The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1760 - 9

COLLEGE ON

PHYSICS OF NANO-DEVICES

10 - 21 July 2006

Physics of Graphene

Presented by:

Andre Geim

The University of Manchester
United Kingdom

MANCHESTER 1824

The University of Manchester

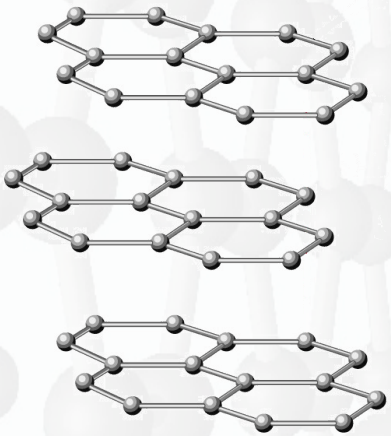
Physics of Graphene

Andre Geim

in collaboration with
K. Novoselov, S. Morozov, D. Jiang, F. Schedin,
V. Falko, J. Meyer, I. Grigorieva & M. Katsnelson

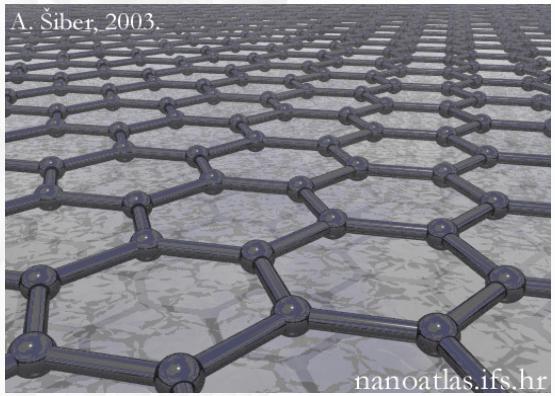
GRAPHENE ALLOTROPES

3D



Graphite

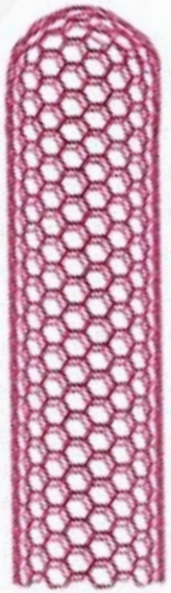
2D



graphene

PRESUMED
NOT
TO EXIST

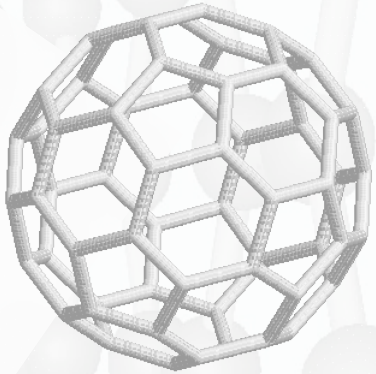
1D



Carbon Nanotube

Multi-wall 1991
Single-wall 1993

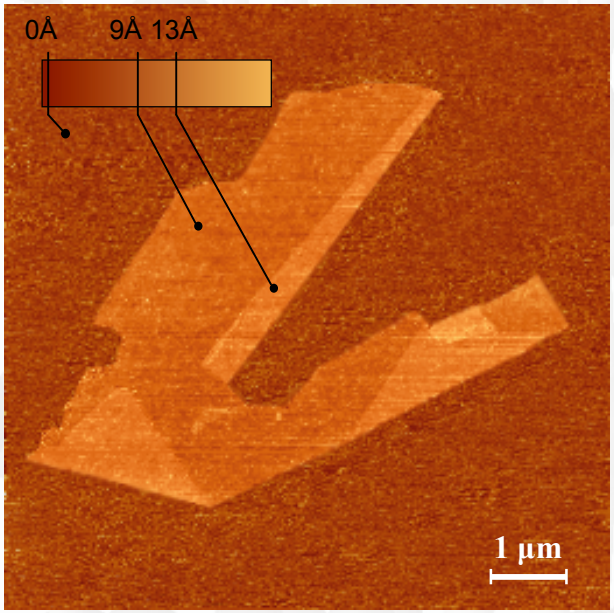
0D



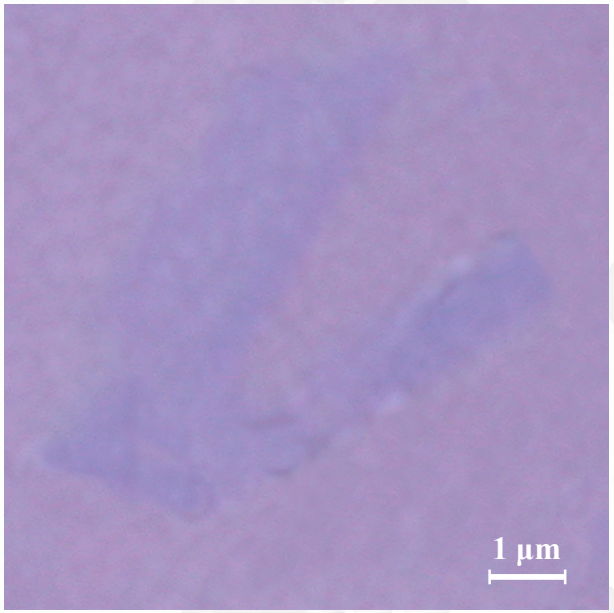
"Buckyball"

1985

Free-Standing Graphene

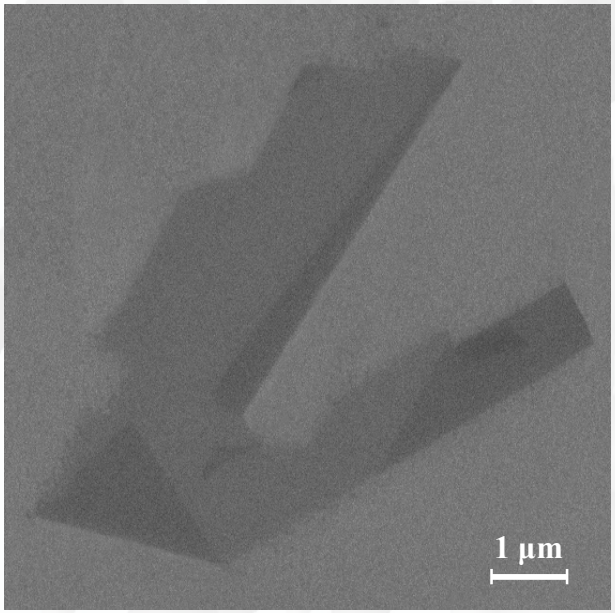


AFM

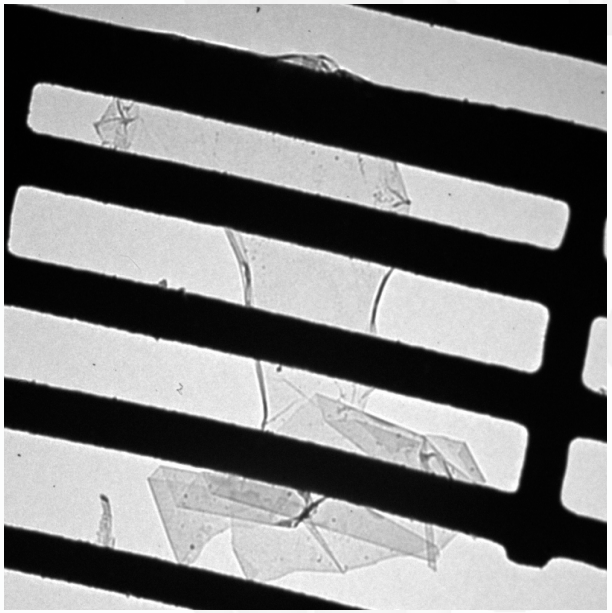


OPTICS

single layer of atoms visible by naked eye



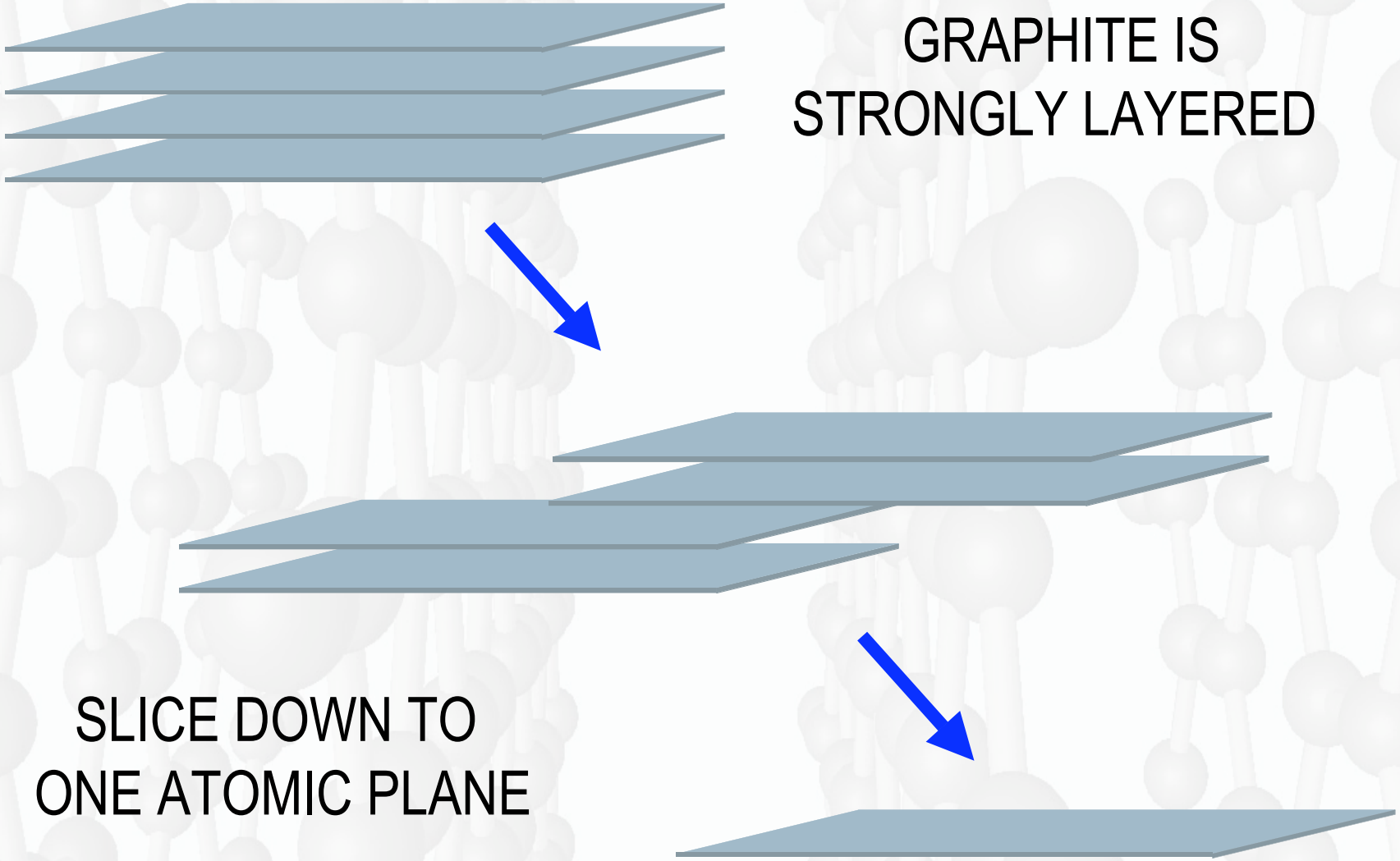
SEM



TEM

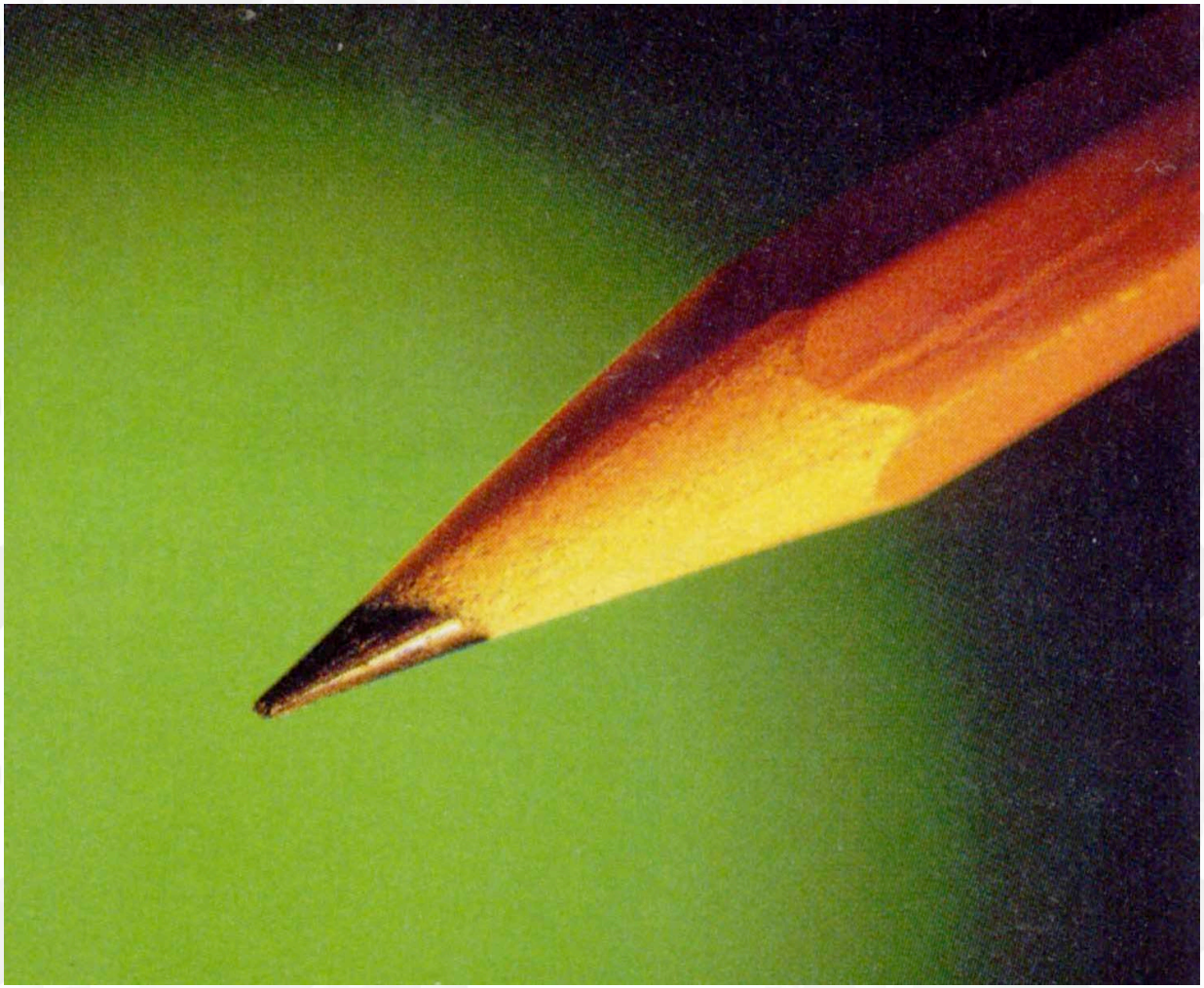
Science **306**, 666 (2004)

Extracting a Single Plane



individual atomic sheets: do they exist?

