Disordered Elastic systems

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Some references

• General reviews (and refs therein)

• Refs to some own papers in the field

• Periodic systems (mostly vortices):

G. Blatter et al. Rev. Mod. Phys 66 1125 (1994).
T. Nattermann and S. Scheidl, Adv. Phys. 49, 607 (2000)
TG and S. Bhattacharya, in High Magnetic Fields: Applications in Condensed Matter Physics and Spectroscopy, edited by C. B. et al. (Springer-Verlag, Berlin, 2002), p.314, cond-mat/0111052

• Interfaces:

TG, A. B. Kolton, and A. Rosso, in Jamming, Yielding and Irreversible deformation in condensed matter, edited by M. C. Miguel and J. M. Rubi (Springer-Verlag, Berlin, 2006), p. 91, cond-mat/0503437

• Quantum systems:

TG, in Quantum phenomena in mesoscopic system, edited by S. I. di Fisica (IOS Press, Amsterdam, 2003), cond-mat/0403531 TG, E. Orignac, In ``Theoretical Methods for Strongly Correlated Electrons'', CRM Series in Mathematical Physics, Eds. D. Senechal et al., Springer, New York, 2003

and references therein...

• Bragg glass:

TG, P. Le Doussal Phys. Rev. B 52 1242 (1995); PRB 55 6577 (1997).
P. Le Doussal TG Physica C 331 233 (2000).
Review: TG, P. Le Doussal In ``Spin Glasses and Random Fields'', ed. A.P. Young, World Scientific (Singapore) 1998, p. 321, cond-mat/9705096

Moving glass: TG, P. Le Doussal Phys. Rev. Lett. 76 3408 (1996).
P. Le Doussal + TG Phys. Rev. B 57 11356 (1998).

• Creep:

P. Chauve, TG, P. Le Doussal Phys. Rev. B 62 624 (2000).A.B. Kolton, A. Rosso, TG, PRL 94 047002 (2005)

• Aging:

A.B. Kolton, A. Rosso, TG, PRL 95 180604 (2005) L.F. Cugliandolo, TG, P. Le Doussal, PRL 96, 217203 (2006)

Plan of the lectures

■ What are disordered elastic systems ? [1]

Fundamental concepts for statics [1]

Fundamental concepts for dynamics [2]

Depinning of Interfaces [3]

Magnetic domain wall







S. Lemerle et al. PRL 80 849 (98)

Ferroelectrics

1





P. Paruch et al. cond-mat/0411178



Vortices in superconductors

$T < T_c$: zero resistance



Flux expulsion (Meissner)







But Vortices



How to see vortices ?

Magneto-optics NbSe₂



MO indicator placed on top of bulk sample. T = 4.3 K

Bitter decoration NbSe₂



Y. Baselevitch T. Johansen Oslo



Scanning SQUID Nb

C. Veauvy, D. Mailly, & K Hasselbach **CRTBT** Grenoble

M. Marchevsky, J. Aarts, P.H. Kes (Kamerlingh Onnes Laboratorium, Leiden University)

Elastic description of Vortex Lattice

$$a = 1.07 \sqrt{\Phi_0 / B}$$





 $\mathbf{J} \qquad \rho \sim \rho_n \left(\frac{H}{H_{c2}} \right)$

Bad conductor

Need to pin the vortices: Disorder



Effect of external disorder on a Solid/Liquid

Classical crystals





Charged spheres: M. Saint Jean, GPS (Jussieu), 2000

Magnetic Bubbles: R. Seshadri et al.

Other classical systems

• Charge density waves

• Contact line of liquid menisci

Crack propagation

Quantum systems



Strong repulsion : Wigner crystal

 Quantum fluctuations instead (in addition to) thermal fluctuations

Wigner Crystal



FIG. 1. Absorption spectrum at 28 T and 60 mK for density 0.77×10^{11} cm⁻² (filling factor v=1/8.7, reduced temperature t=0.33) showing successive resonances and their identification as *p*th spatial harmonics $(q=pq_0)$ of the exciting structure. The values of *p* are chosen for the best alignment with the origin (full line) on the accompanying plot of f_p vs $p^{3/2}$; the dashed line is the zero-order *a priori* calculation of the frequency of the lower hybrid mode of the solid.

E.Y. Andrei, et al PRL 60 2765 (1988)



FIG. 1. (a) Diagonal resistivity ρ_{xx} and (b) Hall resistance ρ_{xy} of a low-density $(n - 4.8 \times 10^{10} \text{ cm}^{-2})$ high-mobility $(\mu = 1.7 \times 10^6 \text{ cm}^2/\text{V sec})$ two-dimensional electron system at various temperatures.

