Lecture III

I. photonic bandgaps

introduction, 3D, disorder, ...

II. mesoscopic light propagation

introduction, localization, ...

III. quantum optics

nanolasers, Purcell, noise

Lecture III:
spontaneous emission and qo

- introduction
- Purcell factor
- few-atom lasers
- quantum noise and correlations in a multiple scattering medium

Introduction

- introduction
- Purcell factor
- few-atom lasers
- quantum noise and correlations in a multiple scattering medium

Coworkers

Femius Koenderink
Peter Lodahl (COM, Denmark)
Karen vd Molen
Allard Mosk
Tom Savels
Martijn Wubs (Augsburg, Germany)

Photonic systems

A dielectric is a material with a dielectric constant

ε

that depends on frequency

$\varepsilon(\omega)$

A *photonic* material has a dielectric constant that depends on position

$\varepsilon(\omega,r)$

and varies in space on a length scale of the order of the wavelength of light

Issues

Sources inside

- **spontaneous emission**
- **lasing**
- **nanoboxes and strong coupling**

Light from outside

- **wave guiding**

LDOS as key parameter

Sources inside:

LDOS is a key parameter for

- **photonic crystals**
- **random systems (probably)**
- **cavities**

local impedance of source

complication:

it is not really the LDOS, but weighed with matrix elements

ImG rather than LDOS

if we have absorption
(which is quite natural for optical systems)

local impedance becomes ImG
no complete set of states any more
not positive definite any longer

With gain?

We have no idea, but we can do the experiments

causality
divergence infinite systems

Why not calculate it?

we only know a complete set of states (required for LDOS) rigorously for a very limited number of nontrivial systems:

- Mie sphere
- dielectric slab (Fabry-Perot)

Numerical methods

DOS is practical for PBG's
LDOS not

FDTD
all modes (guided etc)
not just one incoming mode

Engineers know their cavity

Cavities are built to increase storage capacity for light

What is the relation between cavity Q and LDOS?

Purcell factor ...

- introduction
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Purcell factor

$$F = \frac{Q}{V_{\text{mode}}}(\lambda / n)^3$$

ratio rate in cavity and free space

what is the mode volume?

$$V_{\text{mode}} = \frac{\int_{cavity} \varepsilon \mathbf{E}^2(\mathbf{r}) d\mathbf{r}}{\mathbf{E}_{\max}^2}$$

tautology

where in the cavity?
what polarization?

Few-atom lasers ...

- introduction
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Smallest laser

What is a laser?

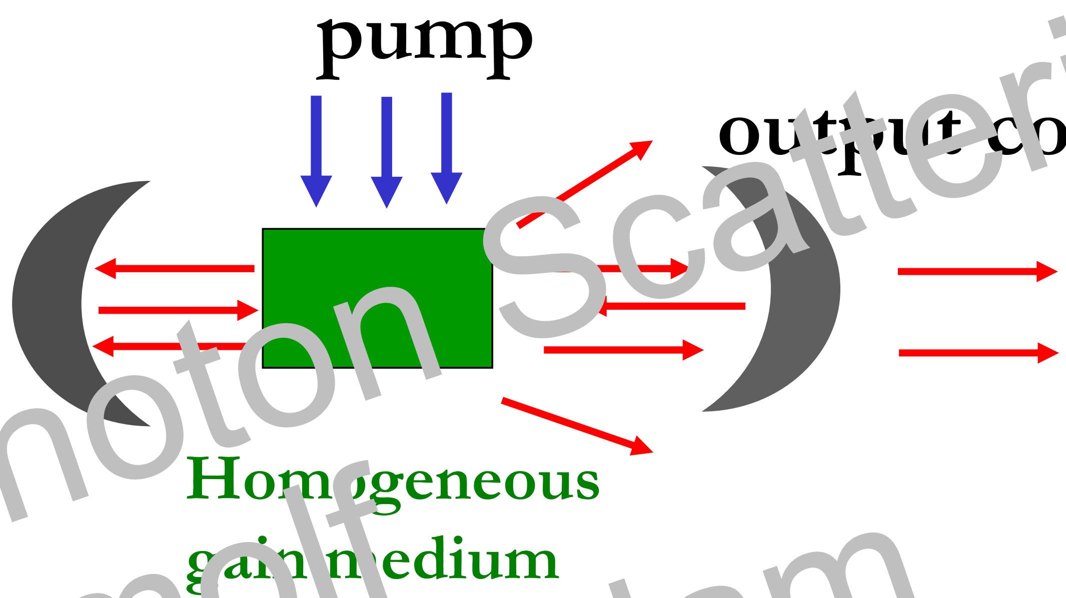
Is there a laser transition?

Siegman

Almost anything is a laser

gain narrowing
mode redistribution

Cavity laser



cavity is macroscopic

gain medium is macroscopic

Microscopic cavity

- dielectric slab (Rikken and Urbach, Wubs)

  infinite system

- dielectric sphere (Mie solution)

Gain in above cavities is

1. inhomogeneous
2. near lasing threshold complicated phenomenological gain dynamics

Cavity exit

Why not do away with the cavity all together and use only atoms?

Bound electron (Lorentz model)

$$\frac{d^2X}{dt^2} + \Gamma\frac{dX}{dt} + \omega_0^2 X = \frac{q}{m}E_0\exp(-i\omega t)$$

$$p \propto X = \frac{1}{2i}\alpha[\omega^+]E_0\exp(-i\omega t)$$

in which

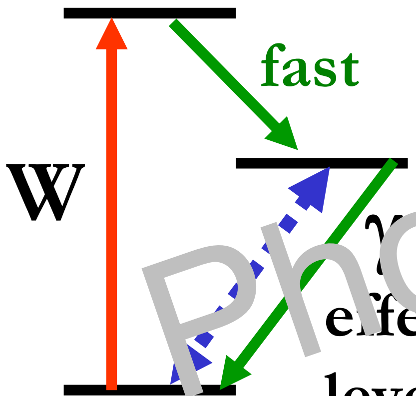
$$\alpha[\omega^+] = \frac{q^2/m}{\omega_0^2 - \omega^2 - 2i\omega\Gamma}$$

linear: no saturation

damped oscillator

Pump an atom/dipole

pumped three-level system



effective two-level system

W is pump rate

γ is spontaneous emission rate

Two limits: W << γ **old situation**

W >> γ **fully inverted population**

$$\alpha[\omega^+] = \frac{\gamma - W}{\gamma + W}\alpha(\omega = \omega_0)\frac{-i\Gamma}{\omega_0 - \omega - i\Gamma}$$

Correct linear response

$$p(t) = \int_{-\infty}^{t} \alpha(t-\tau)E(\tau)d\tau$$

$$\alpha(t) = -i\frac{q^2}{2m\omega_0^2}\omega_0 \sin(\omega_0 t)\exp(-\Gamma|t|)$$ **extinction**

$$\alpha(t) = -i\frac{q^2}{2m\omega_0^2}\omega_0 \sin(\omega_0 t)\exp(\Gamma|t|)$$ **gain: unphysical but very popular**

$$\alpha(t) = -i\frac{\gamma - W}{\gamma + W}\frac{q^2}{2m\omega_0^2}\omega_0 \sin(\omega_0 t)\exp(-\Gamma|t|)$$