APPROACH



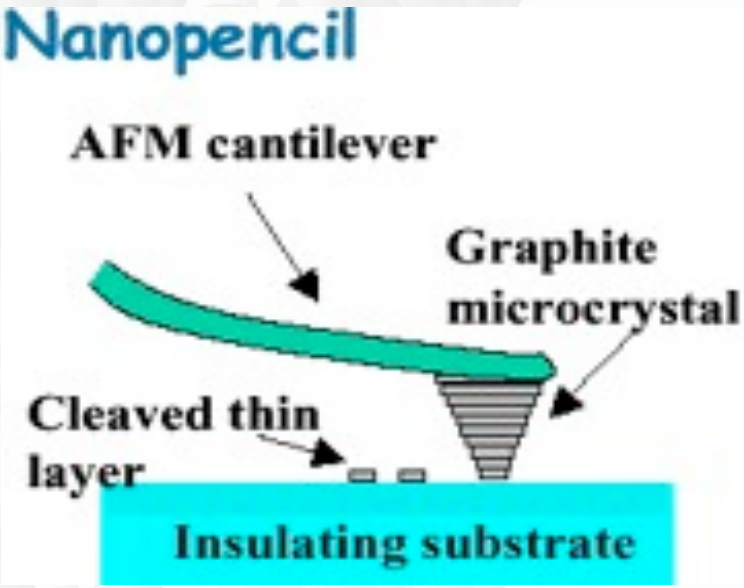
Drawing:
(micro) mechanical cleavage of graphite

mechanical exfoliation: retrospective

EARLIER EFFORTS

Ohashi *et al*, *Tanso* (1997, 2000)
from 1000 down to 50 layers

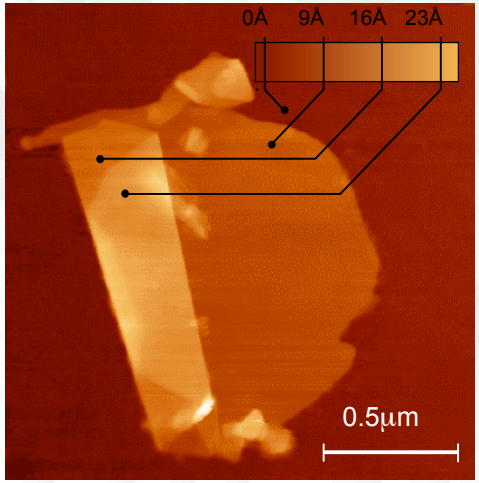
followed by Philip Kim's & Paul McEuen's groups
(*PRL* 2005 & *Nanoletters* 2005) down to 35 layers



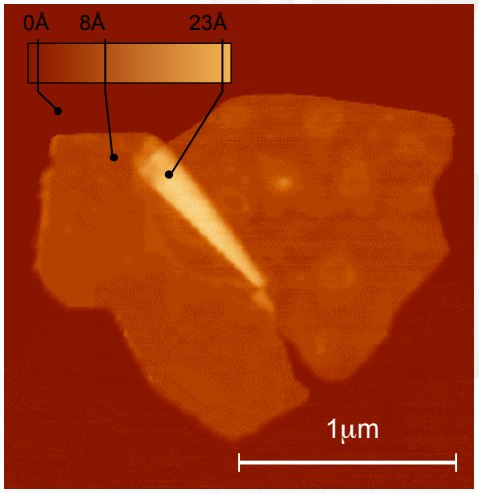
bulk graphite if >10 layers

Other 2D Atomic Crystals

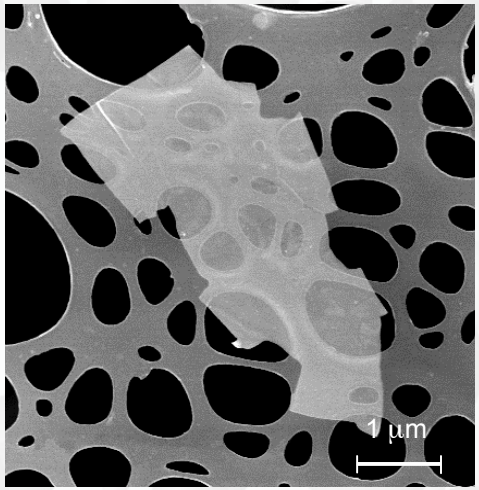
2D boron nitride in AFM



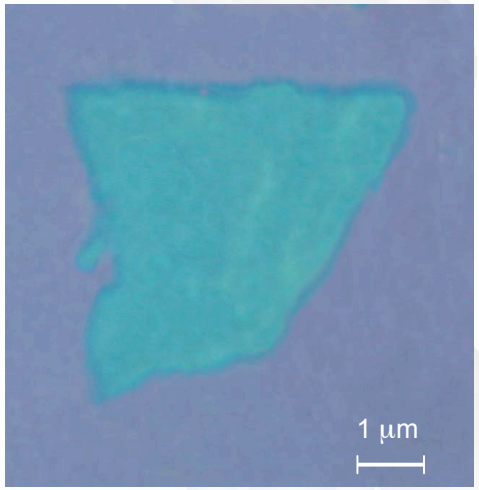
2D $NbSe_2$ in AFM



also,
2,3,4... layers



2D $Bi_2Sr_2CaCu_2O_x$ in SEM

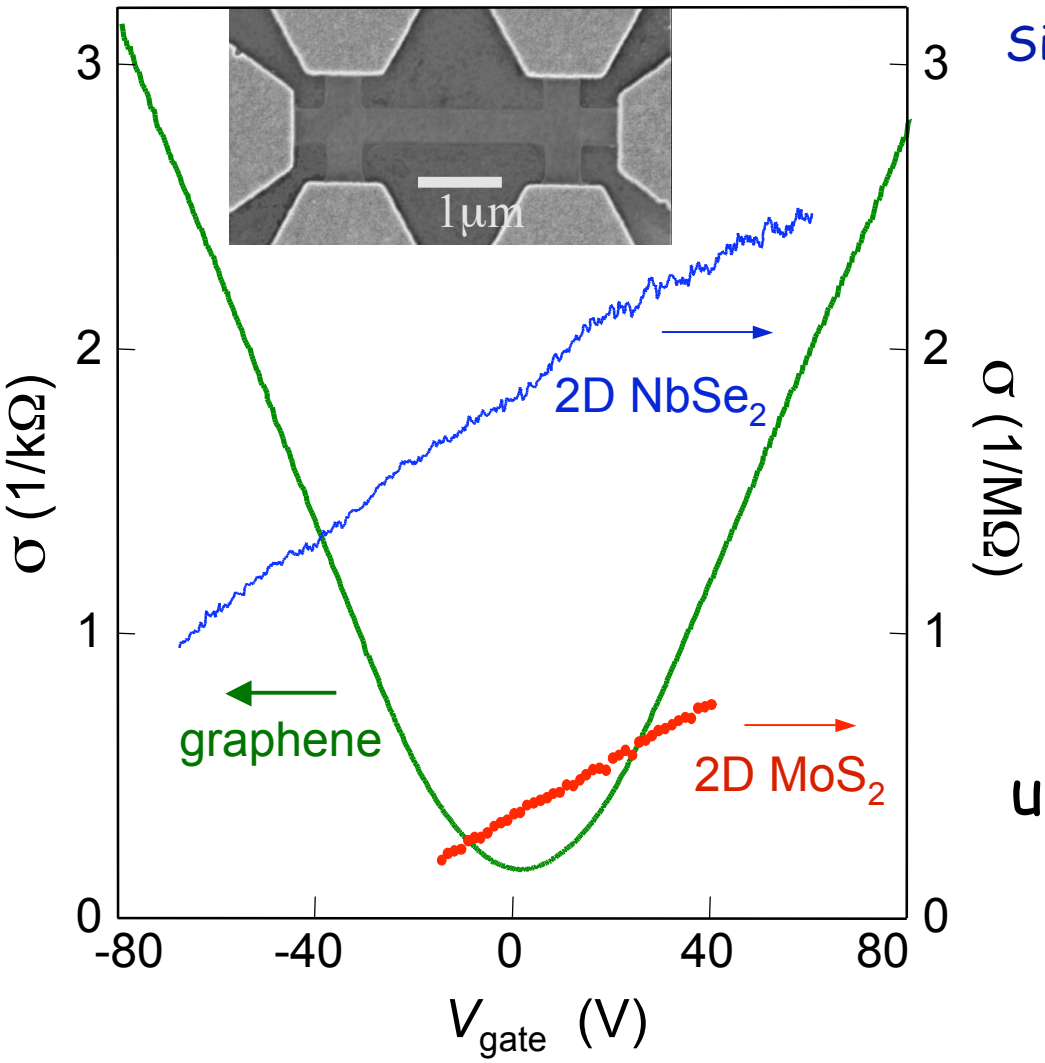


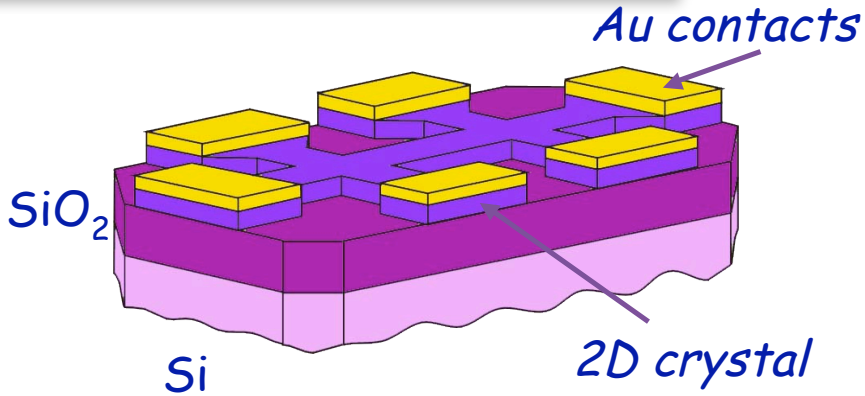
2D MoS_2 in optics

PNAS **102**, 10451 (2005)

why graphene: high quality

Electric Field Effect





$$\sigma = ne\mu = \varepsilon\varepsilon_0 \mu V_g / d$$

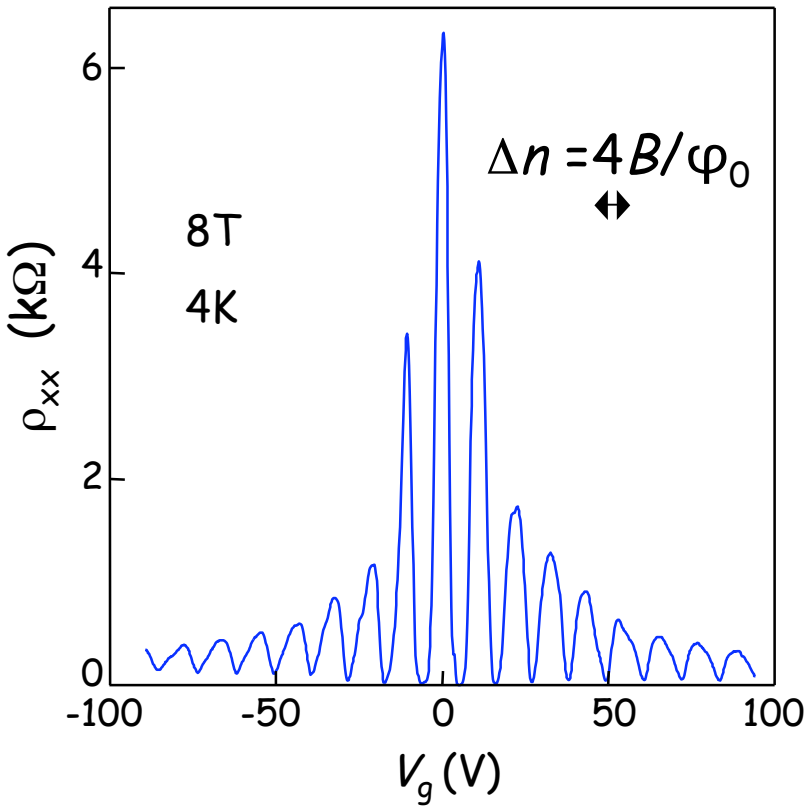
2D $NbSe_2$ & MoS_2:
0.5 to 3 cm^2/Vs
as in bulk at 300K

graphene: <u>currently</u>
up to 6,000 $cm^2/V{\cdot}s$ at 300K

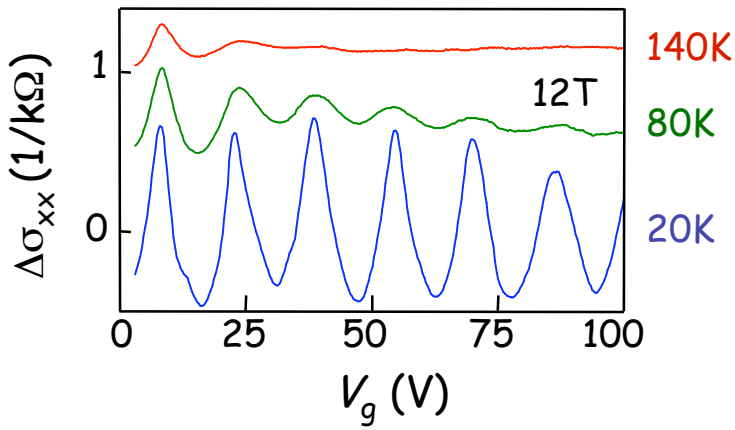
ballistic transport on submicron scale !

