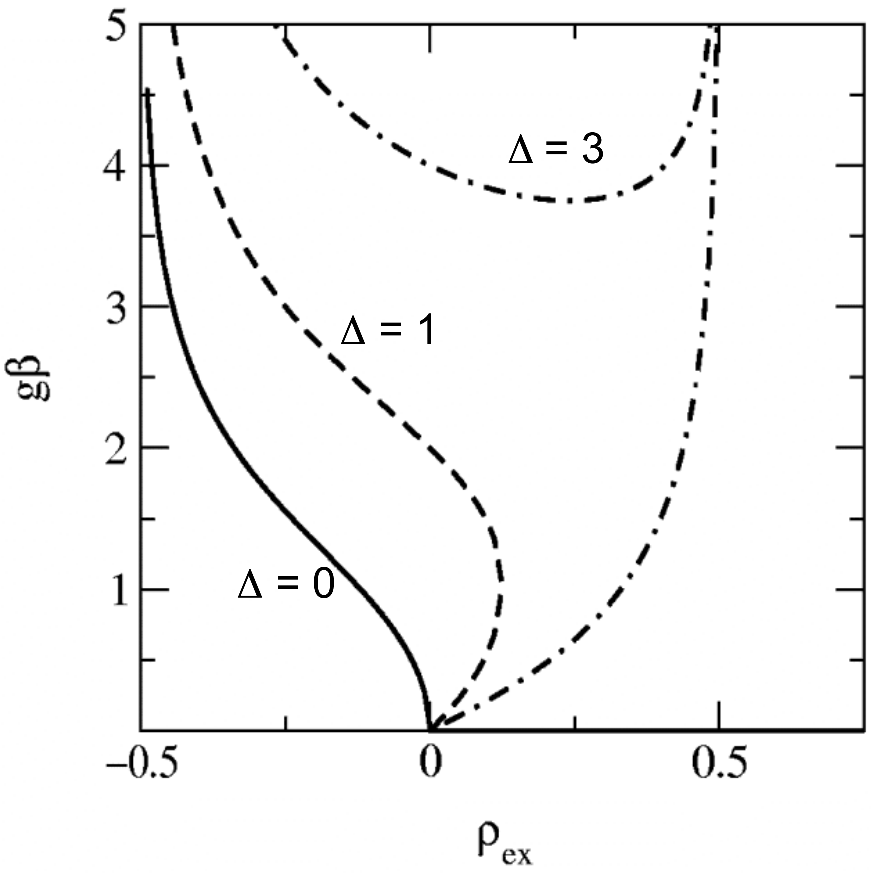


$$\rho_{ex} = \frac{\langle L \rangle}{N} - \frac{1}{2}$$

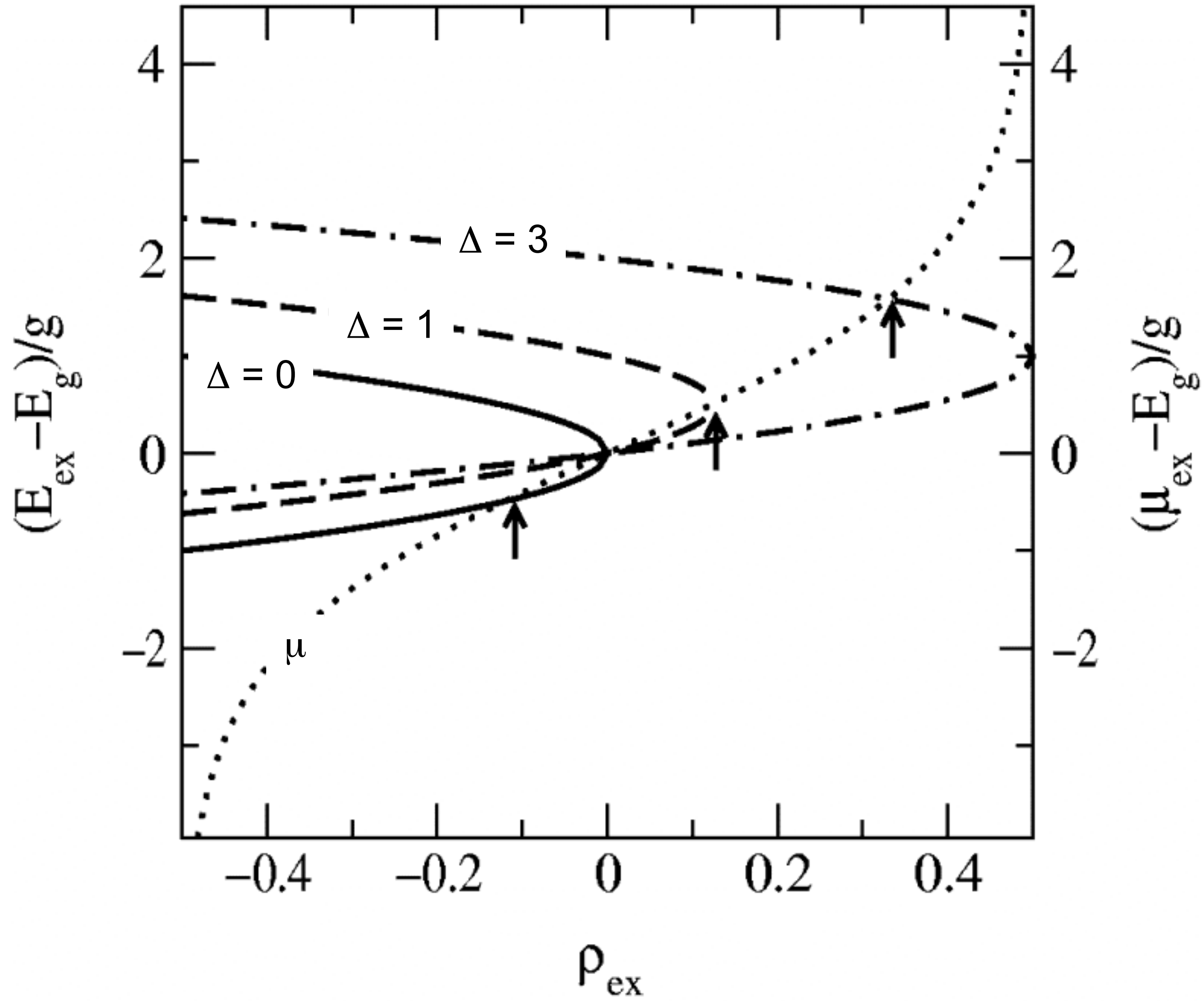
Phase diagram



Detuning $\Delta = (\omega - \varepsilon)/g$



Excitation spectrum for different detunings

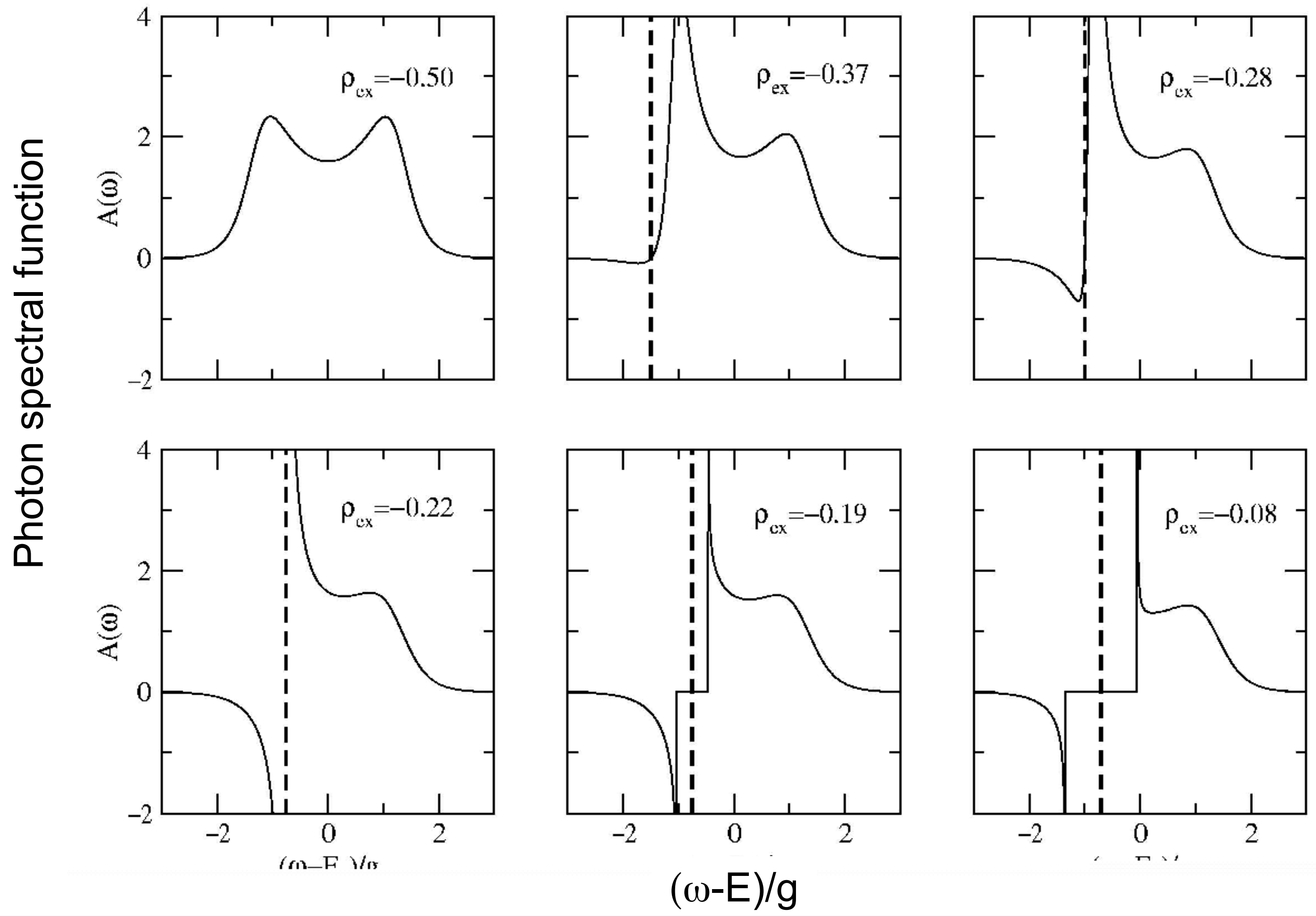


$g\beta = 1$

Excitation spectrum with inhomogeneous broadening

Zero detuning: $\omega = \varepsilon$

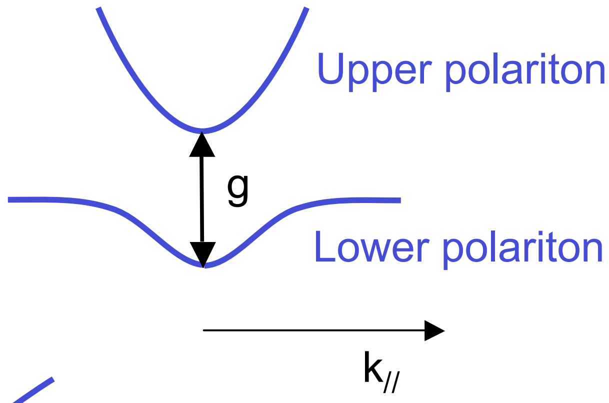
Gaussian broadening of exciton energies σ = 0.5 g

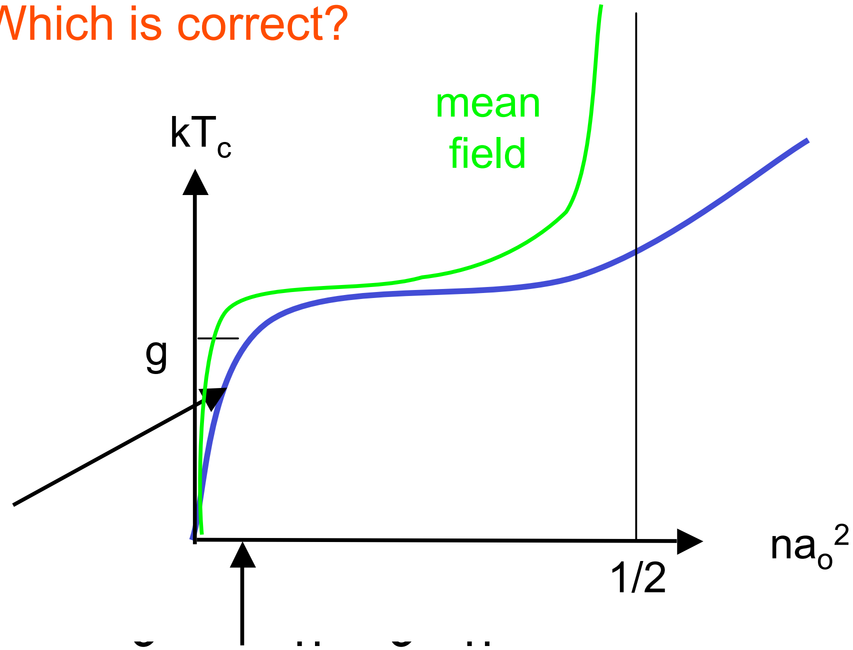


Beyond mean field: Interaction driven or dilute gas?

- Conventional “BEC of polaritons” will give high transition temperature because of light mass m^*
- Single mode Dicke model gives transition temperature ~ g

Which is correct?





$$k_B T_{BEC} \approx \frac{\hbar^2}{2m^*} n$$

$$\frac{g}{\hbar^2/2ma_o^2} \gg \frac{m^*}{m} \approx (\text{few}) \gg (10^{-4})$$

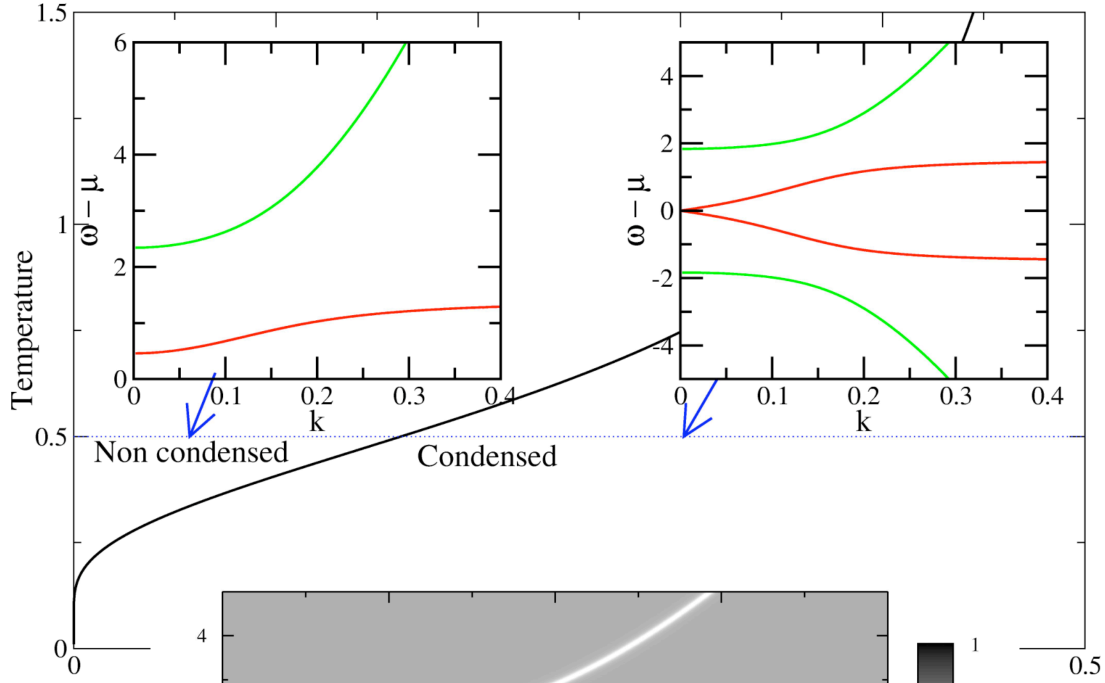
a_o = characteristic separation of excitons
a_o > Bohr radius

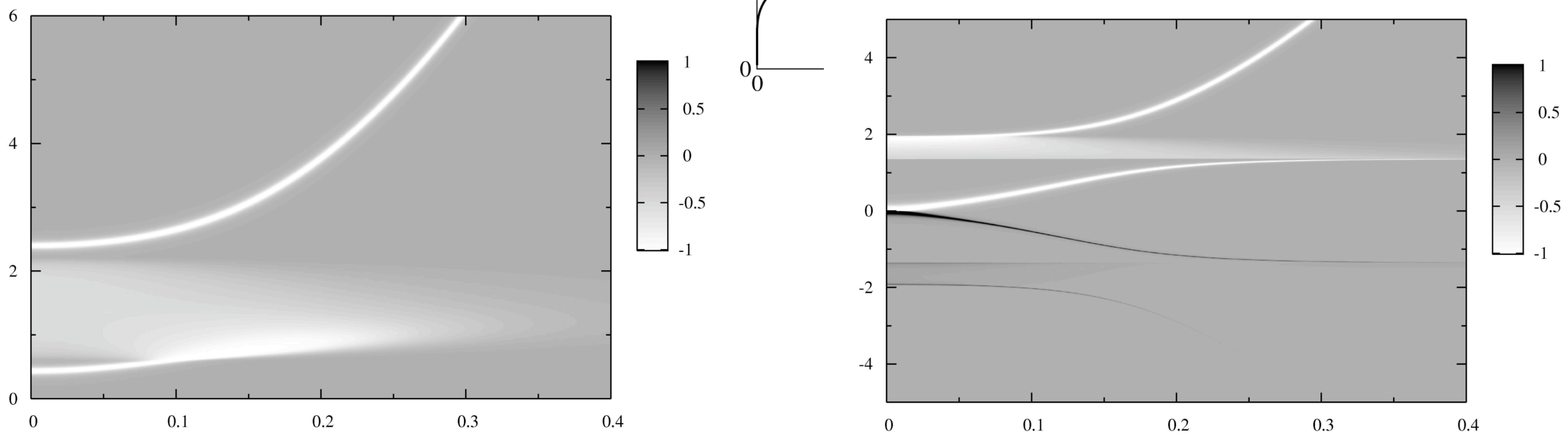
Dilute gas BEC only for excitation levels < 10^9 cm^{-2} or so