gain correct

Dielectric constant from dipoles

dielectric constant

$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$$

$$\varepsilon(\omega) = 1 + \rho\ \alpha(\omega)$$

density **polarizability** **origin of gain**

derivation needs scattering theory

books: long-wavelength limit

Building microscopic laser

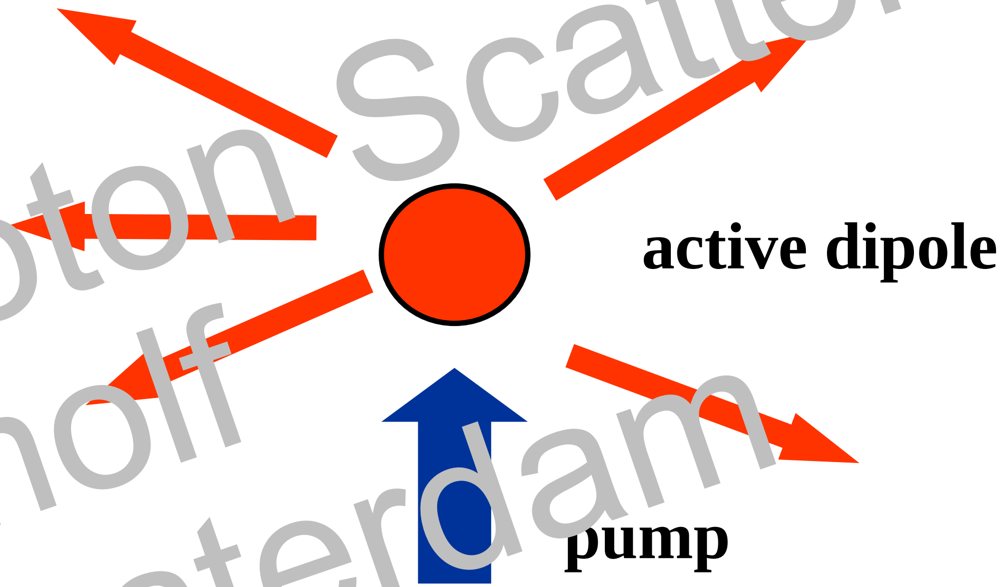
Combine active atom(s) with
passive atom(s)

1. solve full scattering problem with point t-matrices (diagonalization of 3N x 3N matrix)
2. look for laser threshold as a function of pump power and configuraration
3. include saturation to neutralize laser singularity

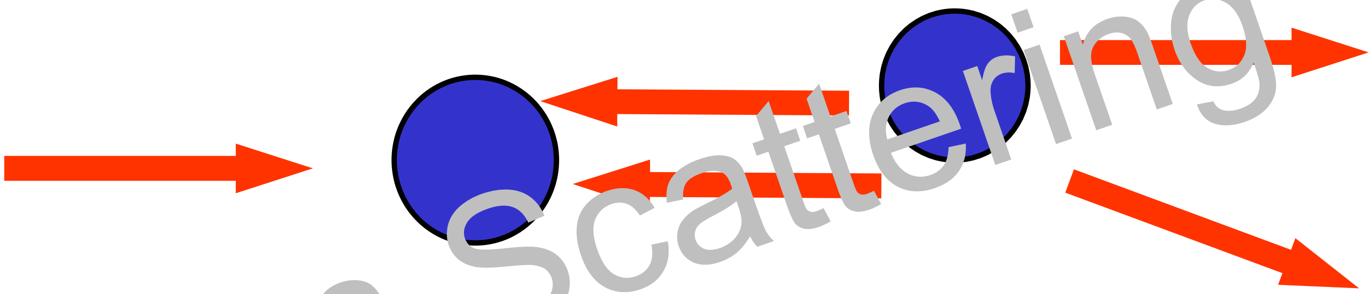
Not just atoms

- real atoms
- quantum dots
- oscillators
- ions
- dye molecules
- ...

One dipole: no laser



Two (and more) dipoles



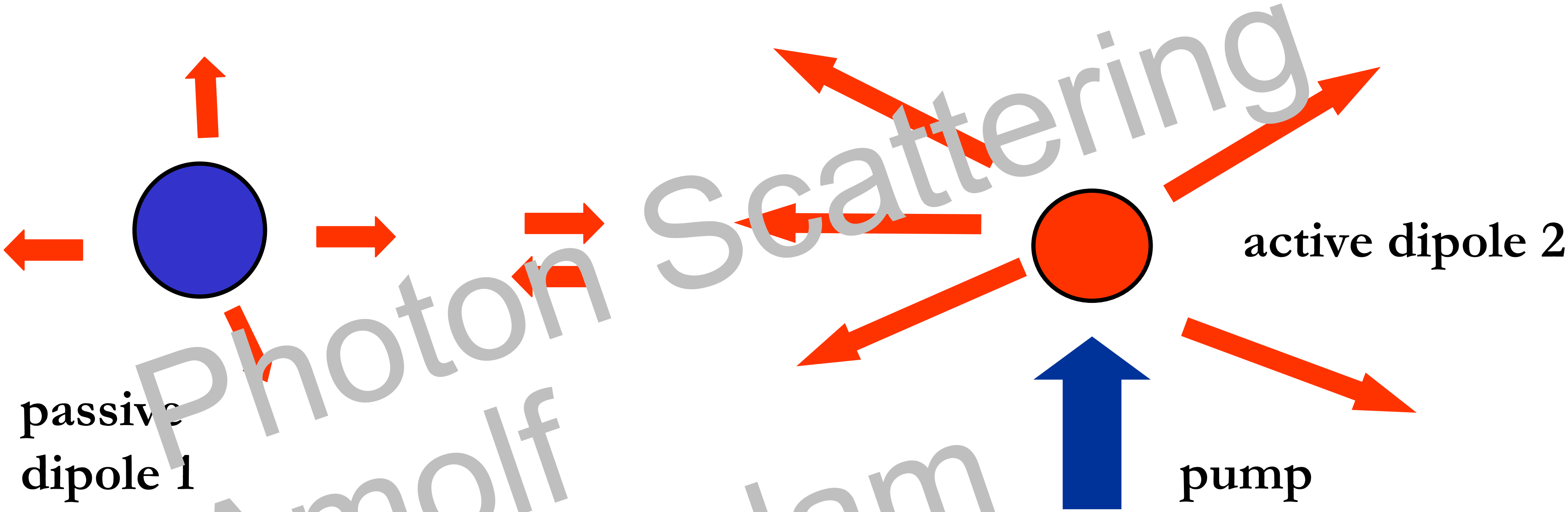
can be solved exactly
we have a t-matrix
up to a few thousands can be solved exactly (vector)

$$t_1(\mathbf{r}_1,\mathbf{r}_2) = \alpha(\omega)\delta(\mathbf{r}_1 - \mathbf{R})\delta(\mathbf{r}_2 - \mathbf{R}_1)$$

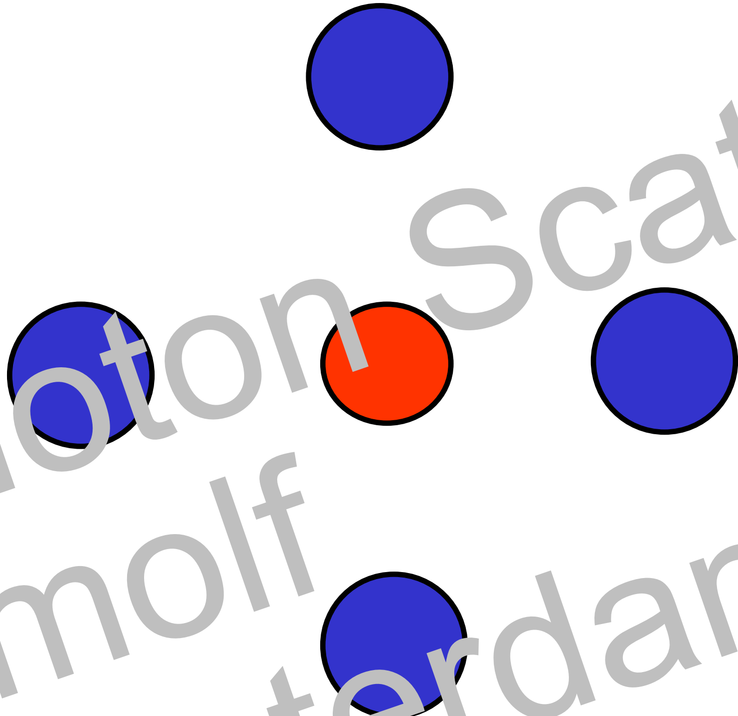
$$t_1 + t_2 + t_1 G_{12} t_2 G_{21} t_1 + \ldots$$

B.A. van Tiggelen and A. L., PRB 50, 16729 (1994).

