



**SMR 1760 - 13**

**COLLEGE ON  
PHYSICS OF NANO-DEVICES**

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*Strongly interacting Fermi gases.  
From few-body to many-body physics*

Presented by:

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# **Strongly interacting Fermi gases. From few-body to many-body physics**

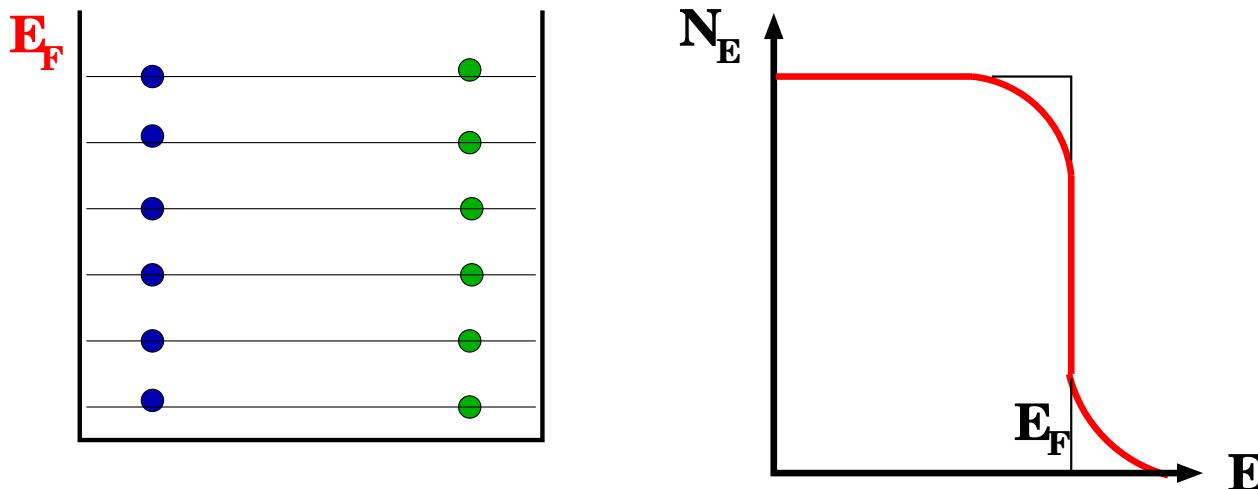
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## **Outline**

- Introduction.
- Feshbach resonance. Molecules in Fermi gases
- Molecule-molecule interaction
- Remarkable collisional stability
- Molecular BEC
- Novel composite bosons ?

## Two-component trapped Fermi gas



$$\int_0^{k_F} \frac{4\pi V k^2 dk}{(2\pi)^3} = \frac{V k_F^3}{6\pi^2} = \frac{N}{2}$$

$$k_F = (3\pi^2 n)^{1/3}; \quad n = \frac{N}{V}; \quad E_F = \frac{\hbar^2 k_F^2}{2m}$$

trapped gas  $\Rightarrow E_F \sim N^{1/3} \hbar\omega$

## Weakly interacting ultracold limit

Weakly interacting gas

$$|a| \ll n^{-1/3}$$

$$n|a|^3 \ll 1 \text{ or } k_F|a| \ll 1$$

Ultracold limit

$$\Lambda_T = \left( \frac{2\pi\hbar^2}{mT} \right)^{1/2} \gg R_e \quad \Rightarrow \text{s-wave scattering}$$

Interspecies interaction only

What does the interaction do?

$a < 0 \rightarrow$  Interspecies attraction  $\rightarrow$  Cooper pairing at low  $T$



Superfluid BCS transition  $\rightarrow T_c \sim E_F \exp\{-\pi/2k_F|a|\}$

$T_c \ll 0.1E_F$  for ordinary  $a$  Very hard to reach

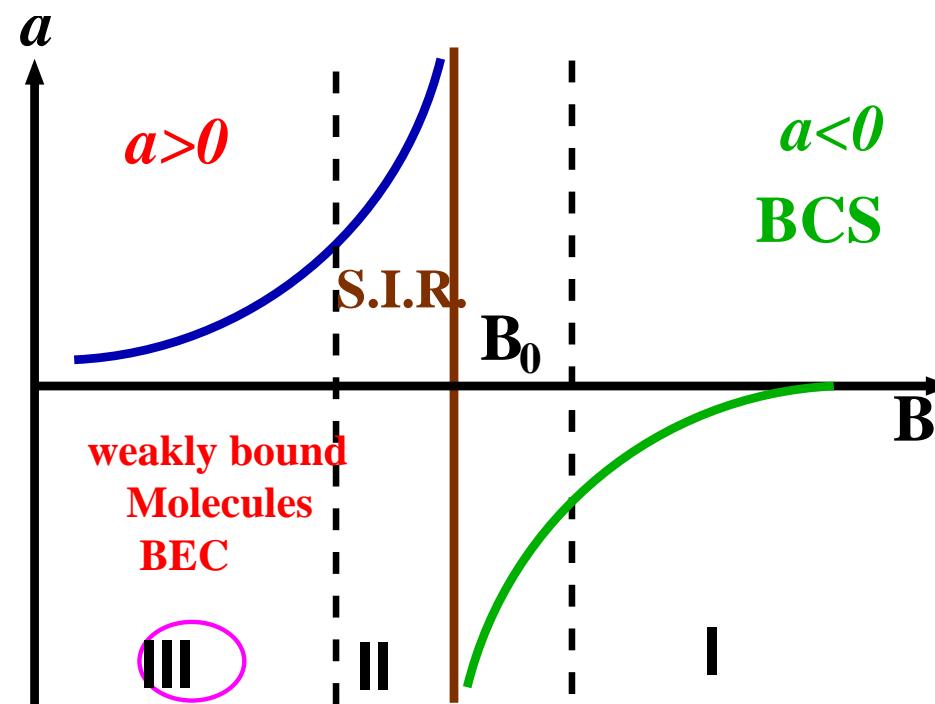
## Experiments $^{40}\text{K}$ $^6\text{Li}$