2D polariton spectrum

Keeling et al PRL 93, 226403 (2004)

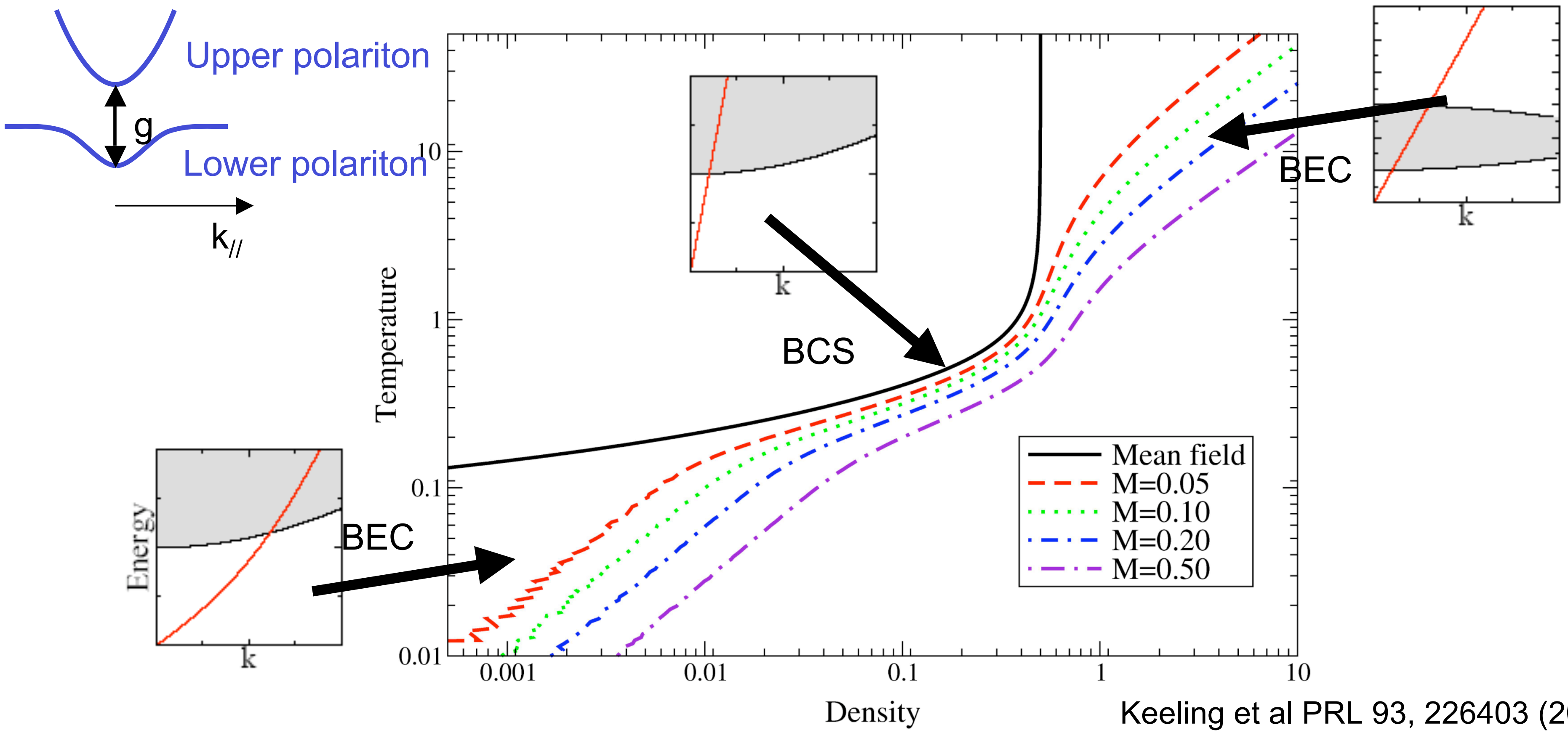
- Excitation spectrum calculated at mean field level
- Thermally populate this spectrum to estimate suppression of superfluid density (one loop)
- Estimate new T_c





Phase diagram

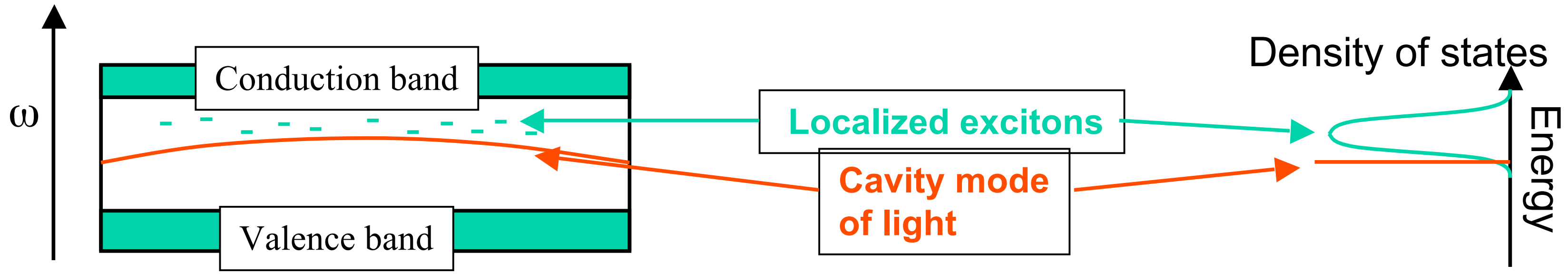
- T_c suppressed in low density (polariton BEC) regime and high density (renormalised photon BEC) regimes
- For typical experimental polariton mass ~ 10^{-5} deviation from mean field is small



Keeling et al PRL 93, 226403 (2004)

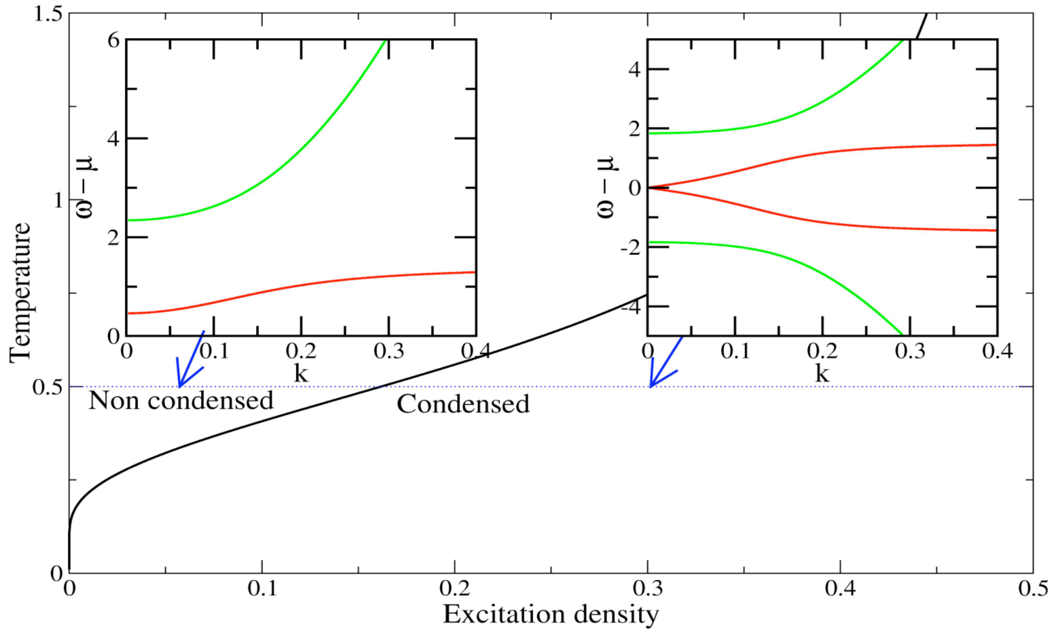
Microcavity polaritons – 2D physics

A simplified model – quantum dot excitons coupled to optical modes of microcavity



In thermal equilibrium, phase coherence – as in a laser – is induced by exchange of photons

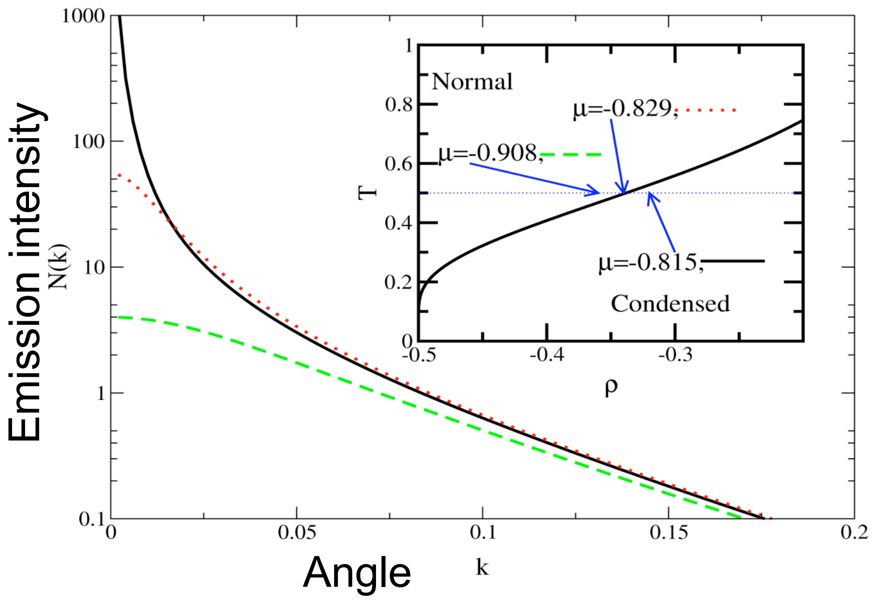
Excitation spectrum in the condensed state has new branches which provide an experimental signature of self-sustained coherence

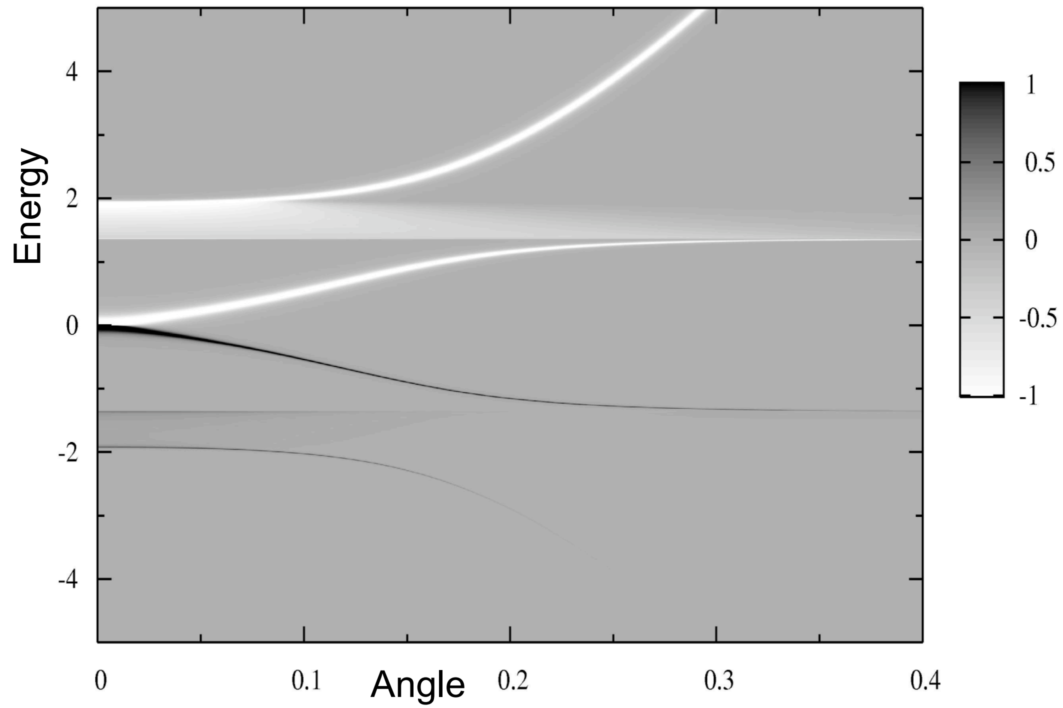


Excitation spectra in microcavities with coherence

Keeling, Eastham, Szymanska, PBL PRL 2004

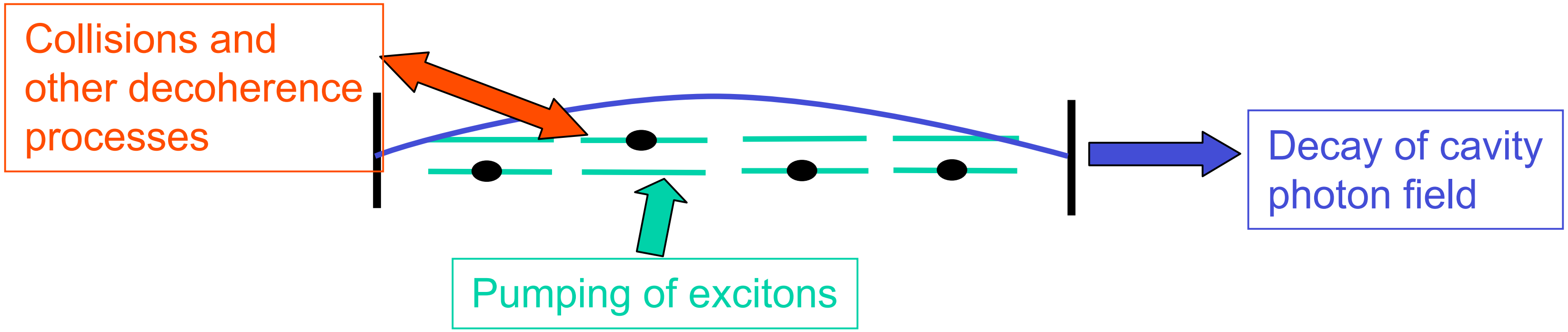
Angular dependence of luminescence becomes sharply peaked at small angles
(No long-range order because a 2D system)





Absorption(white) / Gain(black) spectrum of coherent cavity

Decoherence and the laser



Decay, pumping, and collisions may introduce "decoherence" - loosely, lifetimes for the elementary excitations - include this by coupling to bosonic "baths" of other excitations

Conventional theory of the laser

$$H = H_0 + H_{SB} + H_B$$

$$H_0 = \sum_i \epsilon_i (b_i^\dagger b_i - a_i^\dagger a_i) + \omega_c \alpha^\dagger \alpha + \frac{g}{\sqrt{N}} \sum_i \left[\alpha^\dagger a_i^\dagger b_i + h.c. \right]$$ system

$$H_B = \sum_k \left[\omega_k d_k^\dagger d_k + \omega_{+;k} c_{+;k}^\dagger c_{+;k} + \omega_{-;k} c_{-;k}^\dagger c_{-;k} + \omega_{1;k} c_{1;k}^\dagger c_{1;k} + \omega_{2;k} c_{2;k}^\dagger c_{2;k} \right]$$ bosonic “baths”

$$H_{SB} = \sum_k g_k (\alpha^\dagger d_k + d_k^\dagger \alpha)$$ decay of cavity mode

$$+ \sum_{jk} \left[b_j^\dagger a_j (g_{jk}^{\gamma +} c_{+;k}^\dagger + g_{jk}^{\gamma -} c_{-;k}) + h.c. \right]$$ phase-breaking

$$+ \sum_{jk} \lambda_{jk}^{(1)} (b_j^\dagger b_j + a_j^\dagger a_j)(c_{1;k}^\dagger + c_{1;k})$$ pair-breaking

$$+ \sum_{jk} \lambda_{jk}^{(2)} (b_j^\dagger b_j - a_j^\dagger a_j)(c_{2;k}^\dagger + c_{2;k})$$ non-pair-breaking

From Heisenberg to Langevin equations of motion

$$\frac{d}{dt} = -i\omega_c - ig\sum_i a_i^\dagger b_i - i\sum_k g_k d_k$$

$$\frac{d}{dt}d_k = -i\omega_k d_k - ig_k$$

$$d_k(t) = d_k(t_o)e^{-i\omega_k(t-t_o)} - g_k\int_{t_o}^{t} dt' \,(t')e^{-i\omega_k(t-t')}$$

$$\frac{d}{dt} = -i\omega_c - ig\sum_i a_i^\dagger b_i - i\sum_k g_k d_k(t_o)e^{-i\omega_k(t-t_o)} - \sum_k g_k^2\int_{t_o}^{t} dt' \,(t')e^{-i\omega_k(t-t')}$$

Markov approximation

$$\frac{d}{dt} = (-i\omega_c - \gamma) - ig\sum_i a_i^\dagger b_i + F(t)$$

$$\frac{d}{dt}a_j^\dagger b_j = (-i\epsilon_j - \nu_{?})a_j^\dagger b_j + ig(b_j^\dagger b_j - a_j^\dagger a_j) + \xi_{j-}$$

Polarisation $S^{+,-}$

$$\frac{d}{dt}(b_j^\dagger b_j - a_j^\dagger a_j) = \nu_{jj}(d_o - b_j^\dagger b_j + a_j^\dagger a_j) + 2ig(\ {}^\dagger a_j^\dagger b_j - b_j^\dagger a_j\ ) + \xi_{j;d}$$

