The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
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SMR.1761- 10

SUMMER SCHOOL IN COSMOLOGY AND ASTROPARTICLE PHYSICS

10 - 21 July 2006

String Cosmology

Part 3

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Lecture 3
Inflation in string theory

- Models available so far
- Relation to observations

Crucial data
(from Planck? will fly in 2008, data expected in 2009)

- Precision measurments of the spectral index n_s
- Discovery/non-discovery of light cosmic strings
- Discovery/non-discovery of primordial gravitational waves

1. **Few models of inflation were derived since 2003 within compactified string theory with stabilized moduli. The inflaton field, whose evolution drives inflation is the only field which is not stabilized before the exit from inflation.**

2. **Each of these models relies on particular assumptions.**

3. **Some of these models have clear predictions for observables and are therefore**

FALSIFIABLE BY DATA

OBSERVABLES

A) Tilt of the specturm n_s

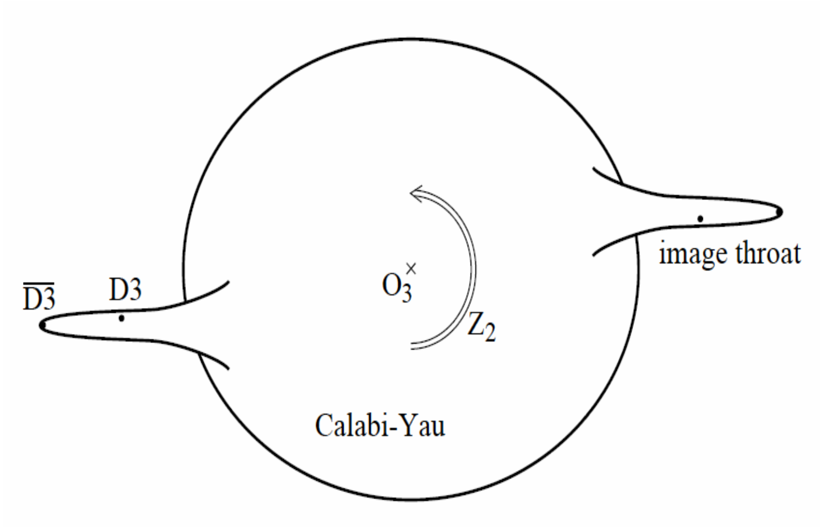
the measure of the violation of the scale invarinace

n_s-1

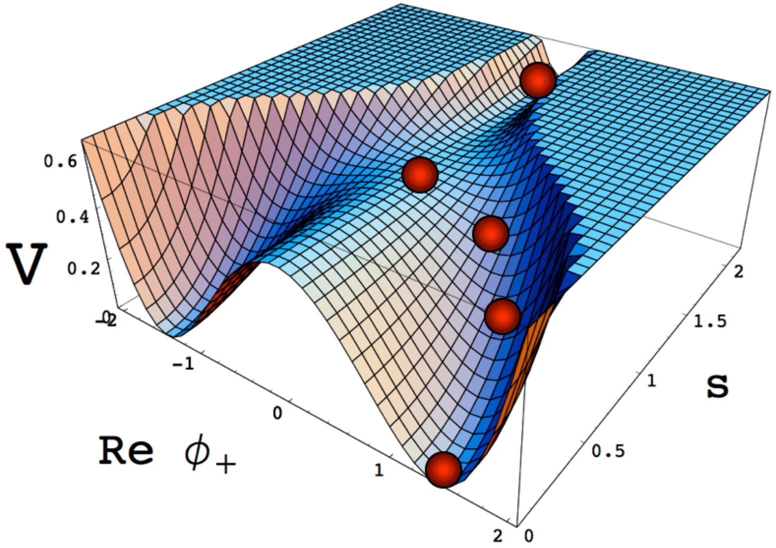
B) Gravitational waves, r= T/S

C) Cosmic strings produced by the end of inflation

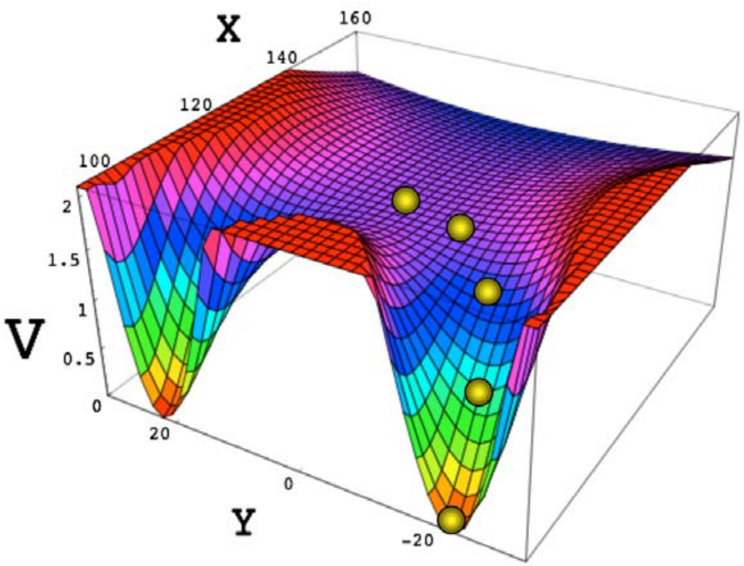
Inflation in string theory



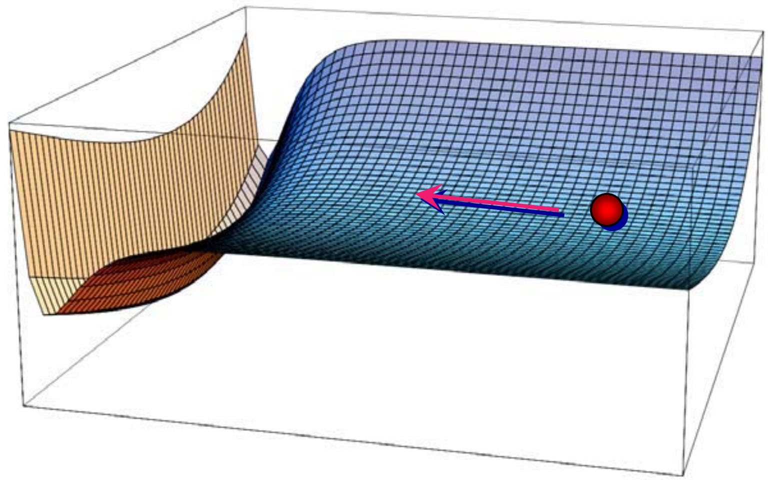
KKLMMT brane-anti-brane inflation



D3/D7 brane inflation



Racetrack modular inflation



Kähler modular inflation

DBI inflation

N-flation

Better racetrack

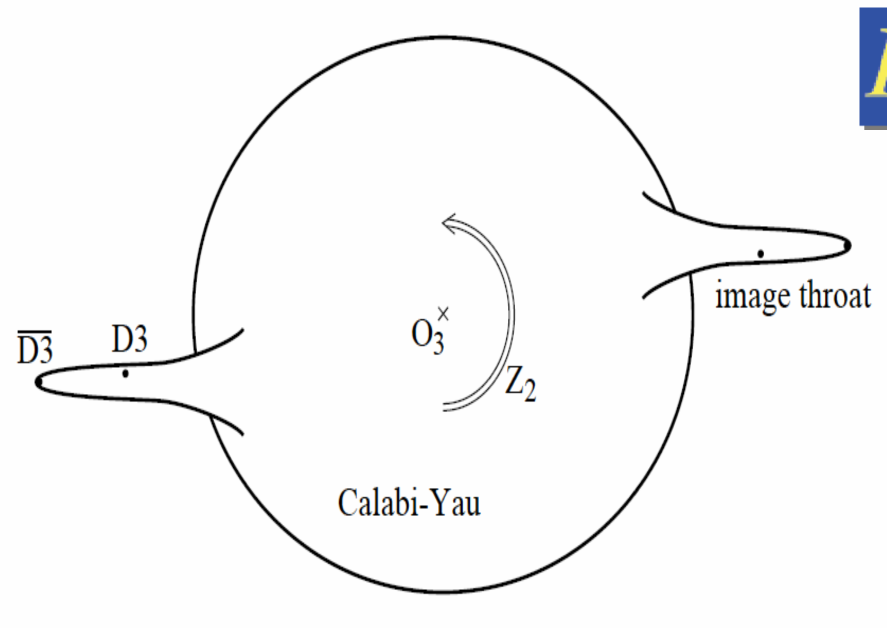
Two types of string inflation models:

- **Moduli Inflation.** The simplest class of models. They use only the fields that are already present in generalized KKLT model.

- **Brane inflation.** The inflaton field corresponds to the distance between branes in Calabi-Yau space.

The KKLMMT model

Kachru, R. K, Linde, Maldacena, McAllister, and Trivedi 2003



$$K = -3\log(\rho + \bar{\rho} - \phi\bar{\phi})$$

$$W = W_0 + e^{-a\rho}$$

$$m_\phi^2 = 2H^2$$

Meanwhile for inflation with a flat spectrum of perturbations one needs

$$m_\phi^2 \sim 10^{-2} H^2$$

This can be achieved by taking W depending on ϕ and by fine-tuning it at the level O(1%)

This model is complicated and requires fine-tuning, but it is based on some well-established concepts of string theory. Its advantage is that the smallness of inflationary parameters has a natural explanation in terms of warping of the Klebanov-Strassler throat

Fine-tuning may not be a problem in the string theory landscape paradigm

Further developed by: Burgess, Cline, Stoica, Quevedo; DeWolfe, Kachru, Verlinde; Iisuka,Trivedi; Berg, Haack, Kors; Buchel, Ghodsi

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$$W = W_0 + A(\phi, z)e^{i\rho}$$

On D3-brane Potentials in Compactifications with Fluxes and Wrapped D-branes

hep-th/0607050

Klebanov, Maldacena et al

$$A(w_1) = A_0 \left(\frac{\mu - w_1}{\mu} \right)^{1/n}$$

Calculated the dependence of the non-perturbative superpotential on the position of the D3 brane in a warped KS geometry

Justification for a fine-tuned solution of the eta-problem

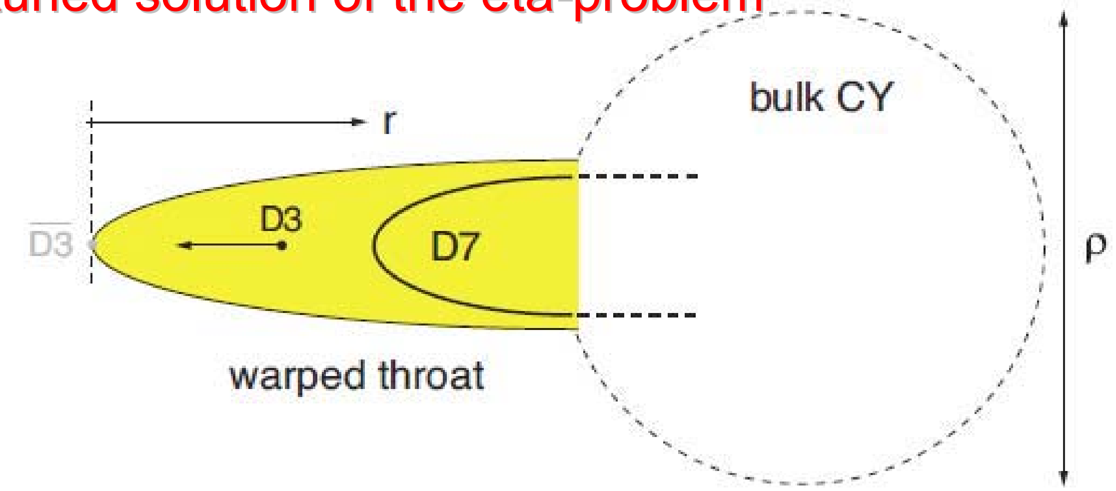


Figure 1: Cartoon of an embedded D7-brane wrapping a four-cycle Σ_4, and a mobile D3-brane, in a warped throat region of a compact Calabi-Yau. In the scenario of [8] the D3-brane feels a force from an anti-D3-brane at the tip of the throat. Alternatively, in [13] it was argued that a D3-brane in the resolved warped deformed conifold background feels a force even in the absence of an anti-D3-brane.

