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**SCHOOL and CONFERENCE  
on  
COMPLEX SYSTEMS  
and  
NONEXTENSIVE STATISTICAL MECHANICS**

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**Random Resistor cum Tunneling-bond Network model :  
a Testing Lab for Percolative Complex Systems**

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# Random Resistor cum Tunneling-bond Network model : a Testing Lab for Percolative Complex Systems

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**Collaborators:** Asok K. Sen, Abhijit Kar Gupta

# Composite Materials (**complex materials**)

❖ Made of **multiple** constituents, with different generalised susceptibilities, placed **randomly** within the host

- transport properties highly dependent on the geometrical connections among the conducting

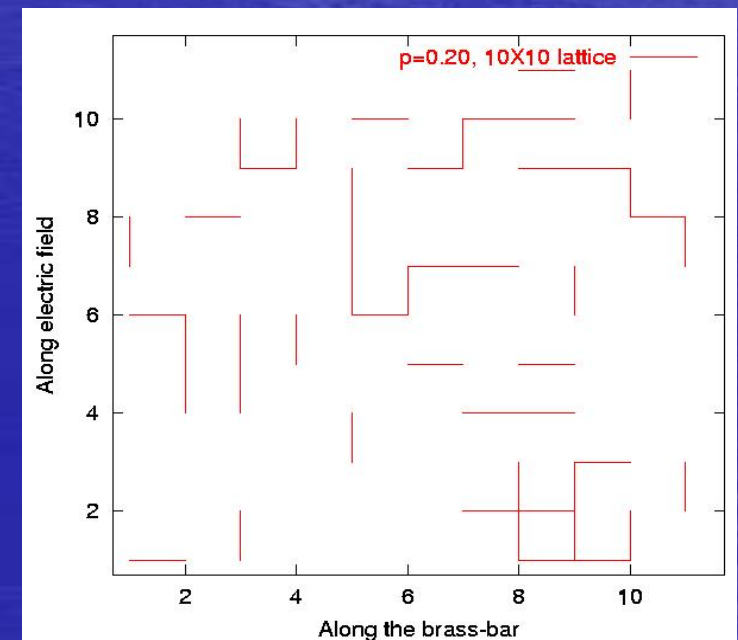
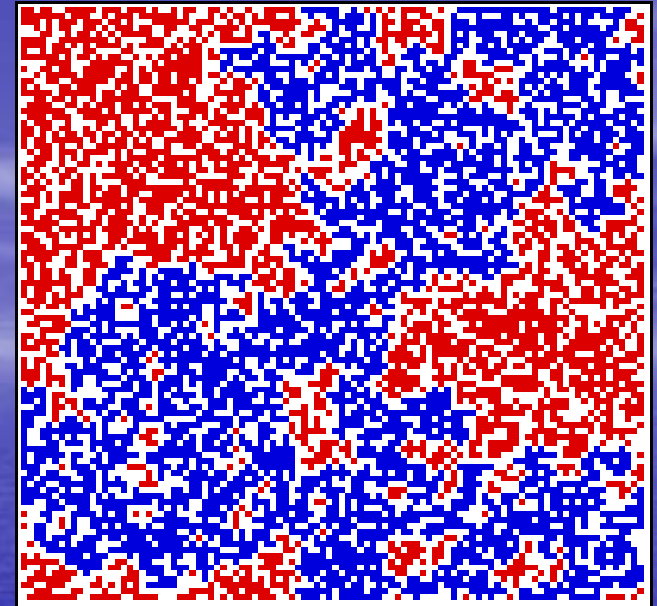


Percolation technique is useful to model such systems

## **Random Resistor Network model (RRN)** (**classical**)

Percolation threshold ,  $p_c = 0.5$

Linear steady response, exponential relaxation, normal distribution of steady response within ensemble



