The Abdus Salam
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United Nations
Educational, Scientific
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International Atomic
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SMR 1760 - 1

COLLEGE ON

PHYSICS OF NANO-DEVICES

10 - 21 July 2006

Nanomechanics in Coulomb Blockade Structures (classical approach)

Presented by:

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Mechanically Assisted Single-Electronics

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- Lecture 1: Nanomechanics in Coulomb Blockade Structures (classical approach)
- Lecture 2: Quantum Nanoelectromechanics Due to Tunneling of a Single Electrons
- Lecture 3: Nanoelectromechanics of Magnetic and Superconducting Tunneling Devices

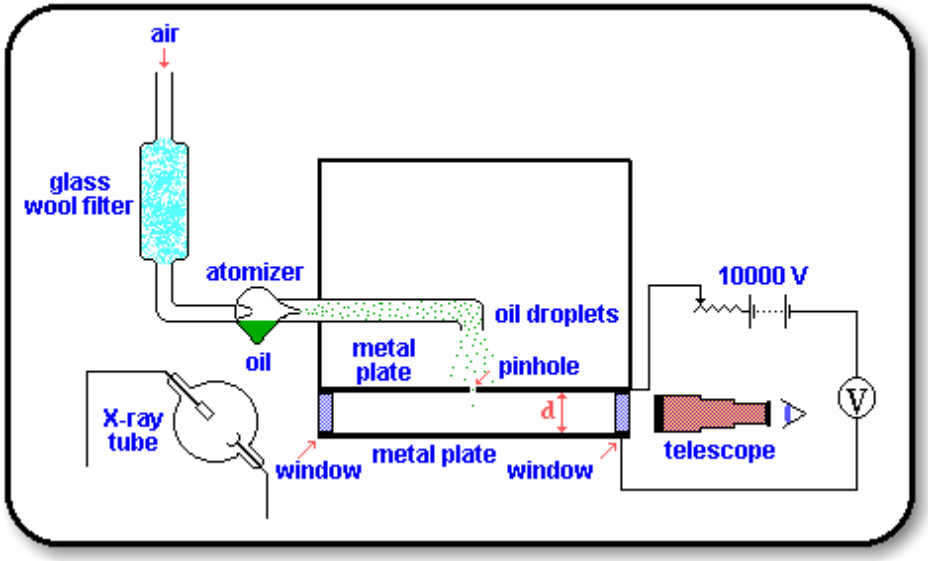
Lecture 1

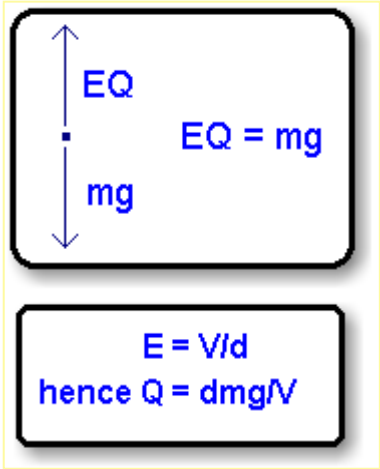
Nanomechanics in Coulomb Blockade Structures (classical approach)

Outline

- *Electric charge quantization in solids (important historical events)*
- *Self-assembled nanocomposites – new materials with special electronic and mechanical properties*
- *Nanoelectromechanical coupling due to tunneling of single electrons*
- *Shuttling of single electrons in NEM-SET devices*
- *Electromechanics of suspended carbon nanotubers (CNT)*

Millikan's Oil-Drop Experiment (Nobel Prize in 1923)





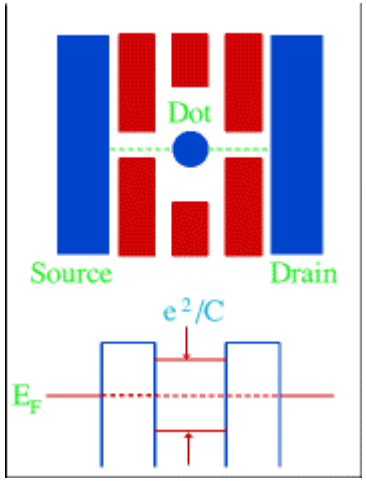
The electronic charge as a discrete quantity:

In 1911, Robert Millikan of the University of Chicago published* the details of an experiment that proved beyond doubt that charge was carried by **discrete** positive and negative entities of equal magnitude, which he called electrons.

The charge on the trapped droplet could be altered by briefly turning on the X-ray tube. When the charge changed, the forces on the droplet were no longer balanced and the droplet started to move.

*Phys. Rev. **32**, 349-397 (1911); se also http://nobelprize.org/physics/laureates/1923/press.html

Electrically Controlled Single-electron Charging



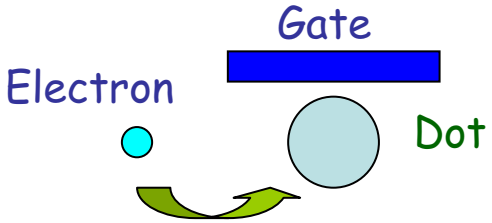
Experiment:

L.S.Kuzmin, K.K.Likharev, JETP Lett. **45**, 495(1987):
T.A.Fulton, C.J.Dolan, PRL, **59**,109(1987);
L.S.Kuzmin, P.Delsing, T.Claeson, K.K.Likharev, PRL,**62**,2539(1989);
P.Delsing, K.K.Likharev, L.S.Kuzmin & T.Claeson, PRL, **63**, 1861, (1989)

Theory:

R.Shekhter., Soviet Physics JETP **36**, 747(1973);
I.O.Kulik, R.Shekhter, Soviet Physics JETP **41**, 308(1975);
D.V.Averin, K.K.Likharev, J.Low Temp.Phys. **62**, 345 (1986)