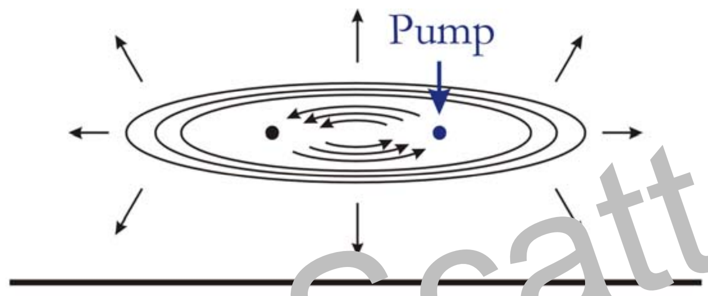
One of the dipoles has gain

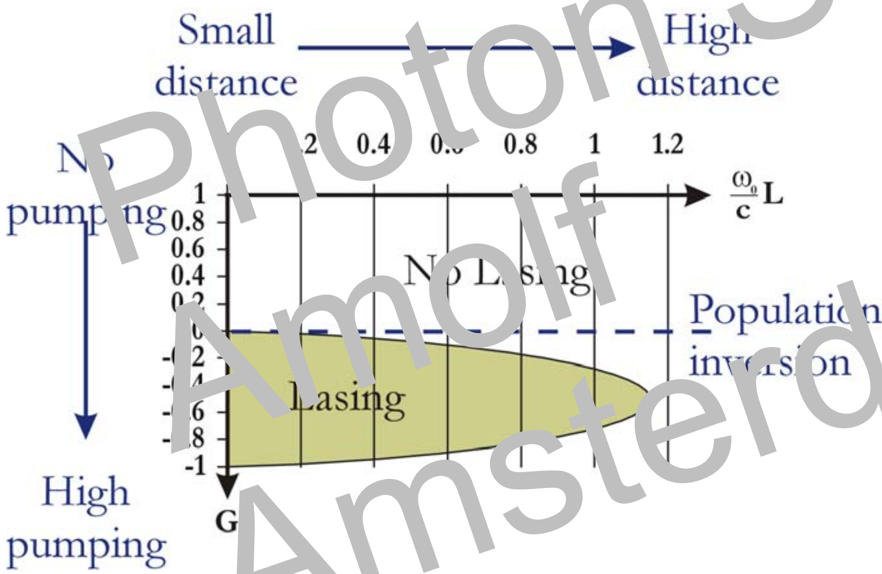


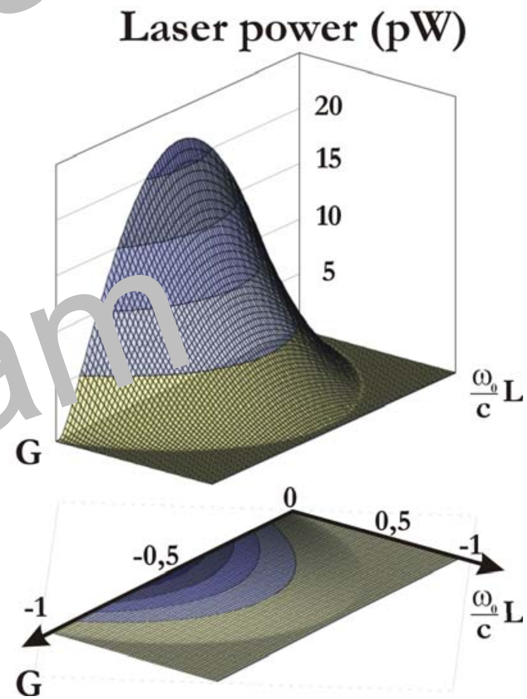
Configurations



Lasing oscillators







Tom Savels

However: saturation spoils it all

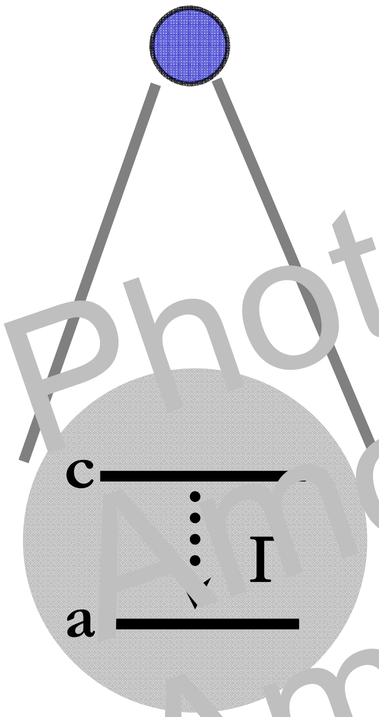
T-matrix describes elastic scattering

Due to saturation inelastic light will be generated:

- passive atoms (= cavity) are spoiled
- active atom is pumped less efficiently

Exit T-matrix

One atom density matrix



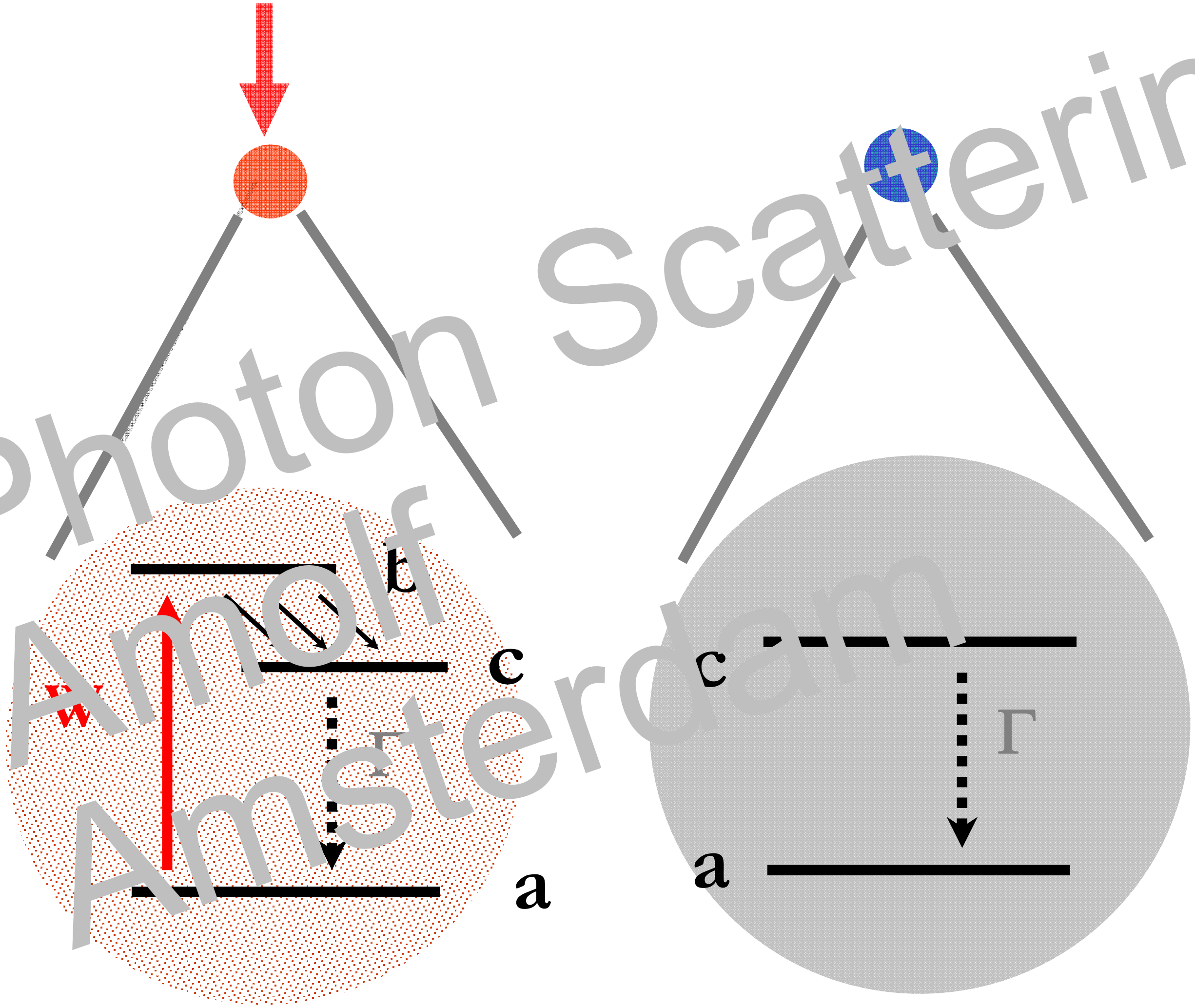
Populations $\sigma_{cc} + \sigma_{aa} = 1$