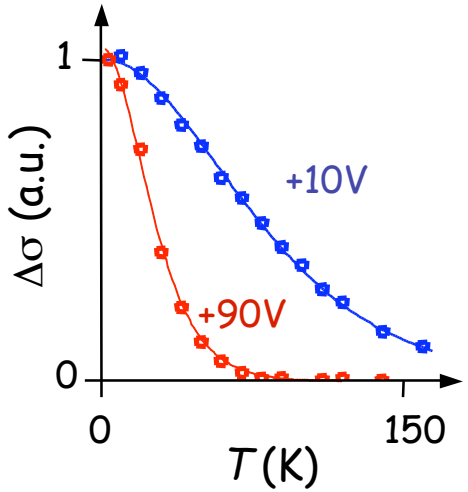
Quantum Oscillations in Graphene

$\omega_c \tau = \mu B = \text{const}$



ShdH oscillations
as a function of
carrier concentration

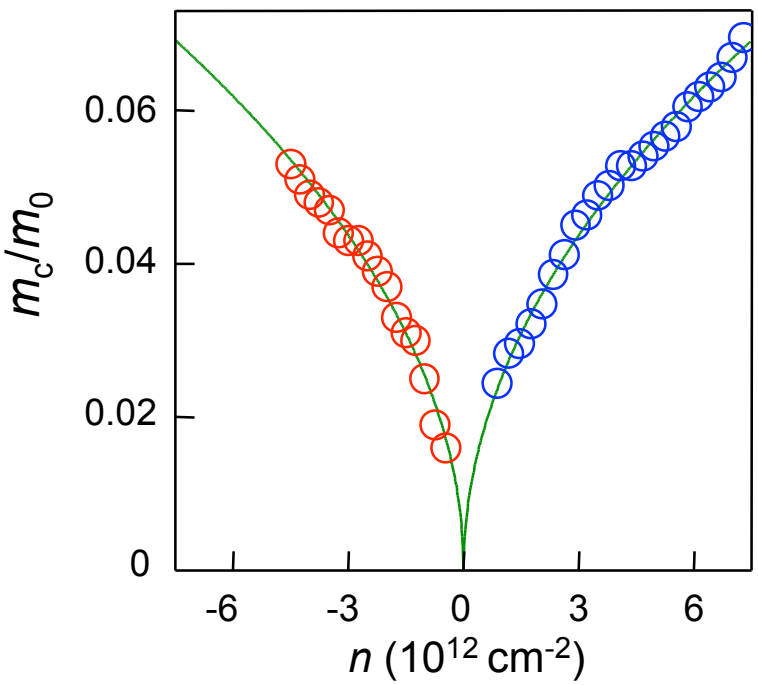




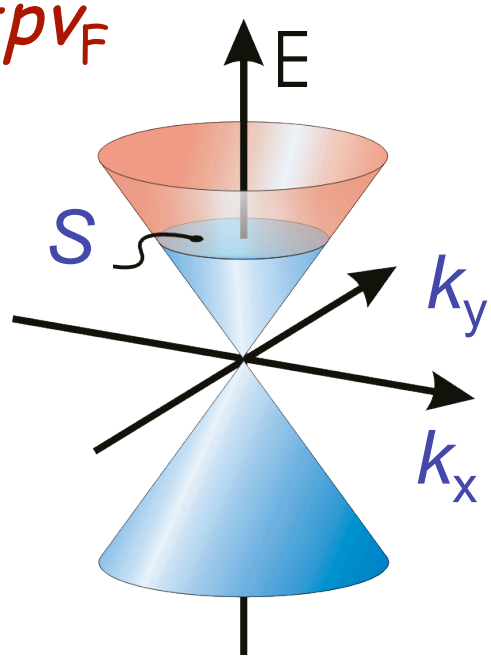
$$\Delta\sigma_{xx} \propto T \sinh\left(\frac{2\pi^2 k_B T m_c}{\hbar e B}\right)$$

