Lecture II

Lecture II: mesoscopic light ...

- introduction
- coherent back scattering
- light localization
- new directions

Introduction ...

- **introduction**
- **coherent back scattering**
- **light localization**
- **new directions**

Photonic matter

A dielectric is a material with a dielectric constant

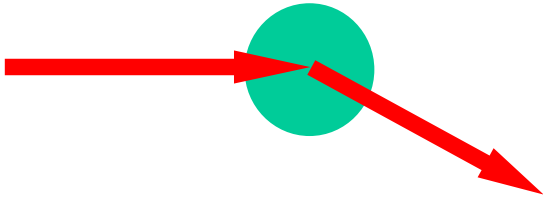$$\varepsilon$$

that depends on frequency

$$\varepsilon(\omega)$$

A *photonic* material has a dielectric constant that depends on position
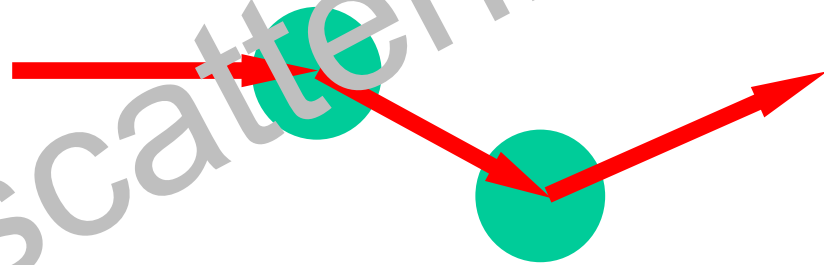
$$\varepsilon(\omega,r)$$

and varies in space on a length scale of the order of the wavelength of light

Multiple scattering

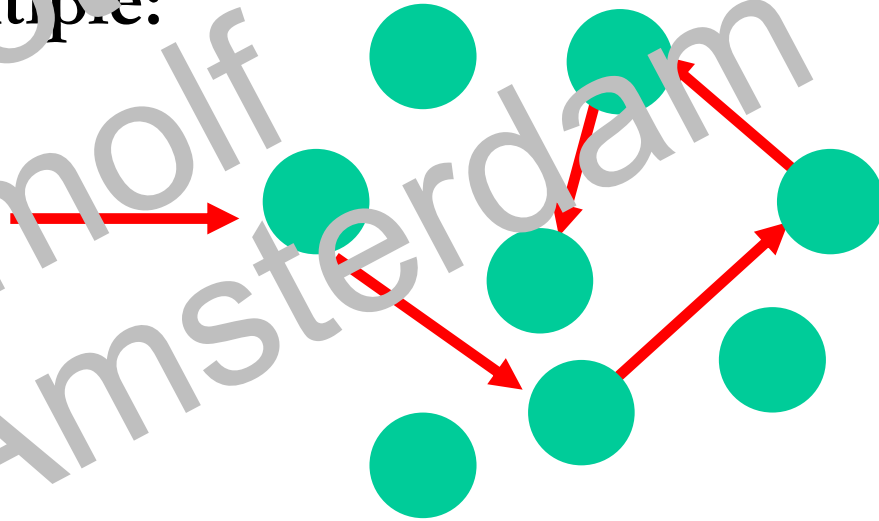**single:**

double:

multiple:

Diverging scattering series

We expect new phenomena when the polarization (polarizability density) becomes of order one

$$\frac{N}{V}\alpha = O(1)$$

- dielectric catastrophy
- light localization
- photonic bandgaps

Is it any interesting?

- diverging phase velocity
- vanishing phase velocity
- diverging group velocity
- vanishing group velocity
- vanishing density of states (photonic bandgap)
- diverging density of states (van-Hove-singularity)
- vanishing diffusion constant
- vanishing mean-free path
- vanishing energy velocity
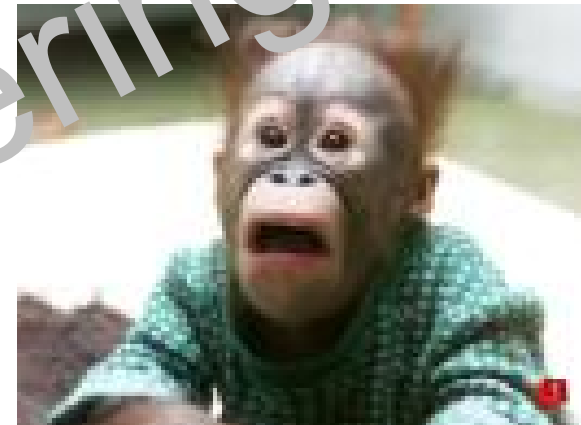
Goal: reduce mean free path to zero

mean free path

ℓ
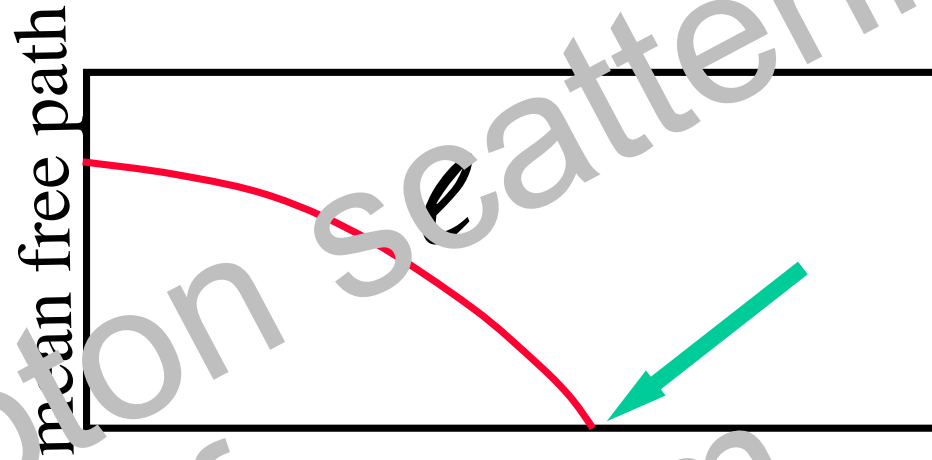
degree of disorder

$k\ell_s \cong$ order(1)

Material aggregation state

State of matter

powders
nanomaterials
supermolecular structures
colloids
sponges
liquid crystals
ultra-cold gasses

Symmetry

non-reciprocal
gyrotropic
anisotropic
bi-anisotropic

Gain

dye + colloids
ground laser crystals

Materials

- high density of scatterers
- large contrast
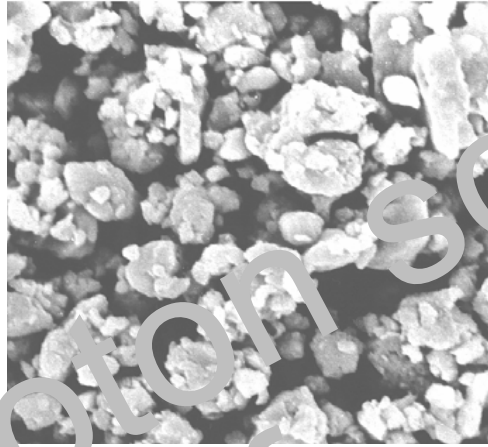- resonant scattering   (size scatterers $\sim \lambda$)

_____

index in the visible:

water   1.3   glass   1.5

diamond   2.4 TiO_2 2.7

GaP   **3.3**

index in the infrared:

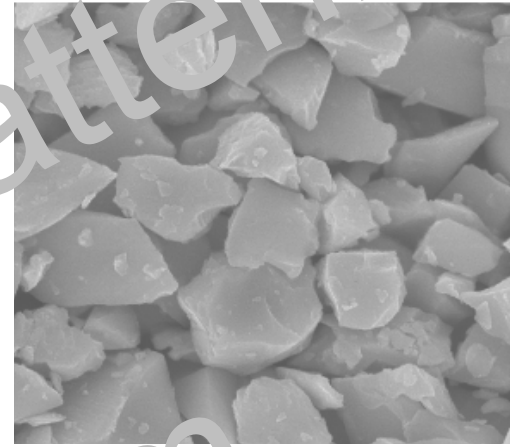GaAs   3.5   (near ir)

Ge   4.1   Si   3.5

Example material: powders

1 66 μm
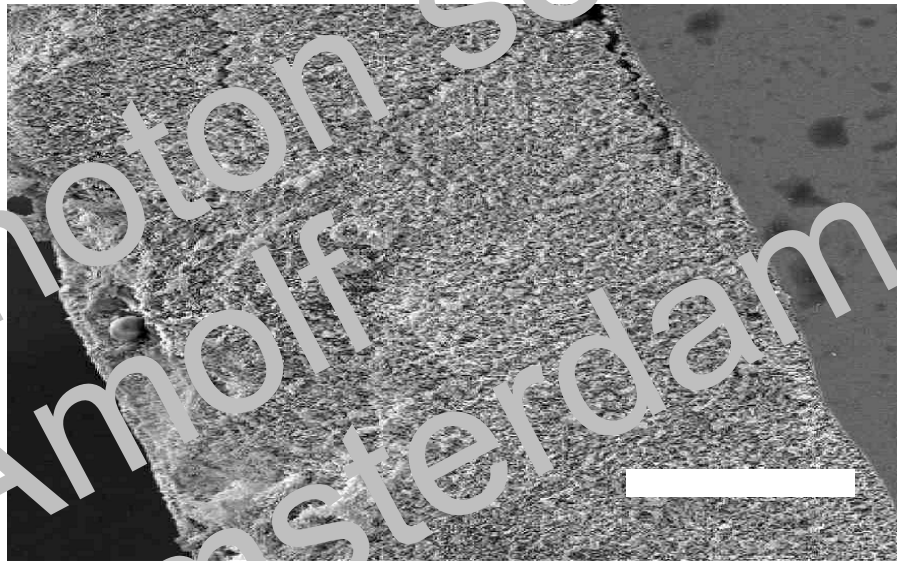**Si**
(a)

1.56 μm
Ge
(b)

Gómez Rivas, Sprik, Soukoulis, Busch, AL, Europhys. Lett. 1999

Example materials: spunge

GaP elecrochemically etched

20 μ

Schuurmans, Vanmaekelbergh, Van de Lagemaat, AL, Science 1999

Is diffusion important?

