Finding Function Fixed Points for Pinned Manifolds: Why & How

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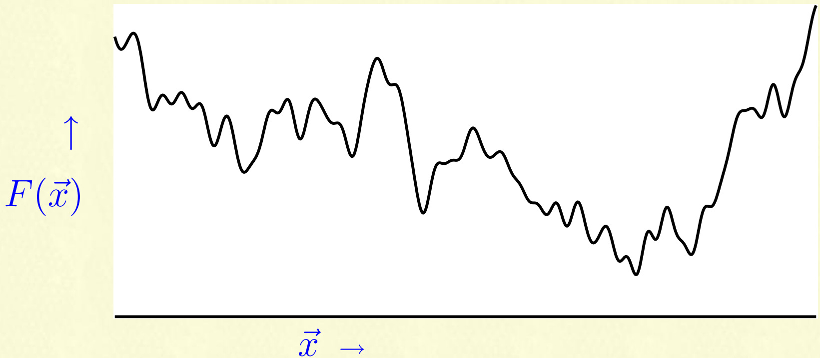
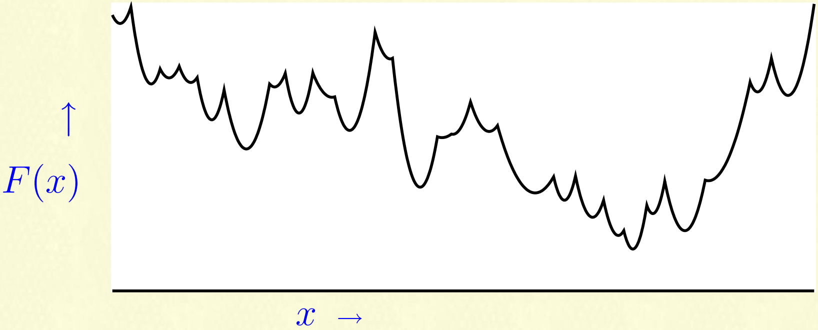
Organization

- Reverse historical approach.
- “Experimental” talk.
- See cond-mat/0606160.
- [Reminded of ancient Greek theater festivals.]

This is a glass talk, so we need this diagram



However, we will mostly see this



Inspiration: A Finite-d Glassy System

- Statics of surfaces pinned by disorder (see T. Giamarchi's talk!)
  - Domain walls in random magnets, contact lines on a rough surface, vortex lines in type-II superconductor, periodic scalar fields, e.g., vortex-free superconductors or vortex-line arrays
- "Simplest" finite-d glassy phases (?)
  - Elastic, no plastic rearrangements.
  - At low T, disorder is irrelevant . . .
    - Only elasticity v. disorder to compete (or elasticity+disorder)
- Characterize by roughness, $w \sim L^{\zeta}$, energy fluctuations $\sim L^{\theta}$. Statics are preliminary to
  - barriers to equilibration
  - dynamics (creep or sliding) in disordered background.

Plot Summary

The effective long wavelength pinning potential for $d < 4$ interface is **universal** (depends on symmetries of pinning potential).

$\Rightarrow$Find fixed points for force-force correlation functions $\Delta(u)$.
$\Rightarrow$Quantitatively confirm shape of $\Delta(u)$.

- First evidence for cusp at zero u (20 yrs)
- “Chaos” (sensitivity to disorder)
- Universal amplitudes.

Production Crew

P. Le Doussal, K. Wiese, and 100 1GHz processors.
$\Rightarrow$C++ code to find **exact ground state** for discrete interfaces $u(x)$ in dimensions $d = 1, 2, 3, 4, \ldots$ with

- User-defined lattices.
- Choice of disorder correlations, corresponding to
  - Random field (RB): $\langle [U(u', x') - U(u, x)]^2 \rangle = |u - u'| \delta(x - x')$
  - Random bond (RF): $\langle [U(u', x') - U(u, x)]^2 \rangle = e^{-|u-u'|} \delta(x - x')$
  - Periodic pinning (RP): $\langle [U(u', x') - U(u, x)]^2 \rangle = \sin[\frac{2\pi(u-u')}{P}] \delta(x - x')$
- Add in a moving harmonic well to the disorder [P. Le Doussal].

$$U_{\text{harmonic}}[u(x)] = \frac{m^2}{2}(u - v)^2$$

Simulation uses rolling disorder and can incrementally find $v \to v + \delta v$.