+

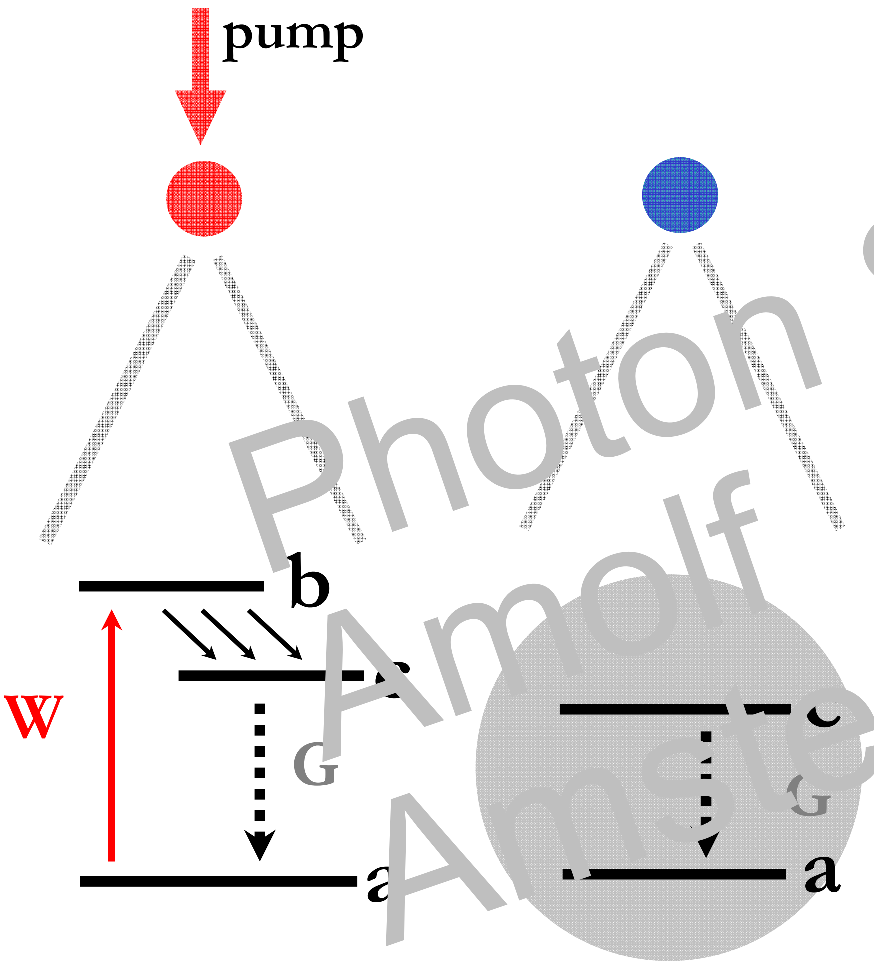
Coherences $\sigma_{ca} = \sigma_{ac}{}^{*}$

$$\begin{pmatrix} \sigma_{cc} & \sigma_{ca} \\ \sigma_{ac} & \sigma_{aa} \end{pmatrix}$$

One pumped and one passive atom



Two-atom density matrix



vector Green's functions for light

$$
\begin{pmatrix}
\sigma_{\substack{cc\\cc}} & \sigma_{\substack{cc\\ca}} & \sigma_{\substack{ca\\cc}} & \sigma_{\substack{ca\\ca}} & \sigma_{\substack{cb\\cc}} & \sigma_{\substack{cb\\ca}} \\
\sigma_{\substack{cc\\ac}} & \sigma_{\substack{cc\\aa}} & \sigma_{\substack{ca\\ac}} & \sigma_{\substack{ca\\aa}} & \sigma_{\substack{cb\\ac}} & \sigma_{\substack{cb\\aa}} \\
\sigma_{\substack{ac\\cc}} & \sigma_{\substack{ac\\ca}} & \sigma_{\substack{aa\\cc}} & \sigma_{\substack{aa\\ca}} & \sigma_{\substack{ab\\cc}} & \sigma_{\substack{ab\\ca}} \\
\sigma_{\substack{ac\\ac}} & \sigma_{\substack{ac\\aa}} & \sigma_{\substack{aa\\ac}} & \sigma_{\substack{aa\\aa}} & \sigma_{\substack{ab\\ac}} & \sigma_{\substack{ab\\aa}} \\
\sigma_{\substack{bc\\cc}} & \sigma_{\substack{bc\\ca}} & \sigma_{\substack{ba\\cc}} & \sigma_{\substack{ba\\ca}} & \sigma_{\substack{bb\\cc}} & \sigma_{\substack{bb\\ca}} \\
\sigma_{\substack{bc\\ac}} & \sigma_{\substack{bc\\aa}} & \sigma_{\substack{ba\\ac}} & \sigma_{\substack{ba\\aa}} & \sigma_{\substack{bb\\ac}} & \sigma_{\substack{bb\\aa}}
\end{pmatrix}
$$

Master equation

$$\sigma_{\substack{cc\\aa}}, \sigma_{\substack{bc\\ac}}, \sigma_{\substack{bb\\cc}}, \ldots, \sigma_{\substack{ac\\aa}}$$

number of variables = (#combined levels)^2

master equation: $$\frac{d}{dt}\sigma = A\sigma$$

dimension of evolution matrix A = (#levels)^4

Scaling with number of atoms

Number of atoms	Matrix elements
2	36^2
3	144^2
4	576^2
10	2359296^2
20	2473901162496^2

Technical details

all matrix elements are symbolically generated (Mathematica)

