Nonextensive statistical mechanics: Present status of theoretical, experimental, observational and computational aspect

Constantino Tsallis
Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA and
Centro Brasileiro de Pesquisas Fisicas, Xavier Sigaud 150, 22290-180 Rio de Janeiro-RJ, Brazil

Since more than one century, it has become gradually well established that Boltzmann-Gibbs (BG) entropy and associated statistical mechanics are of immense value to physics and elsewhere. They appear to be applicable when the microscopic dynamics of the system is strongly chaotic (i.e., positive Lyapunov exponents, for classical systems), hence mixing and ergodic. The situation is more complex when these dynamics are weakly chaotic (typically, vanishing largest Lyapunov exponent, for classical systems). It appears nevertheless that basically the same methods of standard statistical mechanics can be usefully extended and applied if the BG entropy is generalized into the entropy S_q (which recovers the BG one for $q = 1$). Indeed, this entropy preserves the same basic properties of the BG one, such as concavity, Lesche-stability (or experimental robustness), extensivity (in the presence of global correlations), and finiteness of the entropy production per unit time. Also, the standard and Levy-Gnedenko central limit theorems – grounding stones of BG statistical mechanics – have been recently generalized for q different from unity, thus grounding on equal footing nonextensive statistical mechanics. An important difference emerges, however, among the consequences of using the BG entropy or S_q: the stationary-state distribution is of the exponential form in the former case, whereas it is asymptotically a power-law in the latter. Because of this relevant difference, nonextensive statistics has been usefully applied, during the last two decades, to a great variety of natural (physics, geophysics, astrophysics, biology, chemistry), artificial (computational sciences, engineering, networks, signal and image processing), and even social (economics, linguistics, cognitive psychology) complex systems. When the microscopic dynamics is known, the index q (and its related indices) can in principle be calculated a priori. When only mesoscopic information is available, the index q can be calculated from Langevin and Fokker-Planck equations. When not even that is known, q can be determined (within some error bars) by fitting available data with appropriate analytical forms (like astronomers determine the size and eccentricity of the elliptic orbits of the planets, due to the lack of precise information concerning the initial conditions of the planetary system, that would in principle enable them to calculate all the parameters existing in the Keplerian orbital analytical form). All three possibilities for determining q are currently used in practice, and will be exhibited and discussed. A general overview of the present status of various theoretical, experimental, observational and computational aspects will be provided as well. Due to the physical nature of the concepts that nonextensive statistical mechanics calls into question, it is by no means surprising that it is a matter of criticism by some authors. As scientists, we welcome and benefit from healthy controversy and informed debate on such issues. Some time will be reserved to briefly present and discuss the existing criticism.

Bibliography

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