In many languages diffuse also means vague

We often use the word diffusive with a negative connotation

I think the diffusion laws are next to Newton laws the most important laws in science
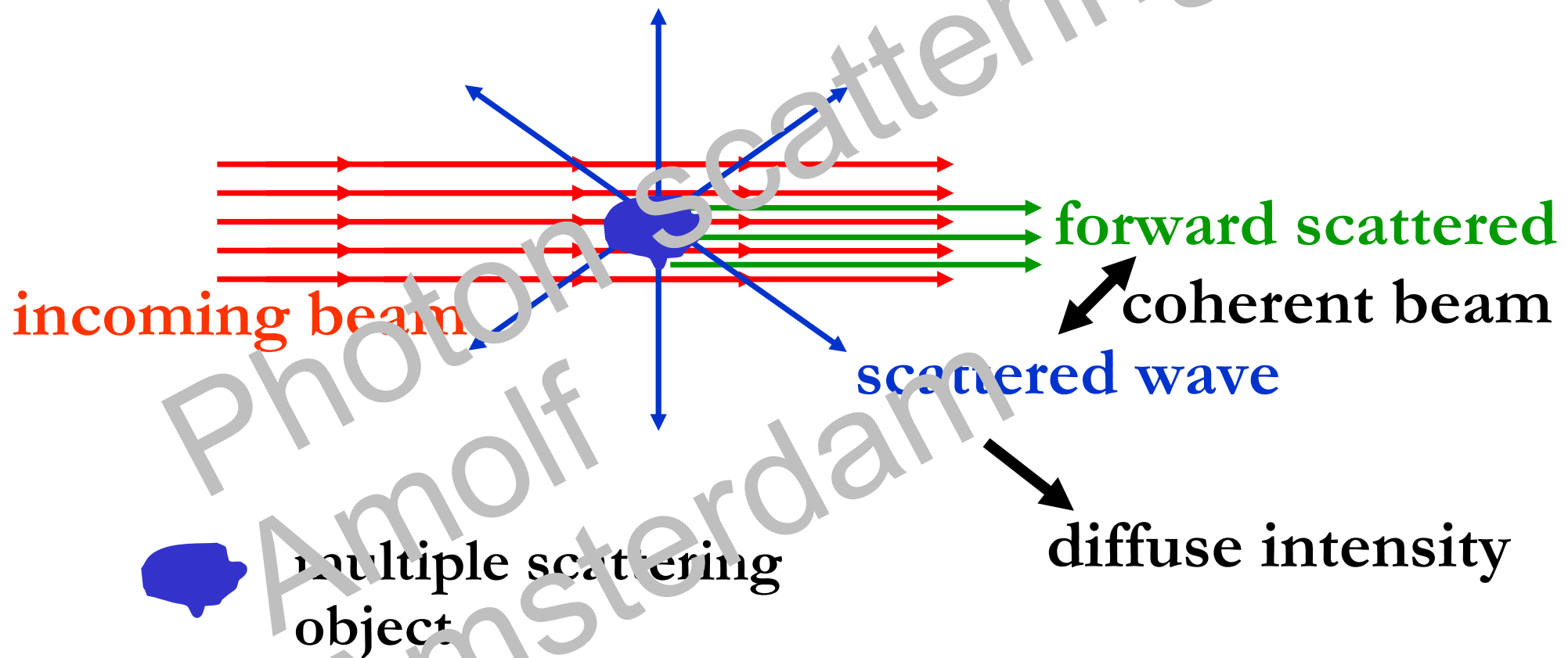
Generality of diffusion

particle diffusion
momentum diffusion
energy diffusion
...

coins
languages

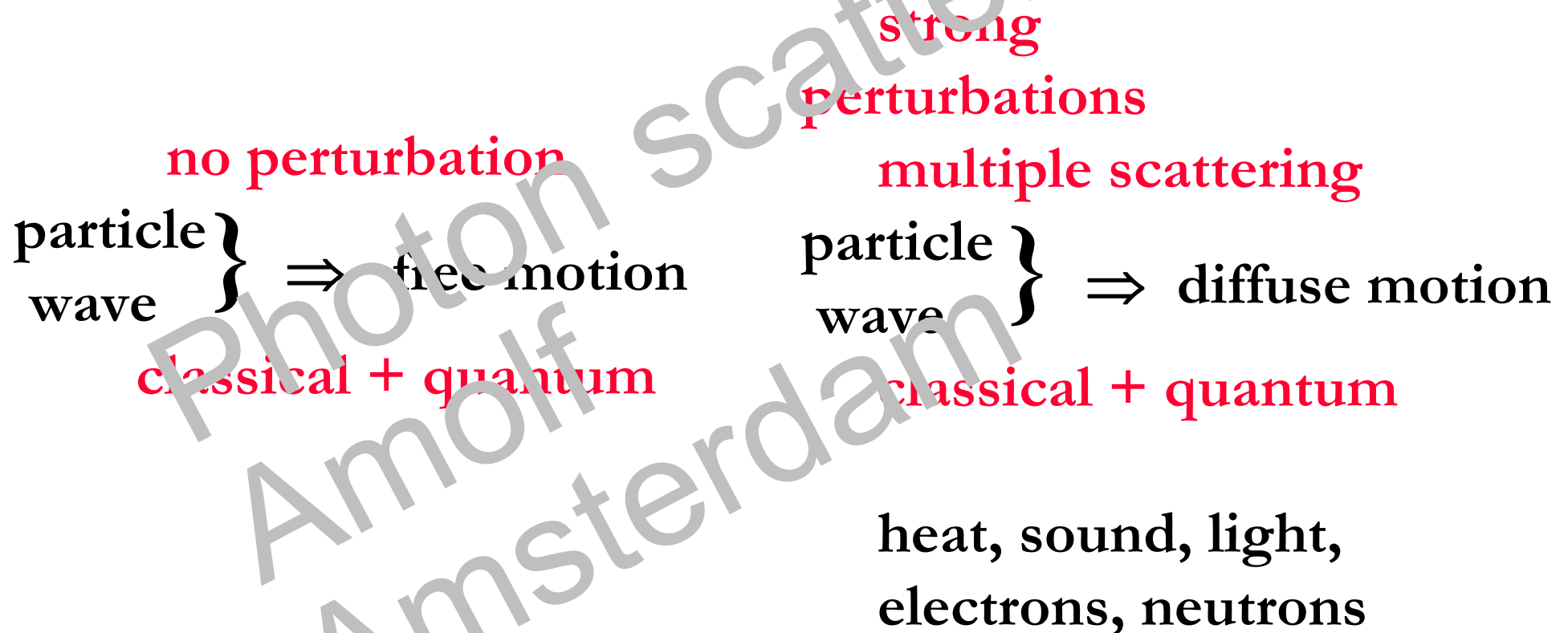*wave diffusion:*

surface waves
sound
light
electron(s)
plasmons
magnons
elastic waves
seismic waves
... many more

Coherent beam

incoming beam

forward scattered

coherent beam

scattered wave

diffuse intensity

multiple scattering object

From scattering to transport

no perturbation

particle $\Big\}$ $\Rightarrow$ free motion
wave

classical + quantum

strong perturbations

multiple scattering

particle $\Big\}$ $\Rightarrow$ diffuse motion
wave

classical + quantum

heat, sound, light, electrons, neutrons

Key diffusion parameters

- mean free path $\quad \ell \quad\quad = \dfrac{1}{\rho\sigma}$

- diffusion coefficients $\quad$ D

- system size $\quad\quad\quad$ L

- conservation law violater $\quad \ell_{\text{abs}}$

Diffusion condition

- global conservation law
  - number of particles
  - total momentum
  - intensity
  - ...

$$\frac{d}{dt}\int A(\mathbf{r})d\mathbf{r} = 0$$

- complicated interactions

imbalance must be transported over space

mild violation of conservation allowed

Transport theory

wave impinges on complex object

Boltzmann theory neglects interference

resulting equation is a balance equation

$$\frac{dI(\mathbf{r},\hat{\mathbf{s}})}{ds} = -\frac{I(\mathbf{r},\hat{\mathbf{s}})}{\ell} + \frac{1}{\ell}\int_{4\pi} p(\hat{\mathbf{s}},\hat{\mathbf{s}}')I(\mathbf{r},\hat{\mathbf{s}}')d\hat{\mathbf{s}}'$$

change   =   loss   +   gain

$I(\mathbf{r},\hat{\mathbf{s}})$ intensity at $\mathbf{r}$ in direction $\hat{\mathbf{s}}$
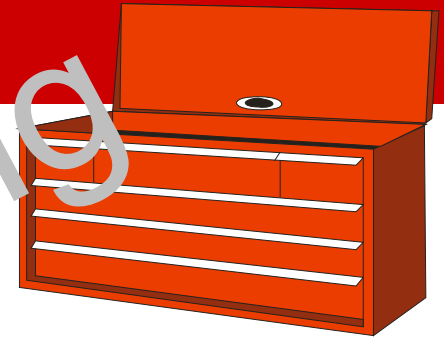
Diffusion theory

long-lenght limit of transport theory
gives diffusion equation

$$\frac{\partial I}{\partial t} = D\left(\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} + \frac{\partial^2 I}{\partial z^2}\right)$$ **dynamic form**

$$0 = D\left(\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} + \frac{\partial^2 I}{\partial z^2}\right)$$ **stationary form**

- mean-free path shows in boundary condition
- radiative transport theory slightly better than diffusion theory
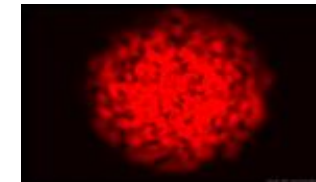
Experimentalist's toolbox