# Experimental observations

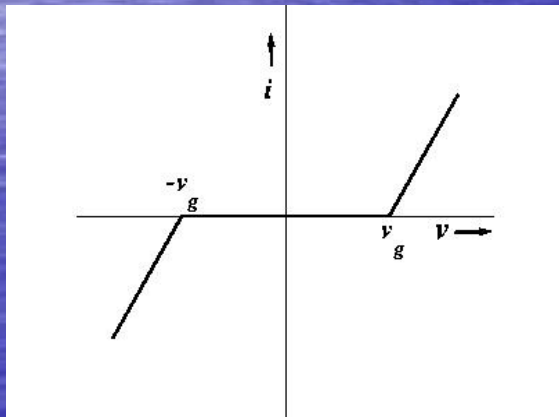
- Ultra-low percolation threshold
- Nonlinearity in the bulk response, on both sides of the classical  $p_c$ ; non-integer power-law in  $G(V)$  for small  $V$
- Low temperature resistance minimum observed in quantum disordered substances
- Nonlinear dynamic response,  $G(\omega) \sim \omega^{0.7}$  for low  $\omega$

# Random Resistor cum Tunneling-bond Network model (RRTN) (Semi-quantum)

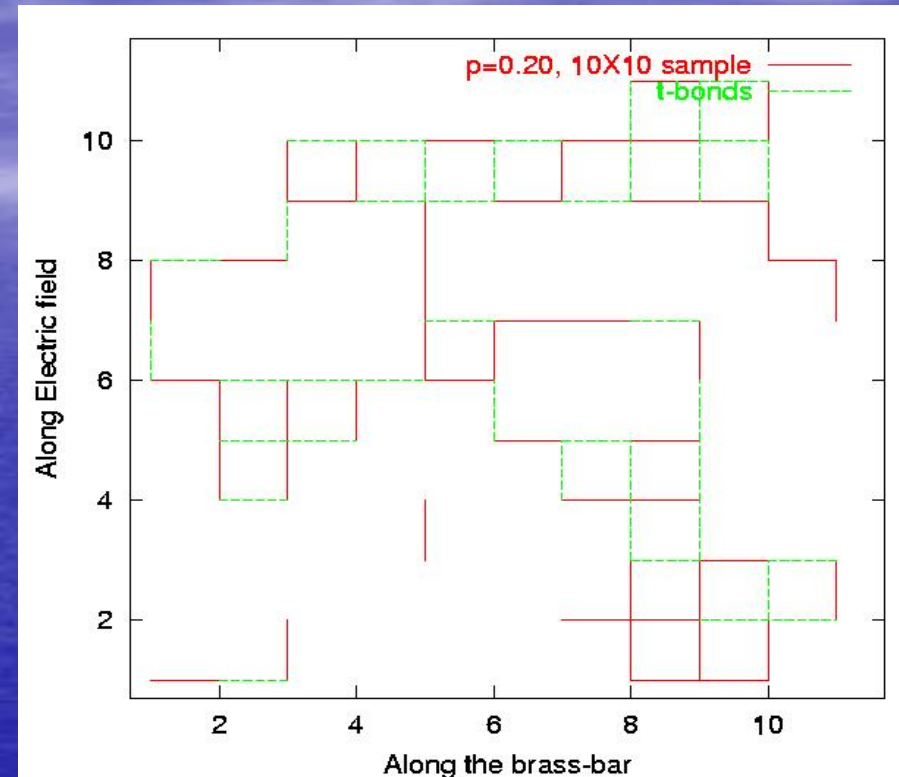
Sen and Kar Gupta, LNP, 1994

Kar Gupta and Sen, PRB, 57, 3375 (1998)

- Correlated-random bonds
- phenomenological tunneling between two nearest -neighbour RRN bonds