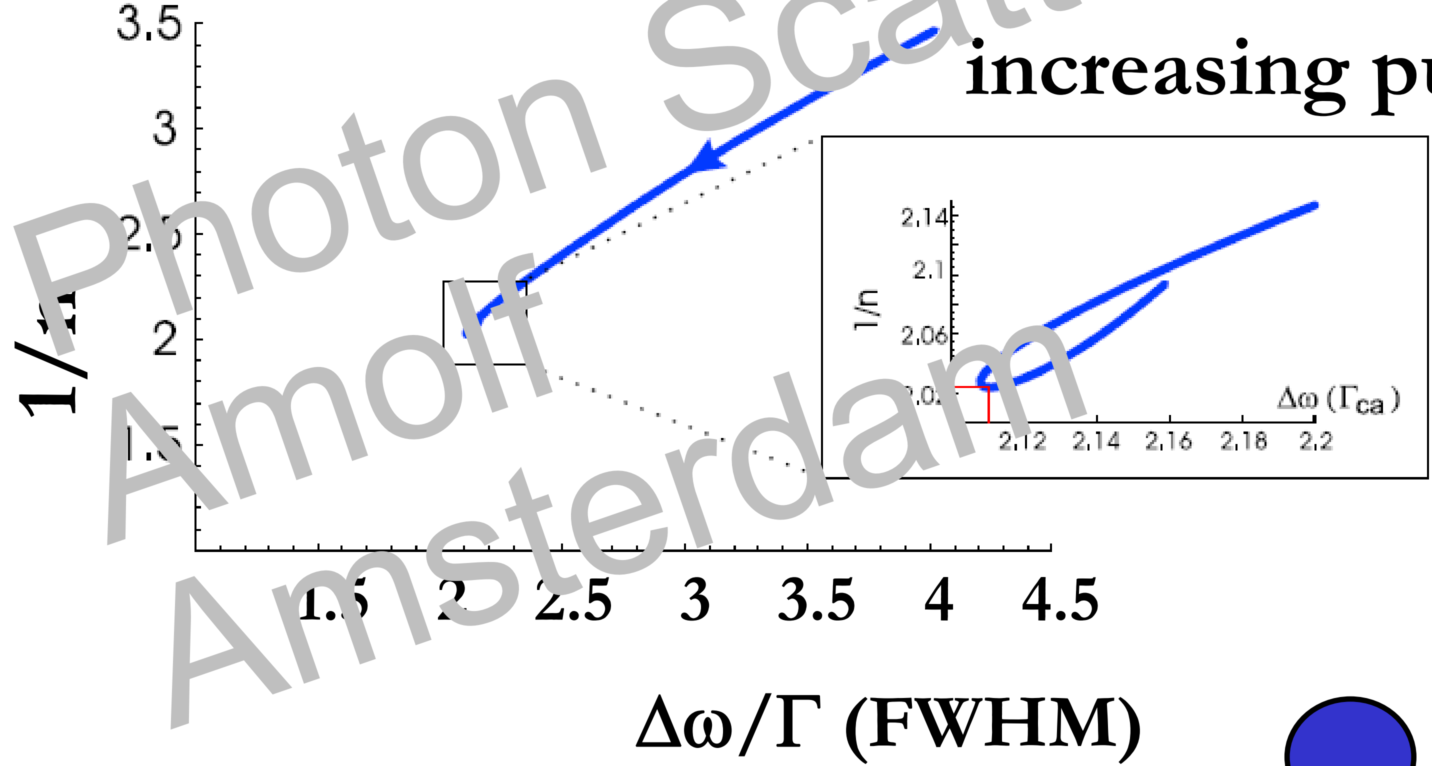
numerical inversion

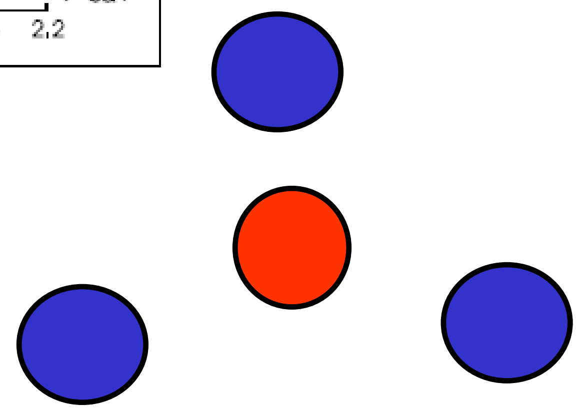
Tom Savels

Laser behavior

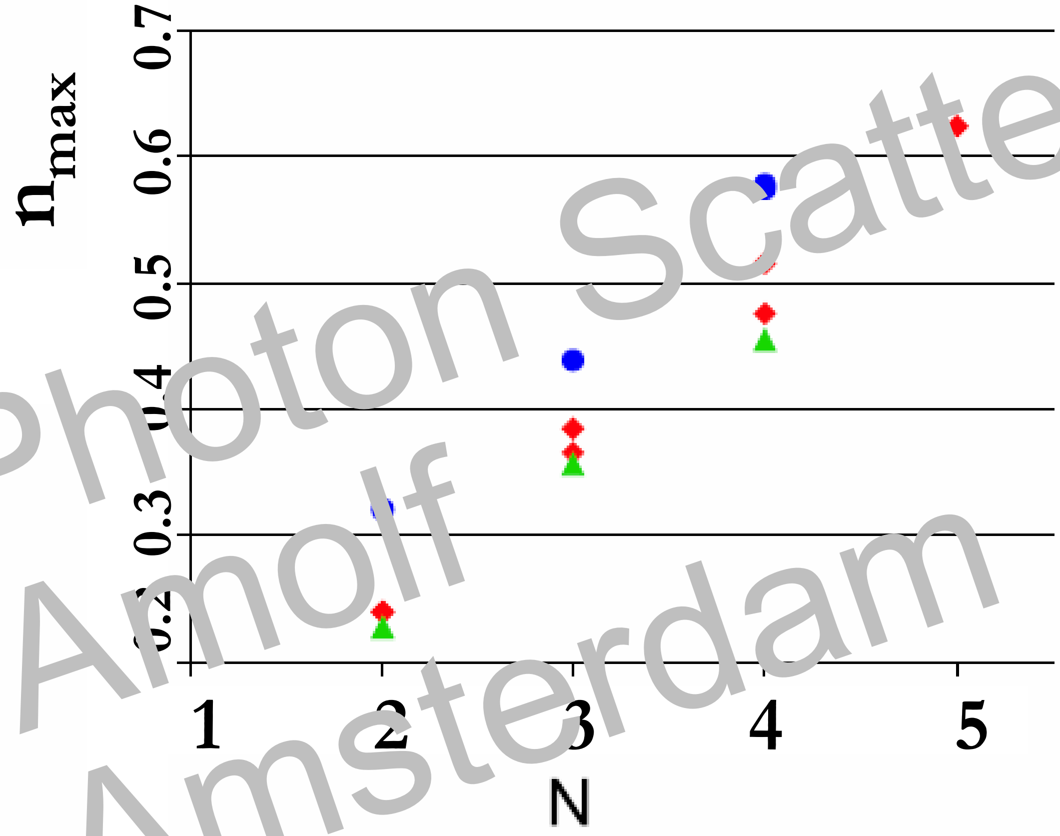
$$\int_{\Delta\omega} I(\omega)\,d\omega \equiv n\Gamma$$

n = number of excitations





Scaling with size



number of atoms

Our results

Our on-resonance atoms gradually behave laser-like if you put more and more atoms together

There does not seem to be a critical number of atoms

Tweezer experiment

Dye pumped gives gain
Sphere with dye
We trap one such sphere with tweezers
We pump the trapped sphere
Observe luminescense

Peter Zijlstra, Karen vd Molen, and Allard Mosk

Quantum noise ...

- introduction
- Purcell factor
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Is quantum optical scattering dull?