- **total (angularly integrated) transmission and reflection**

- **angularly resolved transmission and reflection**

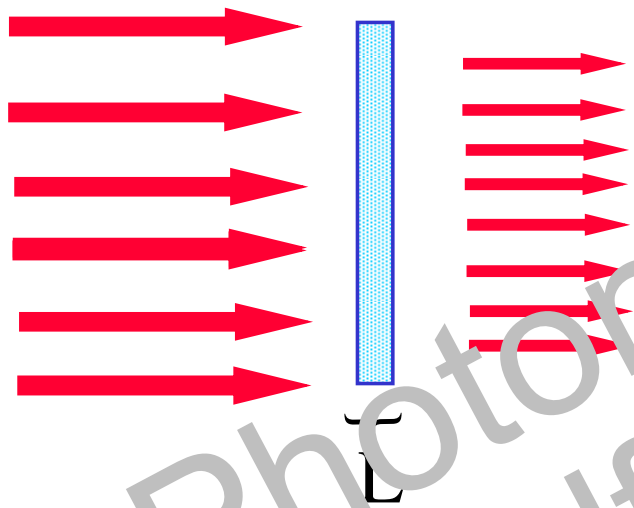- **intensity correlations and fluctuations (speckle)**

- **interferometry (phase and amplitude)**

as a function of sample thickness, wavelength, polarization, time, pulse duration, ...
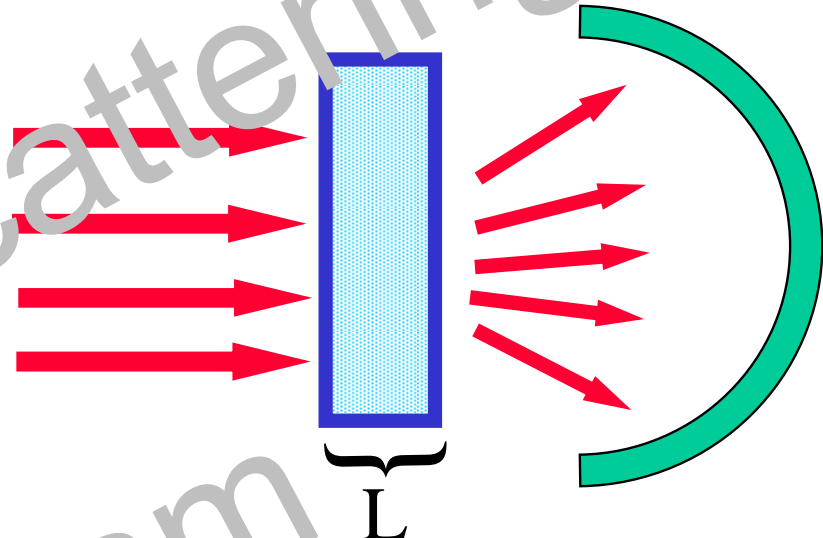
Stationary transmission

coherent stationary | incoherent stationary

L

L

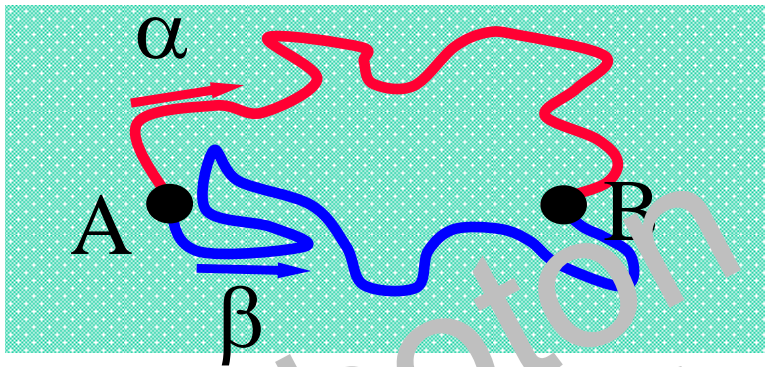not transport

transport

$$T = \exp(-\frac{L}{\ell})$$

$$T = \frac{\ell}{L}$$

Coherent backscattering ...

✓ **introduction**

● **coherent back scattering**
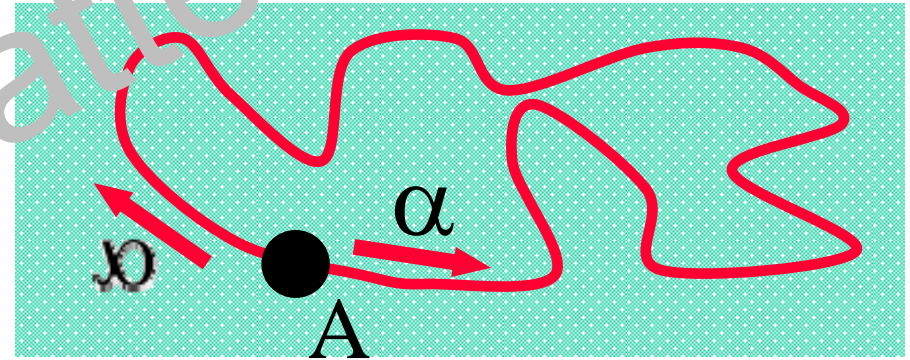
● **light localization**

● **new directions**

Weak localization

arriving probability

returning probability

$$P = (\alpha + \beta + ...)^2 =$$
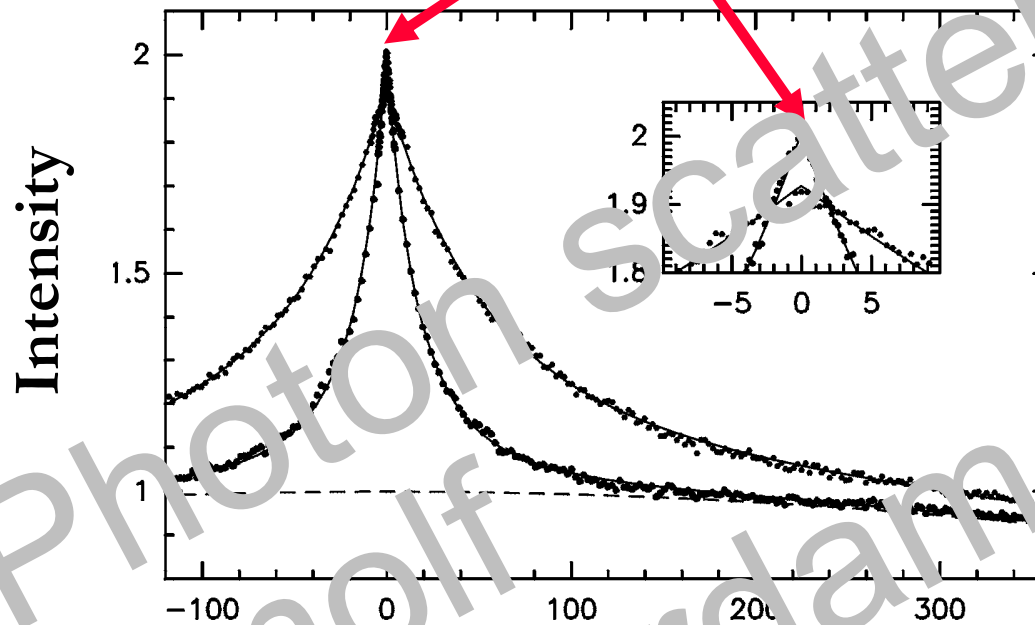
$$= \alpha^2 + \beta^2 + cross - terms$$

$$P = (\alpha + \alpha)^2$$

$$= \alpha^2 + \alpha^2 + 2\alpha^2$$

$$= 4\alpha^2 = 2 \times 2\alpha^2$$

Coherent backscattering
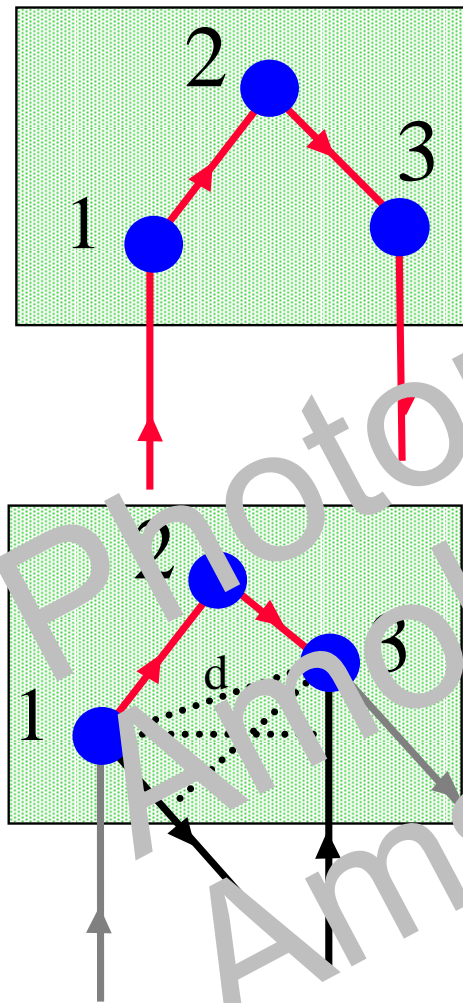
$$\Delta L = 2\text{d}\cos\{\tfrac{1}{2}(2\theta_i + \theta_s)\}\sin(\tfrac{1}{2}\theta_s)$$
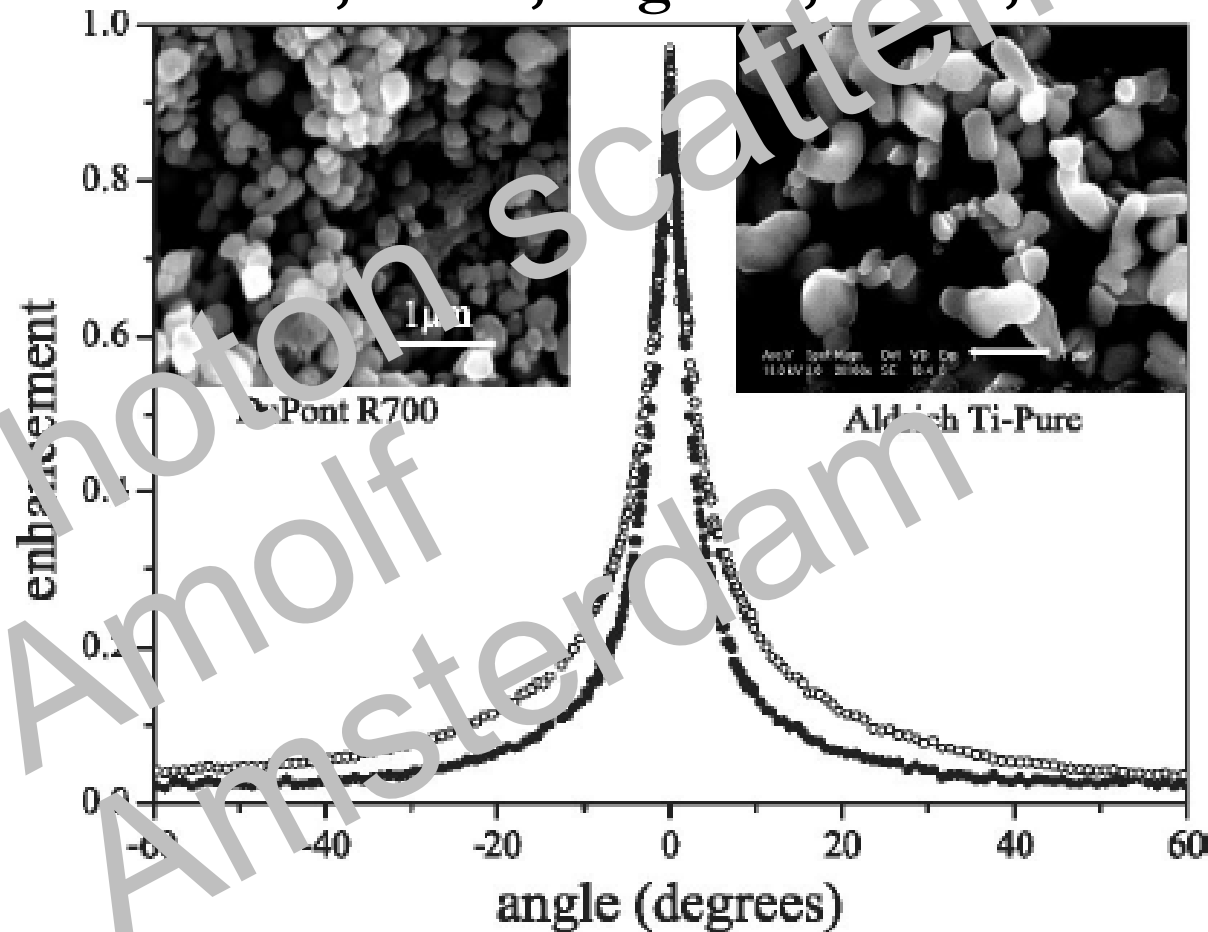
CBS examples