Band Structure of Graphene

cyclotron mass strongly depends concentration



$E=pv_F$



$B_F = (\hbar/2\pi e)S$ and $m_c = (\hbar^2/2\pi)\partial S/\partial E$

experimental dependences

$B_F \sim n$ and $m_c \sim n^{1/2}$

necessitates $S \sim E(k)^2$ or $E \sim k$

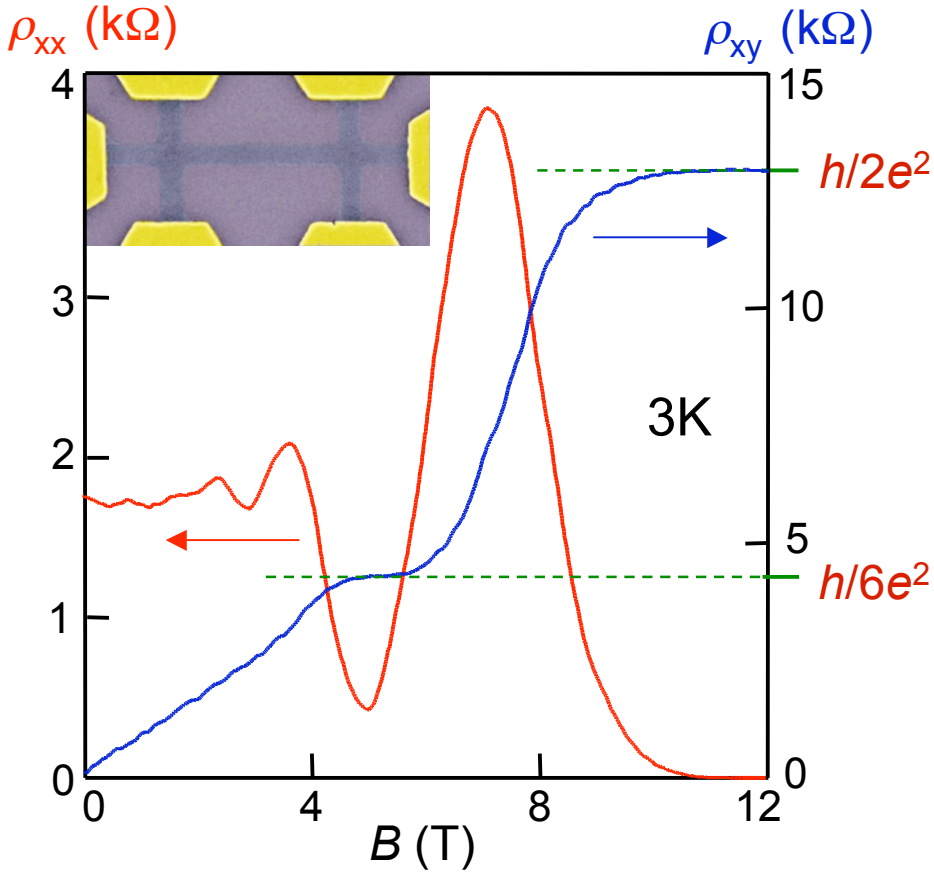
cyclotron mass

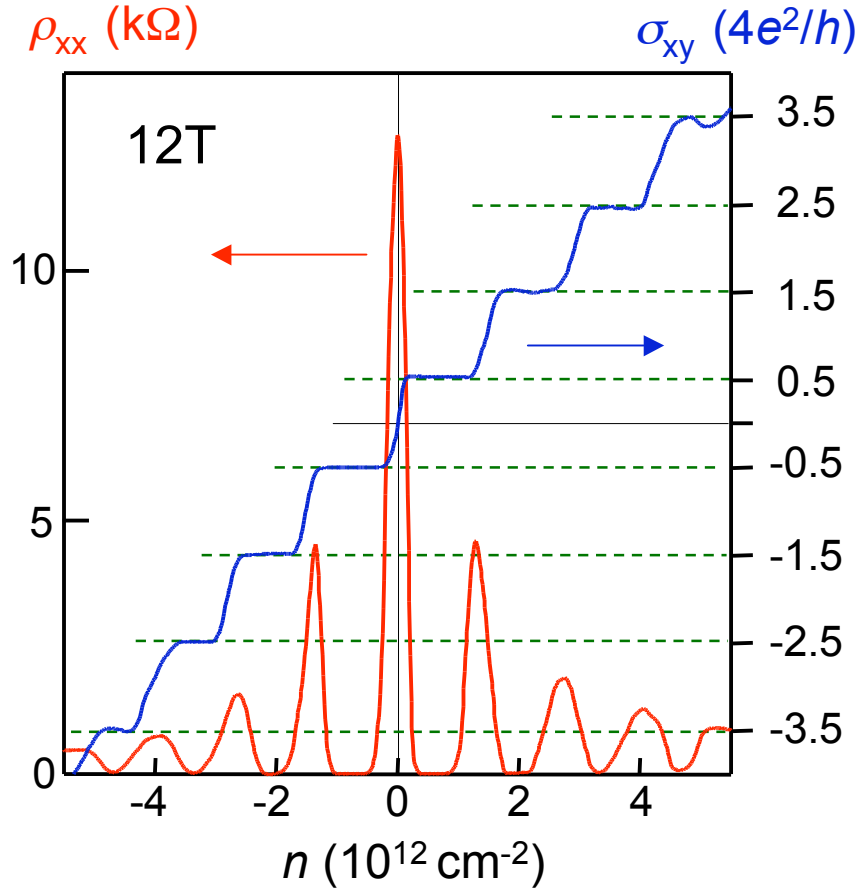
$E = m_c v_F^2$

$v_F \approx 1.05\text{x}10^6$ m/s

Quantum Hall Effect in Graphene

quadruple degeneracy:
plateaus are expected at $h/4Ne^2$



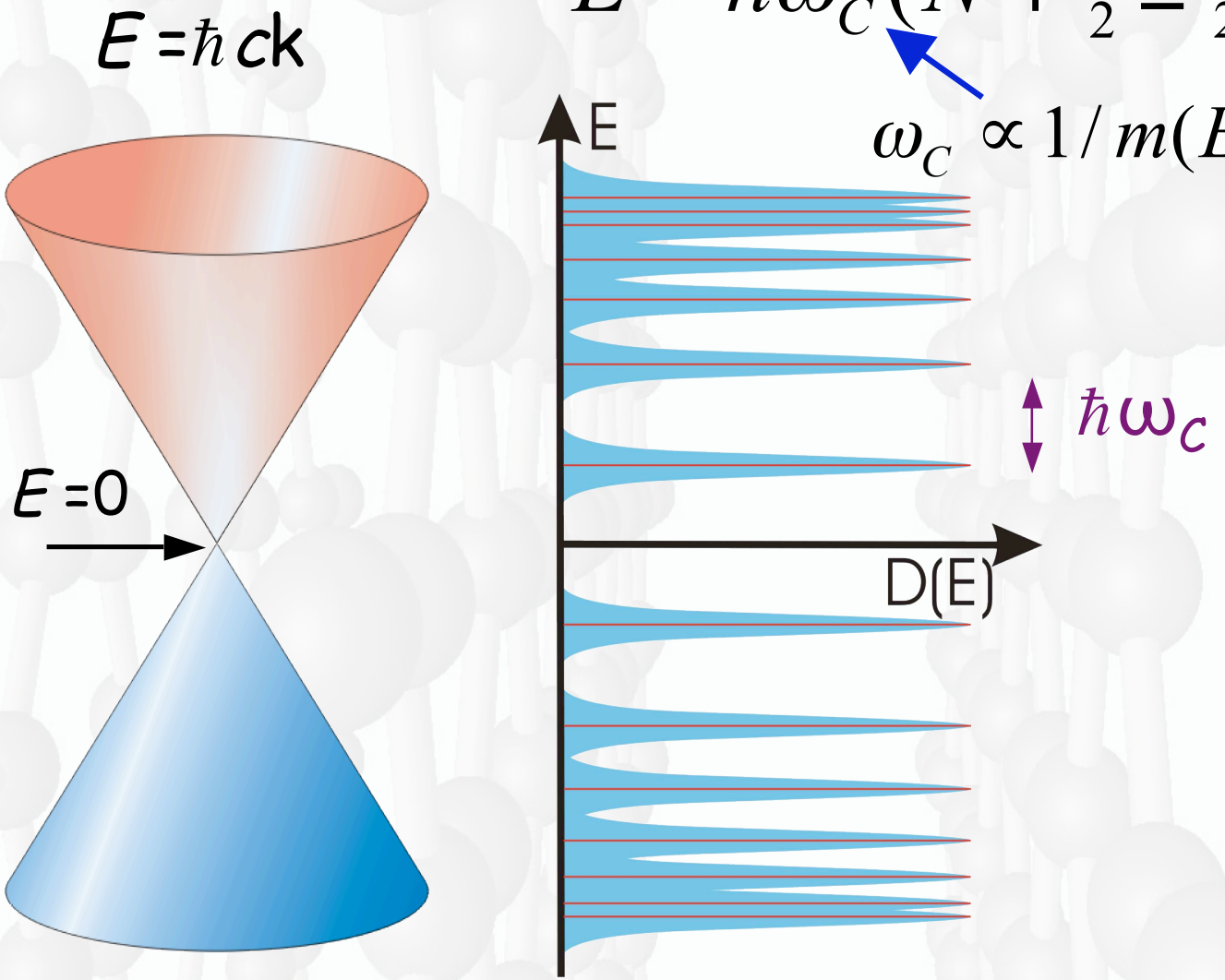


half-integer QHE

quantization of Dirac fermions

$$E = \hbar\omega_C\left(N + \tfrac{1}{2} \pm \tfrac{1}{2}\right)$$

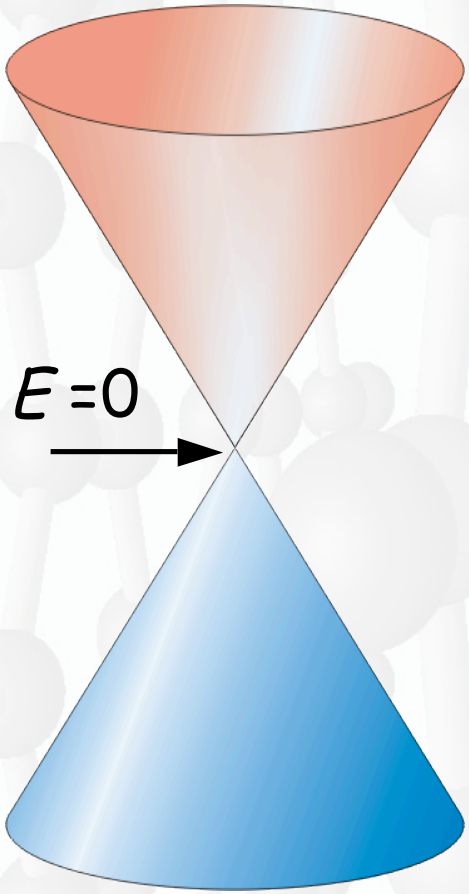
$$\omega_C \propto 1/m(E)$$

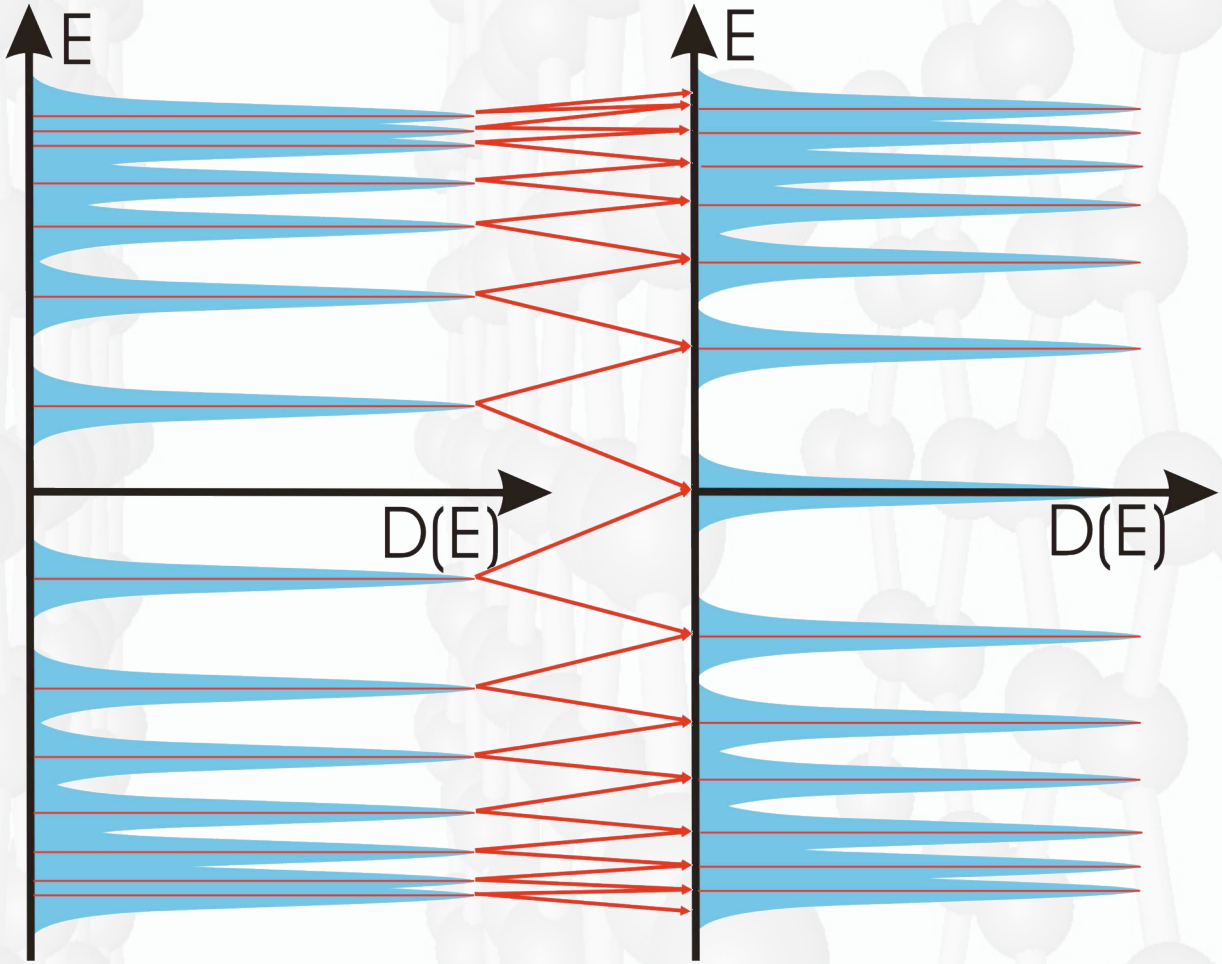


quantization of Dirac fermions

$E = \hbar ck$

$$E = \pm\sqrt{2\hbar c^2 eBN}$$

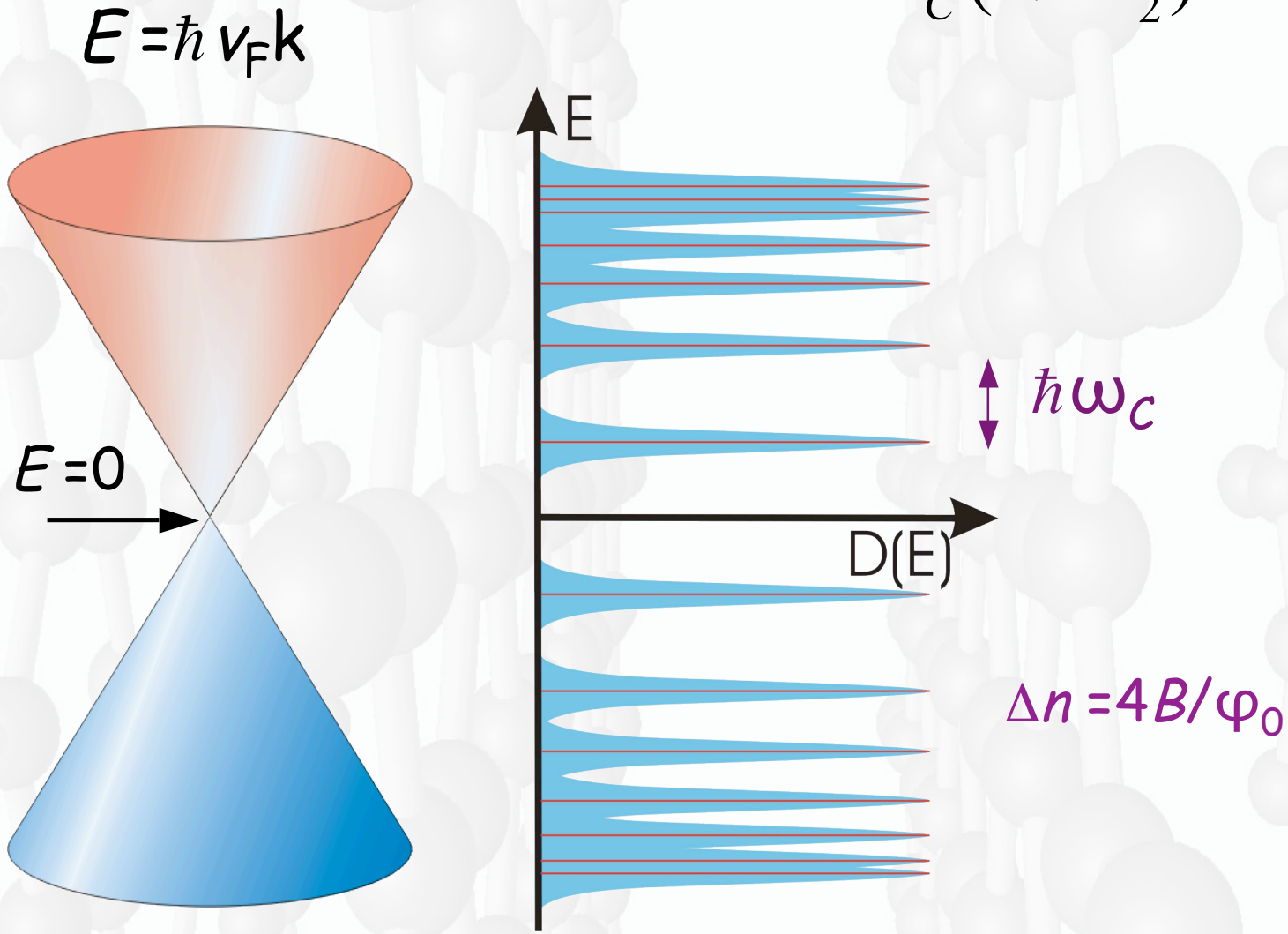




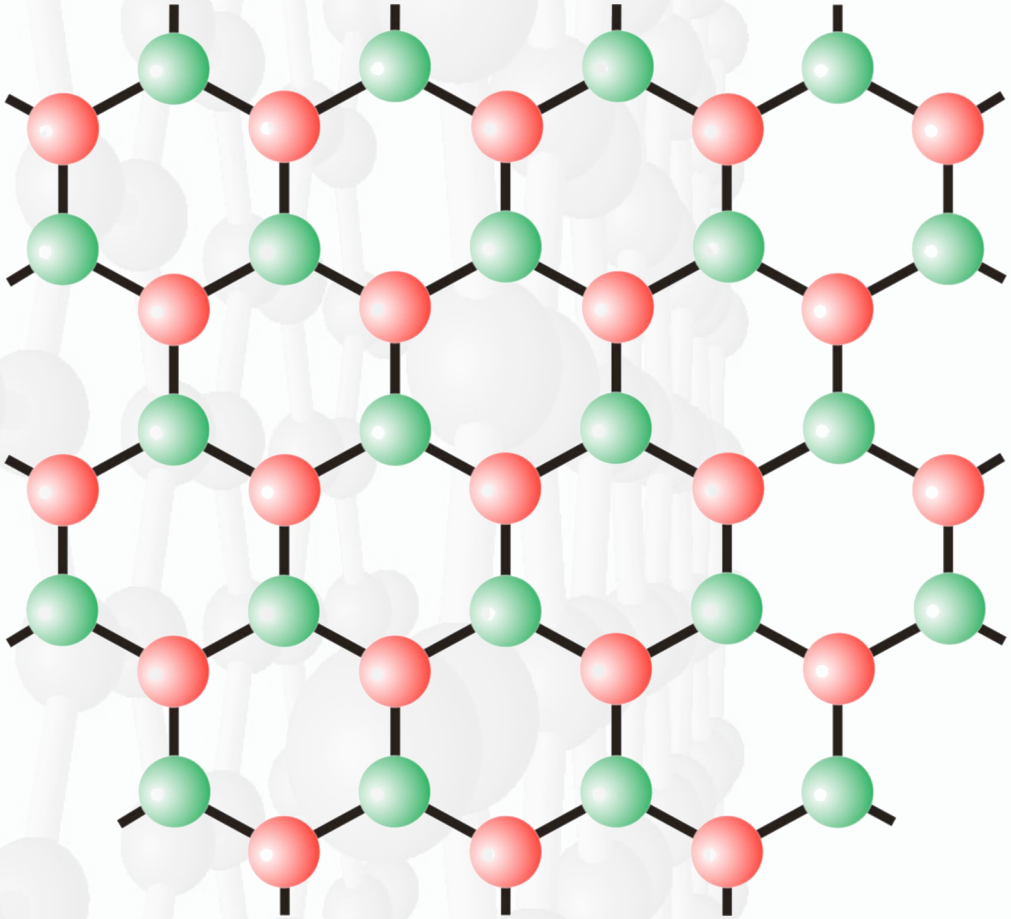