Inversion S^Z

From Langevin equations to mean field

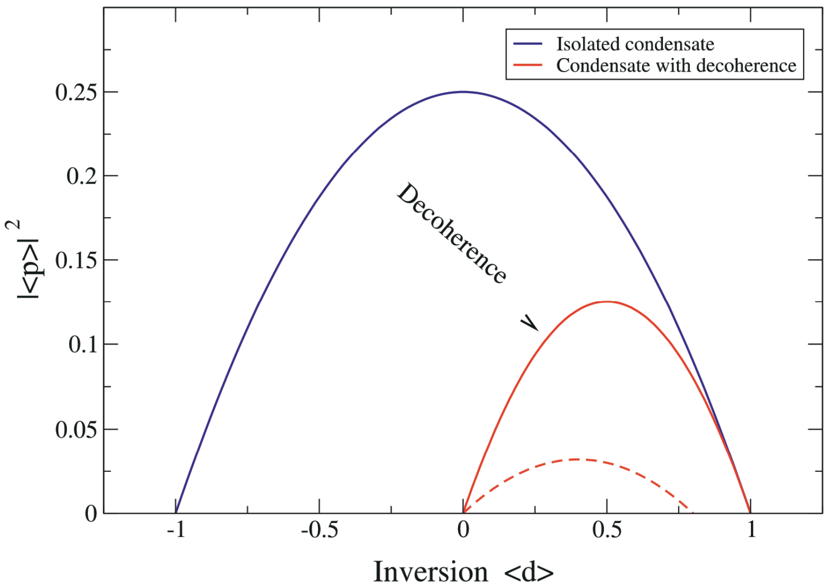
Bloch equations in a self-consistent field

$$\frac{d}{dt}\langle\psi\rangle = (-i\omega_c - \kappa)\langle\psi\rangle - ig\sum_j \left\langle S_j^- \right\rangle$$

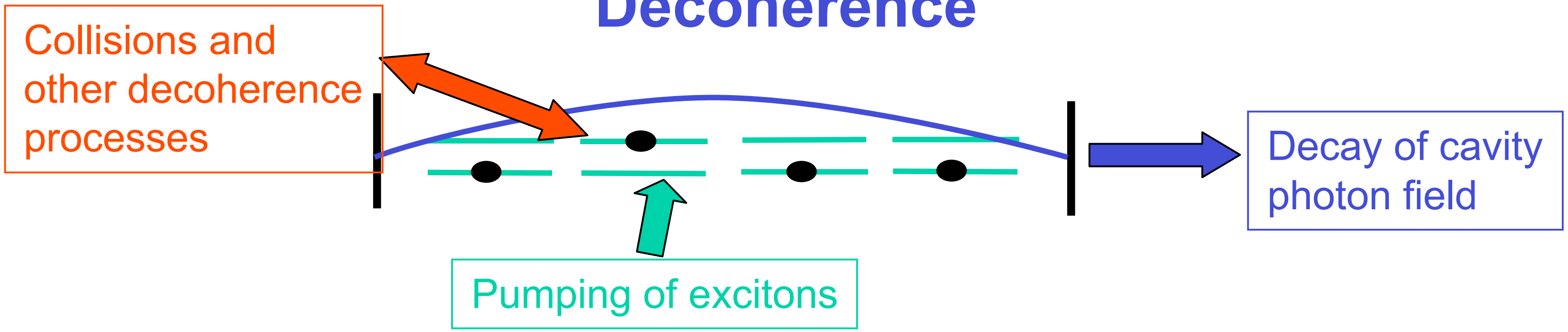
$$\frac{d}{dt}\left\langle S_j^- \right\rangle = (-i\epsilon_j - \gamma_\perp)\left\langle S_j^- \right\rangle + ig\langle\psi\rangle\left\langle S_j^z \right\rangle$$

$$\frac{d}{dt}\left\langle S_j^z \right\rangle = \gamma_\parallel\left(d_0 - \left\langle S_j^z \right\rangle\right) + 2ig\left(\psi^\dagger\left\langle S_j^- \right\rangle - \left\langle S_j^+ \right\rangle\langle\psi\rangle\right)$$

If decay processes are turned off, solutions are identical to BCS mean field equations – but these are unstable to infinitesimal damping



Decoherence



Decay, pumping, and collisions may introduce "decoherence" - loosely, lifetimes for the elementary excitations - include this by coupling to bosonic "baths" of other excitations

> in analogy to superconductivity, the external fields may couple in a way that is "pair-breaking" or "non-pair-breaking"

$$\gamma_1 \sum_{i;k} (b_i^\dagger b_i - a_i^\dagger a_i)(c_{1;k}^\dagger + c_{1;k})$$ non-pairbreaking (inhomogeneous distribution of levels)

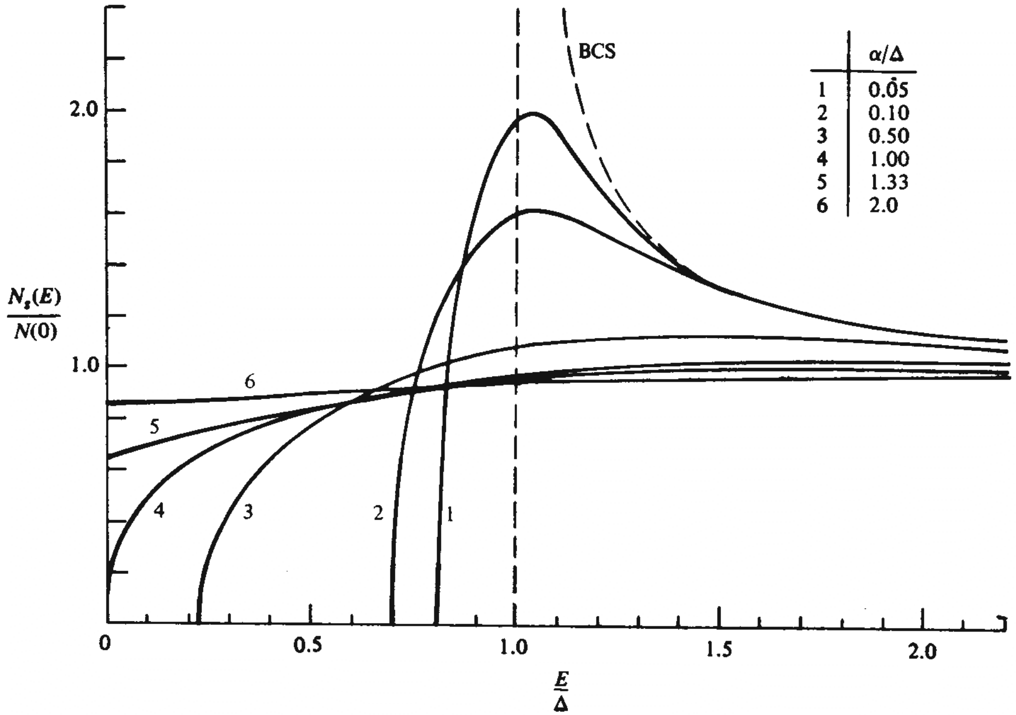
$$\gamma_2 \sum_{i;k} (b_i^\dagger b_i + a_i^\dagger a_i)(c_{2;k}^\dagger + c_{2;k})$$ pairbreaking disorder

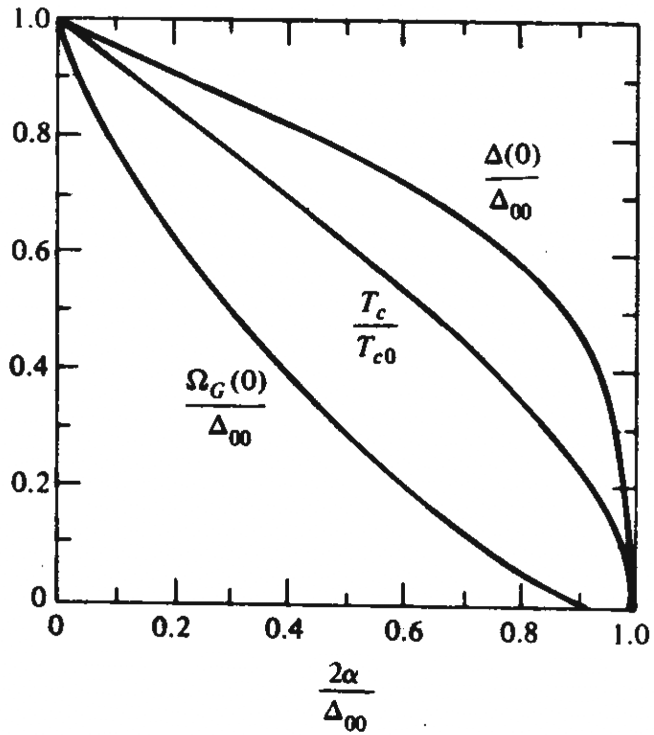
- Conventional theory of the laser assumes that the external fields give rise to rapid decay of the excitonic polarisation - incorrect if the exciton and photon are strongly coupled
- Correct theory is familiar from superconductivity - Abrikosov-Gorkov theory of superconductors with magnetic impurities

$$\gamma_3 \sum_{i;k} (b_i^\dagger a_i c_{3;k}^\dagger + a_i^\dagger b_i c_{3;k})$$ symmetry breaking – XY random field destroys LRO

Detour - Abrikosov-Gorkov theory of gapless superconductivity

- Ordinary impurities that do not break time reversal symmetry are "irrelevant". Construct pairing between degenerate time-reversed pairs of states (Anderson's theorem)
- Fields that break time reversal (e.g. magnetic impurities, spin fluctuations) suppress singlet pairing, leading first to gaplessness, then to destruction of superconductivity
  [Abrikosov & Gorkov ZETF 39, 1781 (1960); JETP 12, 12243 (1961)]





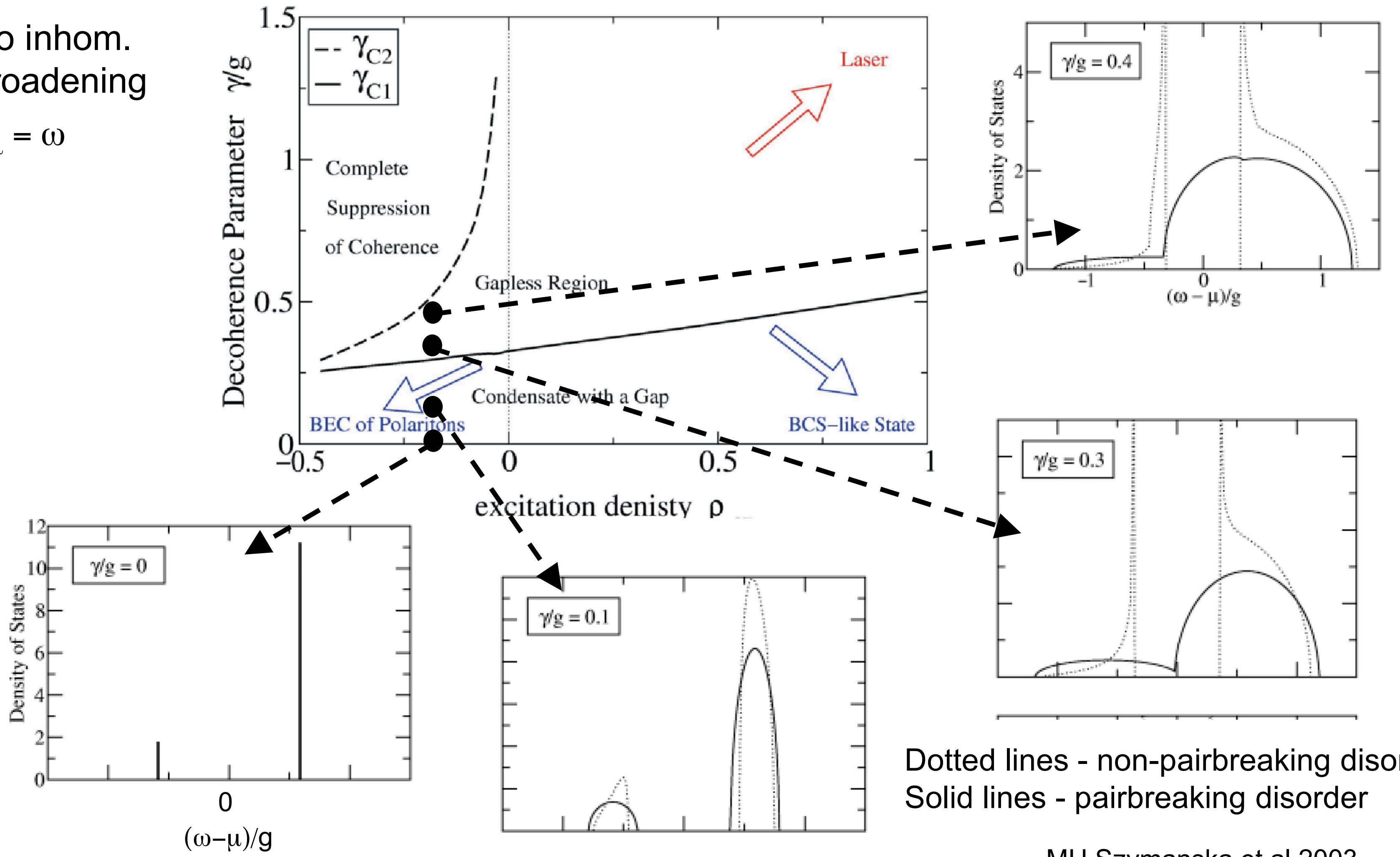
Skalski et al, PR136, A1500 (1964)

Phase diagram of Dicke model with pairbreaking

Pairbreaking characterised by a single parameter $\gamma = \lambda^2 N(0)$

No inhom. broadening

$\varepsilon_\iota = \omega$



Dotted lines - non-pairbreaking disorder
Solid lines - pairbreaking disorder

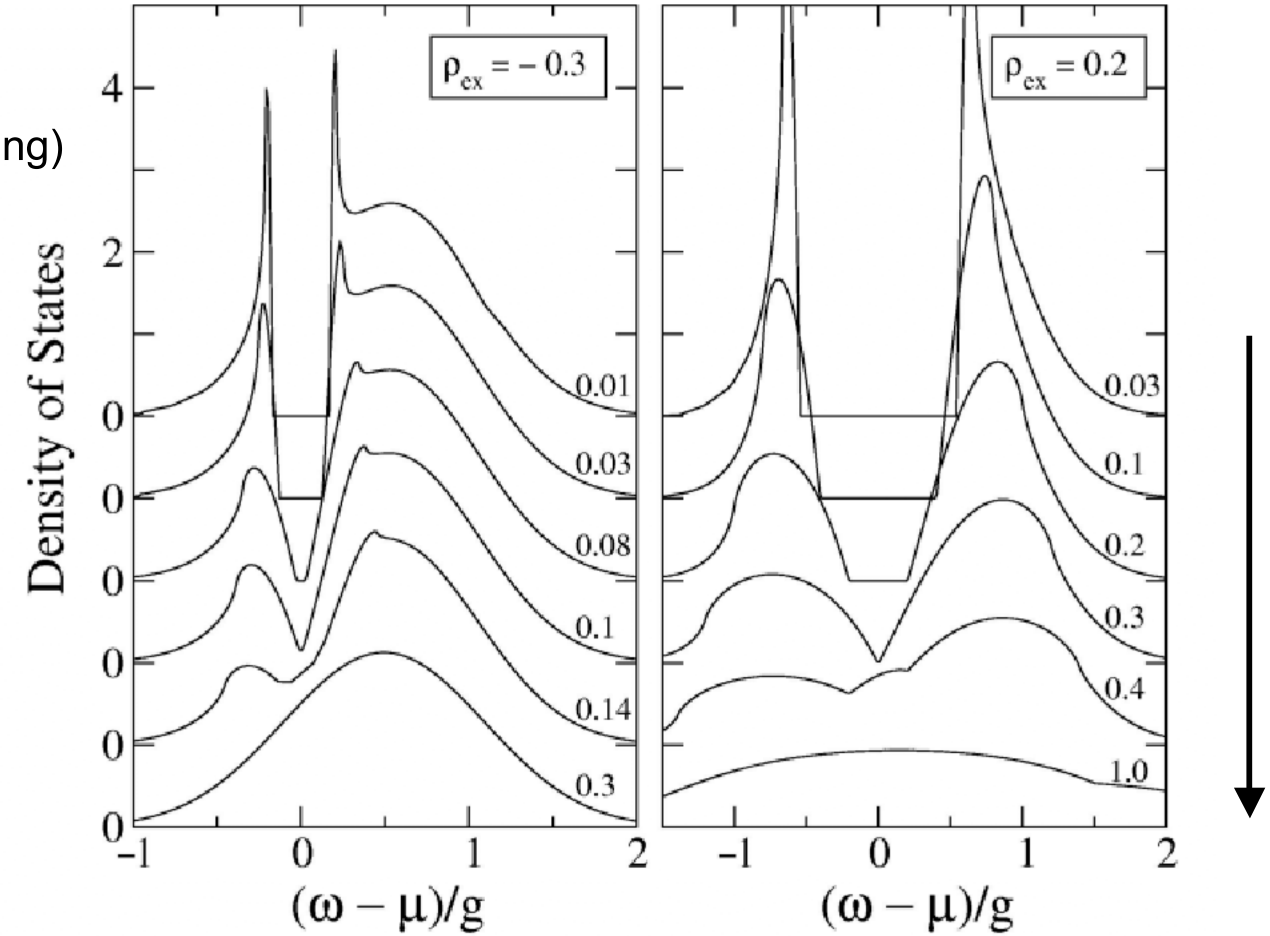
MH Szymanska et al 2003

Transition to gaplessness and lasing

Model with both pairbreaking and non-pairbreaking (inhom. broadening)