R.L. Willett, et al. PRB 38 R7881 (1989)

Other quantum Crystals

• Spin density waves

• Luttinger liquids (1D interacting electrons)

Some examples of **Disordered Elastic Systems** • Interfaces (magnetic domain walls, ferroelectrics, growth interfaces,...) • Periodic systems (vortex lattice, CDW, colloids, magnetic bubbles...) • Quantum systems (Luttinger liquids, Wigner crystal, SDW, Stripes,..) Competition ``Order'' / ``Disorder''

•Basic Features :





(Thermal, quantum) fluctuations



Disorder

`Elasticity'

Questions

Competition ``Order'' / ``Disorder''

- Melting
- Glassy phases
- Statics
- Dynamics



P. Kim PRB 60 R12589 (99)

Statics



T. Klein et al. Nature 413, 404 (2001)



+



H=0,06 Hc Deltat=100s.avi

(V. Repain et al. (Orsay))

New type of physics

• Very controlled (e.g. magnetic field)

- Can pull on on the system
- Plunged in an external disorder





How to model

Elastic description



Elastic description of crystals



R⁰_i: crystal u_i: displacements n=2 d=3 vortices

Elastic hamiltonian

 $H = \frac{1}{2} \sum_{\alpha\beta} \int c_{\alpha\beta}(q) u_{\alpha}(q) u_{\beta}(-q) dq$

Simplest elastic hamiltonian : $c(q) = c q^2$ $H = \frac{1}{2} \int c(\partial_{\alpha} u_{\beta}(x))^2$

Long range forces ; bulk, shear and tilt



Limitations

Interfaces(overhangs, bubbles)



H=0,03 Hc dt=500s

J. P Jamet, V. Repain

Periodicdislocations, etc.



M. Marchevsky, J. Aarts, P.H. Kes

What to measure (statics)





$$B(r) = \left\langle \left[u(r) - u(0) \right]^2 \right\rangle$$

Positional order



Thermal fluctuations : Melting



$$\left\langle u^2 \right\rangle = l_T^2 \propto \frac{T}{c}$$

Lindemann criterion of melting :

$$\langle u^2 \rangle = l_T^2 = C_L^2 a^2$$

 $C_L \approx 0.1 - 0.2$

Vortices





Should we care about disorder?





S. Lemerle et al. PRL 80 849 (98)



 $u \propto L^{\varsigma}$

Disorder (point like defects)



 $H = \int V(x)\rho(x)dx$

$$\rho(x) = \sum_{i} \delta(x - R_i^0 - u_i)$$



 $\rho(x) = \rho_0 - \rho_0 \nabla u(x) + \rho_0 \sum e^{iK(x - u(x))}$

Loss of translational order (Larkin)

 $u(R_a) \approx a$

$$H_{el} = \frac{c}{2} \int (\nabla u(r))^2 d^d r$$

 $cR_a^{d-2}a^2$

 $H_{dis} = \int V(r)\rho(r)d^d r$

 $VR_a^{d/2}\rho_0$

 $R_a \propto a \left(\frac{c^2 a^d}{V^2 \rho_0^2} \right)^{1/(4-d)}$

No crystal below four spatial dimensions
Very difficult stat-mech problem



 Optimization : many solutions

Glass



Larkin Model

$$H_{el} = \frac{c}{2} \int (\nabla u(r))^2 d^d r$$

$$H_{dis} = \int f(r)u(r)d^d r$$

• Exactly solvable

$$B(r) = B_{th} + \frac{\Delta}{c^2} r^{4-d}$$

Exponential loss of translational order $C(r) \approx e^{-r^{4-d}}$

• Not valid at large distance



 $\rho_0 \sum_{u} e^{iK(x-u(x))} V(x) \approx f(x)u(x)$

Not valid when : $K_{MAX} u \approx 1$ $u(R_c) \approx \xi$

• New length Rc

• Larkin model has no metastable states and pinning

• Rc is related to pinning



Interfaces: only one length

• Larkin length

 $u(R_c) \approx \xi$



 $\overline{R} < R_c; u(R) = R^{(4-d)/2}$

 $R > R_c$; u(R) = ?????