lowest Landau level is *at ZERO energy*

quantization of Dirac fermions

$$E = \hbar\omega_C (N + \tfrac{1}{2})$$



Chiral Fermions in Graphene



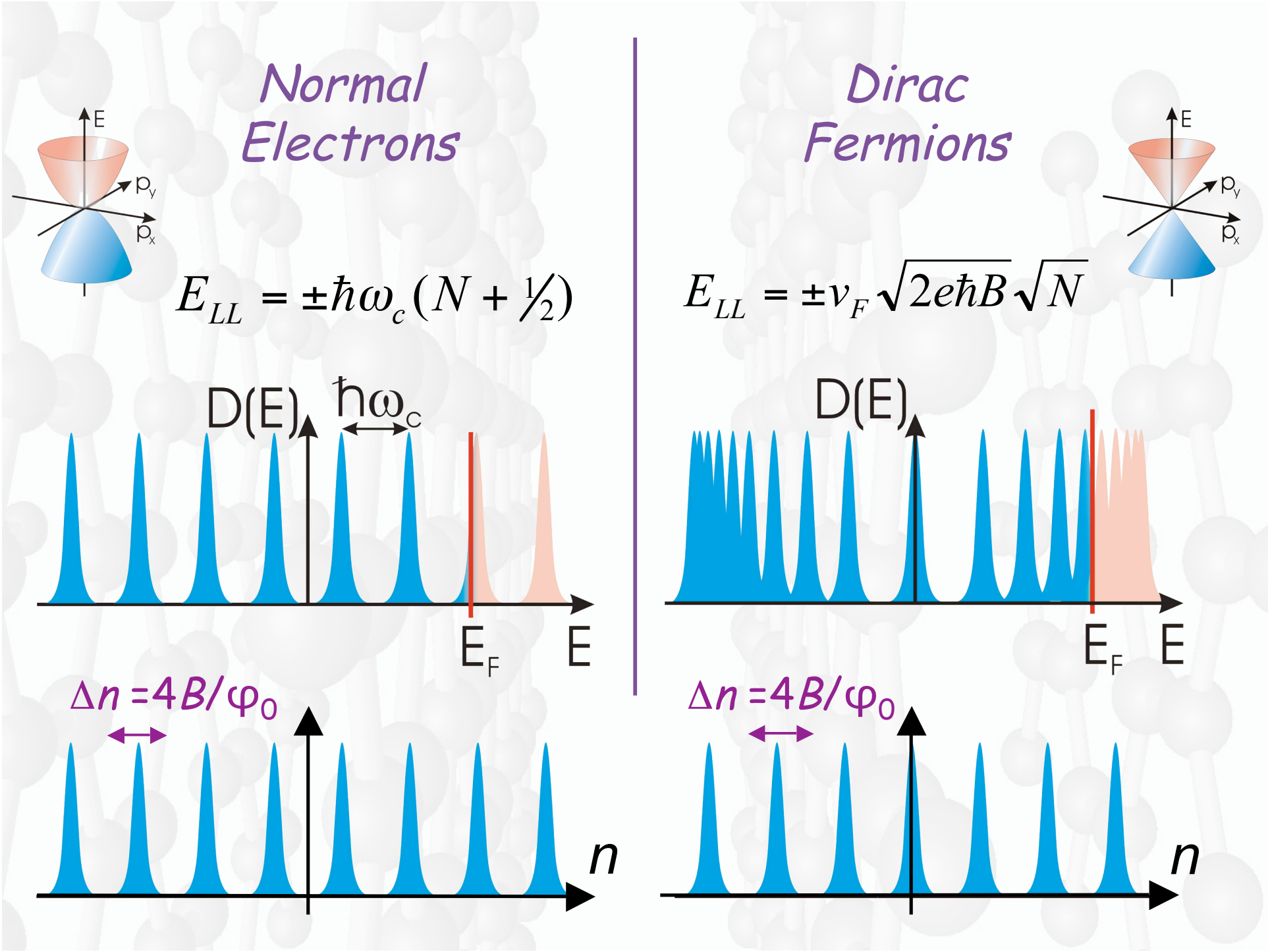
two sublattices

superposition
of their wavefunctions

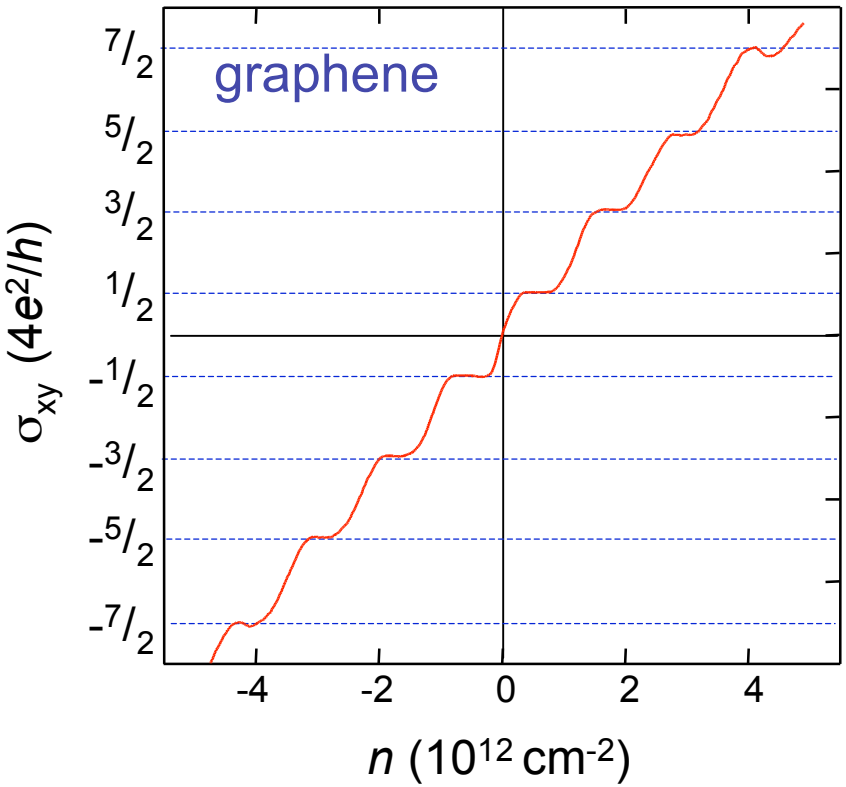
spinors
(2 projections
of pseudospin)

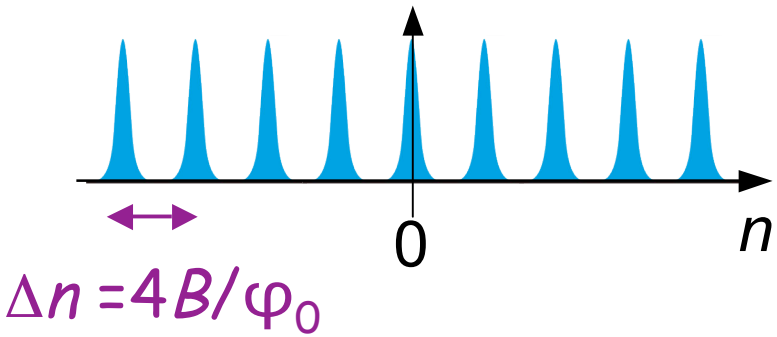
$$\hat{H}_D = \begin{pmatrix} 0 & \hat{p}_x - i\hat{p}_y \\ \hat{p}_x + i\hat{p}_y & 0 \end{pmatrix}$$

Dirac Hamiltonian:
not only spectral shape
but also chirality


Normal Electrons
E
p_y
p_x
$E_{LL} = \pm\hbar\omega_c(N + 1/2)$
D(E)
$\hbar\omega_c$
E_F
E
$\Delta n = 4B/\varphi_0$
n
Dirac Fermions
E
p_y
p_x
$E_{LL} = \pm v_F\sqrt{2e\hbar B}\sqrt{N}$
D(E)
E_F
E
$\Delta n = 4B/\varphi_0$
n

half-integer quantum Hall effect



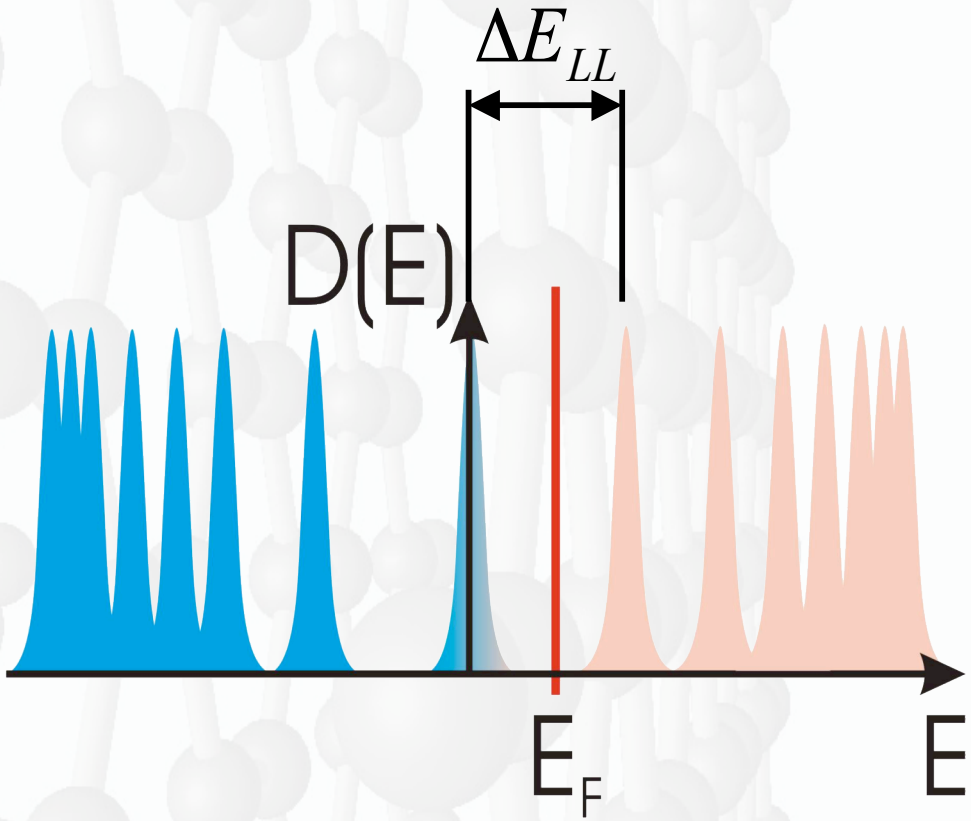


relativistic analogue of the integer QHE

Nature **438**, 197 (2005)

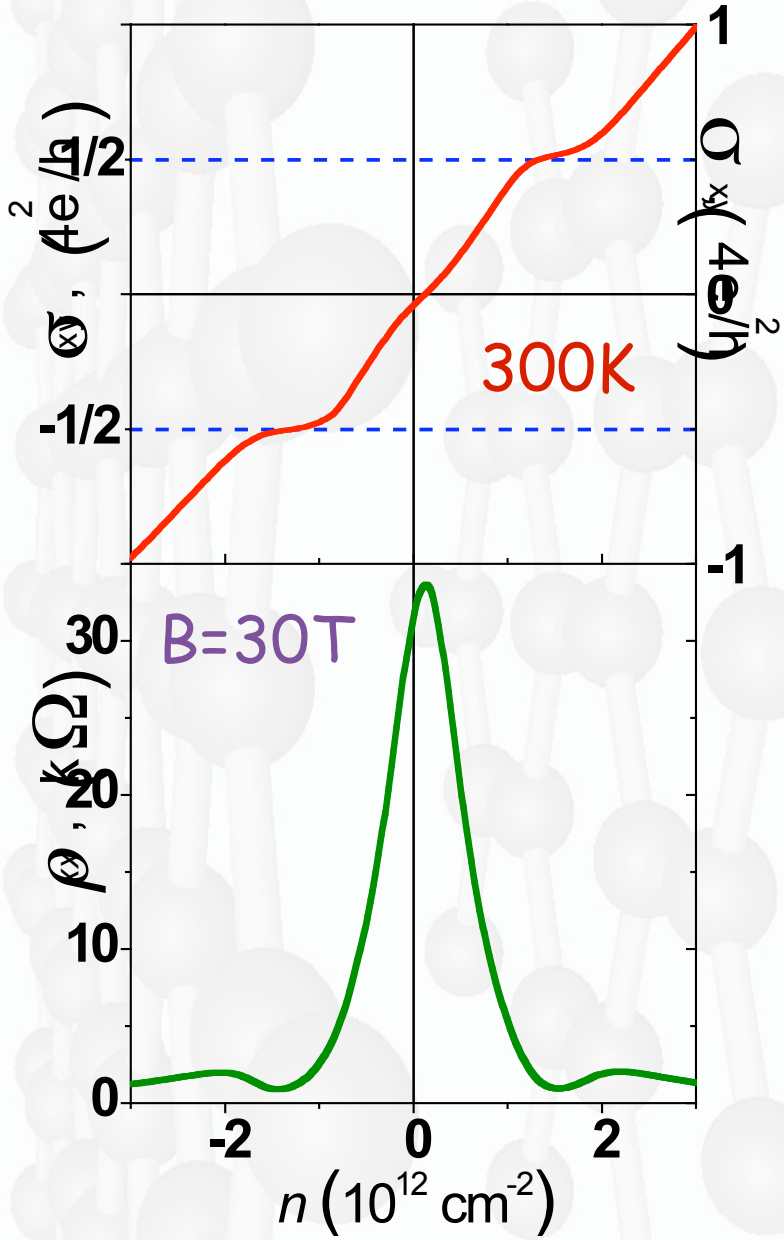
also Zhang *et al*, *ibid* 201 (2005)