Coulomb Blockade

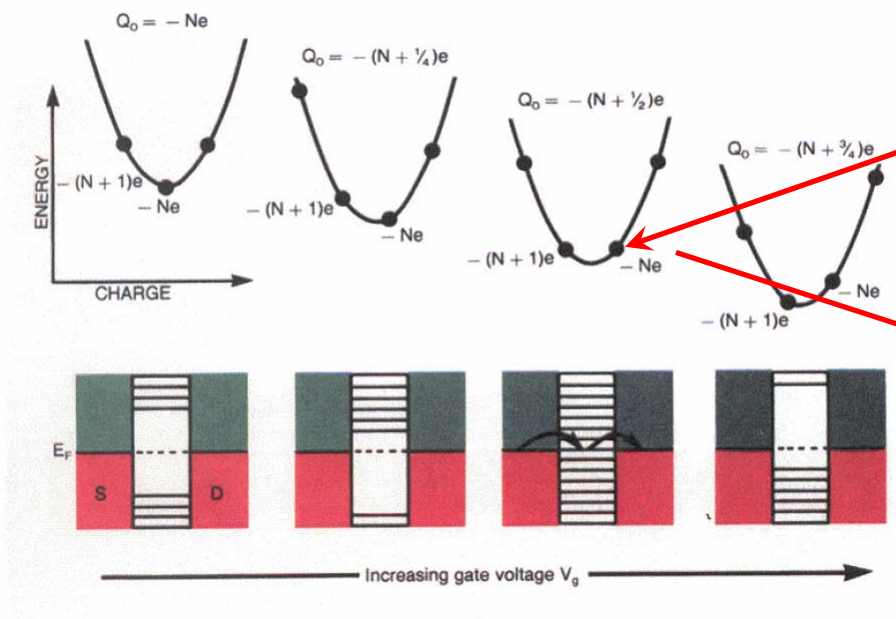


Repulsion at the dot

Cost $$E = QV_g + \frac{Q^2}{2C}$$

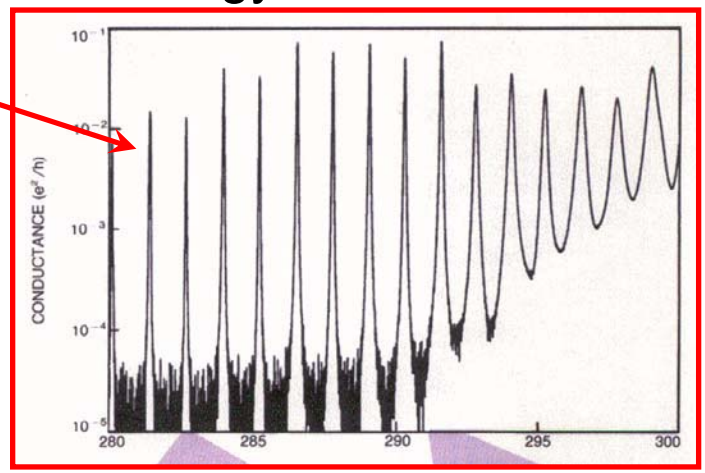
Attraction to the gate

$$Q = -Ne$$



At $$V_g = -\left(N + \frac{1}{2}\right)\frac{e}{C}$$

the energy cost vanishes !



Single-electron transistor (SET)

Submicron SET-Sensors

- CB primary termometer (based on thermal smearing of the CB) in a range 20mK-50K (δT~3%)

  (T.Bergsten et al. Appl.Phys.Lett. **78**, 1264 (2001) Y.Pekola, J.Low Temp.Phys. **135**, (2004))

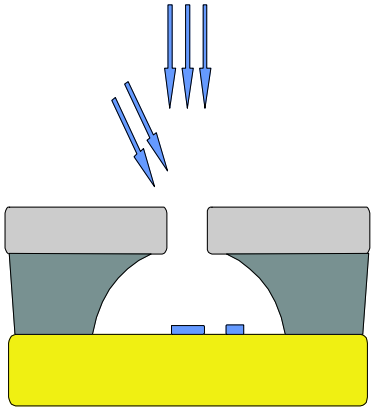
- Most sensitive electrometers (based on SET sensitivity to the gate potential Vg): δq~ $10^{-6} eHz^{-1/2}$

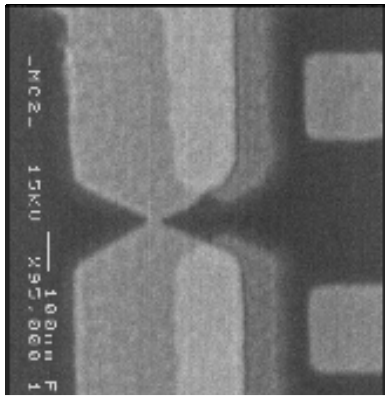
  (M.Devoret et al., Nature **406**, 1039 (2000)).

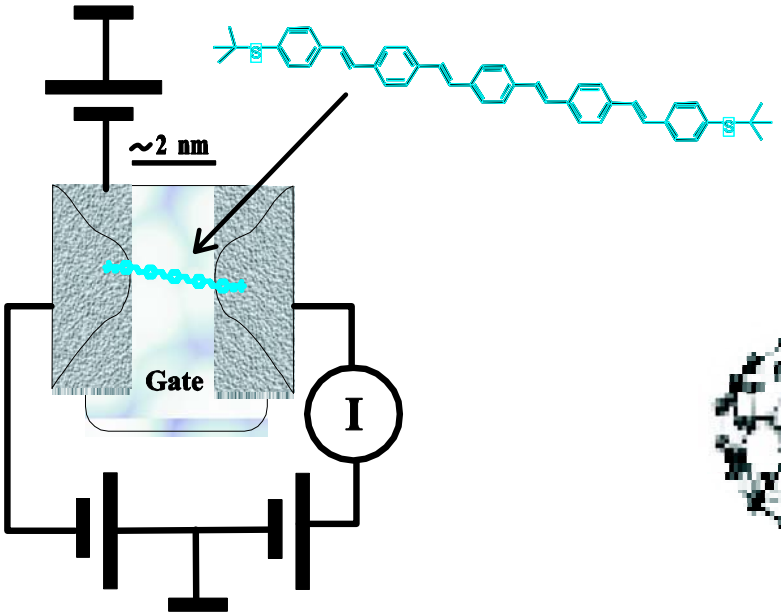
- CB current meter (based on SET oscillations in time)

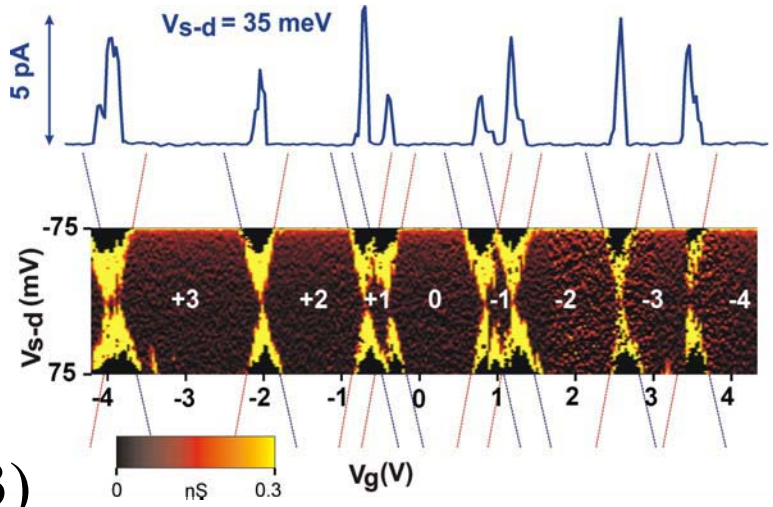
  (J.Bylander et al. Nature **434**, 361 (2005))

Single Molecular Transistors with OPV5 and Fullerenes





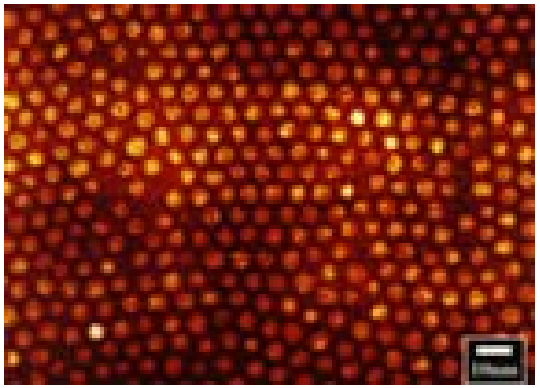




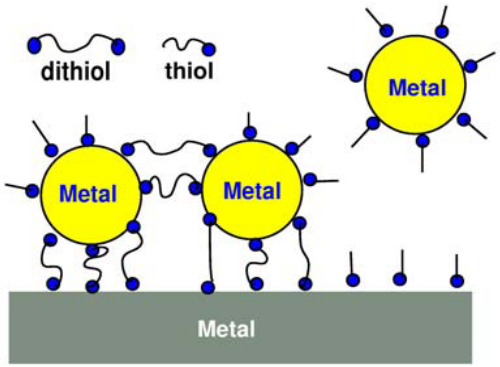
Nature **425**, 628 (2003)

