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SUMMER SCHOOL IN COSMOLOGY AND ASTROPARTICLE PHYSICS

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Inflation

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Inflation

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Plan of the lectures:

- Inflation: a general outlook (today)
- Basic inflationary models (new inflation, chaotic inflation, hybrid inflation)
- Creation of matter after inflation (reheating)
- Quantum cosmology and initial conditions for inflation
- Is our universe a sphere, a torus, or a fractal?
- Eternal inflation and string theory landscape

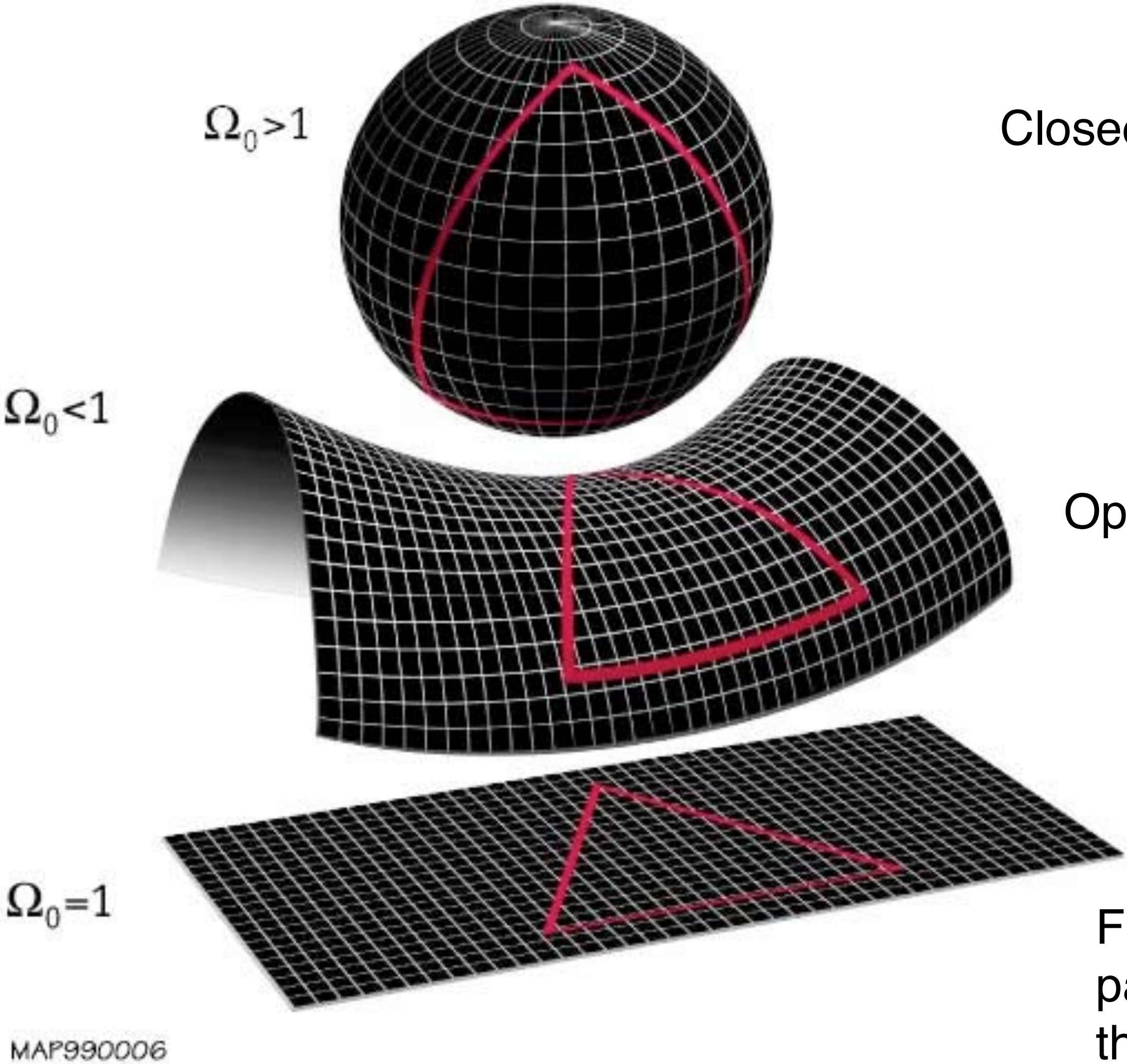
Two major cosmological discoveries:

- The new-born universe experienced rapid acceleration (inflation)

- A new (slow) stage of acceleration started 5 billion years ago (dark energy)

How did it start, and how it is going to end?

Closed, open or flat universe

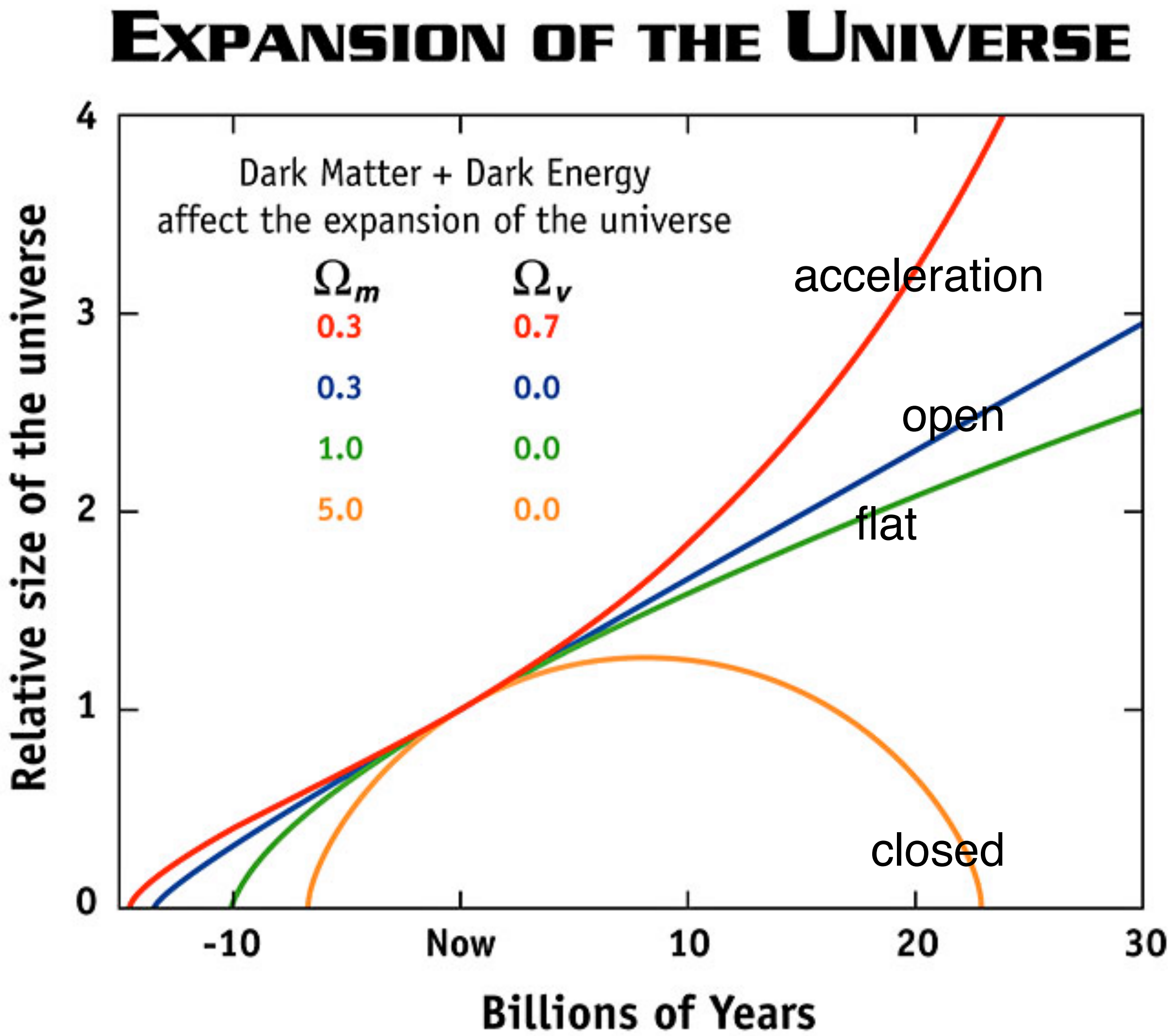


Closed universe. Parallel lines intersect

Open universe. Parallel lines diverge

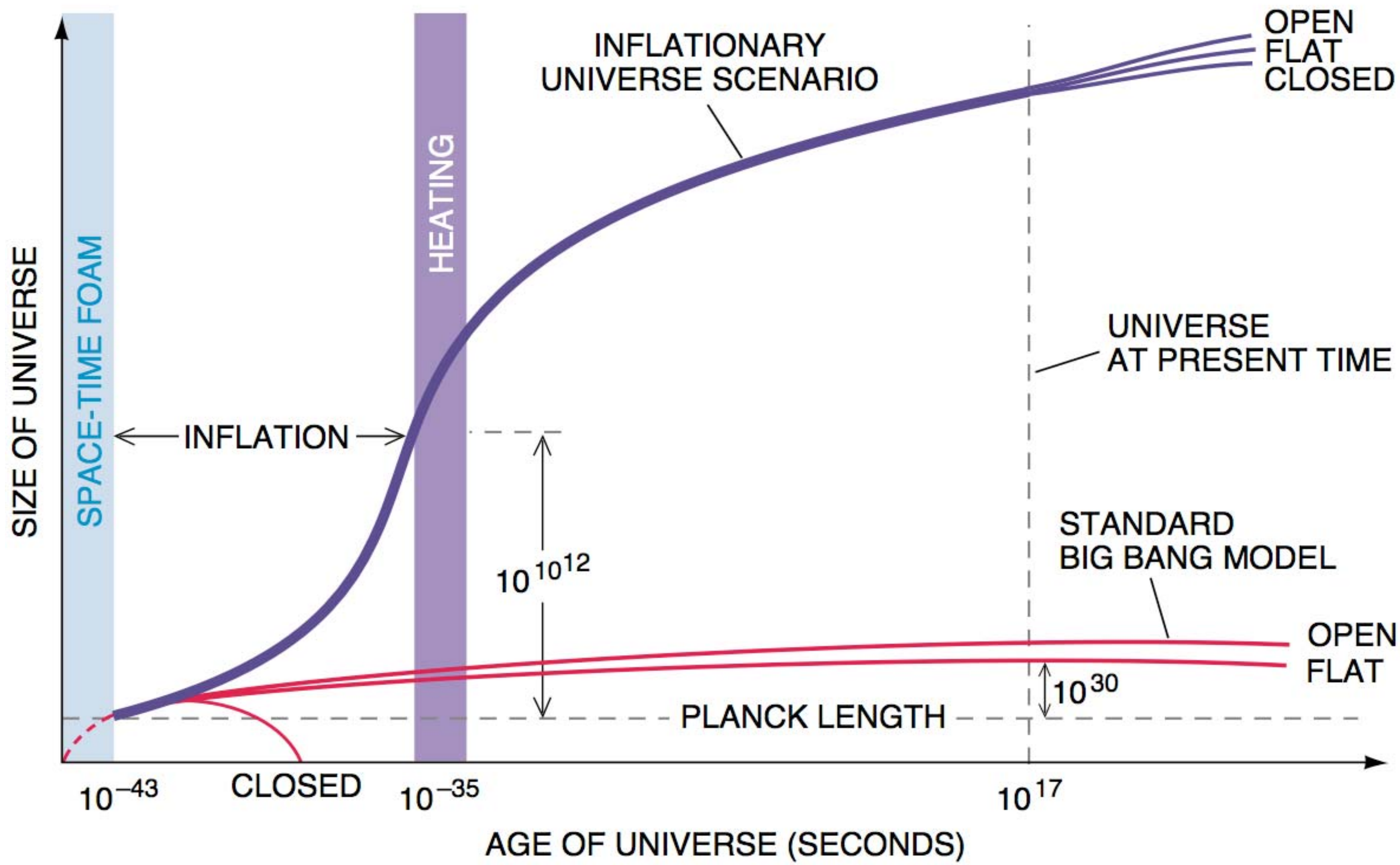
Flat universe. Parallel lines remain parallel, but the distance between them grow with time

Big Bang Theory



If vacuum has positive energy density (dark energy), the universe may accelerate, as it is shown on the upper curve. Such universe may not collapse even if it is closed.

Inflationary Universe



Inflation is an extremely rapid acceleration in the universe soon after its creation.

Problems of the Big Bang theory:

- What was before the Big Bang?
- Why is our universe so **homogeneous** (better than 1 part in 10000)?
- Why is it **isotropic** (the same in all directions)?
- Why all of its parts started expanding simultaneously?
- Why is it **flat**? Why parallel lines do not intersect? Why is the universe so large? Why does it contain so many particles?

Where did the energy come from?

Some basic facts:

1) Energy of matter in the universe **IS NOT CONSERVED:**

$$dE = -p\, dV$$

Volume V of an expanding universe grows, so its energy decreases if pressure p is positive.

2) **Total** energy of matter and of gravity (related to the shape and the volume of the universe) is conserved, but this conservation is somewhat unusual:

The sum of the energy of matter and of the gravitational energy is equal to

zero

Energy of photons in the Big Bang theory

The total energy of radiation in the universe now is greater than **10^{53} g**. According to the Big Bang theory, the total number of photons in the universe practically did not change during its evolution, but the energy of each photon decreased as the temperature of the universe T. The standard classical description of the universe becomes possible at the Planck time, when the temperature of the universe was 10^{32} times greater than now. At that time, the energy of radiation was greater than $10^{53} \times 10^{32}$ = **10^{85} g**

So before the Big Bang there was NOTHING, and then suddenly we got A HUGE AMOUNT OF ENERGY

Where did it come from?

Extending this investigation back to the cosmological singularity, where T was infinite, one finds that in order to create the universe in the Big Bang singularity one should have

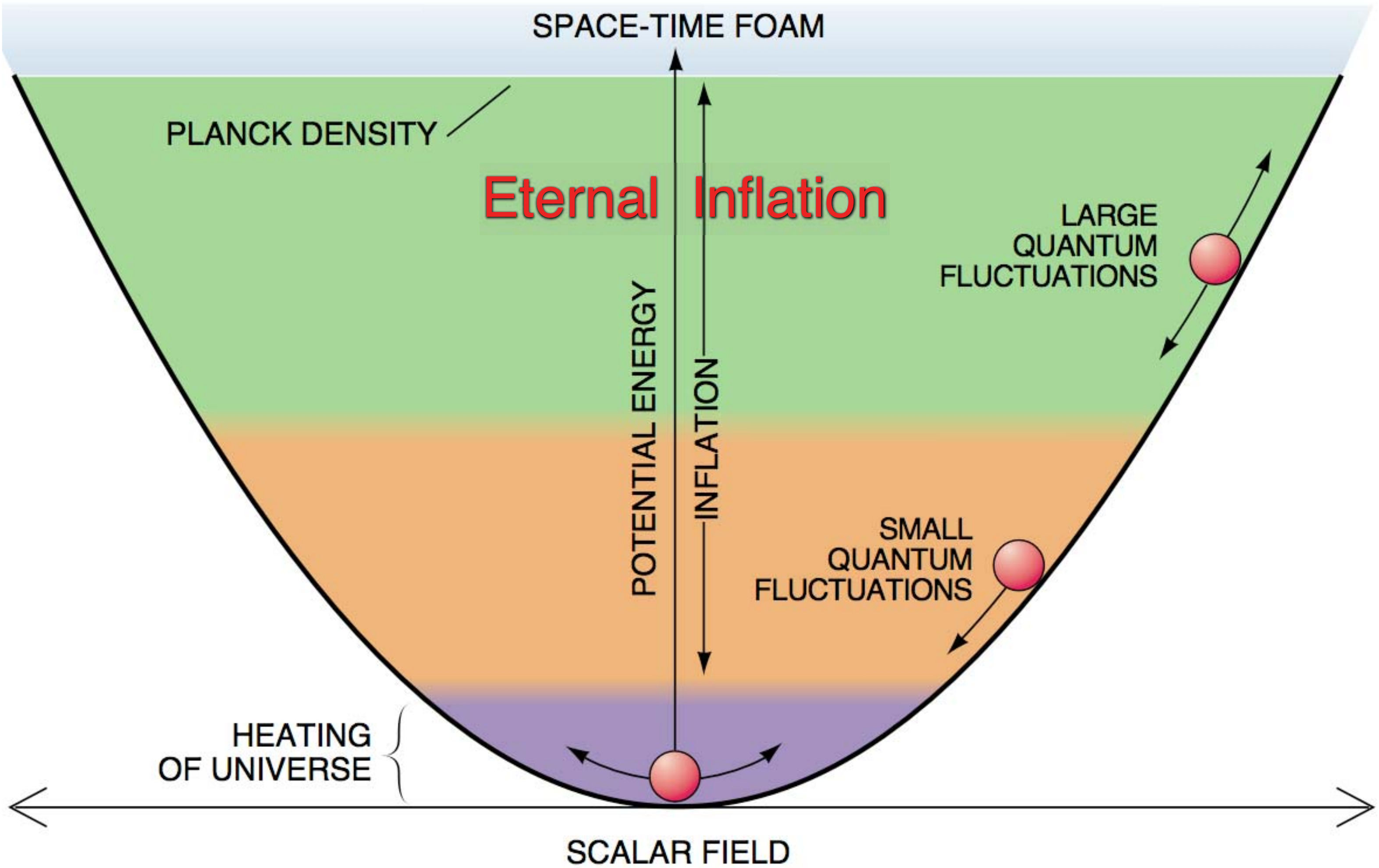
INFINITE AMOUNT OF ENERGY

Inflationary theory

solves many problems of the old Big Bang theory, and explains how the universe could be created from less than one milligram of matter

Inflation as a theory of a harmonic oscillator

$$V(\phi) = \frac{m^2}{2}\phi^2$$



Equations of motion:

- **Einstein equation:**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{m^2}{6}\phi^2$$

- **Klein-Gordon equation:**

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$

Compare with equation for the harmonic oscillator with friction:

$$\ddot{x} + \alpha\dot{x} = -kx$$

Logic of Inflation:

Large φ ⟶ large H ⟶ large friction

field φ moves very slowly, so that its potential energy for a long time remains nearly constant

$$H = \frac{\dot{a}}{a} = \frac{m\phi}{\sqrt{6}} \approx \text{const}$$

$$a \sim e^{Ht}$$

This is the stage of inflation

Inflationary solution for the theory $V = \frac{m^2\phi^2}{2}$ is

$$\phi(t) = \phi_0 - \sqrt{\frac{2}{3}}\, m\, t\,, \qquad a(t) = a_0 \exp\left(\frac{\phi_0^2 - \phi^2(t)}{4}\right) .$$

If $\phi_0 \gg \phi(t) \sim 1$ at the end of inflation, then the total amount of inflation is

$$P \sim \exp \frac{\phi_0^2}{4}.$$

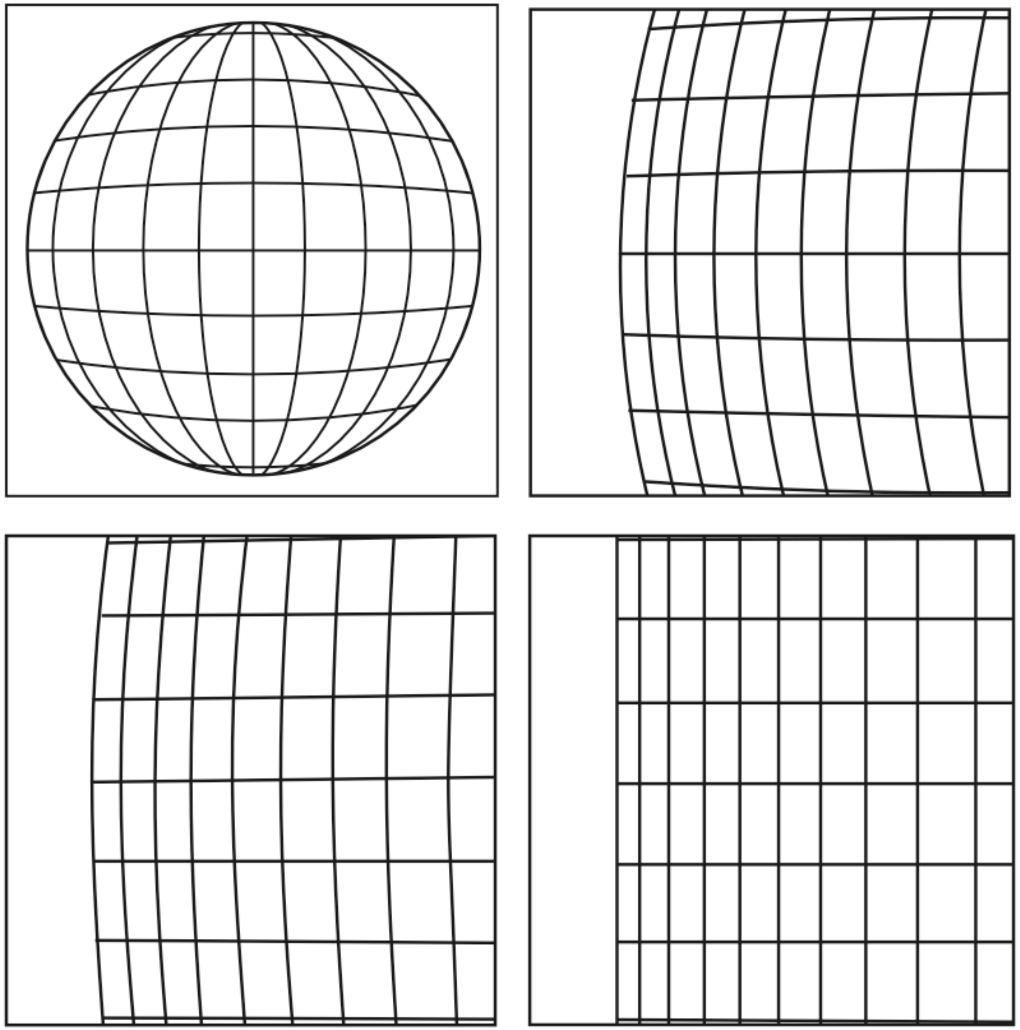
If initially the density was Planckian, $\mathrm{V}(\phi_0) \sim \frac{m^2\,\phi_0^2}{2} \sim 1$, and $m \sim 10^{-5}$, then

$$\mathrm{P} \sim \exp\left(\frac{1}{m^2}\right) \sim 10^{10^{10}} .$$

Inflation makes the universe flat, homogeneous and isotropic

In this simple model the universe typically grows $10^{10000000000}$ times during inflation.