Scattering angle (mrad, 0 = exactly backscattering)

Narrow cone: $BaSO_4$: $k_{med}\, \ell = 22.6$

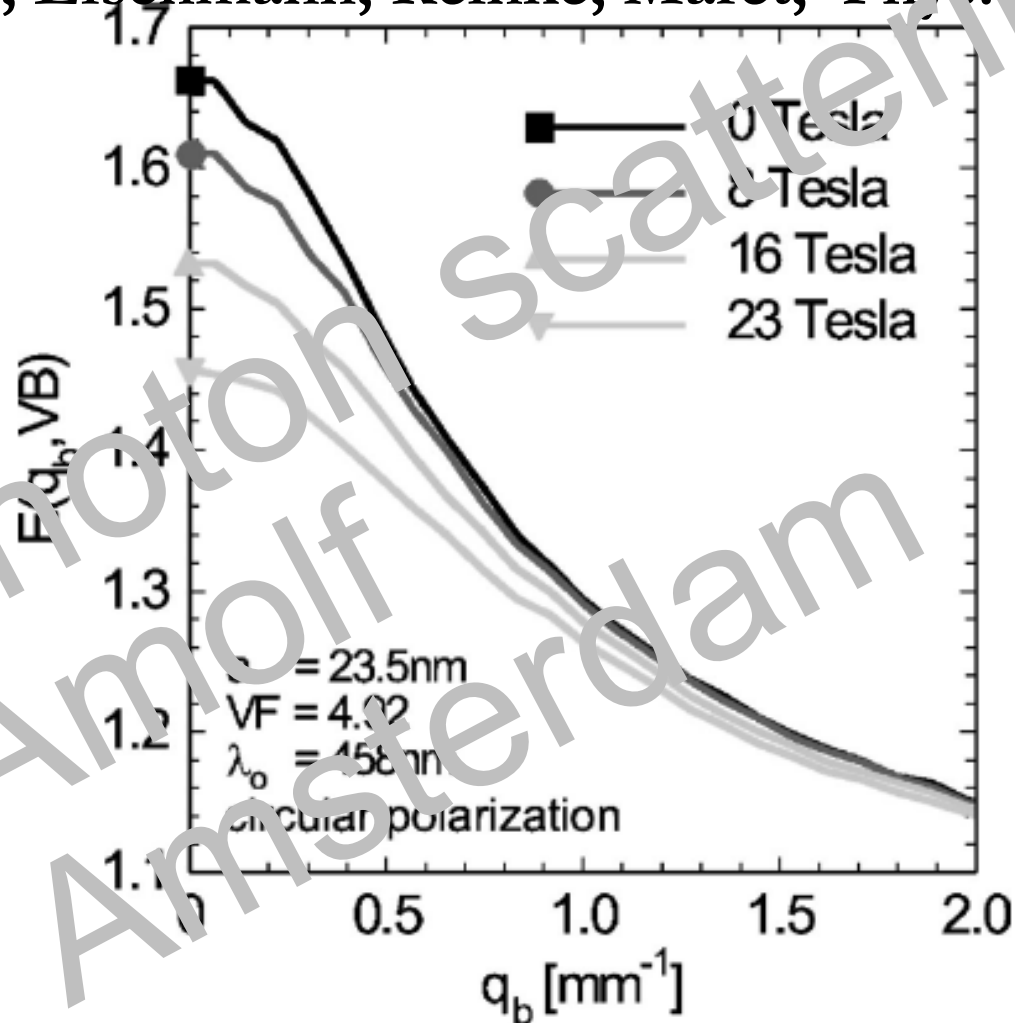Broad cone: TiO_2 : $k_{med}\, \ell = 5.8$

CBS examples (continued)

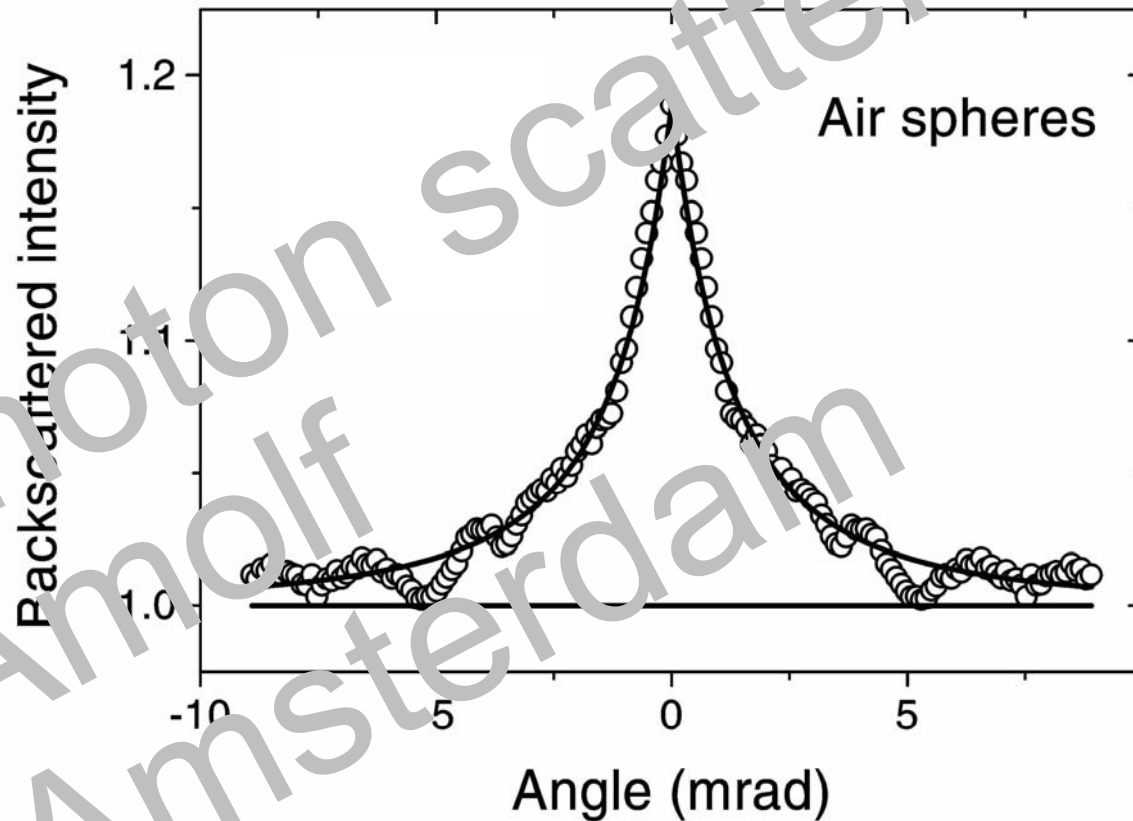Störzer, Gross, Aegerter, Maret, PRL 2006

Magnetic field destroys it

Lenke, Eisenmann, Reinke, Maret, Phys. Rev. E 2002

Disorder in photonic bandgap crystal

air-sphere crystal (wavelength 460 nm)

Koenderink, Megens, Van Soest, Vos, AL, Phys. Lett. A. 2000

Light localization ...