Self-Assembled Metal-organic Composites

Molecular manufacturing – a way to design materials on the nanometer scale



Encapsulated 4 nm Au particles self-assembled into a 2D array supported by a thin film, Anders *et al*., 1995



Scheme for molecular manufacturing

Basic Characteristics

Materials properties:

Electrical – heteroconducting

Mechanical - heteroelastic

Electronic features:

Quantum coherence

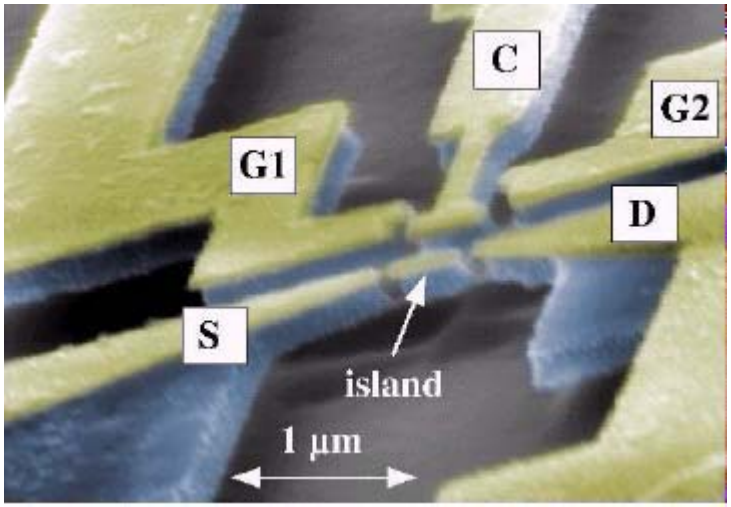
Coulomb correlations

Electromechanical coupling

$$\tau_R = 1/RC, \quad \omega_M \tau_R \sim 1, \; \omega_M \sim 10^{11} - 10^{12}\, s^{-1}$$

Nanoelectromechanical Devices

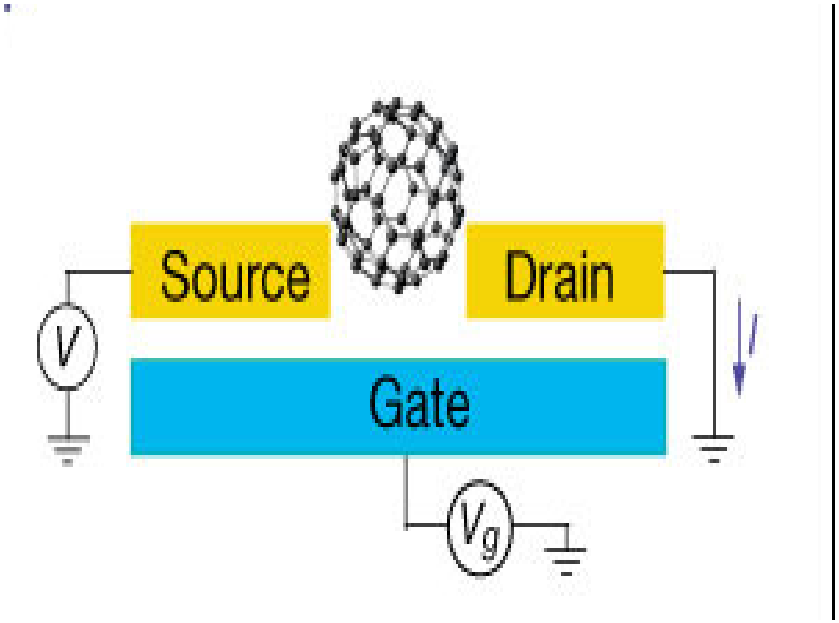
Quantum ”bell”



A. Erbe *et al*., PRL **87**, 96106 (2001);

D. Scheible *et al.* NJP **4**, 86.1 (2002)

Single C_{60} Transistor

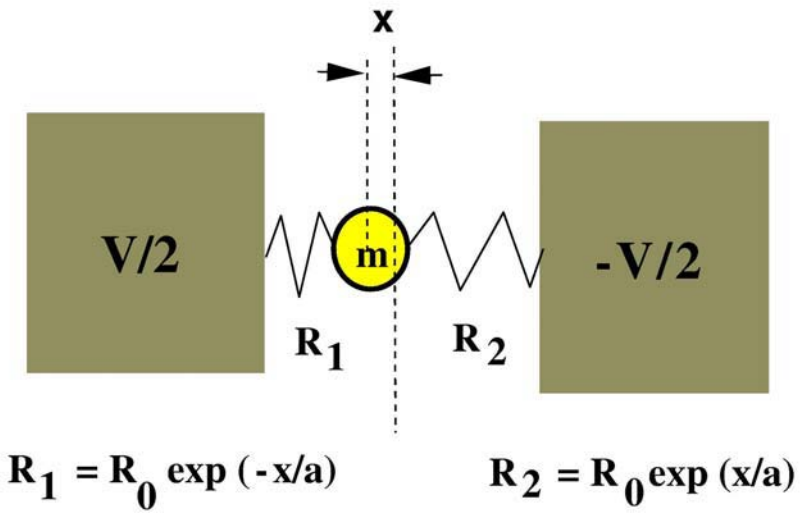


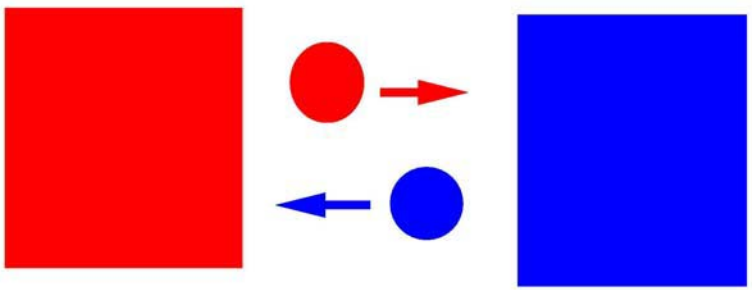
H. Park *et al*., Nature **407**, 57 (2000)

Here: Nanoelectromechanics caused by or associated with single-charge tunneling effects