Now we can see just a tiny part of the universe of size ct = 10^{10} light yrs. That is why the universe looks homogeneous, isotropic, and flat.

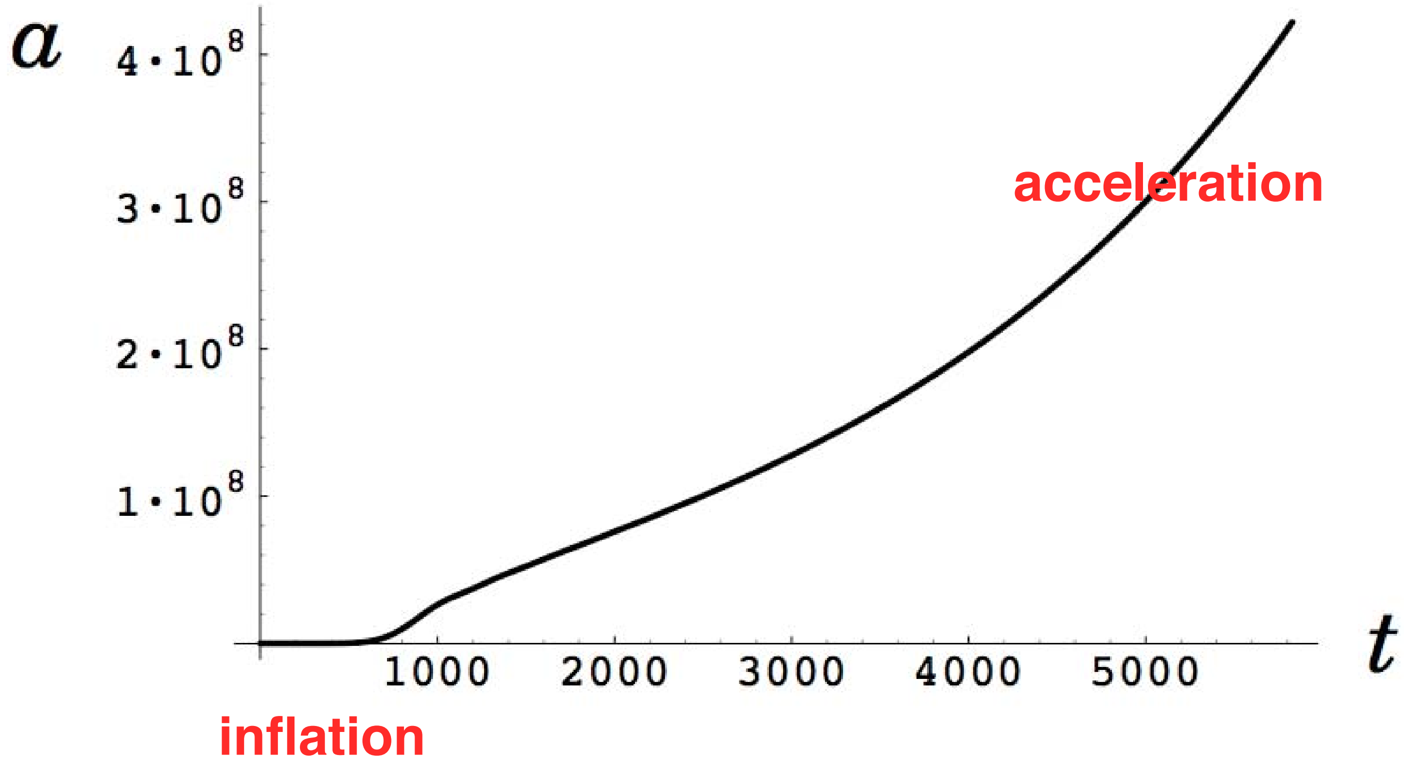


Add a constant to the inflationary potential

- obtain inflation and acceleration

$$V = \frac{m^2}{2}\phi^2 + \Lambda$$

The simplest model of inflation AND dark energy



Note that the energy density of the scalar field during inflation remains nearly constant, because at that stage the field practically does not change.

Meanwhile, the total volume of the universe during inflation grows exponentially, as $a^3(t) \sim e^{3Ht}$.

Therefore the total energy of the scalar field also grows exponentially, as $E \sim e^{3Ht}$.

After inflation, scalar field decays, and all of its energy is transformed into the exponentially large energy/mass of particles populating our universe.

We can start with a tiny domain of the smallest possible size (Planck length $l_P = M_P^{-1} \sim 10^{-33}$ cm) at the largest possible density (Planck density $M_P^4 \sim 10^{94}$ g/cm^3). The total energy of matter inside such a domain is $l_P^3 M_P^4 \sim M_P \sim 10^{-5}$ g. Then inflation makes this domain much larger than the part of the universe we see now.

What is the source of this energy?

Energy density and pressure for the scalar field:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

If the scalar field moves slowly, its pressure is negative,

$$p = w\rho, \qquad w \approx -1$$

Therefore energy of matter grows, $dE = -p\,dV > 0$

Existence of matter with $p < 0$ allows the total energy of matter to grow at the expense of the gravitational energy, which becomes equally large but negative.

$$E_{matter} \sim + e^{3Ht}$$

$$E = 0$$

Total energy of the universe

Exponential instability

Simultaneous creation of space and matter

$$E_{space} \sim - e^{3Ht}$$

If such instability is possible, it appears over and over again. This leads to <u>eternal inflation,</u> which we will discuss later.

So inflation may start in the universe of the Planck mass (energy) $E \sim M_P \sim 10^{-5}$ g, at the Planck time $t_P \sim M_P^{-1} \sim 10^{-43}$ s.

But where did these initial 10^{-5} g of matter come from?

Uncertainty relation (in units $c = \hbar = 1$):

$$\Delta E \cdot \Delta t = M_P \cdot M_p^{-1} = 1$$

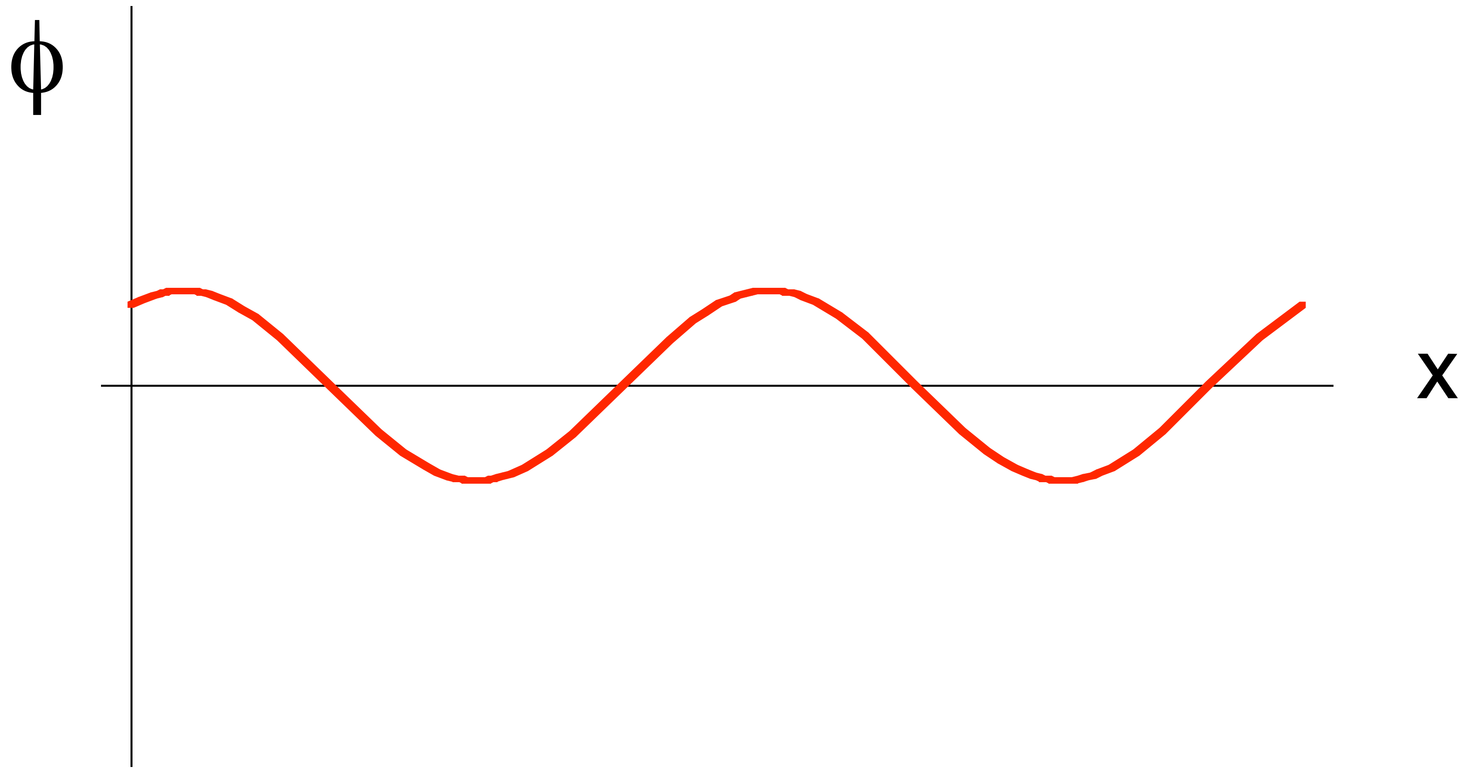
Thus the emergence of the initial 10^{-5} g of matter is a simple consequence of the quantum mechanical uncertainty principle. And once we have 10^{-5} g of matter in the form of a scalar field, inflation begins, and energy becomes exponentially large.

If one can create the whole universe from one milligram of matter, what other miracles are possible?

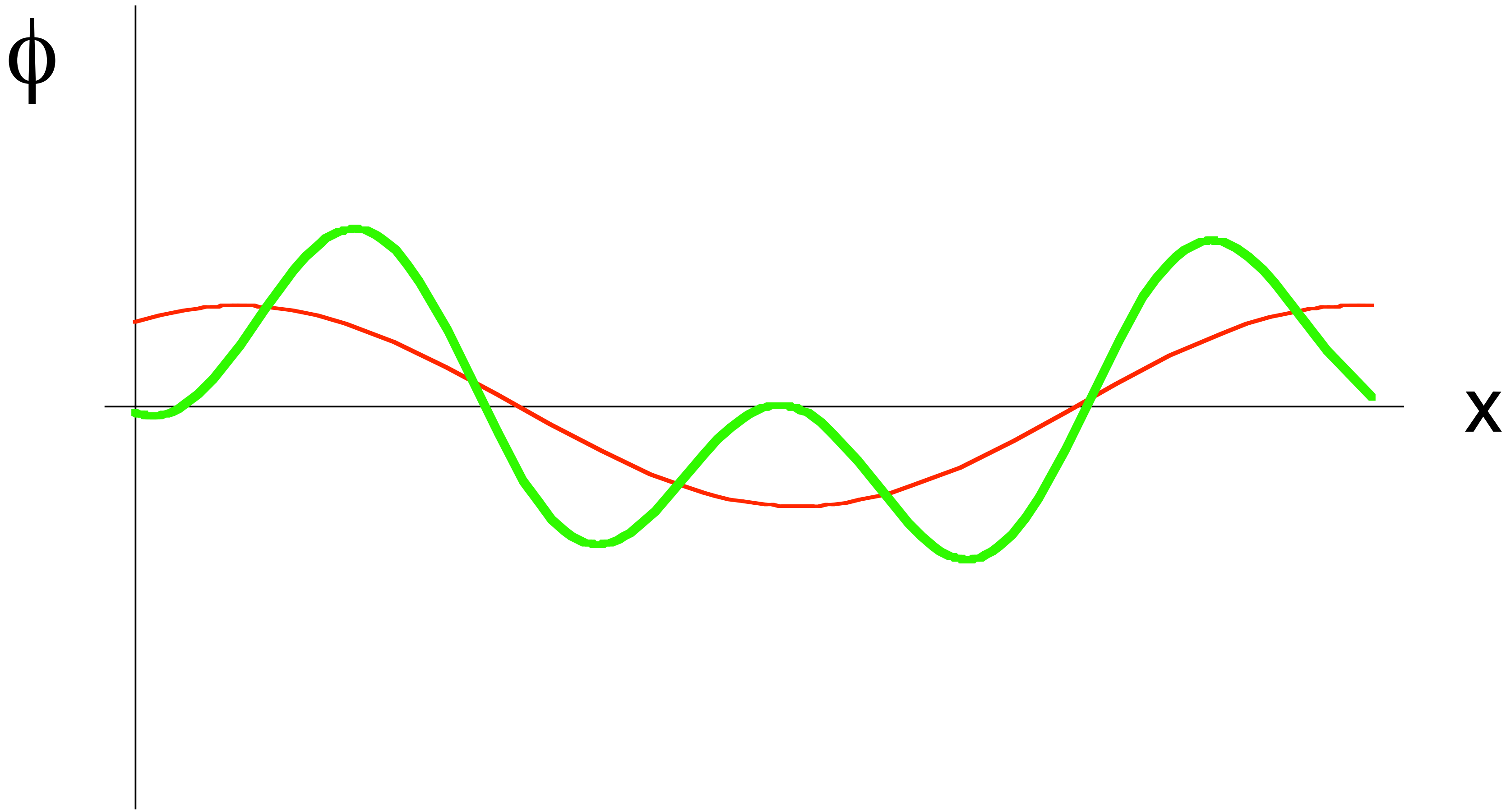
1) Inflation can create galaxies from quantum fluctuations.

2) Inflationary fluctuations can create new exponentially large parts of the universe (eternal inflation).

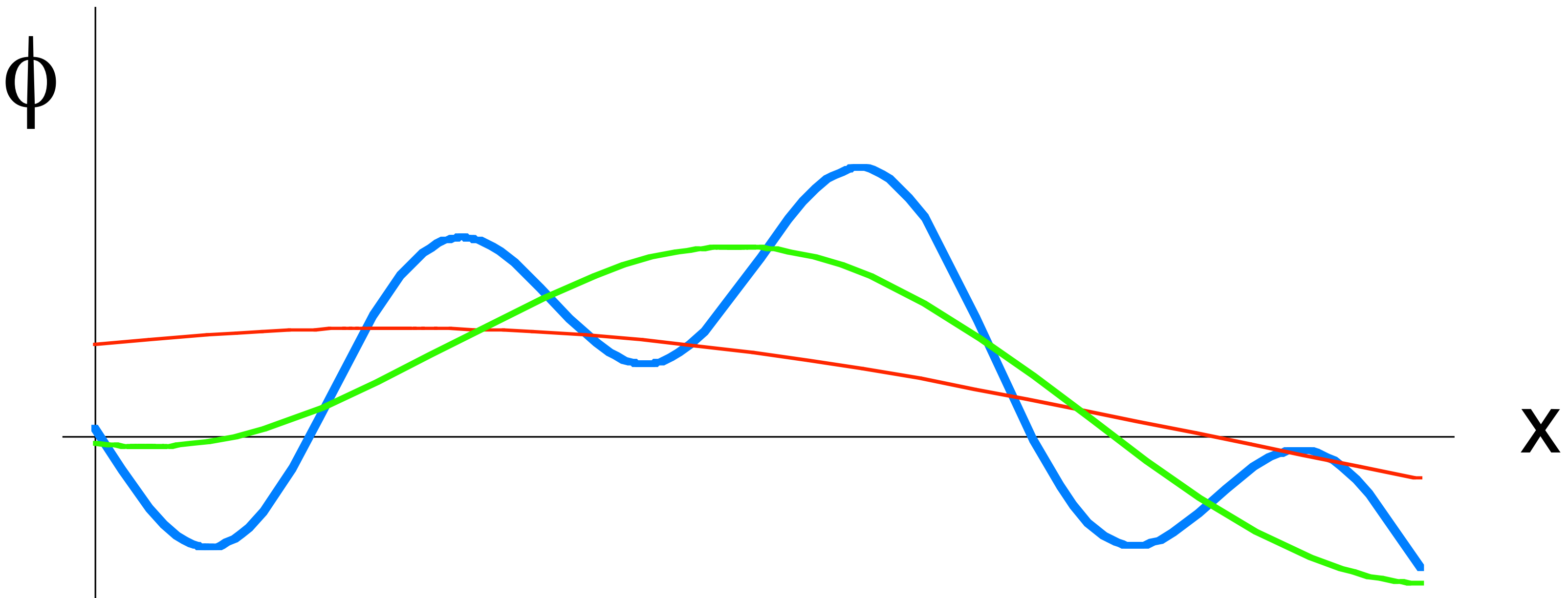
Quantum fluctuations produced during inflation



Small quantum fluctuations of all physical fields exist everywhere. They are similar to waves in the vacuum, which appear and then rapidly oscillate, move and disappear. Inflation stretched them, together with stretching the universe. When the wavelength of the fluctuations became sufficiently large, they stop moving and oscillating, and do not disappear. They look like frozen waves.



When expansion of the universe continues, new quantum fluctuations become stretched, stop oscillation and freeze on top of the previously frozen fluctuations.



This process continues, and eventually the universe becomes populated by inhomogeneous scalar field. Its energy takes different values in different parts of the universe. These inhomogeneities are responsible for the formation of galaxies.

Sometimes these fluctuations are so large that they substantially increase the value of the scalar field in some parts of the universe. Then inflation in these parts of the universe occurs again and again. In other words, the process of inflation becomes eternal.

We will illustrate it now by computer simulation of this process.

Amplitude of perturbations of metric

The average amplitude of inflationary perturbations generated during a typical time interval H^{-1} is given by

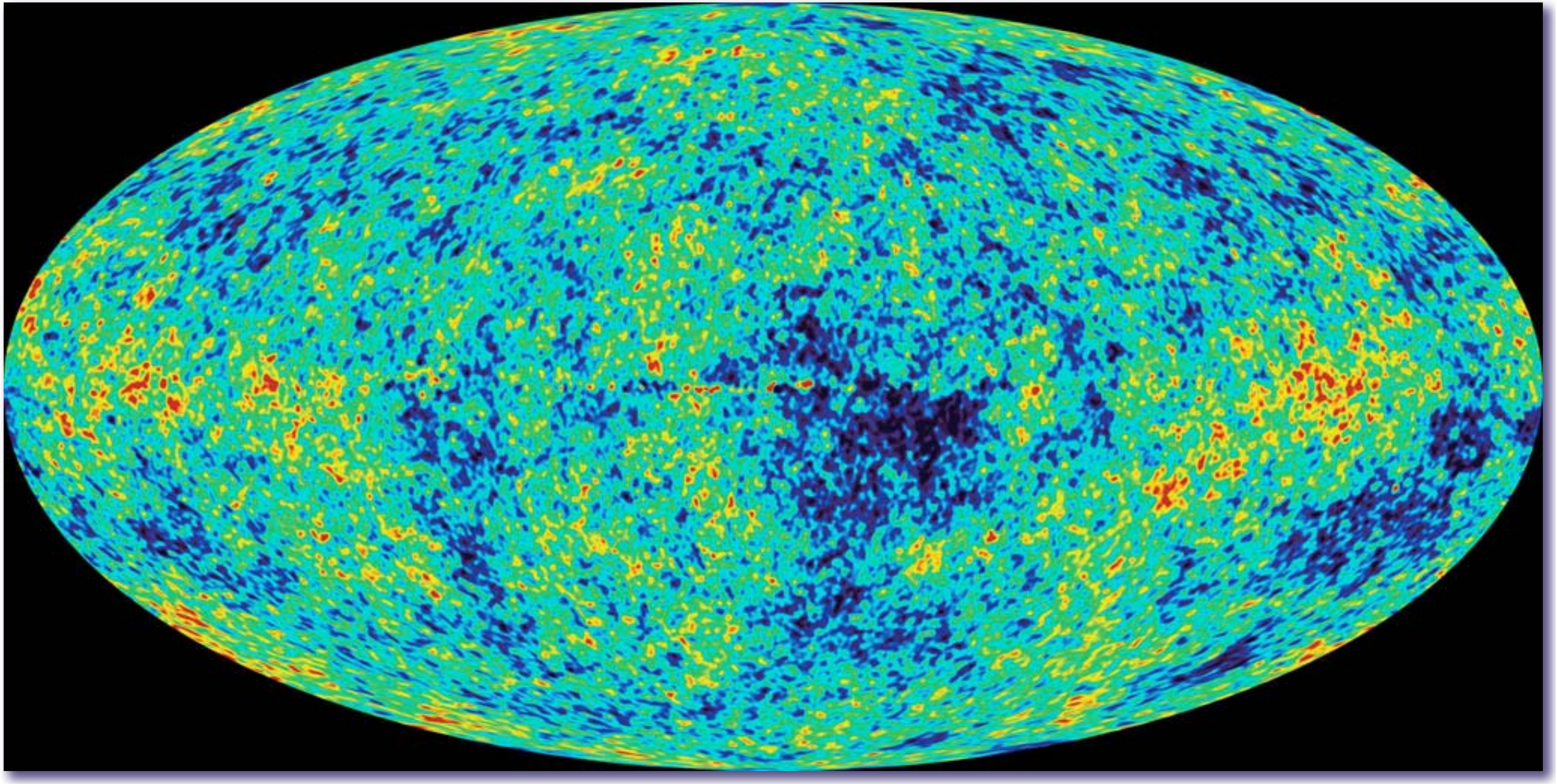
$$|\delta\phi(x)| \approx \frac{H}{2\pi} \ .$$