- ✓ **introduction**
- ✓ **coherent back scattering**
- **light localization**
- **new directions**

Weak to strong localization

enhanced backscattering

$\Rightarrow$ reduces forward propagation

$\Rightarrow$ reduction of mean free path

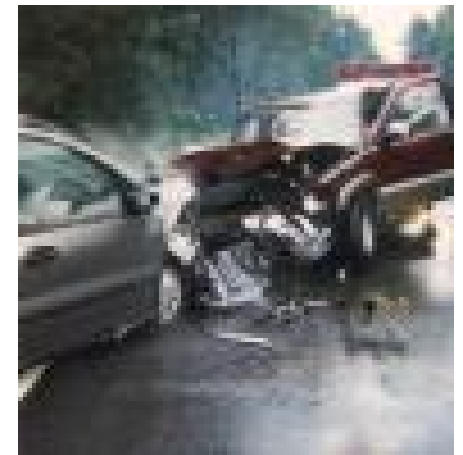$\Rightarrow$ reduction of diffusion   constant

One-liners about localization

- break-down of transport theory
- vanishing of diffusion coefficient
- absence of extended modes

Complications:

- not every breakdown is Anderson localization
- we have a limited under-standing of transport theory in high density systems

Generality of concept of localization

- surface waves
- sound
- light (visible, microwaves etc.)
- electron(s)
- plasmons
- magnons
- elastic waves
- seismic waves
- ...

purple bullets: classical
green bullets: quantum

Scientific merits classical localization

the electron people are very good in
   making their own case

we now understand multiple scattering
   and transport much better
      fluctuations (speckle), I, amplitude, phase
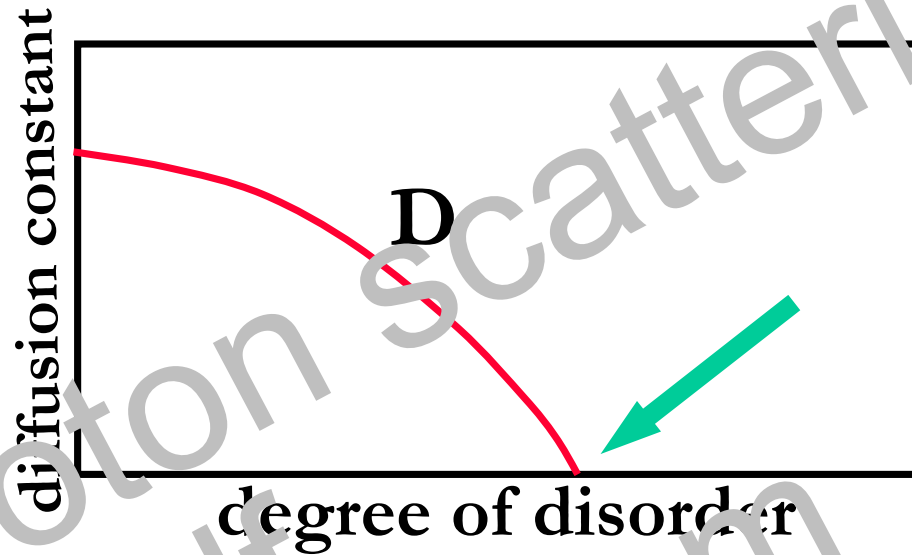
random lasers (scattering + gain)

Is it genuine?

Is absorption mistaken for localization?

Is impurity luminescense mistaken
for localization?

Is a single localized mode mistaken
for localization?

Strong localization

two length scales: ℓ_{scat} and λ

$\ell_{scat} < \lambda/2\pi$ extreme condition

$k\ell_{scat} < 1$

Dimensionality and transport
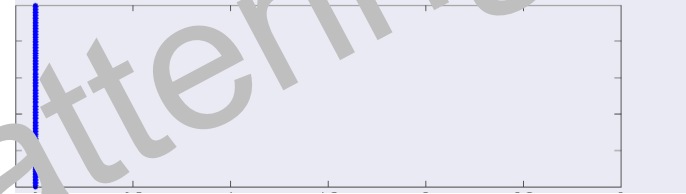

1D

2D

3D

Lower dimensionality slows down

long-time behavior

$$P(t \to \infty) \propto \frac{1}{t^{d/2}}$$

$$P(t \to \infty) \propto \frac{1}{t^{1/2}} \quad \int P(t)\,dt = \infty \qquad \textbf{1D}$$

$$P(t \to \infty) \propto \frac{1}{t^{3/2}} \quad \int P(t)\,dt = \text{finite} \qquad \textbf{3D}$$

lower dimensions are slower

Localization and dimensionality

if system is of infinite size:

- 🟢 in 1D always localization
- 🟢 in 2D always localization

in practice $L > \ell_{loc}$

- 🟢 in 3D critical amount of disorder

Volhardt and Wölfle

Mean-field-type theory

$$\frac{1}{D} = \frac{1}{D_B} + DOS \int\limits_{q_{min}}^{q_{max}} \frac{1}{Dq^2} d\mathbf{q}$$ **general dimensions**

$$\frac{1}{D} = \frac{1}{D_B} + DOS \frac{L}{D}$$ **one dimension**

$$D = D_B(1 - DOS \times L)$$

Extension of localization theory

B. L. Altshuler, A.G. Aronov, and B. Z. Spivak, JETP Lett. 33, 94 (1981)
D.Yu. Sharvin and Yu.V. Sharvin, JETP Lett. 34, 272 (1981).

$$\frac{1}{D(\Omega, \mathbf{r})} = \frac{1}{D_B} + \frac{C_\Omega(\mathbf{r}, \mathbf{r})}{\pi v_E \rho(\omega)}$$
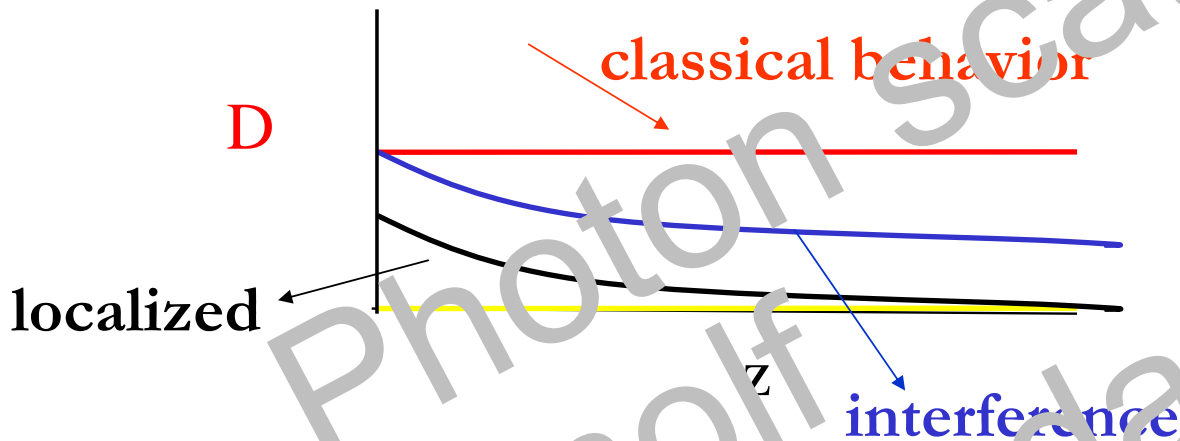
realistic geometry for finding
size dependence
sphere D(r)
slab   D(z)

Inhomogeneous localization

Spatial dependence of mean free path

classical behavior

D

localized

z

interference

Van Tiggelen, Wiersma, AL, PRL (2000)
Skipetrov and Van Tiggelen PRL (2004) dynamic 1D/3D

Other theories

- scaling theory g(L) of gang of four

- numerical simulations
  - systems are always too small
  - what to look for?

- field theories

Outstanding problems in theory

- separation of interference from
    non-interference is questionable
- realistic finite size theory
- critical exponents
- role of absorption
- dynamics
- beyond total transmission
- averaging over disorder
- mathematical definition lacking
- ...

How to observe localization?

$$\ell \implies \ell(L)$$

$$D \implies D(L)$$

Experiments include:
- total transmission (L)
- angular transmission
- pulsed transmission
- speckle correlation (θ, ω, t, phase)
- statistics

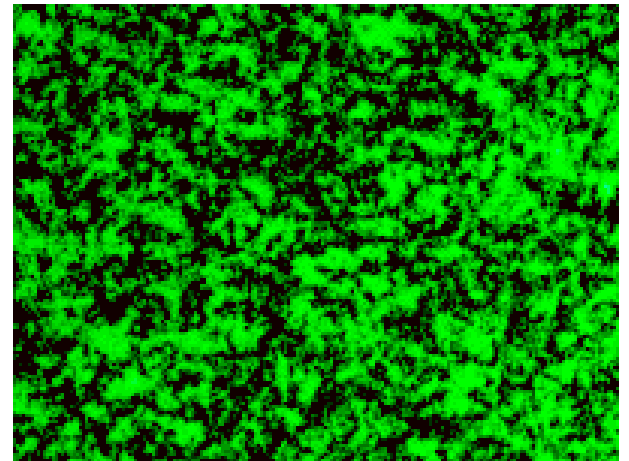# *Parameter space*

- **material**
  form, size of scatterers
- **wavelength**
- **dimensionality**
- **symmetry underlying structure:**
  random, lattice, ..

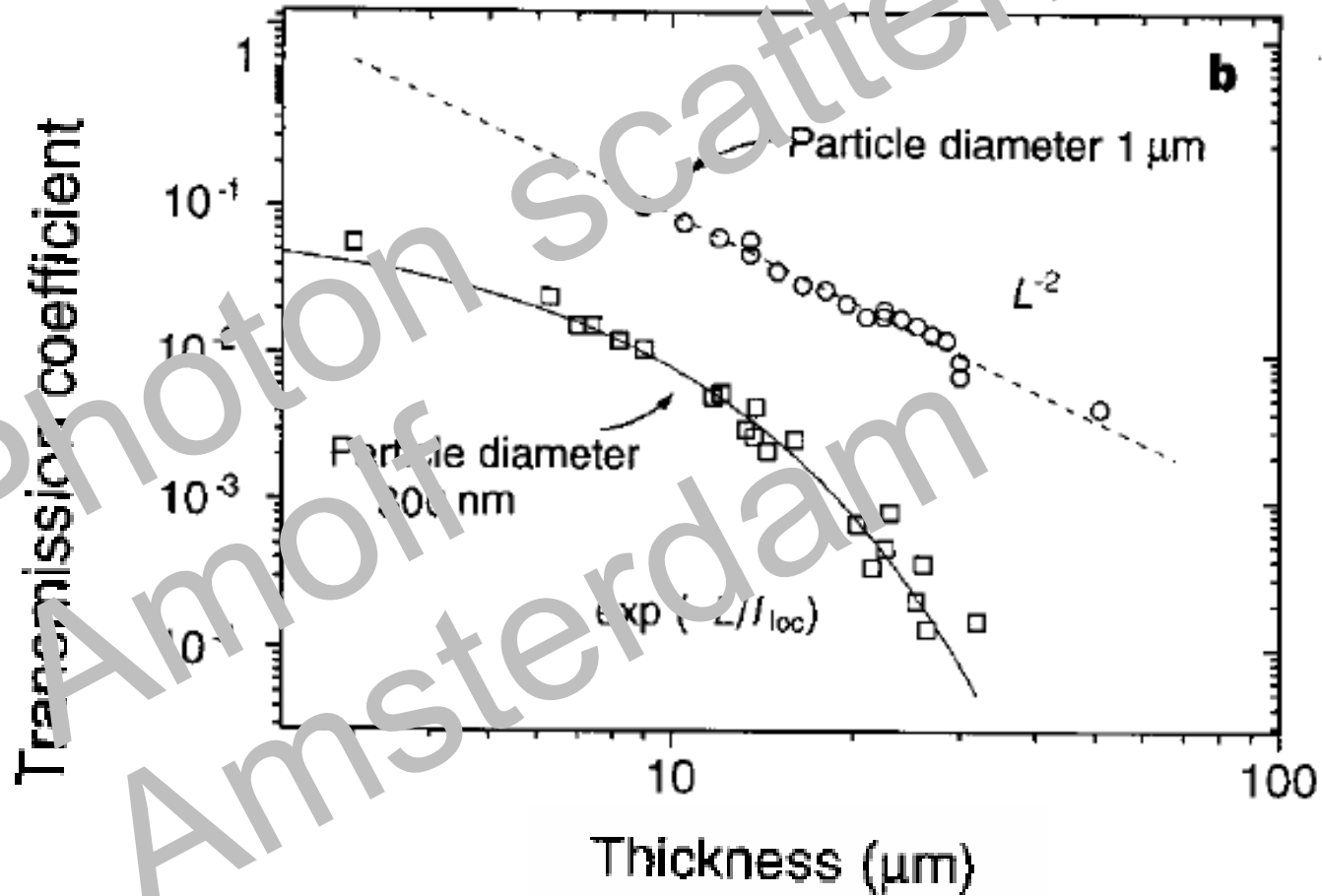 status: $k\ell_{\text{scat}} \approx 3\text{-}4$