Shuttling of Single Electrons in NEM-SET Devices

Millikan's Set-up on a Nanometer Length Scale





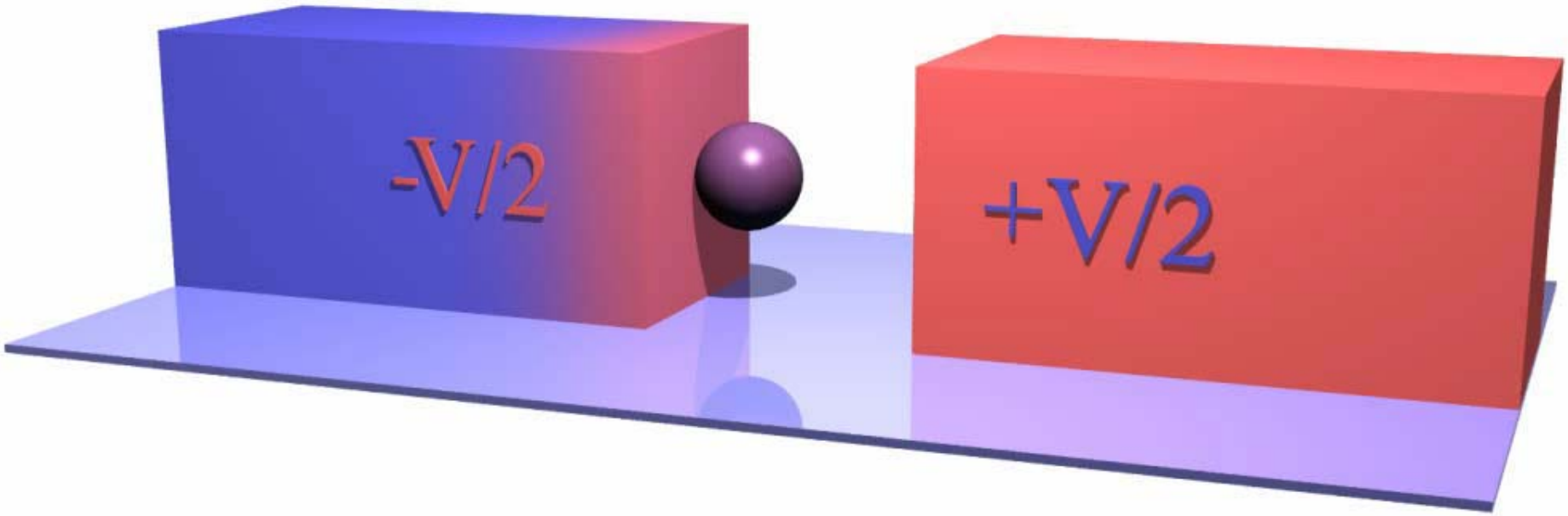
Velocity direction is correlated with the charge sign

$$W = \frac{E}{T}\int_0^T dt\, Q(t)\,\dot{X}(t) > 0$$

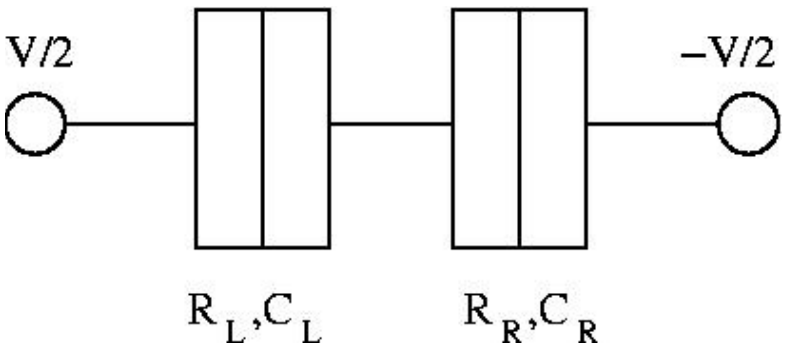
If W exceeds the dissipated power an instability occurs

Gorelik et al., PRL, **80**, 4256(1998)

The Electronic Shuttle



Circuit Model for the Shuttle



Electrostatic energy of the charged grain

$$E_n = Q_n^{\,2}/2C + \frac{V}{2} Q_n \frac{C_L - C_R}{C},$$

$$Q_n = ne$$

$$C = C_L + C_R$$

Electronic tunneling rate

$$G_{n'n}^{s} = \frac{1}{e^2 R_s(x)} \frac{E_{n'} - E_n - eV_s(n'-n)}{1 - \exp\beta\{E_n - E_{n'} - eV_s(n-n')\}}, \quad s = L, R$$

Formulation of the Problem

$$\rho_n = Sp\{\hat{\rho}\}_n$$

$$Q(x) = e\sum_n n\rho_n$$

$$\frac{\partial \rho_n}{\partial t} = \sum_{\substack{n' = n \pm 1 \\ s = 1,2}} \{G^s_{n'n}(x)\rho_{n'} - G^s_{nn'}(x)\rho_n\}$$

$$\frac{d^2 x}{dt^2} = -\omega^2 x - \gamma \frac{dx}{dt} + \frac{Q(x)E}{M}$$

Nanoelectromechanical Instability

$$\frac{dQ}{dt} = I_L + I_R \;\Rightarrow\; \frac{dQ}{dt} = \frac{Q}{C}\left(\frac{1}{R_L} + \frac{1}{R_R}\right) + \frac{V}{C}\left(\frac{C_R}{R_L} - \frac{C_L}{R_R}\right)$$

$$R^{-1}{}_{R,L} = R_0{}^{-1}\left(1 \pm \frac{x(t)}{\lambda}\right)$$

Weak electromechanical coupling

$$\frac{d^2x}{dt^2} = -\omega^2 x - \gamma\frac{dx}{dt} + \frac{Q(x)E}{M}$$

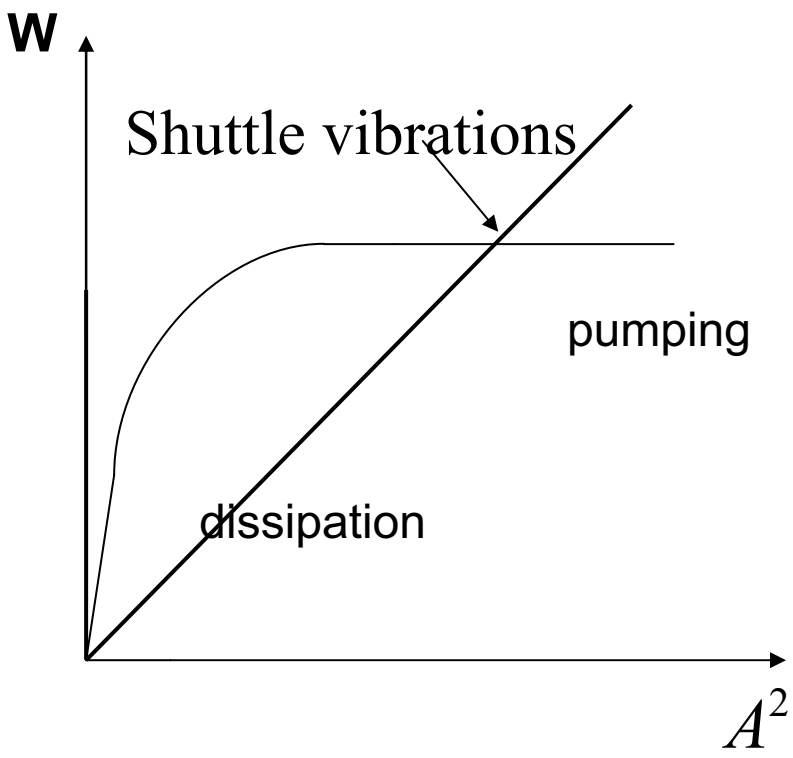
$$\eta = \frac{CVE\lambda}{2M\lambda^2\omega^2} << 1$$

