



The Abdus Salam
International Centre for Theoretical Physics



SMR 1760 - 14

**COLLEGE ON
PHYSICS OF NANO-DEVICES**

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***Strongly interacting mixtures
of heavy and light fermions***

Presented by:

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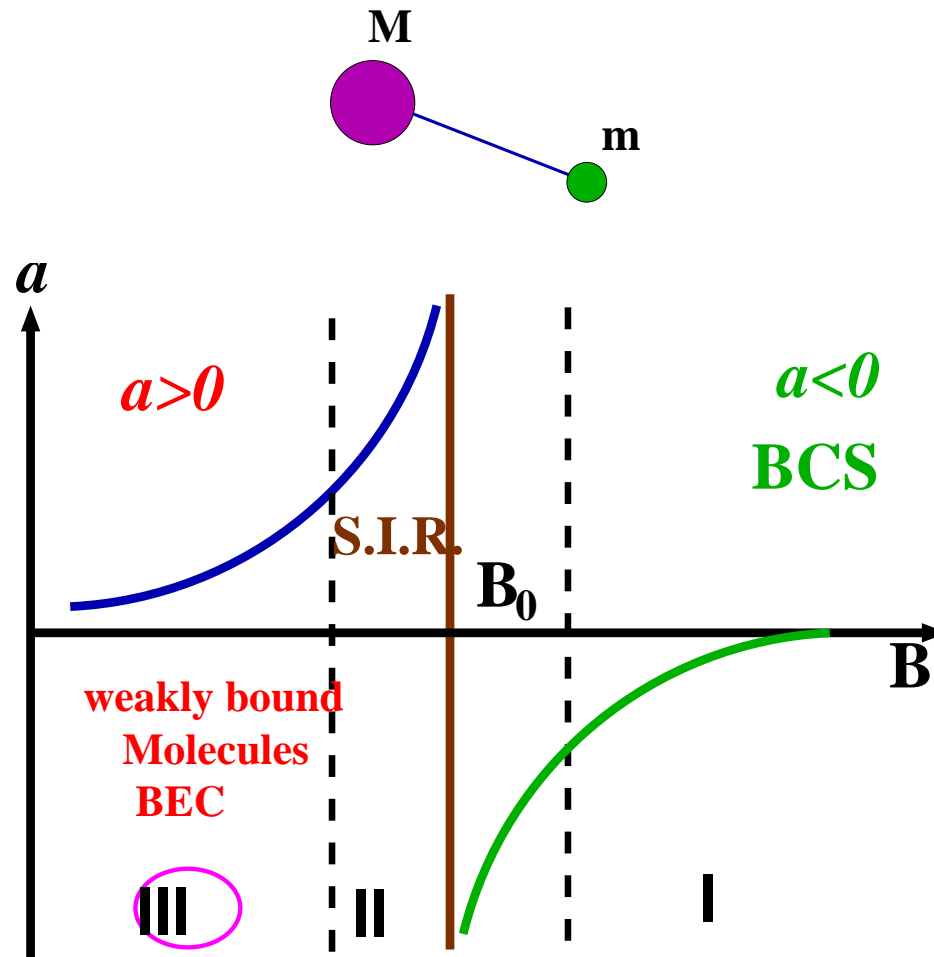
Outline

- Introduction.
- Molecular regime. Interaction between molecules
- Collisional relaxation
- Pauli principle and effect of the mass ratio
- Bose-Fermi molecules
- Ideas for future

Collaborations: D.S. Petrov, C. Salomon (ENS)