room-T quantum Hall effect

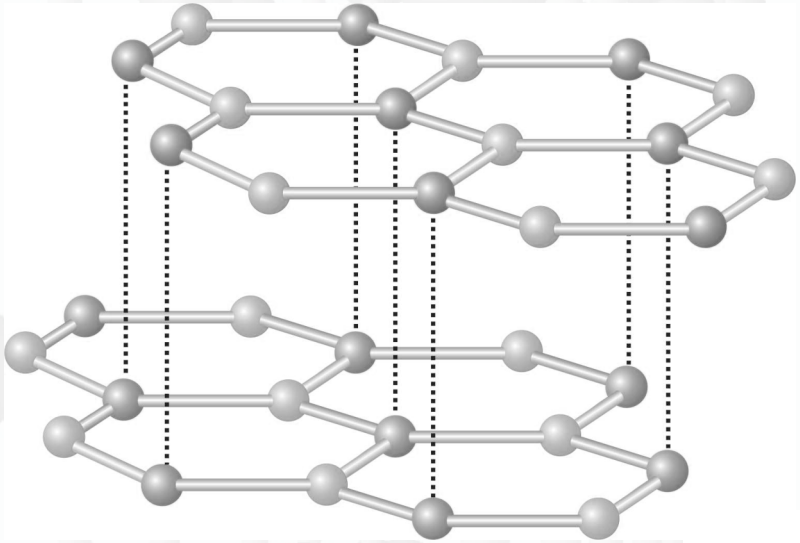


$$\Delta E_{LL} = v_F \sqrt{2e\hbar B}$$

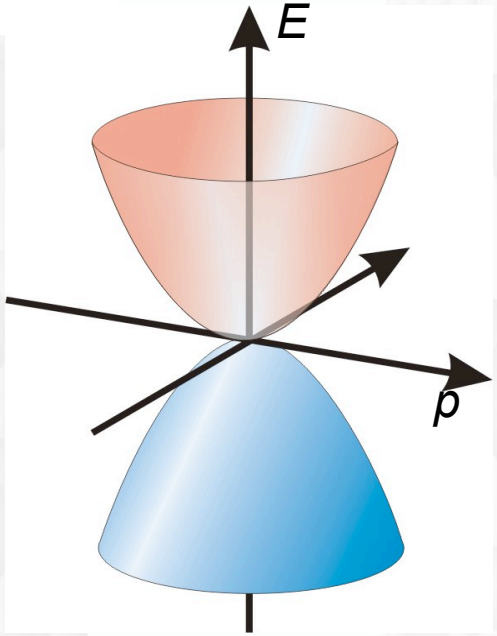
$$\Delta E_{LL}(K) = 400\sqrt{B(T)}$$



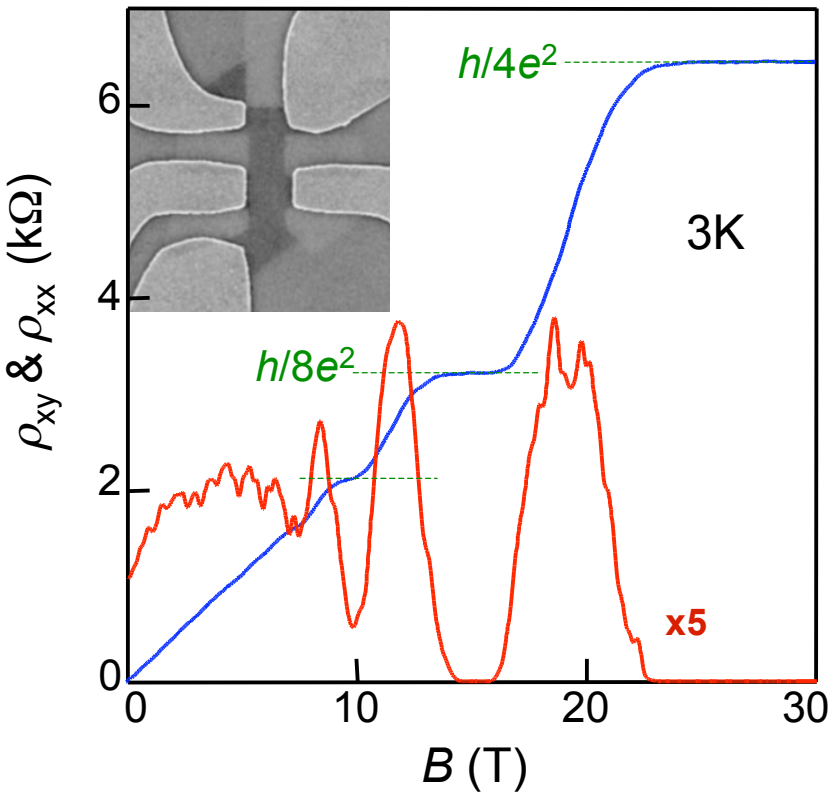
BILAYER GRAPHENE

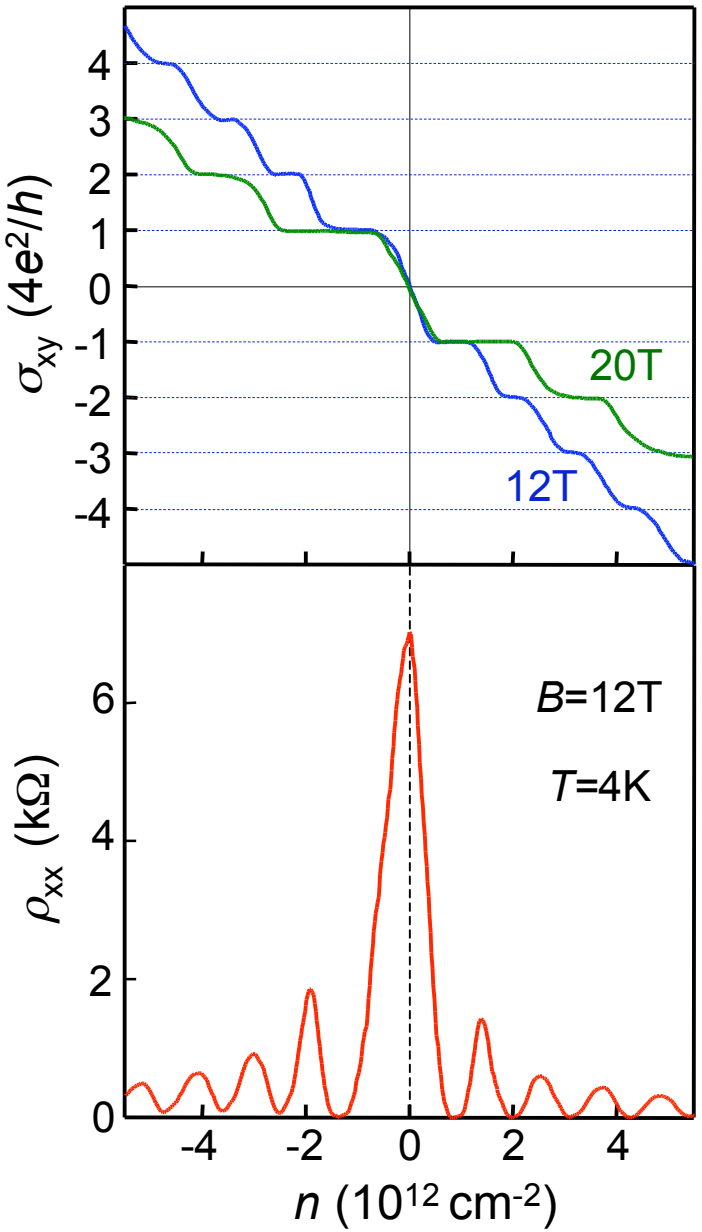


$$E(p) = \pm \tfrac{1}{2}\gamma_1 \pm \sqrt{\tfrac{1}{4}\gamma_1^2 + v_F^2 p^2}$$