Dilute limit  $nR_e^3 \ll 1$       Ultracold limit  $\Lambda_T \gg R_e$

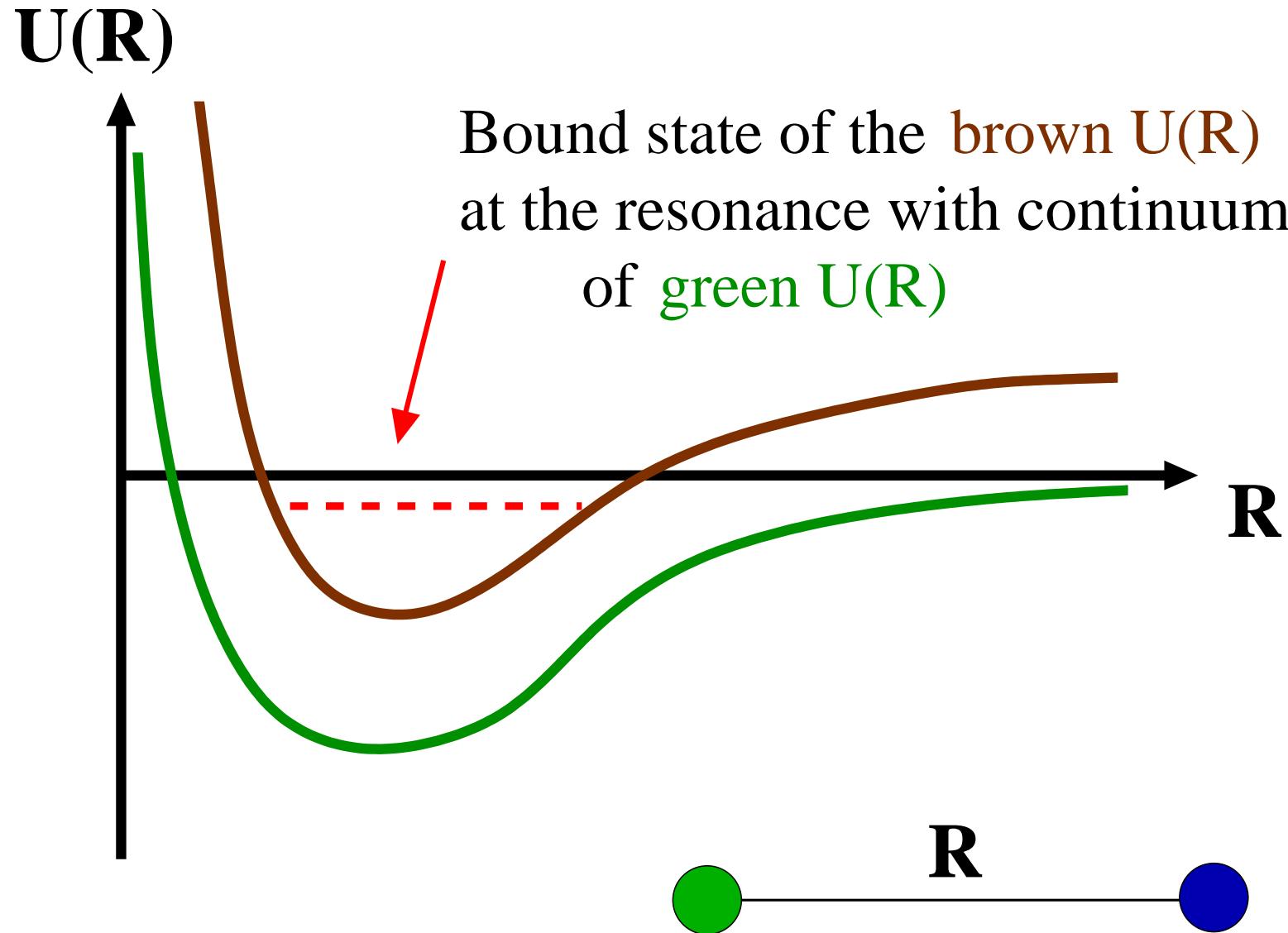
Quantum degeneracy  $\rightarrow$  JILA 1998  $^{40}\text{K}$