Mixtures of Fermi gases

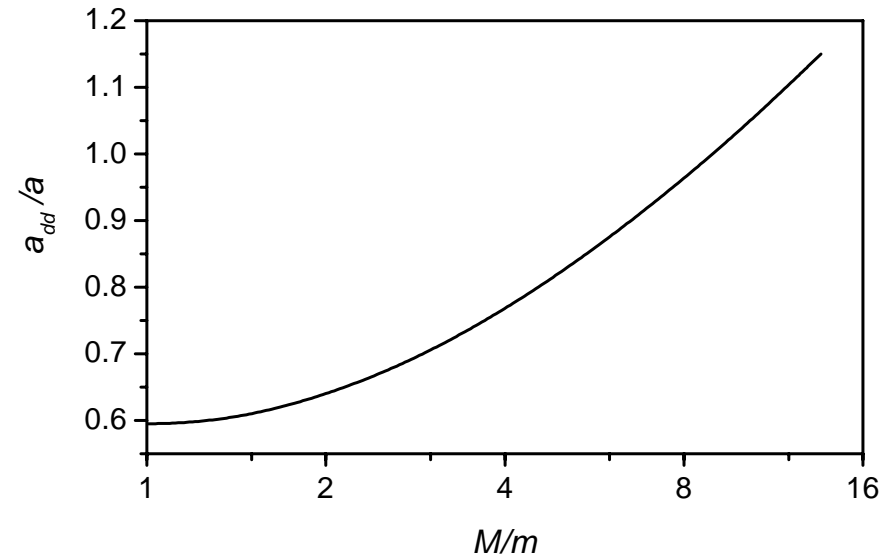
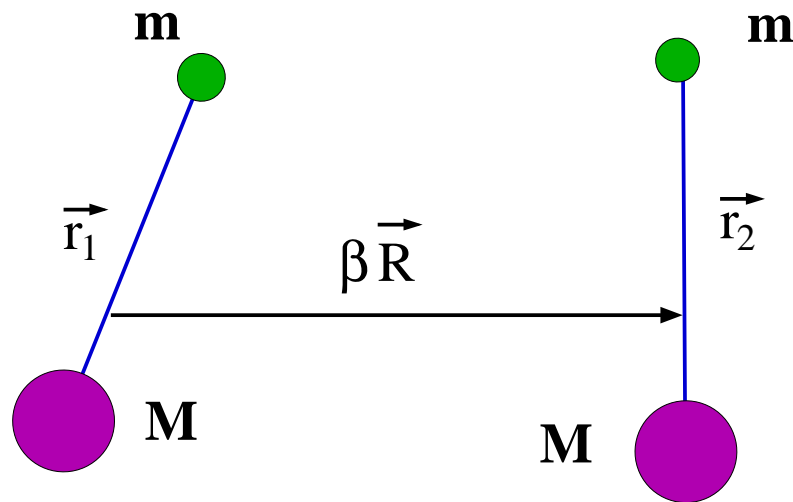
Molecules of different fermionic atoms ${}^6\text{Li}{}^{40}\text{K}$ ${}^6\text{Li}{}^{87}\text{Sr}$



What happens with collisional stability and molecular BEC?
Is there something else interesting ?

Molecule-molecule interaction

Interaction between the molecules (a_{dd}) Petrov et al 2005



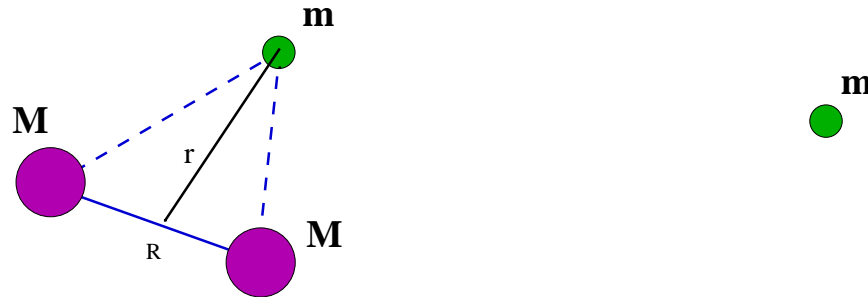
$$\beta = \sqrt{2Mm}/(M + m)$$

Nothing dramatic happens, but why I stop at $M/m = 13.6$?