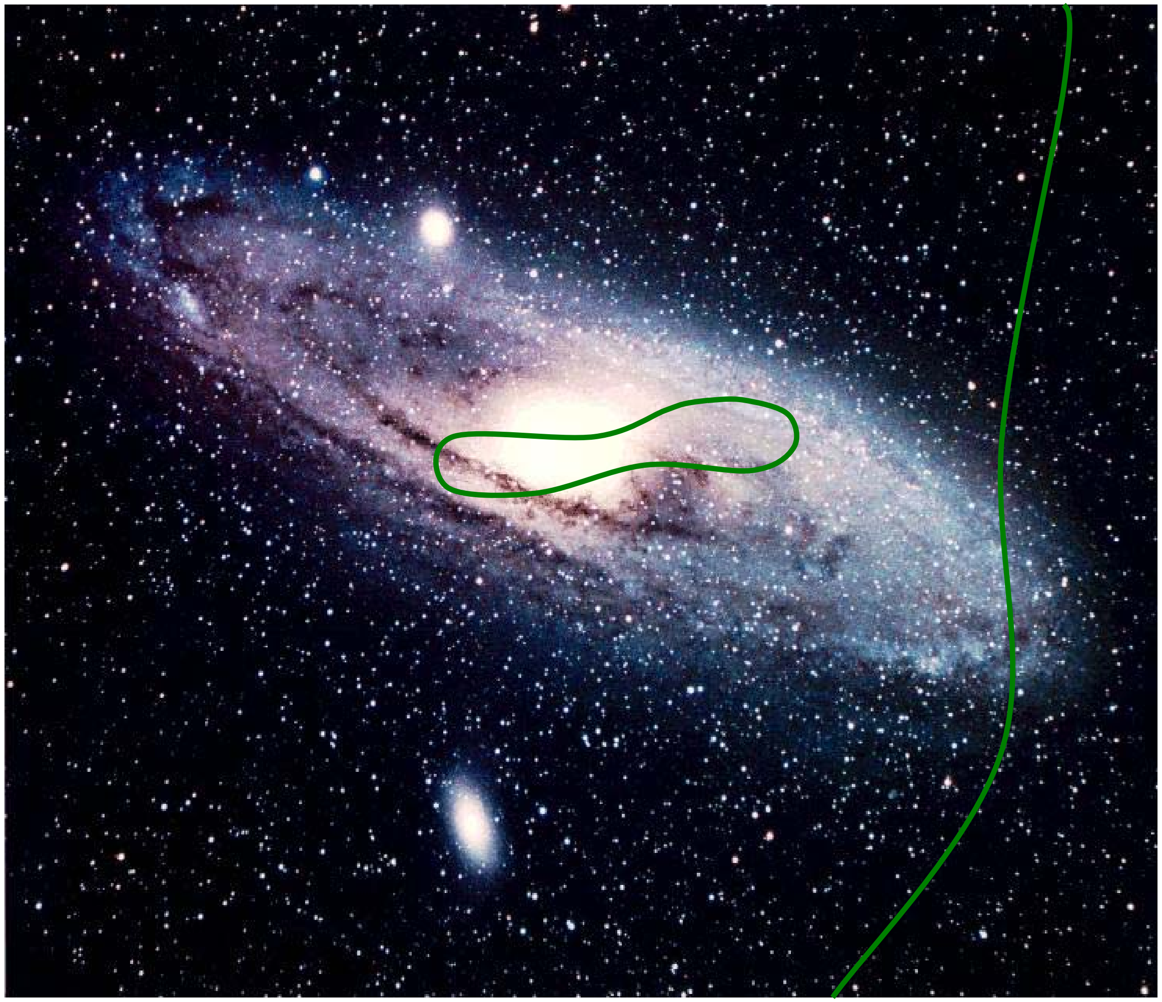
Comments on warped geometry and cosmology.

- After fine tuning (a compensation of Kahler and superpotential contribution to solve the eta-problem) the prediction of spectral index depends on the point of stabilization of complex structure moduli.

$$W = W_0 + A(\phi, z)e^{i\rho}$$

- Here ϕ is the inflaton and z are complex structure moduli. The value of corrections depends on the point where z are stabilized: hopefully can be made of any value to accommodate the future data.
- r=0, n=0.95? n=0.98?
- **Possible problems with reheating**
- Easy to explain the existence of light cosmic strings due to the redshift in the throat

Cosmic Superstrings

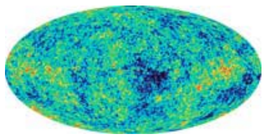


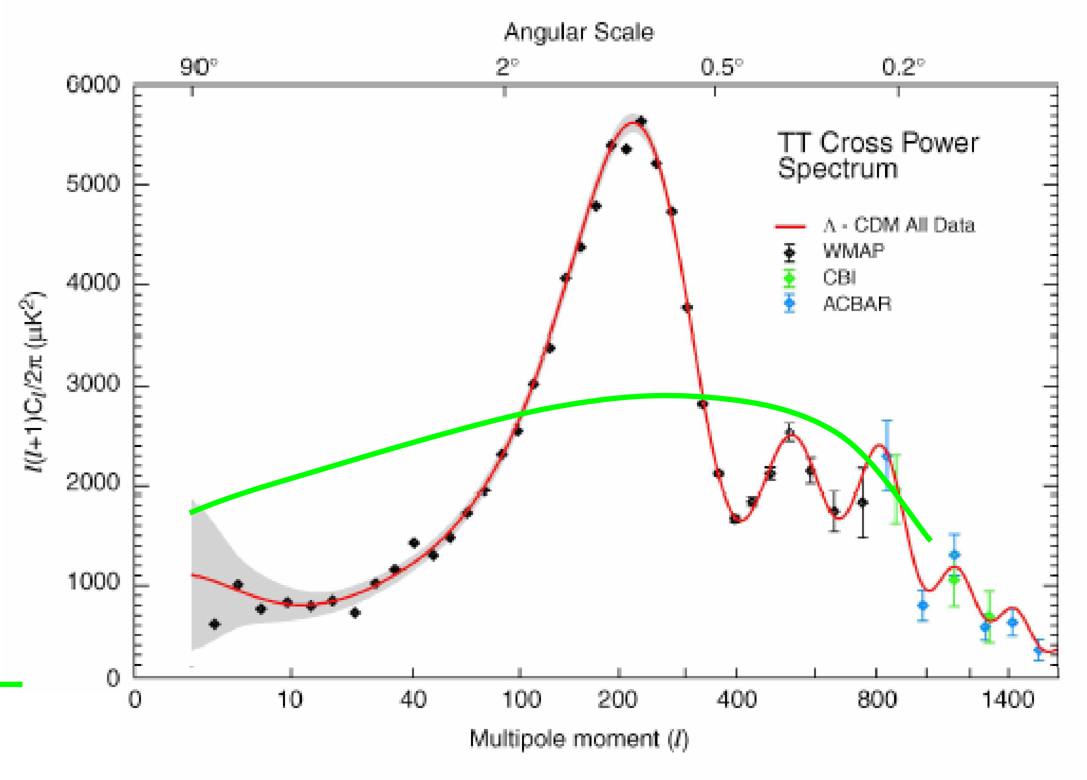
Generic in hybrid inflation models

Strings with $G\mu \sim 10^{-5.5}$ produce observed $\delta T/T$ and $\delta\rho/\rho$ (Zeldovich 1980, Vilenkin 1981).

However, they produce the wrong CMB power spectrum …

CMB power spectrum





WMAP data

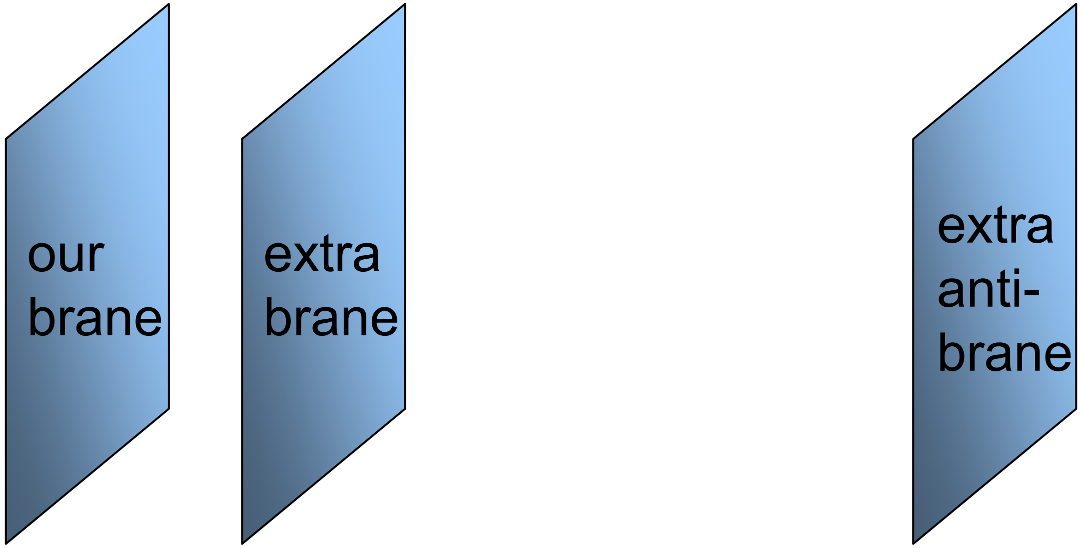
strings

Acoustic peaks come from temporal coherence. Inflation has it, strings don't. String contribution < 10% implies $G\mu \lesssim 5\cdot 10^{-7}$.

Correction in 2006: Tension < $2.7\cdot 10^{-7}$

I. Production:

An attractive model for inflation is that there were extra brane-antibrane pairs in the early universe.



Their energy density induced inflation. Inflaton = brane-antibrane separation. Weak attraction at long distance gives a flat potential, which steepens as the branes come together.

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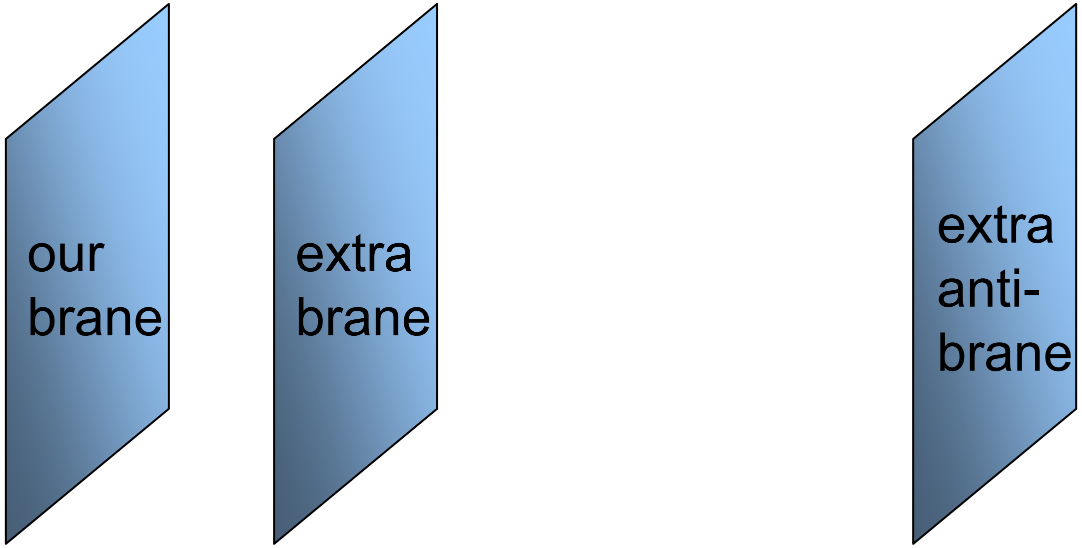

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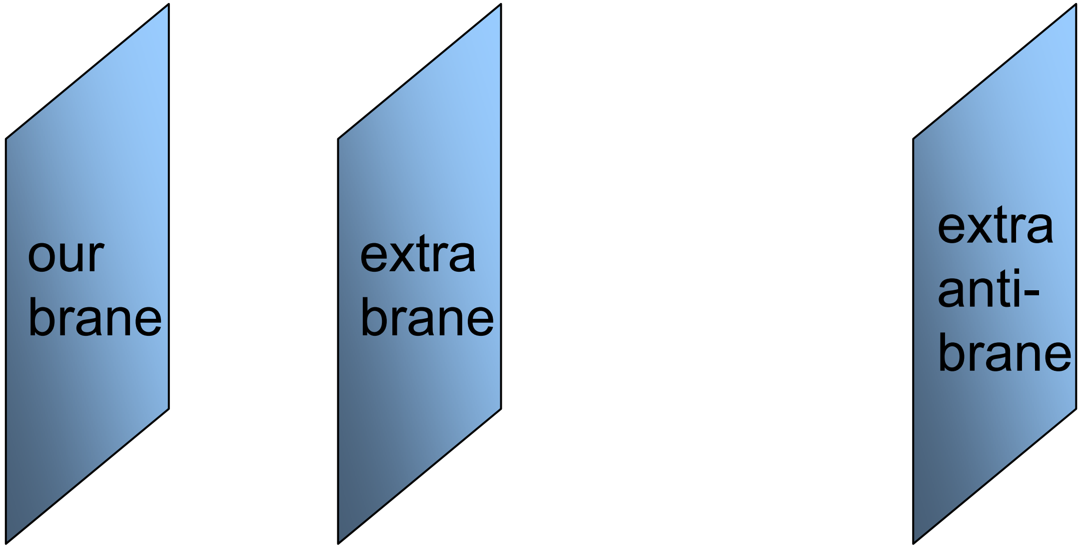