These fluctuations lead to density perturbations that later produce galaxies. To make a rough estimate of these perturbations, note that the fluctuations of the field ϕ lead to a local time delay of the end of inflation, $\delta t = \frac{\delta\phi}{\dot\phi} \sim \frac{H}{2\pi\dot\phi}$. Once the post-inflationary stage begins, the density of the universe decreases as $\rho = 3H^2 \sim t^{-2}$. Therefore a local delay of expansion leads to a local density increase δ_H such that $\delta_H \sim \delta\rho/\rho \sim \delta t/t$. Combining these estimates together yields the famous result

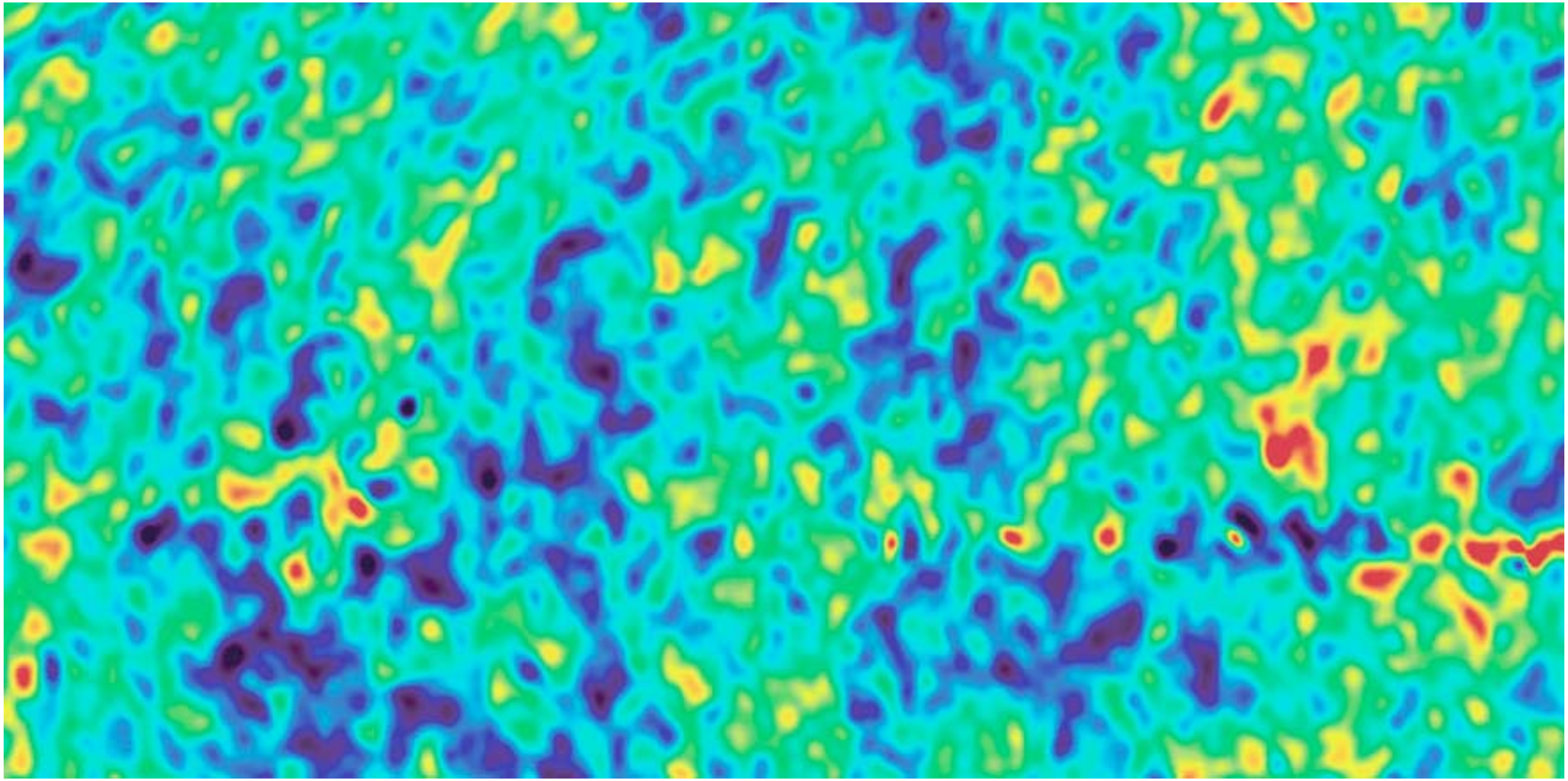
$$\delta_H \sim \frac{\delta\rho}{\rho} \sim \frac{H^2}{2\pi\dot\phi} \ .$$

For the simplest model $m^2\phi^2/2$ the amplitude of perturbations is $\delta_H \sim \frac{m\phi^2}{5\pi\sqrt{6}} \sim m \ln l$ because $l \sim e^{\phi^2/4}$. COBE-WMAP normalization gives $m \sim 3 \times 10^{-6}$.

WMAP and the temperature of the sky



This is a photographic image of quantum fluctuations blown up to the size of the universe

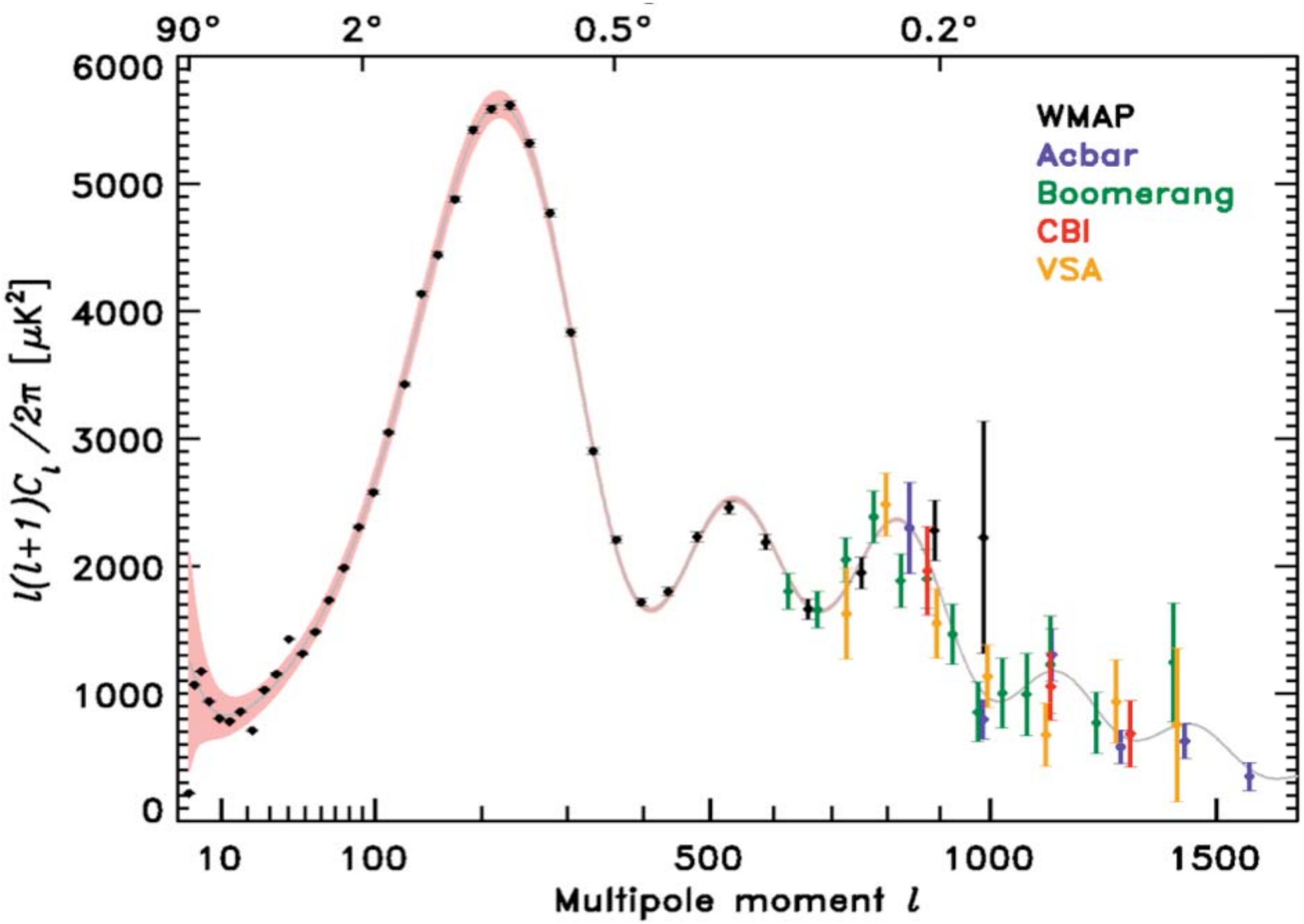


WMAP

and cosmic microwave background anisotropy

Black dots - experimental results (WMAP)

Pink line - predictions of inflationary theory



Consider a more complicated theory :

$$V_0 + \frac{m^2}{2}\phi^2 + \frac{\lambda_n}{n}\phi^n + \frac{\xi}{2}R\phi^2$$

Naively, one could expect that each coefficient in this sum is O(1). However, $|V_0| < 10^{-120}$, otherwise we would not be around.

A.L. 1984, Weinberg 1987

For a quadratic potential, one should have $m \sim 10^{-5}$ to account for the smallness of the amplitude of density perturbations $\delta_H \sim 10^{-5}$

Is there any reason for all other parameters to be small? Specifically, we must have

$$\lambda_n \ll m^2 \qquad \xi \ll 1$$

A simple argument:

Suppose that the upper bound on the inflaton field is given by the condition that the potential energy is smaller than Planckian, V < 1. In addition, the effective gravitational constant should not blow up. In this case

$$\phi^2 < \min\{m^{-2}, \lambda_n{}^{-\frac{2}{n}}, \xi^{-1}\}$$

In these models the total growth of volume of the universe during inflation (ignoring eternal inflation, which will not affect the final conclusion) is

$$e^{\phi^2} < \exp\left[\min\{m^{-2}, \lambda_n{}^{-\frac{2}{n}}, \xi^{-1}\}\right]$$

For a purely quadratic model, the volume is proportional to

$$e^{1/m^2} \sim e^{10^{10}}$$

But for the theory $\lambda\phi^4, \quad \lambda \sim m^{-2}$ the volume is much smaller:

$$e^{\lambda^{-1/2}} \sim e^{1/m} \sim e^{10^5}$$

The greatest growth by a factor of $e^{10^{10}}$ occurs for

$$\lambda_n \ll m^n \qquad \xi \ll m^{-2}$$

But in this case at the end of inflation, when

$$\phi \sim 1 \qquad R \sim m^2$$

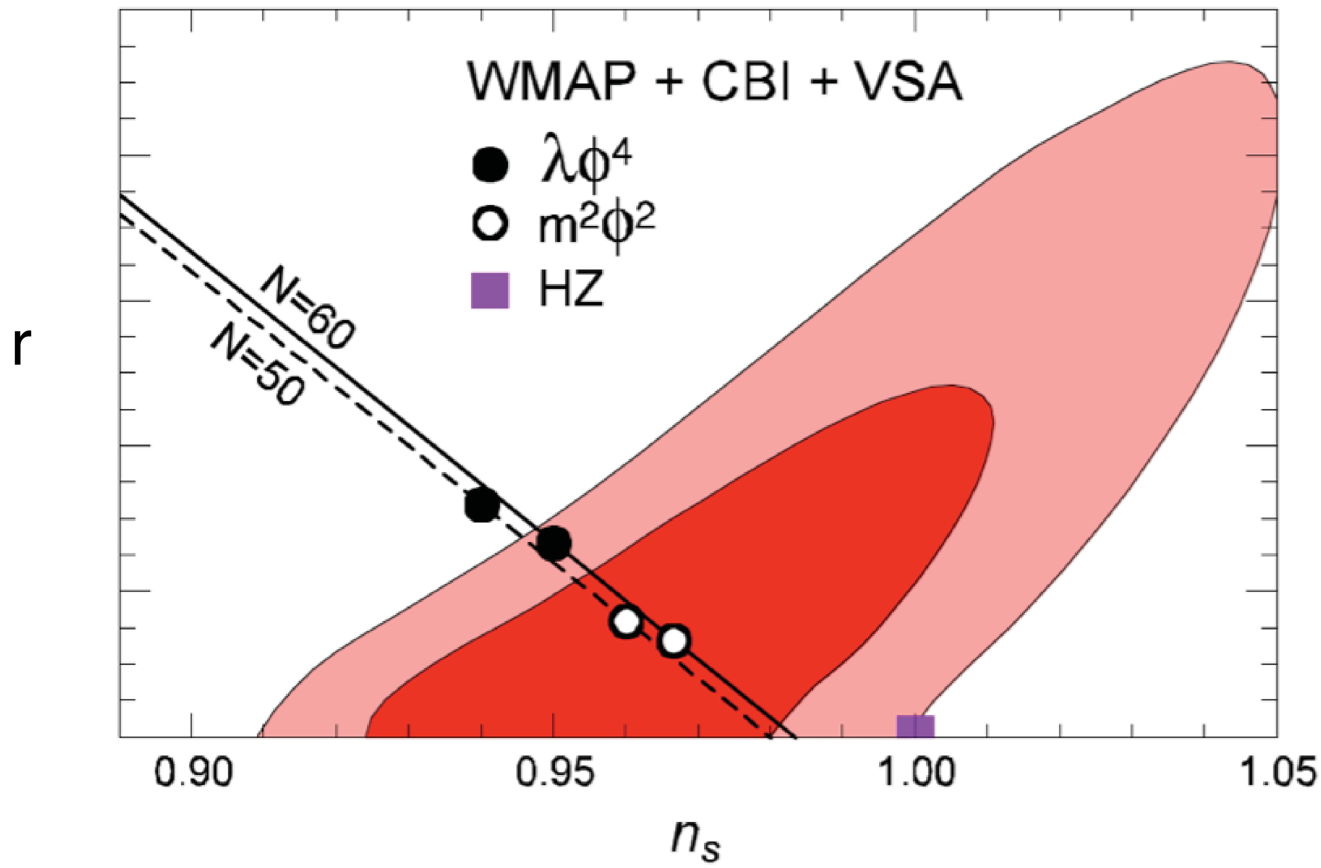
one has

$$\begin{aligned} V &= V_0 + \frac{m^2}{2}\phi^2 + O(m^n) + O(m^4) \\ &\approx V_0 + \frac{m^2}{2}\phi^2 \end{aligned}$$

Thus, if we have a choice of inflationary parameters (e.g. in string landscape scenario), then the simplest chaotic inflation scenario is the best.

This may explain why chaotic inflation is so simple: A power-law fine-tuning of the parameters gives us an exponential growth of volume, which is maximal for a purely quadratic potential

$$V = V_0 + \frac{m^2}{2}\phi^2$$



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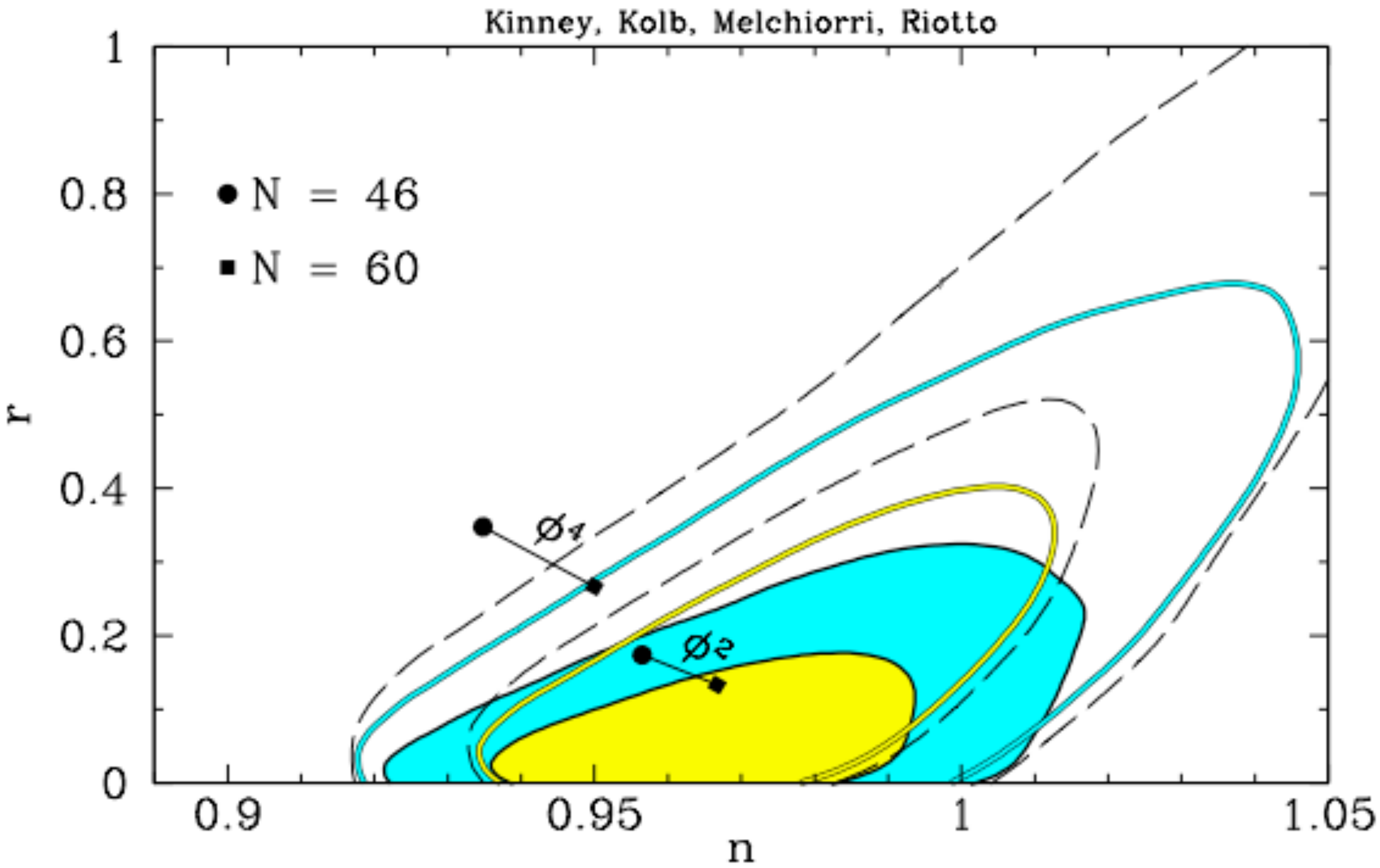


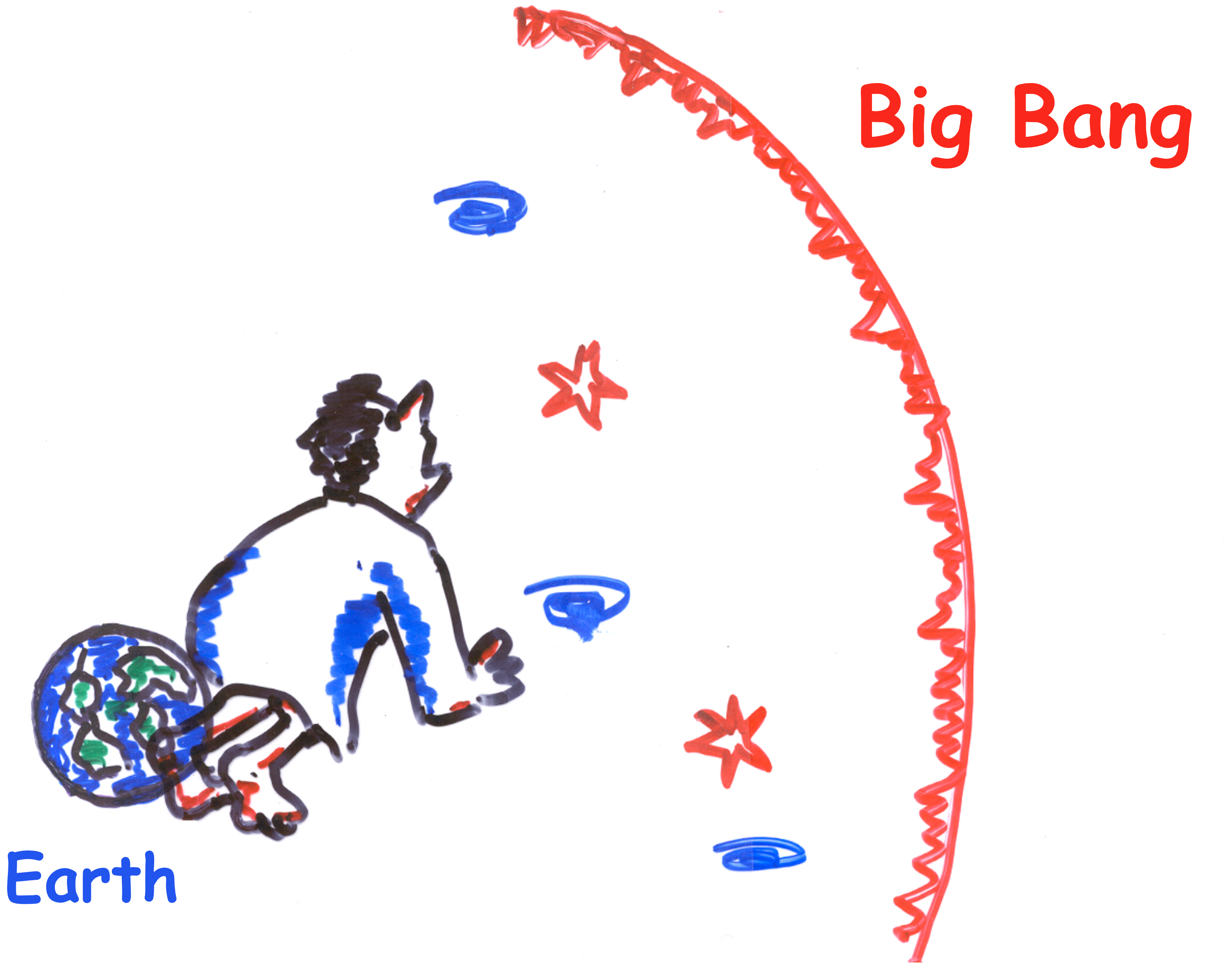
Predictions of Inflation:

1) The universe should be homogeneous, isotropic and flat, $\Omega = 1 + O(10^{-4})$ $[\Omega=\rho/\rho_0]$

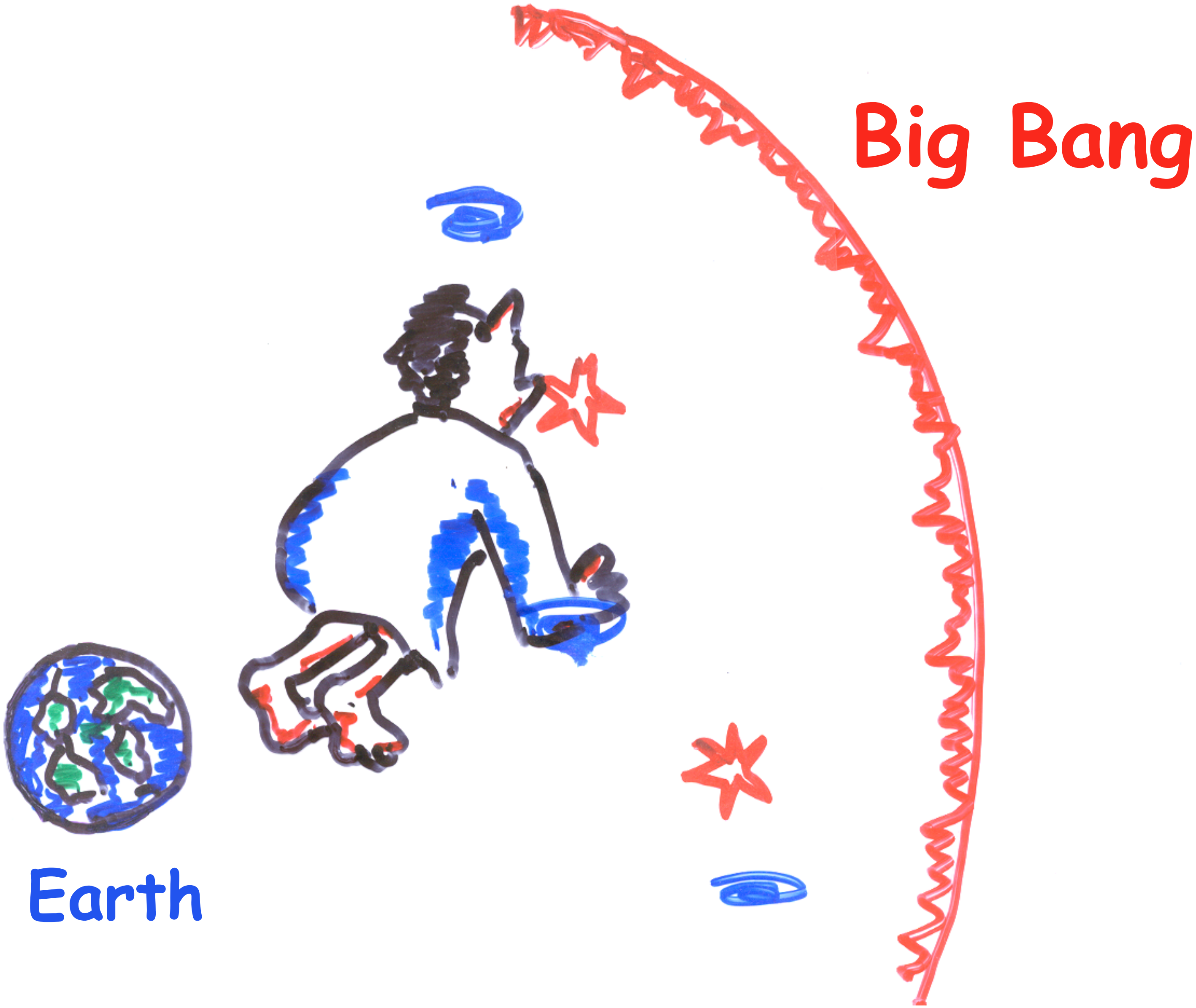
Observations: the universe is homogeneous, isotropic and flat, $\Omega = 1 + O(10^{-2})$

2) Inflationary perturbations should be gaussian and adiabatic, with flat spectrum, $n_s = 1 + O(10^{-1})$

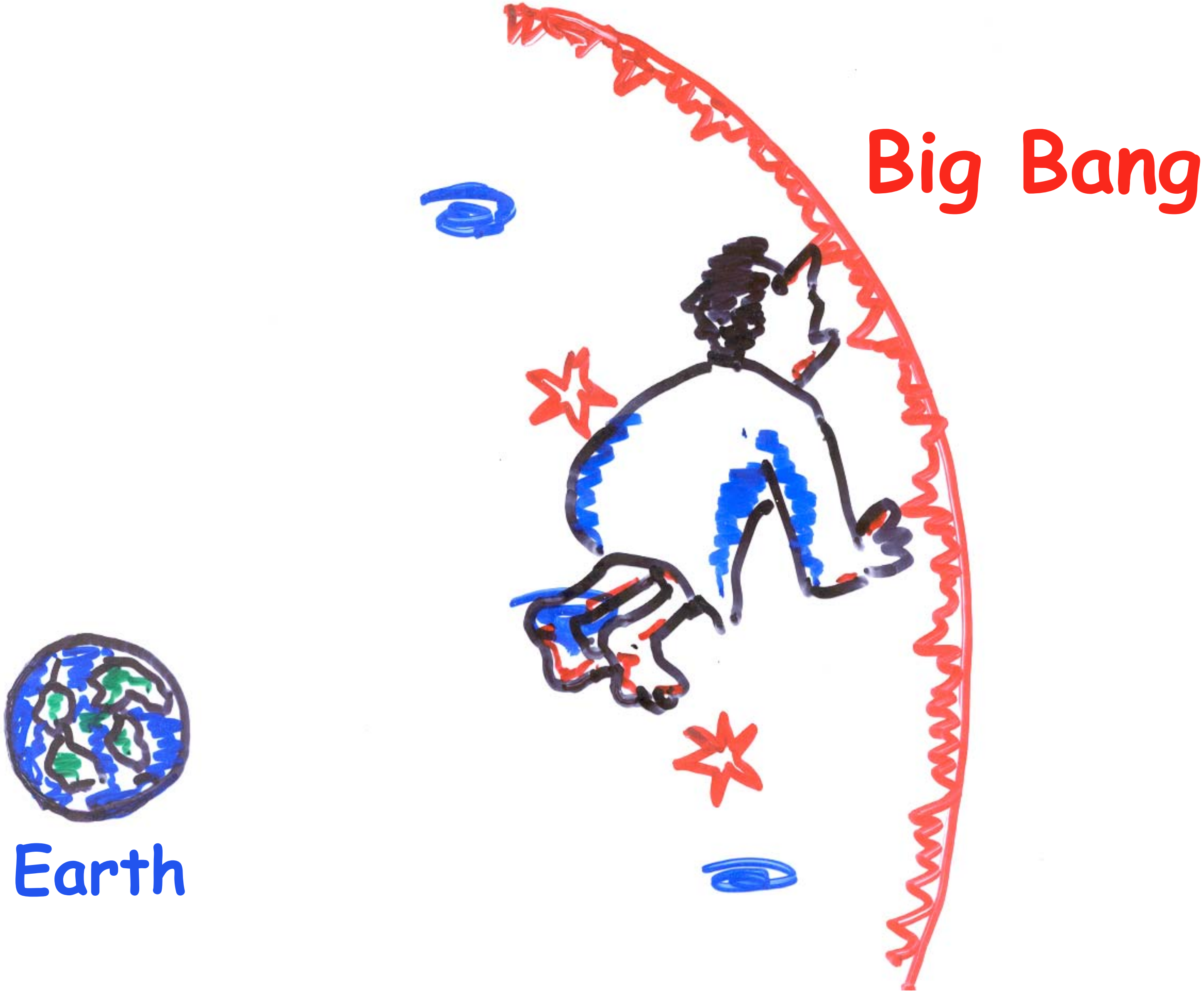
Observations: perturbations are gaussian and adiabatic, with flat spectrum, $n_s = 1 + O(10^{-2})$



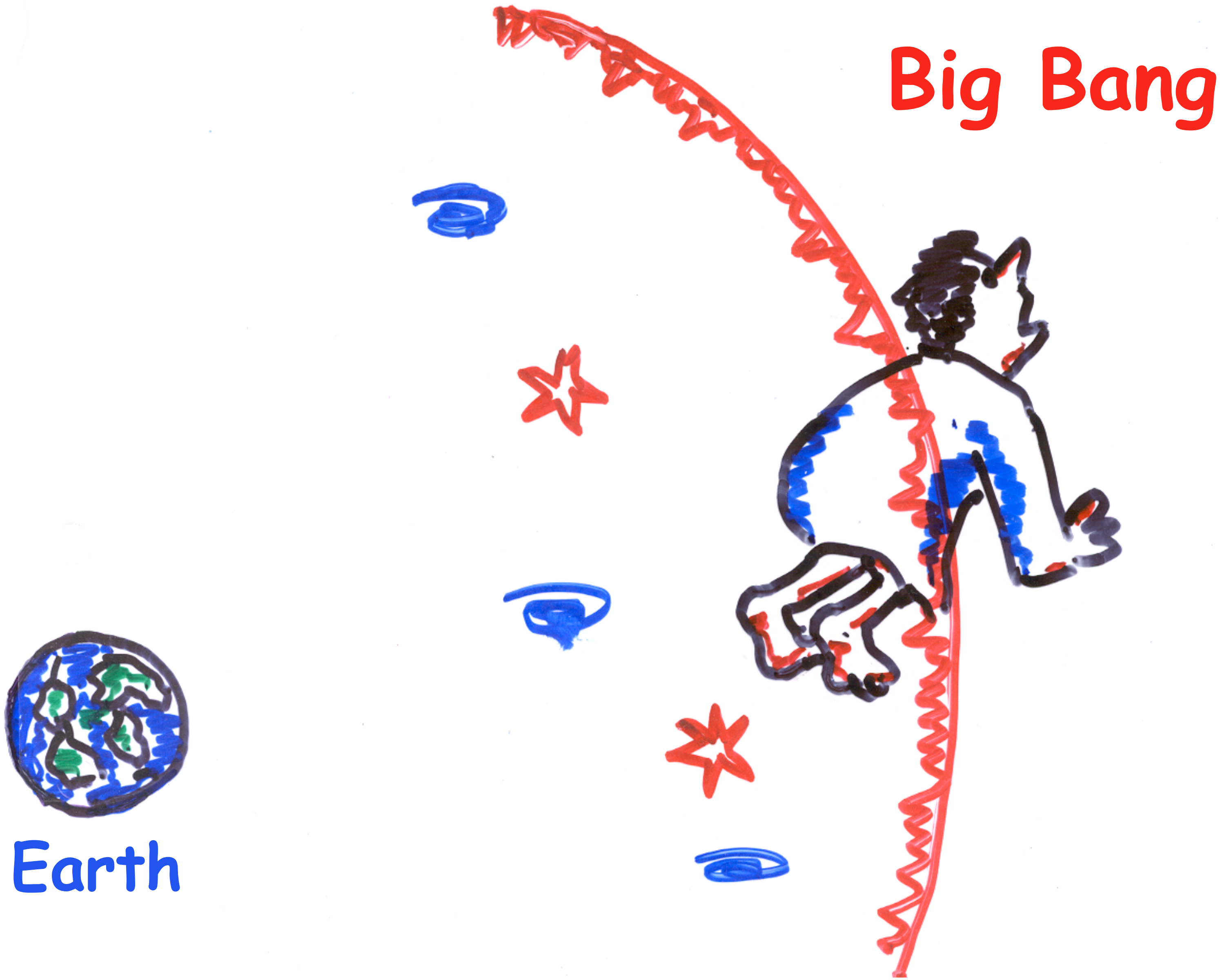
Astronomers use our universe as a “time machine”. By looking at the stars close to us, we see them as they were several hundreds years ago.



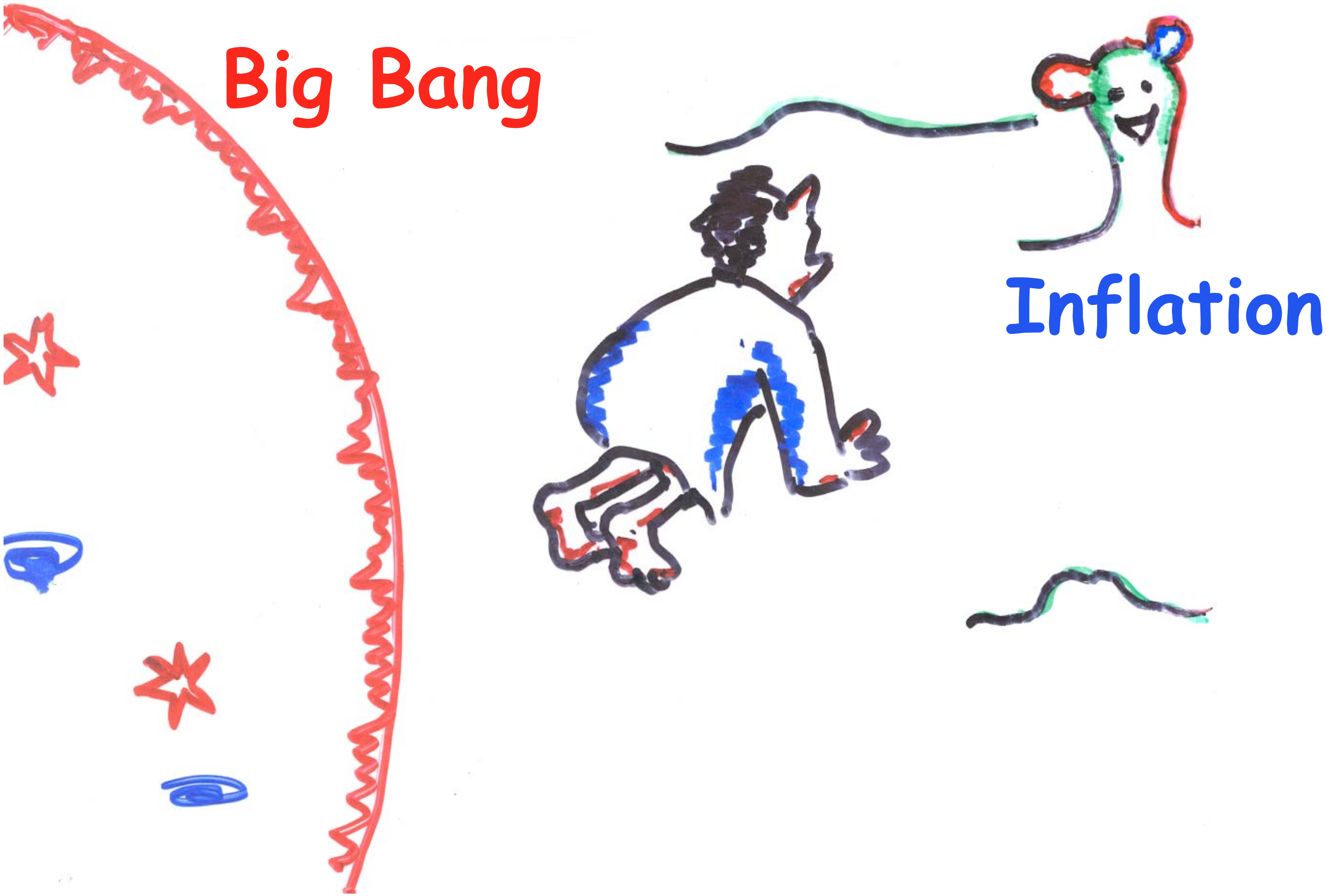
The light from distant galaxies travel to us for billions of years, so we see them in the form they had billions of years ago.



Looking even further, we can detect photons emitted 400000 years after the Big Bang. But 30 years ago everyone believed that there is nothing beyond the cosmic fire created in the Big Bang at the time t = 0.

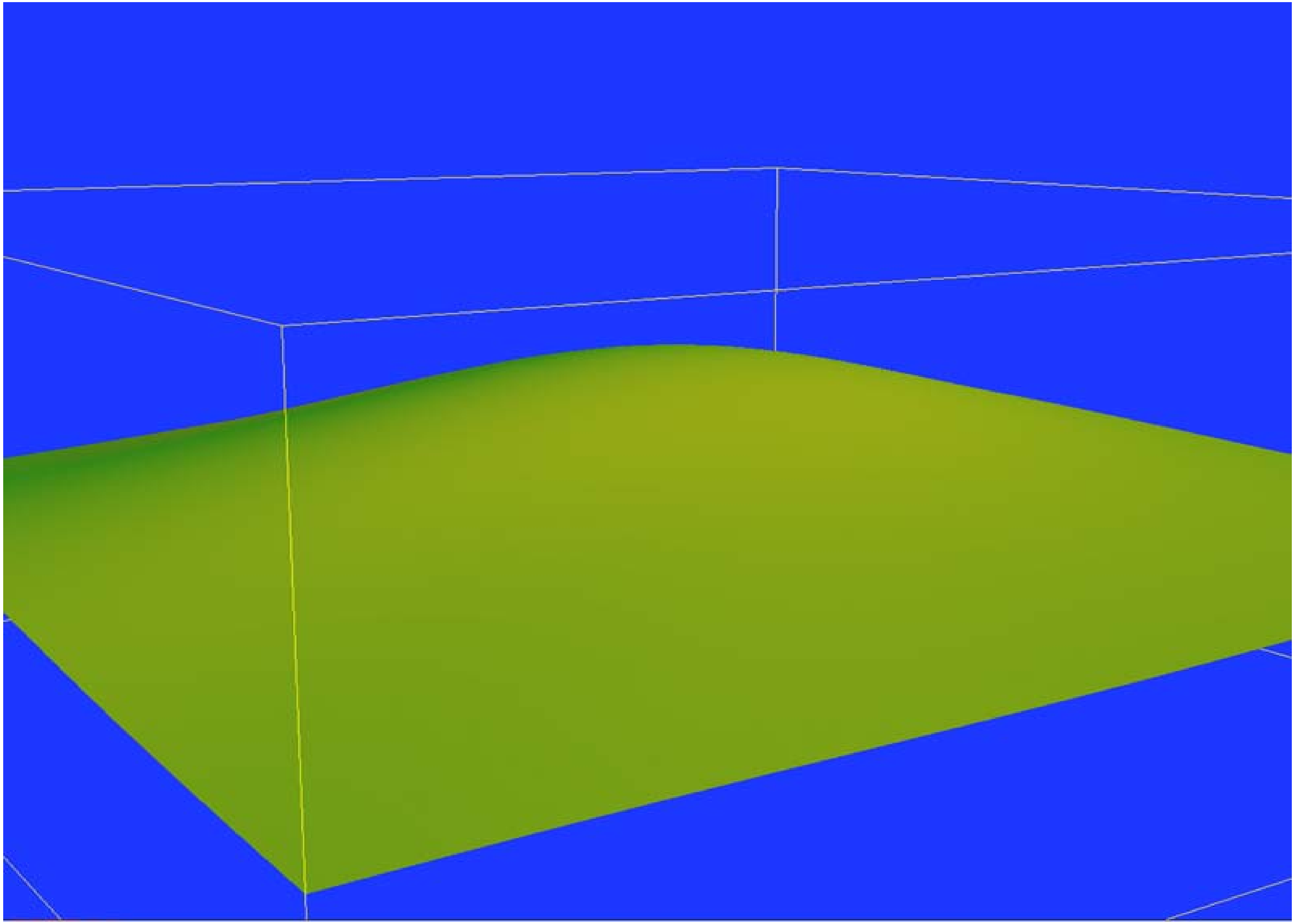


Inflationary theory tells us that this cosmic fire was created not at the time t = 0, but after inflation. If we look beyond the circle of fire surrounding us, we will see enormously large empty space filled only by a scalar field.



If we look there very carefully, we will see small perturbations of space, which are responsible for galaxy formation. And if we look even further, we will see how new parts of inflationary universe are created by quantum fluctuations.

Generation of Quantum Fluctuations



Inflationary perturbations and Brownian motion

Perturbations of the massless scalar field are frozen each time when their wavelength becomes greater than the size of the horizon, or, equivalently, when their momentum k becomes smaller than H.

Each time $t = H^{-1}$ the perturbations with $H < k < e\,H$ become frozen. Since the only dimensional parameter describing this process is H, it is clear that the average amplitude $\delta\phi$ of the perturbations frozen during this time interval is proportional to H. A detailed calculation shows that

$$\delta\phi = \frac{H}{2\pi} = T_H$$

This process repeats each time $t = H^{-1}$, but the sign of $\delta\phi$ each time can be different, like in the Brownian motion. Therefore the typical amplitude of accumulated quantum fluctuations can be estimated as

$$\langle\delta\phi^2\rangle = \left(\frac{H}{2\pi}\right)^2 Ht = \frac{H^3}{4\pi^2}t$$

At any given point, the diffusion of the field ϕ can be described by the probability distribution $P(\phi, t)$. The evolution of $P(\phi, t)$ can be found by solving the diffusion equation:

$$\frac{\partial \mathrm{P}_c(\phi, t)}{\partial t} = D \, \frac{\partial^2 \mathrm{P}_c(\phi, t)}{\partial \phi^2} \ . \tag{1}$$

To find the diffusion coefficient D, we remember that

$$\langle \phi^2 \rangle \equiv \int \phi^2 \, P_c(\phi, t) \, d\phi = \frac{H^3}{4\,\pi^2} \, t \ .$$

Differentiating this relation with respect to t and using (1), we find

$$D = \frac{H^3}{8\,\pi^2} \ .$$

For a massive classical scalar field with $|m^2| \ll H^2$, initially $\langle \phi^2 \rangle$ grows in the same way as for a massless field. But later, the long-wave classical field ϕ, which appears during the first stages of the process, begins to decrease as a result of the slow roll down toward the point $\phi = 0$, in accordance with equation $3H\dot{\phi} = -V' = -m^2\phi$. This leads to stabilization of $\langle \phi^2 \rangle$ at its limiting value $\dfrac{3\,H^4}{8\,\pi^2\,m^2}$ (Bunch, Davies 1978).

