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SCHOOL and CONFERENCE
on
COMPLEX SYSTEMS
and
NONEXTENSIVE STATISTICAL MECHANICS

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Momentum Distributions in Astrophysical Plasmas

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Momentum Distributions in Astrophysical Plasmas

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5 Interesting Processes

- Non-resonant nuclear reactions in stars
- Resonant reactions in stars
- Radiative Recombination (RR) in stars
- Dielectronic recombination (DR)
- Deuteron-deuteron fusion reactions in deuterated metals with deuteron beams

Non-Resonant Nuclear Reactions in Stars

$p + p$ in the Sun

Only very few particles responsible of energy production

- Coulomb repulsion makes fusion difficult
- Tunnel effect (Gamow peak)
- Rate very sensitive to the tail of the distribution
- Small enhancement or depletion leads to major consequences on rates
- Experiments on light ion fusion at Gran Sasso Lab

Described by a Lorentzian cross section

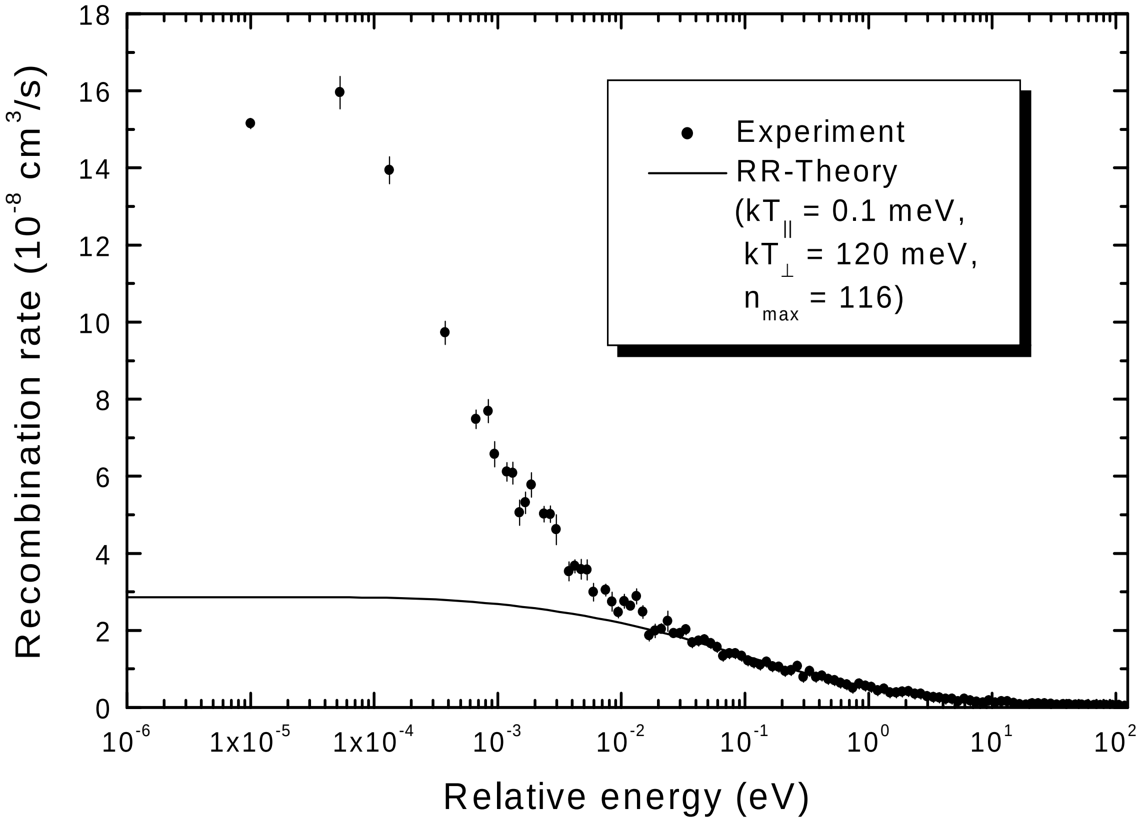
- Rate sensitive to position and width of the resonance with respect to the Gamow peak
- Careful evaluation of $f(E_R)$ close to resonance required
- Small changes from MB lead to great discrepancies (*much more than for non-resonant reactions*)
- Studied in Lab, although measures are prohibitively difficult at stellar temperatures ($< 1\,\mathrm{MeV}$)

Radiative Recombination (RR) in Stars (Atomic)

Acts among electrons and ions in stellar systems

$$A^{Z+} + e^- \rightarrow A^{(Z-1)+} + h\nu$$

- Can be studied in cooling devices in storage rings
- Non-resonant process: high at $E_R \rightarrow 0$
- Very sensitive to the head of distribution



Strong increase of experimental rate respect to theory! [a]

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[a] A.Hoffknecht *et al.*, arXiv: physics/0003088 v1

Dielectronic Recombination (DR) (Atomic)

Resonant process in star atmospheres and storage rings[1]

$$A^{Z+} + e^- \to (A^{(Z-1)+})^* \to A^{(Z-1)+} + h\nu$$

The rate depends on:

- DR peaks' position with respect to temperature
- Head and cutoff of the distribution (for $T < 100\,\mathrm{eV}$)
- Complicated atomic structure
  - 21 resonances for C^{3+} between $0.2 \div 0.6\,\mathrm{eV}$ [2]

Transfer of information from Lab experiment to astrophysical plasmas complicated because:

- In Lab, plasmas **are not** in a global equilibrium state
- Stellar plasmas are in other stationary states
- Hypothesis that everything is Maxwellian **hardly verified**

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[1] E.Schmidt *et al.*, ApJ **641** (2006) 157; N.Bandell, astro-ph/0607412

[2] S.Mannervik *et al.*, PRL **81** (1998) 313

Deuteron-Deuteron Fusion Reactions in Metals

- Accelerated deuteron beams against deuterated metal matrix
- Very important for applications
- Related to problem of hydrogen stocking in cells

Experimental increase (Bochum) **not** explained by atomic screening or other classical effects[3]

- Energy-momentum uncertainty increases the $f(E_R)$ tail
- Looks like a non-extensive deformation, although it is a quantum effect
- The energy-momentum dispersion is Lorentzian
- Enhancement if deformation close to Lorentzian peak

[3] F.Raiola *et al.*, EPJ A **19** (2004) 283

Approach for Treating the Previous 5 Problems

Usually it is based on assumptions of:

- **Global or local thermodynamical equilibrium** (TE, LTE)
- **Maxwellian** distribution of particles (*also when radiation is not described by a Planck distribution*)

MB is a good approximation for describing the 5 problems

Let us consider the Sun

After solving the neutrino oscillation puzzle:

- We forgot about **structural problems**, still to be solved
- We forgot the problem of **choosing a suitable distribution function** $f(v)$

The Solar Model Problem resurrected[4]

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[4] M.A.Asplund *et al.*, Nature **436** (2005) 525

The Choice of the Distribution Function

For passing from MB to another distribution with more particles either in the head or in the tail we must:

- either receive energy from the system
- or give energy

If the system is isolated this energy is *exchanged with the system itself* owing to

- **correlations** among particles (non-linearizable effects)
- presence of an internal **finite energy bath** (related to a cutoff in the distribution)

Reasons for Choosing a Slightly Deformed Maxwellian

In addition to experimental facts, other few reasons call for a deformed Maxwellian distribution:

- Electric *microfields*
- Correlations
- Fluctuations
- Random forces

- Light elements admixture: knock-on perturbations of ion distributions caused by close collisions with fusion products[5]

All previous effects are non-linearizable!