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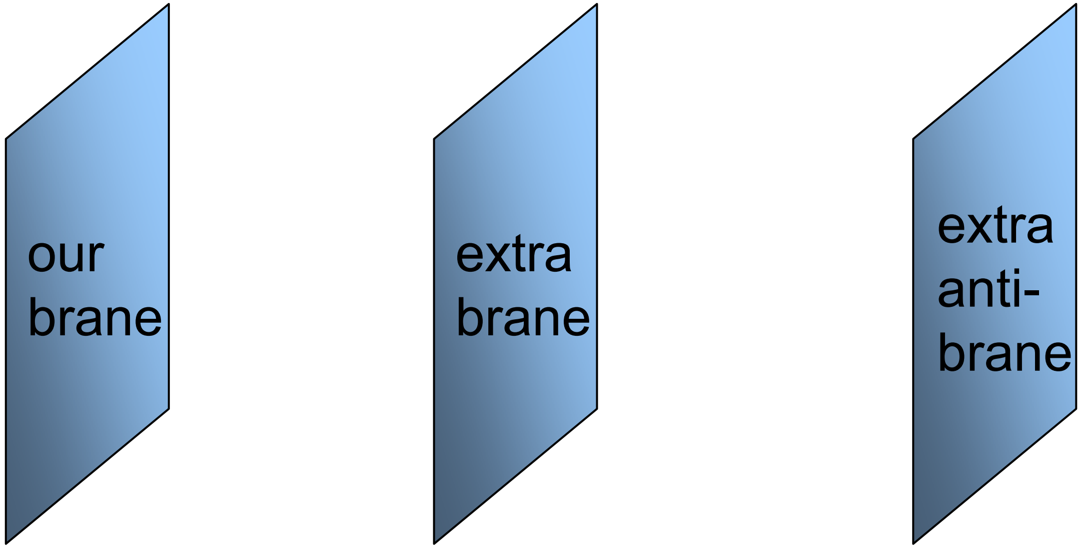

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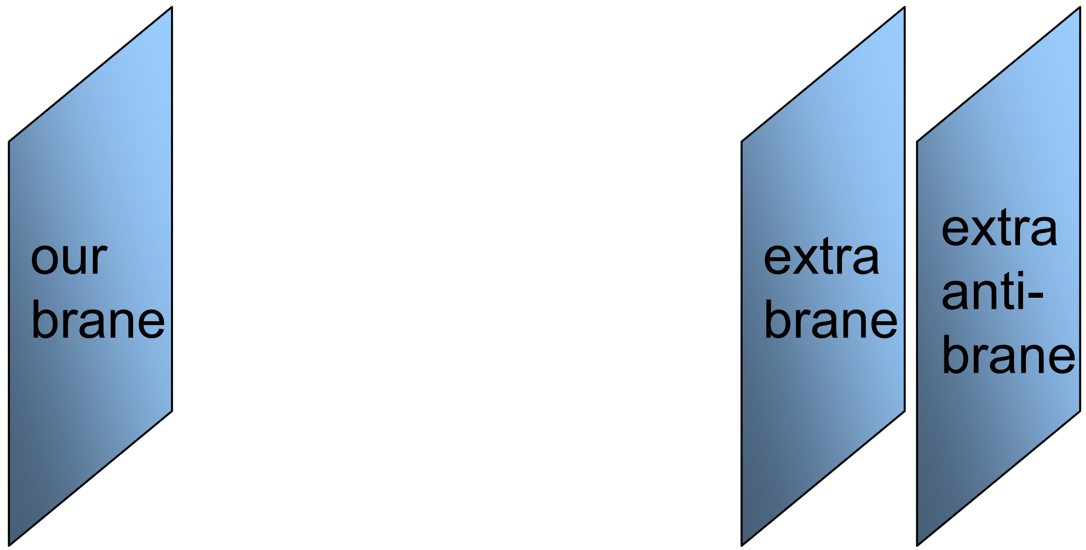


Their energy density induced inflation. Inflaton = brane-antibrane separation. Weak attraction at long distance gives a flat potential, which steepens as the branes come together.

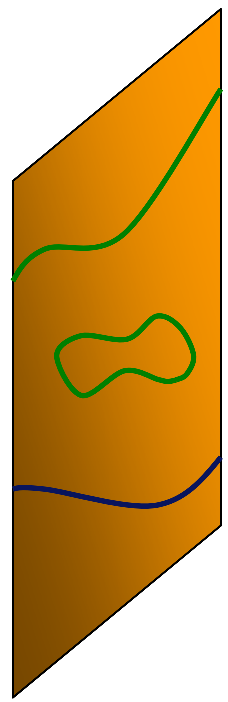
I. Production:

An attractive model for inflation is that there were extra brane-antibrane pairs in the early universe.

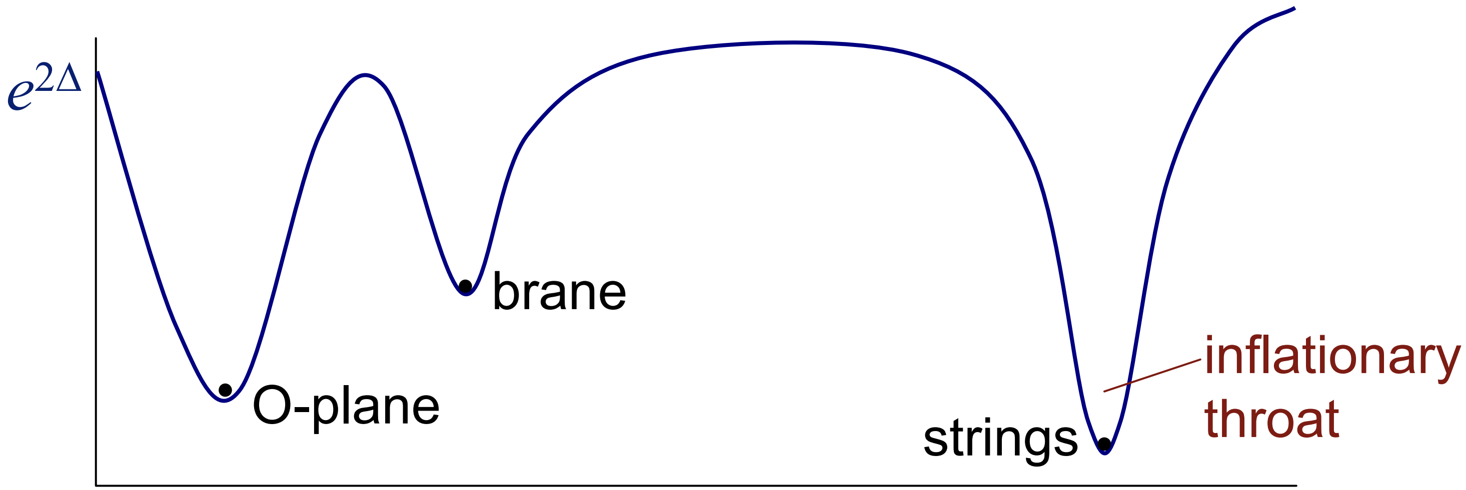
Inflation ends in brane-antibrane annihilation, reheating.

For D-branes/antibrane, there is a $U(1)\mathrm{x}U(1)$ symmetry, which disappears when the branes annihilate. This leads to production of strings just as in field theory. One $U(1)$ gives Dirichlet strings ,the other gives `fundamental' strings

radiation + D-strings + F-strings

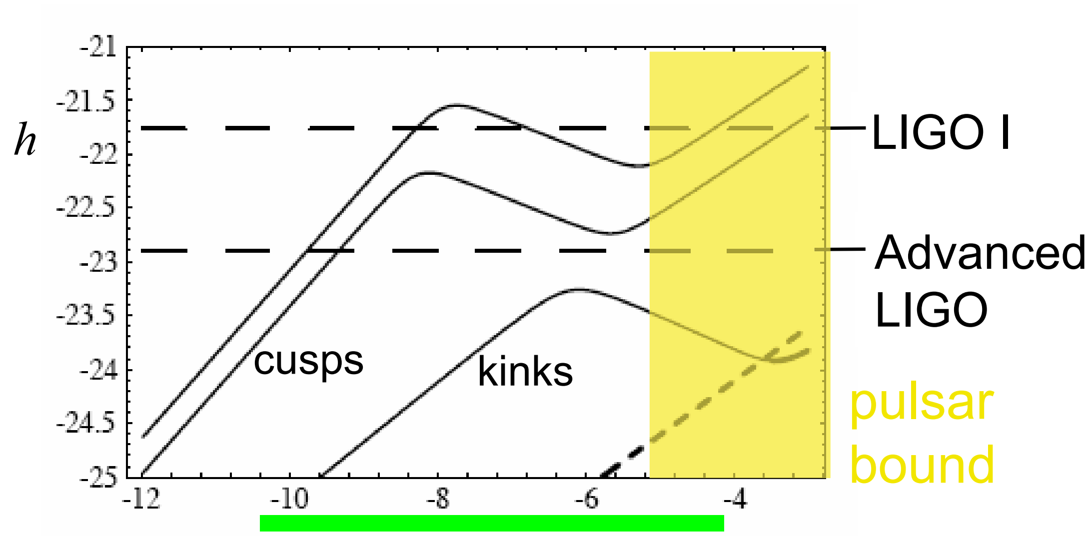


The strings and branes feel a potential due to a gravitational redshift (warp factor) in the compact directions:

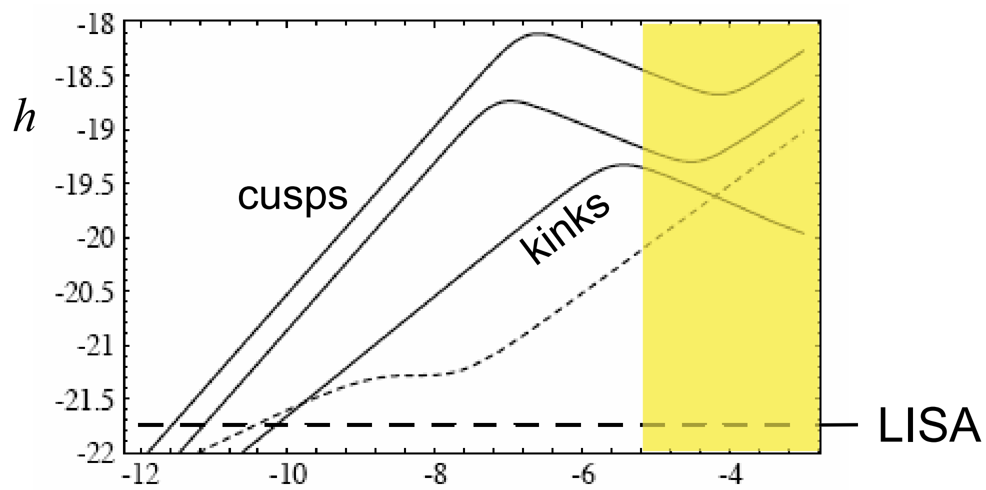


LIGO/LISA signals from string cusps

Polchinski, 2004



$\alpha \sim 50G\mu$ Damour and Vilenkin 2001



Cosmic strings could be the brightest GW sources, over a wide range of $G\mu$

In various brane inflation models one finds

$$10^{-12} \leq G\mu \leq 10^{-6}$$

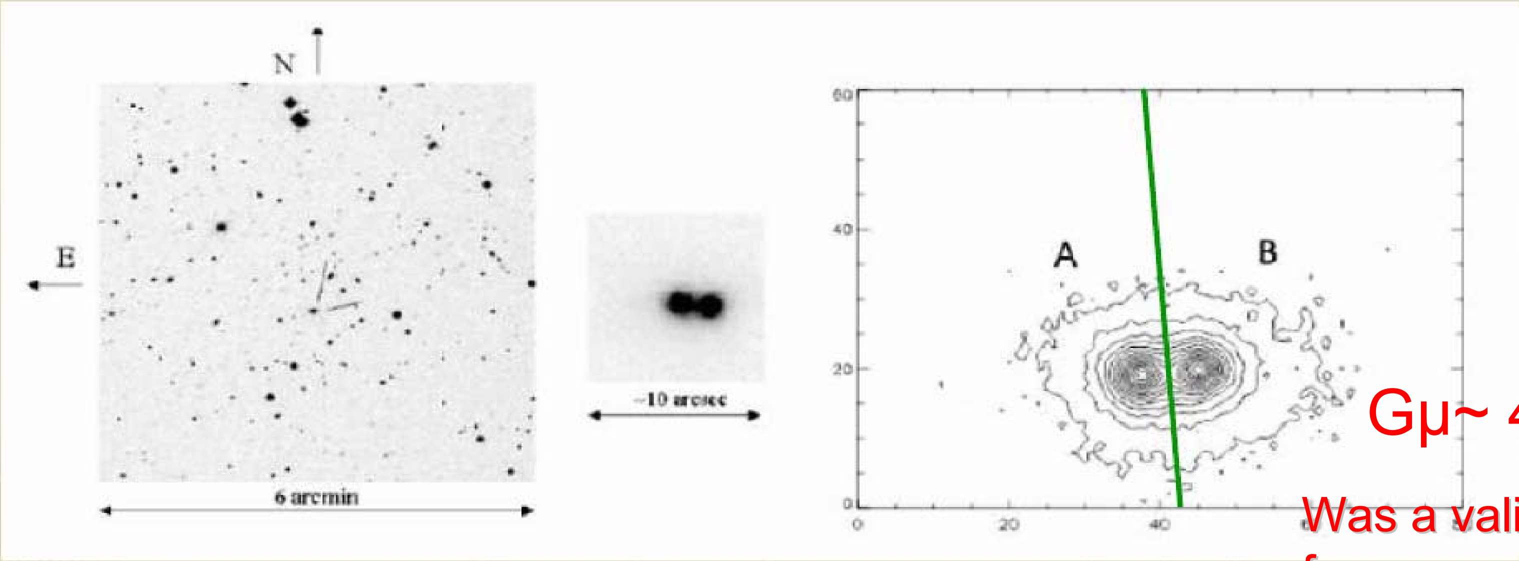
The KKLMMT model is near the middle of this range

Field theory strings?