At present  $n \sim 10^{13} - 10^{14}\text{cm}^{-3}$ ;  $T \sim 1\mu\text{K}$

JILA, LENS Innsbruck, MIT, ENS, Rice, Duke, ETH,  
Hamburg, Tuebingen, Toronto

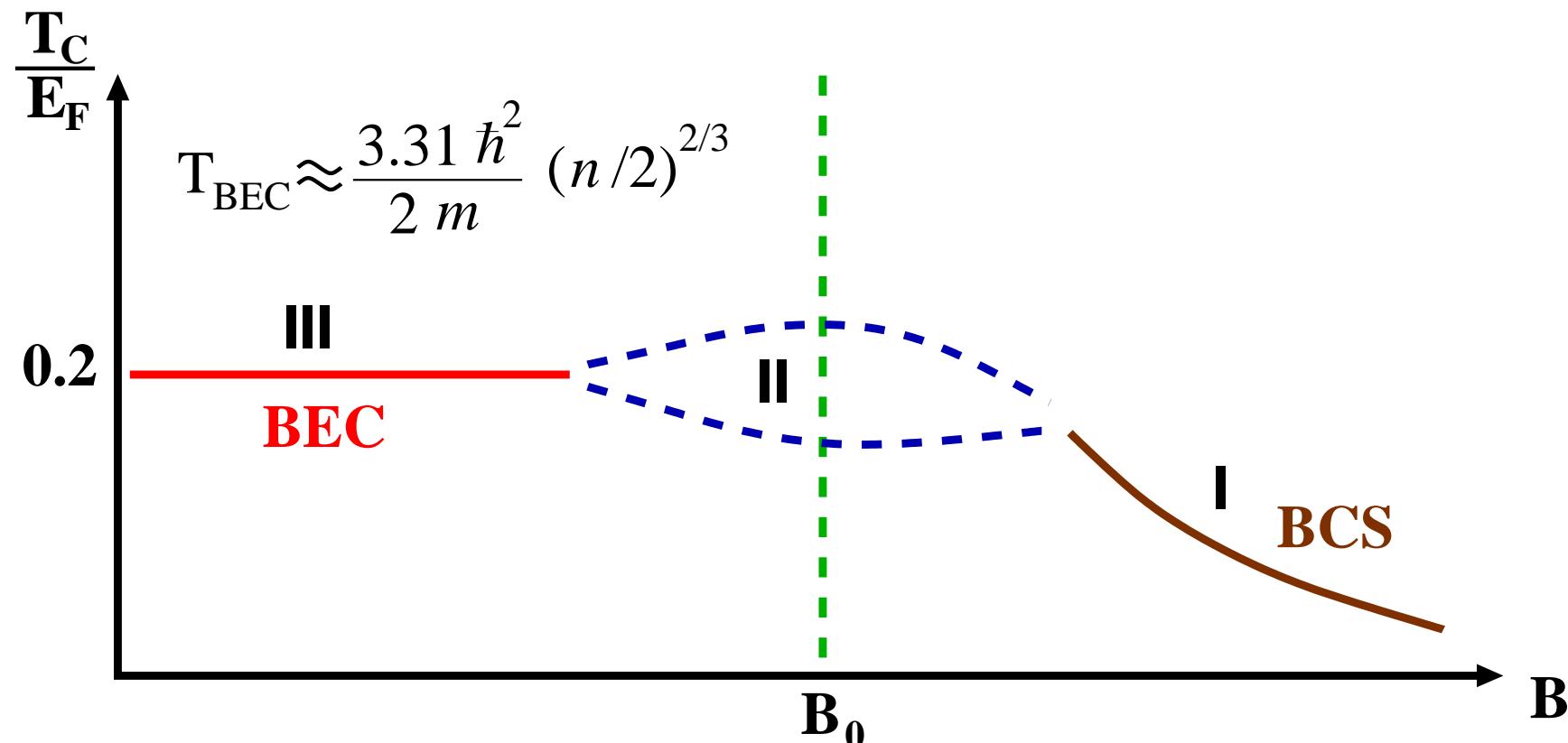


## Feshbach resonance



## Superfluid regimes

- I     $k_F|a| \ll 1 \rightarrow \text{BCS}$
- II     $k_F|a| > 1 \rightarrow \text{Strongly interacting regime}$
- III     $na^3 \ll 1 \rightarrow \text{Gas of bosonic molecules}$   
 $a \gg R_e \rightarrow \text{BEC of weakly bound molecules}$



**BCS-BEC crossover:** Leggett, Nozieres-Schmitt-Rink - p.6/19

## Strongly interacting regime

Wide resonance (single-channel model)

$T = 0 \quad k_F|a| \gg 1 \quad \rightarrow \quad$  Only one distance scale  $n^{-1/3}$

Only one energy scale  $E_F \sim \hbar^2 n^{2/3} / m$

Universal thermodynamics (J. Ho)

Monte Carlo studies  $\rightarrow \mu \approx 0.4E_F$

(Carlson et al, Giorgini/Astracharchik, etc.)