A departure from TE and LTE can go towards
Non-Extensive (NE) generalized statistics

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[5] N.Nakamura, physics/0607127

Stationary Distribution Function Under Random Forces

We start from a kinetic equation in presence of an external/internal random force $\mathcal{F}$, with collision frequency ν:

$$\pm\frac{2}{3}\frac{\mathcal{F}^2}{\mu^2\nu^2}\frac{\mathrm{d}f}{\mathrm{d}v}+\kappa\left(vf+\frac{k_\mathrm{B}T}{\mu}\frac{\mathrm{d}f}{\mathrm{d}v}\right)=0$$

$$f(v)\propto\exp\left[-\int_0^v \mathrm{d}v'\frac{\mu v'}{k_\mathrm{B}T\pm\frac{2}{3}\frac{\mathcal{F}^2}{\mu\kappa\nu^2}}\right]$$

- $\mathcal{F}$ due to electric *microfields* or random forces
- $\nu^2\equiv\nu_0^2+\nu_1^2+\nu_2^2+\dots$ if many competing interactions are present
- $\pm$ due to sub-/super-diffusivity

Interaction Cross Sections

$$\nu(v) \equiv nv\sigma(v)$$

- $\sigma_0(v) = \alpha_0 v^{-1}$: interaction between and ion and an induced-dipole
  - gives MB even in the presence of the external fi eld $\mathcal{F}$
- $\sigma_1(v) = \alpha_1$: reinforced Coulomb interaction
- $\sigma_2(v) = \alpha_2 v$: we shall show it is related to quantum effect

The Long-Life Stationary State ($\frac{T_{\text{eff}}}{T} \leq \frac{\alpha_1^4}{4\alpha_0^2\alpha_2^2}$)

$$f(v) \propto \exp\left(-\frac{\mu v^2}{2k_{\text{B}}T}\right) \times \left(\frac{2c_2v^2 + c_1 - 2\sqrt{|K|c_2}}{2c_2v^2 + c_1 + 2\sqrt{|K|c_2}}\right)^{\frac{\mu\tau}{4k_{\text{B}}T\sqrt{|K|c_2}}}$$

- $c_1 \equiv \left(\frac{\alpha_1}{\alpha_0}\right)^2$ and $c_2 \equiv \left(\frac{\alpha_2}{\alpha_0}\right)^2$
- $\tau \equiv T_{\text{eff}}/T - 1$ and $K \equiv -\frac{c_1^2}{4c_2} + \tau + 1$
- $k_{\text{B}}T_{\text{eff}} = k_{\text{B}}T \pm \frac{2}{3}\frac{\mathcal{F}^2}{\kappa\mu n^2\alpha_0^2}$

Useful, for instance, for low-energy atomic physics

The Long-Life Stationary State ($\frac{T_{\rm eff}}{T} > \frac{\alpha_1^4}{4\alpha_0^2\alpha_2^2}$)

$$f(\varepsilon_p) \propto \exp\left[-\frac{\varepsilon_p}{k_{\rm B}T_{\rm eff}}\right]\exp\left[-\delta\left(\frac{\varepsilon_p}{k_{\rm B}T_{\rm eff}}\right)^2\right]\exp\left[-\gamma\left(\frac{\varepsilon_p}{k_{\rm B}T_{\rm eff}}\right)^3\right]$$

$$\delta = \pm\frac{2}{3}\frac{\mathcal{F}^2}{\kappa\mu^2 n^2}\frac{\alpha_1^2}{\alpha_0^4}$$

$$\gamma = \pm\frac{8}{9}\frac{\mathcal{F}^2 k_{\rm B}T}{\kappa\mu^3 n^2}\frac{\alpha_2^2}{\alpha_0^4}\left(1-\frac{\alpha_1^4}{\alpha_0^2\alpha_2^2}\right)+\frac{16}{27}\frac{\mathcal{F}^4}{\kappa^2\mu^4 n^4}\frac{\alpha_2^2}{\alpha_0^6}$$

- δ-exp (Druyvenstein): if $\varepsilon_p \sim k_{\rm B}T_{\rm eff}/|\delta|$
- γ-exp: if $\varepsilon_p \sim |\delta/\gamma|\, k_{\rm B}T_{\rm eff}$

Connection with Non-Extensive Statistics

Our $f(\varepsilon_p)$ reduces to the NE distribution:

- In the limit $(q-1)\frac{\varepsilon_p}{k_\mathrm{B} T_\mathrm{eff}} \to 0$
- With the position $\delta = (1-q)/2$
  - $q = 1 \mp \frac{4}{3}\frac{\mathcal{F}^2}{\kappa\mu^2 n^2}\frac{\alpha_1^2}{\alpha_0^4}$

In the case of the electric microfields we get $\delta \simeq 12\Gamma^2\alpha^4$:

- Γ, plasma parameter (generally $\Gamma \lesssim 1$ or $\Gamma \geq 1$)
- $0.4 < \alpha < 1$ (for dense stellar plasmas)
  - α parameter related to ion-ion correlation function

The quantum energy-momentum uncertainty with a Lorentz dispersion $\mathcal{D}(E, \varepsilon_p)$ gives a power-like tail on the $f(\varepsilon_p)$ distribution

$$f(\varepsilon_p) \equiv \int \mathrm{d}\varepsilon \mathcal{D}(\varepsilon, \varepsilon_p) \propto \frac{\sqrt{\varepsilon_p}}{(k_\mathrm{B}T)^{3/2}} \left[\exp\left(-\frac{\varepsilon_p}{k_\mathrm{B}T}\right) + \mathrm{const} \cdot \frac{(k_\mathrm{B}T)^{3/2}}{\varepsilon_p^4} \right]$$

$$f(\varepsilon_p) \sim f_{MB}(\varepsilon_p) + \mathrm{const} \cdot \frac{\sqrt{\varepsilon_p}}{(k_\mathrm{B}T)^{3/2}} \frac{(k_\mathrm{B}T)^{3/2}}{\varepsilon_p^4}$$

This behaviour is obtained from the kinetic equation if $\sigma_2 \propto \sqrt{\varepsilon_p}$ is assumed

From Which Interaction Does $\sigma_2 \propto \sqrt{\varepsilon_p}$ Come?

From a dimensional analysis, the interaction that hypothetically links quantum and non-extensive effects is tidal-like

$$F_Q(r) = f_{Q_0} \left(\frac{r}{R_0} \right)^3 \quad r \leq R_0$$

Assuming:

- An entropic parameter $q \sim 0.1$
- A proton plasma
- Density $n \approx 10^{-14}\,\mathrm{fm}^{-3}$
- $R_0 \approx 10^5\,\mathrm{fm}$

We obtain $f_{Q_0} \approx 10^{-12}\,\mathrm{MeV/fm}$

Non-Extensive Approach to Non-Resonant Fusion Reactions in the Sun

Schism between helioseismology and models with revised composition arise because:

- Abundant elements (*C*, *N*, *O*, *Ne*) provide major contributions to the opacity of the solar interior
- This in turn influences internal the structure and the depth at which the interior becomes convective

Problems with SSM:

- Revision of abundances
- Neutrino fluxes (B^8, *Be*, hep)[6]
- CNO flux

Little cracks in the solar neutrino physics?[7]

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[6] B.Aharmin *et al.* (SNO), hep-ex/0607010

[7] G.Fogli *et al.*, hep-ph/0605186

The Thermonuclear Reaction Rate

Represents the frequency of fusion reactions occurring among the nuclear species *i* and *j*

$$r_{ij} = \frac{N_i N_j \langle v\sigma \rangle_f}{1+\delta_{ij}} = \frac{N_i N_j}{1+\delta_{ij}} \int_0^{+\infty} f(E)\, v(E)\, \sigma(E)\, \mathrm{d}E$$

- N: Number density
- $\langle v\sigma \rangle$: Thermal average
- $f(E)$: Distribution function
- $v(E)$: Relative velocity of reacting nuclei
- $\sigma(E)$: Reaction cross section

Thermonuclear Reaction Cross Section

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-bE^{-1/2}\right)$$

- $\exp\left(-bE^{-1/2}\right)$: Coulomb tunnelling factor
  - $b = \sqrt{2\mu_{ij}}\pi Z_i Z_j \mathrm{e}^2/\hbar$
  - μ_{ij}: reduced mass
  - Z: atomic number
- $S(E)$: astrophysical factor
  - contains the details of the reaction matrix element
  - experimentally measured for many reactions

The Astrophysical Factor $S(E)$

- For non-resonant reactions (e.g. *pp* reaction):

$$S(E) \sim \mathrm{const}$$

- For resonant reactions (e.g. $^{12}C + p \rightarrow ^{13} N + \gamma$):

$$S(E) = \frac{\pi}{2} \omega_{ij} \frac{\hbar^2}{\mu_{ij}} \frac{\Gamma_{in} \Gamma_{out}}{(E - E_R)^2 + (\Gamma_T/2)^2}$$

  - E_R: Resonance energy
  - Γ_T: Total resonance width
  - $\Gamma_{in}, \Gamma_{out}$: Input and output channel widths
  - ω_{ij}: Quantum statistical weighting factor