The Play

Act 1: Random field pinning, $D = 2{+}1$ interface, $m^2 = 0.1$, $L{\times}W = 20 \times 20$, $\delta v = 0.04$, 100 steps.

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Act 4: The shocking events from scene 2.

Safety

Have checked code against

- Previous numerics
- Exact lattice results [Johansson, CMP 209, 437 (2000)] for $D = 1 + 1$
- Equalities for coarse-grained correlations for RF
- Finite-size and lattice effects (latter vanishes for small enough m).

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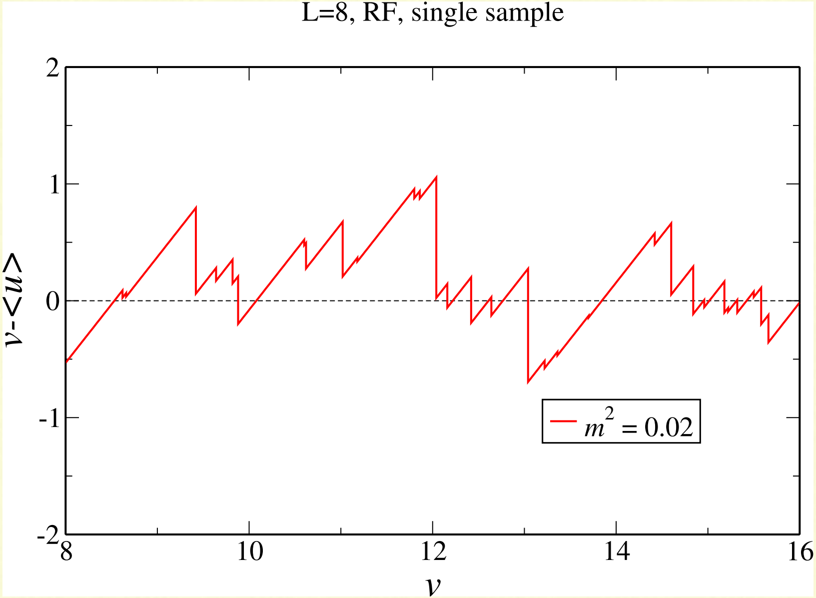


Theory - Functional Renormalization Group

FRG seems to be a controlled verifiable approach to manifolds in a disordered potential.

- Below $d = 4$, ∞ number of relevant operators and metastability.
- Writing $\langle [V_\ell(u,\vec{x}) - V_\ell(0,\vec{0})]^2 \rangle = -2R_\ell(u)\delta(\vec{x})$, D. S. Fisher (1986) derived flow equations, using $\Delta(u) = -R''(u)$,

$$\frac{d\Delta(u)}{d\ell} = (\epsilon - 4\zeta)\Delta(u) + \zeta u \Delta'(u) + \frac{1}{2}\left[\Delta''(u)\right]^2 - \Delta''(u)\Delta''(0)$$

- *Non-analytic fixed points:* $\Delta(u)$, force-force correlations, have a **cusp** at $u = 0$.

Relevance

$R(u)$ and its derivatives $\Rightarrow$ the physical picture of pinned interfaces:

- Fisher, Narayan, Balents; Balents, Bouchaud, Mezard (1986-1996): sequence of scalloped potentials [singularity in $R(u)$] due to hopping between metastable states, suggestive connections to Burgers equation.
- Le Doussal, recently: scallops derived from harmonic well + disorder; precise connection to Burgers equation.
- Fixed points for flow of $R(u)$ gives exponents for roughness, etc.
- Finite drive, changing disorder [”chaos”], and temperature round out the singularity at different scales [zero pinning force $\Delta'''(0)$].