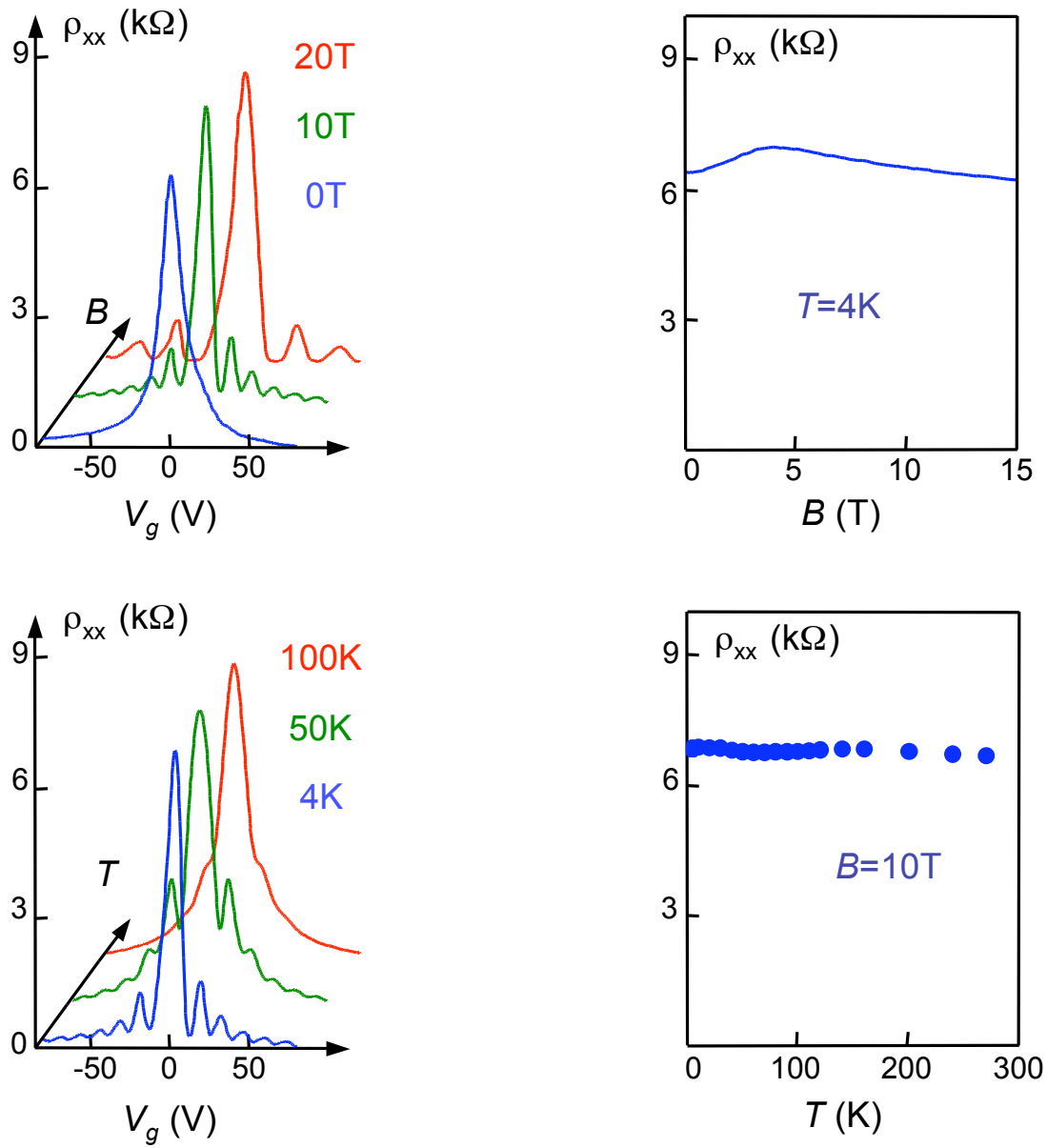


QHE in bilayer graphene

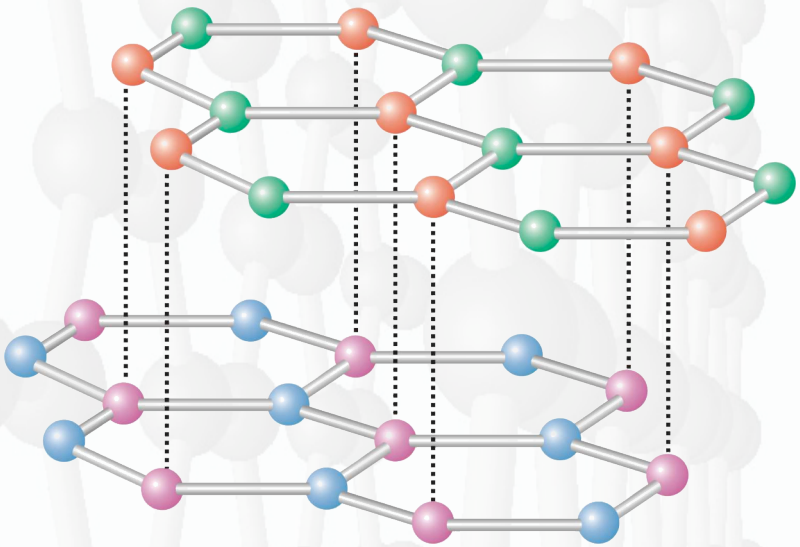




metallic state at ν≈0

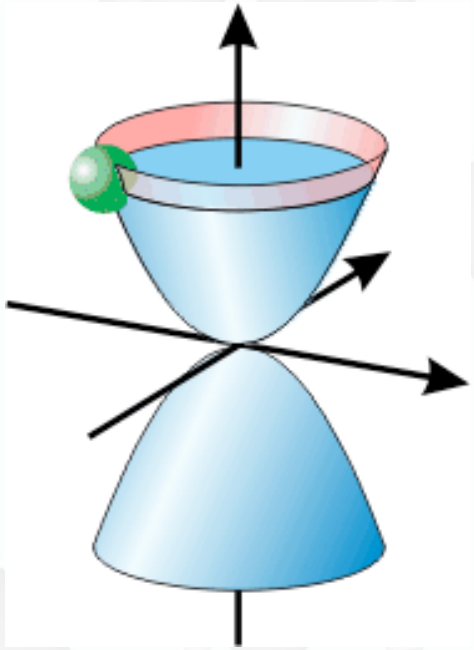


chiral massive fermions



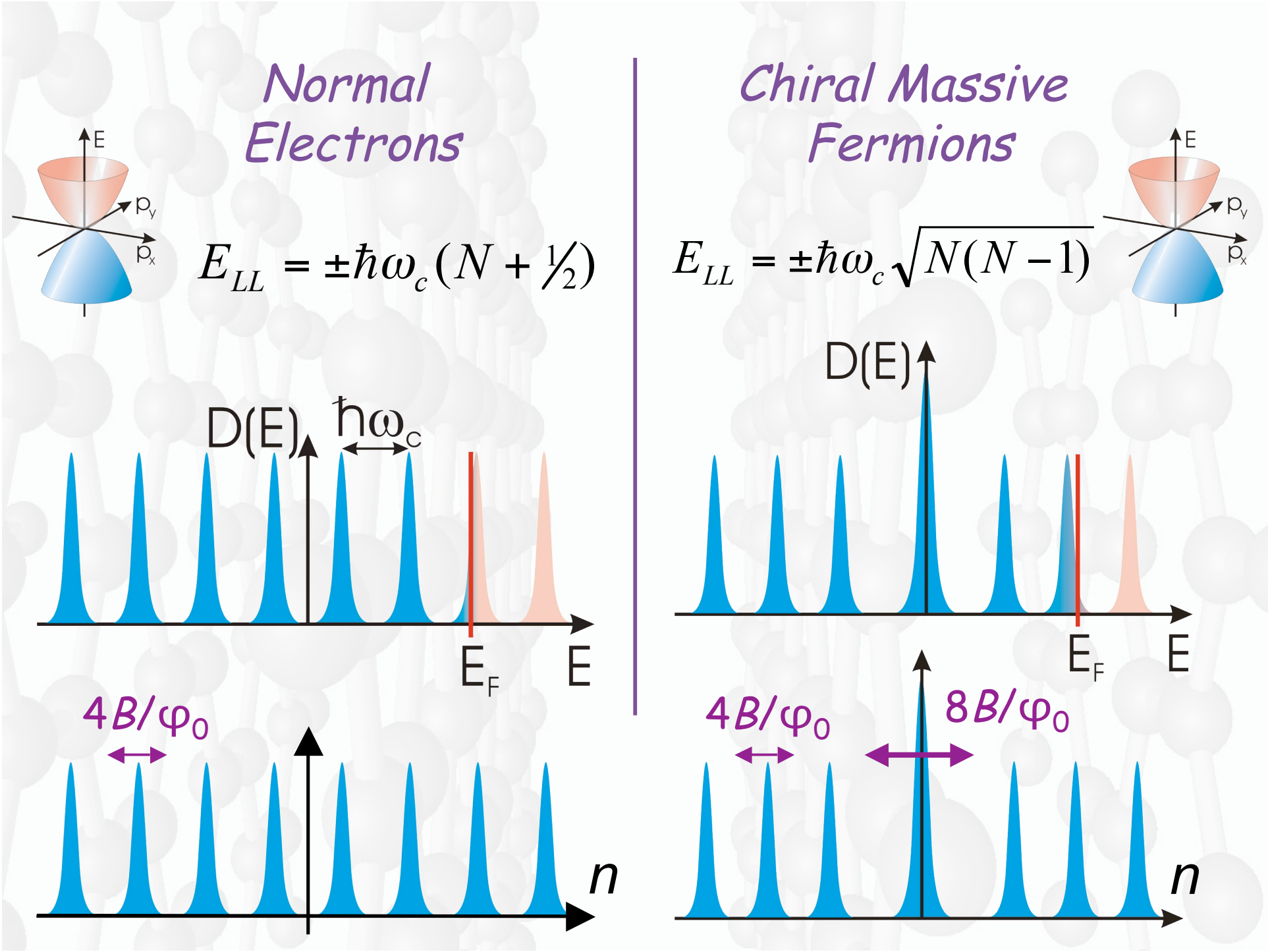
$$E(p) = \pm\tfrac{1}{2}\gamma_1 \pm \sqrt{\tfrac{1}{4}\gamma_1^2 + v_F^2 p^2}$$

$$\hat{H}_2 = -\frac{1}{2m}\begin{pmatrix} 0 & (\hat{p}_x + i\hat{p}_y)^2 \\ (\hat{p}_x - i\hat{p}_y)^2 & 0 \end{pmatrix}$$

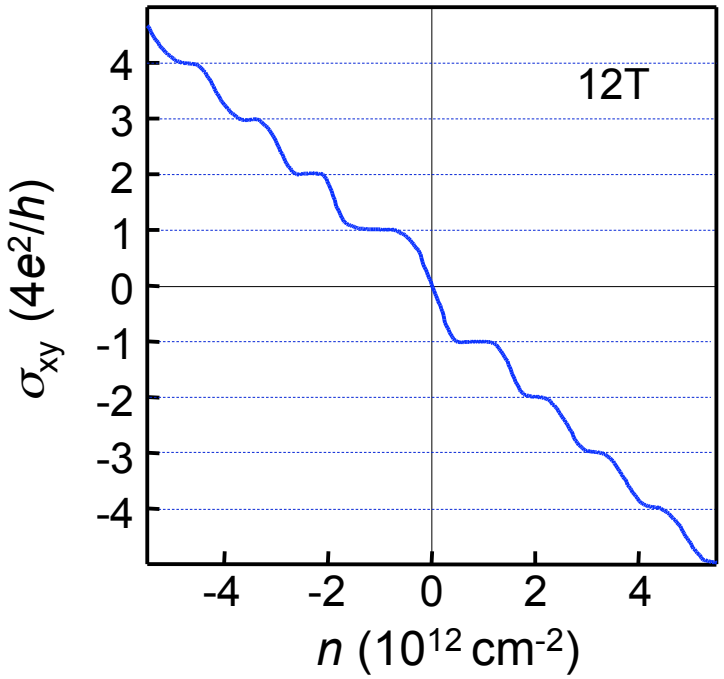


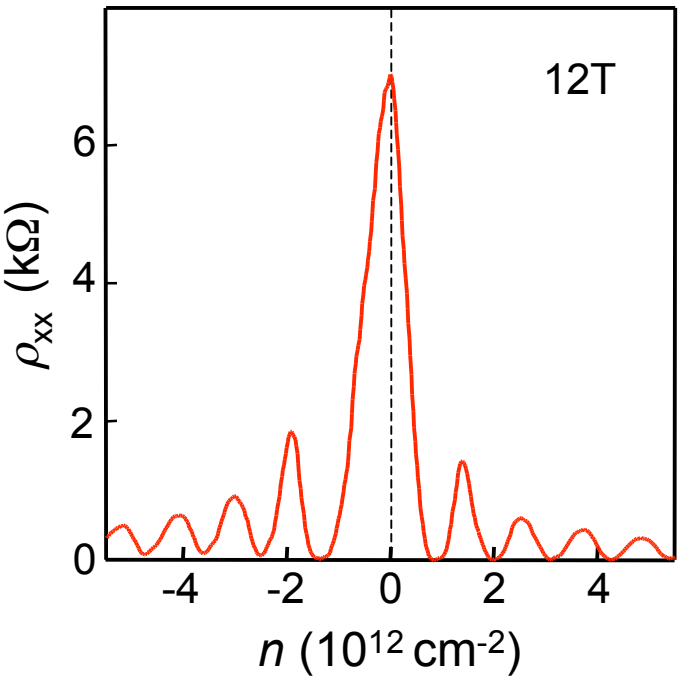
$$E_N = \pm\hbar\omega_c\sqrt{N(N-1)}$$

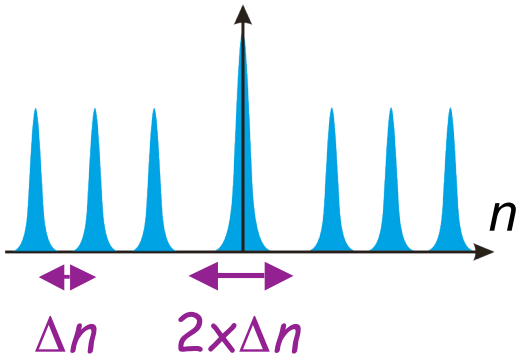
McCann & Falko, *PRL* 2006

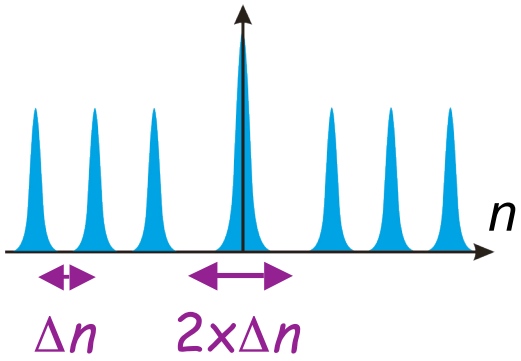

Normal Electrons
E
p_y
p_x
$E_{LL} = \pm\hbar\omega_c(N + 1/2)$
D(E)
$\hbar\omega_c$
E_F
E
$4B/\varphi_0$
n
Chiral Massive Fermions
E
p_y
p_x
$E_{LL} = \pm\hbar\omega_c\sqrt{N(N-1)}$
D(E)
E_F
E
$4B/\varphi_0$
$8B/\varphi_0$
n

chiral QHE in bilayer graphene



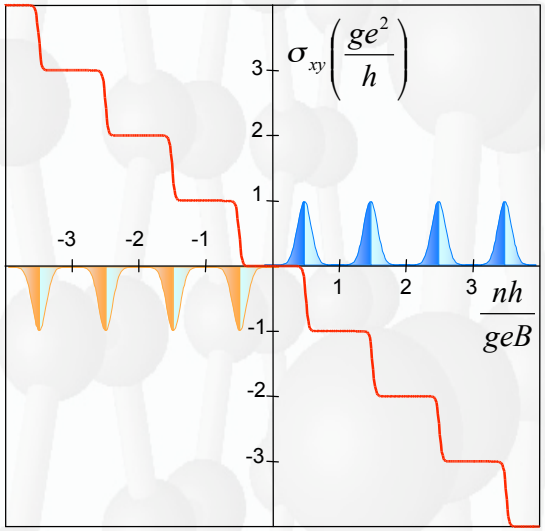






three types of IQHE

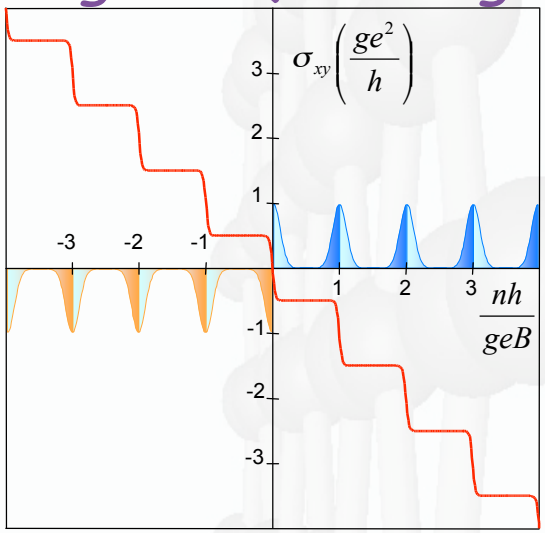
conventional IQHE



all LL at non-zero E
zero Berry phase

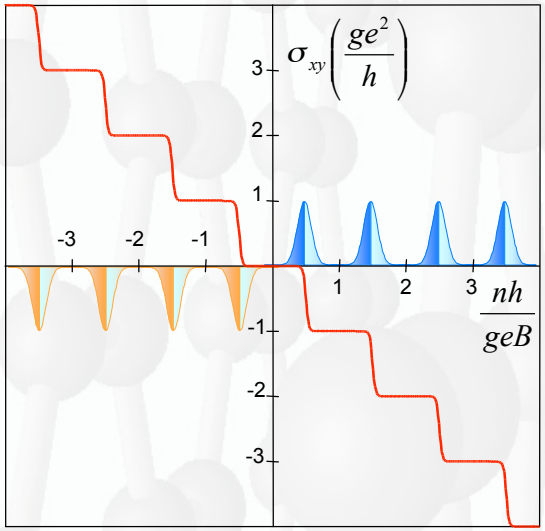
three types of IQHE

"half-integer" QHE in graphene



one LL at zero E
Berry phase π
metallic at $\nu \rightarrow 0$

conventional IQHE



all LL at non-zero E
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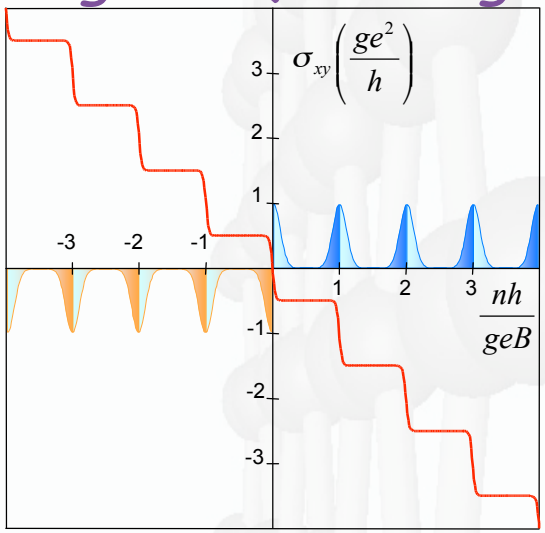
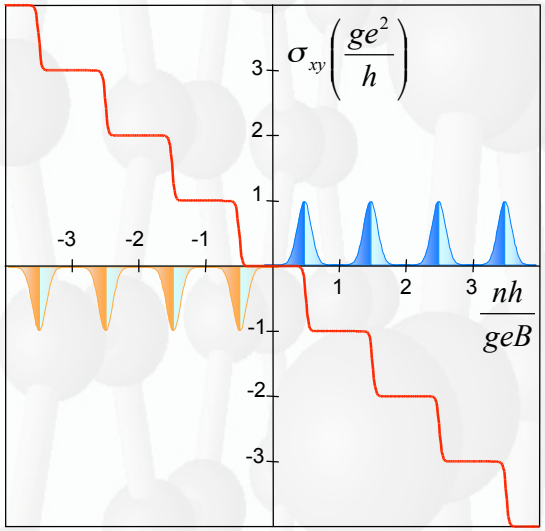
three types of IQHE