The Energy Distribution Function $f(E)$

- For plasmas in a Global Thermodynamical Equilibrium (GTE):
  - Maxwell-Boltzmann (MB):

$$f_{MB}(E) = \frac{2}{\pi^{1/2}} \frac{E^{1/2}}{(k_{\mathrm{B}} T)^{3/2}} \exp\left(-\frac{E}{k_{\mathrm{B}} T}\right)$$

- For plasmas in a Metaequilibrium State:
  - Generalized statistical theories (e.g. non-extensive Tsallis-Clayton):

$$f_{NE}(E) \propto \left[1 - (1-q)\frac{E}{k_B T}\right]^{\frac{1}{1-q}} \sim f_{MB}(E) \cdot \exp\left[-\delta\left(\frac{E}{k_B T}\right)^2\right]$$

  - $\delta \sim (1-q)/2$, for *small deformations*

The Gamow Peak

The integrand of the thermal average in MB case is:

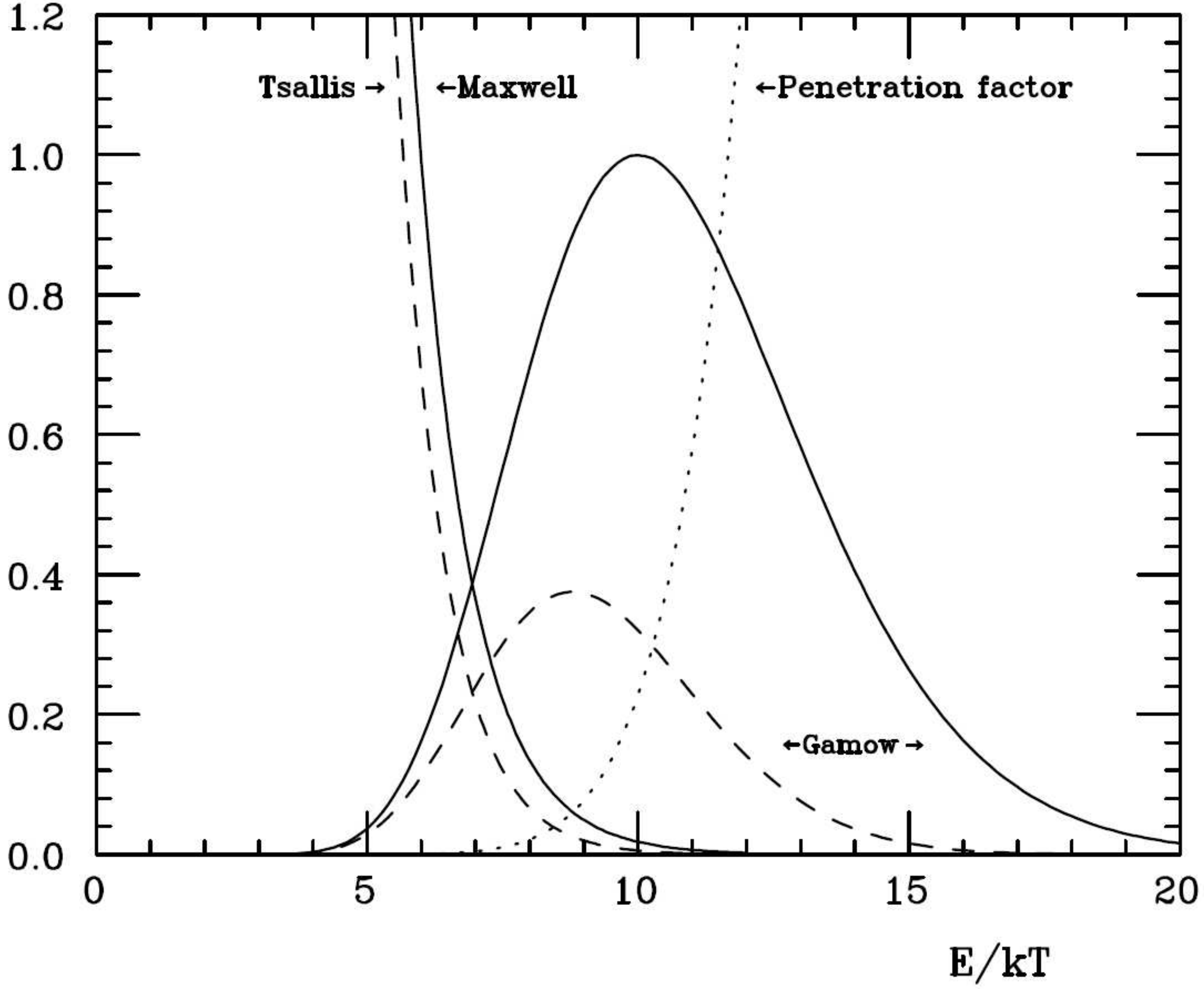
$$v(E)\sigma(E)f(E) \propto S(E) \cdot \exp\left(-\frac{E}{k_B T} - \frac{b}{\sqrt{E}}\right)$$

For non-resonant reactions, it shows a rather narrow peak at:

- $E_G = (bk_B T/2)^{2/3}$ **(MB case)**
- $\widetilde{E}_G \approx E_G \cdot [1 - 4\delta E_G/(3k_B T)]$ **(NM case)**

- Only very few particles responsible of energy production in astrophysical plasmas

The Gamow Peak



Non-Resonant Reaction Rate

- The Maxwell-Boltzmann rate (MB)

$$r_{ij}^{M}[NR] = \frac{2^{5/2}}{3^{1/2}} N_i N_j \mu_{ij}^{-1/2} \frac{S(E_G)}{(k_{\rm B}T)^{1/2}} \left(\frac{E_G}{k_{\rm B}T}\right)^{1/2} \exp\left(-\frac{3E_G}{k_{\rm B}T}\right)$$

- The non-extensive rate (NM)

$$\begin{cases} r_{ij}^{NM}[NR] = r_{ij}^{M}[NR] \cdot \frac{S(\widetilde{E}_G)}{S(E_G)} \left(1 + \frac{15}{4}\delta - \frac{7}{3}\delta \frac{E_G}{k_{\rm B}T}\right) \exp(-\Delta_{ij}) \\ \Delta_{ij}(\delta, \widetilde{E}_G) = -\frac{3E_G}{k_{\rm B}T} \left[1 - \left(1 + \frac{5}{3}\delta \frac{\widetilde{E}_G}{k_{\rm B}T}\right) \left(1 + 2\delta \frac{\widetilde{E}_G}{k_{\rm B}T}\right)^{-2/3}\right] \end{cases}$$

Why Using Generalized Statistics in Astrophysical Systems?

Ideal hypotheses:

1. purely binary collisions
2. instantaneous collisions
3. dilute plasma
4. Markovian system
5. no correlations
6. $\Gamma \ll 1$

Real conditions:

1. many-body collisions
2. dense plasma
3. memory effects
4. weakly-coupled plasma

Neutrino Luminosity Constraint

From experiments and the SSM we get the *upper limit*

$$\left(\frac{L_{CNO}}{L_\odot}\right)_{max} \simeq 7.3\%$$

The NE luminosity reads:

$$(\Phi_{CNO}^{max})_{NM} \equiv (\Phi_{CNO}^{max})_M \ \exp(-338.5\delta)$$

$$(\Phi_{CNO}^{max})_M \simeq 2.49 \cdot 10^9\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

This imposes an **upper limit** to the deformation:

$$|\delta| \simeq 0.0045 \qquad \Longleftrightarrow \qquad 0.991 \lesssim q \lesssim 1.009$$

Narrow Resonances at Low Energy (**r**)

$$\begin{cases} E_R \approx E_G \\ \Gamma_T \ll E_R \end{cases}$$

- Only the cross section around the resonance contributes to the rate

$$r_{ij}[r] \approx N_i N_j f(E_R) v(E_R) \int_0^{+\infty} \sigma(E)\,\mathrm{d}E$$

- The Maxwell-Boltzmann rate:

$$r_{ij}^{MB}[r] = (2\pi)^{3/2} N_i N_j \frac{\hbar^2}{(\mu_{ij} k_B T)^{3/2}} \frac{\Gamma_{in}(E_R)\Gamma_{out}}{\Gamma_T} \exp\left(-\frac{E_R}{k_B T}\right)$$