Two types of disorder



$\begin{array}{c} \uparrow \\ \uparrow \\ \downarrow \end{array}$

Random bond

$$\int dx dz V(x, z) \rho(x, z)$$
$$= \int dz V(u(z), z)$$

$$\int dz \int_0^{u(z)} V(x,z)$$

Interfaces

$$H_{el} = \frac{c}{2} \int (\nabla u(r))^2 d^d r \qquad H_{dis} = \int V(r, u(r)) d^d r$$

 $V(z, x)V(z', x') = D\delta(x - x')\delta(z - z')$

 $Cu^2 L^{d-2}$ $D^{1/2} L^{d/2} u^{-m/2}$

$$u_{RB} \propto L^{4-d}$$

 $u_{RF} \propto L^{4-m}$

Flory argument (mean field)

$u_{RB} \propto L^{\varsigma} \qquad \zeta$: roughness exponent

d = 1; $\zeta = 2/3$ (random bond)



Ferroelectics





P. Paruch et al. PRL 94 194601 (05)

How to solve ?

• Average over disorder (replica trick)

• Two main methods :

Variational approach

Renormalization (functional RG)

Replicas

$$\overline{\langle O \rangle} = \int \mathcal{D}V p(V) \langle O \rangle_V = \int DV p(V) \frac{\int \mathcal{D}\phi O[\phi] e^{-S_V[\phi]}}{\int D\phi e^{-S_V[\phi]}}$$

$$\int D\phi_1 D\phi_2 \dots D\phi_n O[\phi_1] e^{-\sum_{i=1}^n S_V[\phi_i]} = \int D\phi O[\phi] e^{-S_V[\phi]} \left[\int D\phi e^{-S_V[\phi]} \right]^{n-1}$$

Average over disorder

$$H = \frac{c}{2} \int (\nabla u)^2 d^d x + \rho_0 \sum_{K} \int e^{iK(x - u(x))} V(x)$$

Classical systems

$$H = \sum_{a} c \int (\nabla u_{a})^{2} d^{d} x - \rho_{0} \Delta \sum_{a,b} \sum_{K} \int \cos(K(u_{a}(x) - u_{b}(x))) d^{d} x$$

• Quantum problem (disorder is time independent)

$$S = \sum_{a} c \int (\nabla u_{a})^{2} d^{d+1} x$$

$$- \rho_{0} \Delta \sum_{a,b} \sum_{K} \int \cos(K(u_{a}(x,\tau) - u_{b}(x,\tau'))) d^{d} x d\tau d\tau'$$

Variational Method

Find the best quadratic Hamiltonian $H_0 = \sum_{ab} \int G_{ab}^{-1}(q) u_q^a u_{-q}^b d^d q$

Minimize G_{ab} is a 0x0 matrix

$$F_{\rm var} = F_0 + \left\langle (H - H_0) \right\rangle_H$$

$$\frac{dF_{\rm var}}{dG_{ab}(q)} = 0$$

$$G_{ab}^{-1}(q) = q^2 + \int ...e^{\sum \int G_{ab}...}$$

0x0 limit

• Replica symetric

Unstable

• Replica symetry broken solution

 $G_{a\neq b}$



 G_{aa}

Hierarchical structure

a,b continuous in [0,1]



• Signals metastability and Glassy properties

• Disordered elastic system = glass

• RSB from d=4 to d=2 (or 1+1) above a lengthscale Rc

Functional Renormalization Group $\beta H = \sum_{a} \frac{c}{T} \int (\nabla u_{a})^{2} d^{d} x$ $- \rho_{0} \frac{\Lambda}{T^{2}} \sum_{a,b} \sum_{K} \int \cos(K(u_{a}(x) - u_{b}(x))) d^{d} x$

In usual RG : $\Delta(u) \approx a + bu^2 + cu^4 + \dots$ Needs only to keep b and c (higher powers are irrelevant)

Disordered System

 $\sum \frac{c}{T} \left[(\nabla u_a)^2 d^d x \right]$

 $u \propto L^0$ $T \propto L^{2-d}$

 $-\frac{1}{T^{2}}\sum_{a,b}\sum_{K}\int \Delta(K(u_{a}(x)-u_{b}(x)))d^{d}x$

 $\Delta \propto L^{4-d}$

Needs to keep the *whole* function



Nonanalyticity at a finite lengthscale Rc such that u(Rc) ~lc
 (A. Larkin, D. Fisher)

• Cusp signals metastability and glassy states

Interfaces

• Power law growth of the displacements: $u \sim L^{\zeta}$

• L < L_c No metastabiliy (Larkin model): $\zeta = (4-d)/2$

• L > L_c Metastability, glassy properties $\zeta \sim 0.208(4-d) + \dots [FRG]$

• $\zeta \sim (4-\delta)/(4+\mu)$ [Flory]

Crystals

Identical to interfaces ?