Nature of superfluid pairing, Transition temperature, Excitations

$$T_c = 0.15E_F \quad \text{UMASS-ETH}$$

Experiments

BEC-type behavior of fermionic atom pairs (JILA, MIT)

Excitation frequencies and damping rates (Innsbruck, Duke)

Pairing gap (Innsbruck), Heat capacity (Duke)

Correlations (JILA)

Vortices (MIT)

## Scattering amplitude and bound state

$$f = \frac{1}{a^{-1} + ik + k^2 R_*} \Rightarrow \text{scattering amplitude}$$

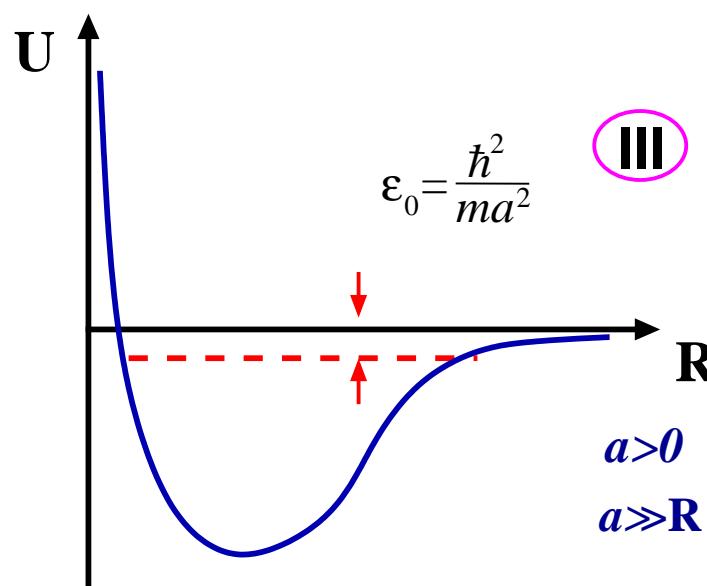
Bound state  $\Rightarrow$  pole of  $f$

$$k = i\eta \Rightarrow \eta^2 R_* + \eta - a^{-1} = 0; \quad \eta = -\frac{1}{2R_*} + \left( \frac{1}{4R_*^2} + \frac{1}{aR_*} \right)^{1/2}$$

$$a > 0 \text{ and } R_* \ll a \Rightarrow \eta = a^{-1}$$

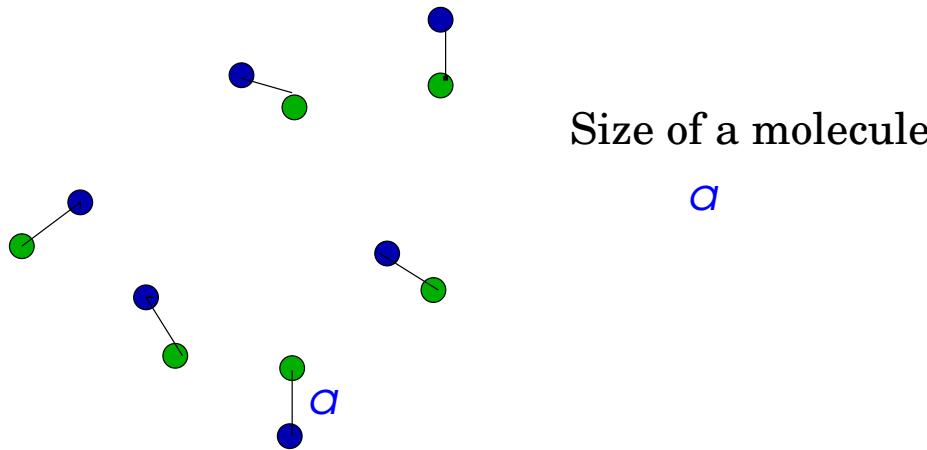
$$\text{Binding energy} \Rightarrow \epsilon_0 = \frac{\hbar^2 \eta^2}{m} = \frac{\hbar^2}{ma^2}$$

$R_* \ll a \Rightarrow$  wide resonance/single-channel model



## Gas of bosonic molecules (dimers)

Region III ( $a > 0$ )  $\Rightarrow$  gas of weakly bound bosonic molecules



$a \ll n^{-1/3}$  or  $na^3 \ll 1$   $\Rightarrow$  weakly interacting Bose gas

$$\text{Interaction energy } E_{int} = \frac{N(N-1)}{2} \varepsilon_{int}$$

$$\varepsilon_{int} = \frac{g}{V}; \quad g = ?$$

$g < 0$   $\Rightarrow$  collapse of a Bose-Einstein condensate

$g > 0$   $\Rightarrow$  stable BEC

## Molecule-molecule elastic interaction

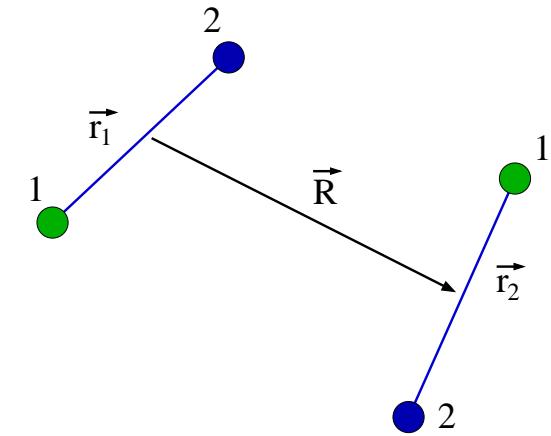
Interaction constant  $g = 4\pi\hbar^2 a_{dd}/2m$  "Old answer"  $\rightarrow 2a$