- The non-extensive rate:

$$r_{ij}^{NM}[r] = r_{ij}^{MB}[r] \left(1 + \frac{15}{4}\delta\right) \exp\left[-\delta\left(\frac{E_R}{k_B T}\right)^2\right]$$

Wide Resonances at Higher Energy (R)

$$\begin{cases} E_R > E_G \;\; \text{(or } E_R \gg E_G\text{)} \\ \Gamma_T > E_G \end{cases}$$

- The reaction occurs in the low energy tail of the resonance
- The Breit-Wigner formula expresses the S-factor at E_G
- The Maxwell-Boltzmann rate:

$$r_{ij}^{MB}[R] = \frac{2^{3/2}\pi}{3^{1/2}} \frac{N_i N_j \hbar^2 E_G^{1/2}}{\mu_{ij}^{3/2} k_B T} \frac{\Gamma_{in}\Gamma_{out}}{(E_G - E_R)^2 + (\Gamma_T/2)^2} \exp\left(-\frac{3E_G}{k_B T}\right)$$

- The non-extensive rate:

$$\begin{aligned} r_{ij}^{NM}[R] &\approx r_{ij}^{MB}[R] \left[1 + \frac{15}{4}\delta - \frac{7}{3}\delta \frac{E_G}{k_B T} + \right. \\ &+ \left. \frac{8}{3} \frac{(E_G - E_R)E_G}{(E_G - E_R)^2 + \Gamma_T^2/4} \delta \frac{E_G}{k_\mathrm{B} T} - \left(\frac{E_G}{k_\mathrm{B} T}\right)^2 \delta \right] \end{aligned}$$

The CNO-Cycle Scheme

$$\begin{cases} {}^{12}C(p,\gamma)^{13}N(e^+\nu_e)^{13}C \\ {}^{13}C(p,\gamma)^{14}N \\ {}^{14}N(p,\gamma)^{15}O(e^+\nu_e)^{15}N \\ {}^{15}N(p,\alpha)^{12}C \end{cases}$$

- Physically equivalent to

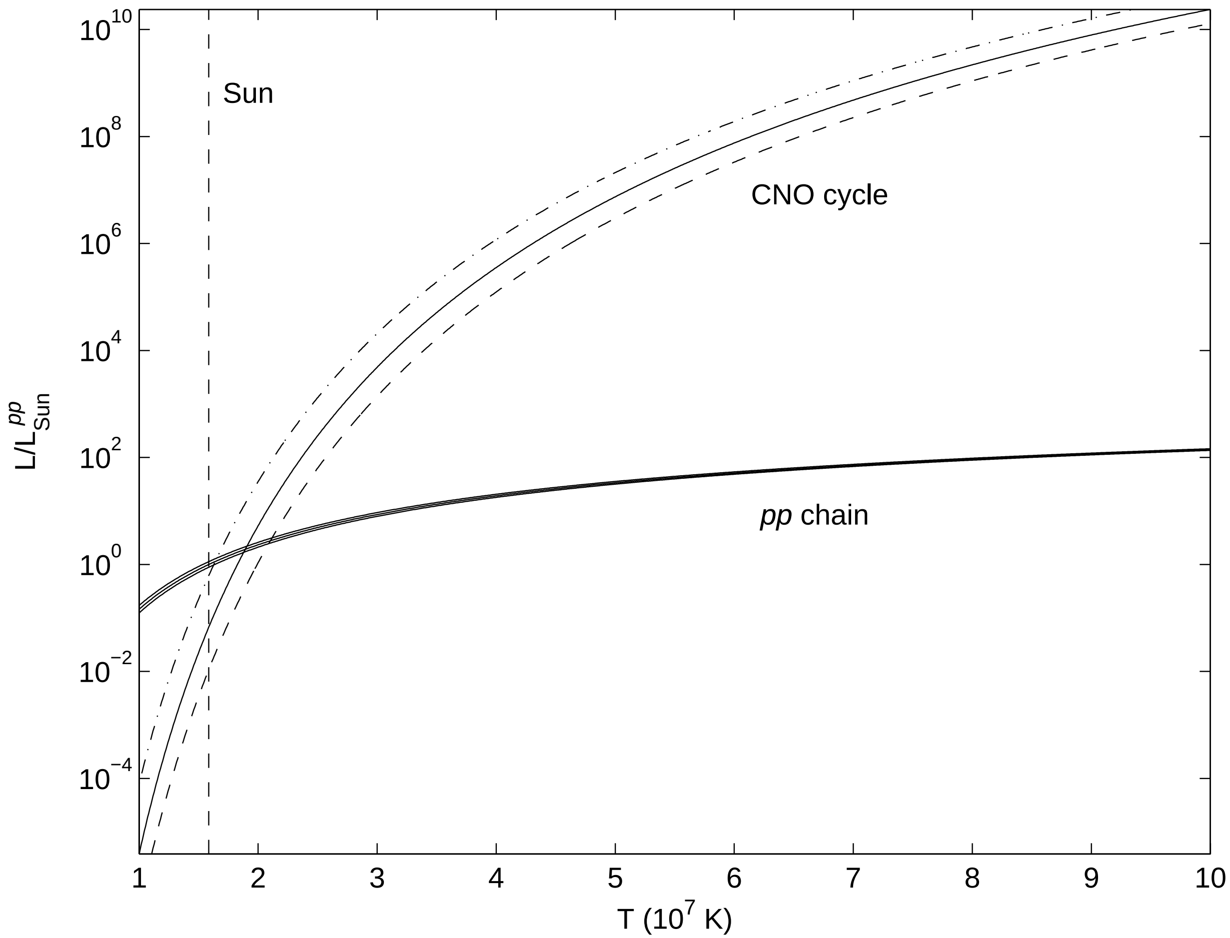
$$X + 4p \rightarrow X + \alpha + 2e^+ + 2\nu_e + 25\mathrm{MeV}$$

- ${}^{14}N(p,\gamma)^{15}O$ is the slowest[8]
- ${}^{12}C(p,\gamma)^{13}N$ resonant at $E_R \simeq 460\mathrm{keV}$
- ${}^{14}N(p,\gamma)^{15}O$ resonant at $E_R \simeq 278\mathrm{keV}$

[8] A. Lemut *et al.* (LUNA), PLB **634** (2006) 483;
G. Imbriani *et al.* (LUNA), Astron. Astrophys. **420** (2004) 625

Non-Extensive Approach to CNO-Cycle Reactions

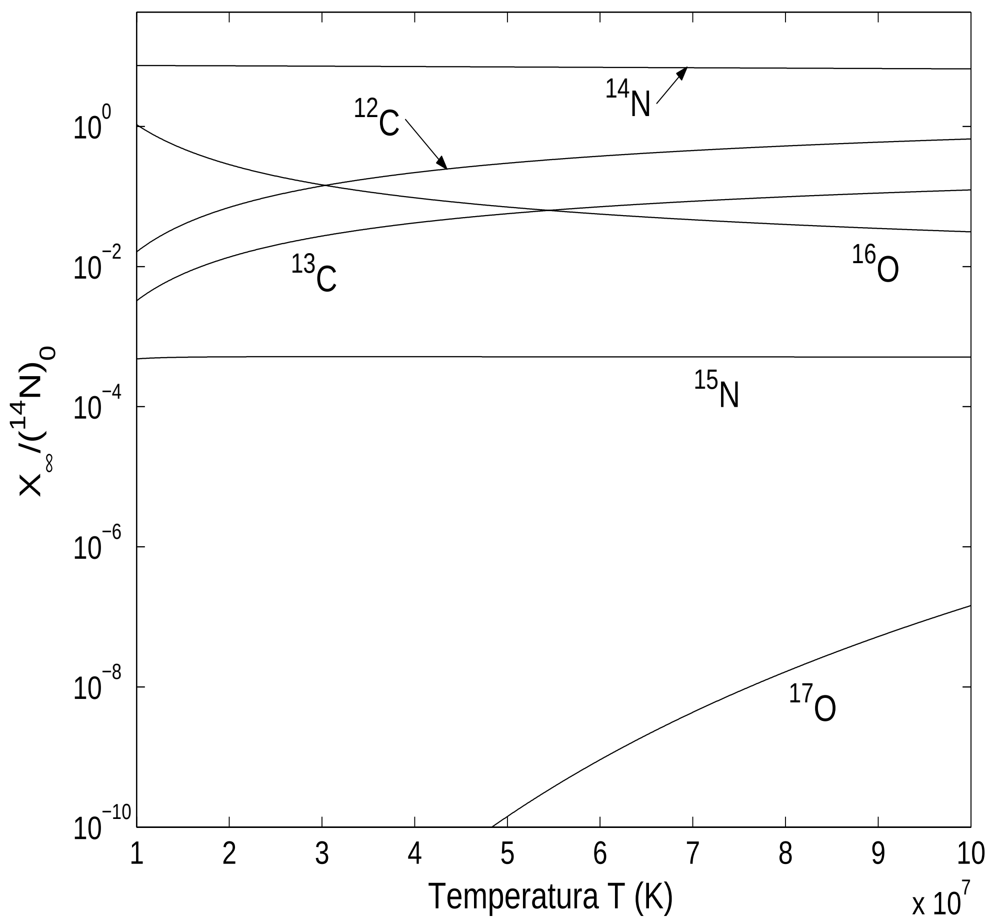
The star luminosity ratio $L/L_{\odot}$ versus plasma temperature
$(q = 0.991 \div 1.009)$