Collisional relaxation. Effective potential

Exact solution for the dependence on a and M/m

$M \gg m \rightarrow$ Born-Oppenheimer picture



$r \ll a \rightarrow$ One bound state of a light atom with two fixed heavy ones

$$\psi(\vec{r}) \propto \left(\frac{\exp(-\lambda|\vec{r}-\vec{R}/2|/R)}{|\vec{r}-\vec{R}/2|} + \frac{\exp(-\lambda|\vec{r}+\vec{R}/2|/R)}{|\vec{r}+\vec{R}/2|} \right)$$

Bethe-Peierls boundary condition

$$|\vec{r} \pm \vec{R}/2| \rightarrow 0 \quad \psi \propto (1 - a/|\vec{r} \pm \vec{R}/2|)$$

$$\lambda \approx 0.567 \quad \varepsilon(R) = U_{eff}(R) = -\hbar^2 \lambda^2 / 2mR^2$$

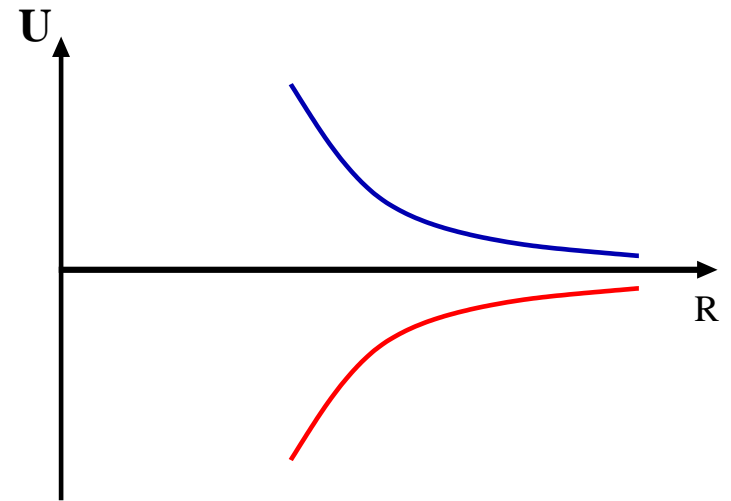
Mediated attractive potential $U_{eff} \approx -0.16\hbar^2/mR^2$