**4-body problem    Exact solution for  $a \gg R_e$  (Petrov et al 2003)**

$\Psi \rightarrow 9$  variables

$a \gg R_e \Rightarrow$  Zero-range approximation

$$\Psi_{r_1 \rightarrow 0} \rightarrow f(\vec{r}_2, \vec{R})(1/4\pi r_1 - 1/4\pi a)$$



$$R \gg a \quad \Psi = \phi_0(r_1)\phi_0(r_2)(1 - a_{dd}/R); \quad \phi_0(r) = \frac{1}{\sqrt{2\pi a}} \exp(-r/a)$$

$$R \gg a \quad f(\vec{r}, \vec{R}) = (2/rR) \exp(-r/a)(1 - a_{dd}/R);$$

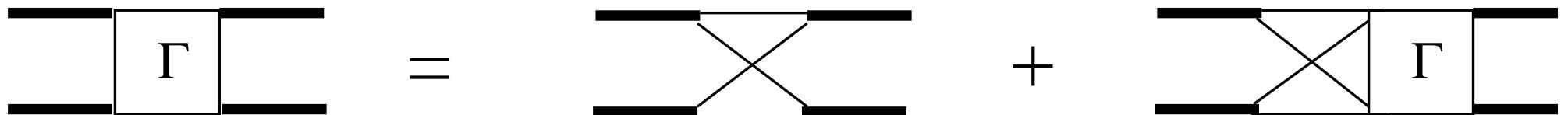
## Derivation of $a_{dd}$

Limit  $r_1 \rightarrow 0$     Integral equation for  $f$  (3 variables)

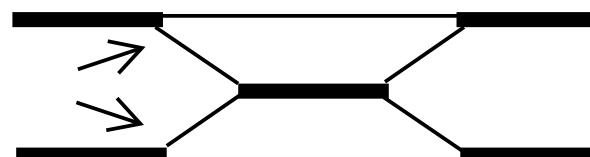
$$a_{dd} = 0.6a$$

Monte Carlo (Giorgini/Astracharchik, 2004)

Diagrammatic approach (M.Kagan et al, 2005; V. Gurarie et al, 2006)