To describe diffusion for an arbitrary potential $V(\phi)$, one should write the diffusion equation in a more general form:

$$\frac{\partial P_c}{\partial t} = \mathrm{D}\,\frac{\partial^2 P_c}{\partial \phi^2} + b\,\frac{\partial}{\partial \phi}\left(P_c\,\frac{d\mathrm{V}}{d\phi}\right) \ ,$$

where as before $\mathrm{D} = \dfrac{H^3}{8\,\pi^2}$ and b is the mobility coefficient, defined by the equation $\dot{\phi} = -b\,V'$. Using equation $3\,H\,\dot{\phi} = -V'$ one finds (Starobinsky 1985)

$$\frac{\partial \mathrm{P}_c}{\partial t} = \frac{\mathrm{H}^3}{8\,\pi^2}\,\frac{\partial^2 P_c}{\partial \phi^2} + \frac{1}{3\,H}\,\frac{\partial}{\partial \phi}\left(\mathrm{P}_c\,\frac{dV}{d\phi}\right) \ .$$

In deriving this equation, we assumed that H is independent of the field ϕ. More generally, one has

$$\frac{\partial P_c}{\partial t} = \frac{\partial^2}{\partial \phi^2}\left(\frac{H^3\,\mathrm{P}_c}{8\,\pi^2}\right) + \frac{\partial}{\partial \phi}\left(\frac{P_c}{3\,\mathrm{H}}\,\frac{dV}{d\phi}\right) \ .$$

In fact, there are two different diffusion equations: The first one (Kolmogorov forward equation) describes the probability to find the field ϕ if the evolution starts from the initial field ϕ_o. The second equation (Kolmogorov backward equation) describes the probability that the initial value of the field is given by ϕ_o if the evolution eventually brings the field to its present value ϕ.

For the stationary regime $\frac{\partial P}{\partial t} = 0$ the combined solution of these two equations is given by

$$P = \exp\left(-\frac{24\pi^2}{V(\phi_0)}\right) \times \exp\left(\frac{24\pi^2}{V(\phi)}\right)$$

The first of these two terms is the square of the tunneling wave function of the universe, describing the probability of initial conditions. The second term is the square of the Hartle-Hawking wave function describing the ground state of the universe.

Eternal Chaotic Inflation

Consider an inflationary domain of size H^{-1}. Equation $3H\dot{\phi} = -m^2\phi$ implies that during a typical time interval $\Delta t = H^{-1}$ the field inside this domain will be reduced by

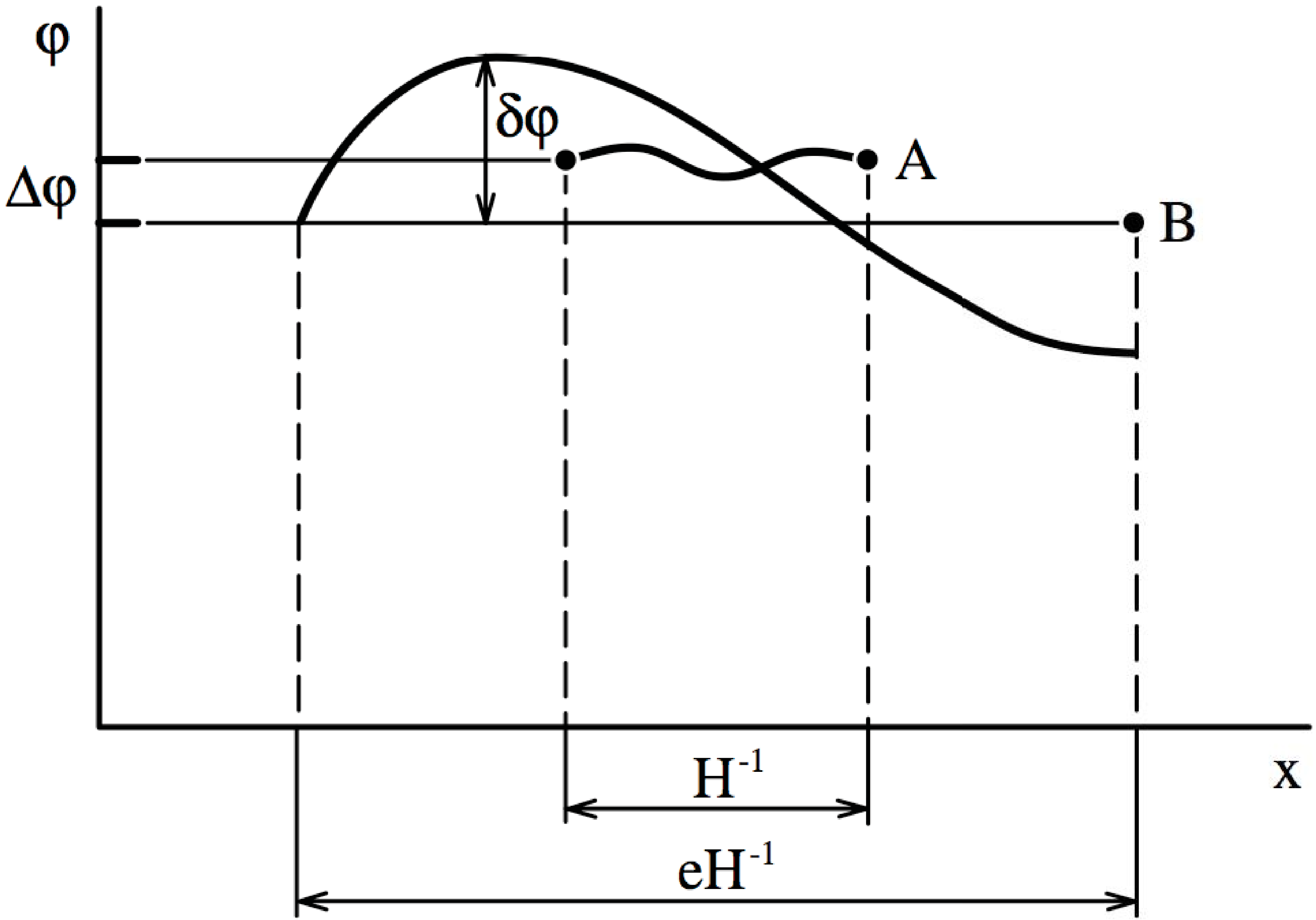
$$\Delta\phi = \frac{2}{\phi}$$

For $\phi \gg \frac{5}{\sqrt{m}}$ one has

$$\Delta\phi = \frac{2}{\phi} \ll \delta\phi(x) \sim \frac{H}{2\pi} = \frac{m\phi}{2\pi\sqrt{6}}$$

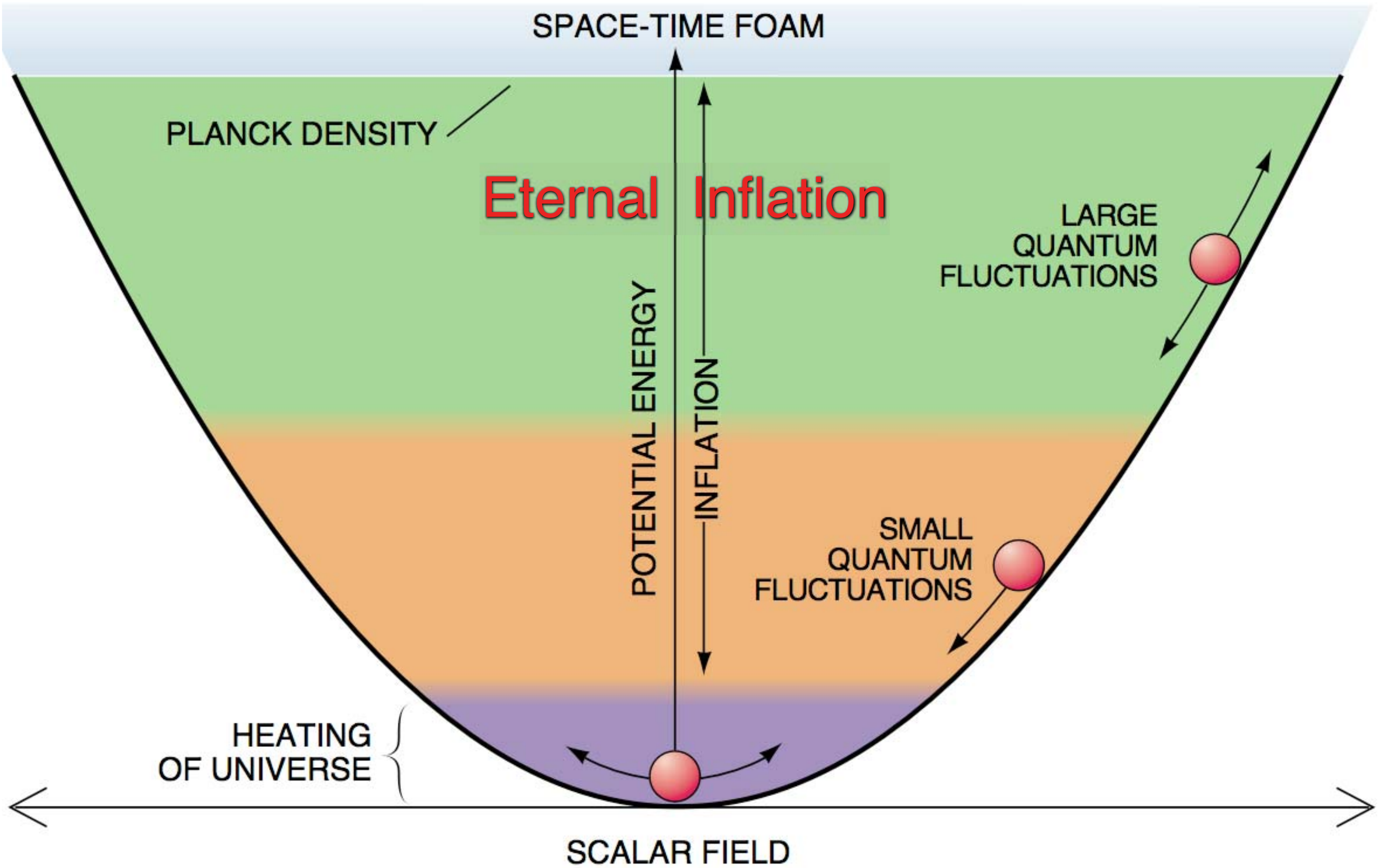
Because the typical wavelength of the fluctuations $\delta\phi(x)$ generated during the time is H^{-1}, the whole domain after $\Delta t = H^{-1}$ effectively becomes divided into $e^3 \sim 20$ separate domains of size H^{-1}, each containing almost homogeneous field $\phi - \Delta\phi + \delta\phi$. In almost a half of these domains the field ϕ **grows** by $|\delta\phi(x)| - \Delta\phi \approx |\delta\phi(x)| = H/2\pi$. This means that the total volume of the universe containing **growing** field ϕ increases 10 times. During the next time interval $\Delta t = H^{-1}$ this process repeats. Thus, after the two time intervals H^{-1} the total volume of the universe containing the growing scalar field increases 100 times, etc. The universe enters eternal process of self-reproduction.

Eternal Chaotic Inflation



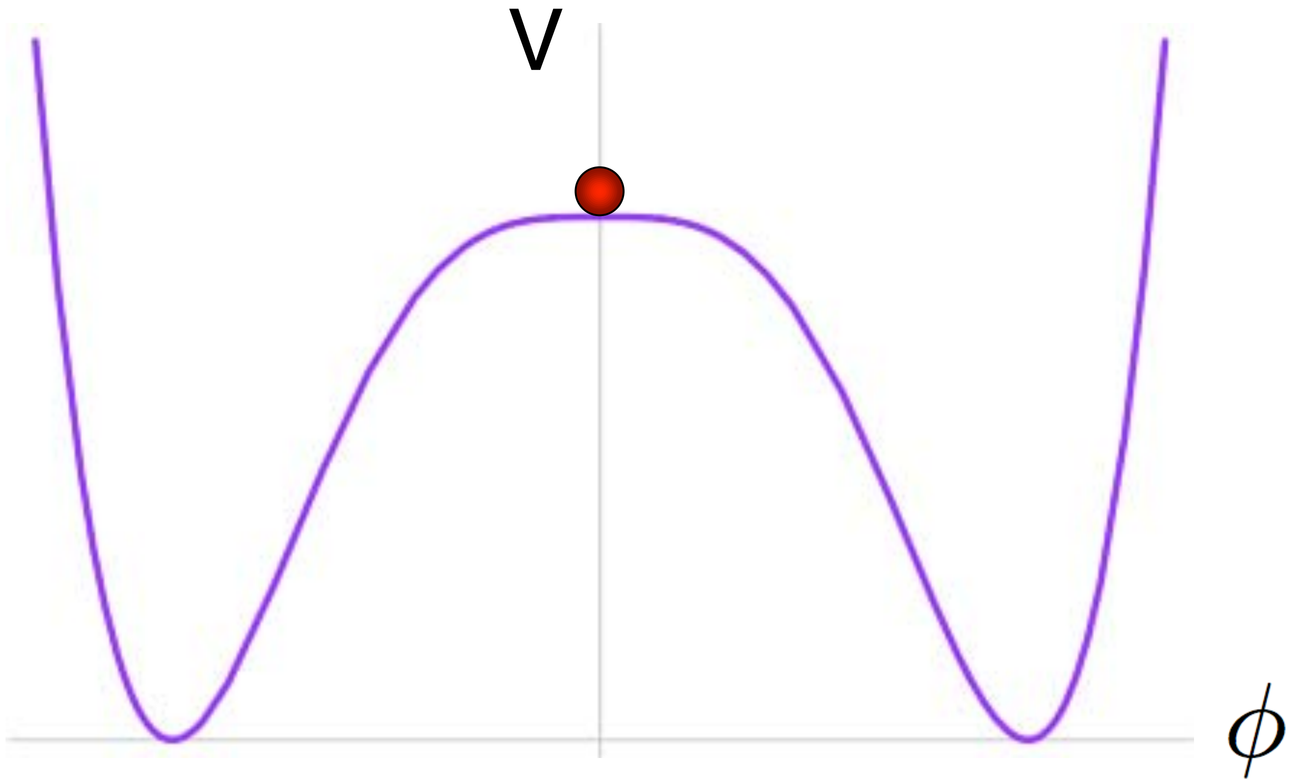
Inflation as a theory of a harmonic oscillator