Collisional relaxation. Role of the Pauli principle

Pauli principle \Rightarrow Centrifugal potential $U_c = 2\hbar^2/MR^2$

Mediated attraction competes
with Pauli principle

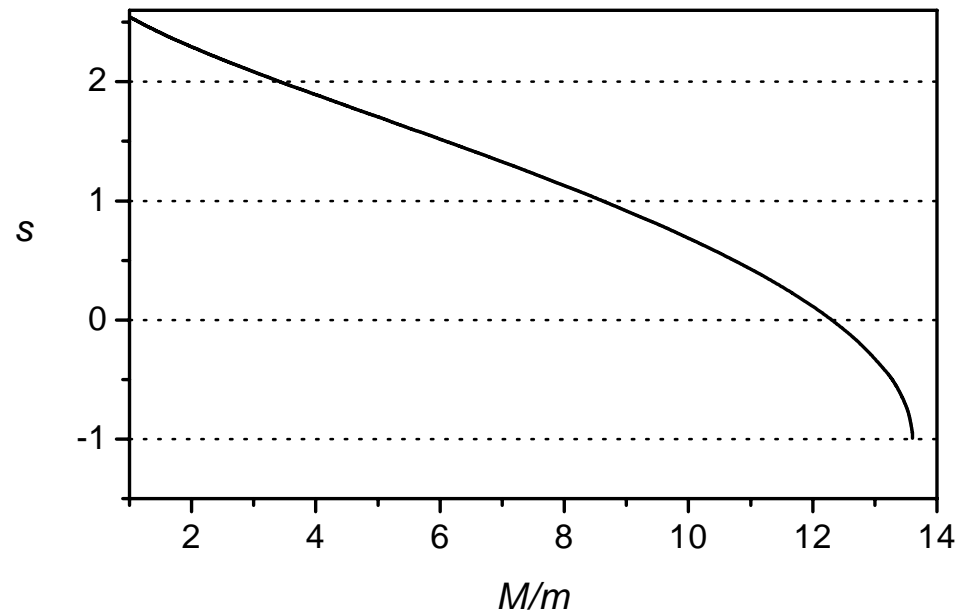
$$\alpha_{rel} \sim a^{-s}$$



$$\frac{M}{m} < 12.33 \quad \Rightarrow \quad U_{eff} + U_c > 0; \quad s > 0$$

α_{rel} decreases with increasing a , but
 s decreases with increasing M/m

Relaxation rate



$$\alpha_{rel} \sim \frac{1}{a^s}$$

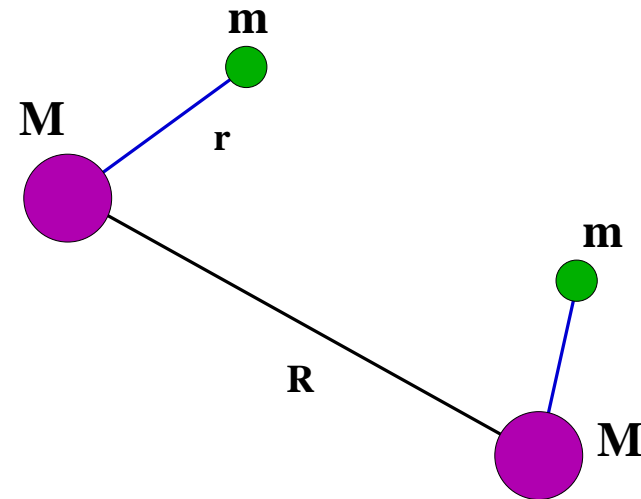
$M/m > 12.33 \rightarrow U_{eff} + U_c < 0, s < 0 \rightarrow \alpha_{rel}$ increases with **a**

$M/m = 13.6 \rightarrow s = -1 \rightarrow \alpha_{rel} \sim a$

$M/m > 13.6 \rightarrow$ **fall into center** **short-range physics**

Long-range repulsion between the molecules

Born-Oppenheimer
picture



$R > a \Rightarrow 2$ bound states of a light atom with two fixed heavy ones

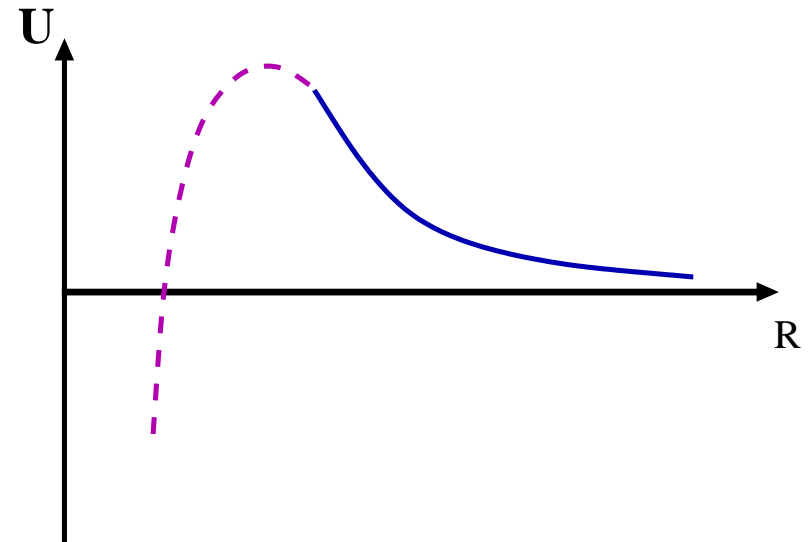
Suppression of relaxation ?