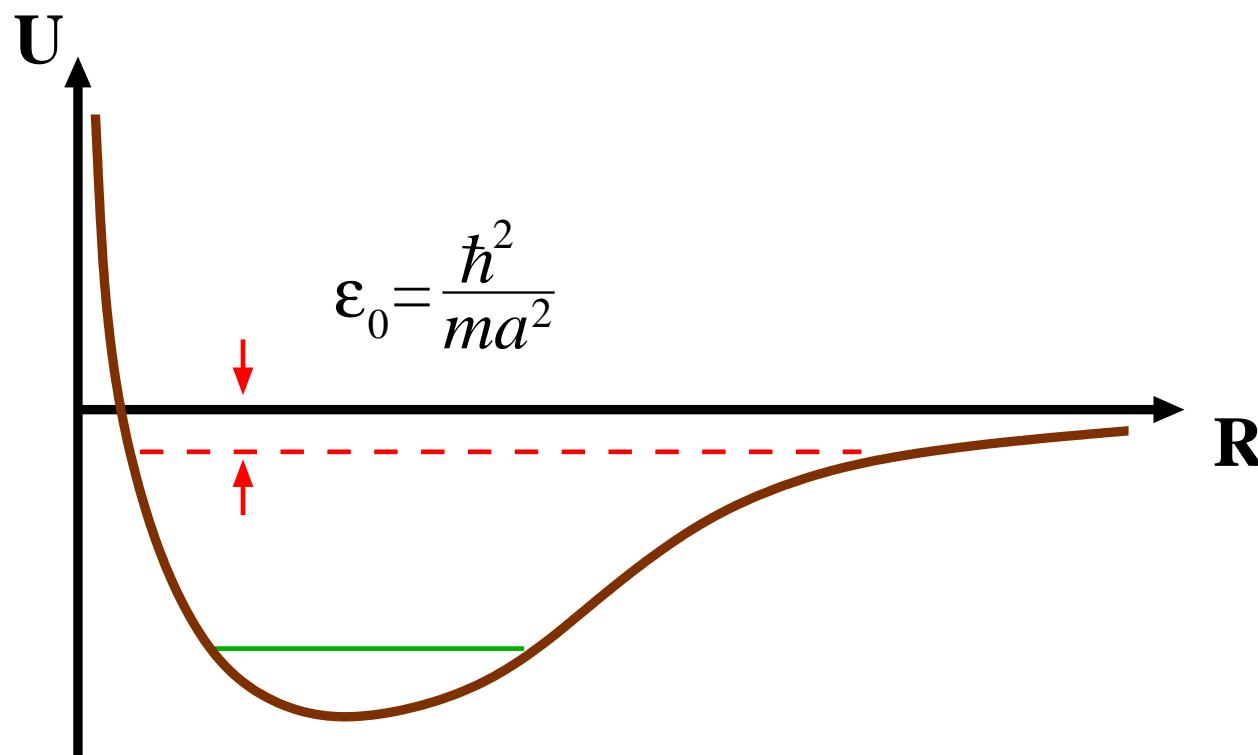


$$\Gamma = 0.75a$$



## Weakly bound dimers

Weakly bound dimers → The highest rovibrational state of the diatomic molecule



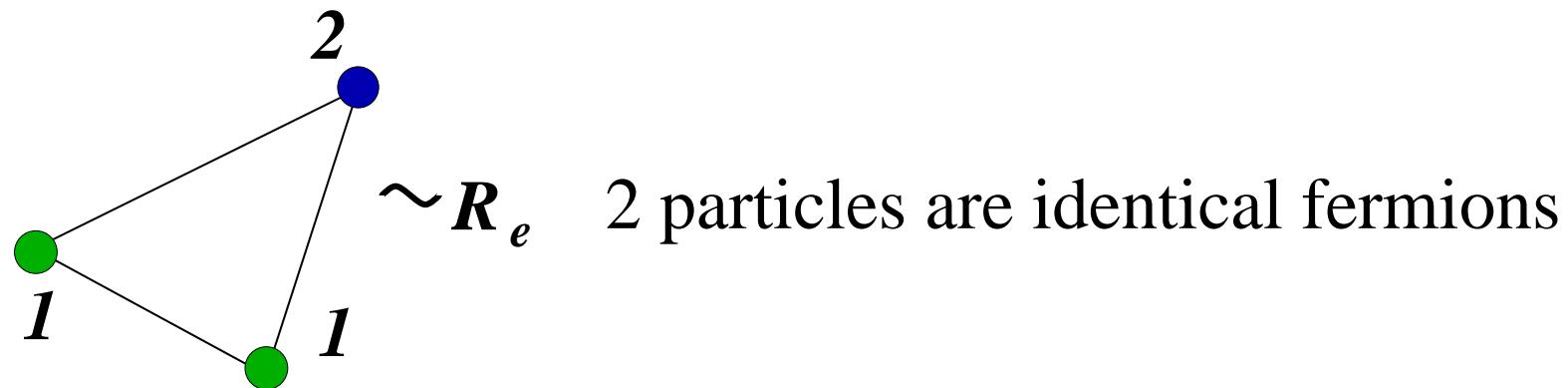
Collisional relaxation to deep bound states  
(~ 1ms for  $\text{Rb}_2$  at  $n \sim 10^{13} \text{cm}^{-3}$ )

## Atom-dimer collisions. Physical picture

Weakly bound dimer  $\sim a$

Size →

Deep bound state  $\sim R_e$  (50 Å)  $\ll a$



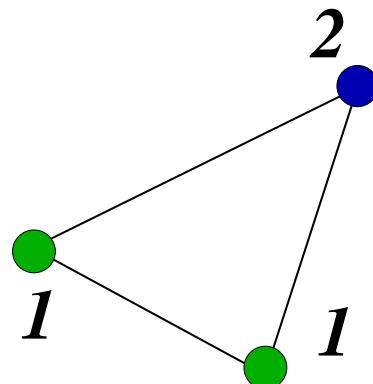
Pauli principle

$$\alpha_{rel} \sim (k_{eff} R_e)^{2?} \sim (R_e/a)^{2?}$$

## Atom-dimer collisions. Relaxation rate

The released binding energy is  $\sim \hbar^2/mR_e^2$

Establish the dependence of  $\alpha_{rel}$  on  $a$



$\sim R_e$  2 particles are identical fermions

$r \sim R_e \Rightarrow \Psi = A(a)\tilde{\psi} \rightarrow$  valid at any  $r \ll a$

$R_e \ll r \ll a \Rightarrow$  zero-range approximation

as well as at any  $r \gg R_e$

The only distance scale is  $a \Rightarrow \Psi = B(a)F(\vec{r}_i/a)$

$$\rho = \sqrt{x^2 + y^2} \ll a \Rightarrow \Psi \approx B(a)(\rho/a)^\gamma \Phi_\gamma(\Omega)$$

$$A(a) = B(a)a^{-\gamma} \quad \gamma \approx 0.1662$$

## Atom-dimer collisions. Relaxation rate

Long distance behavior  $\Rightarrow$

$$\Psi = \left(1 - \frac{a_{ad}}{R}\right) \frac{1}{\sqrt{2\pi}a^{3/2}(r/a)} \exp(-r/a)$$

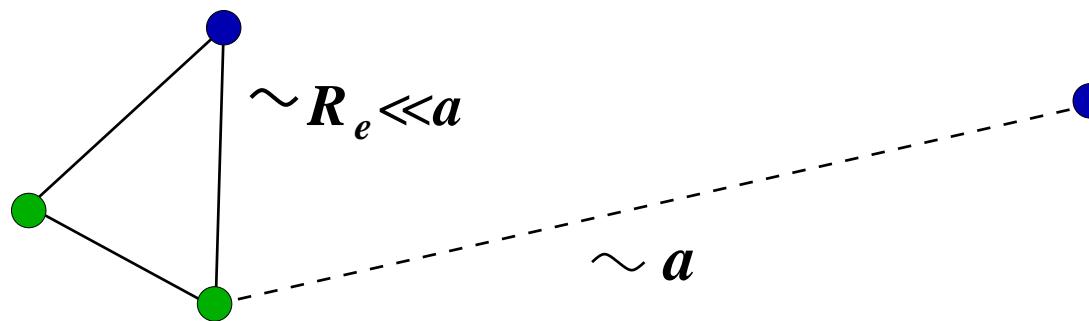
$\Rightarrow B \propto a^{-3/2}$ . Hence,  $A(a) \propto a^{-3/2-\gamma}$