Measured correlations vs. 1-loop predictions

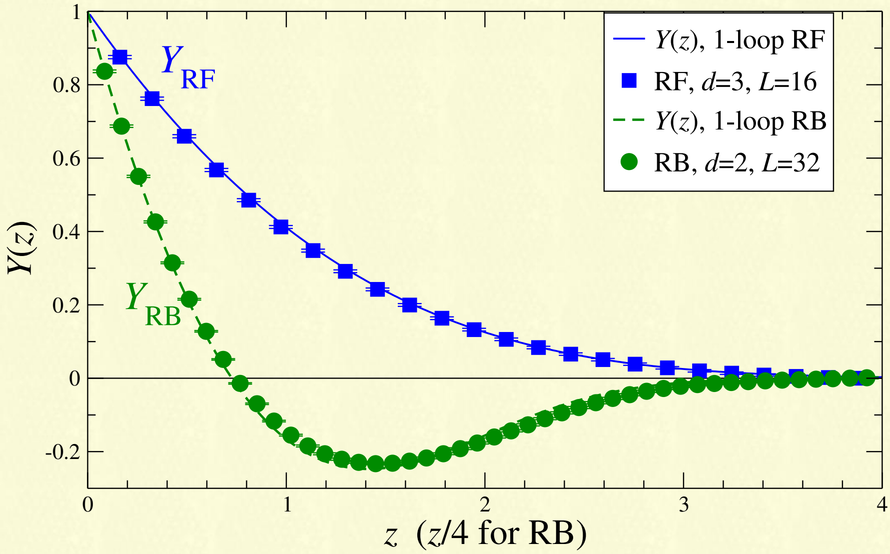
- Compute fixed point: large enough L, small enough m, so that

$$\tilde{\Delta}(mu^{\zeta}) = m^{\epsilon-4\zeta-d}\overline{[v' - \langle u\rangle(v')][v - \langle u\rangle(v)]}$$

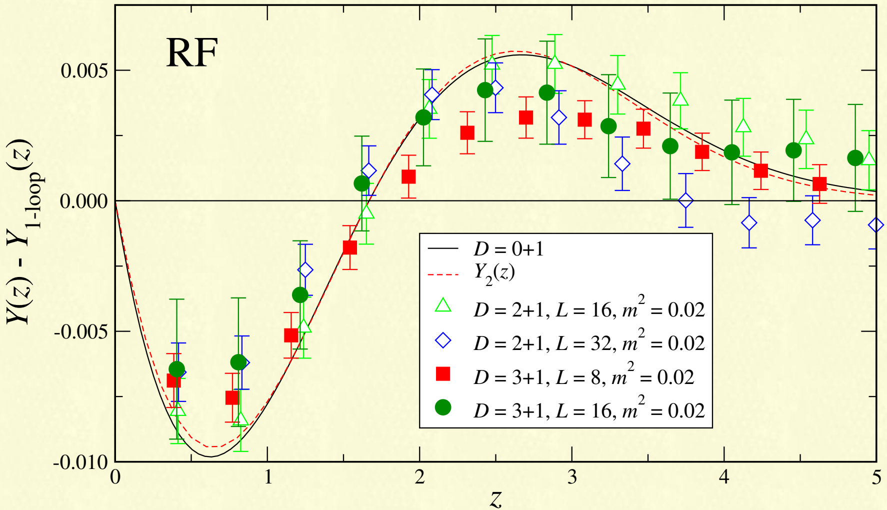
  is converged.

- Rescale to $Y(u) = \tilde{\Delta}(u)/\tilde{\Delta}(0)$ and scale $z = um^{\zeta}$ to get $\int Y = 1$ (RF), $\int Y^2 = 1$(RB).