Yes, because

- trivially the same as classical case
- scattering into many modes
  will dilute any quantum effects
- scattering will lose coherence

No, because

- experiments can detect many modes
- elastic scattering fully coherent

study of propagation of non-classical states

Quantization

$r^{aa'}_\omega$ reflection coefficient $\quad t^{ab}_\omega$ transmission coefficient

$$E^b_\omega = t^{ab}_\omega E^a_\omega$$ E_ω classical field

$$\hat{a}^b_\omega = \sum_a t^{ab}_\omega \hat{a}^a_\omega + \sum_{b'} r^{bb'}_\omega \hat{a}^{b'}_\omega$$ $\hat{a}_\omega$ quantum operator

Scaling of transmission

T = Total (angularly integrated) transmission

$$T^{Tech.Noise} \propto \left(\frac{\ell}{L}\right)^2$$

ℓ is mean free path

L is sample thickness

$$T^{Shot\ Noise} \propto \frac{\ell}{L}$$

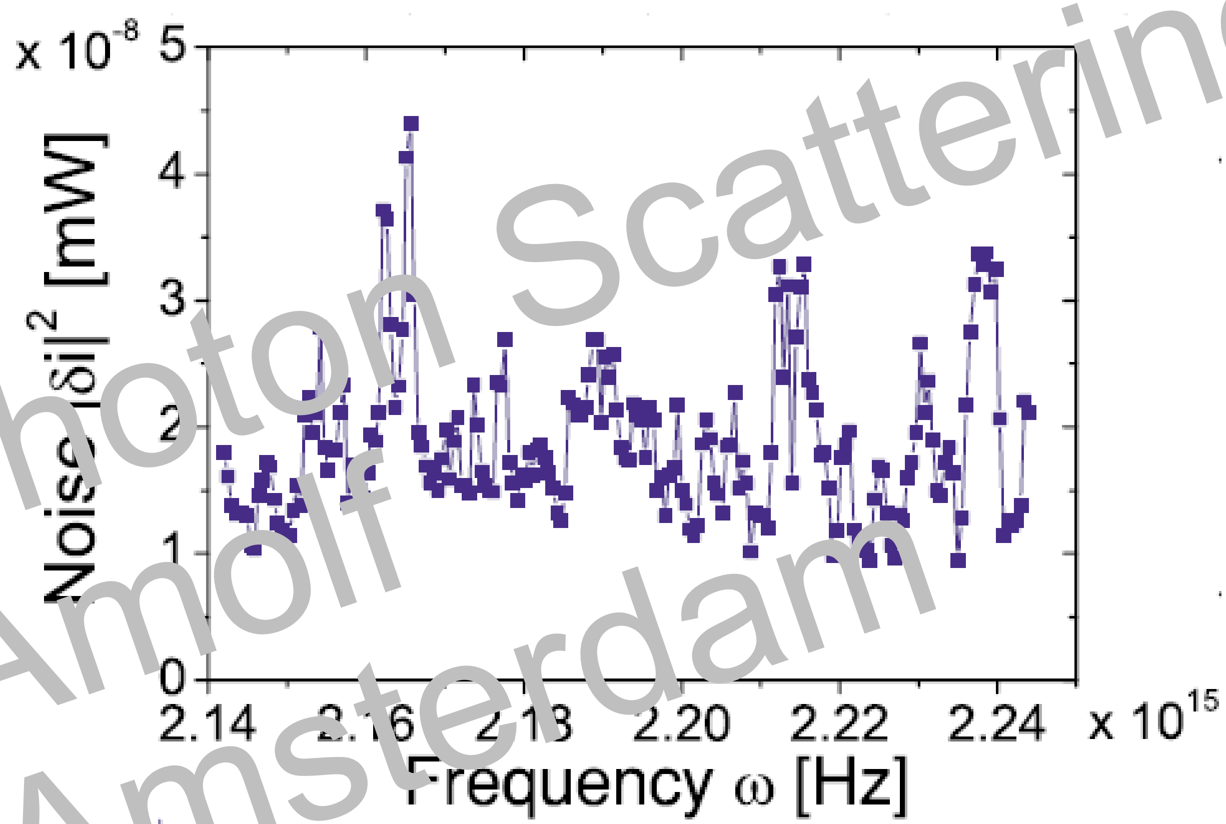
Additional other correlations predicted

Lodahl and A.L. PRL 2005

Lodahl, Mosk, and A. L. PRL 2005

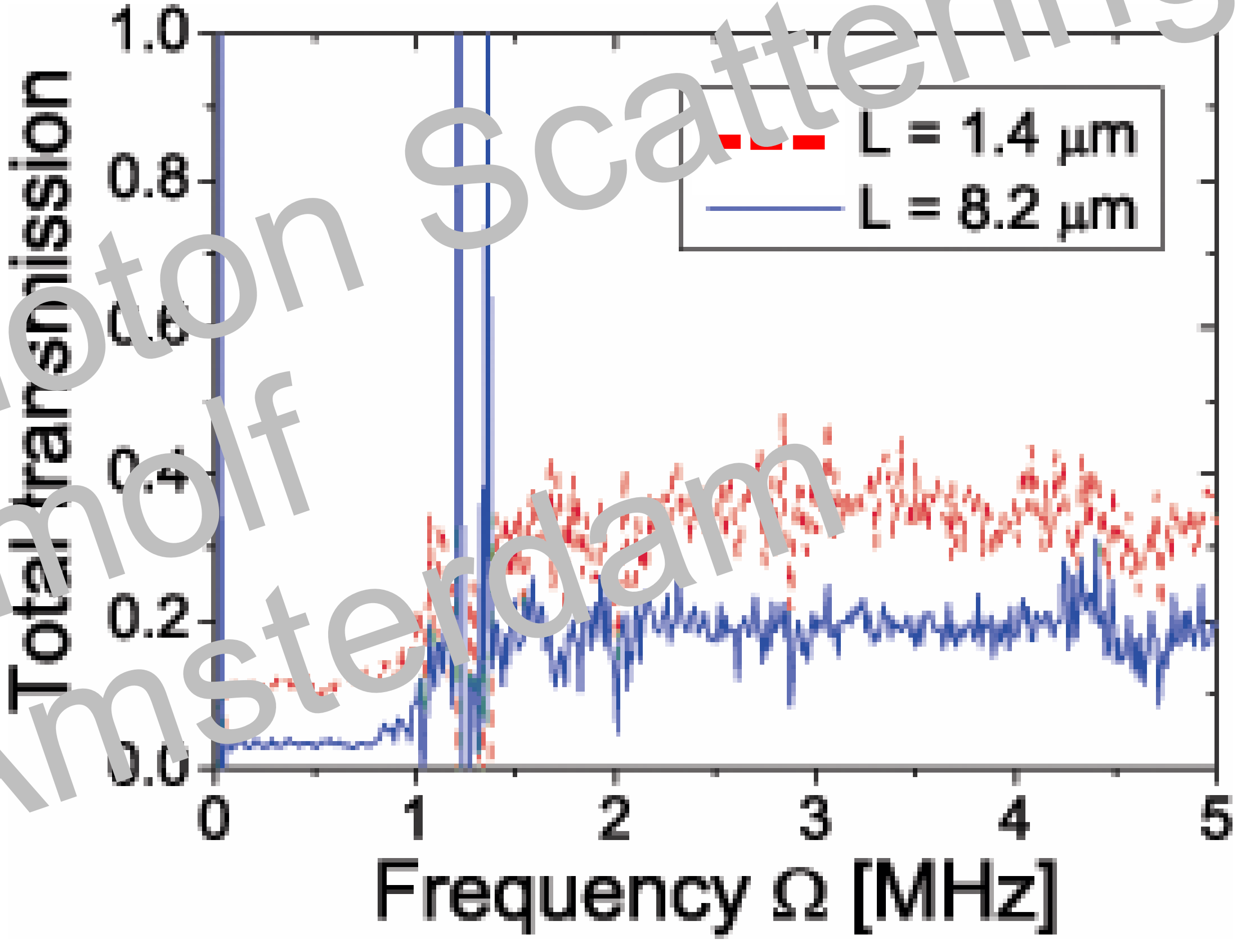
Experimental setup noise

Raw noise data

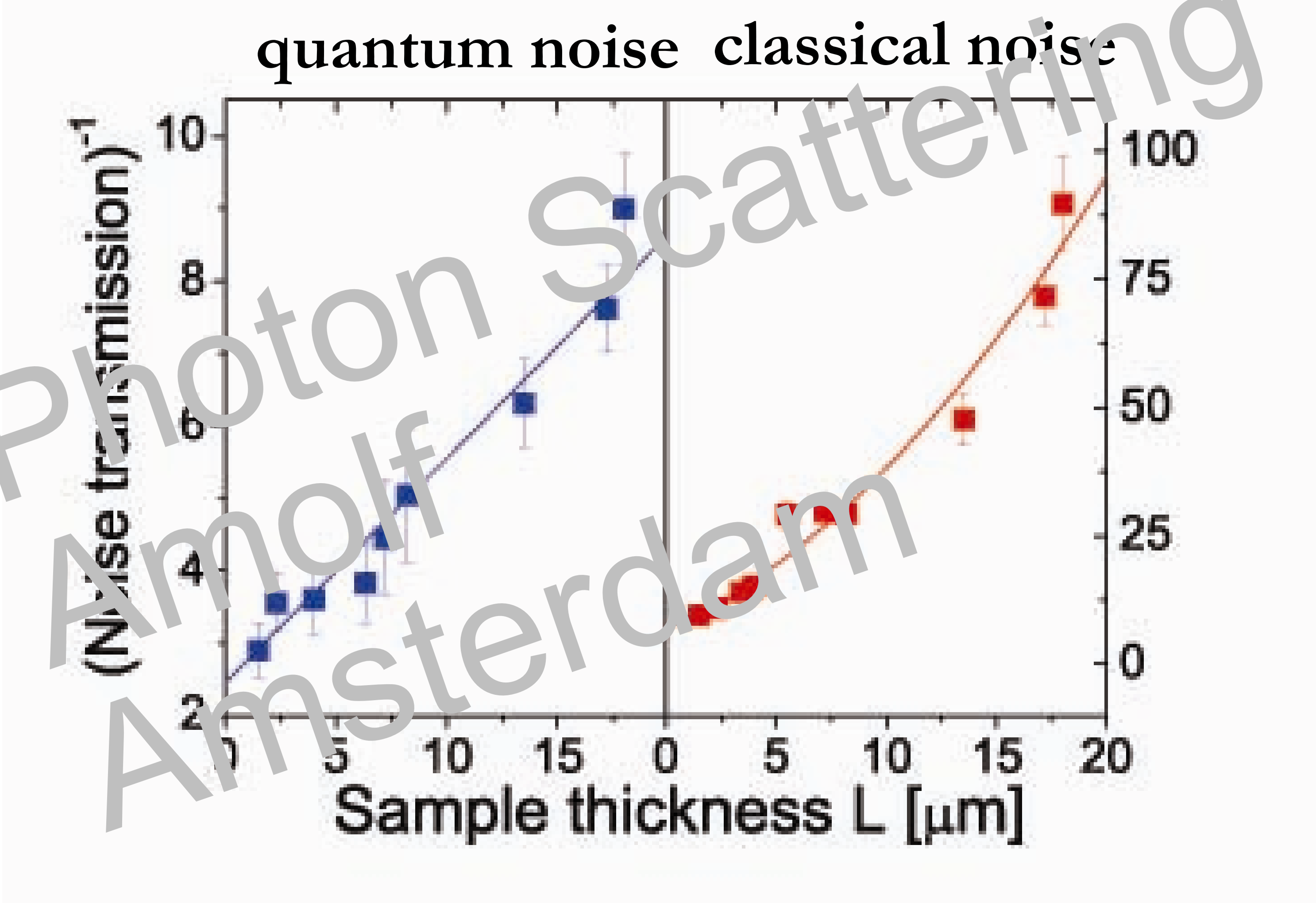


Noise measurement

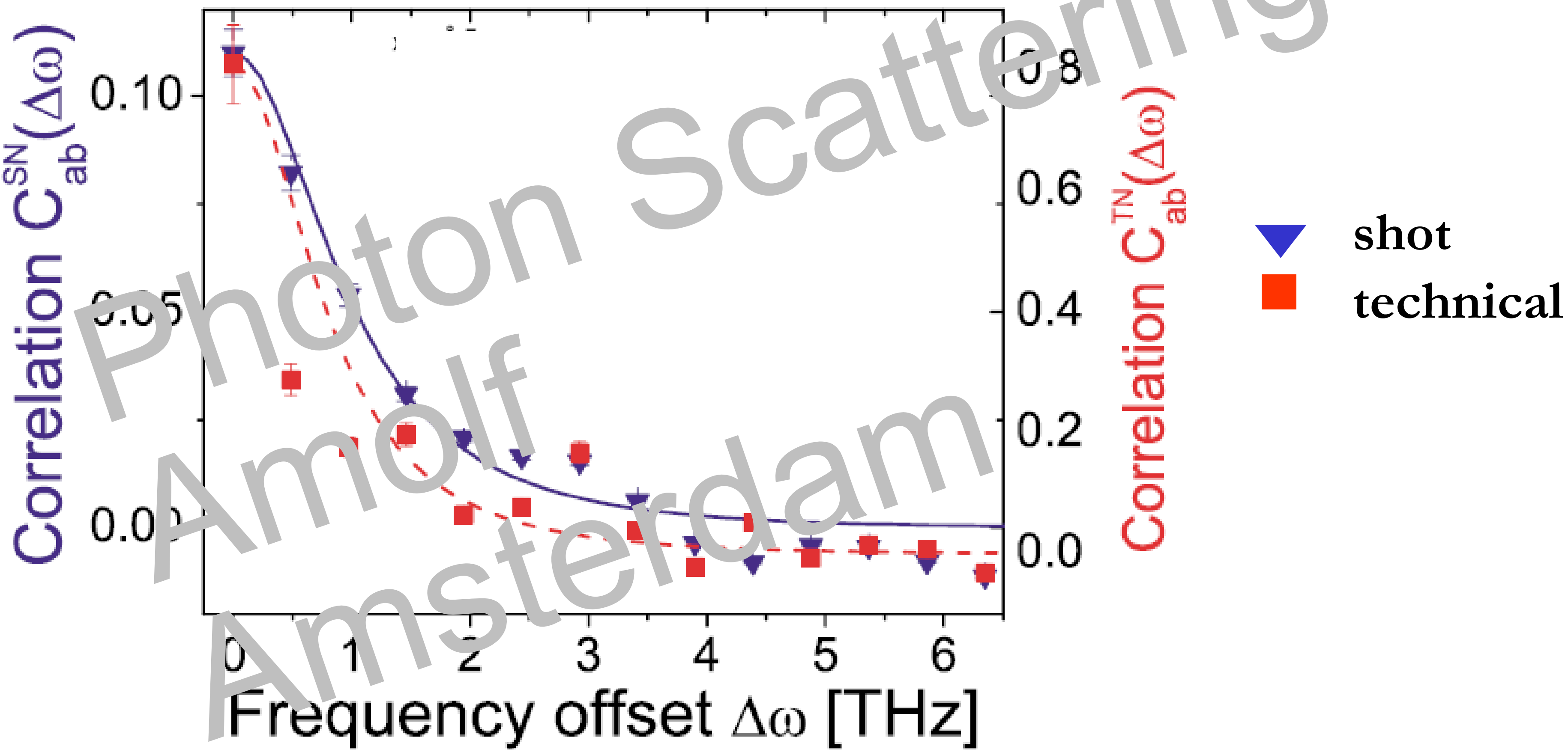