There is a suppression of relaxation for a large M/m

$$R \gg a$$

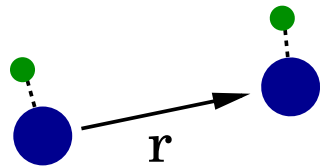
$$U_{eff}(R) \approx \left(\frac{\hbar^2}{2maR} \right) \exp(-2R/a)$$

$$P \sim \exp(-0.6\sqrt{M/m})$$



Fermionic molecules in Bose-Fermi mixtures

Heavy bosons and light fermions → long range repulsion



$$U(r) = \frac{\hbar^2}{2mar} \exp\left(-\frac{2r}{a}\right)$$

p-wave scattering → both elastic scattering and relaxation are suppressed

$$\alpha_{in} \sim (ka)^2 \frac{\hbar R_e}{M} P$$

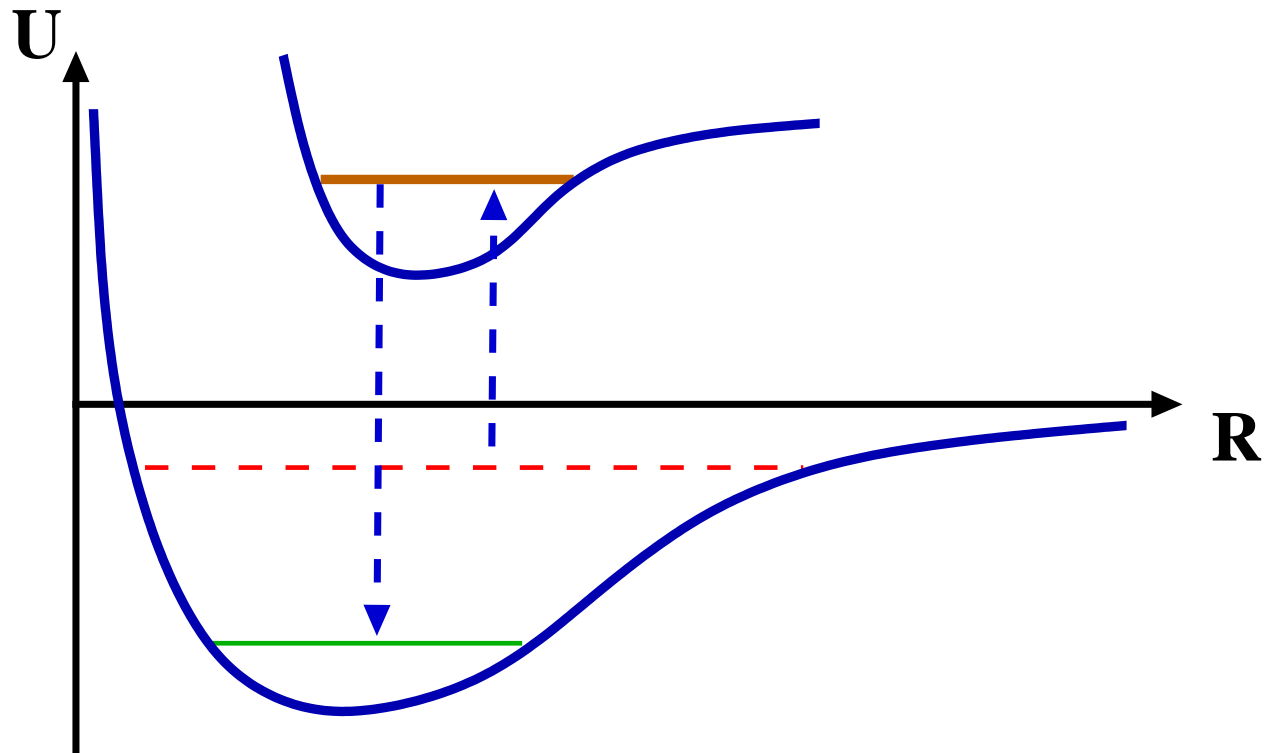
large life time even at high densities

$$\alpha_i \ll \alpha_{el} \rightarrow ?$$

Dipolar Fermi gas!