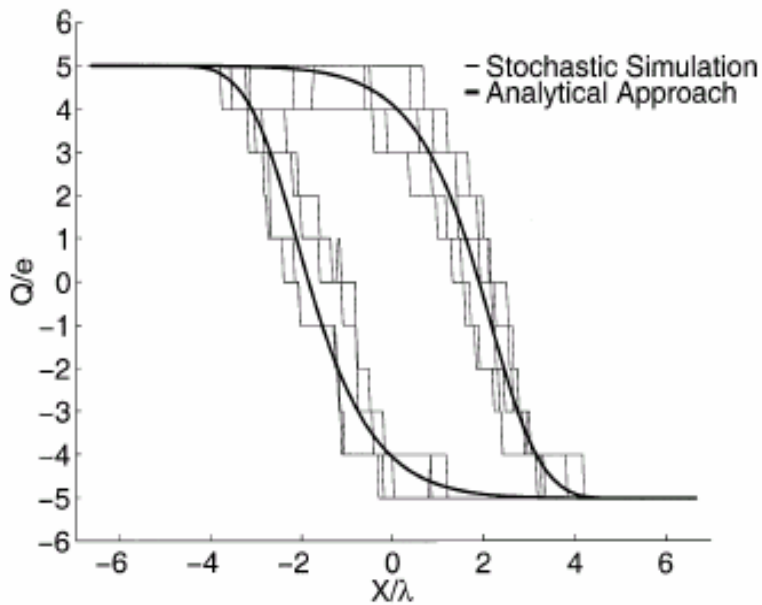
$$x(t) = x_0 \exp\{i\omega t + \alpha t\}$$

$$\alpha = \frac{1}{2}\{\gamma_{thr} - \gamma\}$$

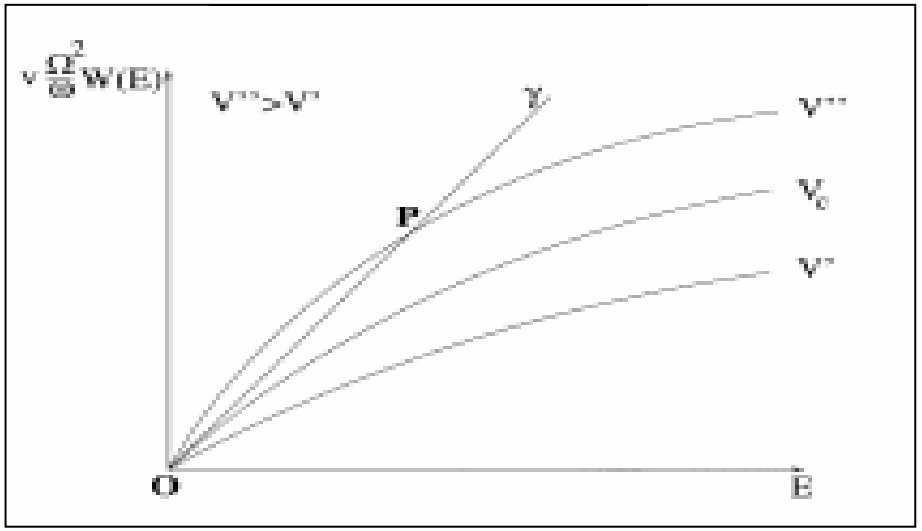
$$\gamma_{thr} = \frac{\eta\omega}{2} f(\omega_R/\omega); \quad \omega_R{}^{-1} = R_0 C; \quad f(x) = \frac{x}{x^2+1}$$