String theory strings?

Sazhin?

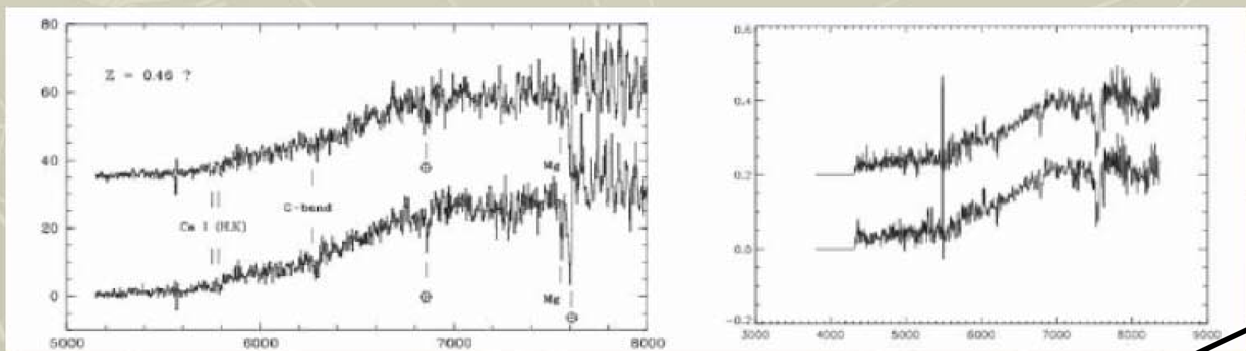
A cosmic string lens (CSL1)?



$G\mu \sim 4 \times 10^{-7}$

Was a valid candidate for a cosmic string event. Ruled out by Hubble Space Telescope in 2006

Sazhin, et al. 2003. $\delta\phi \sim 2''$.



With correct CMB bound would be ruled out long time ago!

$G\mu < 2.7 \times 10^{-7}$

B polarization of cosmic microwave background as a tracer of strings

Seljak and Slosar

(Dated: May 15, 2006)

String models can produce successful inflationary scenarios in the context of brane collisions and in many of these models cosmic strings may also be produced. In scenarios such as KKLMMT the string contribution is naturally predicted to be well below the inflationary signal for cosmic microwave background (CMB) temperature anisotropies, in agreement with the existing limits. We find that for B type polarization of CMB the situation is reversed and the dominant signal comes from vector modes generated by cosmic strings, which exceeds the gravity wave signal from both inflation and strings. The signal can be detected for a broad range of parameter space: future polarization experiments may be able to detect the string signal down to the string tension $G\mu = 10^{-9}$, although foregrounds and lensing are likely to worsen these limits. We argue that the optimal scale to search for the string signature is at $\ell \sim 1000$, but in models with high optical depth the signal from reionization peak at large scales is also significant. The shape of the power spectrum allows one to distinguish the string signature from the gravity waves from inflation, but only with a sufficiently high angular resolution experiment.

Racetrack Inflation

$\rho = iT$

the first working model of the **moduli inflation**

Blanco-Pilado, Burgess, Cline, Escoda, Gomes-Reino, Kallosh, Linde, Quevedo

Superpotential: $$W = W_0 + A\,e^{-aT} + B\,e^{-bT}$$

Kähler potential: $$K = -3\log(T + T^*)$$

KKLT Uplifting term: $$\delta V = \frac{E}{(T+T^*)^2}$$

Rescaling (same slow-roll etc)

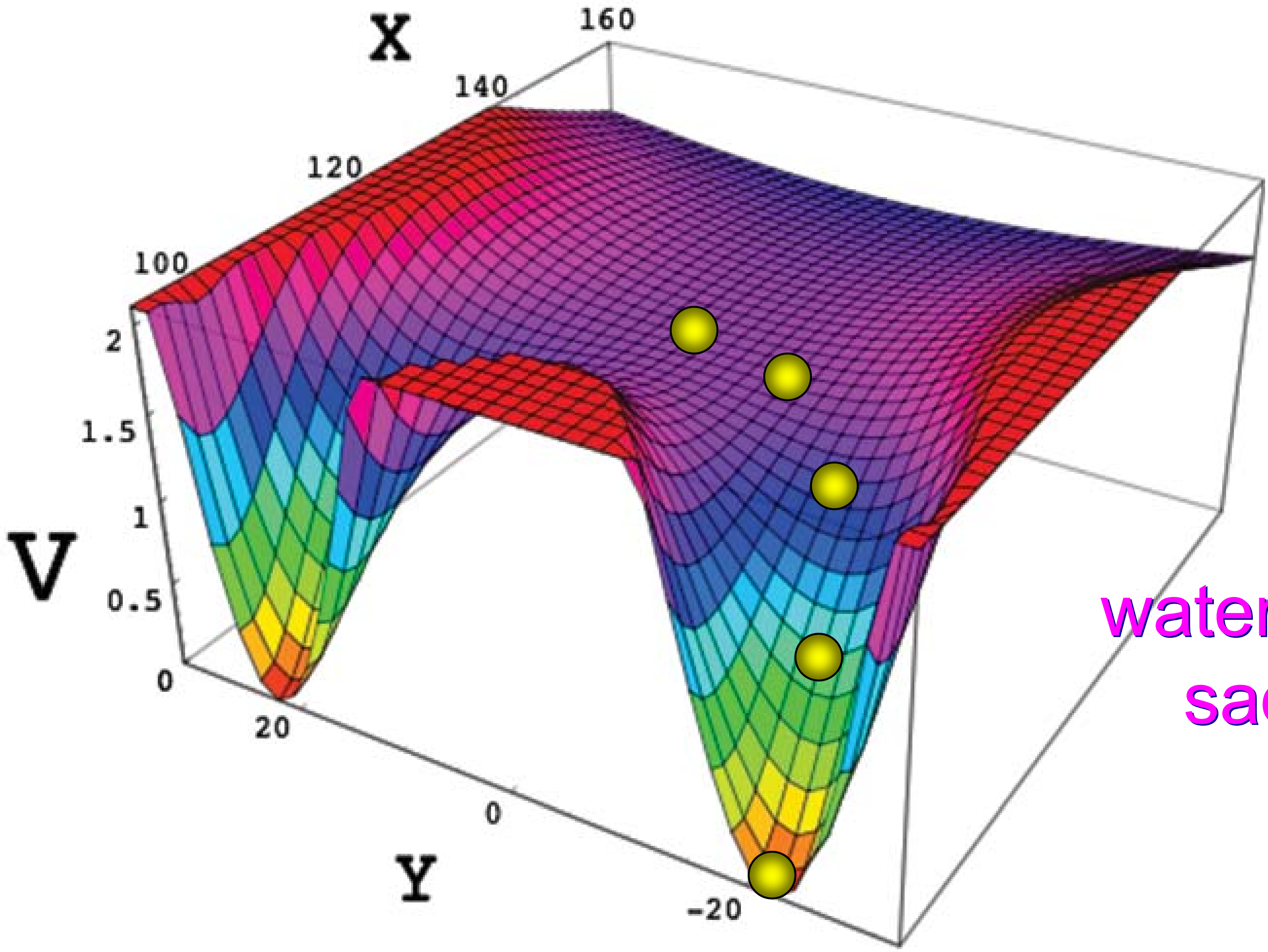
$$a \to \frac{a}{\lambda}, \quad b \to \frac{b}{\lambda}, \quad E \to \lambda^2 E$$

$$T \to \lambda T$$

$$A \to \lambda^{3/2} A, \quad B \to \lambda^{3/2} B, \quad W_0 \to \lambda^{3/2} W_0$$

Parameters and Potential

$$A = \frac{1}{50}, \qquad B = -\frac{35}{1000}, \qquad a = \frac{2\pi}{100}, \qquad b = \frac{2\pi}{90}, \qquad W_0 = -\frac{1}{25000}$$



$n_s = 0.95$

waterfall from the saddle point

Alpha' corrections to Kahler potential

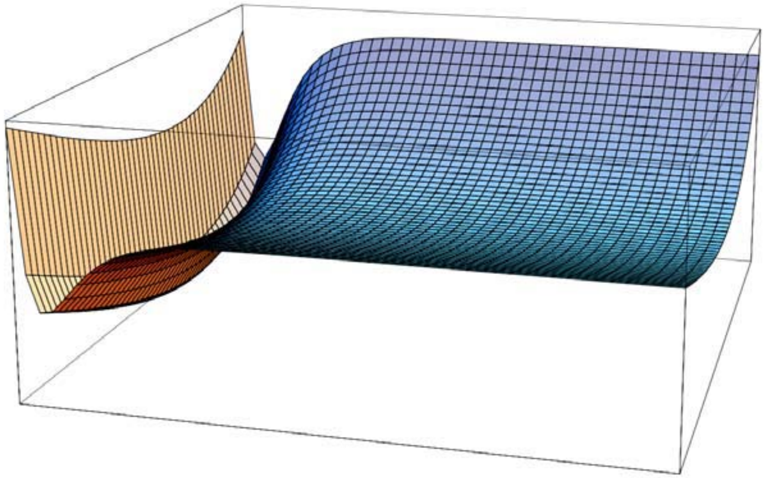
$$S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_s}\,e^{-2\phi}\left[R_s + 4(\partial\phi)^2 + \alpha'^3\frac{\zeta(3)}{3\cdot 2^{11}}\,J_0\right]$$

$$J_0 \sim R^4_{....}$$

$$K = -2\cdot\ln\left(\mathcal{V} + \frac{1}{2}\,\hat{\xi}\right) \ , \ \hat{\xi} = \xi e^{-3\phi/2} \ , \ \xi = -\frac{1}{2}\,\zeta(3)\chi$$

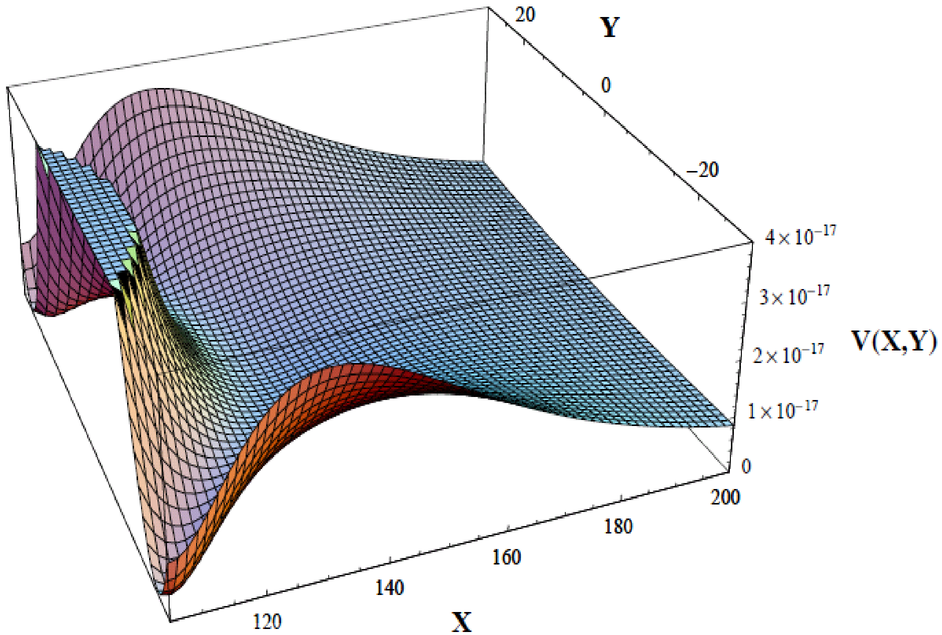
$$= \underbrace{-3\cdot\ln\left(T+\bar{T}\right)}_{K^{(0)}} - 2\cdot\ln\left(1 + \frac{\hat{\xi}}{2(2\,\mathrm{Re}\,T)^{3/2}}\right) \ .$$

May be used for uplifting to dS



Kähler modular inflation

Berglund, Balasubramanian, Conlon, Quevedo



Alpha' corrections to racetrack inflation

Westphal

B. Greene,
A. Weltman

In the following table we have evaluated η for a range of values of λ and L and compared the classical and α' corrected results[5]. The values of L we have chosen correspond to the following cases. $L = 2$ corresponds to $\chi = -6$ and $g_s \approx 0.9$; this represents a value of L on the low end of physical interest. $L = 60$ and $L = -60$ correspond to the quintic and the mirror quintic respectively, with $g_s \approx 1$. $L = 2500$ corresponds to the quintic and $g_s \approx 1/12$.