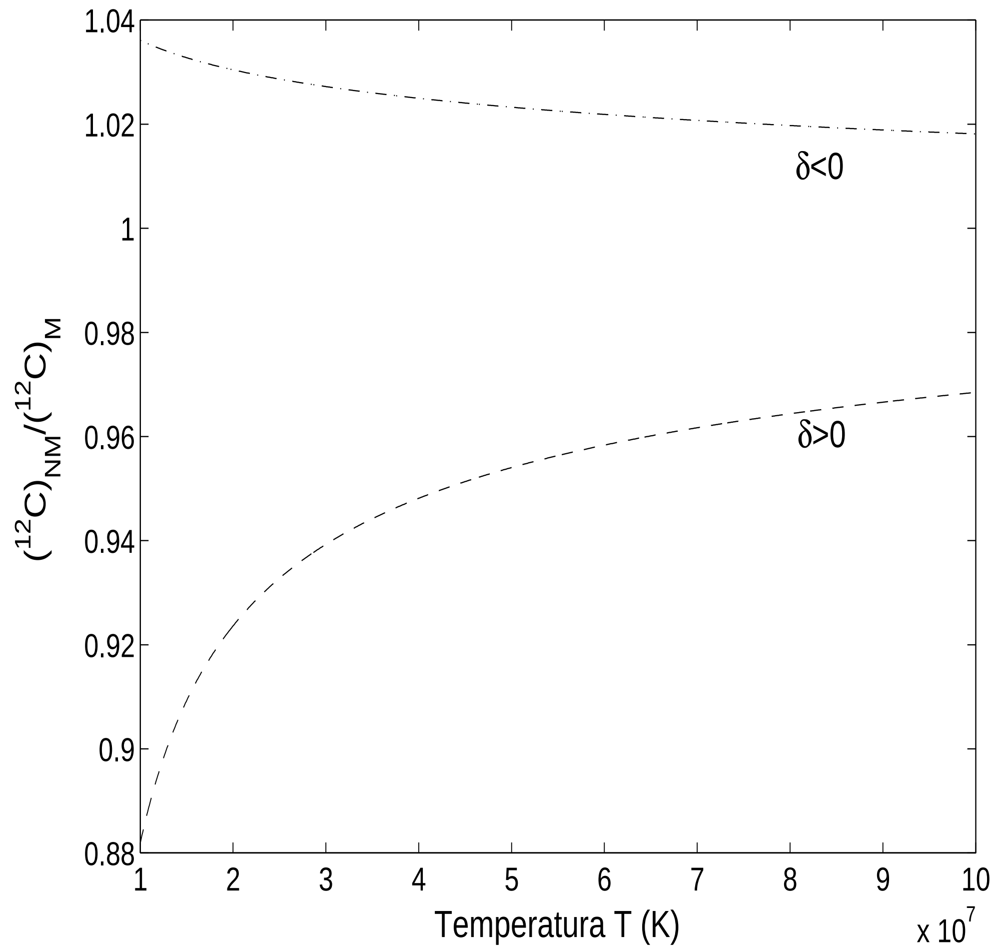


- Larger NE effects on CNO than on *pp*
- Only slight deformations allowed in the Sun
- CNO provides nearly all luminosity at higher *T*

C, N, O Concentrations at Equilibrium

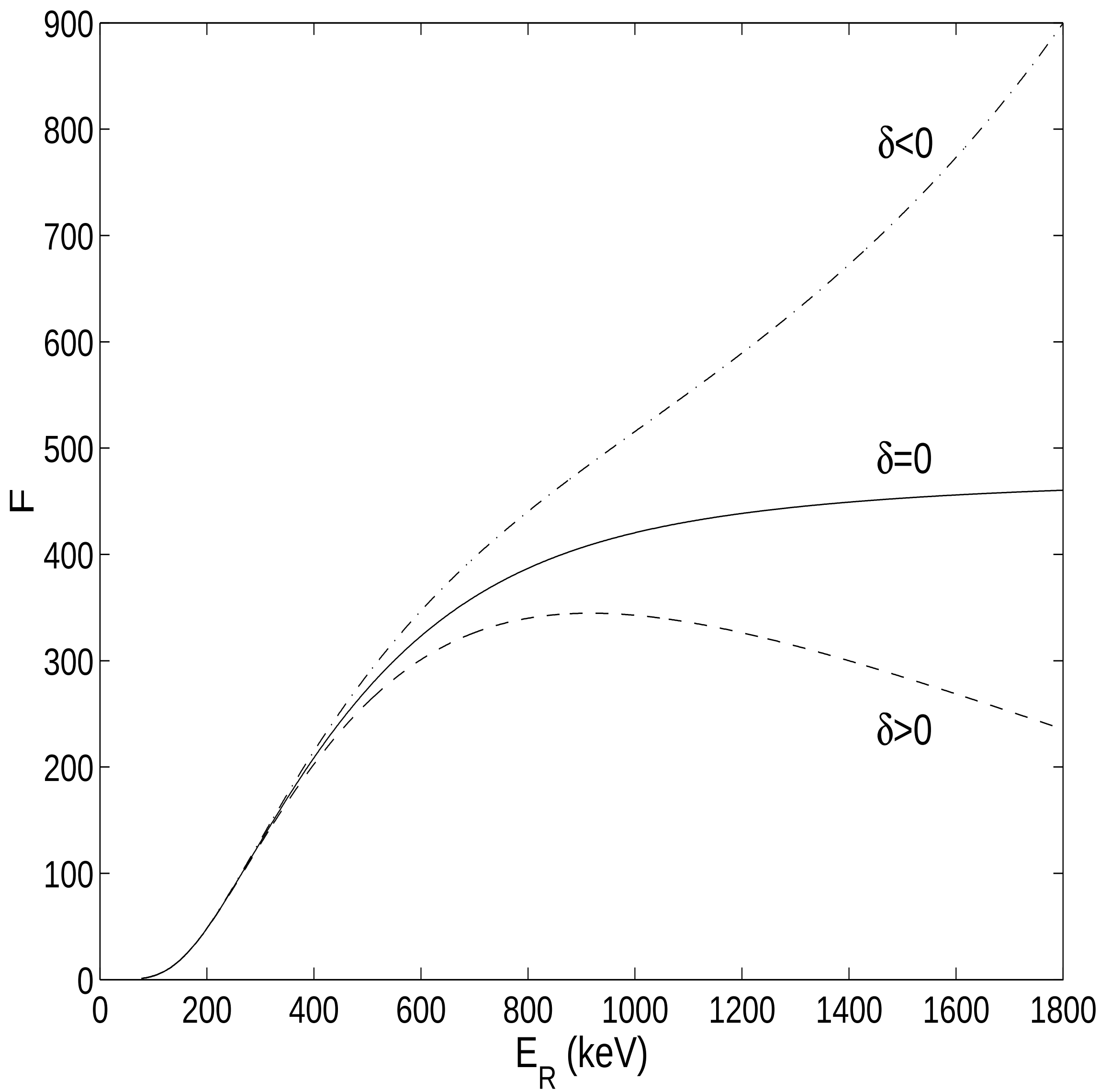
A slight deformation modifies the nuclide concentrations without affecting the bulk properties of the stellar system





The $^{12}C + ^{12}C$ Resonant Reaction

- Occurs in white dwarfs
  - $M \simeq 1.4 M_\odot$
  - $\rho \simeq 10^9 \, \mathrm{g/cm^3}$
  - $\Gamma \sim 5$
- Ignition temperature $T \sim 10^8 \, \mathrm{K}$
- Experimental cross section resonant at $E = 2.4 \, \mathrm{MeV}$ and possibly also far below
  - $\Gamma_{in}(E_R) \ll \Gamma_T \ll E_R$
- Screening effects
- Big NE corrections!