$$V(\phi) = \frac{m^2}{2}\phi^2$$



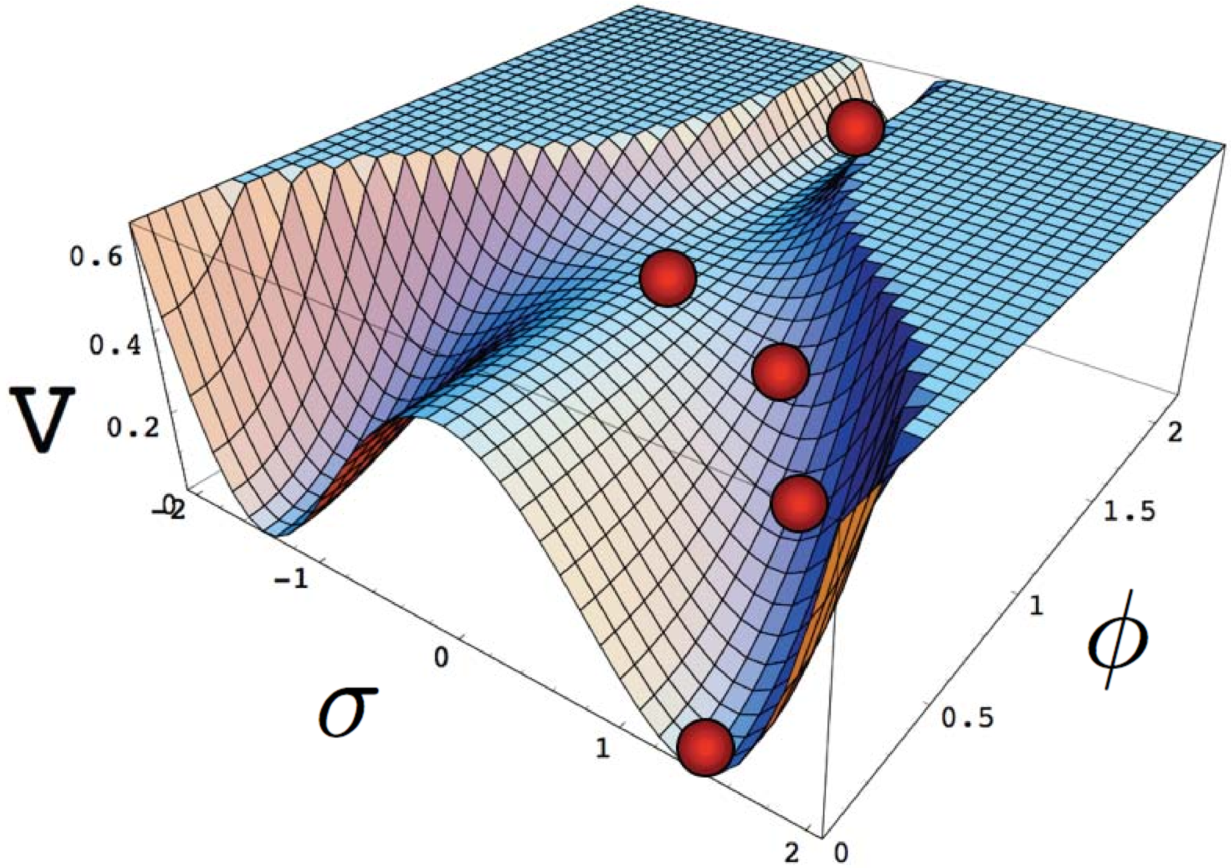
New Inflation

$$V = g^4 \left(\phi^4 \ln \phi - \phi^4/4 + 1/4\right)$$

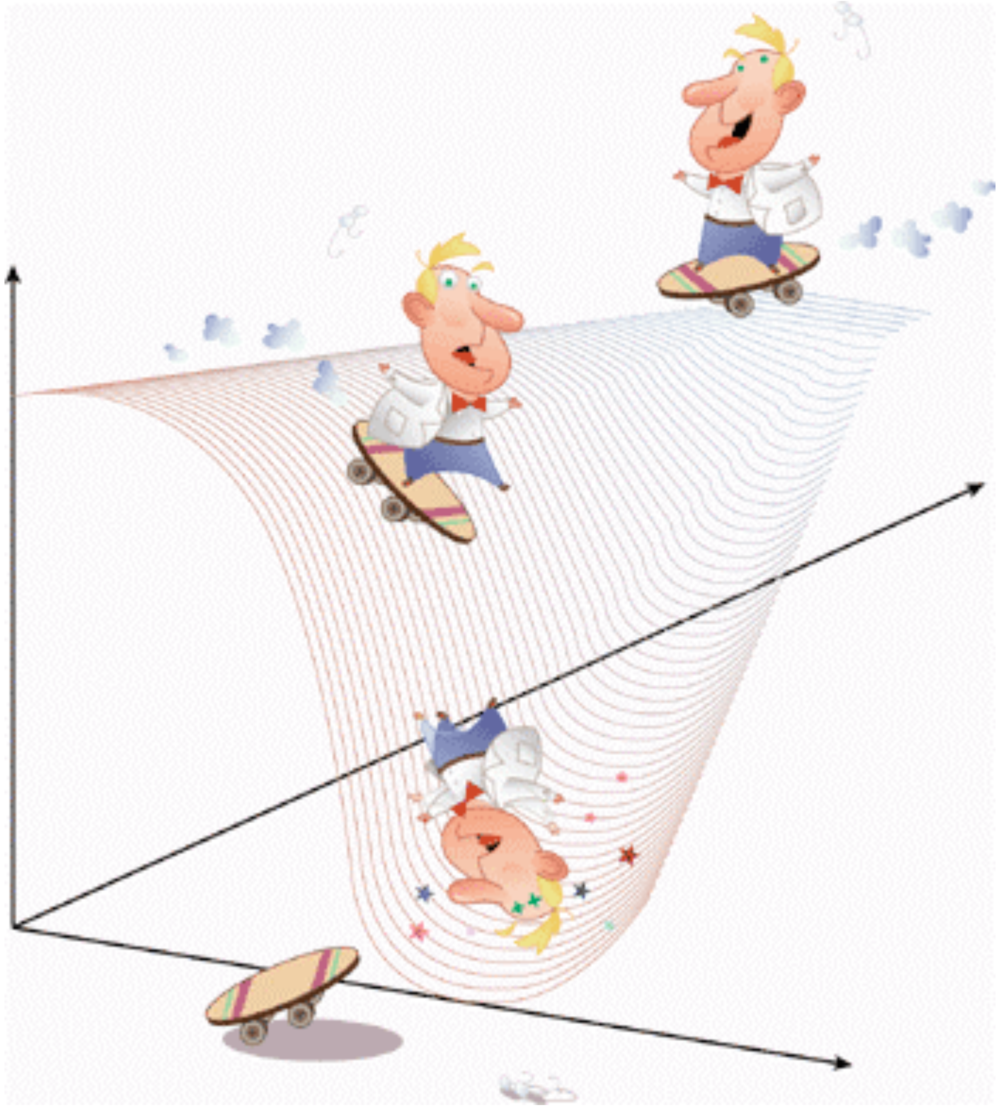


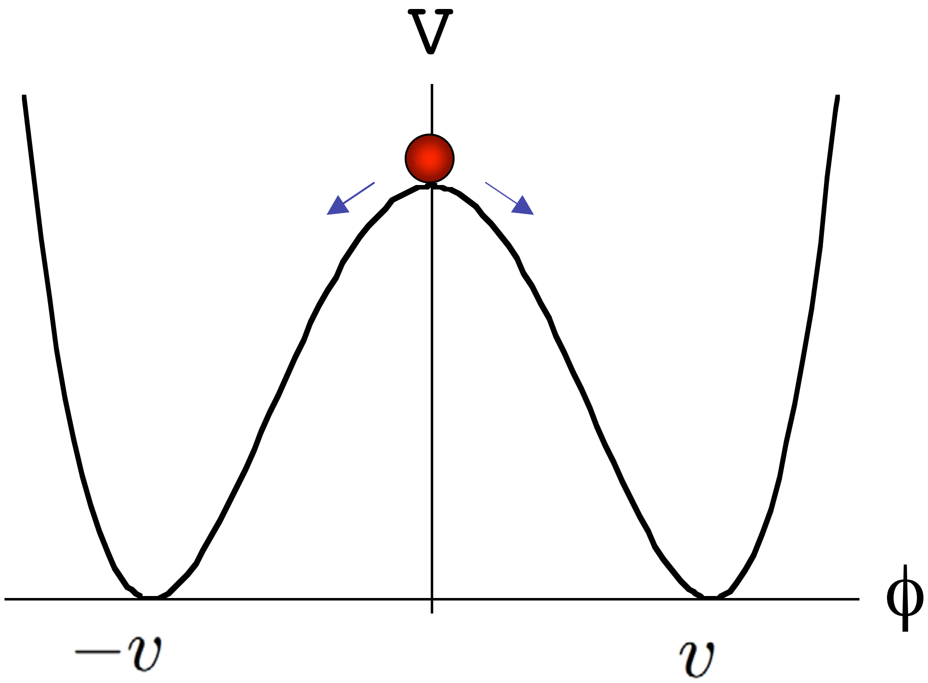
Hybrid Inflation

$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2$$



Warm-up: Dynamics of spontaneous symmetry breaking





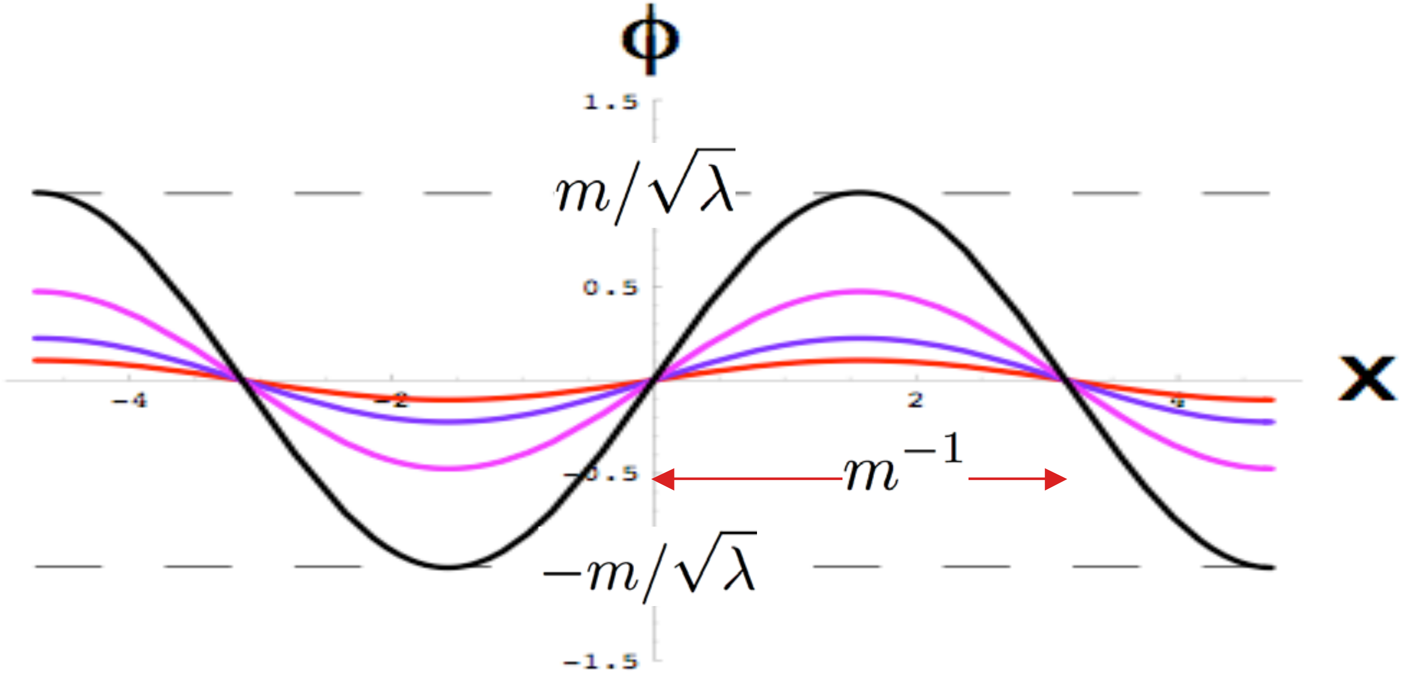
$$V = \frac{\lambda}{4}(\phi^2 - v^2)^2$$
$$= -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{m^4}{4\lambda}$$

$$v = m/\sqrt{\lambda}$$

How many oscillations does the field distribution make before it relaxes near the minimum of the potential V ?

1 $\quad 1/\lambda$ $\quad e^{1/\lambda}$ $\quad$ other

Answer: 1 oscillation



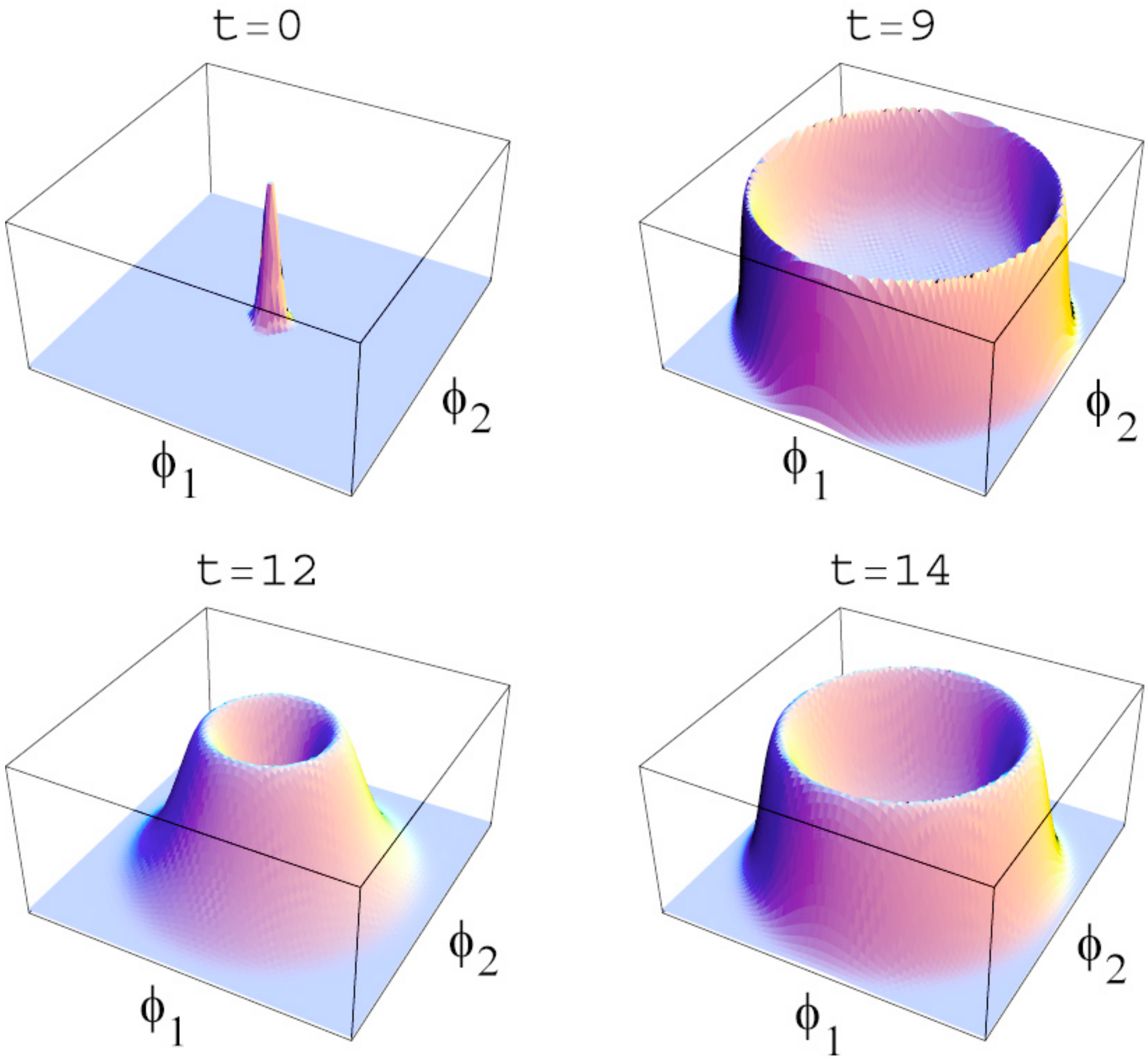
All quantum fluctuations with k < m grow exponentially:

$$\delta\phi \sim e^{iwt} \sim e^{i\sqrt{k^2-m^2}\,t} \sim e^{\sqrt{m^2-k^2}\,t}$$

When they reach the minimum of the potential, the energy of the field gradients becomes comparable with its initial potential energy.

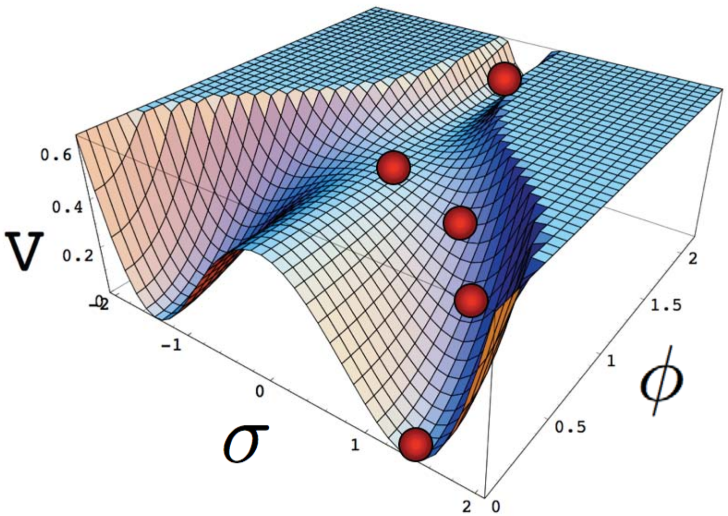
$$E_{\text{gradient}} \sim \frac{1}{2}(\partial_i\phi)^2 \sim \left(m\cdot\frac{m}{\sqrt{\lambda}}\right)^2 \sim \frac{m^4}{\lambda}$$

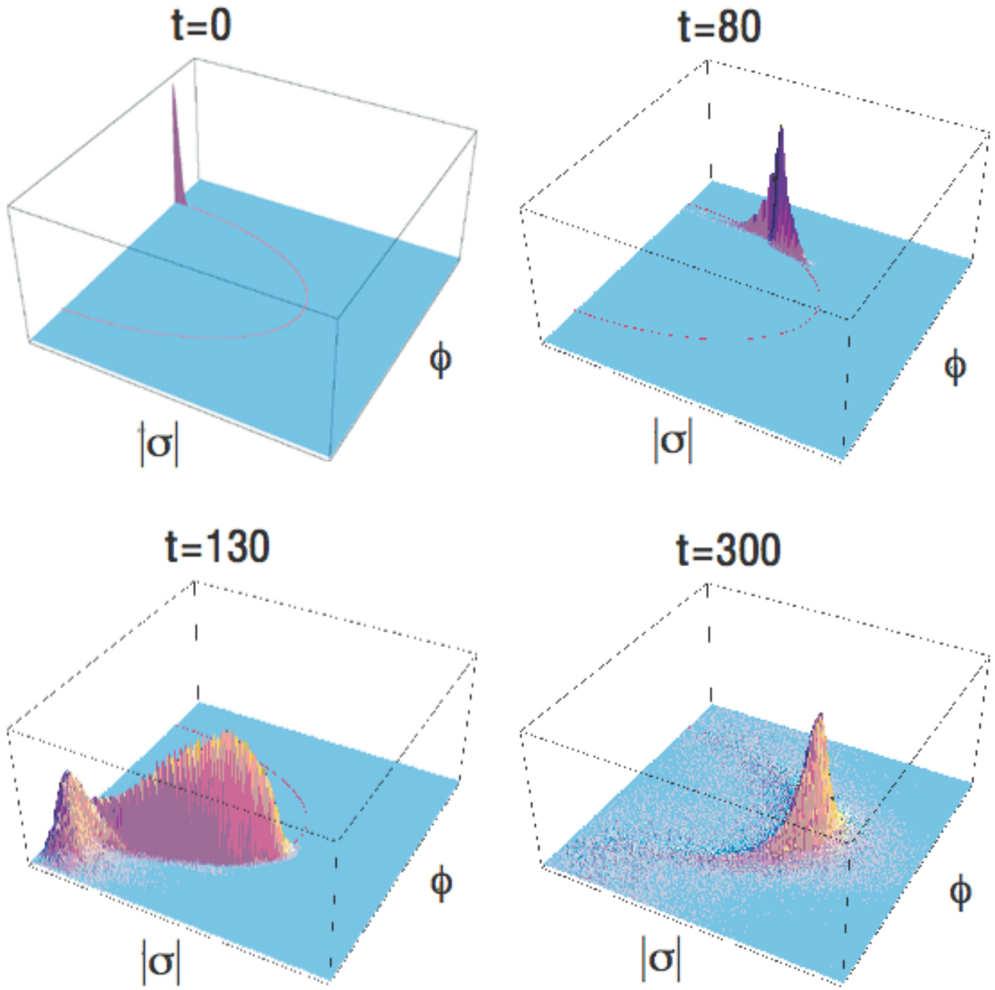
Not much is left for the oscillations; the process of spontaneous symmetry breaking is basically over in a single oscillation of the field distribution.


t=0
t=9
t=12
t=14
ϕ_1
ϕ_2

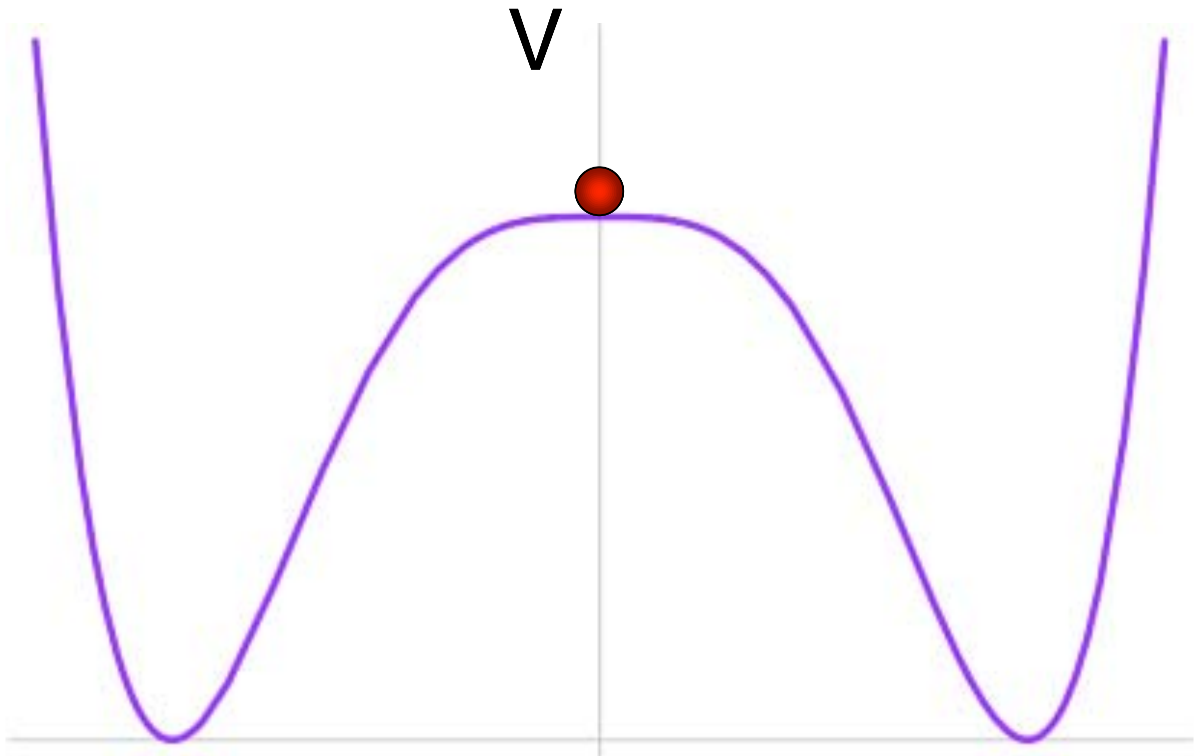
After hybrid Inflation

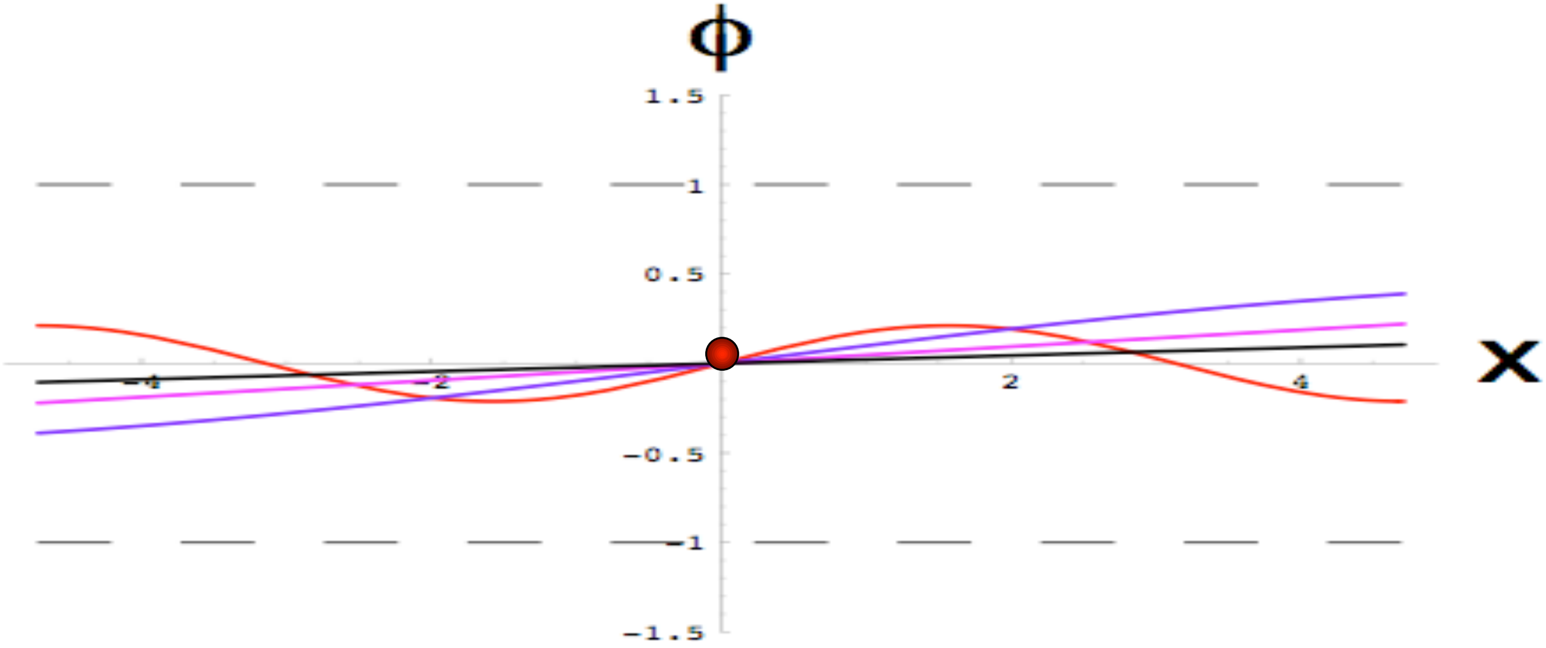
$$V(\sigma,\phi)=\frac{1}{4\lambda}(M^2-\lambda\sigma^2)^2+\frac{m^2}{2}\phi^2+\frac{g^2}{2}\phi^2\sigma^2$$





Inflating topological defects in new inflation





During inflation we have two competing processes: growth of the field

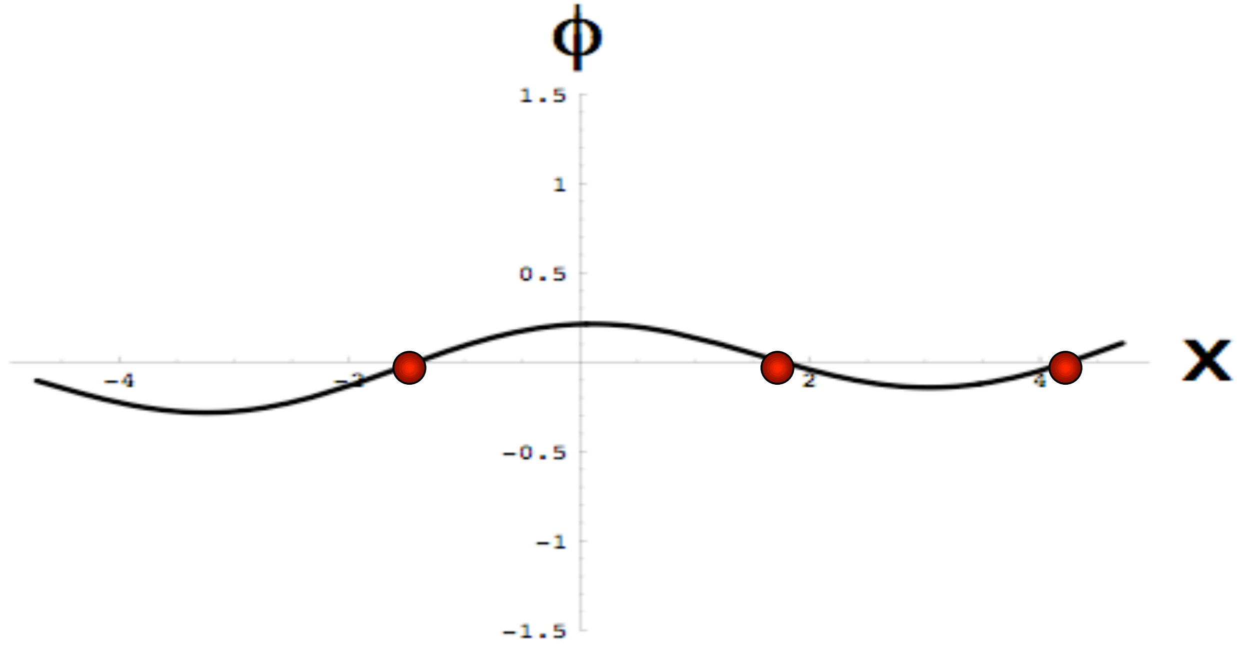
$$\delta\phi \sim \exp\left(\frac{m^2 t}{3H^2}\right)$$

and expansion of space

$$x \sim \exp\left(Ht\right)$$

For H >> m, the value of the field in a vicinity of a topological defect exponentially decreases, and the total volume of space containing small values of the field exponentially grows.