 $u \sim L^{\zeta}$

 \square Above R_c

 $C(r) \propto e^{-L^{2\zeta}}$

Exponentential loss of positional order ??

Naive vision of a D.E. crystal

- Loss of translational order beyond Ra
- (Wrong) argument: disorder induces dislocations at Ra



Periodic systems: new universality class





 $u \sim I^{\varsigma}$

 $u \sim Log(L)^{1/2}$

(Nattermann, Korshunov, TG+Le doussal)

Variational and/or FRG

$$\begin{split} \partial \widetilde{\Delta}(u) &= (\epsilon - 2\zeta) \widetilde{\Delta}(u) + \zeta u \widetilde{\Delta}'(u) + \widetilde{T} \widetilde{\Delta}''(u) \\ &+ \widetilde{\Delta}''(u) [\widetilde{\Delta}(0) - \widetilde{\Delta}(u)] - \widetilde{\Delta}'(u)^2, \\ \partial \ln \widetilde{T} &= \epsilon - 2 - 2\zeta. \end{split}$$

(TG, P. Le Doussal)

• Periodic system (crystal): $\Delta(u) = A \cos(u)$

• Fixed point:
$$\zeta = 0$$

$$\Delta^*(ax) = \frac{\epsilon a^2}{6} \left(\frac{1}{6} - x(1-x) \right)$$



Bragg Glass

(TG, Le Doussal; Nattermann)





 quasi long range translational order (powerlaw Bragg peaks)

- perfect topological order (no free defects)
- Glassy properties



P. Kim PRB 60 R12589 (99)

Unified phase diagram





B. Khaykovich et al. PRL 76 2555 (96)



Hardy et al. Physica C 232 347 (94)





K. Deligiannis et al. PRL 79 2121 (97)



N. Avraham et al. Nature 411 451 (2001)

Y. Paltiel et al. PRL 85 3712 (2000)

BSCCO

NBSe2

Bragg glass : « melts » like a crystal (first order melting)



T. Klein et al. Nature 413 404 (2001)











Questions for dynamics

• Competition between disorder and elasticity: glassy properties

• Dynamics ?



Pinning (Fc) and Larkin length (Rc)



 $F_c = \frac{c\xi}{R^2}$

 $H_{el} = \frac{c}{2} \int (\nabla u(r))^2 d^d r \qquad H_{el} = \int Fu(r) d^d r$

 $cR_c^{d-2}\xi^2$



Depinning (T=0)

$$(u_{r,t}-u_{0,0})^2 = r^{2\zeta} C(t/r^z),$$

ζ for F ~ F_c differs from ζ for F=0

$$v \sim (f - f_c)^{\beta},$$

 $\xi \sim (f - f_c)^{-\nu}.$

$$\nu = \frac{1}{2-\zeta} = \frac{\beta}{(z-\zeta)}.$$

Only RF universality class

Response to a small force





TAFF : typical barrier Linear response

 $v \propto e^{-\beta\Delta}F$
• Glassy system: no typical barrier

(Ioffe + Vinokur; Nattermann)

$$H_{el} = \frac{c}{2} \int (\nabla u(r))^2 d^d r$$
$$c R^{d-2+2\varsigma}$$

$$H_{el} = \int Fu(r)d^{d}r$$
$$FR^{d+\zeta}$$

$$L_{opt} \approx F^{\varsigma-2} \frac{d+2\varsigma-2}{\varsigma-2}$$
$$U(L_{opt}) \approx F^{\frac{\varsigma-2}{\varsigma-2}}$$

 $v \propto e^{-\beta U_c (F_c/F)} rac{d-2+2\zeta}{2-\zeta}$

Strong assumptions

Motion so slow that static properties can be used

 Scaling of barriers is identical to the one of metastable states

Dominated by typical barriers

Microscopic calculation ??

How to study

 $\eta \partial_t u = c \nabla^2 u + F_{pin}[u] + f + \zeta$

u

$$\int Du D\hat{u} e^{i\hat{u}(\partial_t u - c\nabla^2 u -)}$$

$$S_{\rm uns}(u,\hat{u}) = \int_{rt} i\hat{u}_{rt}(\eta\partial_t - c\nabla^2)u_{rt} - \eta T \int_{rt} i\hat{u}_{rt}i\hat{u}_{rt}$$