*Almost a theorem* for observing  
**nonlinearity**

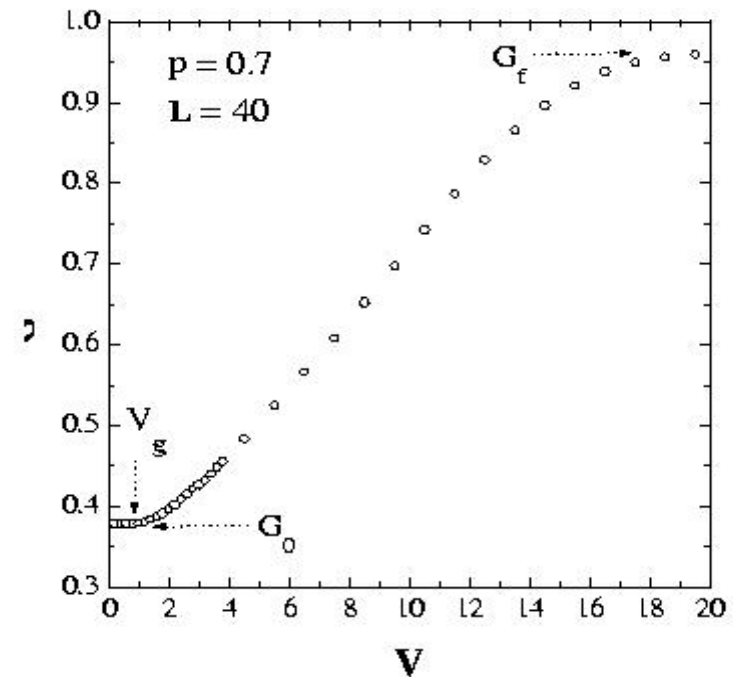


- disorder + nonlinear
- $p_{ct}=0.181$
- $v_g$  develops  $V_g$  for the bulk

## Static $G$ - $V$ characteristics

- Lower linear region for  $V < V_g$
- Sigmoidal region for  $V > V_g$
- Upper linear region for  $V \gg V_g$

Kar Gupta and Sen, PRB, 57, 3375 (1998)