Radiative Recombination

$$A^{Z+} + e^{-} \rightarrow A^{(Z-1)+} + h\nu$$

Only recombination channel with naked ions!

$$\sigma_n^{RR}(E_k) = 2.10 \cdot 10^{-22} \cdot g_n(E_k) \cdot \frac{Z^4 E_{1s}^2}{n E_k (Z^2 E_{1s} + n^2 E_k)} [\mathrm{cm}^2]$$

$$\alpha^{RR} \propto \int_0^{+\infty} f(E_k) \sigma_{tot}^{RR}(E_k) \sqrt{E_k}\, \mathrm{d}E_k$$

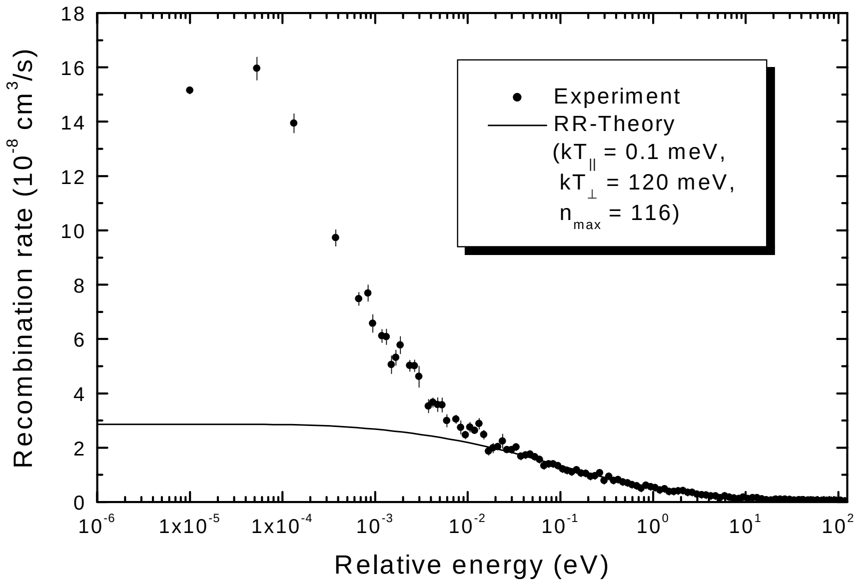
- Is the usual assumption $f = f_{MB}$ correct? $\Longrightarrow$ experiments at storage rings
- In electron coolers, cold parallel electron beam merged with a hot ion beam, along B-field lines ("freezing" of radial motion)

Experimental Results on Radiative Recombination

- Different conditions in experiments imply isotropic $f_{MB} \Longrightarrow$ anisotropic (flattened) f_{MB}:

$$f(E) = \frac{m}{2\pi kT_\perp}\left(\frac{m}{2\pi kT_\parallel}\right)^{\frac{1}{2}} \times$$

$$\times \exp\left[-\frac{E_\perp}{kT_\perp}\right] \exp\left[-\frac{E_\parallel}{kT_\parallel}\right]$$



[a] A. Hoffknecht *et al.*, arXiv: physics/0003088 v1 (2000)

- Approaching zero relative E_k, α^{RR} increases more than predicted (RR enhancement); no complete theoretical explanation

Explanations and Non-Extensive Approach

Note: α^{RR} is theoretically calculated in free space, but in experimental conditions, external forces ($\overrightarrow{E}$, $\overrightarrow{B}$ field)!

$$\Delta\alpha^{RR} \propto Z^{2.5} B^{0.5} T_{\perp}^{-1.0} T_{\parallel}^{-0.3} \equiv \left| f(Z, B, T_{\perp}, T_{\parallel}) \right|$$ [9]

α^{RR} sensitive to low relative energy particle, i.e. head of the distribution $\Longrightarrow f_q$ with $q < 1$; in the isotropic case:

$$\alpha_q^{RR} = B_q \frac{1}{(Mc^2)^2} c^4 \int_0^{E_{cut}} \left[1 - (1-q) \frac{E}{k_B T_q} \right]^{\frac{q}{1-q}} \sigma_{tot}^{RR}(E_k) E_k \, \mathrm{d}E_k$$

with the correct "non-extensive" temperature $k_B T_q = \frac{2}{5-3q} k_B T_{MB}$ $(q < 1)$ and the cutoff $E_{cut} = \frac{k_B T_q}{1-q}$

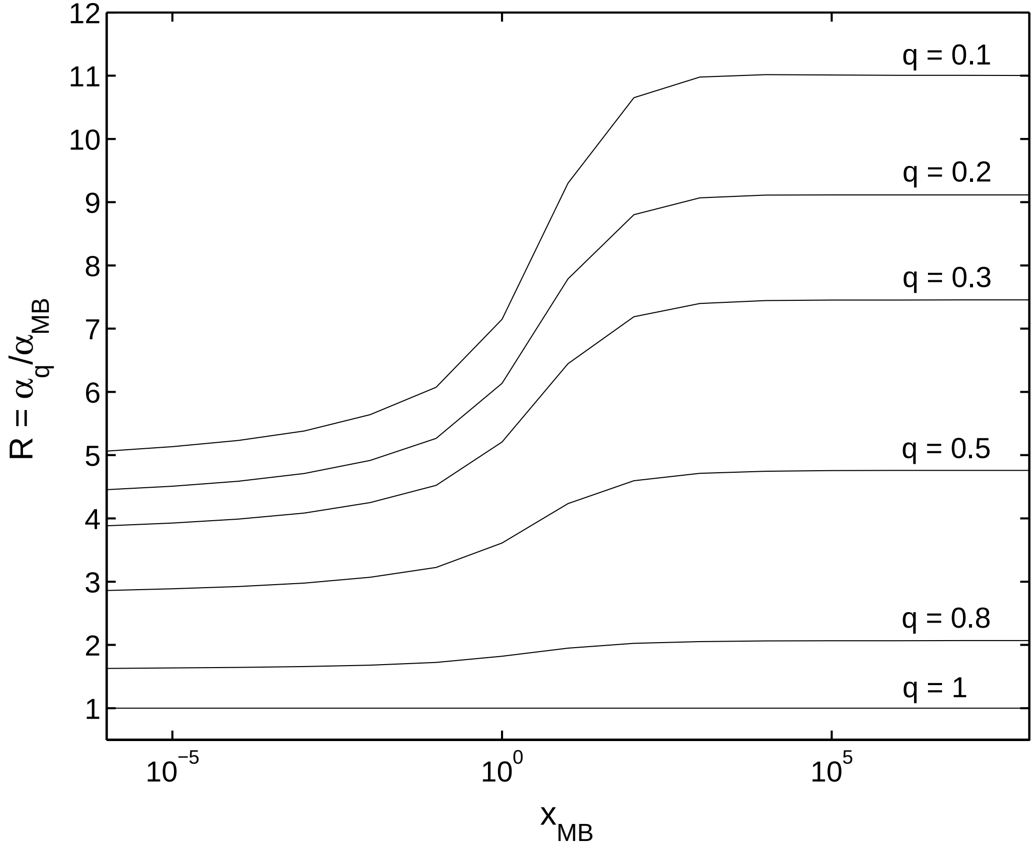
$$q \approx 1 - f(Z, B, T_{\perp}, T_{\parallel})$$

[9] C. Heerlein, G. Zwicknagel and C. Toepffer, NIM B **205** (2003) 395

Results of the Non-Extensive Approach

At zero relative energy, $\frac{\alpha_q}{\alpha_{MB}}$:

- Major effects at high T ($x_{\mathrm{MB}} = k_B T / E_{1\mathrm{s}}$) on deformed-to-MB rate ratio ($R = \alpha_q / \alpha_{MB}$)
- Cutoff experimental observation: test of validity
- Cutoff presence in stars: finite heat reservoir