Topological inflation, A.L. 1994, Vilenkin 1994



Small quantum fluctuations of the scalar field freeze on the top of the flattened distribution of the scalar field. This creates new pairs of points where the scalar field vanishes, i.e. new pairs of topological defects. They do not annihilate because the distance between them exponentially grows.

Then quantum fluctuations in a vicinity of each new inflating monopole produce new pairs of inflating monopoles.

Thus, the total volume of space near inflating domain walls (strings, monopoles) grows exponentially, despite the ongoing process of spontaneous symmetry breaking.

Inflating `t Hooft - Polyakov monopoles serve as indestructible seeds for the universe creation.

If inflation begins inside one such monopole, it continues forever, and creates an infinitely large fractal distribution of eternally inflating monopoles.

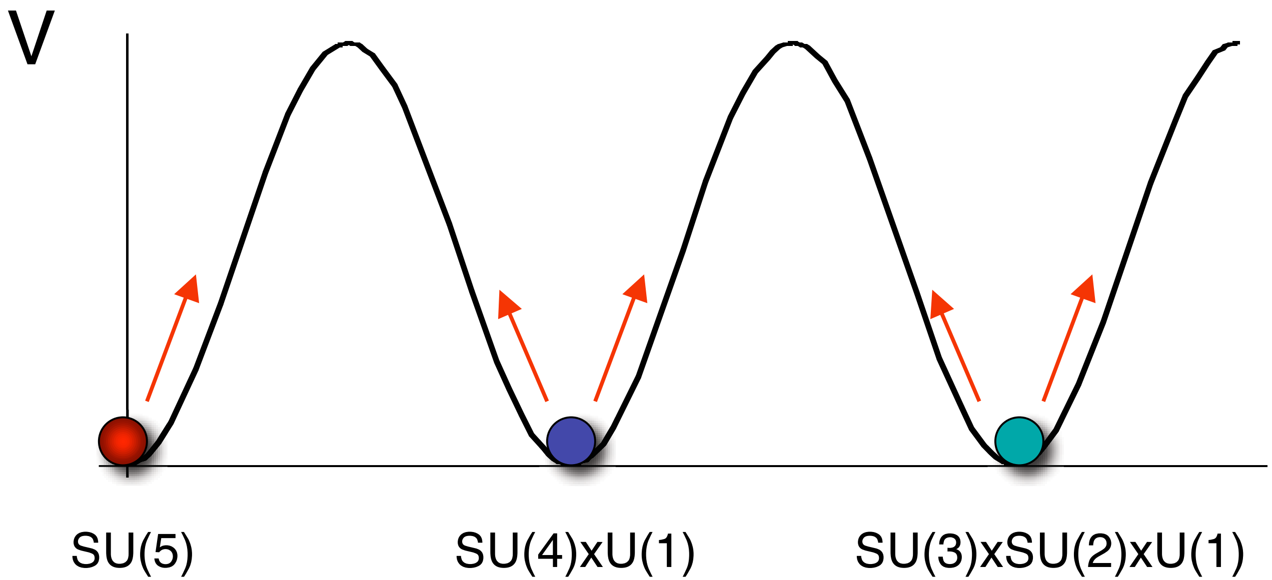
From the Universe to the Multiverse

In realistic theories of elementary particles there are **many scalar fields**, and their potential energy has **many different minima**. Each minimum corresponds to different masses of particles and different laws of their interactions.

Quantum fluctuations during eternal inflation can bring the scalar fields to different minima in different exponentially large parts of the universe. The universe becomes divided into many exponentially large parts with **different laws of physics** operating in each of them. (In our computer simulations we will show them by using different colors.)

Example: SUSY landscape

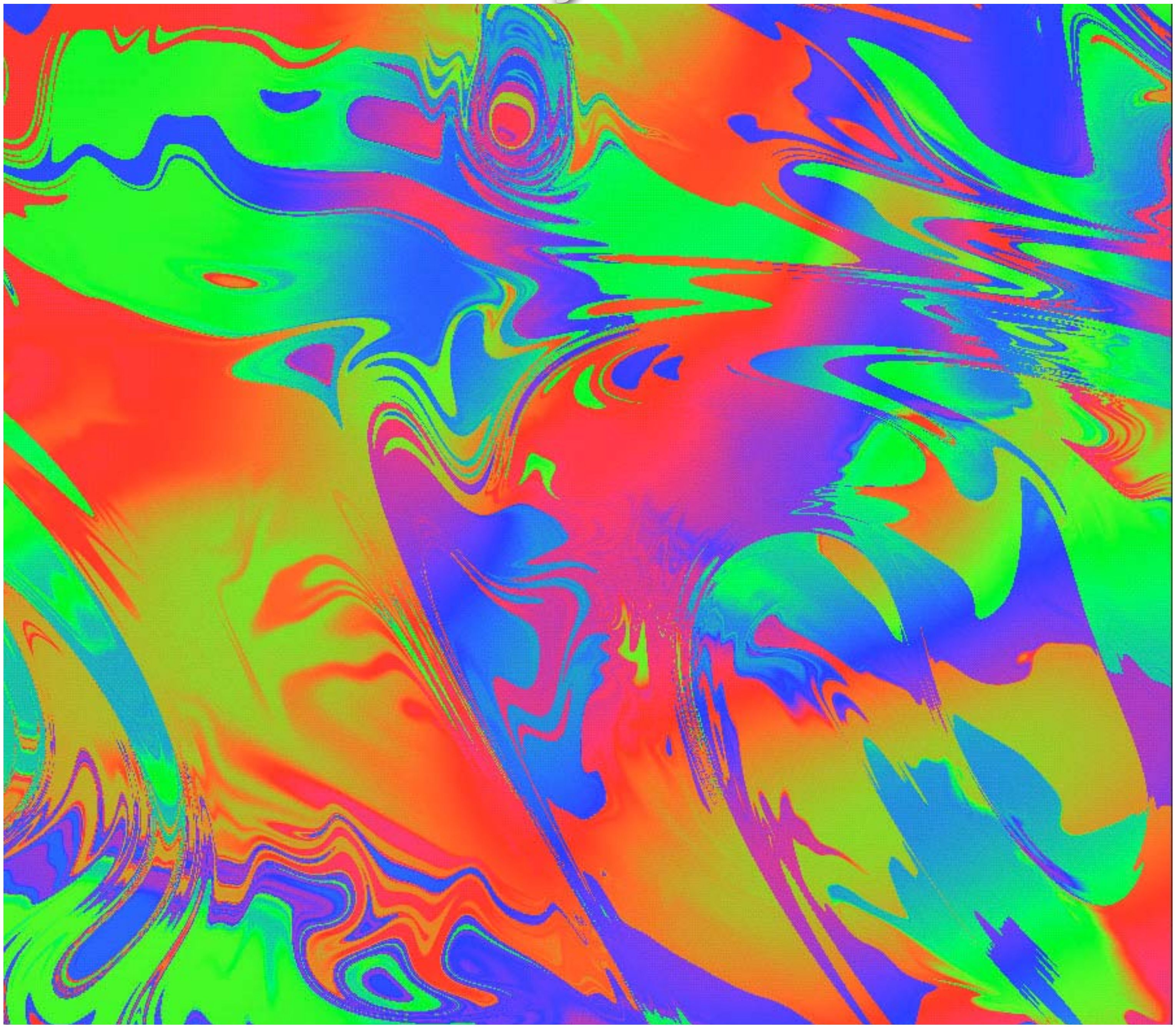
Supersymmetric SU(5)



Weinberg 1982: Supersymmetry forbids tunneling from SU(5) to SU(3)xSU(2)XU(1). This implied that we cannot break SU(5) symmetry.

A.L. 1983: Inflation solves this problem. Inflationary fluctuations bring us to each of the three minima. Inflation make each of the parts of the universe exponentially big. We can live only in the SU(3)xSU(2)xU(1) minimum.

Kandinsky Universe



Genetic code of the Universe

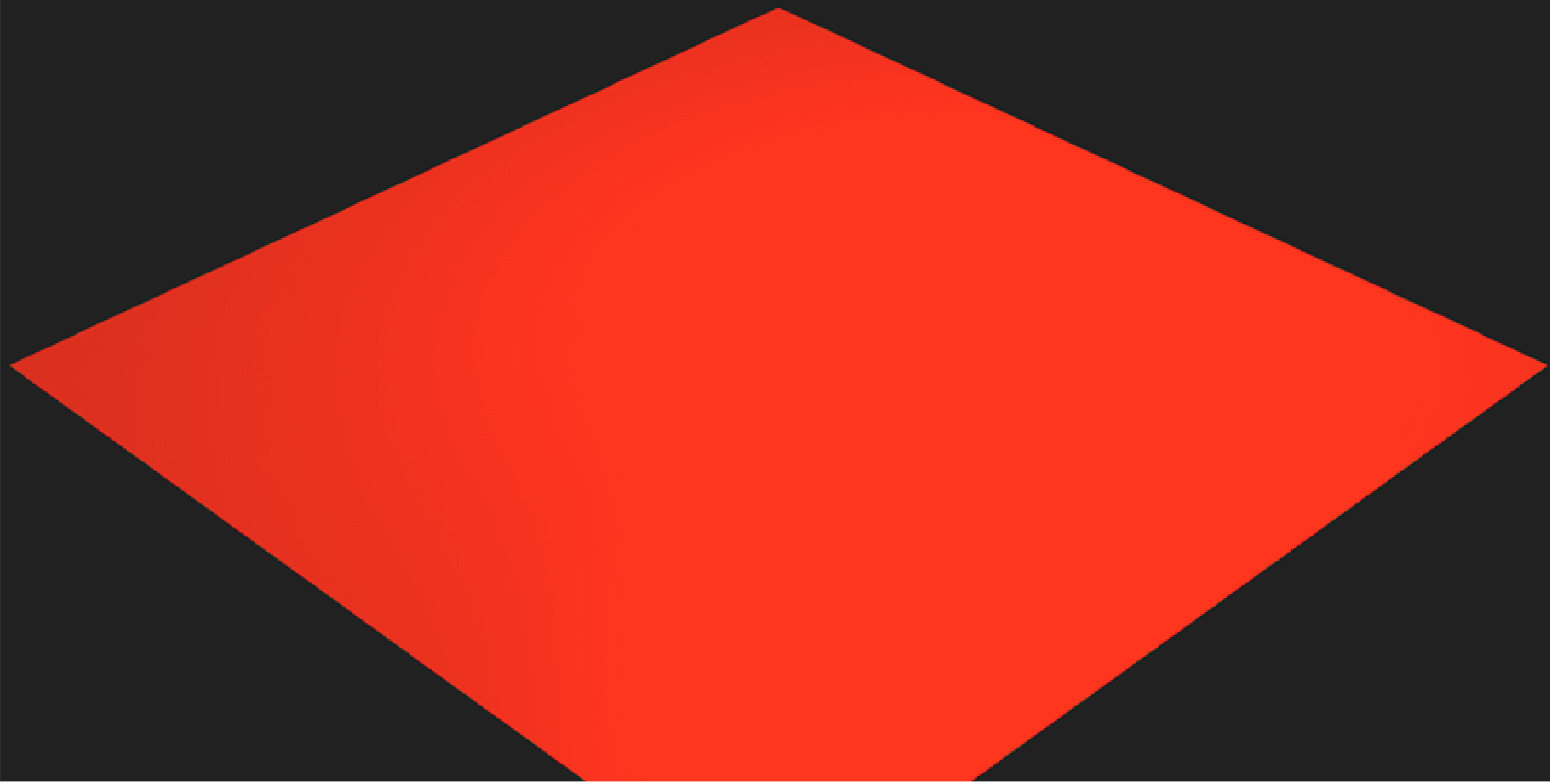
To be more accurate, one may have **one** fundamental law of physics, like a single genetic code for the whole Universe. However, this law may have different realizations. For example, water can be liquid, solid or gas. In elementary particle physics, the effective laws of physics depend on the values of the scalar fields.

Quantum fluctuations during inflation can take the scalar fields from one minimum of their potential energy to another, altering its genetic code. Once it happens in a small part of the universe, inflation makes this part exponentially big.

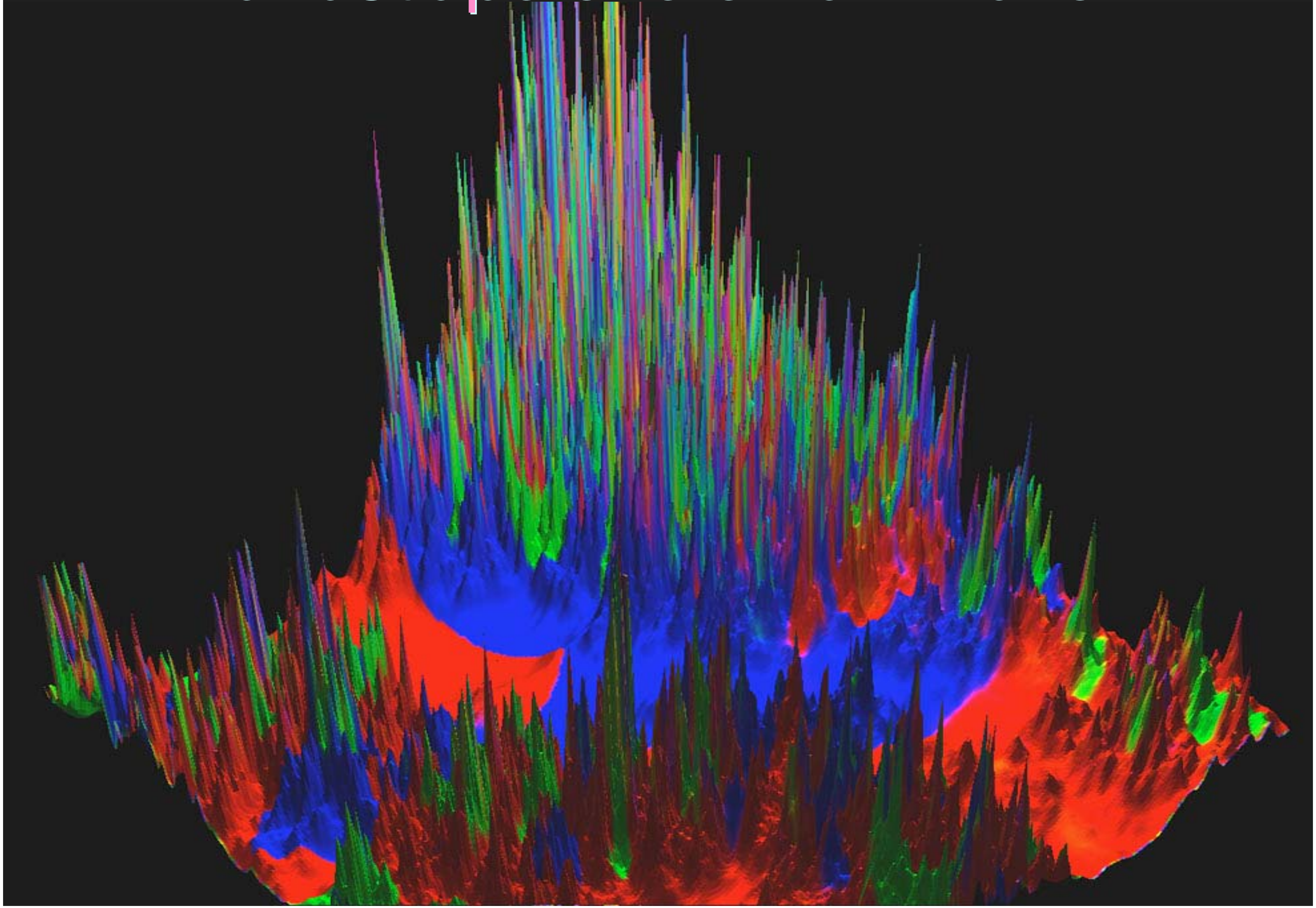
This is the cosmological mutation mechanism

String Theory Landscape

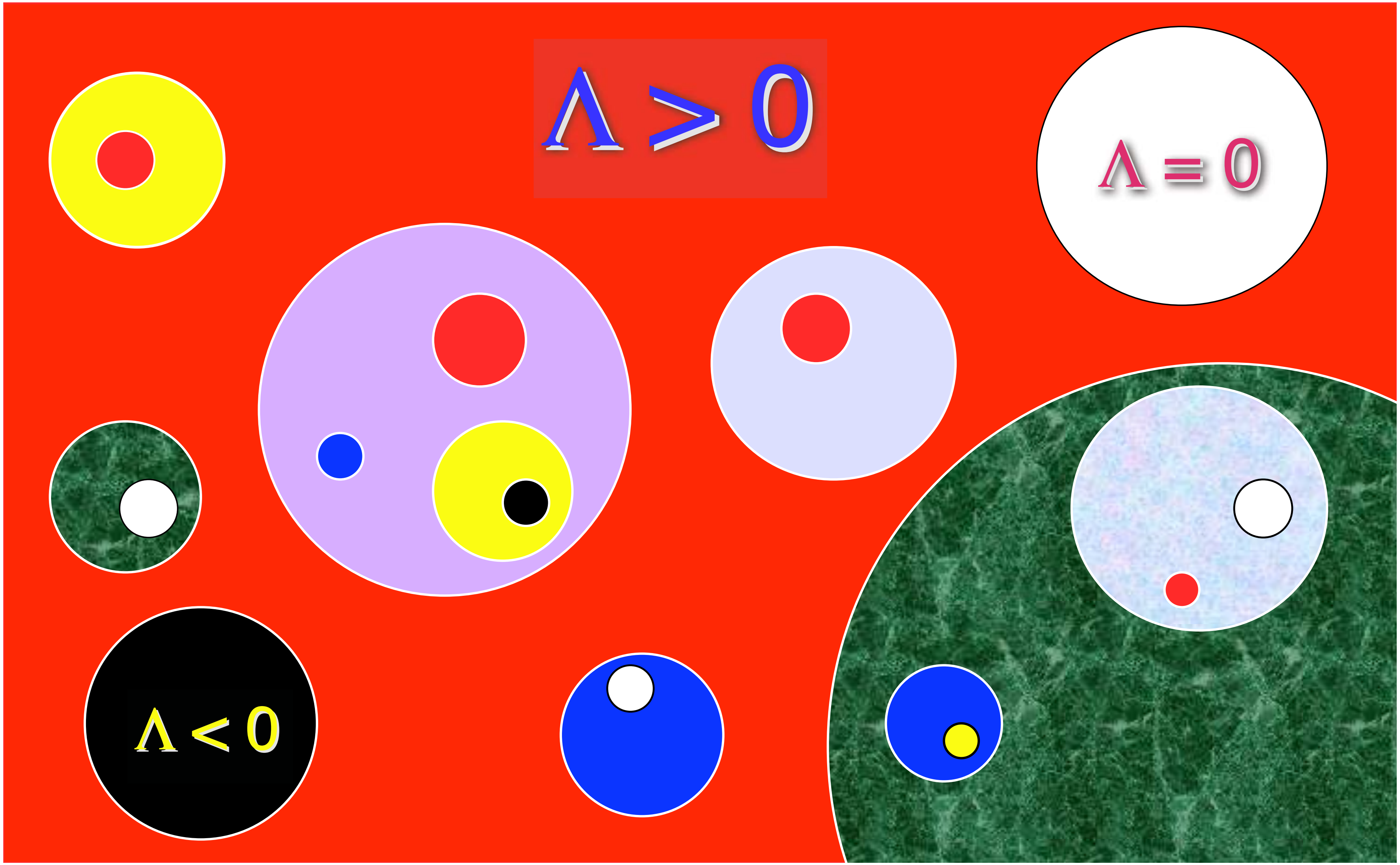
Populating the Landscape



Landscape of eternal inflation



Let 10^{1000} flowers blossom



Probabilities in the Landscape

We must find all possible vacua (statistics), and all possible continuous parameters (out-of-equilibrium cosmological dynamics).

Douglas 2003

We must also find a way to compare the probability to live in each of these states.