Ideas for future

Idea from Yalle studies of molecules of bosonic atoms



Replace bosons by fermions

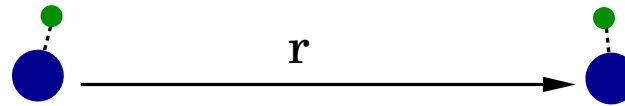
Large n

Large τ



Dipolar gas

It is a gas?



$$U(r) = \frac{\hbar^2}{2ma^2(r/a)} \exp\left(-\frac{2r}{a}\right)$$

$$K = -\frac{\hbar^2}{2Ma^2} \Delta_{(r/a)} = -\frac{m}{M} \frac{\hbar^2}{2ma^2} \Delta_{(r/a)}$$

$$H = \sum_i K(r_i) + \frac{1}{2} \sum_{i \neq j} U(r_{ij}) \Rightarrow \frac{\hbar^2}{2ma^2} \sum_i \tilde{H}\left(\frac{r_i}{a}; \frac{M}{m}\right)$$

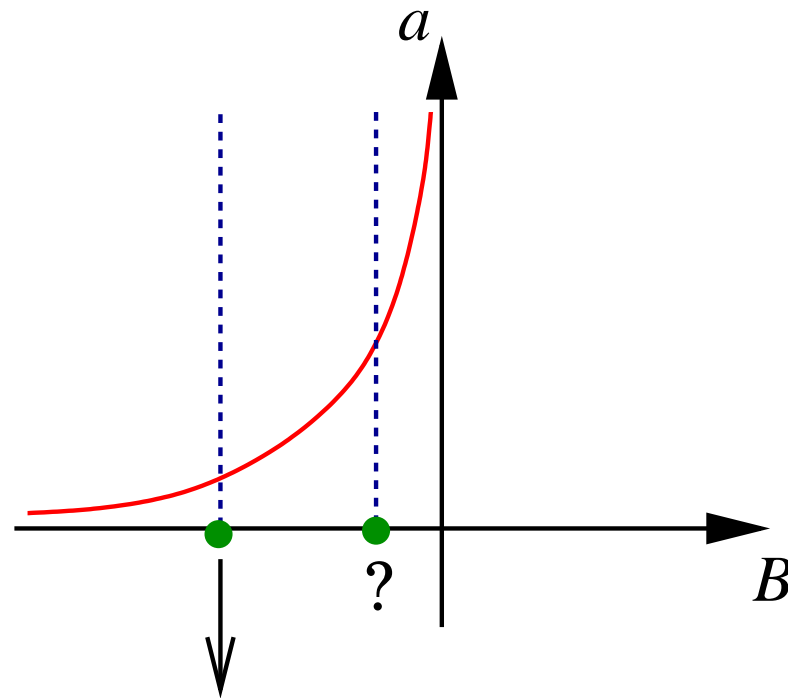
$$\text{Large } \frac{M}{m} \Rightarrow \text{small } \frac{K}{U}$$

$$\rightarrow \text{Wigner crystal for } \frac{M}{m} > \left(\frac{M}{m}\right)_c$$

Quantum transitions

$$\frac{M}{m} > \left(\frac{M}{m} \right)_c \quad \text{and } n \text{ fixed}$$

Increase a



depends on $\frac{M}{m}$ but always $na^3 \ll 1$

Crystalline phase

$$\frac{M}{m} \sim 100$$

How to obtain the crystalline phase?

Optical lattice for heavy fermions \Rightarrow

Increase of M/m

Formation of a superlattice

Conclusions

- Remarkable physics of weakly bound molecules in cold Fermi gases
- Novel physics of molecular collisional stability in mixtures of Fermi gases
- Possibilities to create new macroscopic quantum systems