Localization effects

- coherent backscattering (industry)
- resonance delay
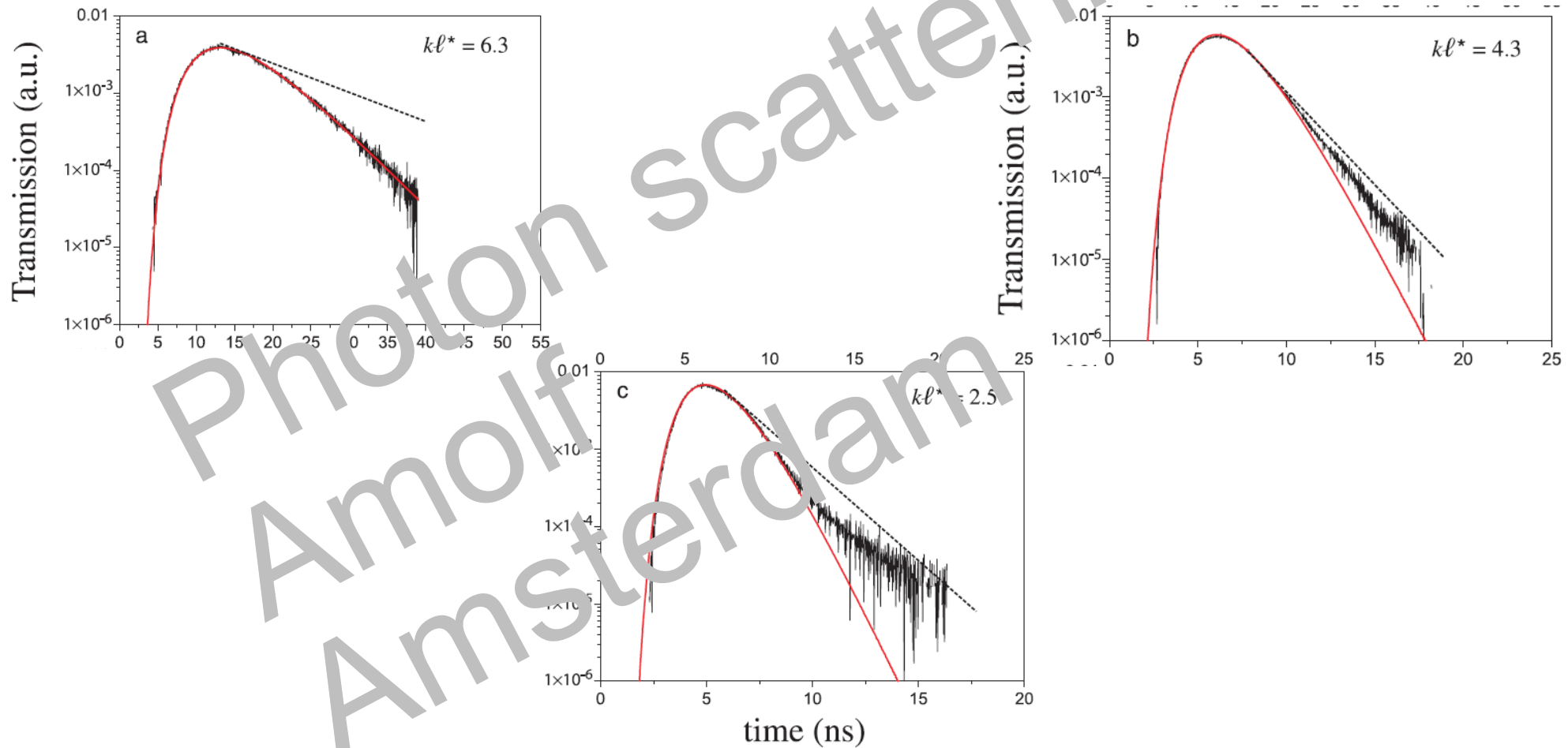- long-range speckle (UCF)
- phase statistics

In GaAs (infrared)
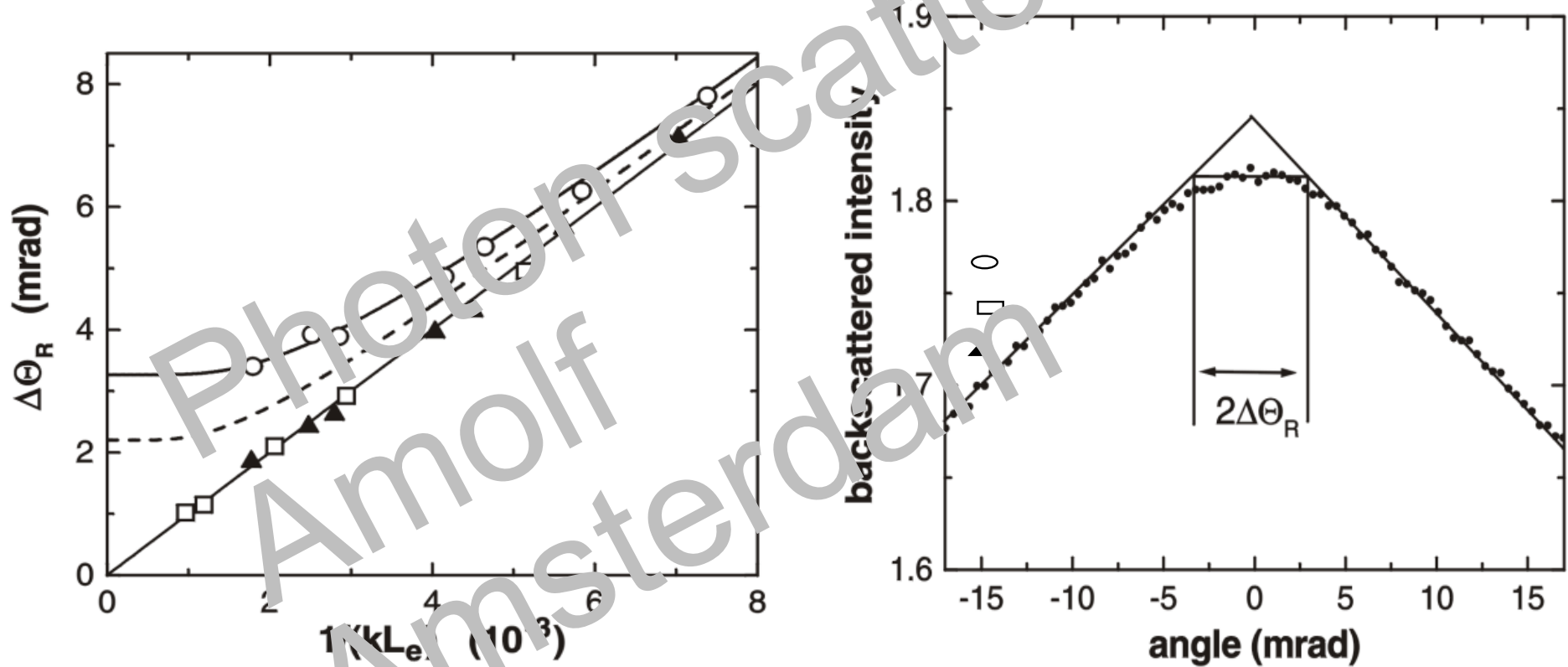
Wiersma, Bartolini, A.L. , Righiri, Nature 1997

Long-time tails

Störzer, Gross, Aegerter, Georg Maret, PRL 2006
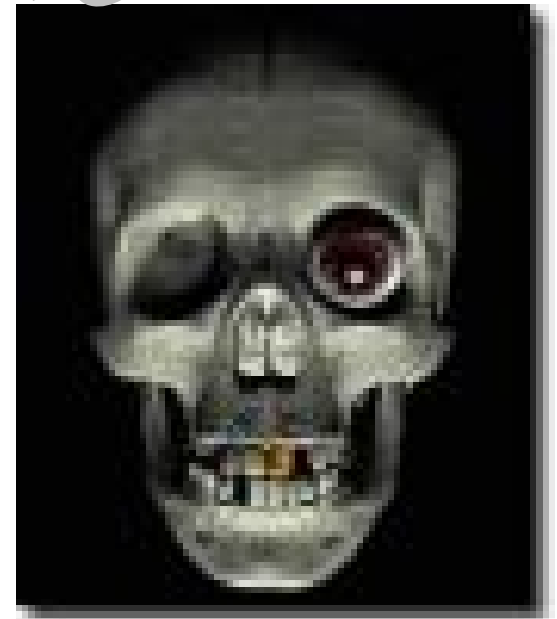
Critical backscattering