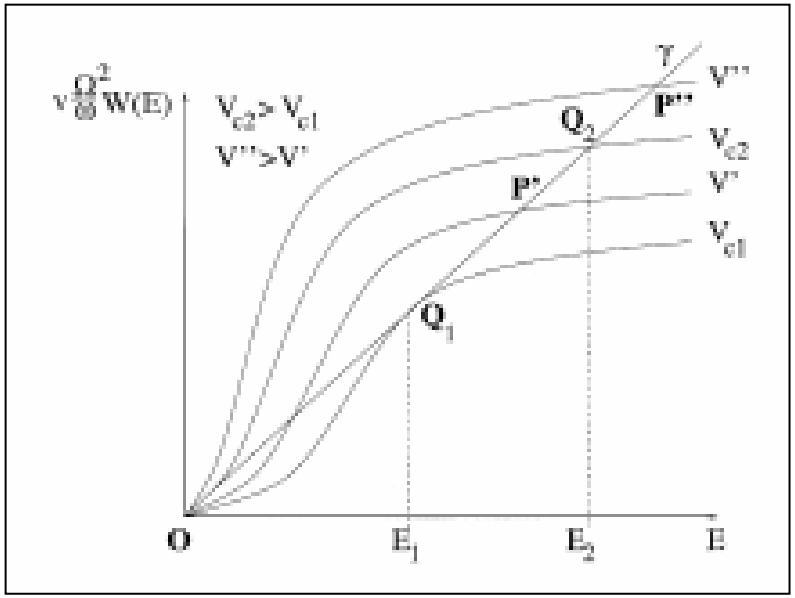
Stable Shuttle Vibrations





Different Scenarios of a Shuttle Instability





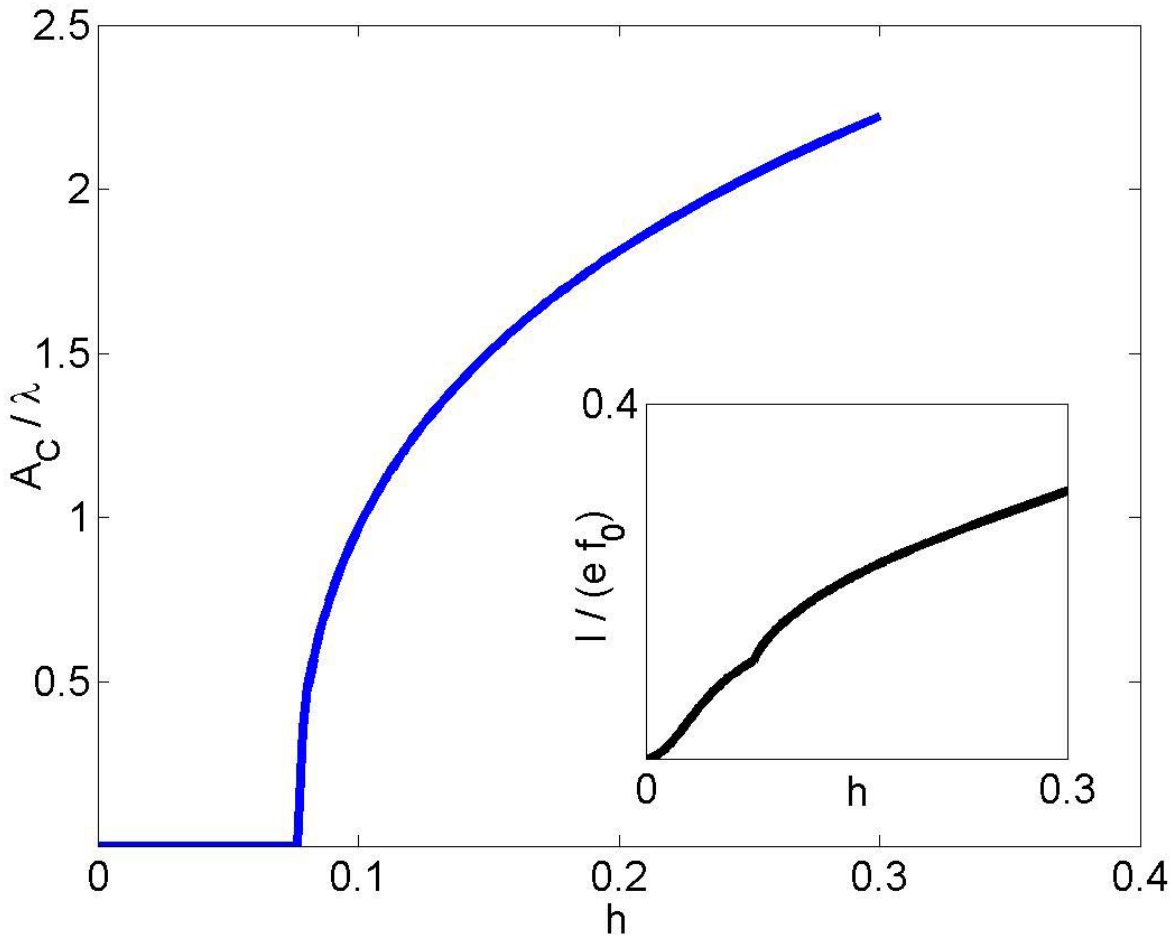
"SOFT" instability

"HARD" instability

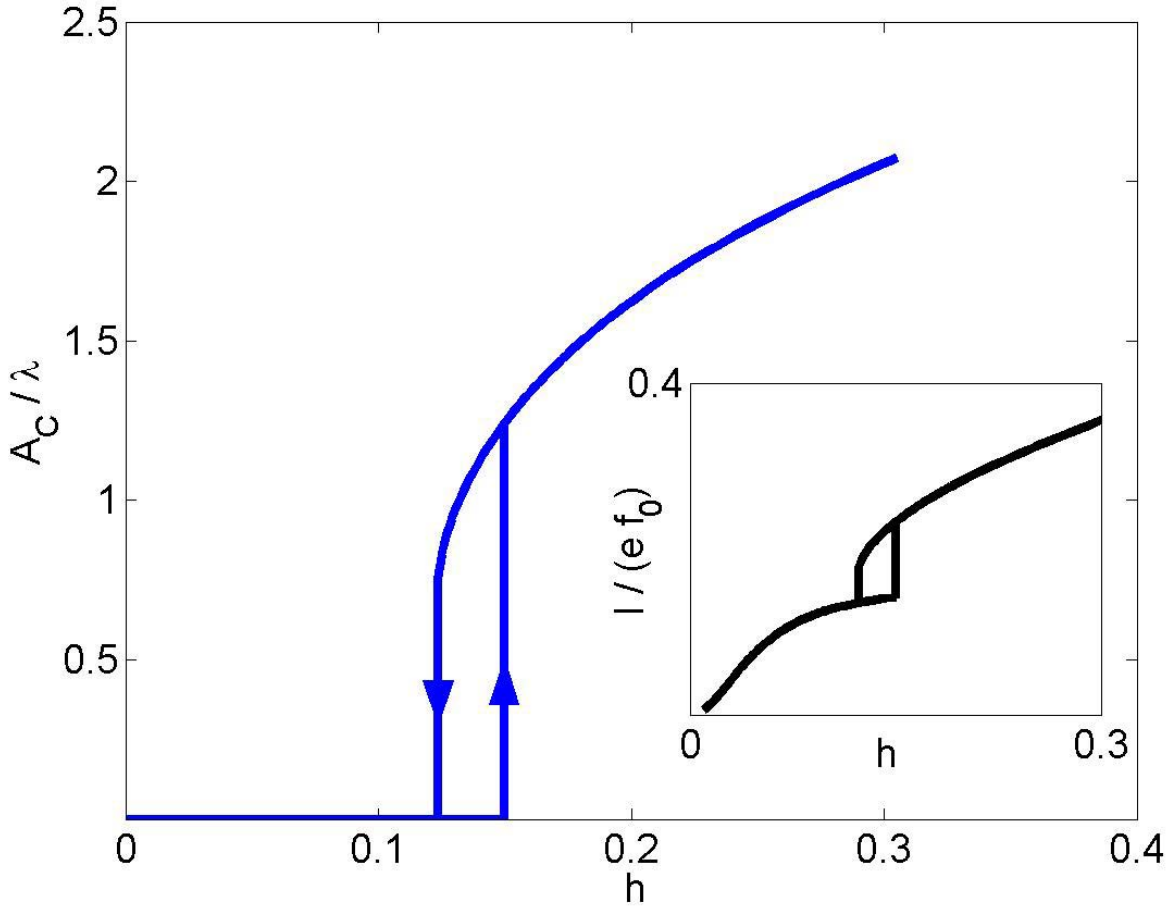
$\omega_R / \Gamma < \frac{\sqrt{3}}{2}$ *Only **one** stable mechanical regime is possible*

$\omega_R / \Gamma > \frac{\sqrt{3}}{2}$ ***Two** regimes of locally stable mechanical operations*