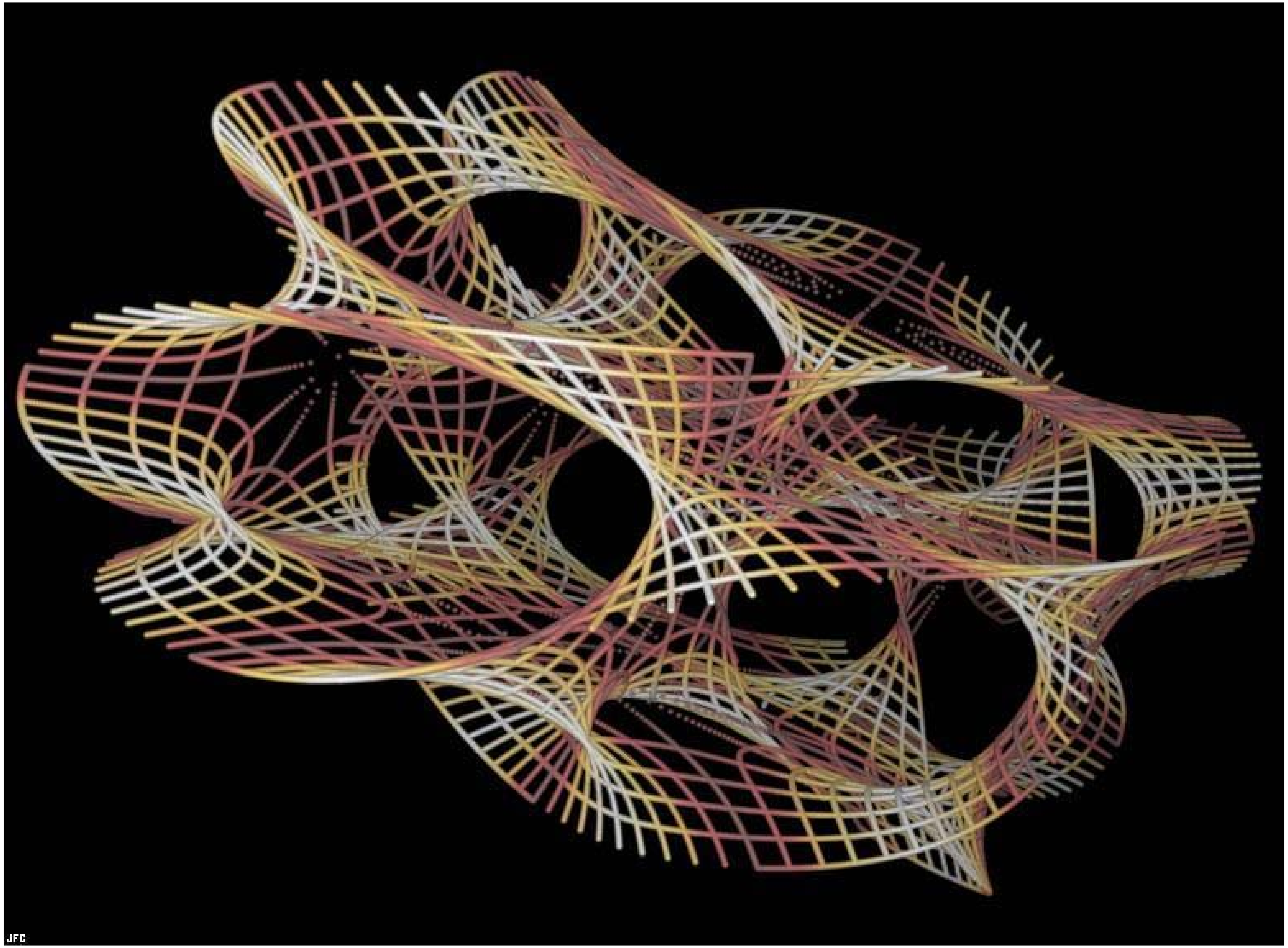
	$L=0$		$L=2$		$L=60$		$L=-60$		$L=2500$	
λ	X_{saddle}	η	X_{saddle}	η	X_{saddle}	η	X_{saddle}	η	X_{saddle}	η
100000	12321632	−0.0061	12321632	−0.0061	12321632	−0.0061	12321632	−0.0061	12321632	−0.0061
4000	492865.3	−0.0061	492865.3	−0.0061	492865.3	−0.0061	492865.2	−0.0061	492867.0	−0.0056
400	49286.53	−0.0061	49286.53	−0.0061	49286.66	−0.0057	49286.40	−0.0064	49291.92	0.0085
80	9857.305	−0.0061	9857.315	−0.0059	9857.594	−0.0022	9857.017	−0.0099	9869.600	0.1582
50	6160.816	−0.0061	6160.828	−0.0058	6161.182	0.0018	6160.451	−0.0140	6176.740	0.3310
20	2464.326	−0.0061	2464.346	−0.0050	2464.906	0.0252	2463.751	−0.0372	2494.857	1.4940
10	1232.163	−0.0061	1232.190	−0.0032	1232.989	0.0827	1231.355	−0.0935	no min	–
2	246.4326	−0.0061	246.4938	0.0268	248.5787	1.0784	244.7928	−0.9355	no min	–
1	123.2163	−0.0061	123.3035	0.0875	no min	–	121.1970	−2.4635	no min	–
1/2	61.60816	−0.0061	61.73434	0.2622	no min	–	59.34719	−6.3833	no min	–
1/4	30.80408	−0.0061	30.99693	0.7882	no min	–	28.45786	−18.4781	no min	–
1/8	15.40204	−0.0061	no min	–	no min	–	12.81586	37.7937	no min	–

Table 1: Effect of α' corrections on stabilisation and η when we move the minimum using λ.

Notice that for reasonable values of L, say $\Re(T) \sim 10^7$ in Planck units with $2\pi\alpha' = 1$, the α' corrections become negligible and supergravity analysis is both qualitatively and quantitatively unaffected. However, at smaller values of $\Re(T)$–but large enough for perturbation theory to be justified–the corrections not only change the details of the inflationary model but ultimately prevent inflation from

Successful moduli stabilization and inflation may place restriction on Euler number of the compactification (number of handles)

A slice of quintic



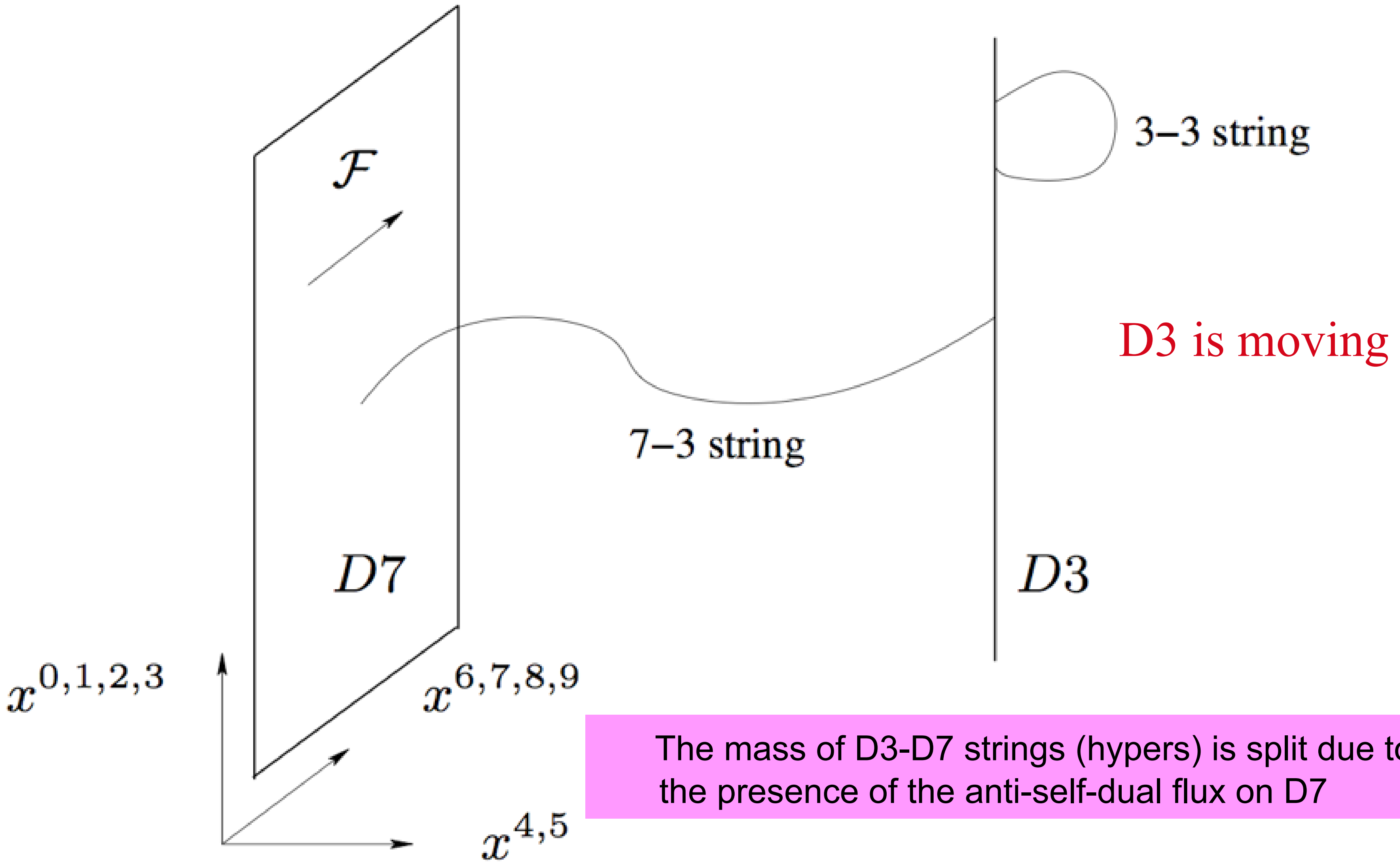
$T = \sigma + i\alpha$ Total volume σ fixed by gaugino condensation/instantons