$$\alpha_{\text{rel}} \propto A^2(a) \Rightarrow \alpha_{\text{rel}} \propto a^{-3-2\gamma} = a^{-3.33}$$

$$\alpha_{\text{rel}} = C \left(\frac{\hbar R_e}{m}\right) \left(\frac{R_e}{a}\right)^s; \quad s = 3.33$$

Strong decrease of relaxation on approach to resonance

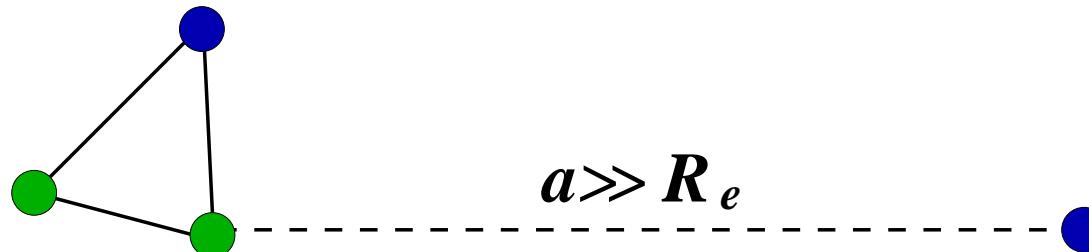
## Molecule-molecule relaxation collisions



$$\alpha_{rel} = C \frac{\hbar R_e}{m} \left( \frac{R_e}{a} \right)^s ; \quad s = 2.55$$

$$\tau \sim (\alpha_{rel} n)^{-1} \sim \text{seconds}$$

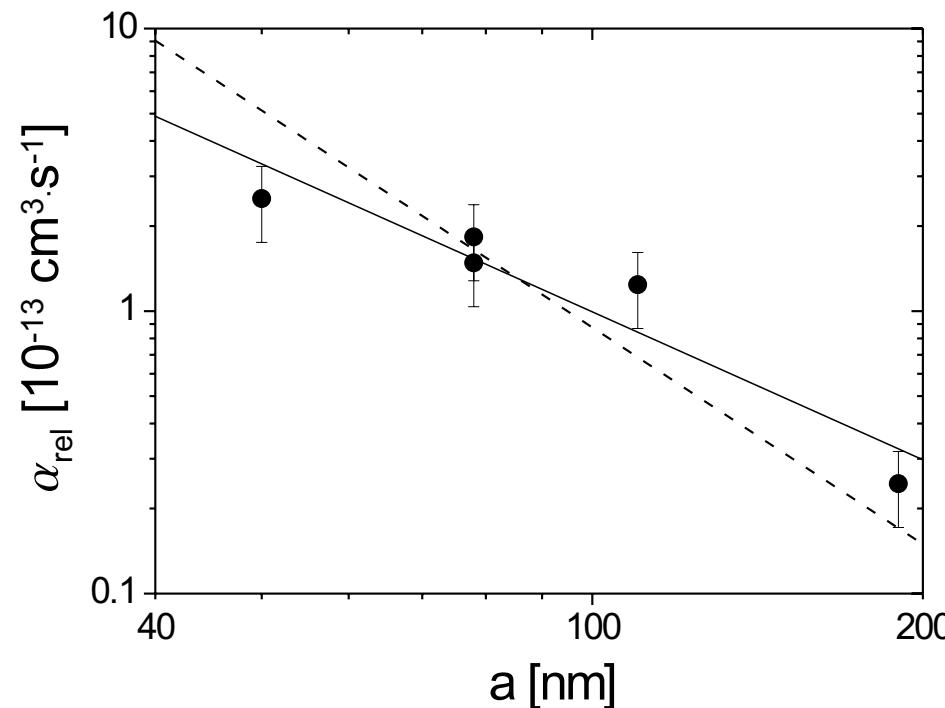
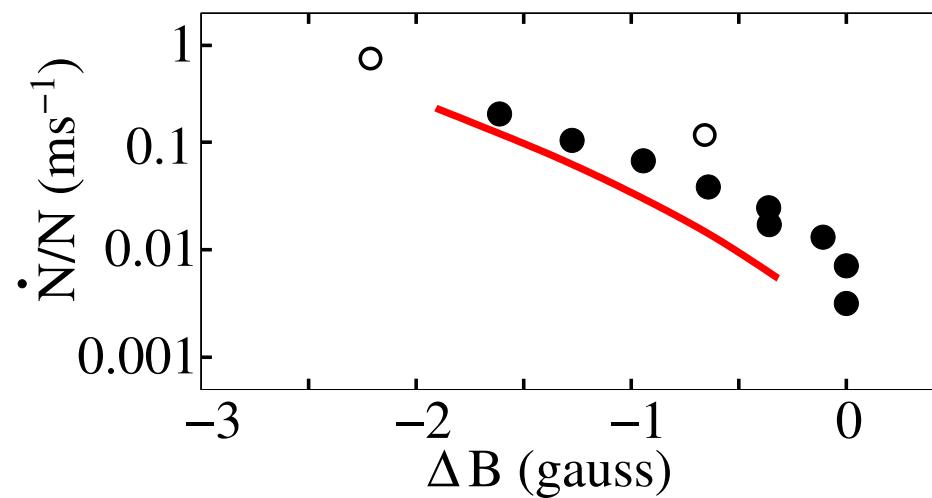
## Molecules of bosonoc atoms