"half-integer" QHE in graphene



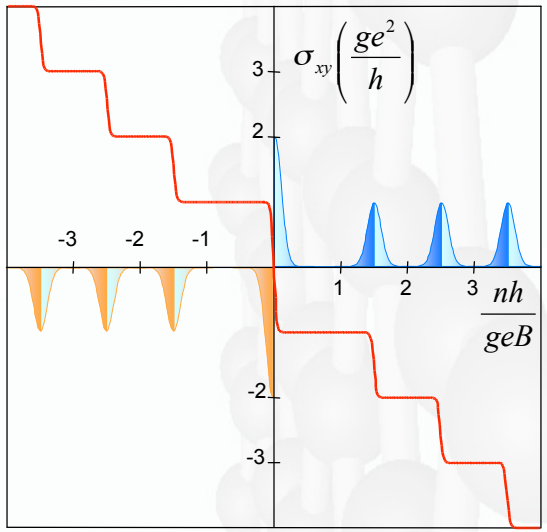
one LL at zero E
Berry phase π
metallic at $\nu \rightarrow 0$

conventional IQHE



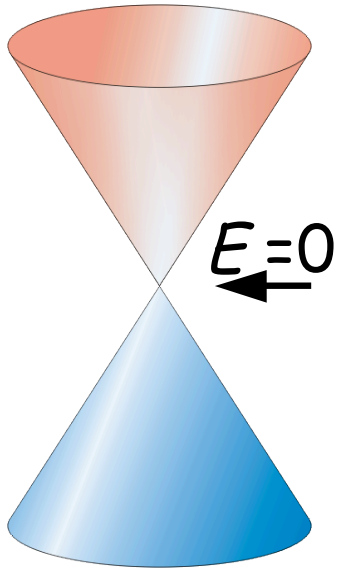
all LL at non-zero E
zero Berry phase

chiral IQHE in bilayer graphene

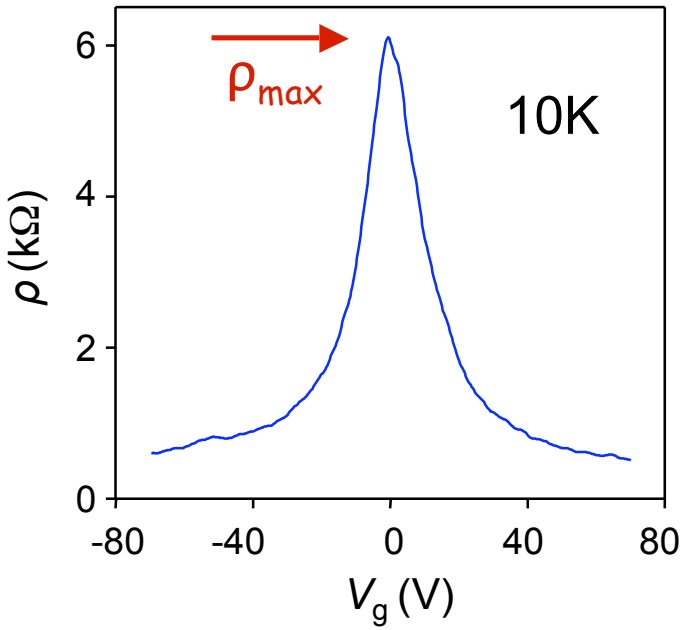


two LLs at zero E
Berry phase 2π
metallic at $\nu \rightarrow 0$

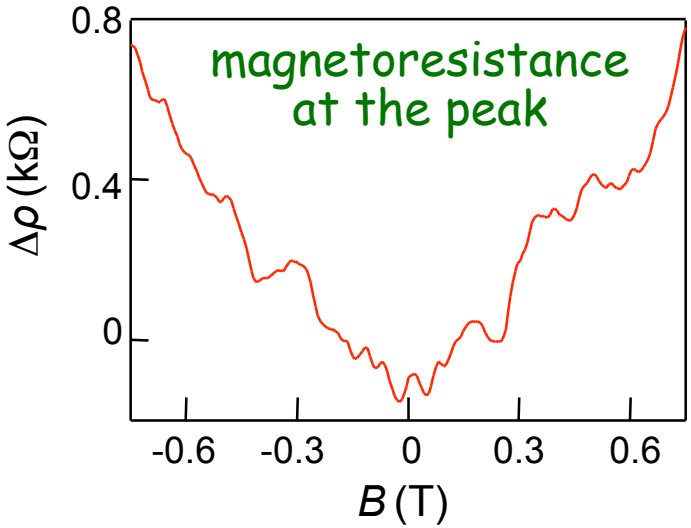
Minimum Quantum Conductivity



zero-gap semiconductor

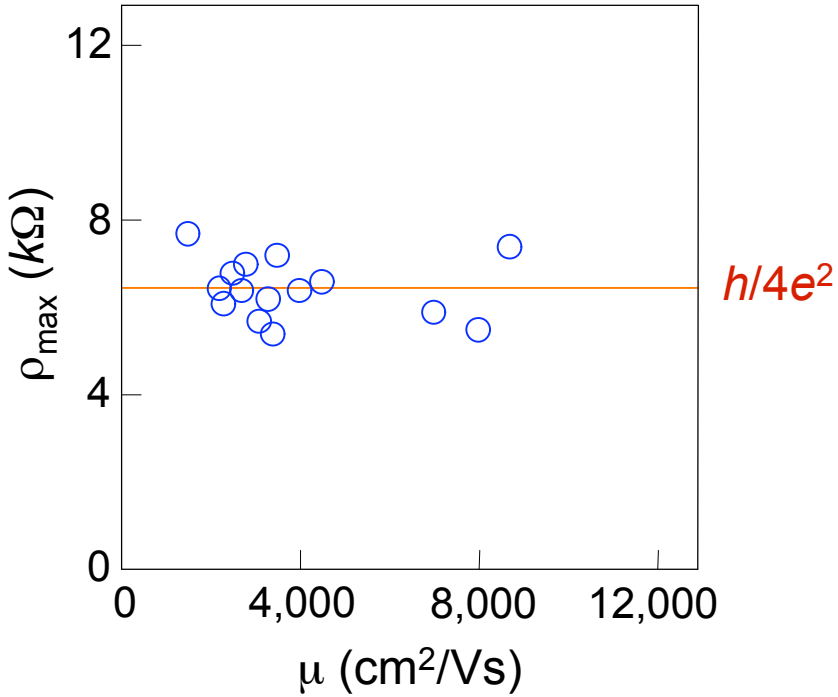


no temperature dependence in the peak between 3 and 80K



NO field-induced localization (classical magnetoresistance)

Minimum Quantum Conductivity

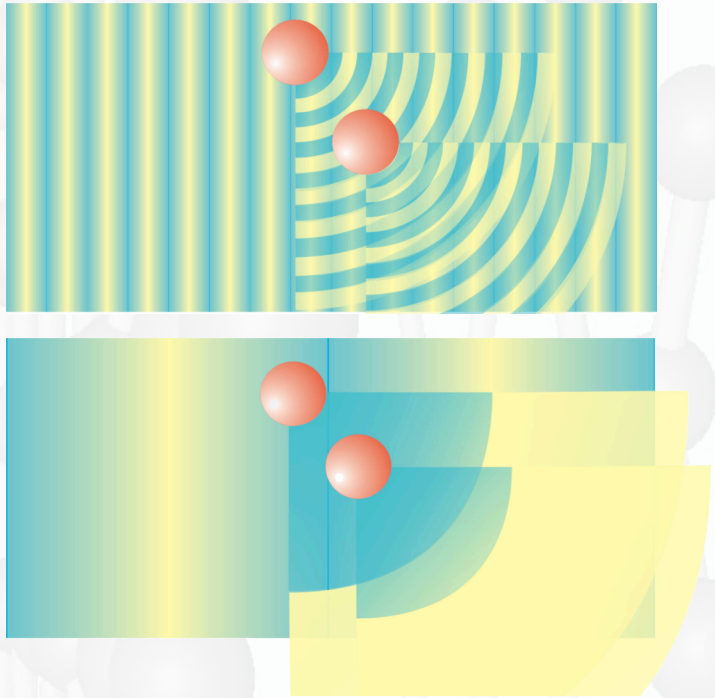


quantized resistivity NOT resistance

(h/e^2 per spin and valley)

Minimum Quantum Conductivity

Mott's argument: $l \geq \lambda_F$



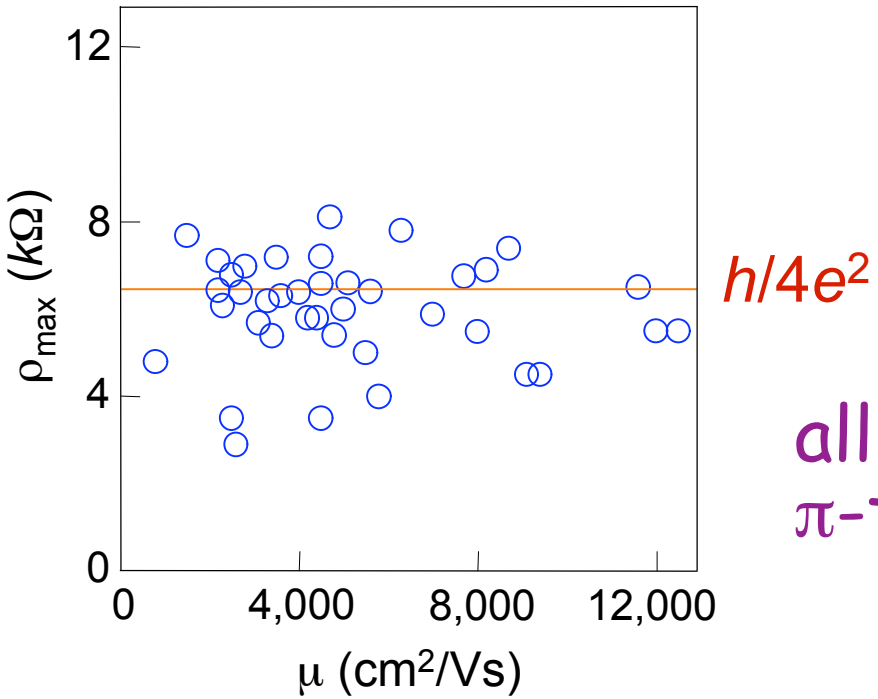
$$\sigma = ne\mu = \frac{e^2}{h} k_F l \geq \frac{e^2}{h}$$

requires remaining a metal
NO LOCALIZATION

minimum conductivity also in bilayer
(parabolic spectrum)

due to chirality of charge carriers

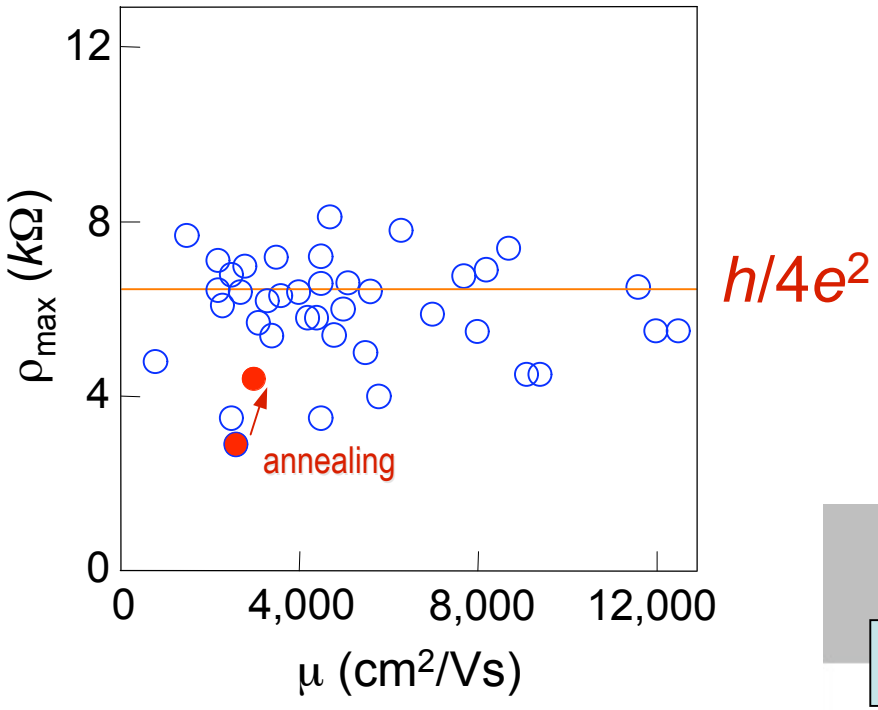
Minimum Quantum Conductivity



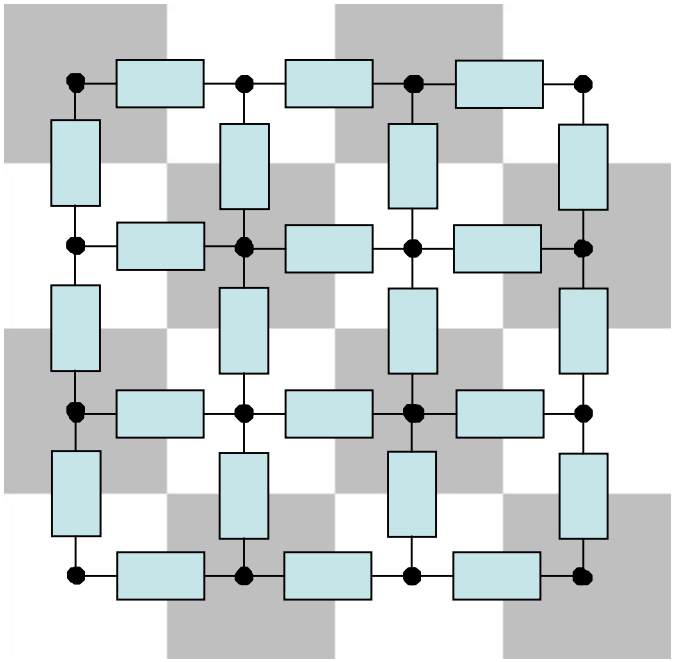
all theories predict
π-times larger value

is still missing

Minimum Quantum Conductivity



network of
h/e^2-resistors



macroscopic inhomogeneity
leads to lower measured ρ_{xx}

$h/4e^2 \pm 15\%$

CONCLUSIONS

Two Chiral Quantum Hall Effects

metallic conductivity $\approx e^2/h$
in zero-concentration limit

RELATIVISTIC-LIKE
CONDENSED MATTER PHYSICS