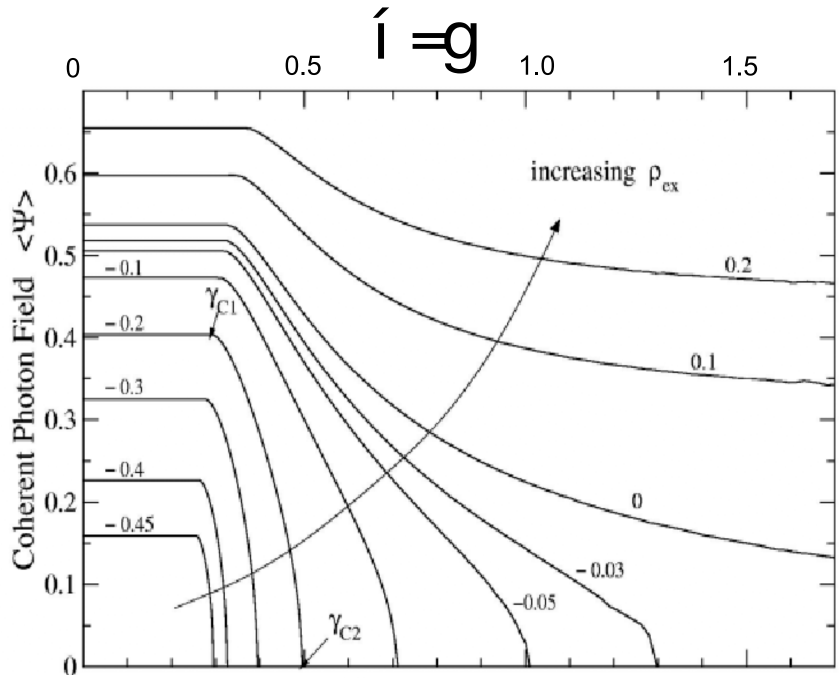
Low density: below inversion

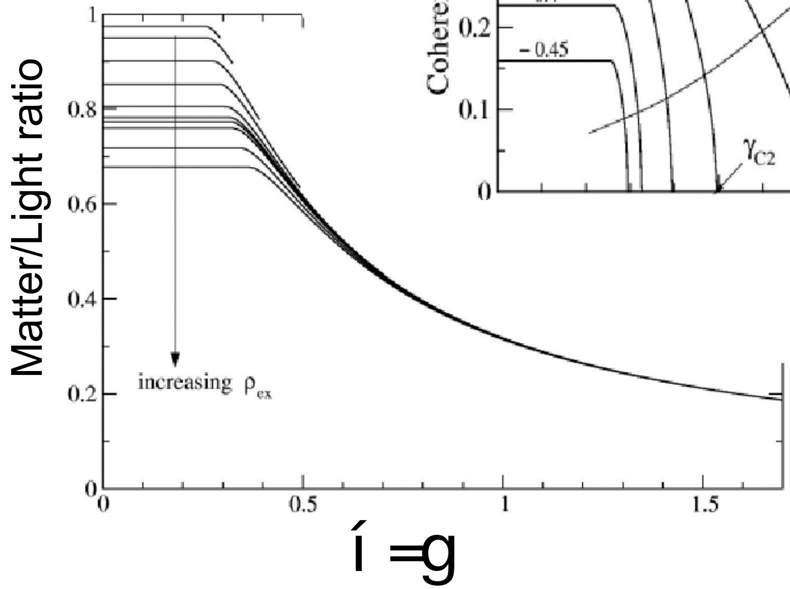
Higher density: above inversion



Increasing pairbreaking

This "laser" is indeed an example of BEC





Large decoherence -- "laser"

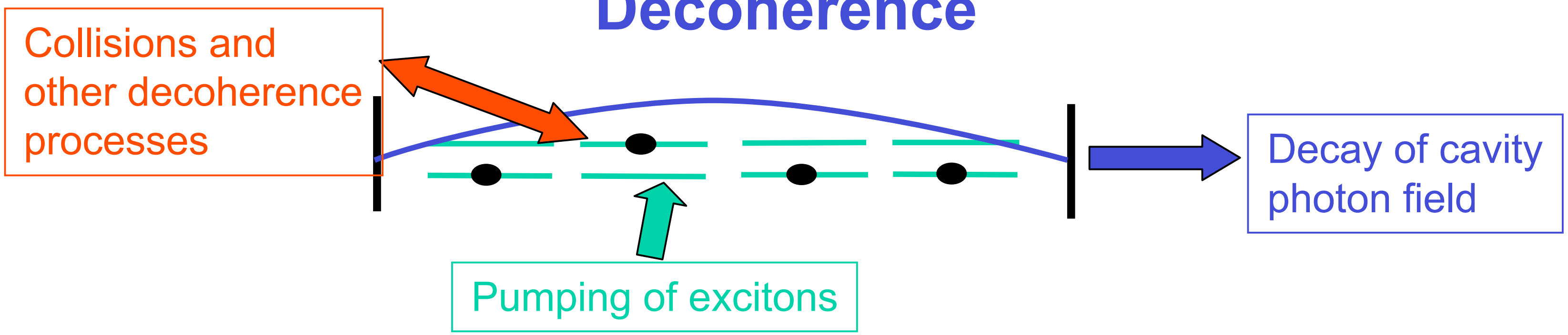
- order parameter nearly photon like
- electronic excitations have short lifetime
- excitation spectrum gapless

real lasers are usually far from equilibrium

Small decoherence -- BEC of polaritons

- order parameter mixed exciton/photon
- excitation spectrum has a gap

Decoherence



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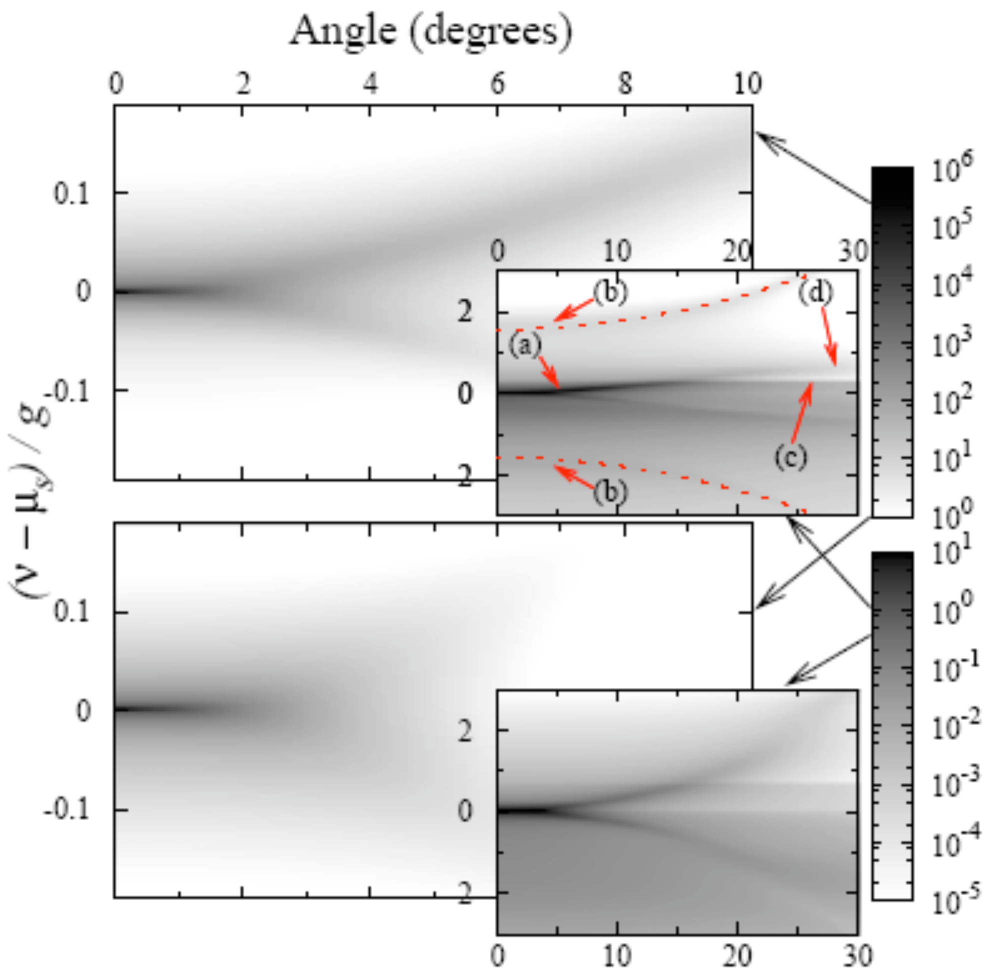
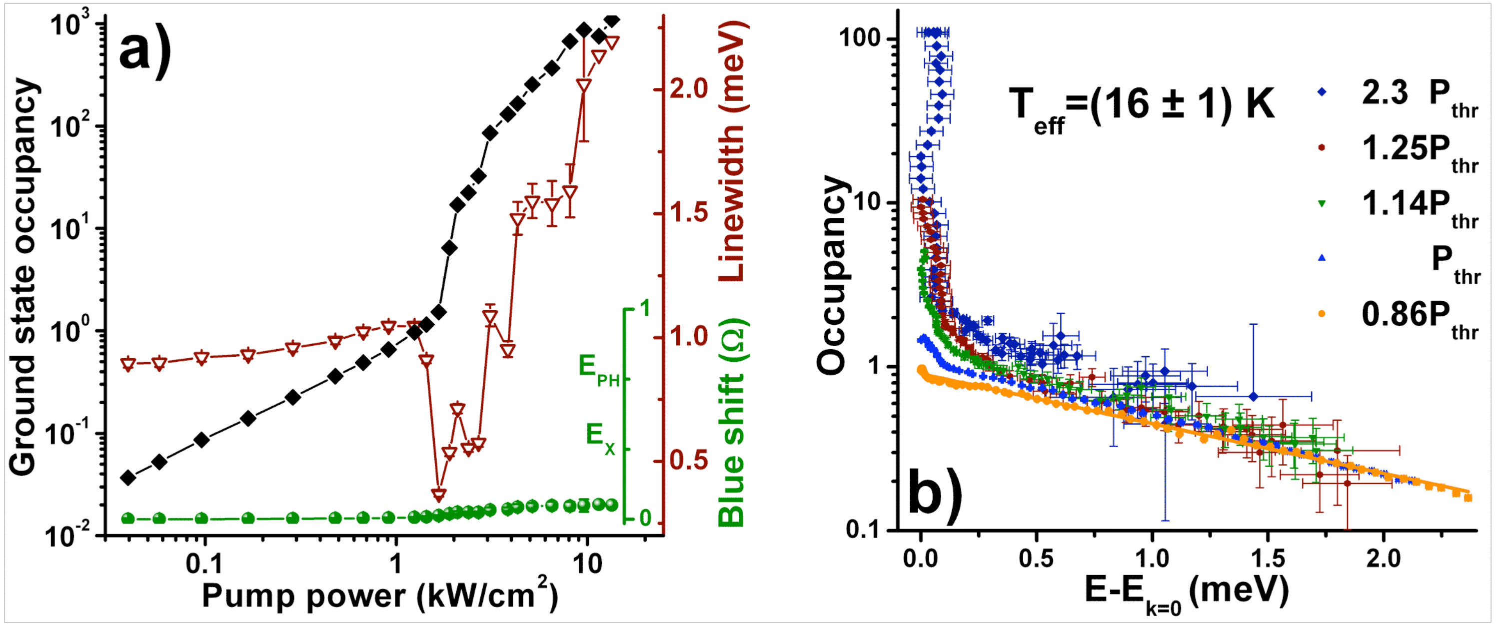
Steady state system of pumped polaritons

- Simplest dynamical model for driven condensate
- Decay of photon mode
- Separate pumping of electron and hole by fermion baths (like an LED)
- Bogoliubov mode becomes diffusive at long length scales – merges with quasi-LRO of condensed system



Szymanska et al cond-mat/06

Distribution at varying density



Blue shift used to estimate density
High energy tail of distribution used to fix temperature
Onset of non-linearity gives estimate of critical density
Linewidth well above transition is *inhomogeneous*

Comparison to recent experiments - density

Appears to be well inside mean-field regime

7

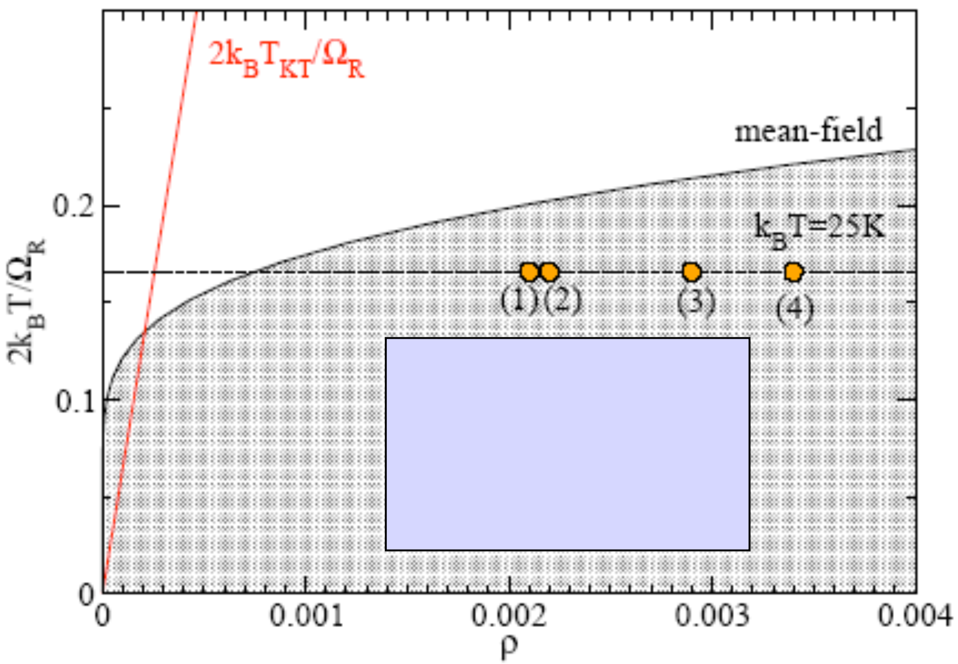


FIG. 9: Mean-field phase diagram with superimposed data from the $T_{\text{cryo}} = 25$K measurements for $\omega_0 - E_x = 5.06$meV (effective detuning $\delta = +6$meV). The Kosterlitz-Thouless phase boundary (red) is explicitly plotted for a photonic mass $m^*_{\text{ph}} = 3.96 \times 10^{-5}$ ($m^*_{\text{pol}} = 1.022 \times 10^{-4}$).

Linewidth

- Calclulation includes dephasing from pumping and decay
- Below threshold, linewidth narrows and intensity grows (critical fluctuations)
- Measured linewidth is consistent with dephasing that is weak enough to permit effects of condensation

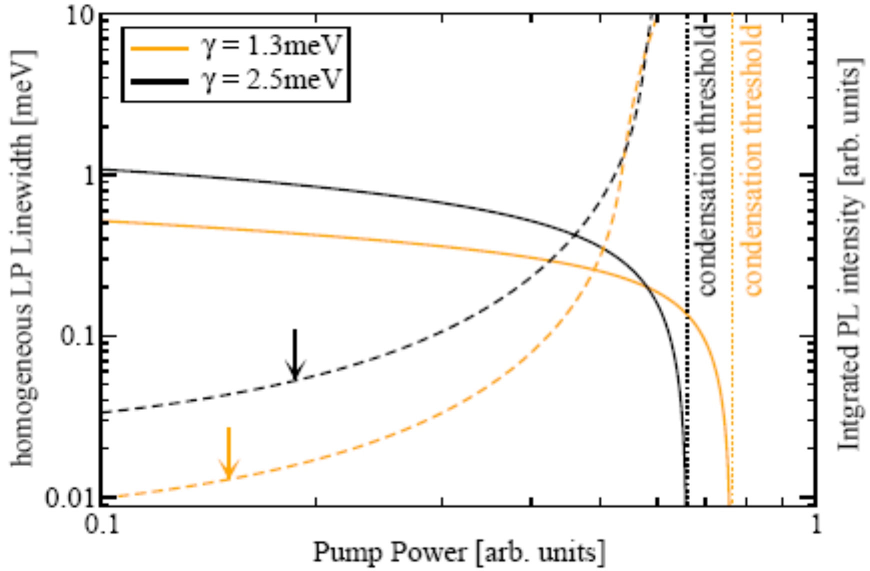
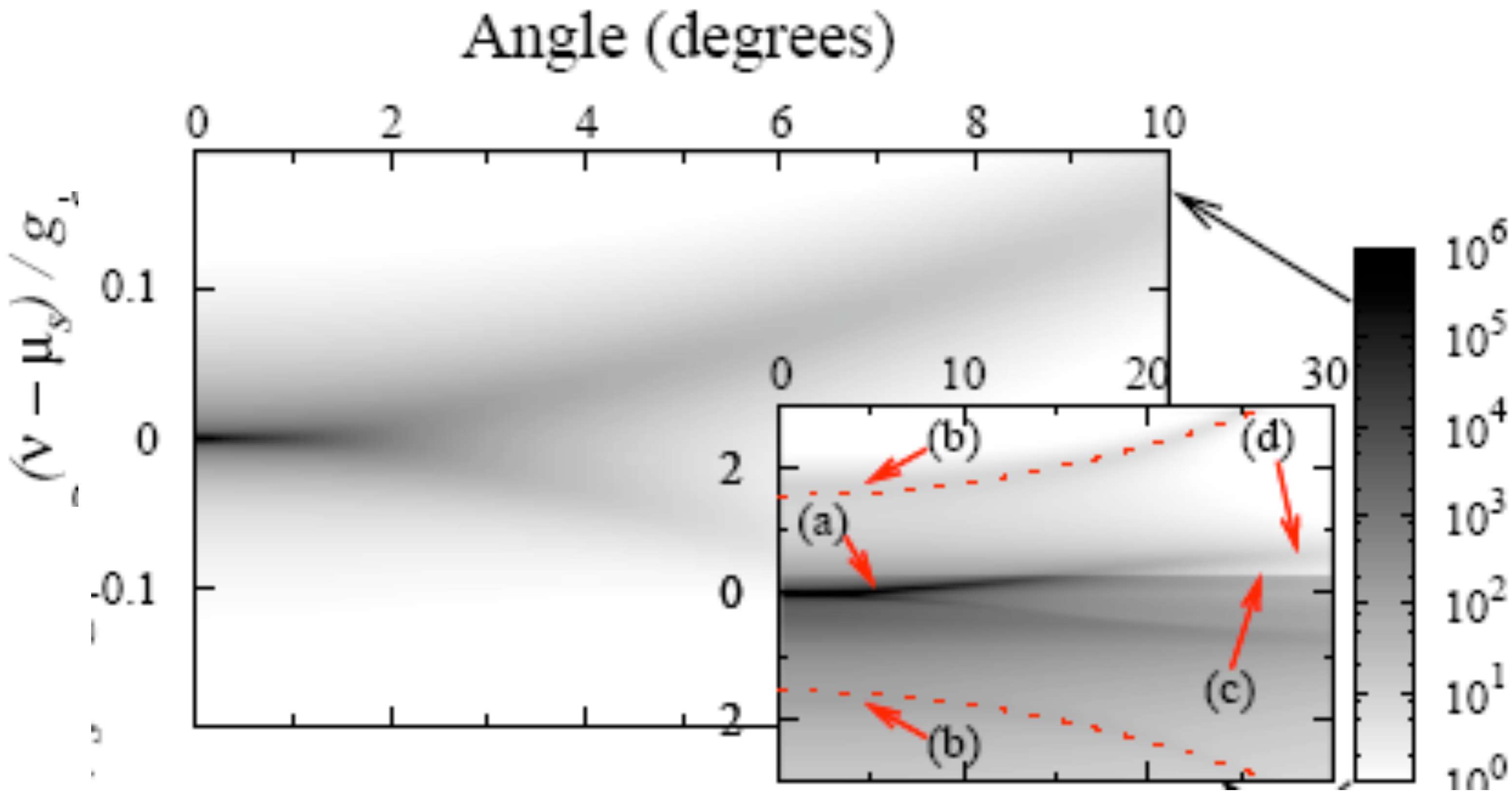


FIG. 5: Calculated homogeneous line-width of the $\mathbf{k}_{\|} = 0$ lower polariton (solid line) and the integrated $\mathbf{k}_{\|} = 0$ PL intensity as a function of the pump intensity for two different dephasing parameters γ. The decay rate of the photon is determined from the homogeneous photon linewidth, measured to be around 1meV. The threshold for non-linear emission is explicitly shown.