Shape moduli and axion-dilaton fixed by fluxes

D3/D7 Inflation

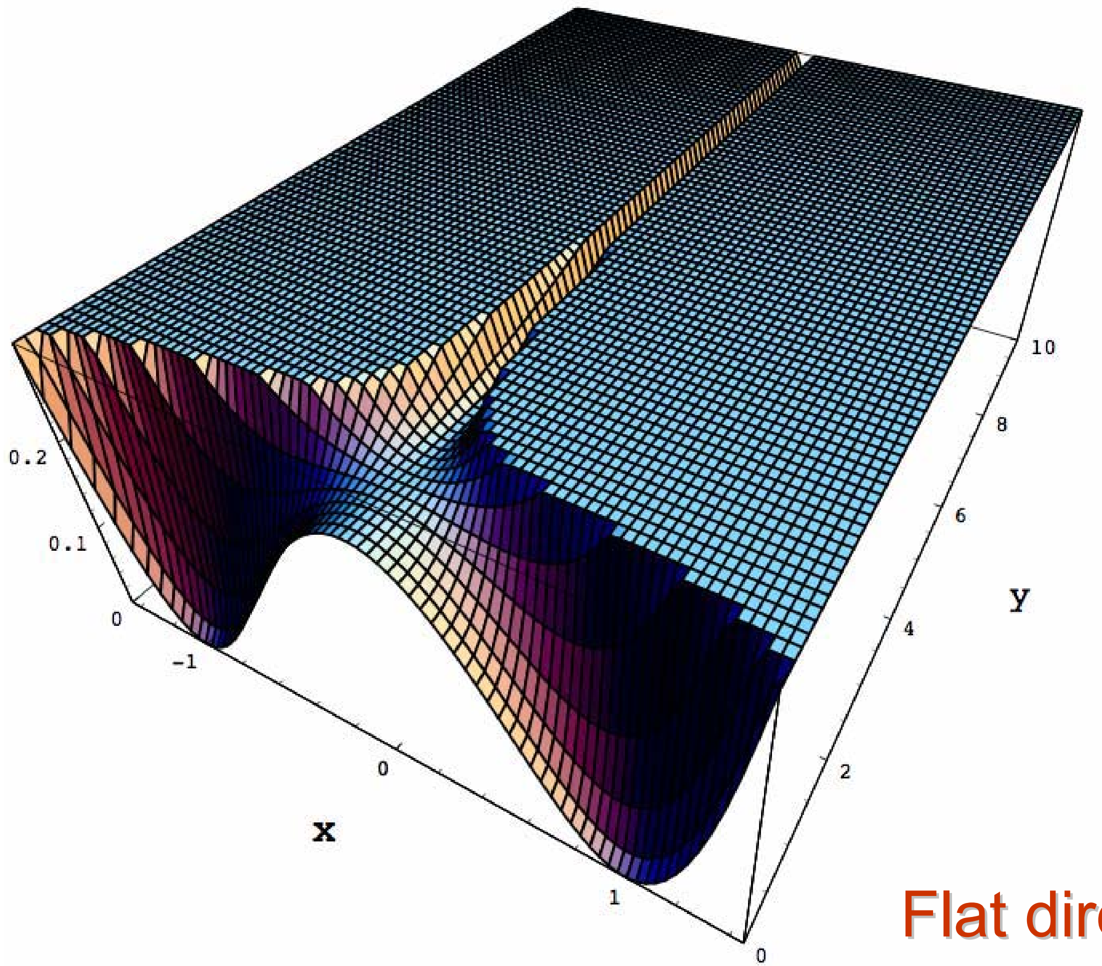
Dasgupta, Herdeiro, Hirano, Kallosh

This is a stringy version of D-term Inflation



The mass of D3-D7 strings (hypers) is split due to the presence of the anti-self-dual flux on D7

Hybrid D3/D7 Inflation Model

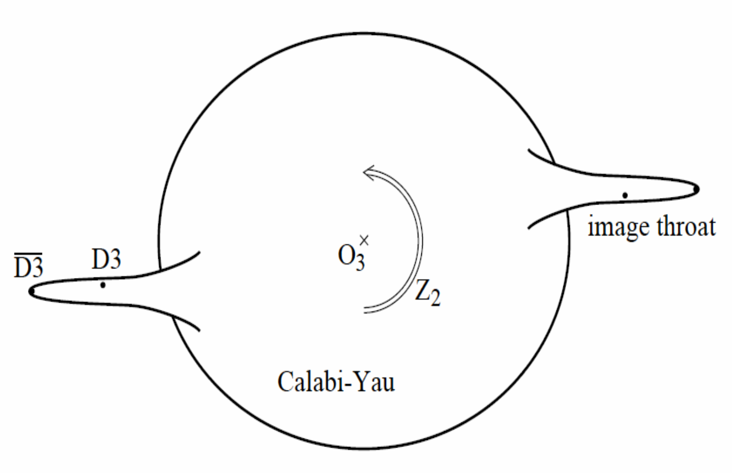


How to make this model valid in string theory with the volume stabilization

Shift Symmetry?

Flat direction, slow-roll ?

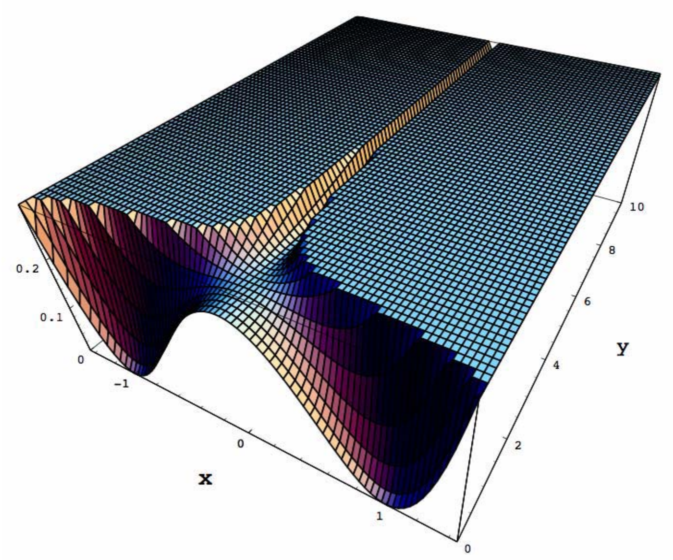
Inflationary models using mobile D3 branes



KKLMMT brane-anti-brane inflation

Fine-tuning n_s ?

Easy to explain light cosmic strings



D3/D7 brane inflation

with volume stabilization and

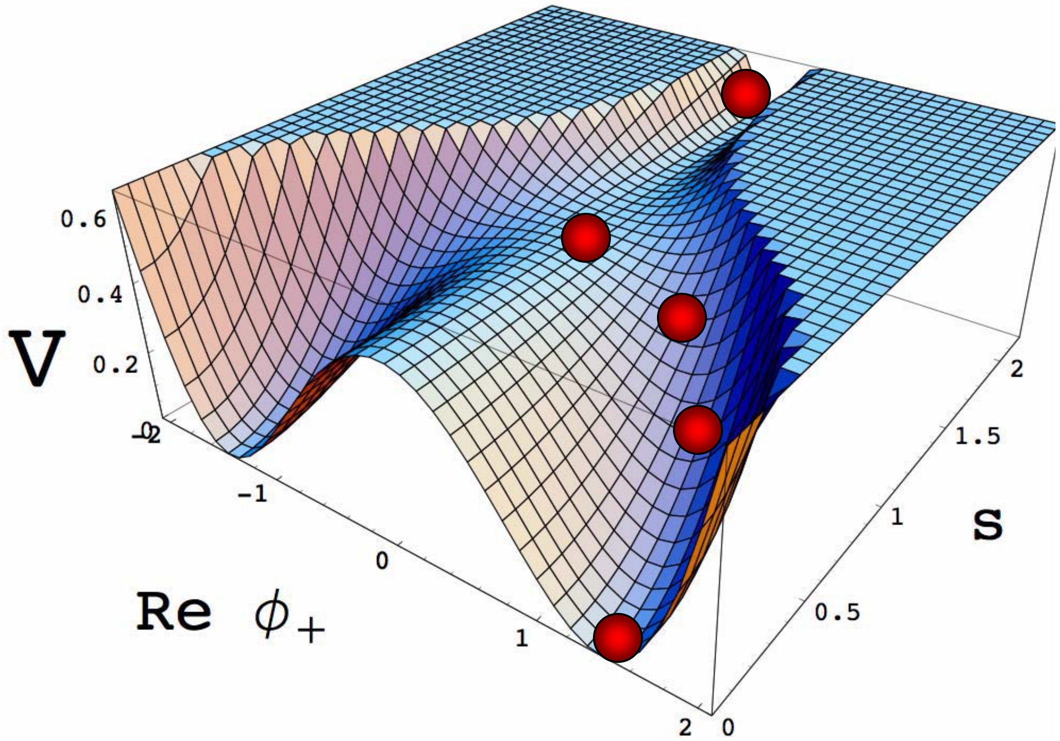
shift symmetry, slightly

broken by quantum corrections

Stringy D-term inflation

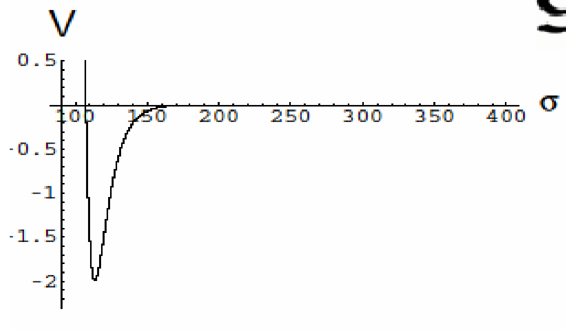
Potential of D3/D7 hybrid inflation with a stabilized volume modulus