Measured correlations vs. 1-loop predictions

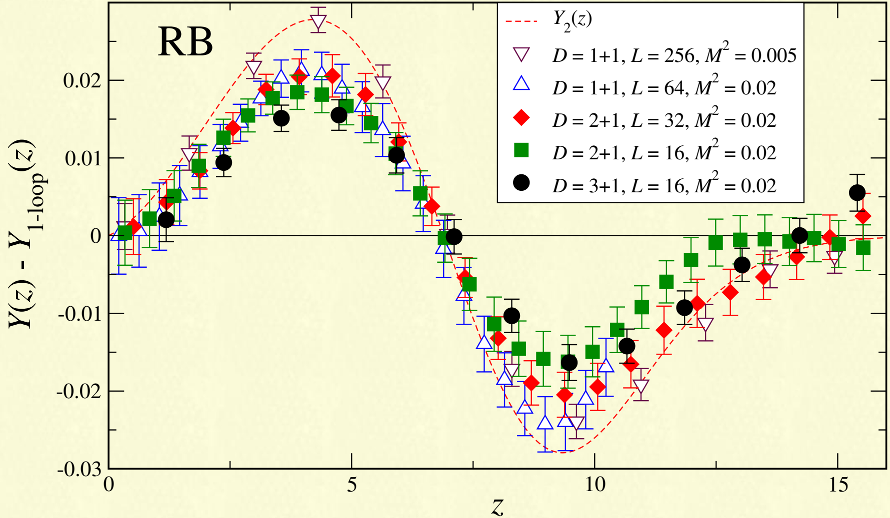


Residuals, RF



Where *one form* of the *2*-loop prediction is $Y(z) = Y_1(z) + (4-d)Y_2(z)$

Residuals, RB



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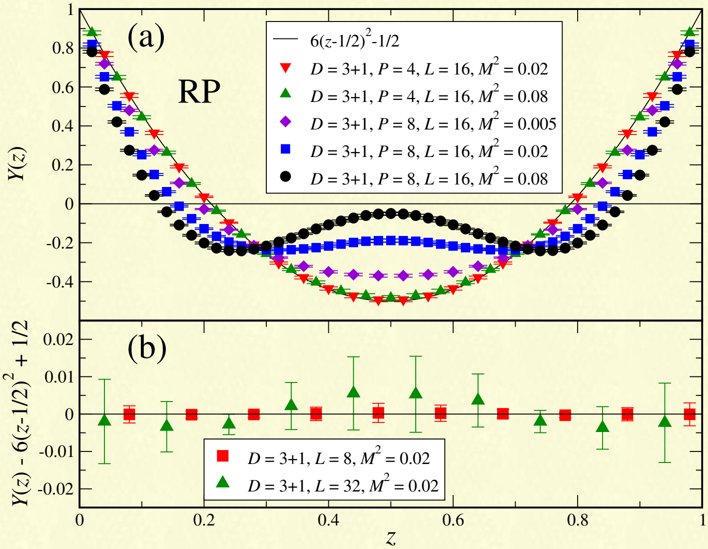
RP: crossover from RB to RP



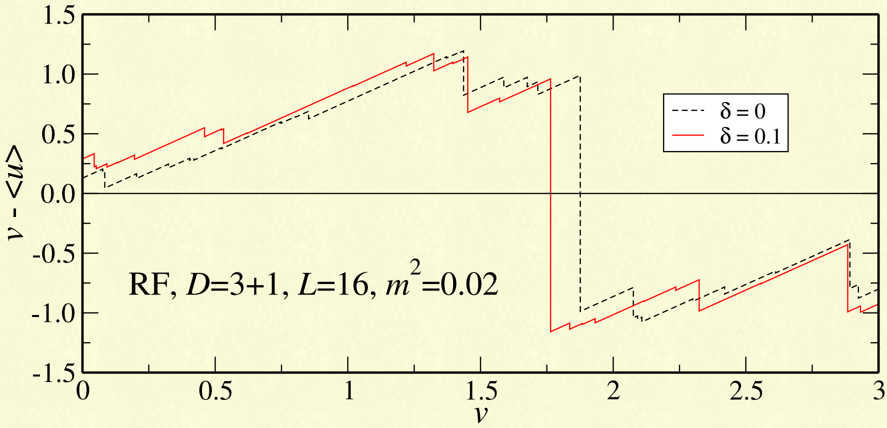
General prediction: $Y(z)$ is a parabola with zero mean (i.e., $6(z-\frac{1}{2})^2-\frac{1}{2}$).