Optical emission above threshold

Keeling et al., cond-mat/0603447

At low momenta, Goldstone-Bogoliubov mode becomes dissipative
Non-linear emission dominates in experiment – no dynamical modes observed

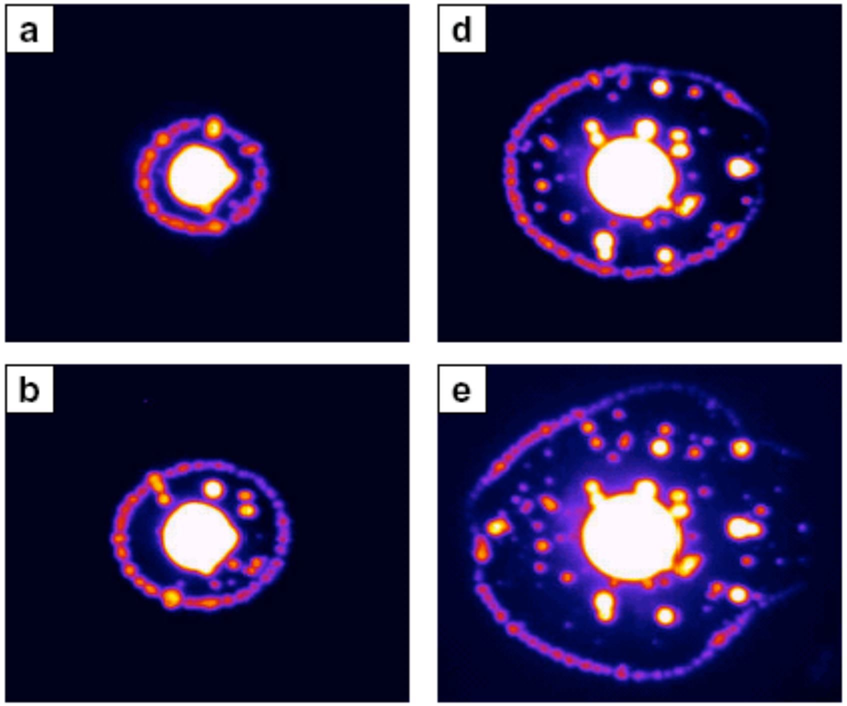


Conclusions

- Excitonic insulator is a broad concept that logically includes CDW's, ferromagnets, quantum Hall bilayers as well as excitonic BEC
- Excitonic coherence – oscillator phase-locking
  - enemy of condensation is decoherence
  - excitons are not conserved so *all* exciton condensates are expected to show coherence for short enough times only
  - condensates will either be diffusive (polaritons) or have a gap (CDW)
- BCS + pairbreaking or phasebreaking fluctuations gives a robust model that connects exciton/polariton BEC continuously to
  - semiconductor plasma laser (pairbreaking) or
  - solid state laser (phase breaking)
  - is a laser a condensate? – largely semantic
- Now good evidence for polariton condensation in recent experiments

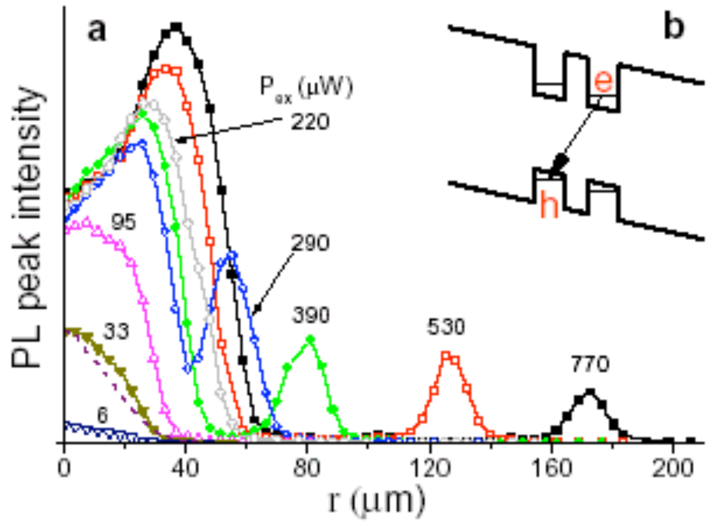
AlGaAs CQW – rings, droplets and beads

Butov et al. Nature, Aug 2002; similar data from Snoke and others



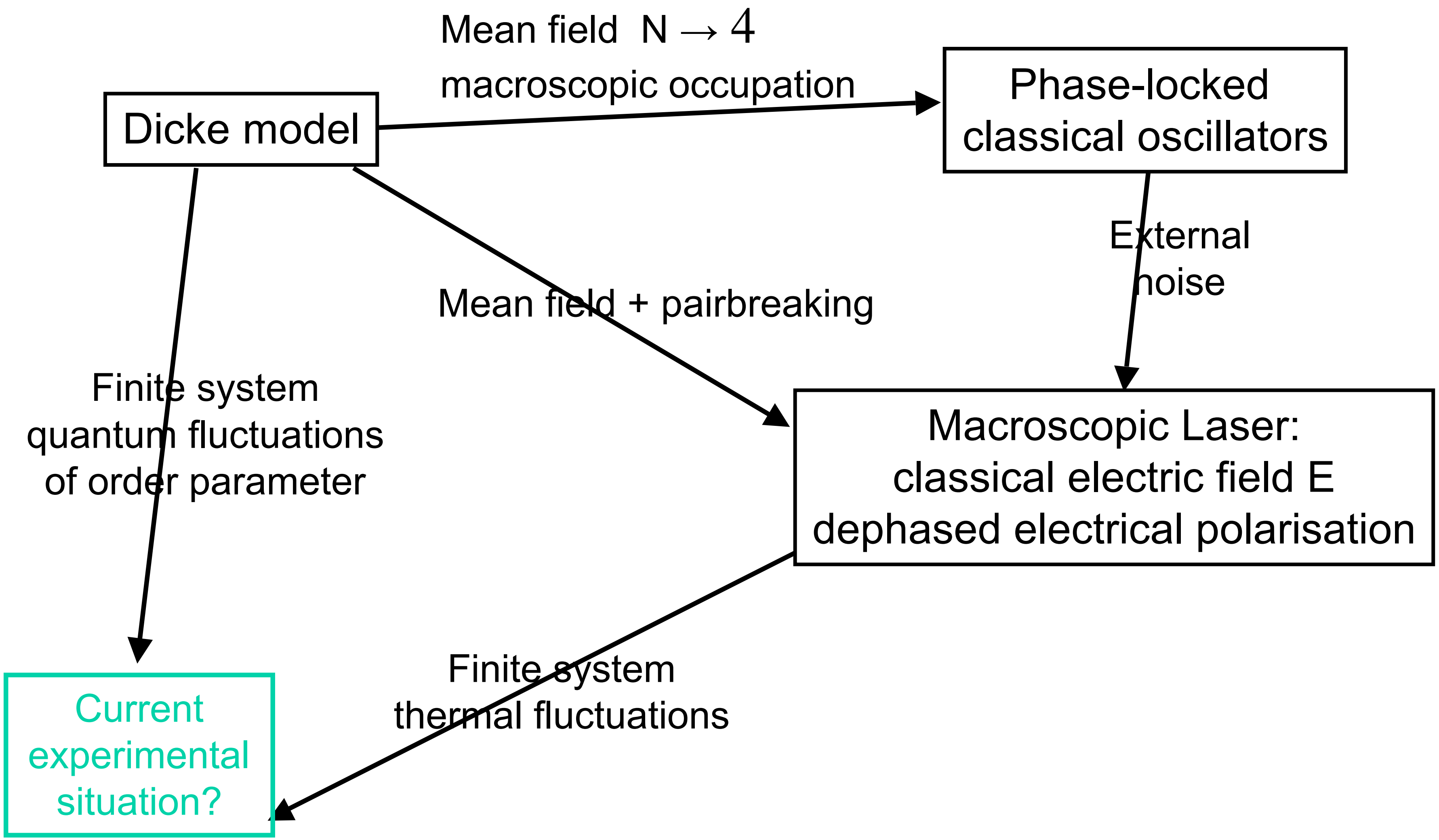
100 μm T=1.8 K

Outer ring is >100 μm from excitation – cold exciton formation due to independent e-h recombination
Inner ring moves out more slowly with power
Localised bright spots fixed in sample – pinholes?

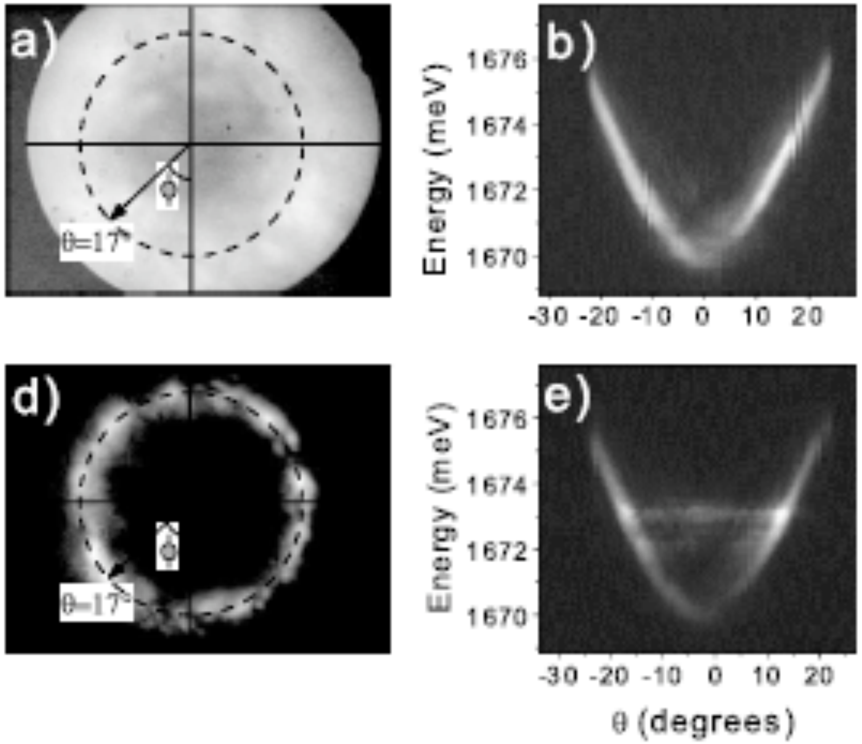


Ring is produce by separate electron-hole recombination
Beads due to classical nonlinear instability?

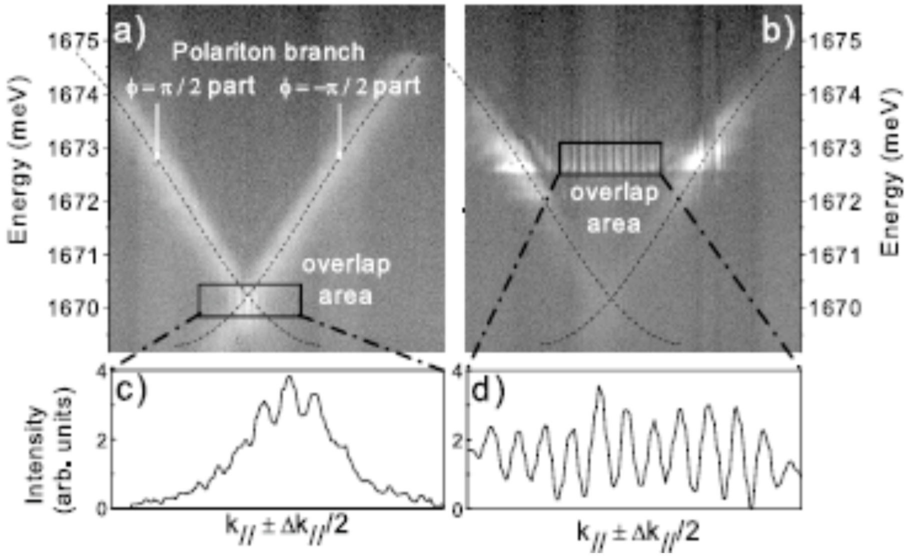
Connections



Spontaneous phase coherence in CdTe microcavities



Richard et al PRL 94, 187401, 2005



Conclusions

- Exciton condensation

How do you make a BEC wavefunction based on pairs of fermions?
BCS

In the dense limit (and always in 2D) the transition is driven by interactions and is better thought of as phase-locking of excitons

If recombination is disallowed, this is a true superfluid.

Experimental situation is interesting but somewhat confused ...

- Coupled excitons and photons - polaritons

What happens to the light field if the "matter" field is coherent? Still BCS

Two order parameters have phases that are entrained. In the low density regime, this "looks like" BEC of polaritons.

- Open systems

How do you treat coupling to the environment? BCS + pairbreaking (AG)

Weak pairbreaking, gap is robust, and BEC persists.

Strong pairbreaking, gap closes, order parameter becomes almost entirely photon-like

No fundamental distinction between BEC of polaritons and a laser.

Multiband superconductivity in ultracold atoms, polaritons, and superconductors

Peter Littlewood, University of Cambridge
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Cold Atoms

Meera Parish, Francesca Marchetti, Marzena Szymanska, Ben Simons,
Bogdan Mihaila, Eddy Timmermans, Darryl Smith, Sasha Balatsky (Los Alamos)

MM Parish et al cond-mat/0410131 Phys.Rev. B71 (2005) 064513
MM Parish et al.,cond-mat/0409756 Phys.Rev.Lett. 94 (2005) 240402
B Mihaila et al, cond-mat/0502110 Phys.Rev.Lett. 95 (2005) 090402

Excitons and Polaritons

Anson Cheung, Paul Eastham, Jonathan Keeling, Francesca Marchetti, Ben Simons, Marzena Szymanska,
Pablo Lopez Rios, Richard Needs

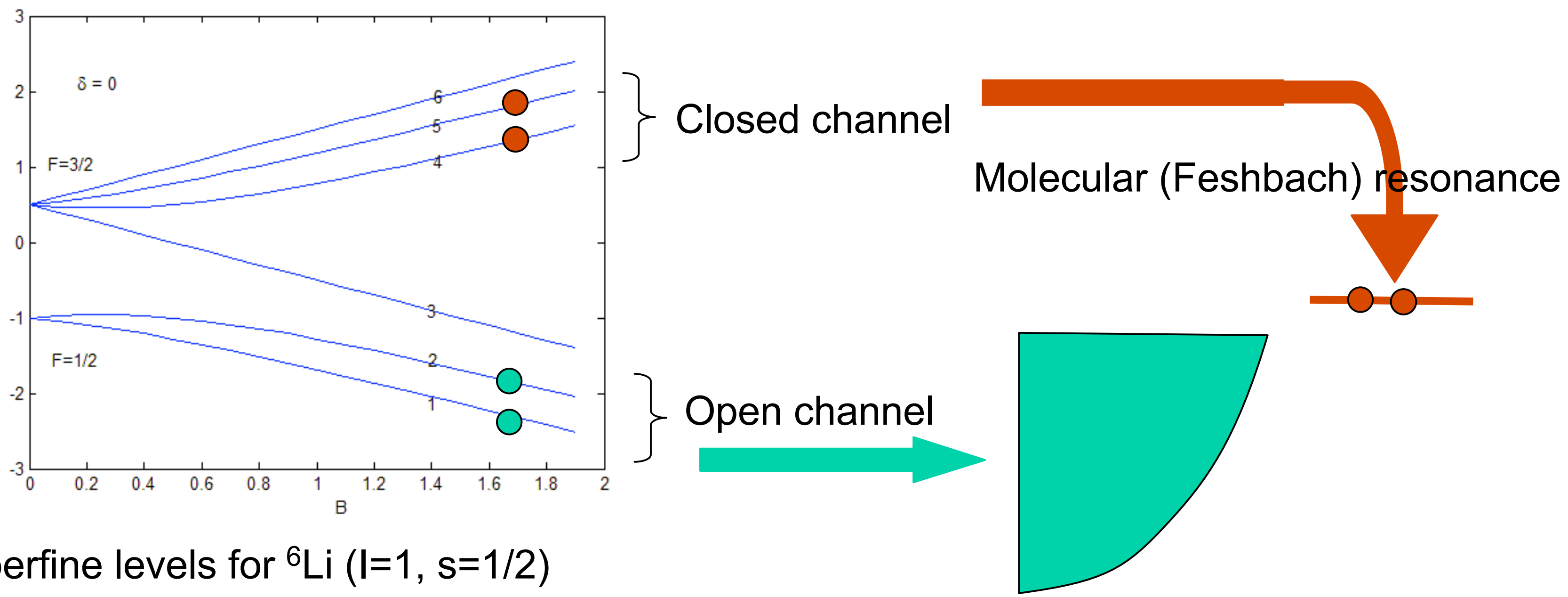
PR Eastham and PBL, Phys. Rev. B **64**, 235101 (2001)
MH Szymanska, PBL and BD Simons, Phys. Rev. A **68**, 13818 (2003)
J Keeling, L Levitov and PBL, Phys.Rev.Lett **92**, 176402, (2004)
F Marchetti, BD Simons and PBL, Phys Rev B **70**, 155327 (2004).
J Keeling, MH Szymanska, PR Eastham and PBL, Phys Rev Lett **93** 226403 (2004)

Cold atomic fermi gases

- Superconductivity in fermi gases tuned through the BCS-BEC crossover.