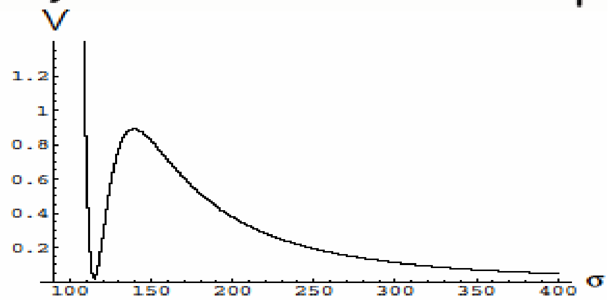
$$n_s = 0.98$$

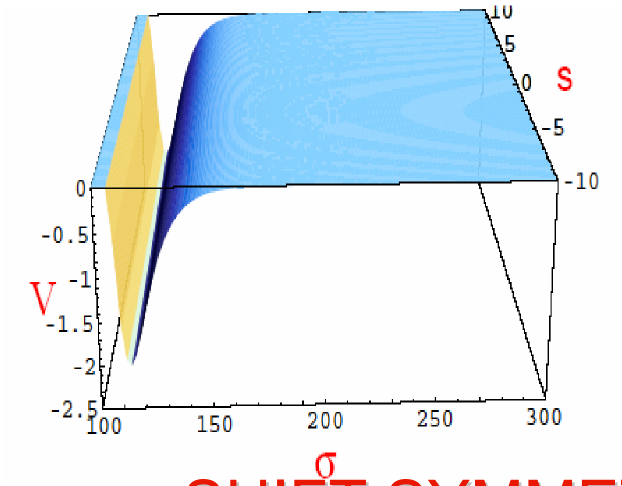


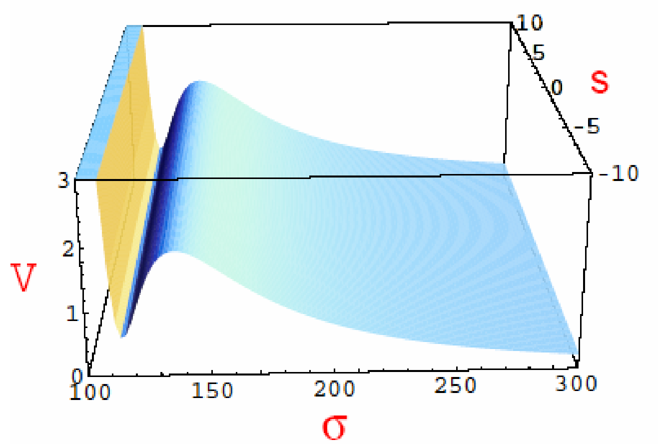
Inflaton Trench

$$\mathcal{G}(\rho, \bar{\rho}, \bar{s} + s) = K + \ln |W|^2$$









SHIFT SYMMETRY, slightly broken by quantum corrections

Hsu,R.K.,Prokushkin;
Firouzjahi,Tye

The motion of branes does not destabilize the volume

String inflation and shift symmetry

$$K = -3\log(\rho + \bar{\rho} - \frac{1}{2}(s + \bar{s})^2)$$

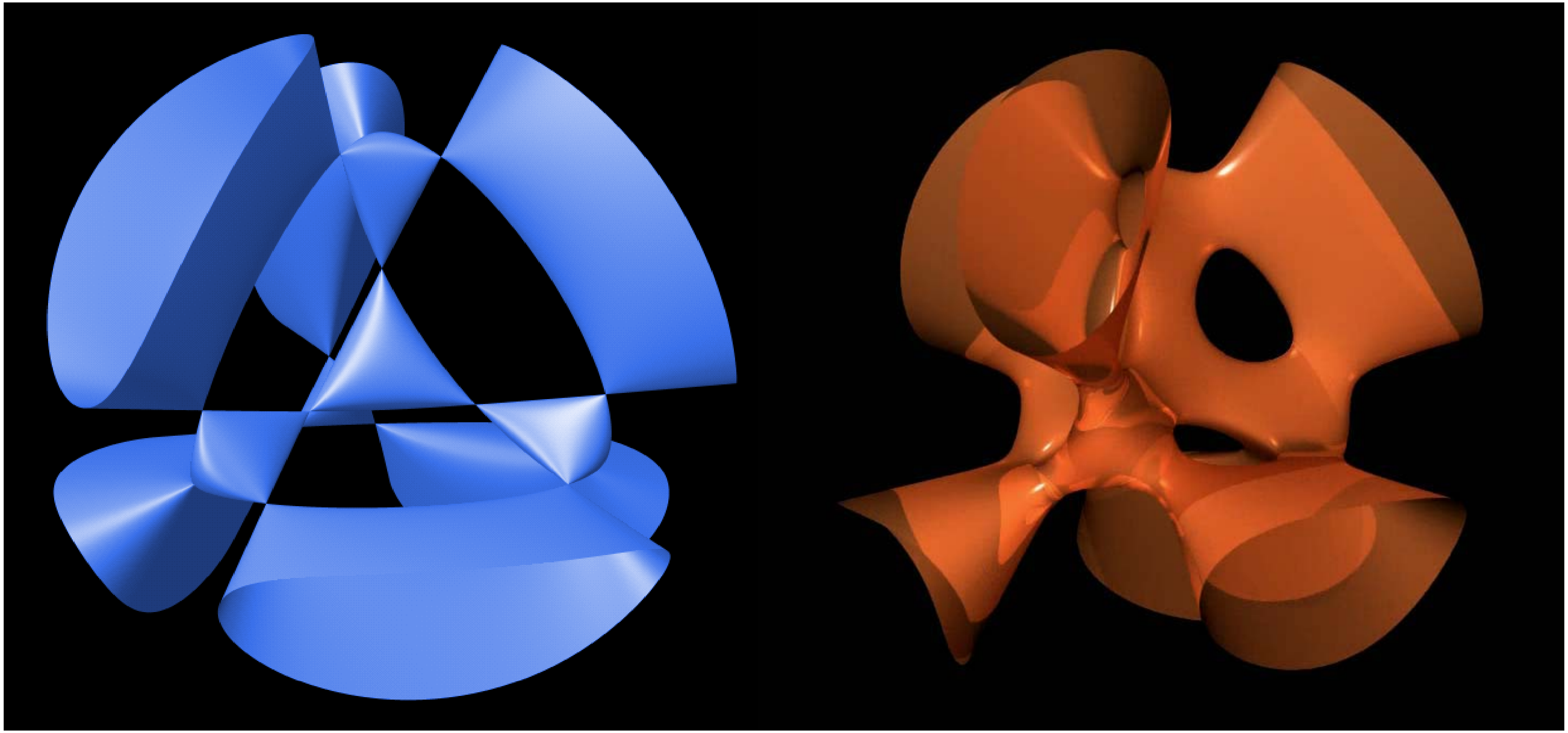
$$W = W_0 + Ae^{-a\rho} + W_{Dterm}$$

Shift symmetry protects flatness of the inflaton potential in the $s - \bar{s}$ direction. This is not just a requirement which is desirable for inflation, but, in a certain class of string theory models, it may be a consequence of a classical symmetry slightly broken by quantum corrections. Example, K3x $\frac{T^2}{Z_2}$

work in progress

The orientifold on K3 x $\frac{T^2}{Z_2}$

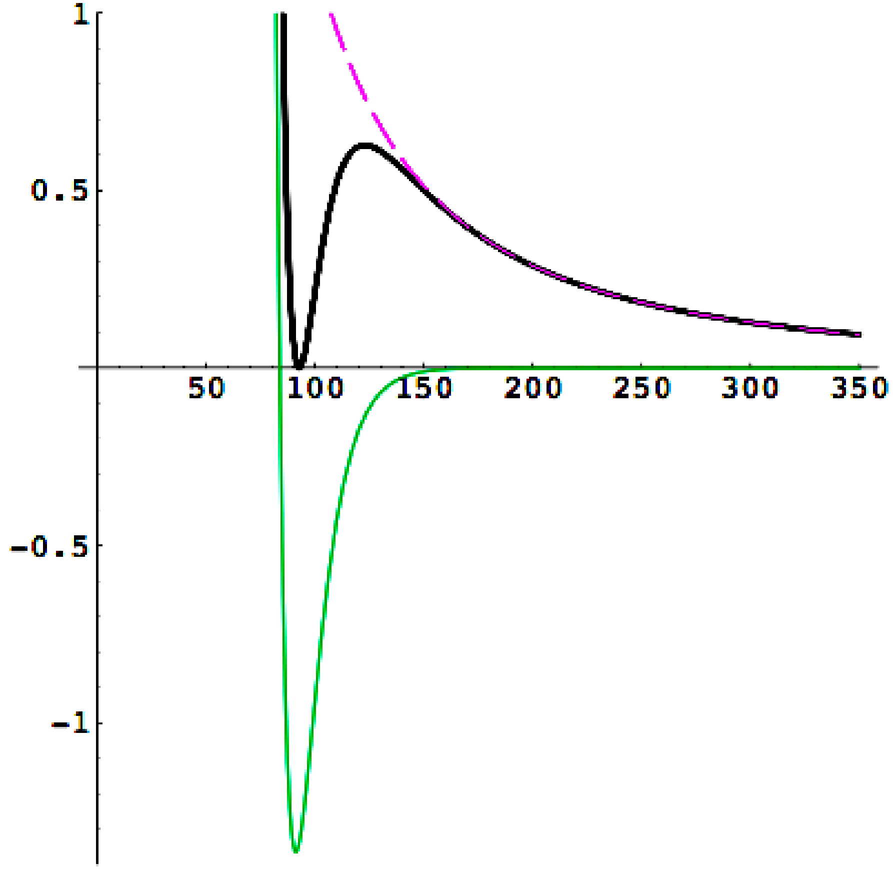
- In F-theory compactifications on K3 x K3 one of the attractive K3 must be a Kummer surface to describe an orientifold in IIB, the second attractive K3 can be regular.



Double Uplifting

$$V_{\text{tot}} = V_{\text{KKLT}}(\sigma) + V_2(\sigma, s, \phi_\pm)$$

Kallosh, A.L., in progress



First uplifting: KKLT

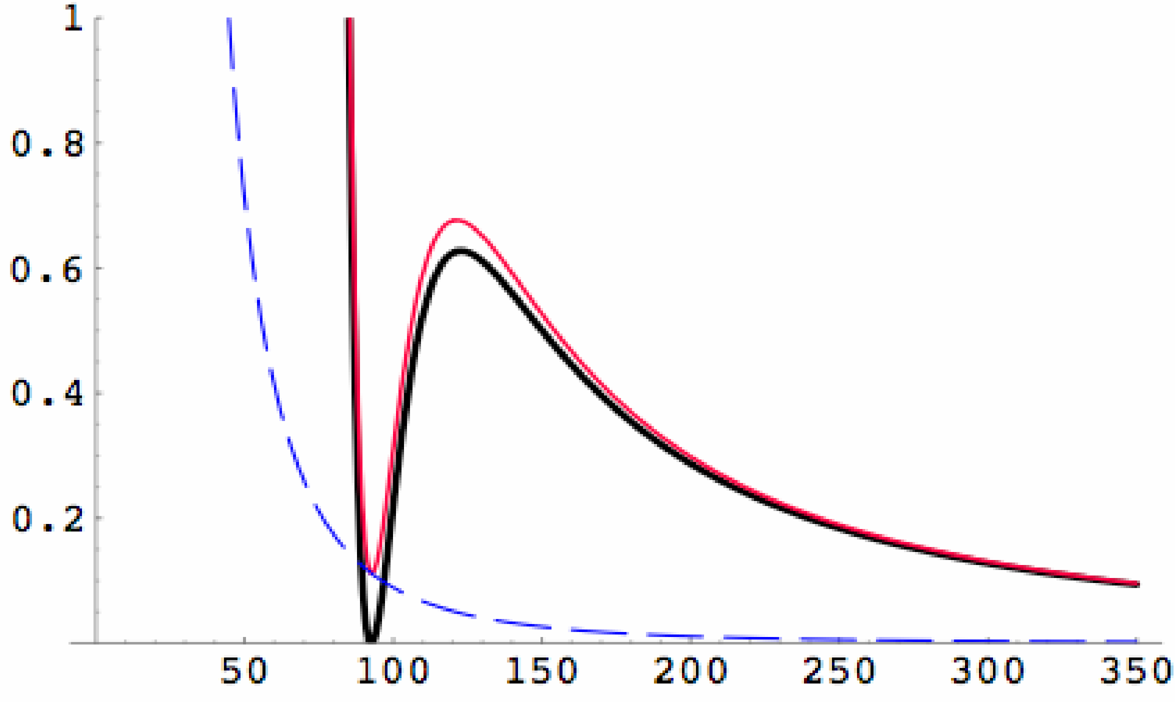
$$V_{\text{KKLT}} = V_{\text{AdS}} + \frac{D}{\sigma^2}$$

Second uplifting in D3/D7 model

$$V(s,\sigma,\phi_{\pm}=0) = V_{AdS} + \frac{D}{\sigma^2} + \frac{C}{\sigma^3}$$

D3 contribution

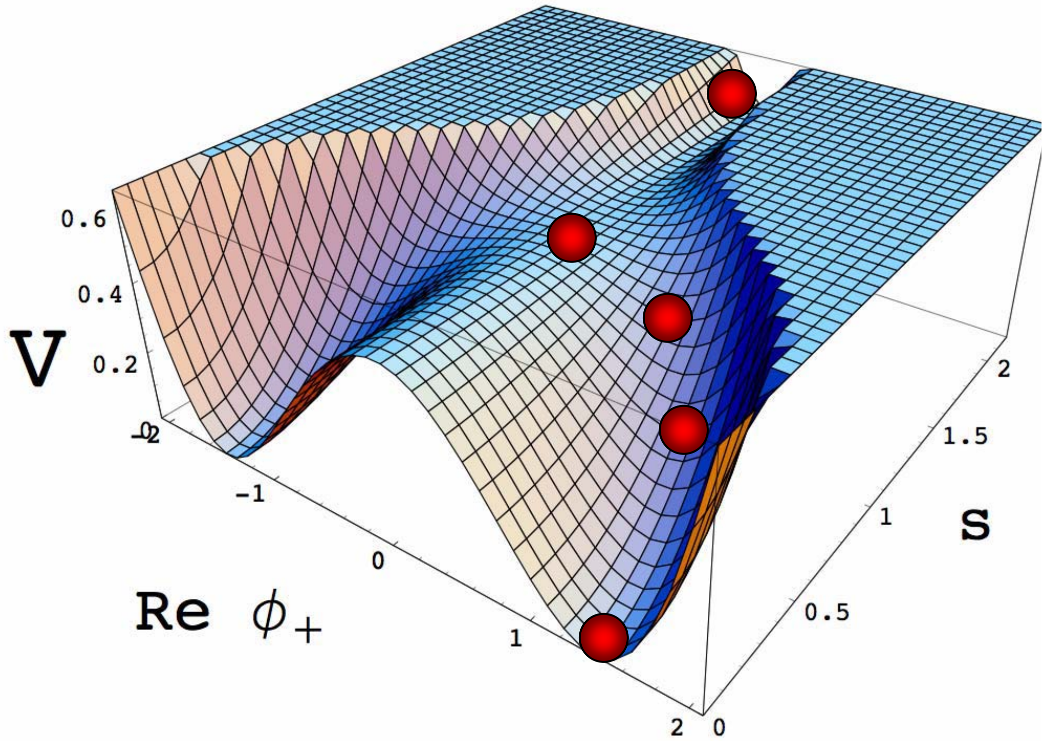
effective FI



Potential of D3/D7 inflation with a stabilized volume modulus

$$V_2(\phi_+, \phi_-, s, \sigma_{\rm cr}) = \frac{g^2}{2}\,|\phi_+\phi_-|^2 + \frac{g^2}{2} s^2 \left[|\phi_+|^2 + |\phi_-|^2\right] + \frac{g^2}{8}\left[|\phi_+|^2 - |\phi_-|^2 - 2\xi\right]^2$$

$$V_{1-\rm loop} = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\ln\frac{|S^2|}{|S_c^2|}\right)$$



Unlike in the brane-antibrane scenario, inflation in D3/D7 model may not require fine-tuning because of the shift symmetry

Mobile D3 brane?

- In our direct approach only the positions of D3 branes are not fixed by either fluxes or known instantons from wrapped branes

Berg, Haack, Kors ???
problem with calculations
1.Valid only in absence of flux
2. All 16 D7 on top of each other
3. We have no gaugino condensation

Ori Ganor???

Assumption about the use of the worldsheet instantons and duality in presence RR fluxes with N=1 susy ???

Berglund, Myer ???