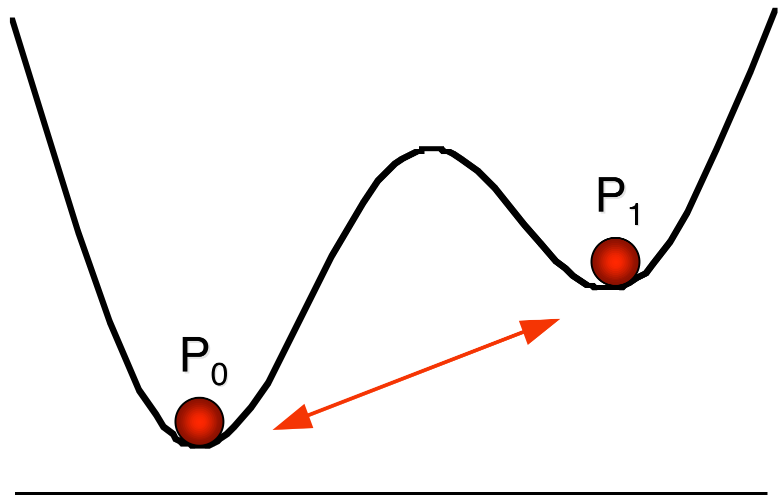
A.Linde, D. Linde, Mezhlumian, Bellido 1994;
Vilenkin 1995; Garriga, Schwarz-Perlov, Vilenkin, Winitzki, 2005

Example: Two dS vacua

$$P_0\, e^{-S_0+B} = P_1\, e^{-S_1+B}$$

$$S = \frac{1}{V}$$ is dS entropy

$$\frac{P_1}{P_0} = e^{S_1 - S_0} = e^{\Delta S}$$



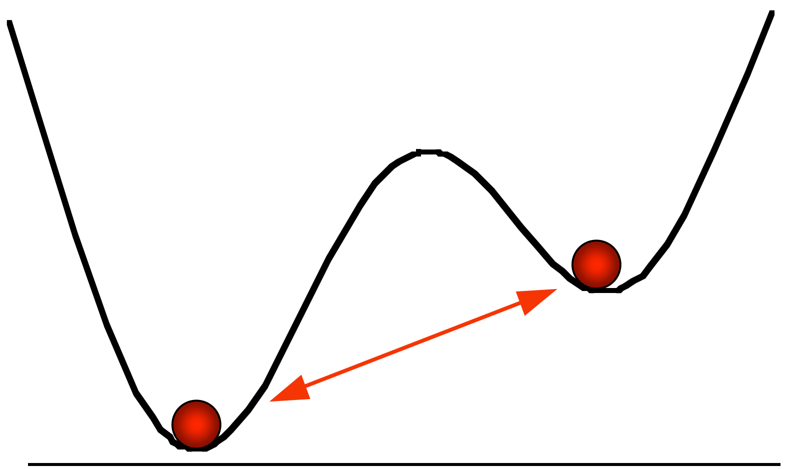
P_i is the probability to find a given point in the vacuum dS_i

This is the square of the Hartle-Hawking wave function, which tells that the fraction of the comoving volume of the universe with the cosmological constant $V_i = \Lambda$ is proportional to

$$P = e^{\frac{1}{\Lambda}}$$

In this context the HH wave function describes the ground state of the universe, it has no relation to creation of the universe "from nothing", and it does not require any modifications recently discussed in the literature.

In this scenario we should live in the lowest of all dS spaces compatible with the existence of our life



However, this would also mean that instead of inflation we would have eternal recycling of dead dS spaces. This would disagree with observations.

Dyson, Goheer, Kleban, Susskind 2002

Fortunately, this problem disappears in the KKLT scenario because of metastability of dS vacua.

Volume stabilization

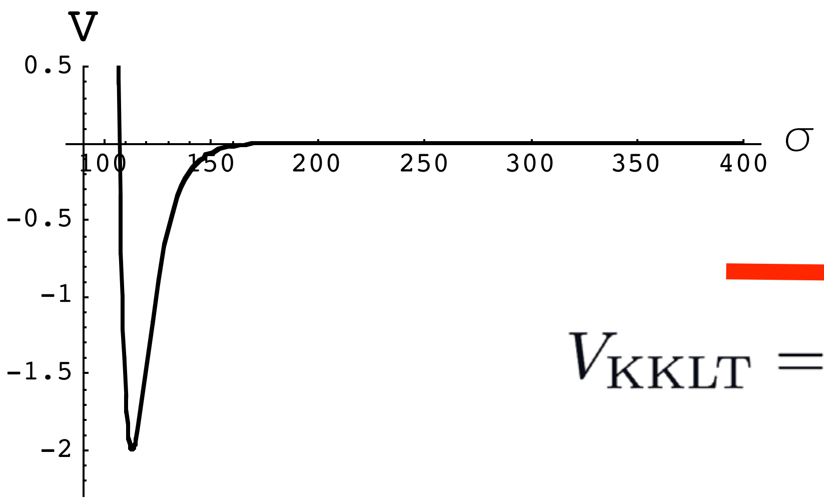
Kachru, Kallosh, A.L., Trivedi 2003

Basic steps of the KKLT scenario:

1) Start with a theory with runaway potential discussed above

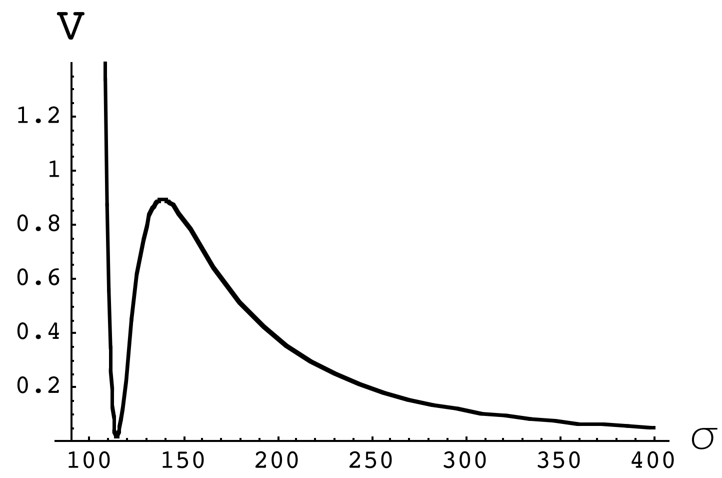
2) Bend this potential down due to (nonperturbative) quantum effects

3) Uplift the minimum to the state with positive vacuum energy by adding a positive energy of an anti-D3 brane in warped Calabi-Yau space



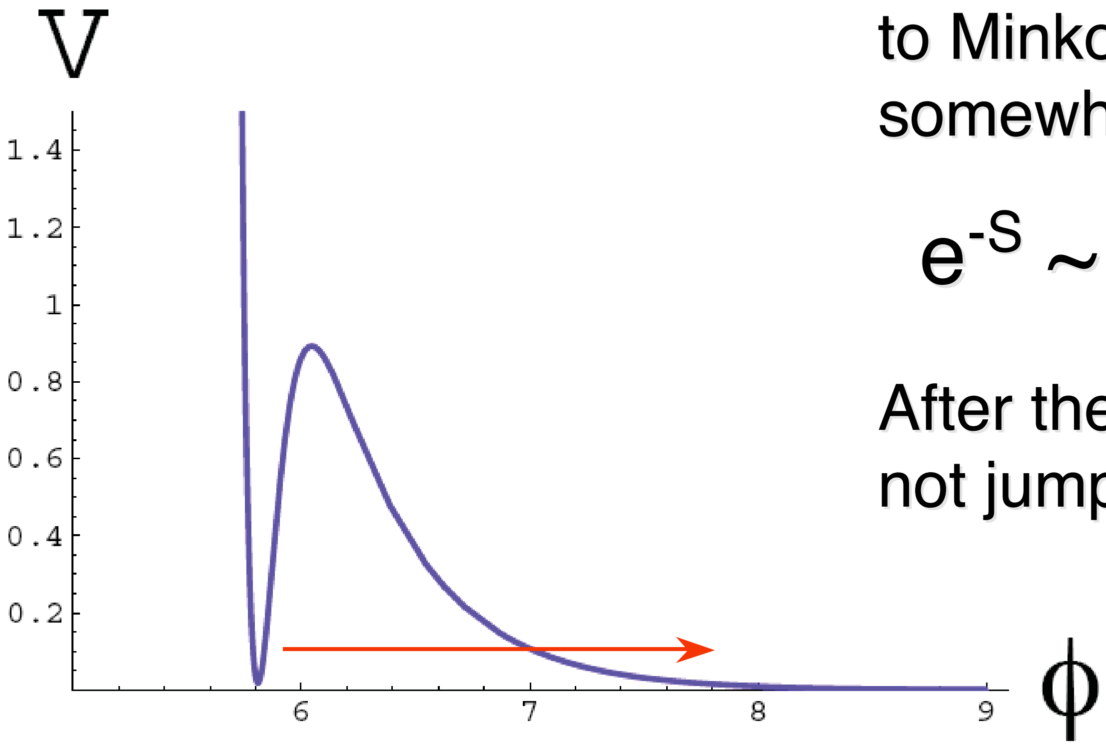
AdS minimum

$$V_{\rm KKLT} = V_{\rm AdS} + \frac{D}{\sigma^2}$$



Metastable dS minimum

KKLT potential always has a Minkowski minimum (Dine-Seiberg vacuum)



Probability of tunneling from dS to Minkowski space typically is somewhat greater than

$$e^{-S} \sim e^{-1/\Lambda} \sim e^{-10^{120}}$$

After the tunneling, the field does not jump back - no recycling.

Now we will study decay from dS to the collapsing vacua with negative vacuum energy.

Decay of the metastable dS space

The decay probability is

$$P(\phi) = e^{-B} = e^{-S(\phi)+S_0}$$

Here $S(\phi)$ is the Euclidean action for the tunneling, and $S_0 = S(\phi_0)$ is the action for the initial dS state with $\phi = \phi_0$:

$$S_0 = -\frac{24\pi^2}{V_0} < 0 \ .$$

This action has a simple sign-reversal relation to the dS entropy $\mathbf{S_0}$:

$$\mathbf{S_0} = -S_0 = +\frac{24\pi^2}{V_0} \ .$$

The decay time of the metastable dS vacuum $t_{\rm decay} \sim P^{-1}(\varphi)$ is:

$$t_{\rm decay} = e^{S(\phi)+\mathbf{S_0}} = t_r \ e^{S(\phi)} \ .$$

Here $t_r \sim e^{\mathbf{S_0}}$ is the so-called recurrence time.

The tunneling probability is given by e^{-B}, where (Parke, 1982)

$$B = \frac{27\pi^2\sigma^4}{2(V_0 - V_1)^3}\, r(x,y)\ .$$

Here σ is the wall tension. The first term is the no-gravity thin-wall result, $r(x,y)$ is the gravitational correction,

$$r(x,y) = \frac{2[(1+xy) - (1+2xy+x^2)^{\frac{1}{2}}]}{x^2(y^2-1)(1+2xy+x^2)^{\frac{1}{2}}}\ .$$

where

$$x = \left(\frac{\rho_0}{2\Lambda}\right)^2, \qquad y = \frac{\Lambda^2}{\lambda^2},$$

and

$$\rho_0 = \frac{3\sigma}{V_0 - V_1} \qquad \lambda^2 = \frac{3}{V_0 + V_1} \qquad \Lambda^2 = \frac{3}{V_0 - V_1}$$

The tension of the wall in the BPS limit is

$$\sigma = \frac{2}{\sqrt{3}}(|V_1|^{1/2} - |V_0|^{1/2})$$

For the tunneling from the supersymmetric Minkowski space, the infinite bubble size corresponds to $x = 1$, $y = -1$. In this case $r(x, y)$ blows up and the tunneling is impossible.

A small uplifting of the Minkowski minimum to $V(\phi_0) > 0$ makes the tunneling probability finite. The final result is surprisingly simple:

$$B = \frac{12\pi^2}{V_0} = \frac{\mathbf{S_0}}{2} \ .$$

where $\mathbf{S_0}$ is the entropy of dS space, $\mathbf{S_0} = \frac{24\pi^2}{V_0}$. This means that the tunneling probability is given by the universal model-independent equation which has a simple geometric interpretation in terms of dS entropy:

$$P \sim e^{-\mathbf{S_0}/2} \ .$$

An unexpected feature of this result is that the tunneling is suppressed not by a factor $e^{-\mathbf{S_0}}$, but by a factor $e^{-\mathbf{S_0}/2}$. This means that the decay time of an uplifted Minkowski vacuum will be shorter than the recurrence time $t_r \sim e^{\mathbf{S_0}}$ by a huge factor $e^{\mathbf{S_0}/2} \sim e^{10^{120}/2}$.

Tunneling to the sink

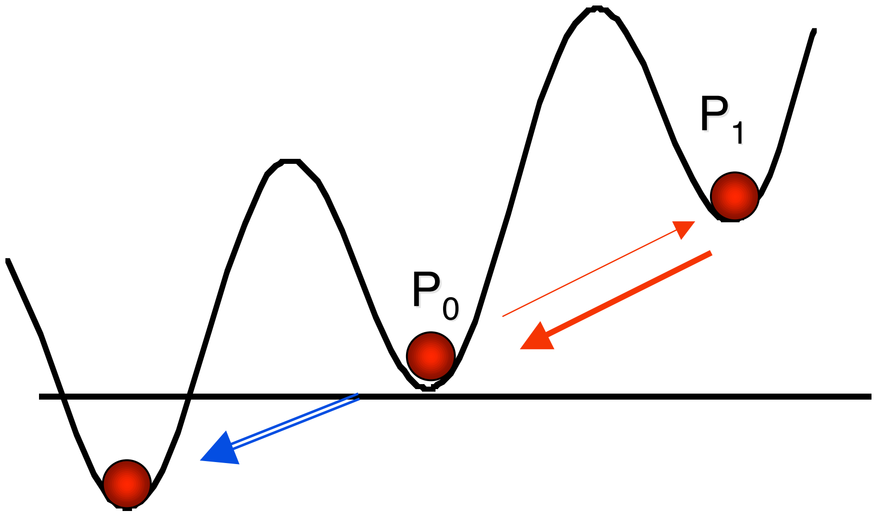
Ceresole, Dall'Agata, Giryavets, Kallosh, A.L., hep-th/0605266

The probability of decay to the vacuum with negative energy depends on the vacuum energy of the uplifted vacuum $|V_{AdS}|$ prior to the uplifting. In the class of the KKLT models that we explored, $|V_{AdS}|$ is related to SUSY breaking (to the square of the gravitino mass) after the AdS uplifting, and is of the order

$$\Gamma \sim e^{-\frac{1}{|V_{AdS}|}} \sim e^{-m_{3/2}^{-2}} \gg e^{-\frac{1}{\Lambda}}$$

In the simplest SUSY models, the rate of a decay to a sink is $\sim e^{10^{120}}$ times greater than the probability to jump from our vacuum to a higher dS vacuum.

Two dS vacua and AdS sink

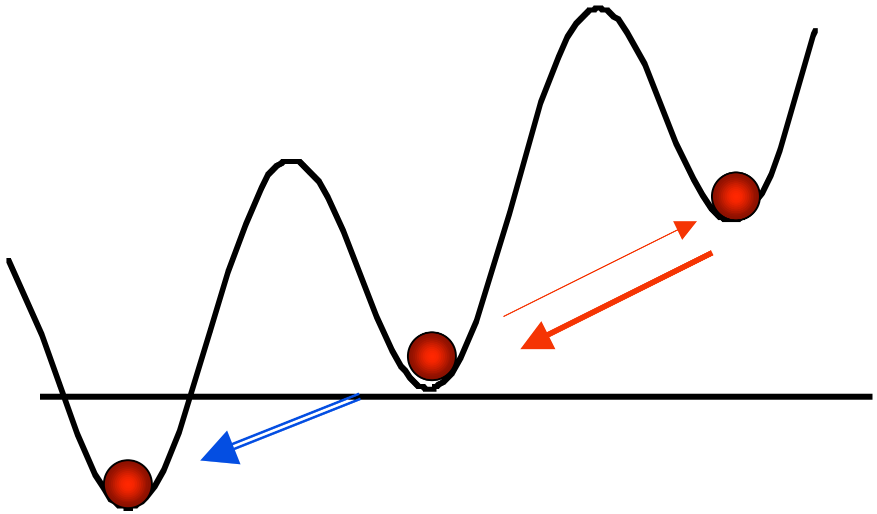


Parts of dS space tunneling to space with negative V rapidly collapse and drop out of equilibrium (one-way road to hell). Therefore instead of detailed balance equations, one has flow equations:

$$\frac{dP_0}{dt} = -P_0\, e^{-C_0} - P_0\, e^{-S_0+B} + P_1\, e^{-S_1+B} \ ,$$

$$\frac{dP_1}{dt} = P_0\, e^{-S_0+B} - P_1\, e^{-S_1+B} \ .$$

Narrow and Wide Sinks



If the decay to the sink is slower than the decay of the upper dS vacuum to the lower dS vacuum, then the probability distribution is given by the Hartle-Hawking expression, despite the vacuum instability and the general probability flow down. On the other hand, if the decay to the sink is very fast, one will have an inverted probability distribution.

The comoving volume remaining in both of the dS spaces exponentially decreases in time due to the sink in the lower dS vacuum, but the ratio $P_1(t)/P_0(t) = p$ remains constant.

Suppose first that $e^{-C_0} \ll e^{-\mathbf{S_1}+|S(\phi)|}$, i.e. the probability to fall to the sink from the lower vacuum is smaller than the probability of the decay of the upper vacuum. In this case one recovers the result related to the square of the Hartle-Hawking wave function:

$$p = \frac{P_1}{P_0} = e^{\mathbf{S_1}-\mathbf{S_0}} \ll 1 \ .$$

Now let us consider an opposite regime, and assume that the decay rate of the uplifted dS vacuum to the sink is relatively large, $e^{-C_0} \gg e^{-\mathbf{S_1}+|S(\phi)|}$. In this case

$$p = \frac{P_1}{P_0} = e^{\mathbf{S_1}-|S(\phi)|-C_0} \approx e^{\mathbf{S_1}-|S(\phi)|} \gg 1 \ ,$$

i.e. one has an **inverted probability distribution**.

Compare two currents, J_{01} and J_{10}, describing the probability flows from the lower dS state to the higher one, and other way around. In the absence of the sink,

$$J_{01} = J_{10} \ ,$$

This regime remains valid even in the presence of the sink, for $e^{-\mathbf{S_0}+|S(\phi)|} < e^{-C_0} \ll e^{-\mathbf{S_1}+|S(\phi)|}$.

If the decay rate to the sink is large, $e^{-C_0} \gg e^{-\mathbf{S_1}+|S(\phi)|}$, one has a completely different result:

$$\frac{J_{01}}{J_{10}} = \frac{P_0\, e^{-\mathbf{S_0}+|S(\phi)|}}{P_1\, e^{-\mathbf{S_1}+|S(\phi)|}} = e^{-\mathbf{S_0}+|S(\phi)|+C_0} \approx e^{-\mathbf{S_0}} \sim e^{-10^{120}} \ .$$

Thus we have a crucial regime change when the decay rate of the lower vacuum to the sink starts competing with the decay rate of the upper dS vacuum.

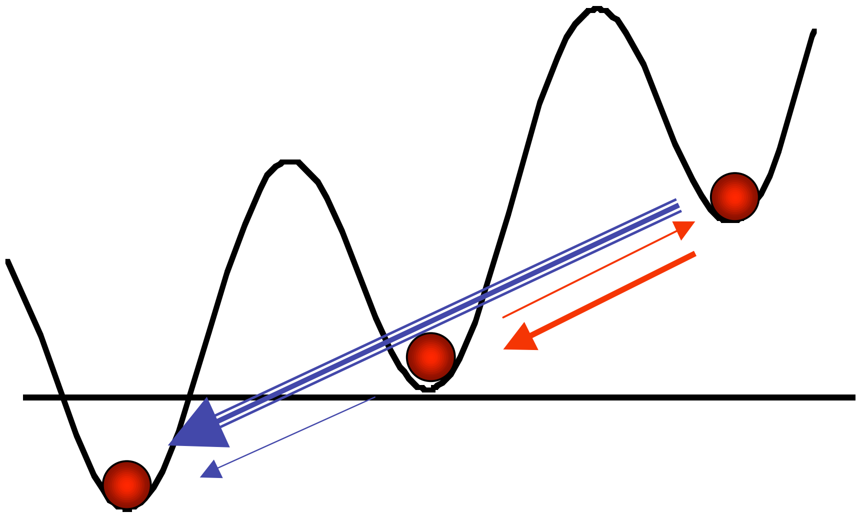
If the probability leak occurs only in the upper dS vacuum, for $e^{-C_1} \ll e^{-\mathbf{S_1}+|S(\phi)|}$ one again recovers the Hartle-Hawking distribution, whereas for $e^{-C_1} \gg e^{-\mathbf{S_1}+|S(\phi)|}$ one finds

$$p = \frac{P_1}{P_0} = e^{-\mathbf{S_0}+|S(\phi)|+C_1} \approx e^{-\mathbf{S_0}} \sim e^{-10^{120}} \ .$$

Note that in this regime we have a flat probability distribution, which does not depend on $V(\phi_1)$.

More generally, one should consider a dynamical equilibrium of a system of many dS, Minkowski and AdS vacua, where the probability leaks may occur at each level. This makes the investigation of the probabilities in the landscape much more involved, but also much more interesting.

Generic probability leaks



In general, the probability leaks from dS to a collapsing space or to a Minkowski space may occur from all dS vacua.

The resulting probability distribution may differ dramatically from the Hartle-Hawking distribution.

Conclusion:

In the string landscape scenario we do not study the ground state of the universe, as we did before. Instead of that, we study the universe with many holes in the ground.

Instead of studying static probabilities, like in a pond with still water, we study probability currents, like in a river dividing into many streams.

In other words, in addition to exploring vacuum statistics, we also explore vacuum dynamics, including irreversible vacuum decay and the growth of the volume of different parts of the universe.

All vacuum states in string theory are METASTABLE. After a very long time they will decay. At that time, our part of the universe will become ten-dimensional, or it will collapse and disappear.

But because of eternal inflation,
the universe as a whole is immortal

Self-reproducing Inflationary Universe