C. A. Regal, M. Greiner and D. S. Jin, Phys. Rev. Lett. 92, 040403 (2004); M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman and W. Ketterle, Phys. Rev. Lett. 92, 120403 (2004).

$$\hat{H}_{\rm atom} = A\,\mathbf{s}\cdot\mathbf{I} + \mathbf{B}\cdot(2\,\mu_{\rm e}\,\mathbf{s} - \mu_{\rm n}\,\mathbf{I})\,.$$



Hyperfine levels for ^{6}Li (I=1, s=1/2)

Outline - Superconductivity in fermionic atomic gases

- Pairing mediated by Feshbach resonance (molecular exciton)
- Tuning near the resonance used to mediate weak-strong coupling crossover.
- BCS-BEC crossover ?
  - "single channel" (2 fermionic states paired by effective interaction)
  - "Bose-Fermi" (2 fermionic states paired by exchange with a bosonic molecule)
  - "multi-level" (n fermionic states with realistic interactions, especially n=3)
- Parallel to solid state systems?
  - BEC of exciton polaritons
  - multi-band pairing ??
- Signatures of the different states
  - measuring excitation spectrum by monitoring ground state fluctuations – Kerr spectroscopy

BCS-BEC crossover in one-channel model

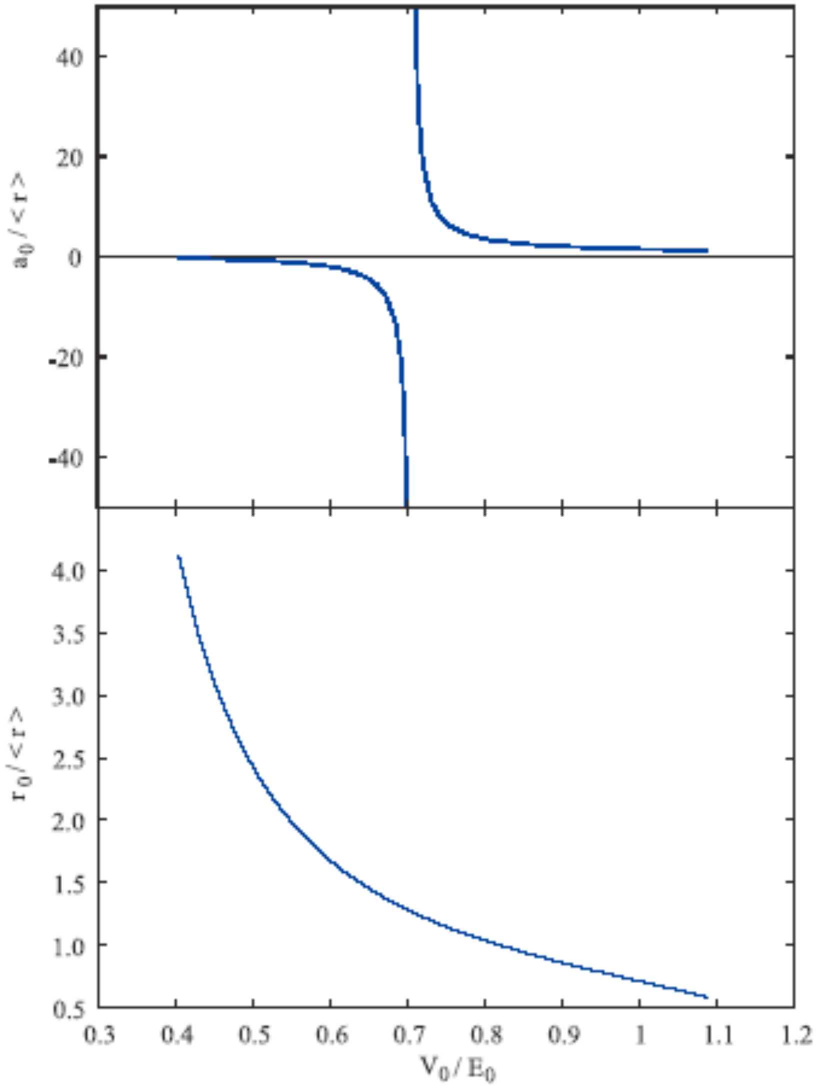
- Natural parameter in cold atom problem

$$\tilde{n} = (k_F\ a_o)^{à\ 1}$$

  - a_o is scattering length
- Compare to excitons

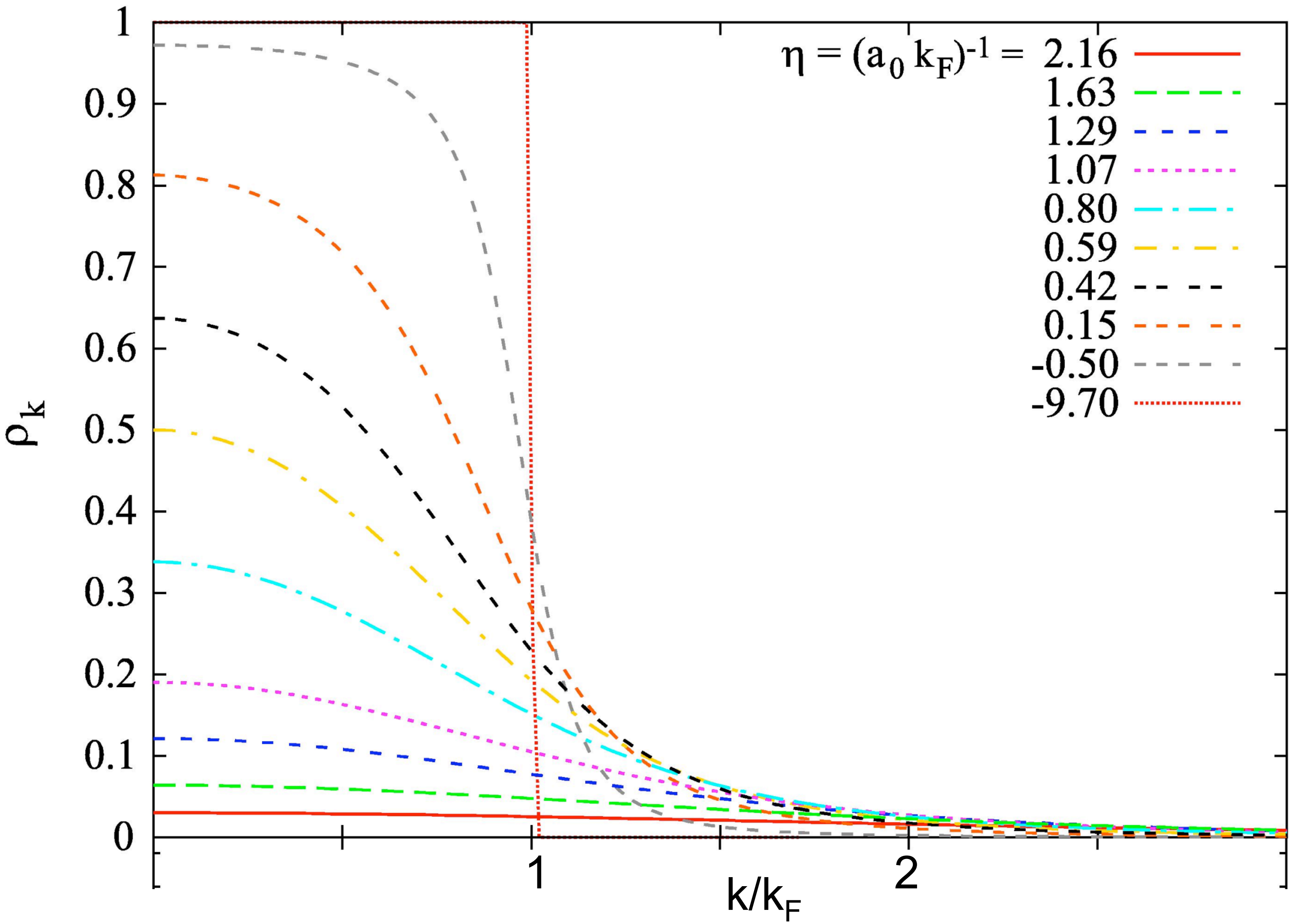
$$r_s = ð\ \frac{9ù}{4}\ n^{1=3}\ (k_F\ a_{Bohr})^{à\ 1}$$

- Choose model potential of a short-range gaussian with depth V_o , and range r_0



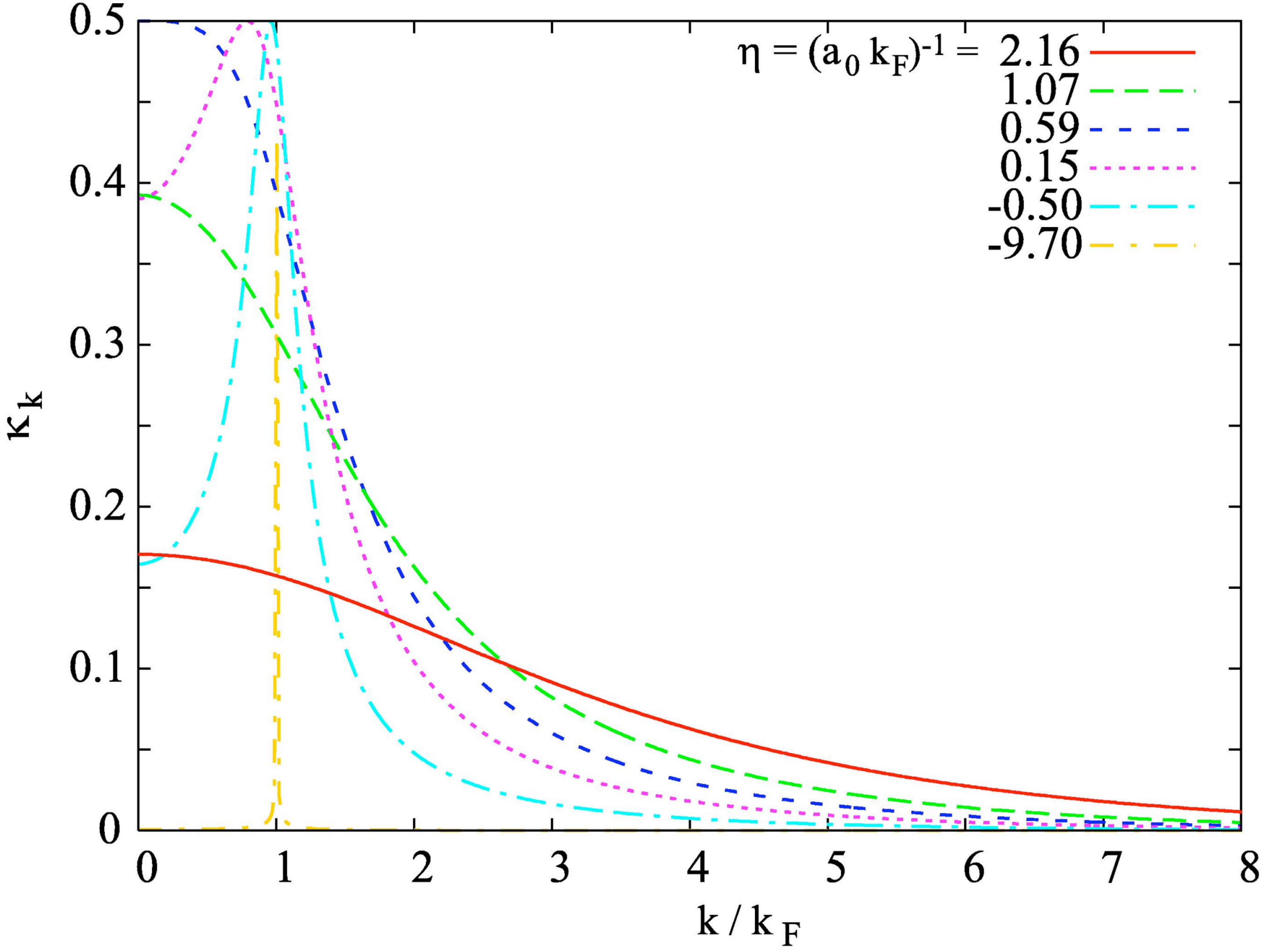
Well-known physics – Leggett; Nozieres & Schmitt-Rink; Randeria

Occupancy

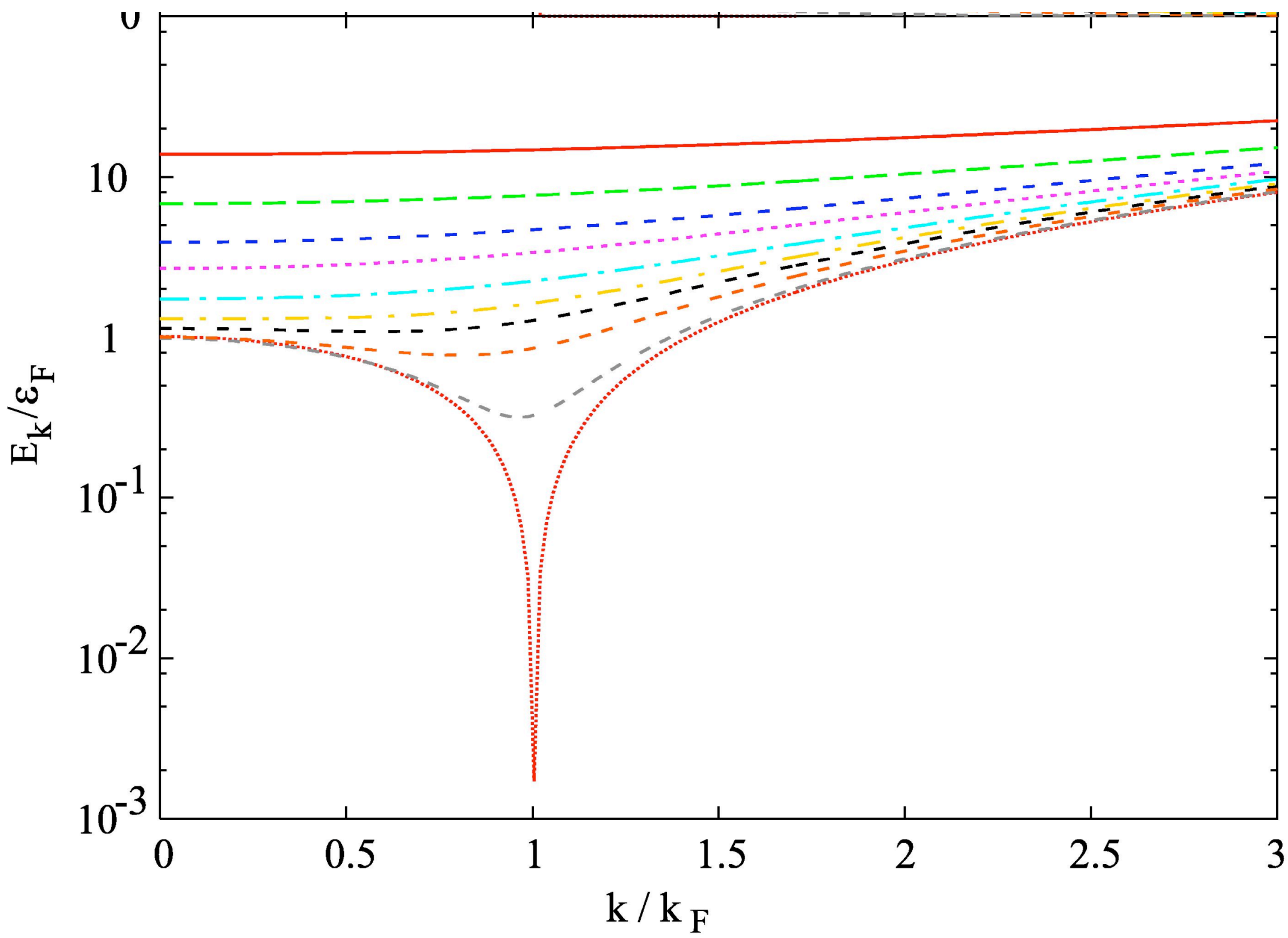


$\eta = (a_0 k_F)^{-1} =$ 2.16
1.63
1.29
1.07
0.80
0.59
0.42
0.15
-0.50
-9.70
ρ_k
k/k_F
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1
2