”Soft” Onset of Shuttle Vibrations

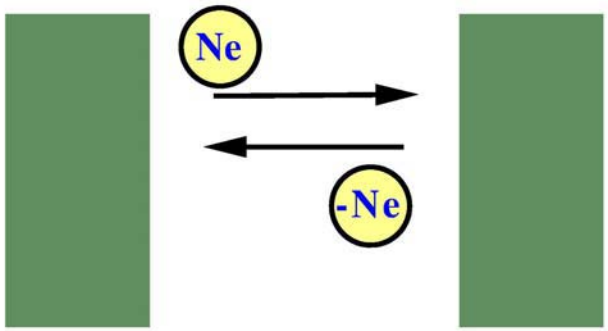


”Hard” Onset of Shuttle Vibrations



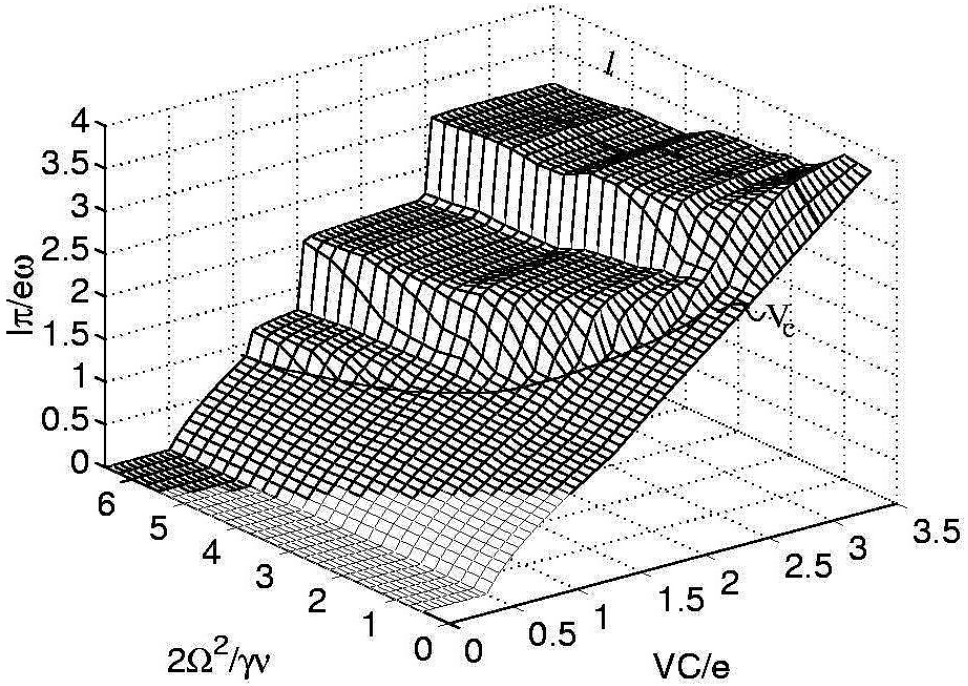
Shuttling of Electronic Charge

Instability occurs at $V > V_c$ and develops into a limit cycle of dot vibrations. Both V_c and vibrational amplitude are determined by dissipation.



$$I = 2eN\omega$$

$$N = \mathrm{Int}\left[\frac{VC}{2e}\right]$$

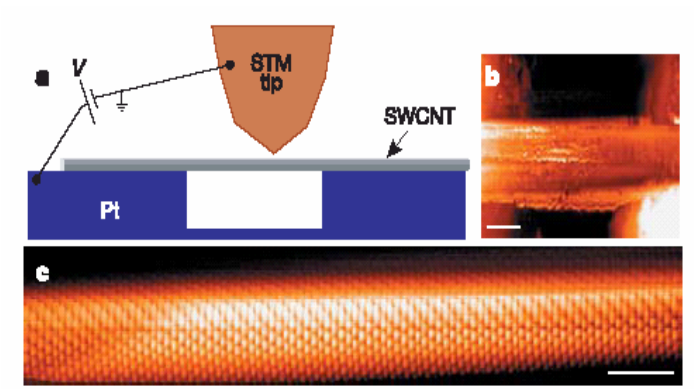


Electromechanical Instabilities of Suspended CNTs

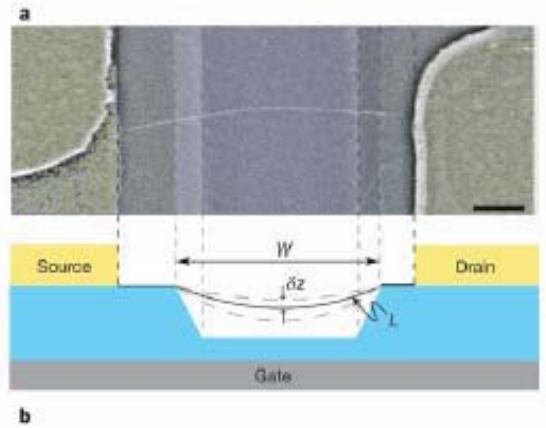
L. M. Jonsson *et al.* Nano Lett. **5**, 1165 (2005)

CNT-Based Nanoelectromechanics

A suspended CNT has mechanical degrees of freedom => study electromechanical effects on the nanoscale.

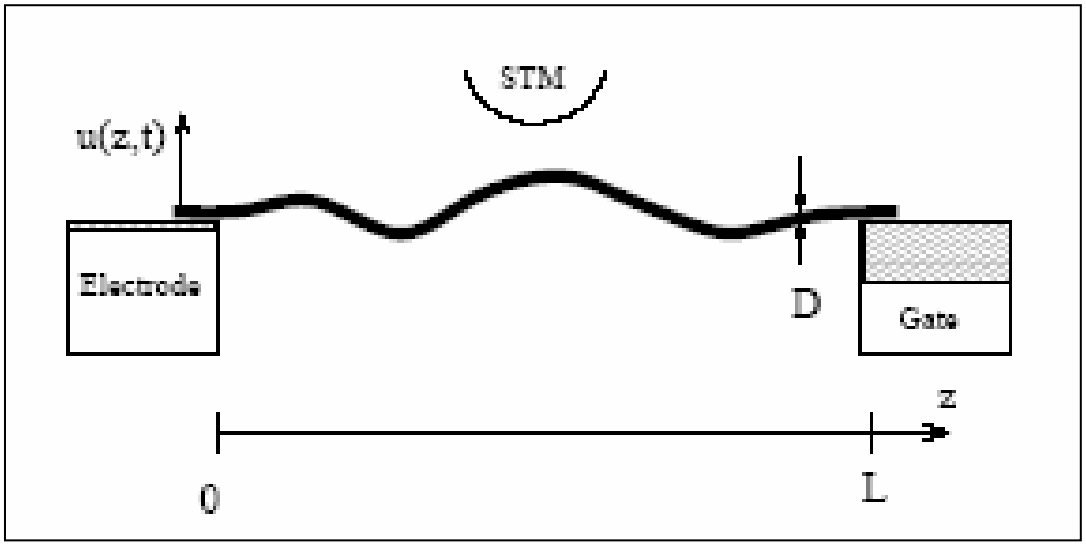


B. J. LeRoy et al., Nature **432**, 371 (2004)



V. Sazonova et al., Nature **431**, 284 (2004)

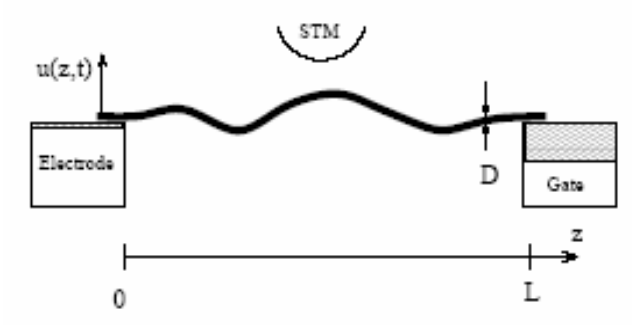
Model



The model system.

- An STM-tip, biased at $-V/2$ is placed above a suspended CNT with diameter D and length L.
- One end of the nanotube is connected to an electrode biased at $V/2$.
- A gate electrode with potential Vg, used to control the electronic levels on the nanotube.
- Only deformation of the tube in the plane is considered and shape of the nanotube is given by $u(z; t)$.
- The nanotube is considered to be a metallic island between the STM and the electrode.
- Tunnelling from the STM to the nanotube depends on tube deformation whereas tunneling matrix elements connecting the nanotube and the electrode are constant.