“Chaos” (sensitivity to disorder)

Recent predictions by P. Le Doussal [PRL **96**, 235702 (2006)] for correlations

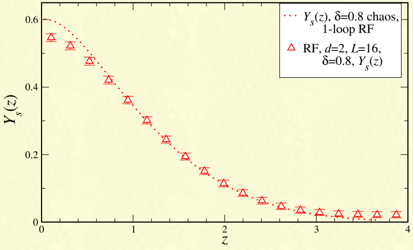
$$\Delta_{12}(y) = \langle [v + y - u_1(v + y)][v - u_2(v)] \rangle$$

between samples with disorders U_1 and U_2, with difference measured by δ.
We can check this - shapes of curves (1 adjustable parameter).

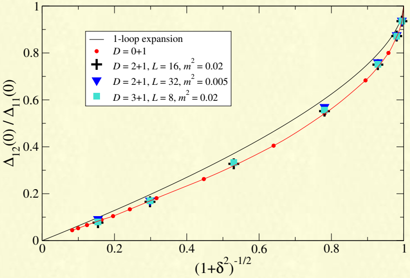


Chaos

Normalized $\Delta_{12}(y)$, fixed perturbation δ



$\Delta_{12}(0)/\Delta_{11}(0)$, varying δ [parameter free ratio]



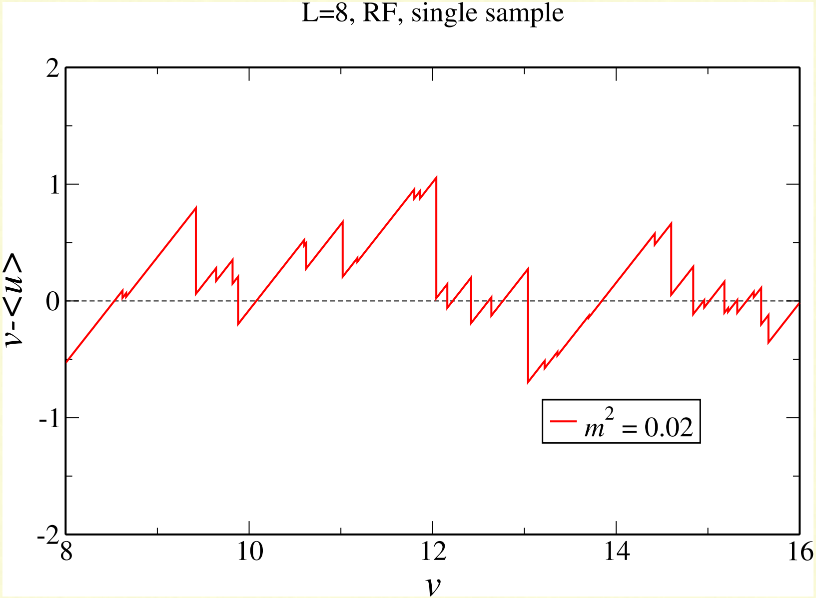
Functional Burgers Equation

$d = 0$: particle in a single $V(u)$ given by a random walk + $\frac{m^2}{2}(u-v)^2$. *Exact correspondence* between $v - \langle u \rangle$ and velocity in Burgers equation, given $t \to m^{-2}$, $V \to v - \langle u \rangle$, $\nu \to t$: jumps in $\langle u \rangle$ are shocks in 1D decaying Burgers equation.

$$\partial_t V + V \partial_x V = \nu \partial_x^2 V$$

Functional equation: formally similar.
Consequences:
In a single sample, see coalescence of jumps as decrease m^2.