D3/D7 D-brane inflation model, hopefully, can be derived from string theory: all moduli stabilized, small controllable breaking of shift symmetry

D3/D7 Phenomenology with Stabilized Volume and Inflation

$$V = \frac{g^2\xi^2}{2}[1 + \frac{g^2}{8\pi^2} \ln \frac{S^2}{S_{cr}^2}]$$

The conditions for successful slow-roll inflation require

$$\xi \sim 1.5 \times 10^{-5}$$

This is possible for quantized fluxes, and realistic values of volume and string coupling

Dasgupta, Hsu, R.K., Linde, Zagermann

$$\mathcal{F} \sim 2\pi 10^{-7} \frac{\sigma^3}{g_s} \sim 2\pi$$

$$n_s \sim 0.98$$

Supersymmetric Hybrid Inflation with Non-Minimal Kahler potential

Bastero-Gil, King, Shafi

- Minimal supersymmetric hybrid inflation based on a minimal Kahler potential predicts a spectral index n_s\gsim 0.98. On the other hand, WMAP three year data prefers a central value n_s \approx 0.95. We propose a class of supersymmetric hybrid inflation models based on the same minimal superpotential but with a non-minimal Kahler potential. Including radiative corrections using the one-loop effective potential, we show that the prediction for the spectral index is sensitive to the small non-minimal corrections, and can lead to a significantly red-tilted spectrum, in agreement with WMAP.

If indeed n_s=0.95 will be confirmed for r=0 theories, can we use the loop corrections to adjust the index of the stringy version of D-term inflation, D3/D7 brane inflation from 0.98 to 0.95?

- Cosmic strings have to be suppressed, they can be made light but only for $n_s \sim 1$. But they can be completely removed.

No problems with reheating, well understood

2006

New "Better Racetrack" Model of Inflation is based on explicit construction of string theory, where the KKLT-type stabilization of moduli was perfermed by Denef, Douglas, Florea (DDF) in 2004

The orientifold of $\mathbb{P}^4_{[1,1,1,6,9]}$

The model is a Calabi-Yau threefold with 2 Kahler moduli and 272 complex structure moduli. The moduli space admits an orientifold action which allows to reduce the moduli space of the Calabi-Yau complex structures to just 2 parameters. The model was studied by string theorists for a long time, starting with Candelas, Font, Katz and Morrison in 1994.

The defining equation for the Calabi-Yau 2-parameter subspace of the complex structure moduli space is

$$f = x_1^{18} + x_2^{18} + x_3^{18} + x_4^3 + x_5^2 - 18\psi x_1 x_2 x_3 x_4 x_5 - 3\phi x_1^6 x_2^6 x_3^6$$

DDF have stabilized the axion-dilaton, τ
and the comples structures ψ, ϕ

They have also stabilized two Kahler moduli by explicit construction of the KKLT-type non-perturbative instanton corrections to the superpotential.

We have taken this model out of the minimum where all moduli are stabilized and have shown that it inflates into the stabilized vacuum of string theory

Two complex moduli: the simplest known case of realistic KKLT stabilization

- Kähler potential

$$K = -2\ln[(\tau_2 + \bar{\tau}_2)^{3/2} - (\tau_1 + \bar{\tau}_1)^{3/2}]$$

- Superpotential

$$W = W_0 + Ae^{-a\tau_1} + Be^{-b\tau_2}$$

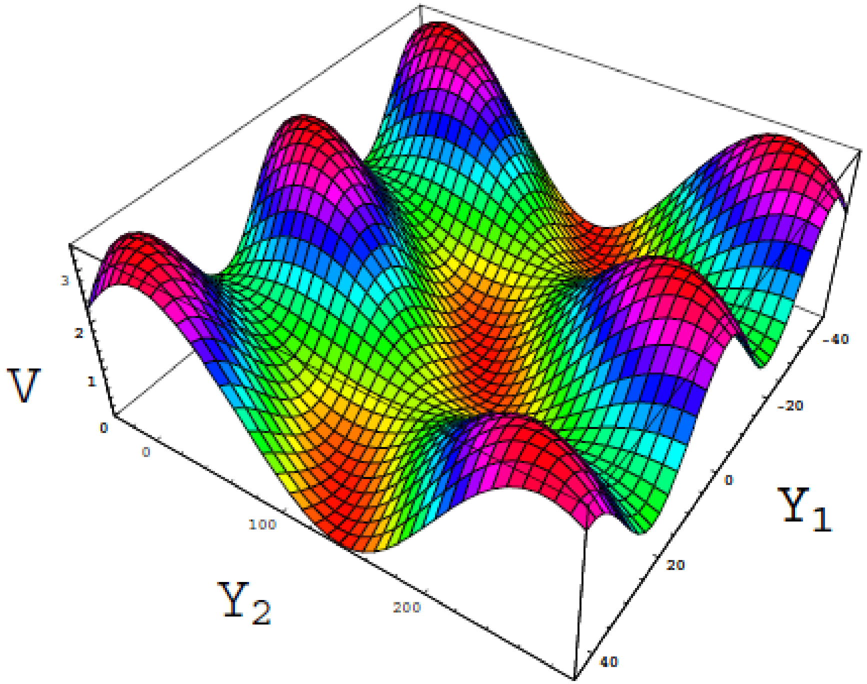


Figure 1: The potential as a function of the axion variables Y_1, Y_2 at the minimum of the radial variables X_1, X_2, , in units 10^{-15} of the Planck density.

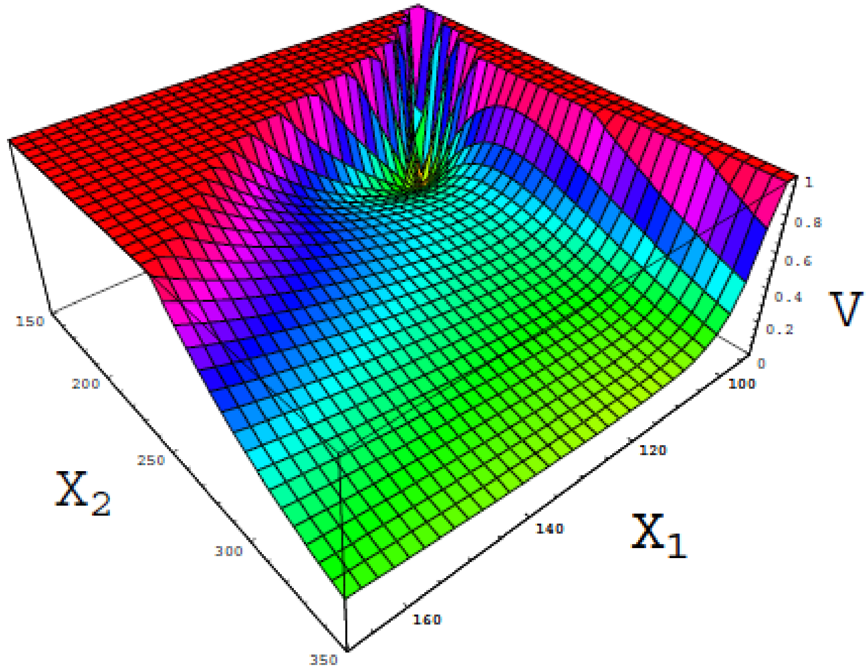
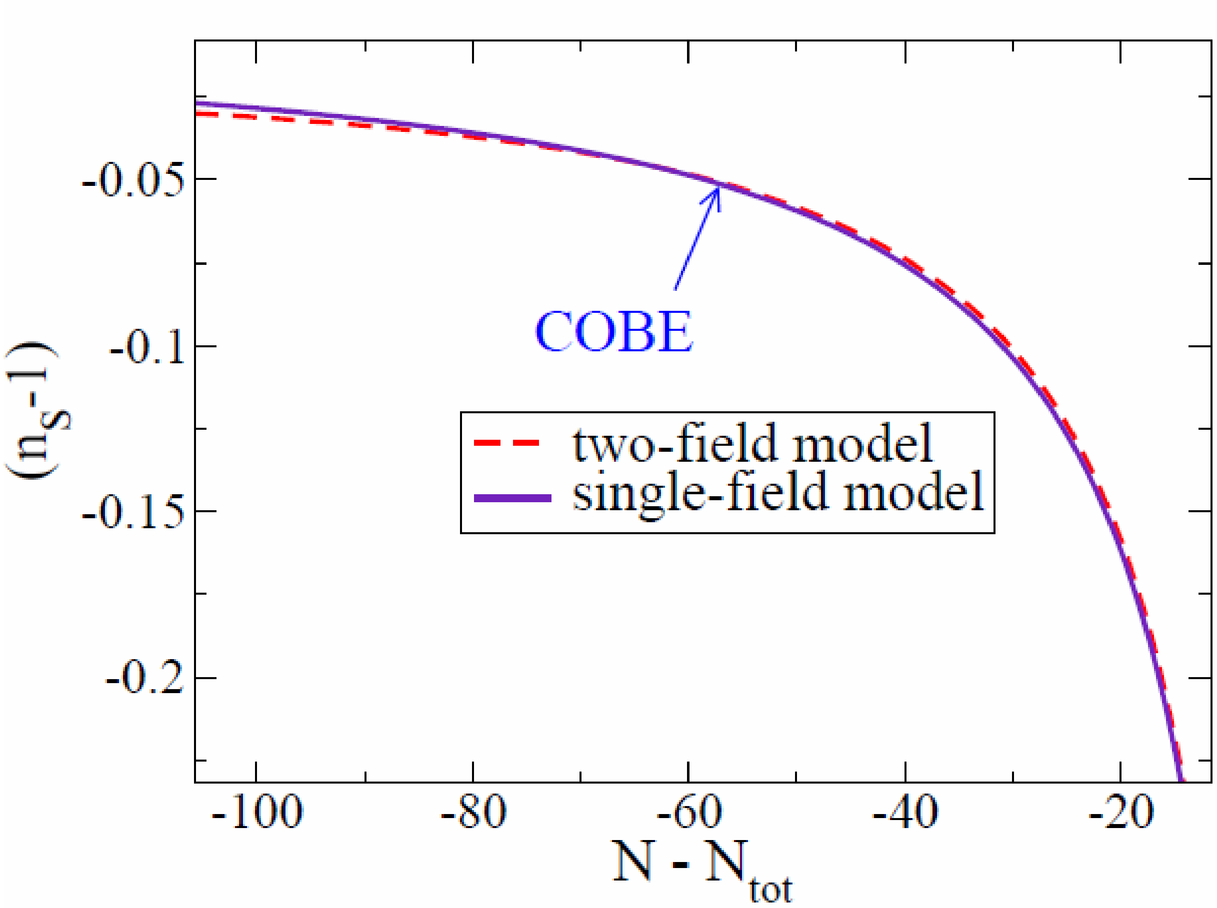


Figure 2: The potential as a function of the radial variables X_1, X_2 at the minimum of the angular variables Y_1, Y_2, in units 10^{-14} of the Planck density.



No cosmic strings

n_s=0.95

No grav. waves

Spectral index as a function of the number of e-foldings of inflation (minus the total number of e-foldings), for the original racetrack model, and the two-Kähler modulus model of this paper.

DBI INFLATION (grav. waves, non-gaussianity)

Silverstein et al, 2003

- A

$$S = -\int d^4x\sqrt{-g}\left(f(\phi)^{-1}\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} - V(\phi) - f(\phi)^{-1}\right)$$

- Restriction on velocity

$$\dot{\phi}^2 \leq \frac{1}{f(\phi)}.$$

- Recent study, G. Shiu et al. hep-th/060518
- To match the data, D3 must move close to the tip of the warped throat. In models with 60-e-folds in KS throat the non-gaussianity is above current bounds. Other models?

N-flation 2005 (assited inflation, Liddle, Mazumdar, Schunk, 1998)

S. Dimopoulos, S. Kachru, J. McGreevy, and J.G. Wacker

Many axions in string theory

The presence of many axion fields in four-dimensional string vacua can lead to a simple, radiatively stable realization of chaotic inflation.

$$V_n(\phi_n) = \Lambda_n^4 \cos\left(\frac{2\pi\phi_n}{f_n}\right) + \Lambda_n^{(2)\,4} \cos\left(\frac{4\pi\phi_n}{f_n}\right) + \cdots \qquad m_n = m$$

- More recent, Easther, McAllister
- Actually performed KKLT-type stabilization of f_n

$$V = V_0 + \sum_i \alpha_i \cos(2\pi\phi_i) + \sum_{i,j} \beta_{ij} \cos(2\pi\phi_i - 2\pi\phi_j)$$

$\phi > M_{PL}$ is possible as a collective effect

- This class of models needs much more work
- The hope is that the discovery of grav. waves will not falsify string theory…

Summary on String Cosmology

- Over the last few years we were able to construct first few models of the cosmological constant/dark energy in the context of non-perturbative string theory

- Several models of string theory inflation are available now, much more work is required

- Future cosmological and particle physics data will help us to test the new ideas in string theory and cosmology

If the inflationary model is derived in string theory by reliable methods and assumptions stated clearly

If the model has unambigous prediction for observables

A) B) C) …

When the precision data will come in we will be able to test the string theory assumptions underlying the derivation of the corresponding inflationary models