Non-Extensive Fit to DR Astrophysical Rate for C^{3+}

- The Maxwell-Boltzmann rate[10]

$$\alpha_{\text{fit}}^{\text{DR}}(k_B T) = \frac{1}{T^{3/2}} \sum_i c_i \exp\left(-\frac{E_i}{k_B T}\right)$$

- The non-estensive case

$$\alpha_{\text{fit}}^{\text{DR}}(k_B T) = \frac{1}{T^{3/2}} \left(1 + \frac{15}{4}\delta\right) \sum_i c_i \exp\left[-\frac{E_i}{k_B T} - \delta\left(\frac{E_i}{k_B T}\right)^2\right]$$

- For C^{3+}: $i = 1 \div 5$

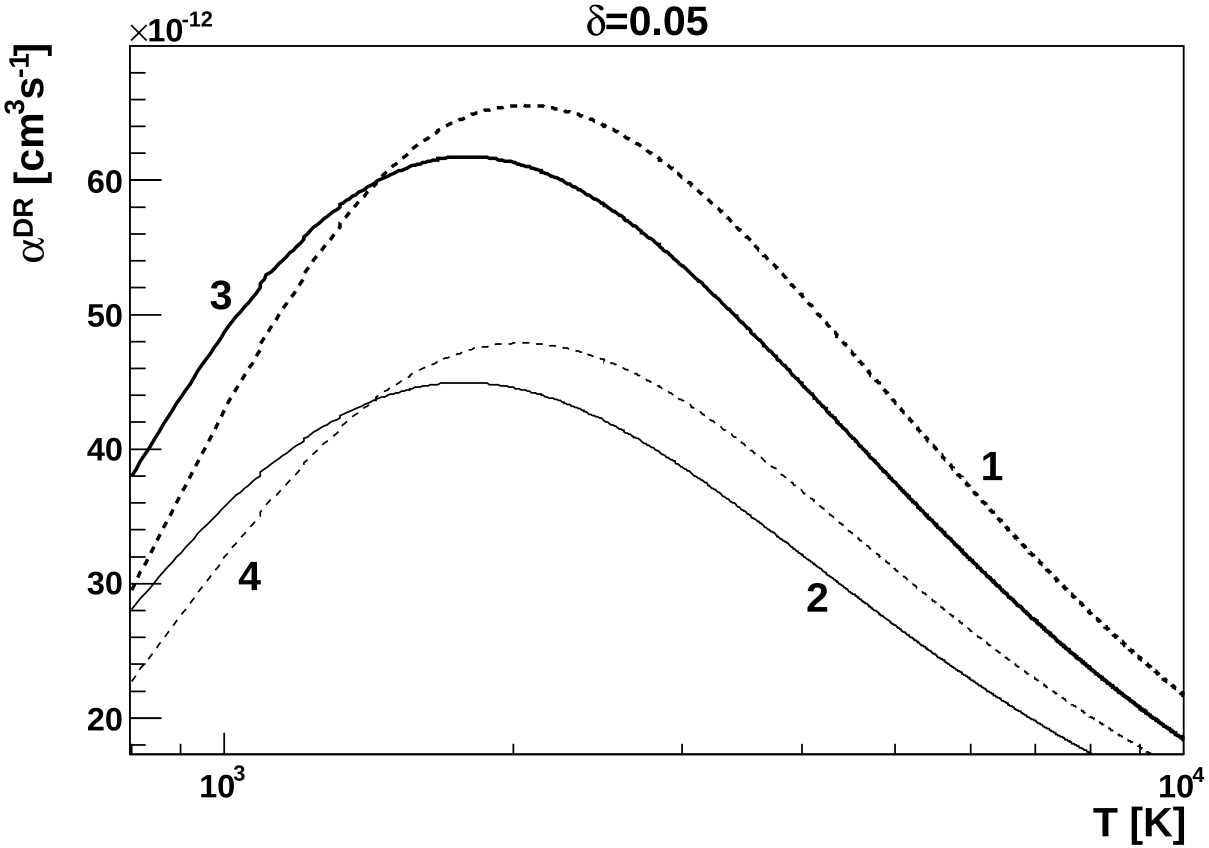
[10] S.Schippers *et al.*, ApJ **555** (2001) 1027

The Non-Extensive DR Astrophysical Rate for C^{3+}

$$\alpha^{DR}(k_B T) \simeq \sum_i \frac{\bar{\sigma}_i E_{R,i}\, c}{(k_B T)^{\frac{3}{2}} \sqrt{\mu c^2}} \left[1 - \frac{\delta}{2}\left(\frac{\Gamma_i}{k_B T}\right)^2\right] \times$$
$$\times \quad \exp\left[-\frac{E_{R,i}}{k_B T}\left(1 + \delta \frac{E_{R,i}}{k_B T} - \frac{\Gamma^2}{4 E_{R,i} k_B T}\right)\right] \times$$
$$\times \quad \exp\left[-\delta\left(\frac{\Gamma_i}{k_B T}\right)^2 \left(\frac{\Gamma_i^2}{4\,(k_B T)^2} - \frac{E_{R,i}}{k_B T}\right)\right]$$

- The Γ parameter stongly varies with T
- Correlations different for $T = 10^6\,\mathrm{K}$ and $T = 10^2\,\mathrm{K}$
- The plasma does not preserve its Maxwellian behaviour
- For $T > 10^5\,\mathrm{K}$ we assume $\delta = 0$
- For $T < 10^3\,\mathrm{K}$ we assume $\delta = 0.2$

Non-Extensive Approach to DR for C^{3+}



1. α_{NE}
2. Experimental fit[a]
3. α_{MB}
4. Non-extensive fit

[a] S.Schippers *et al.*, ApJ **555** (2001) 1027

Quantum Dispersion Effect (1)

Due to many body collision effects, particle energy E and momentum

$$p = \sqrt{2m\epsilon_p}$$

can be affected by a dispersion relation characterized, in first approximation, by the Lorentzian[11]:

$$\delta(E - \epsilon_p)_\gamma = \frac{1}{\pi} \frac{\gamma}{(E - \epsilon_p)^2 + \gamma^2}$$

As a consequence a given particle population can be Maxwellian distributed in energy but not in momentum

[11] V. Galitskii and V. Yakimets, Sov. Phys. JETP **24** (1967) 637; A. Starostin *et al.*, PLA **274** (2000) 64

Quantum Dispersion Effect (2)

Since momentum rather than energy determines the scattering amplitude, the reaction cross section must be averaged over momentum distribution

As a consequence quantum dispersion effect can be extremely relevant, for instance, in fusion reaction between charged particles

The interacting particles energy momentum dispersion relation has been proposed recently by us to explain the strong enhancement observed for the low energy $d(d,p)t$ fusion reaction experiment performed using a deuterated target[12]

[12] F. Raiola *et al.*, PLB **547** (2002) 193; F. Raiola *et al.*, EPJA **27** (2006) 79

Quantum Dispersion Effect (3)

Low energy fusion reaction experiment

- Monochromatic beam particles at energy E_b
- Target particles Maxwellian distributed in energy: $n(E_t) \propto \exp(E_t/k_B T)$