Formulation of the Problem: Mechanics of Suspended CNT



Lagrangian of the vibrating string

$$\tilde{L} = \int_0^L dz \left\{ \frac{\rho S}{2} \dot{u}^2(z,t) - \frac{EI}{2}\left(u''(z,t)^2\right) \right\}$$

E*-the Youngs modulus;* ***I****-moment of inertia of the cross section*

Equation of motion:

$$\rho S \ddot{u}(z,t) + EIu''''(z,t) = 0$$

$$u(z=o) = u(z=L) = u'(z=0) = u'(z=L) = 0$$

Vibrational modes

$$u_n(z,t) = e^{i\omega_n t} u_n(z)$$

$$u_n(z) = A_n \left\{ \left(\sin k_n L - \sinh k_n L\right)\left(\cos k_n z - \cosh k_n z\right) - \left(\cos k_n L - \cosh k_n L\right)\left(\sin k_n z - \sinh k_n z\right) \right\}$$

Spectrum of vibrations

$$\cos k_n L \cosh k_n L = 1$$

$$\omega_n = \frac{c_n}{L^2}\sqrt{\frac{EI}{\rho S}}\,; \quad c_{n=1;2;3\ldots} = 22.4;\, 61.7;\, 121.9;\, \ldots$$

Formulation of the Problem: Electrostatics of Suspended CNT

The electrostatic energy Ec should be included in the Lagrangian of vibrating CNT.
Deformation u(z,t) causes an extra charge Q on the tube.
To linear order in Q the electrostatic energy Ec({u(z,t)},Q) is:

$$Ec\left(\{u(z,t)\},Q\right) = Q\int dz K(z) u(z,t)$$

In the limit of STM size **d** much smaller than the length **L** of the CNT one can model K(z) as:
(E-electric field in te vicinity of STM tip)

$$K(z) \approx Ed\delta(z)$$

$$u(z,t) = \sum_n X_n(t) u_n(z,t)$$

Nanoelectromechanics of Suspended CNT

Modified circuit model for the charge dynamics

$$\frac{dQ}{dt} = I_L + I_R \quad \Rightarrow \quad \frac{dQ}{dt} = \frac{Q}{C}\left(\frac{1}{R_L} + \frac{1}{R_R}\right) + \frac{V}{C}\left(\frac{C_R}{R_L} - \frac{C_L}{R_R}\right)$$

$$R^{-1}{}_R = R_0{}^{-1}\left(1 \pm \frac{u(z=0,t)}{\lambda}\right)$$

Equation for the mechanical vibrations

$$\rho S\ddot{u}(z,t) + EIu''''(z,t) = EQ(\{u(z=0,t)\})$$

$$\ddot{X}_n(t) + \omega_n^2 X_n(t) - \gamma \dot{X}_n(t) = \varsigma \frac{EQ(\sum_m u_m X_m(t))}{\rho S}$$

γ- damping of the mechanical vibrations

- *Coupling of electrons to a set of vibrators*
- *Vibrations in different modes are coupled*

$$u(z,t) = \sum_n X_n(t) u_n(z,t)$$

Nanoelectromechanical Instability

- *Linearization over Xn(t)*
- *Weak electromechanical coupling:*

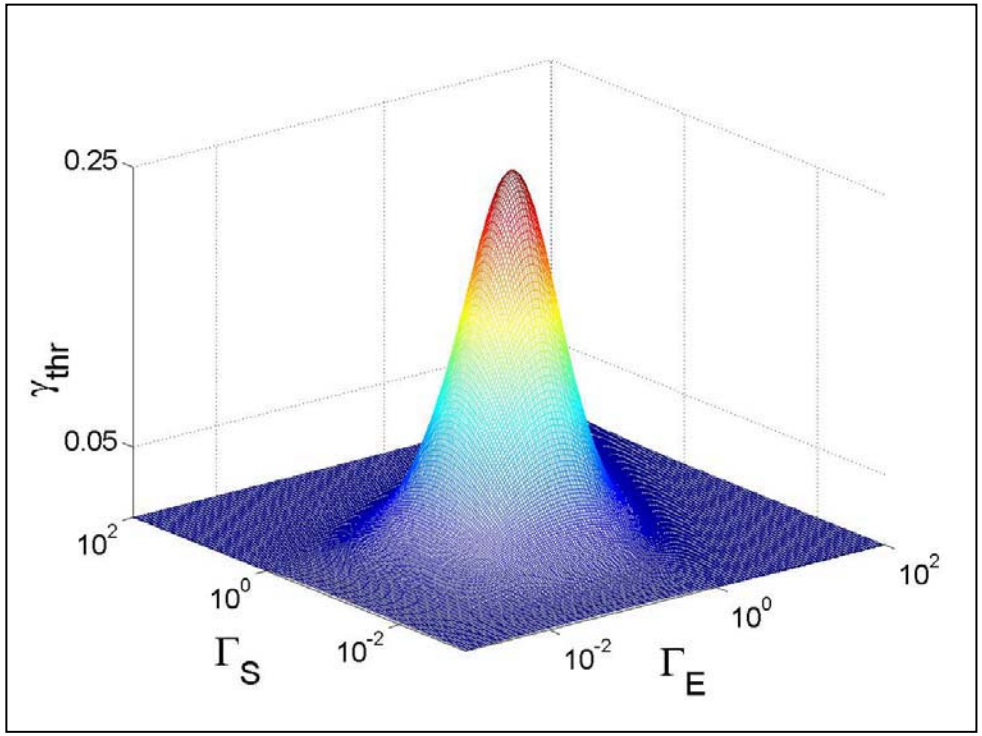
$$\eta = \frac{CVE\lambda}{2M\lambda^2\omega^2} << 1$$

Dynamics of different vibrating modes decouples

$$X_n(t) = X_{n0} \exp\{i\omega_n t + \alpha_n t\}$$

$$\alpha_n = \frac{1}{2}\{\gamma_{thr\,n} - \gamma\}$$

Optimal Conditions for NEM Instability



Optimal conditions:

- $\Gamma = \omega 1$ and
- Symmetric tunneling rates

Quantum Nanoelectromechanics of Shuttle Systems

$$\delta X \delta P \cong \hbar$$

$$\delta X \cong 2X_0 \equiv \sqrt{\frac{2\hbar}{M\omega}}$$

If $\dfrac{R(X+\delta X)}{R(X)} >> 1$ then quantum fluctuations of the grain significantly affect nanoelectromechanics.

Conditions for Quantum Shuttling

$$\frac{2X_0}{\lambda} \gtrsim 1$$

λ – Tunneling length

1. Fullerene based SET

$$\omega \cong 1 \text{ THz}$$

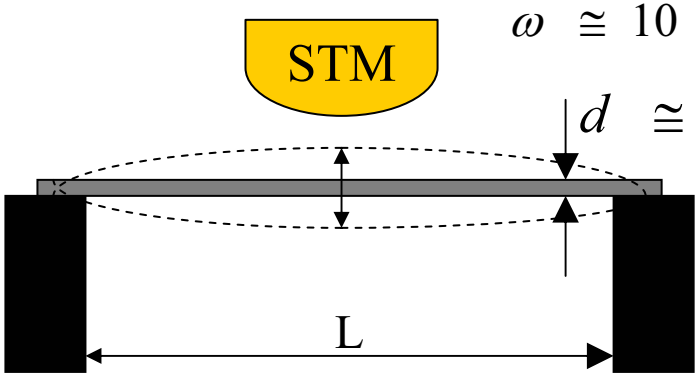
$$\frac{X_0}{\lambda} \cong 0.1$$

Quasiclassical shuttle vibrations.

2. Suspended CNT

$$\omega \cong 10^{14} \text{ Hz} \left(\frac{d}{L}\right)^2$$

$$d \cong 1 \text{ nm}$$



$\omega \cong 10^8 - 10^9$ Hz for SWNT with $L \cong 1\,\mu m$