noise transmission in TiO_2 slab



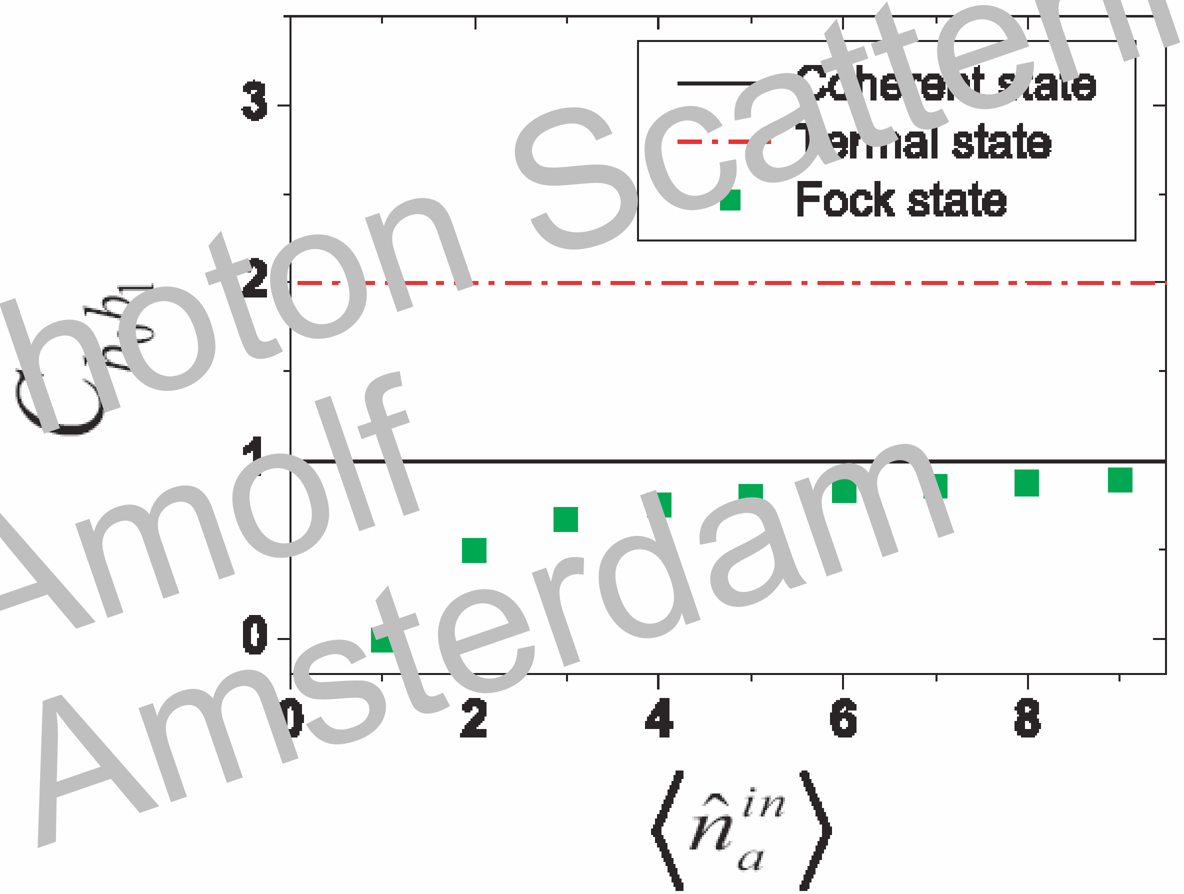
Observation of scaling



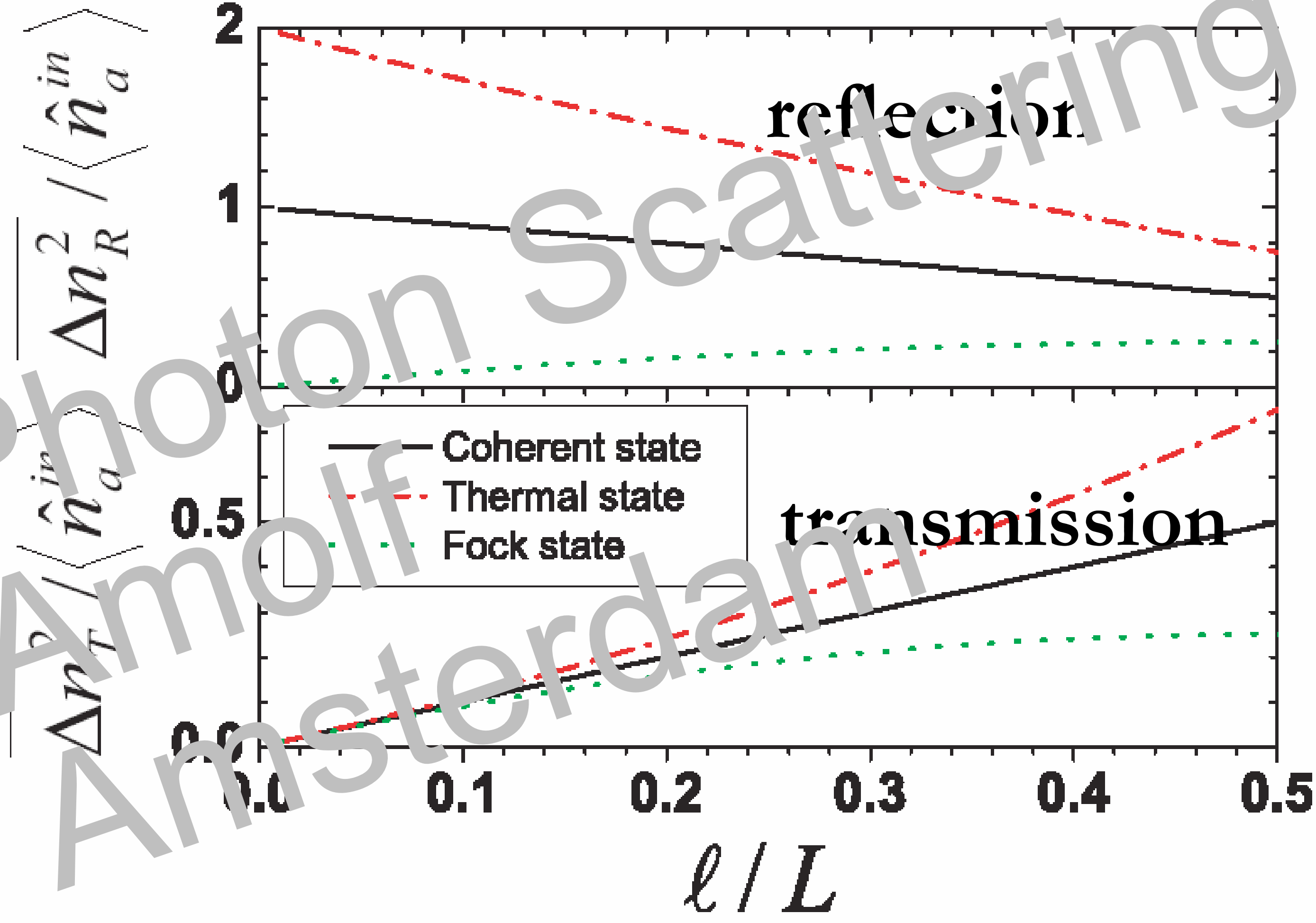
Noise correlation



Spatial correlation function



Photon number fluctuations



Future quantum optics

- we have predicted a number of quantum correlations (Beenakker et al.)
- we intend to experiment with squeezed and entangled states

Lecture III:
spontaneous emission and qo

- ✓ introduction
- ✓ Purcell factor
- ✓ few-atom lasers
- ✓ quantum noise and correlations in a multiple scattering medium