Sequence of m^2 in a single sample



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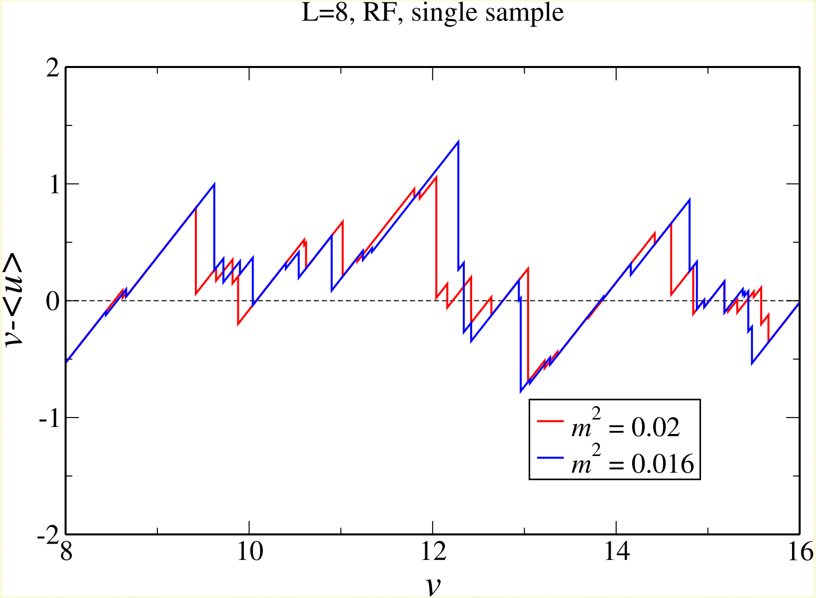

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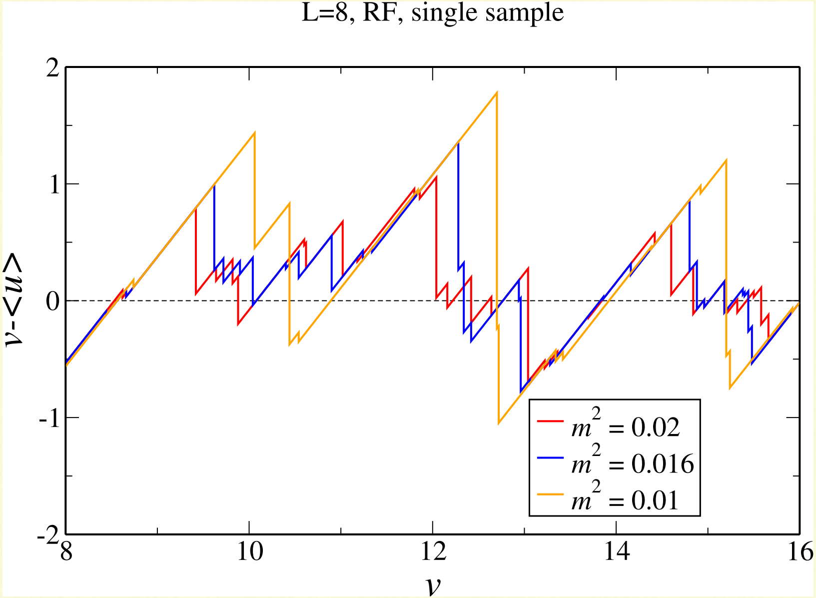

Highlights & Sequels

- Confirmed prediction for nonanalytic form, ***linear cusps*** in *force-force correlator* $\Delta(u)$, for pinned manifold.
- One-loop calculation appears to be very good, not full story; RP shows expected exact parabola.
- Supports exponent values, validates approach, physical picture.
- Functional decaying Burgers eqn. for $v - u(x)$.
- Amplitudes have also been obtained, using measurements of elasticity.