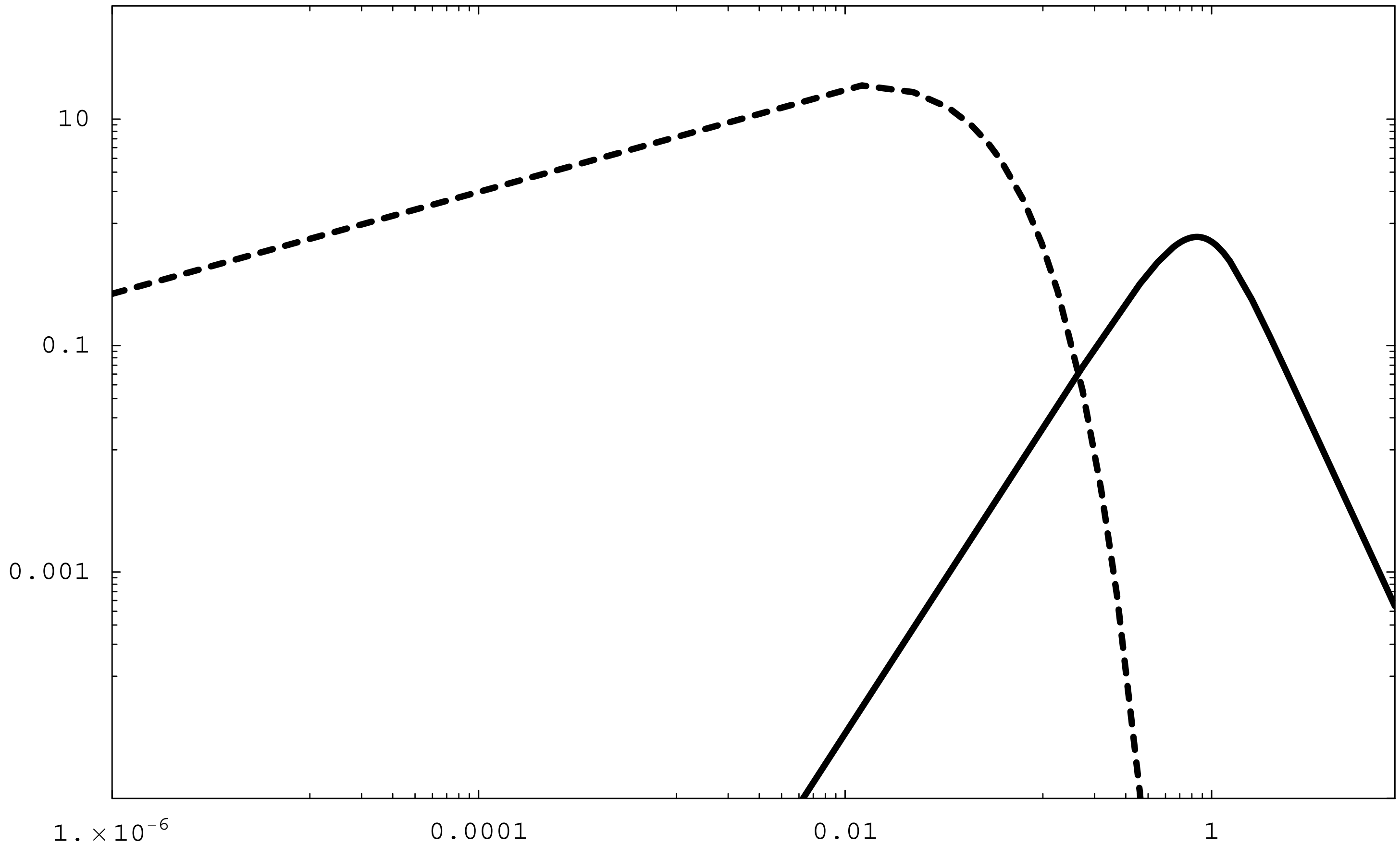
Target particles momentum distribution:

$$f(\epsilon_{p_t}) = \frac{\int \mathrm{d}E_t \delta_\gamma(E - \epsilon_{p_t}) \exp(E_t/k_B T)}{\int \mathrm{d}E_t \int \mathrm{d}^3\mathbf{p}_t \delta_\gamma(E - \epsilon_{p_t}) \exp(E_t/k_B T)}$$

The reaction rate is given by: $n_p n_t \langle \sigma v_{rel} \rangle$

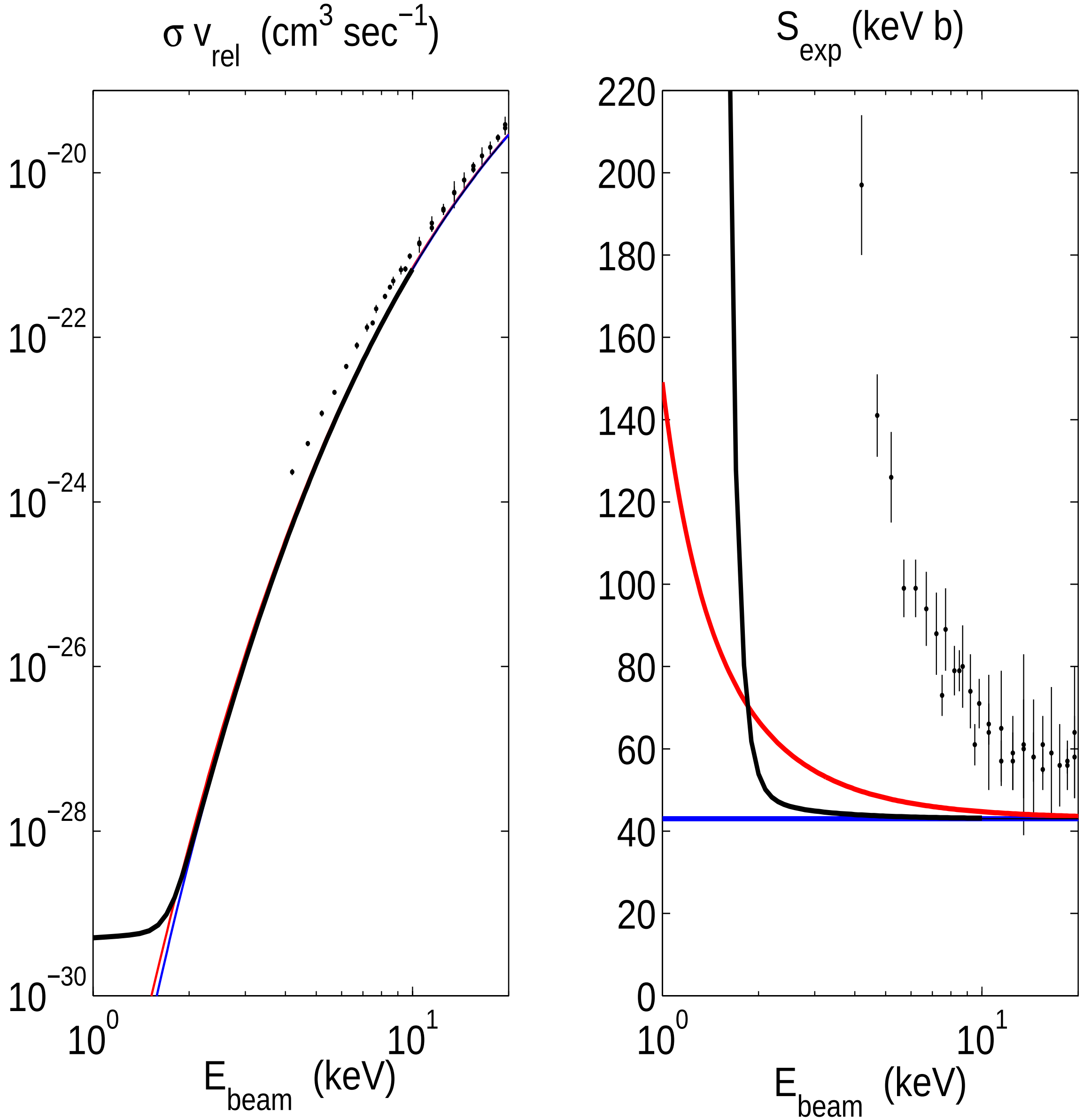
$$\langle \sigma v_{rel} \rangle = \int \mathrm{d}^3\mathbf{p}_t f(\epsilon_{p_t}) v_{rel} \sigma(E_{CM})$$

Quantum Dispersion Effect (4)



Momentum distribution function $f(\epsilon_{p_t})$ in eV^{-1} (full line) compared with the Maxwellian distribution (dashed line) for $T = 10^o\,\mathrm{C} = 0.0244\,\mathrm{eV}$

Quantum Dispersion Effect (5)



Experimental data[a] compared with teoretical curves

- Bare nuclei curve (blue)
- Electron screened with screening potential $U_e = 28\,\mathrm{eV}$ (red)
- Quantum tail thermal effect (**black**)

[a] F. Raiola *et al.*, PLB **547** (2002) 193

Conclusions

- In stellar plasmas, MB is only a first-order approximation and corrections originate from the microscopical dynamics
- Quantum corrections may be related to a $\sigma(\varepsilon_p) \propto \sqrt{\varepsilon_p}$ cross section
- All deformations may be understood within NE statistical mechanics
- Momentum distributions other than MB may be interpreted as long-life stationary states
- If both Lab and stellar plasmas are not in a Maxwellian state, one must be very careful in transferring info obtained in Lab to interpret astrophysical observations

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