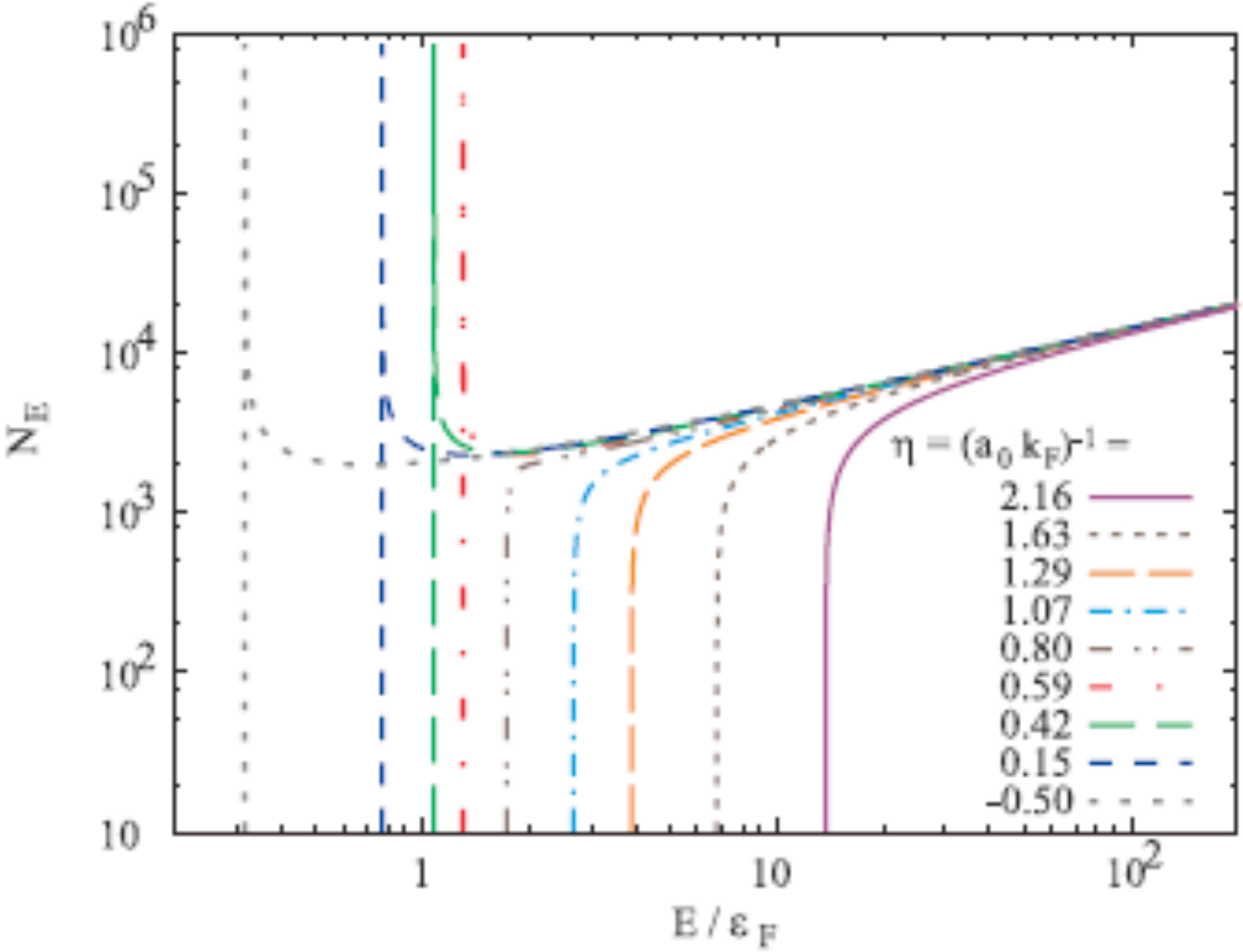
Condensate wavefunction



Excitation spectrum



Density of states

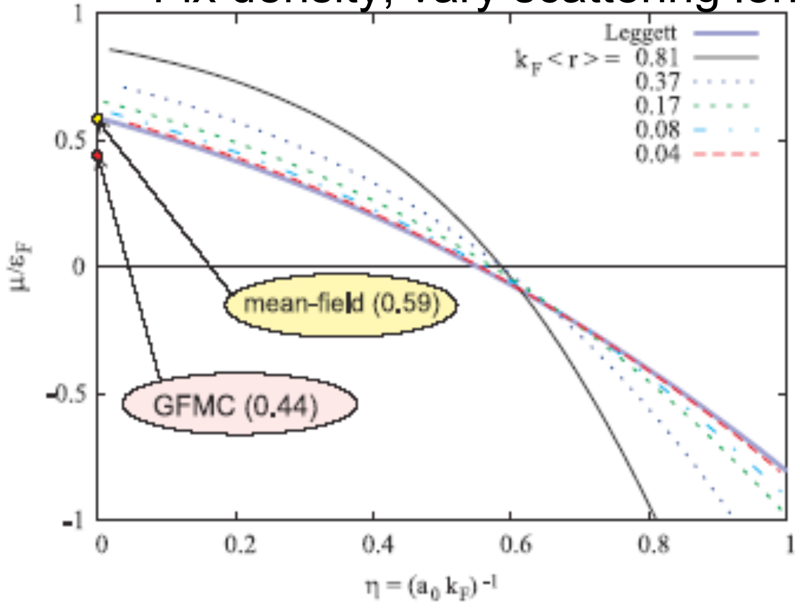


Comparison to low density limit

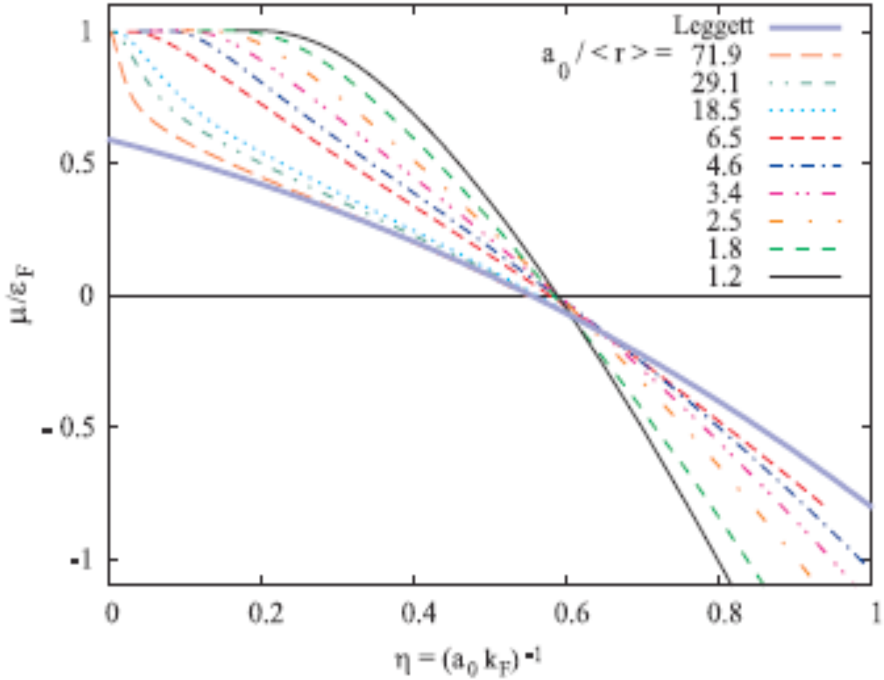
- “Universal” result in terms of single parameter η in the low density limit (Leggett)

ñ = $(k_F\ a_o)^{à\ 1}$

Fix density, vary scattering length



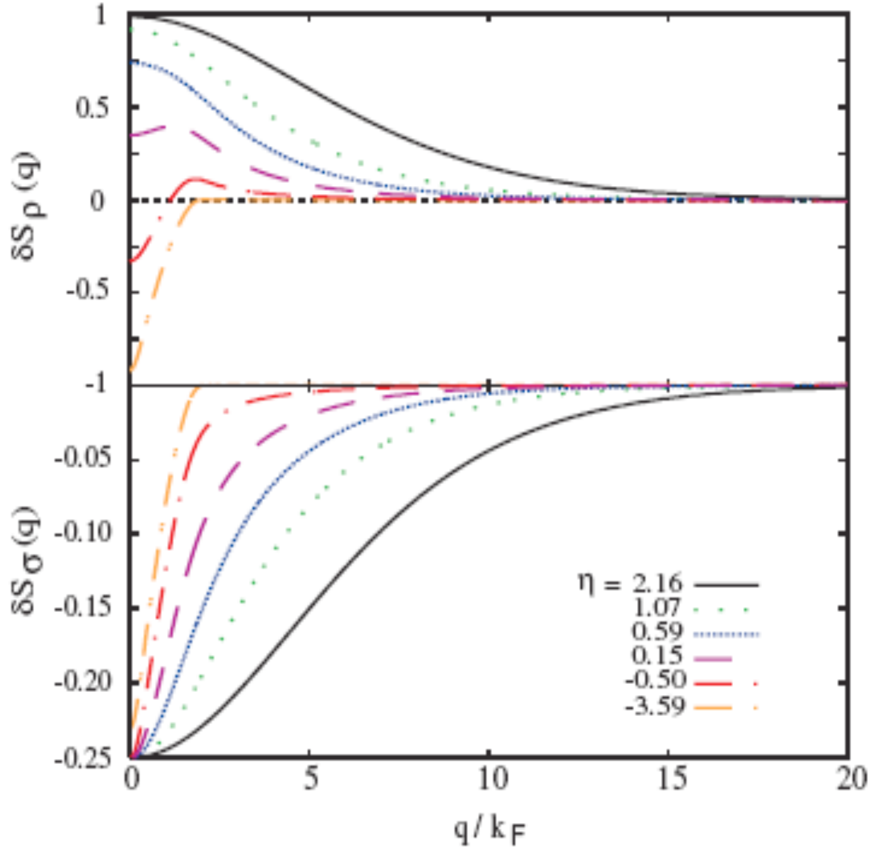
Fix scattering length, vary density



Response and correlation functions

$$S_\rho(q) = \langle e^{i q \cdot r} \rho(r)\rho(0) \rangle = 1 + \delta S_\rho(q)$$

$$S_\sigma(q) = \langle e^{i q \cdot r} \sigma_z(r)\sigma_z(0) \rangle = 1/4 + \delta S_\sigma(q)$$

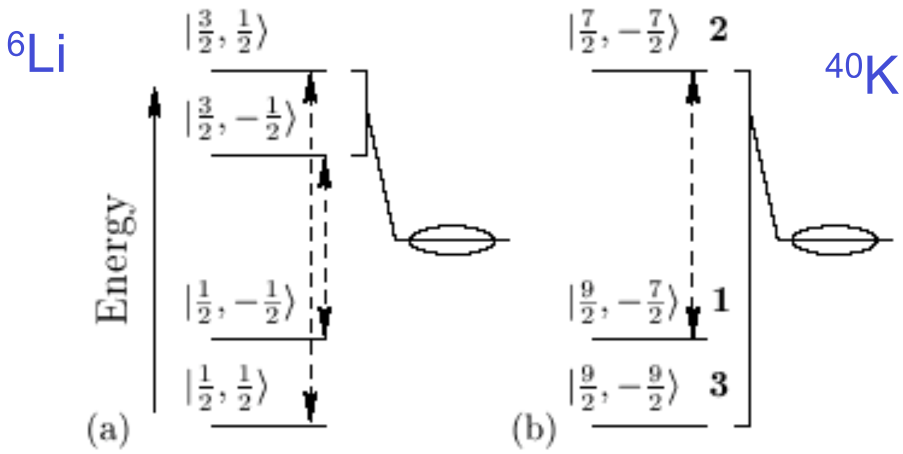


Fermi-Bose model

Replace closed channel by a molecular state – interaction mediated by molecular boson
Holland et al PRL 87, 120406 (2001); Timmermans et al. Phys.Lett A 285, 228 (2001)

$$H = \sum_{i\sigma} \epsilon_i a^\dagger_{i\sigma} a_{i\sigma} + g \sum_i \left[a_{i\uparrow} a_{i\downarrow} \phi^\dagger_i + h.c. \right] + \omega \sum_i \phi^\dagger_i \phi_i$$

Identical to model of polaritons: excitons (as 2-level systems) + photon
Is it adequate to treat the molecular boson as featureless?



In ^{40}K the closed and open channels share a hyperfine level
a 3-level fermion system

How to treat a model with three fermionic levels ?

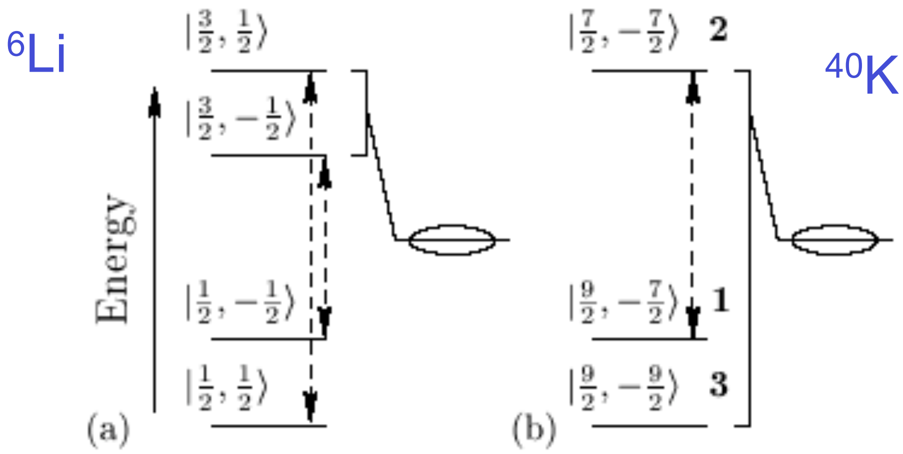
Replace closed channel by a molecular state – interaction mediated by molecular boson

Holland et al PRL 87, 120406 (2001); Timmermans et al. Phys.Lett A 285, 228 (2001)

$$H = \sum_{k\sigma} \epsilon_k a^\dagger_{k\sigma} a_{k\sigma} + g \sum_k \left(a_{k\uparrow} a_{k\downarrow} \phi^\dagger_k + h.c. \right) + \omega \sum_k \phi^\dagger_k \phi_k$$

Identical to polariton Hamiltonian

- but is it adequate to treat the molecular boson as featureless?

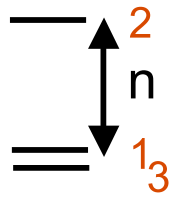


In ^{40}K the closed and open channels share a hyperfine level

a 3-level fermion system

Minimal model – 3 state fermi system

Open channel 1-3
Feshbach molecule 2-3



$$\hat{H} - \sum_{i=1}^{3} \mu_i \hat{N}_i = \sum_{\mathbf{k}i} (\epsilon_{\mathbf{k}i} - \mu_i)\, a^\dagger_{\mathbf{k}i} a_{\mathbf{k}i} \qquad (3)$$

$$+ \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} U_\mathbf{q}\, a^\dagger_{\mathbf{k}2} a^\dagger_{\mathbf{k}'3} a_{\mathbf{k}'-\mathbf{q}3} a_{\mathbf{k}+\mathbf{q}2}$$

Direct interaction - Feshbach

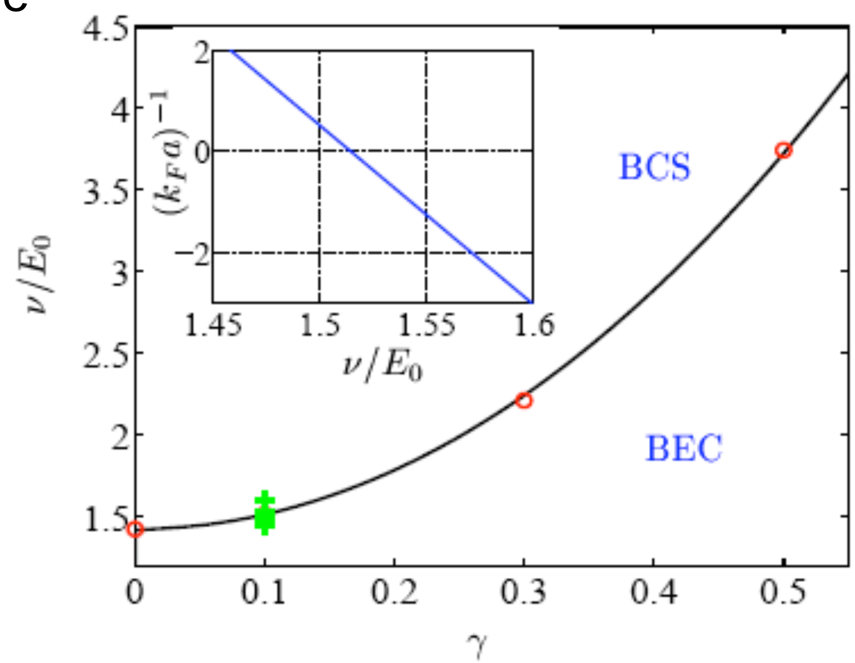
$$+ \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \left[g_\mathbf{q}\, a^\dagger_{\mathbf{k}1} a^\dagger_{\mathbf{k}'3} a_{\mathbf{k}'-\mathbf{q}3} a_{\mathbf{k}+\mathbf{q}2} + \mathrm{h.c.} \right],$$

Exchange between 1-2

Conserves $(N_1 + N_2)$, N_3 separately. Prepare system so that these are equal
Short range interactions with a range $1/k_0$,
Three dimensionless parameters

Detuning	ν/E_0; $E_0 = \hbar^2 k_0^2/2m$
Interaction	$u_0 = U_0 N(E_0)$
Mixing	$\gamma = g_q/U_q$

Effective two body scattering length ***a*** defines crossover



Generalised BCS variational solution

Minimise Free energy with generalised Bogoliubov transformation

$$\left\langle \hat{H} - \mu \hat{N} \right\rangle$$

$$a_{ki} = \sum_j \left[u_{ij}(k)\alpha_{kj} + v_{ij}(k)\alpha^{\dagger}_{-kj} \right]$$