$$-f \int_{rt} i\hat{u}_{rt} \qquad (4.1)$$

$$-\frac{1}{2} \int_{rtt'} i\hat{u}_{rt}i\hat{u}_{rt'}\Delta(u_{rt} - u_{rt'})$$

Martin-Siggia-Rose, Keldysh

Correlator of disorder

Creep from FRG

$$\begin{split} \partial \widetilde{\Delta}(u) &= (\epsilon - 2\zeta) \widetilde{\Delta}(u) + \zeta u \widetilde{\Delta}'(u) + \widetilde{T} \widetilde{\Delta}''(u) \\ &+ \int_{s > 0, s' > 0} e^{-s - s'} (\widetilde{\Delta}''(u) \{ \widetilde{\Delta}[(s' - s)\lambda]] \\ &- \widetilde{\Delta}[u + (s' - s)\lambda] \} - \widetilde{\Delta}'(u - s'\lambda) \widetilde{\Delta}'(u + s\lambda) \\ &+ \widetilde{\Delta}'[(s' + s)\lambda] [\widetilde{\Delta}'(u - s'\lambda) - \widetilde{\Delta}'(u + s\lambda)]), \end{split}$$

$$(4.11)$$

$$\partial \ln \lambda = 2 - \zeta - \int_{s>0} e^{-s} s \widetilde{\Delta}''(s\lambda),$$

$$\partial \ln \tilde{T} = \epsilon - 2 - 2\zeta + \int_{s>0} e^{-s} s \lambda \tilde{\Delta}^{\prime\prime\prime}(s\lambda),$$

$$\partial \tilde{f} = e^{-(2-\zeta)l} c \Lambda_0^2 \int_{s>0} e^{-s} \tilde{\Delta}'(s\lambda),$$

$$\begin{split} \widetilde{\Delta}_{l}(u) &= \frac{S_{D}\Lambda_{l}^{D}}{(c\Lambda_{l}^{2}e^{\zeta l})^{2}} \Delta_{l}(ue^{\zeta l}), \\ \widetilde{T}_{l} &= \frac{S_{D}\Lambda_{l}^{D}}{c\Lambda_{l}^{2}e^{2\zeta l}}T_{l}, \\ \lambda_{l} &= \frac{\eta_{l}v}{c\Lambda_{l}^{2}e^{\zeta l}}, \\ \widetilde{f}_{0} &= f - \eta_{0}v, \end{split}$$

(Chauve, TG, Le doussal)

Dynamics: rounding of cusp



$$\frac{\eta v}{f_c} \approx \exp\left[-\frac{U_c}{T} \left(\frac{f}{f_c}\right)^{-\mu}\right]$$
$$\mu = \frac{D - 2 + 2\zeta_{\text{eq}}}{2 - \zeta_{\text{eq}}}$$

New lengthscale: avalanches

Motion quite different from phenomenological picture (two regimes)



Vortices



Bragg glass

 $\zeta = 0, d=3$

 $\mu = 0.5$

D.T. Fuchs et al. PRL 81 3944 (98)



S. Lemerle et al. PRL 80 849 (98) $\zeta = 2/3$; $\mu = 1/4$

Ferroelectics



 $\mu \sim 0.58$

 $\zeta \sim 0.26$

 $\mu = \frac{d-2+2\zeta}{d\sim 2.49\zeta}$

T. Tybell et al. PRL 89 097601 (02)P. Paruch et al. PRL 94 197601 (05)

Compatible with d=2 + dipolar interactions



$H = 159 \,\mathrm{A \, m^{-1}}$ and $L_{\rm C} = 40 \,\mathrm{nm}$.

 $R_T \sim 1 \ \mu \ m$

V. Repain et al. EPL 68 460 (04)

Large V

Interfaces reorder at large V





Thermal Roughening for $R > R_v$

Crystal vs Interfaces Disorder remains in perp. direction

• Motion via static channels



• Moving glass (TG, P. Le Doussal)





F. Pardo et al. Nature, 396 348 (1998)

A. Kolton et al PRL 83 3061 (1999)

Dynamical Phase Diagram



Transverse critical force

Fy

Crystal without disorder



Moving glass





Absence of Hall voltage



• Flor < Ftran : no hall voltage

Compatible with the existence of a transverse threshold

> F. Perruchot et al. Physica B 256 587 (1998)

Not the end of the story





Influence on dynamics ?

Roughness of the irradiated layer :

(V. Repain et al. (Orsay))



 $210 \ \mu m$

Pt/Co(0,5 nm)/Pt/SiO₂

Out of equilibrium

Creep: Molecular dynamics





Exponents larger than equilibrium value !

Glasses : Aging



f(t1,t2)

f : Depends on both times

Aging of the Bragg glass or the interfaces ?

Aging in interfaces



(A. B. Kolton, A. Rosso, TG)

Conclusions ?

... It is a magical world

... Let's go exploring !