Schuurmans, Megens, Vanmaekelbergh, AL PRL 1999

Absorption

absorption is a real killer

- it kills long light paths
- causes exponential decay (L)
- localization is characterized by exponential decay (L)

experimentalists do everything they can to minimize absorption (typical 1 out of a million) but often not enough
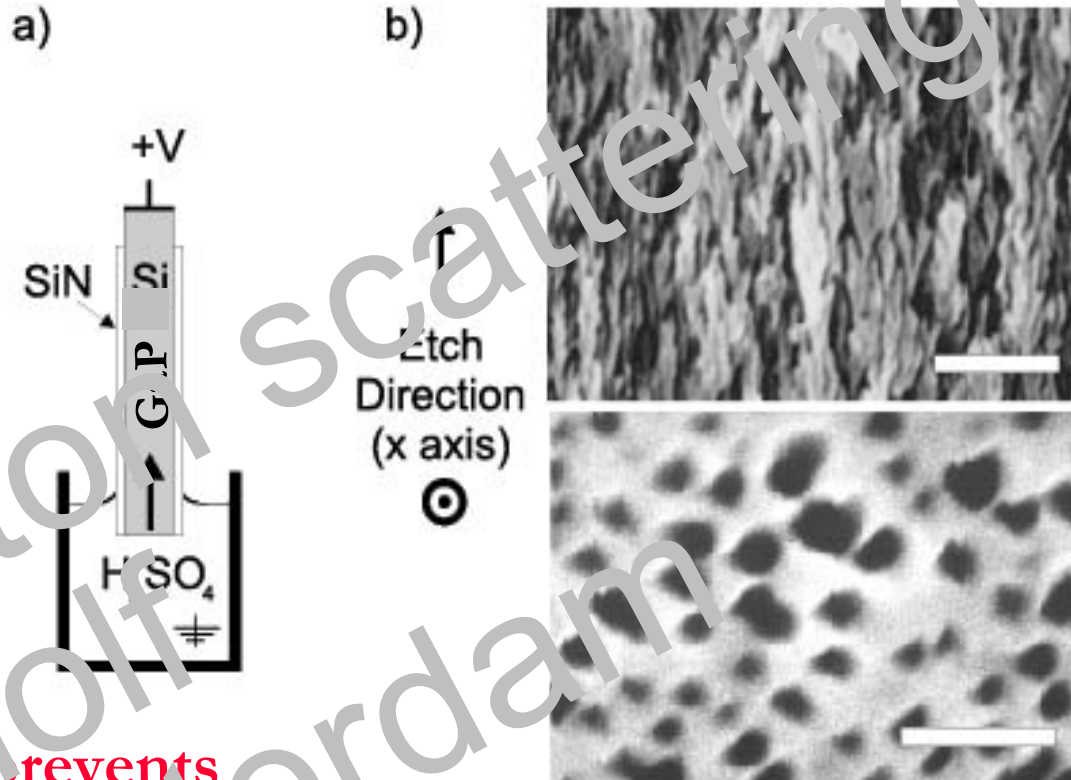
New directions ...

✓ **introduction**

✓ **coherent back scattering**

✓ **light localization**

● **new directions**

Anisotropy

Localization in lower dimensionality

is easier to obtain

Anisotropy might be a way

Anisotropic diffusion

a)

+V

SiN   Si

G.P

H₂SO₄

b)

Etch
Direction
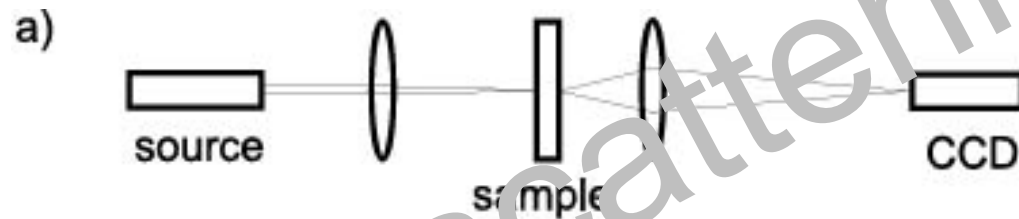(x axis)

Silicon nitride layer prevents etching at the polished surface of the wafer, pores grow from the bottom edge up.

Both parallel (top, scale bar=1 μm) and perpendicular (bottom, scale bar=300 nm) cross sections with respect to the etch direction
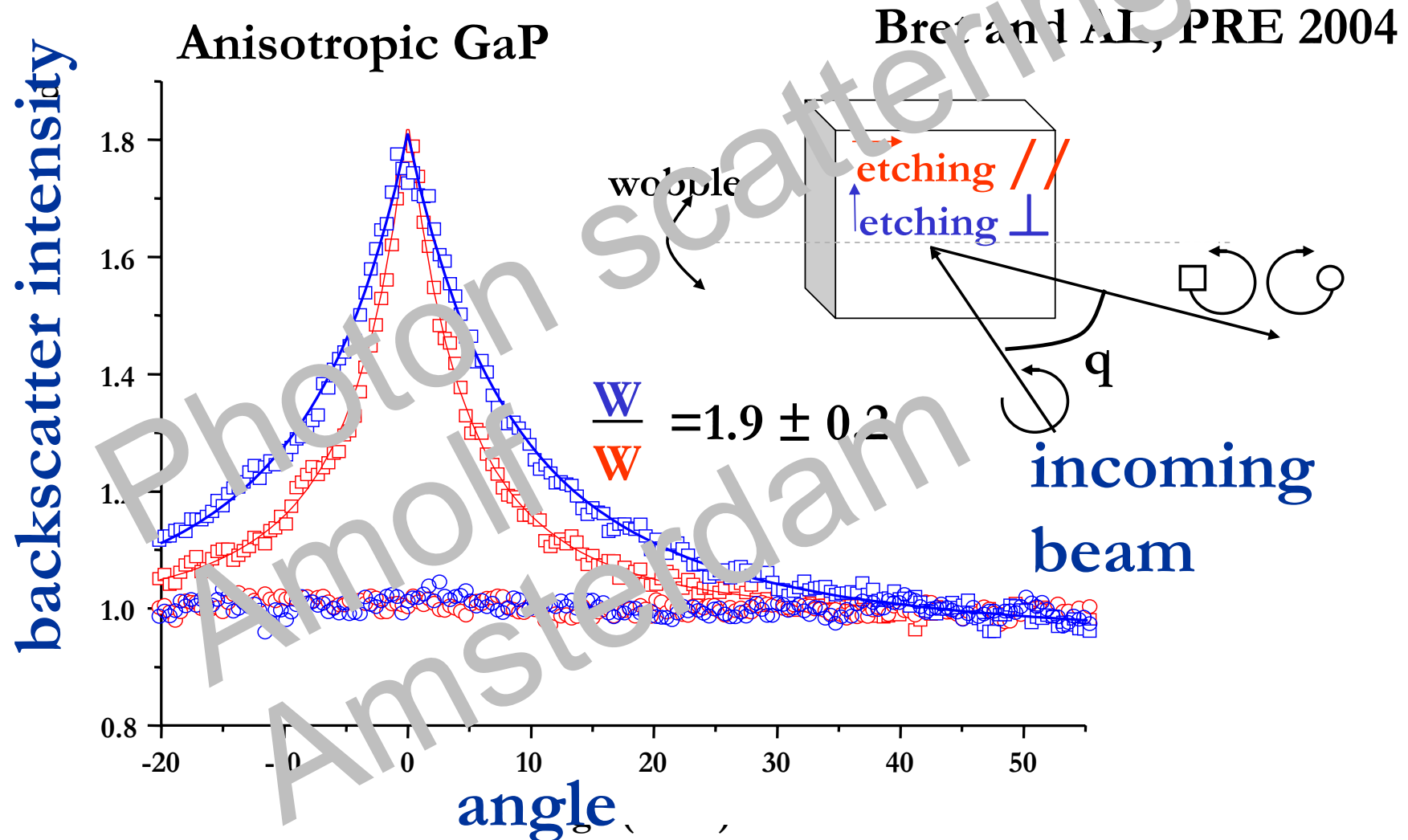
Asymmetric spot

a)

source   sample   CCD

b)