Normal density $\quad \rho_{ij}(k) = \sum_m v^{*}_{im}(k) v_{jm}(k)$

Anomalous density $\quad \kappa_{ij}(k) = \sum_m v^{*}_{im}(k) u_{jm}(k)$

In practice, numerical, but there is an easy interpretation of results

State 3 pairs with *either* state 1 *or* state 2

Choose "optimal" linear combination for pairing

$$b^{\dagger}_{k1'} = \cos\theta_k \, a^{\dagger}_{k1} + \sin\theta_k \, a^{\dagger}_{k2}$$

$$b^{\dagger}_{k2'} = -\sin\theta_k \, a^{\dagger}_{k1} + \cos\theta_k \, a^{\dagger}_{k2}$$

Pair with state 1' ; 2' unoccupied $\quad |D\rangle = \prod_k \left[\cos\phi_k + \sin\phi_k \, a^{\dagger}_{k3} b^{\dagger}_{-k1'} \right]$

ϕ_k : strength of pairing ; θ_k mixing angle

Mixing produced by Pauli blocking

- Effective single particle spectrum of mixed states
  - Occupy state 1 for $k < k_F$ (free particle like)
  - Occupy state 2 or $k > k_F$ (quasimolecular)
- "Pauli blocking" of molecular state by the fermi sea

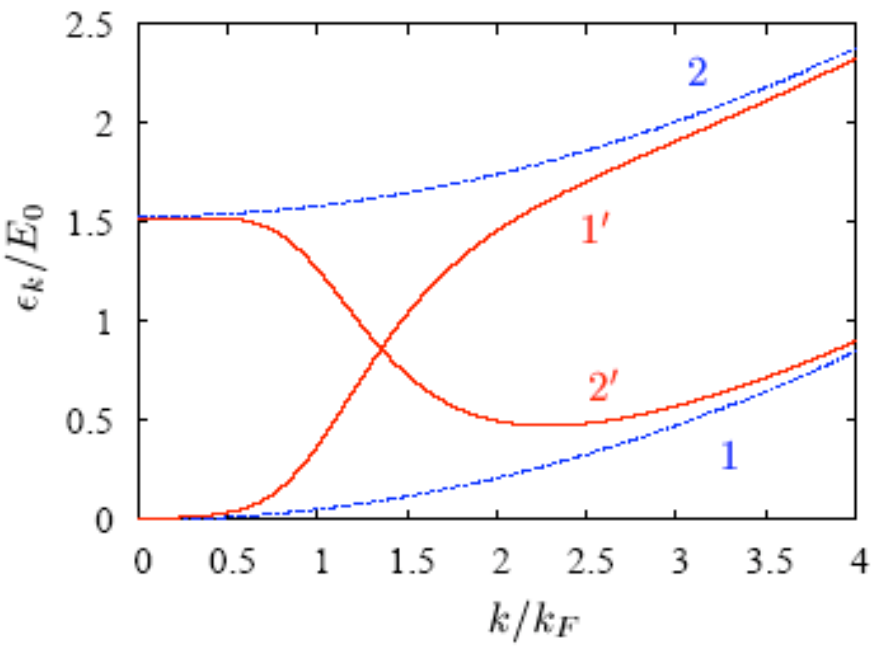
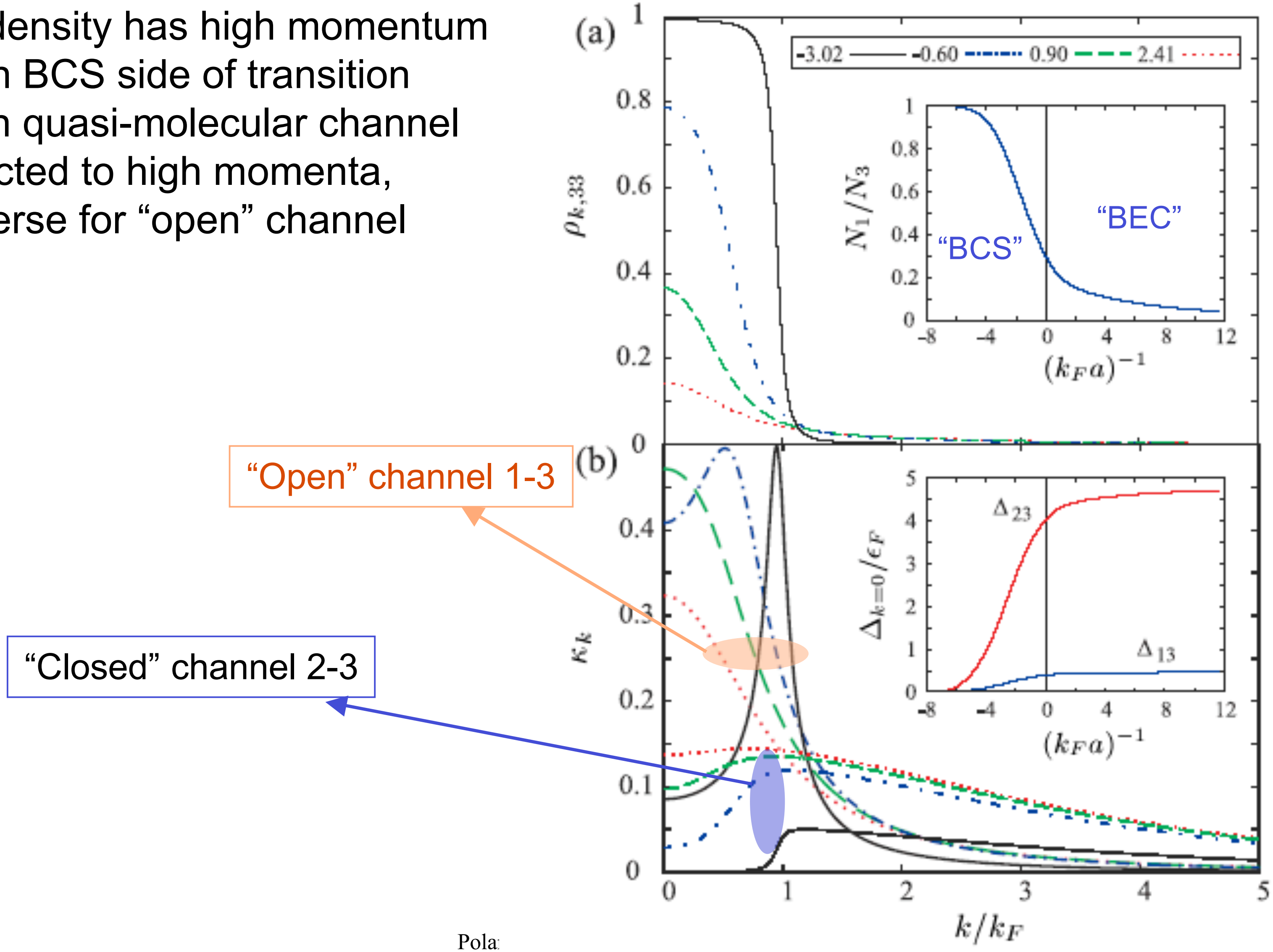


FIG. 3: (Color online) Spectrum of the parent (1, 2) and hybrid ($1'$, $2'$) states as inferred from the numerical analysis for $\nu/E_0 = 1.53$, $u_0 = 3.76$ and $\gamma = 0.1$.

Numerical results

Normal density has high momentum tail on BCS side of transition

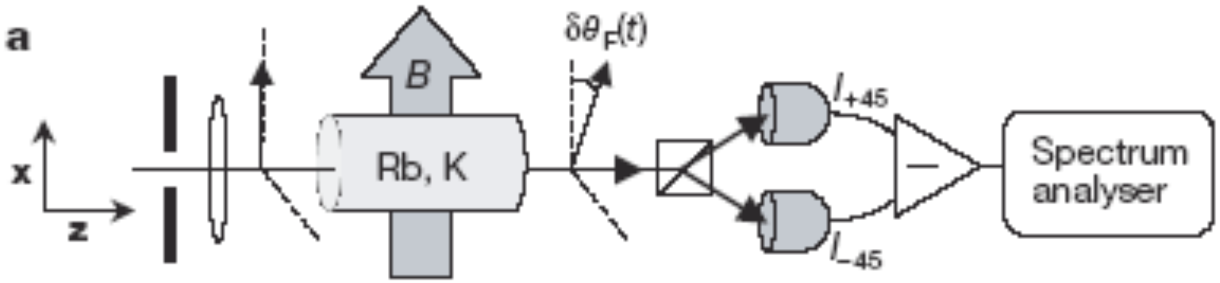
Pairing in quasi-molecular channel restricted to high momenta, converse for "open" channel



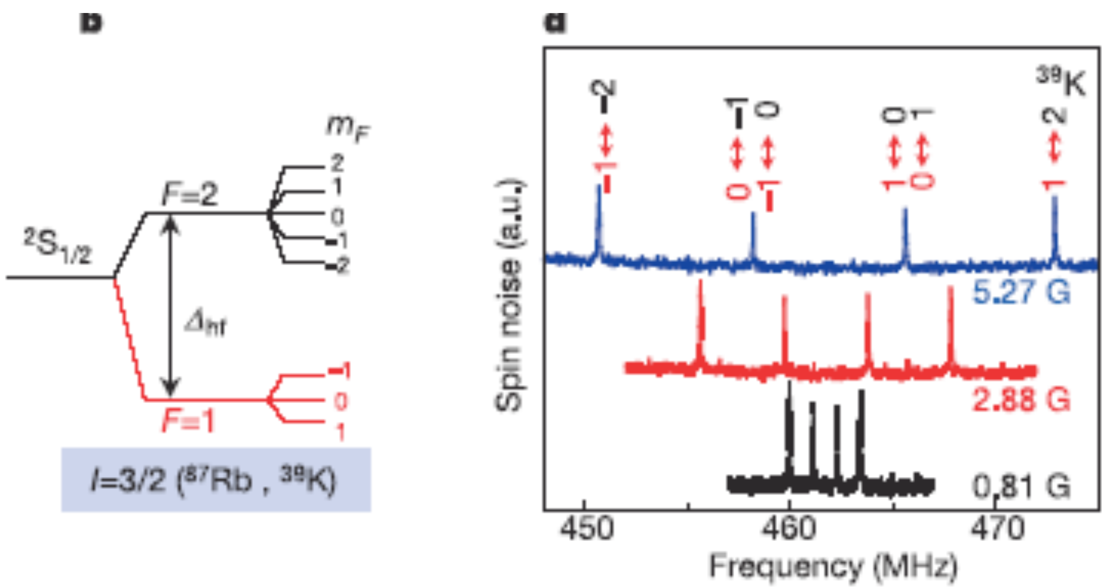
Remarks

- Higher level 2’ unoccupied for reasonable physical parameters
  - however, if the energy separation not so big, start to occupy this pairbreaking state
  - close analogy to singlet superconductivity in FM at the Pauli paramagnetic limit → will give Fulde-Ferrell-Larkin-Ovchinnikov state?
- Bose-Fermi theory is not the appropriate model near the crossover
- Away from the crossover, a single-channel model is the right effective theory
- Experimental signatures?
  - Current experiments largely focus on determining “molecular fraction”
  - Quantum numbers of the ground state change at the crossover, so magnetic susceptibility is different (Kerr fluctuation spectroscopy)
  - Excitation spectroscopy – transitions into excited states
  - Collective modes

Measurement of response functions by Kerr rotation



Thermal fluctuations in finite sample provide a measurement of the response function



Crooker et al Nature 2004

Measurement of spin-fluctuation spectrum

In principle can measure quantum fluctuations this way.

In single channel model, ground state is a (pseudo)-singlet

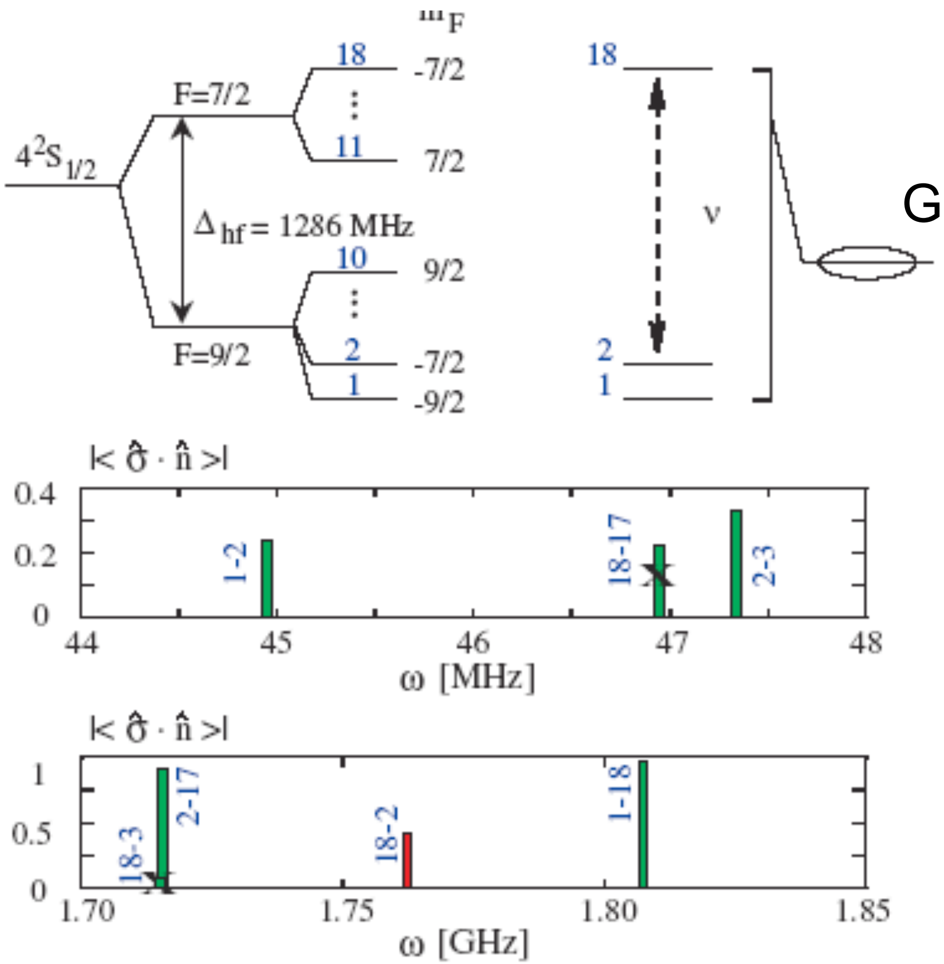
$$S_{\hat{u}}(q = 0) = 0$$

Finite system measures fluctuations at q ~ 1/L

$$S_{\hat{u}}(q) = (qø)^2$$

Multichannel models are different – ground state mixes several hyperfine levels
Spin fluctuations can distinguish BCS/BEC crossover from mixing with closed channel

3-level model for ^{40}K

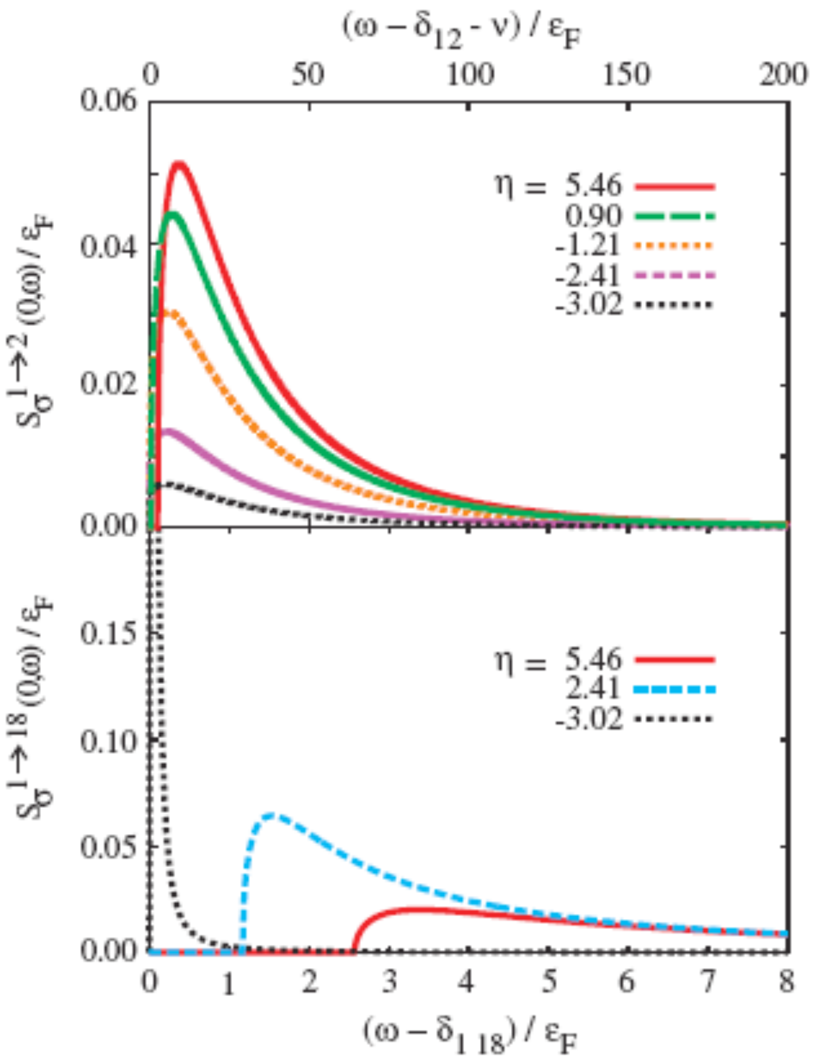


In single channel model,
many transitions are disallowed
e.g. 1->2
Ground state is eigenstate of total “spin”

Allowed transitions in single-channel model marked with X

Mihaila, Crooker, Smith et al. in preparation

3-level model for ^{40}K



High resolution spectroscopy shows characteristic features of spin response at BCS-BEC crossover

Ground state is not an eigenstate of electron spin, so quantum fluctuations exist