Resonant enhancement

$$\alpha_{rel} \sim \hbar a / m \quad \tau < 1\text{ms}$$

## Suppressed collisional relaxation



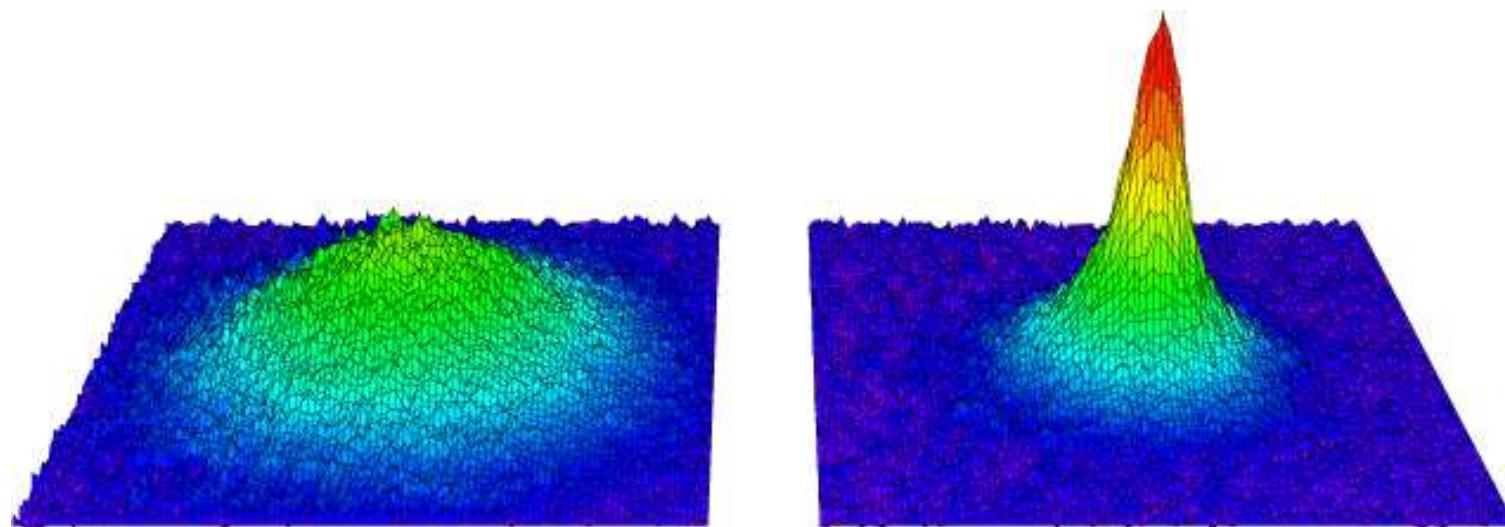
# Bose-Einstein condensates of molecules

Suppressed relaxation    Fast elastic collisions  $a_{dd} = 0.6a$

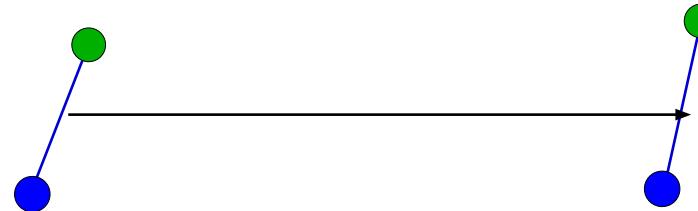
$$^6\text{Li}_2 \rightarrow \frac{\alpha_{rel}}{\alpha_{el}} \leq 10^{-4}$$

Efficient evaporative cooling       $\rightarrow$  BEC

JILA, Innsbruck, MIT, ENS, Rice



## Composite bosons



Bosonic behavior at large separations    **BEC**

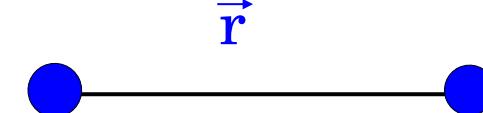
Small separations ?



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$$\Psi \propto (\exp\{i\vec{k}\vec{r}\} - \exp\{-i\vec{k}\vec{r}\})$$
$$\rightarrow 2\vec{k}\vec{r} \text{ for } kr \ll 1$$

$$|\Psi|^2 \propto k^2$$



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$$\Psi \propto (\exp\{i\vec{k}\vec{r}\} + \exp\{-i\vec{k}\vec{r}\})$$
$$\rightarrow \text{const for } kr \ll 1$$

bosons