y
x

$$\frac{D_{xx}}{D_{yy}} = 4.0$$

P. M. Johnson, B. P. J. Bret, J. Gómez Rivas, J.J. Kelly, and A. L. PRL 89 (2002)

More anisotropic CBS

Anisotropic GaP

$$\frac{W}{W} = 1.9 \pm 0.2$$

wobble

etching //

etching ⊥

q

incoming beam

backscatter intensity

angle

Photon scattering
Amolf
Amsterdam

Transverse Localization of Light

Hans De Raedt, Ad Lagendijk, and Pedro de Vries

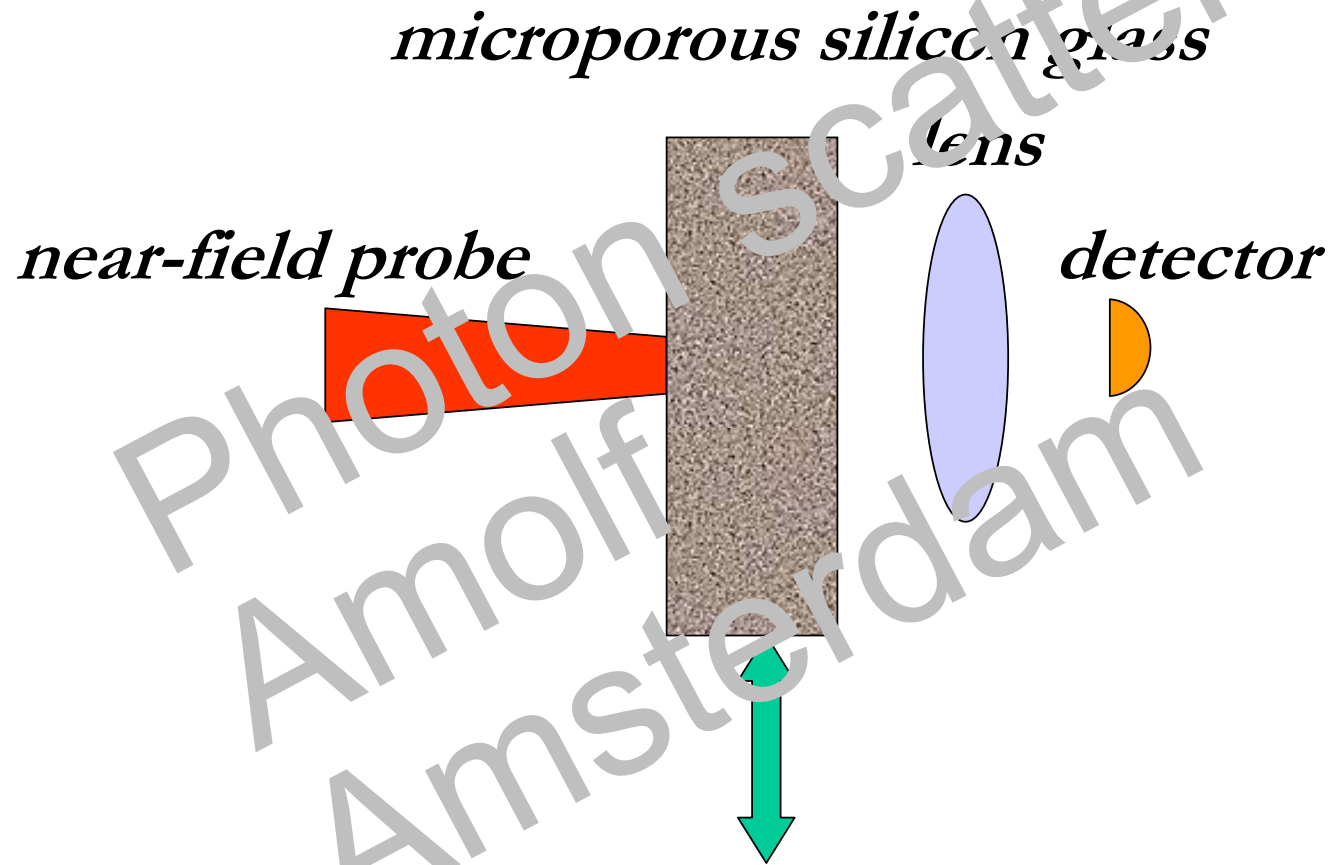solution φ of elliptic problem (3). Apart from a missing minus sign in the definition of H, (5) is nothing but the time-dependent Schrödinger equation (TDSE) for a particle moving in the 2D potential $-V(x,y)$, z playing the role of time. The initial state $\psi(x,y,z=0)$

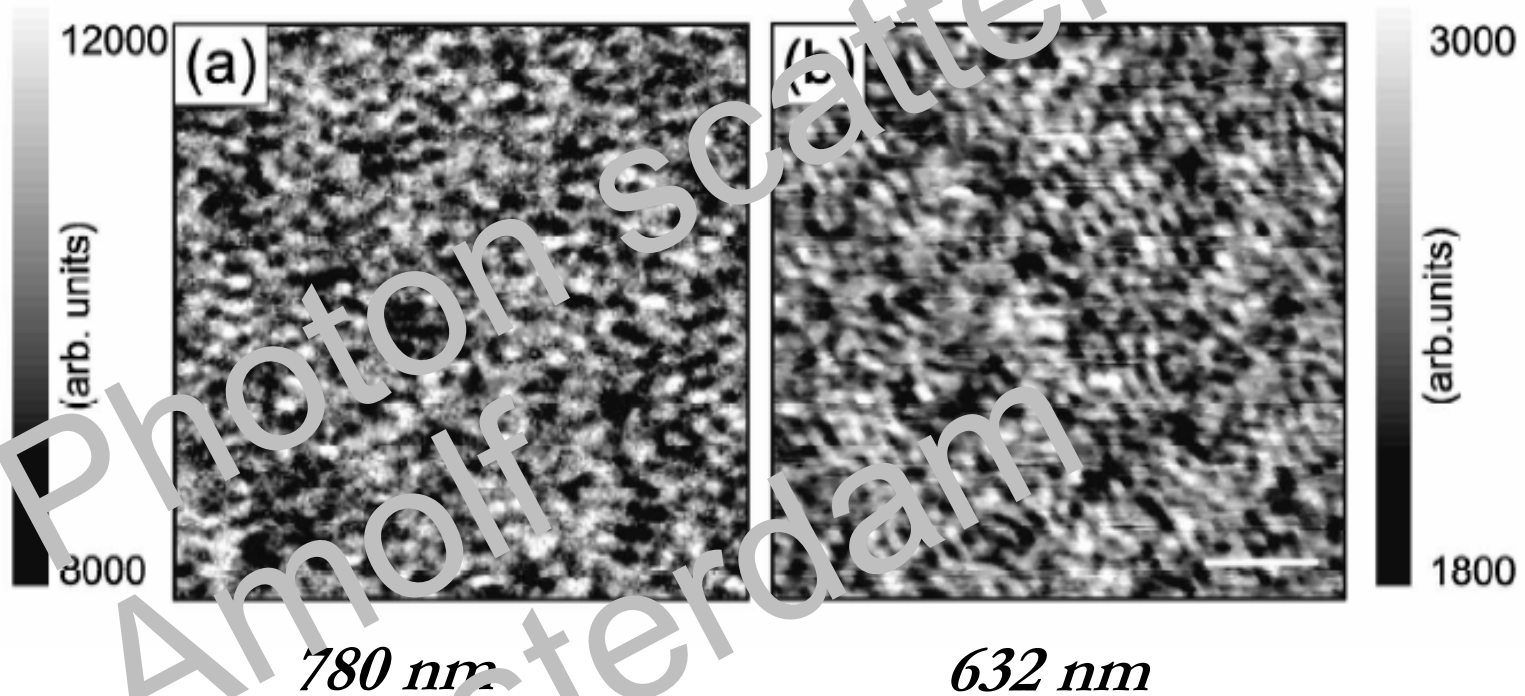# *Beam confinement*

De Raedt, AL, De Vries, PRL (1989)
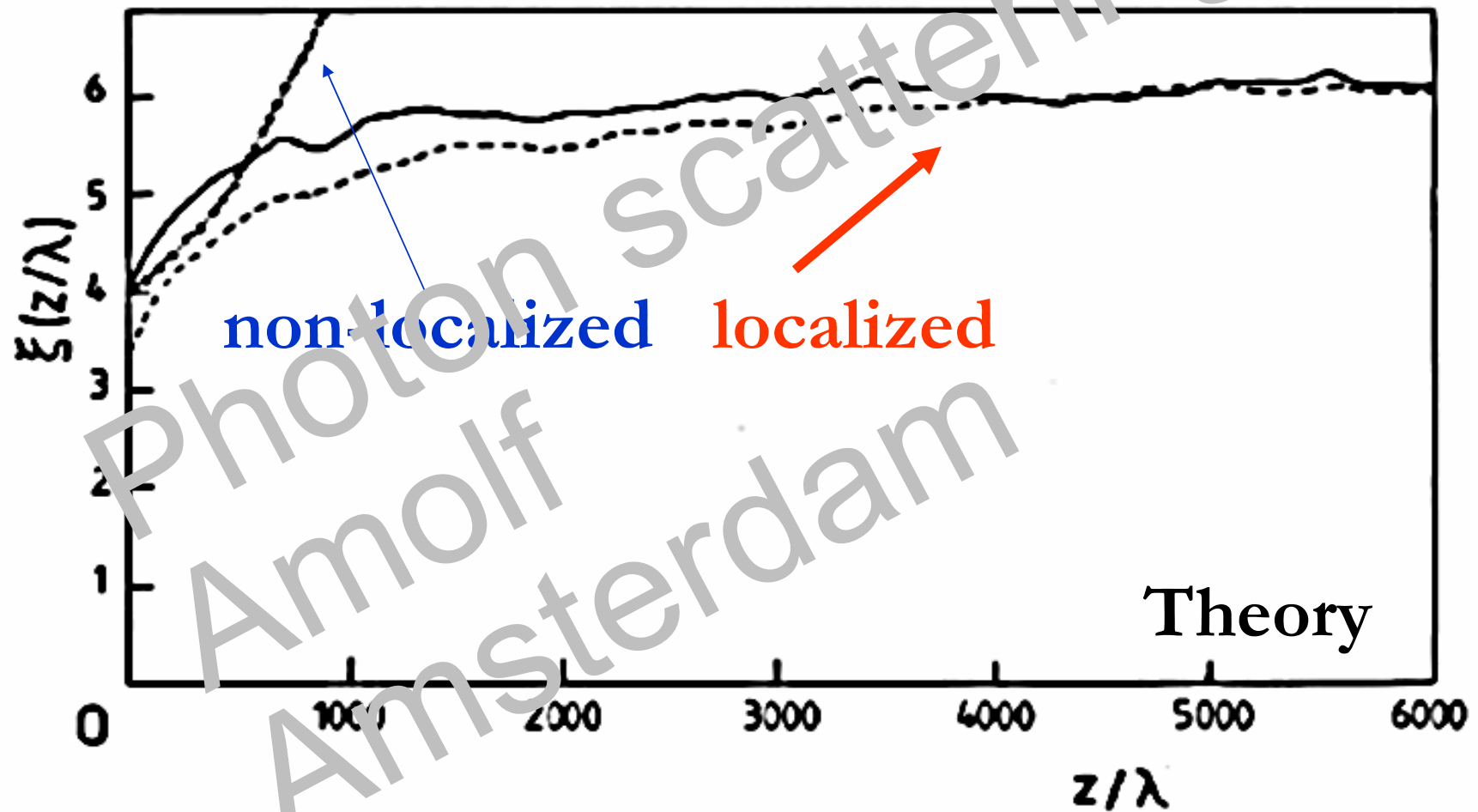
non-localized   localized

Theory

z/λ

microporous silicon glass

lens

near-field probe

detector

Near-field speckle

2d image

780 nm   632 nm

Spin-off

- **better understanding role of disorder**
  in nano-optical material (like PBG)
- **diffusing wave spectroscopy (DWS)**
- **medical imaging**
- **interdisciplinary interests**
- **applications**

Lecture II: mesoscopic light ...

- ✓ introduction
- ✓ coherent back scattering
- ✓ light localization
- ✓ new directions