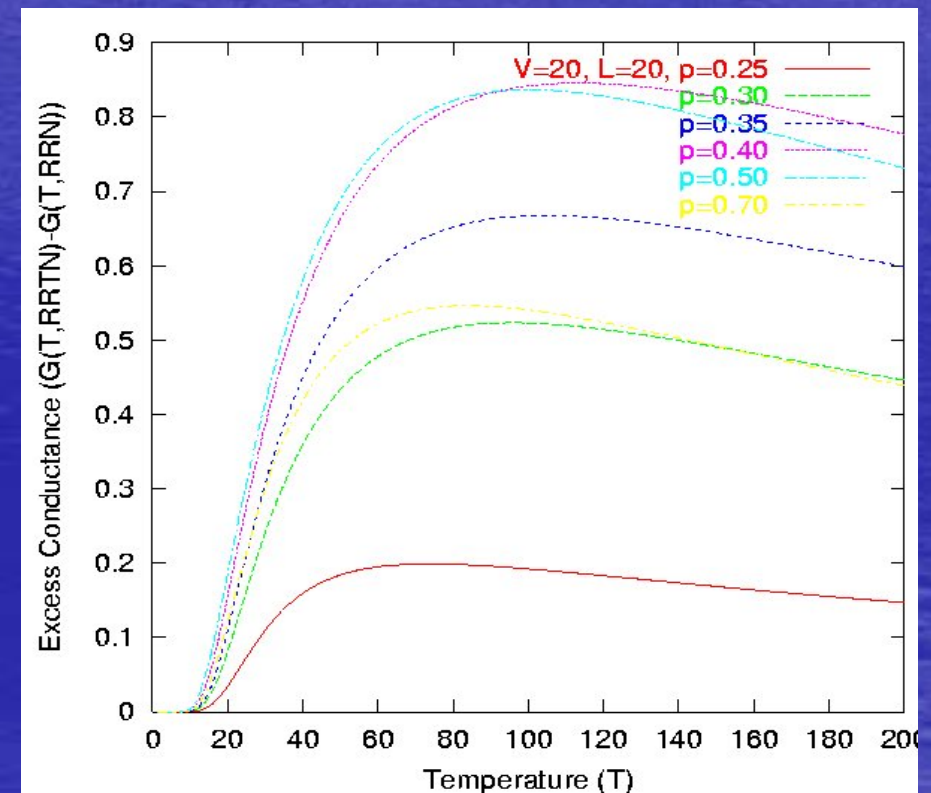
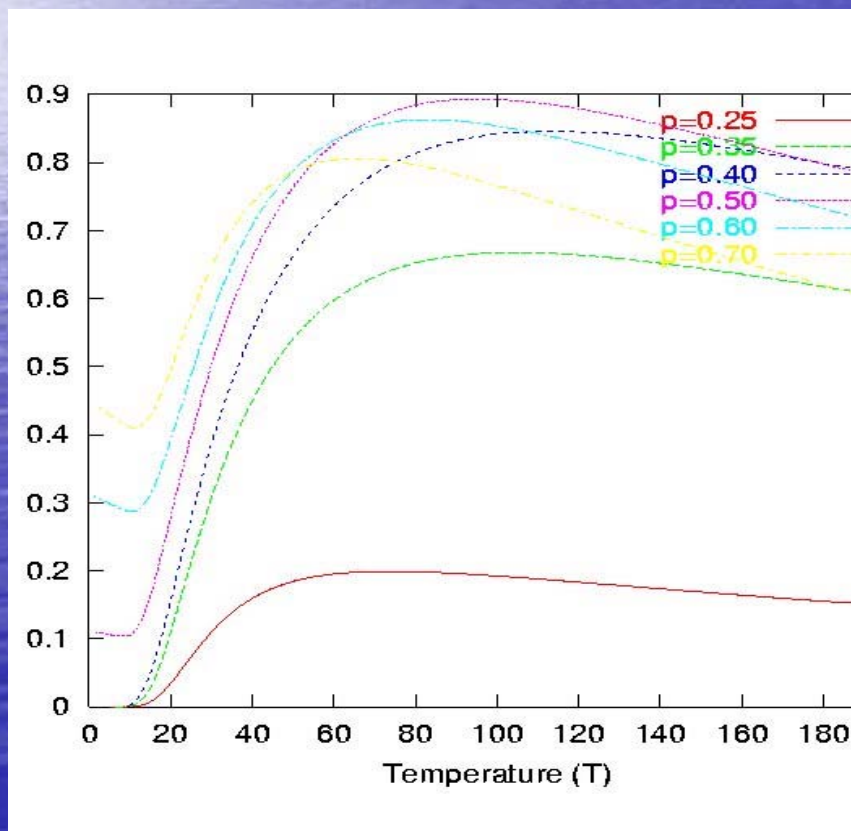


# Variable Range Hopping Conduction study through RRTN model

**A K Sen** and **S Bhattacharya**, in Proc. NATO Adv. Res. Workshop, CMDS10, NATO Sci. Ser. II. Math. Phys. Chem., 158, Eds. D Bergman, E Inan (Kluwer Acad. Publ., Dordrecht 2004), pp. 367-373.

Explicit temperature dependence in the microscopic bonds:

$$g_o = 1/(1+a_0T), \quad g_t = 10 \exp(-100/T)$$



## Ensemble averaging:

**One needs to handle an ensemble averaged quantity in statistical mechanics.**

✓ Ergodic hypothesis

✓ **Question is:** whether averaging over all sorts of disorder is meaningful?

✓ **Procedure:** Calculate (**normalised**) Fluctuation, :  $\kappa^2 = (\langle I^2 \rangle - \langle I \rangle^2) / \langle I \rangle^2$

✓ Investigate on **self-averaging** property: :

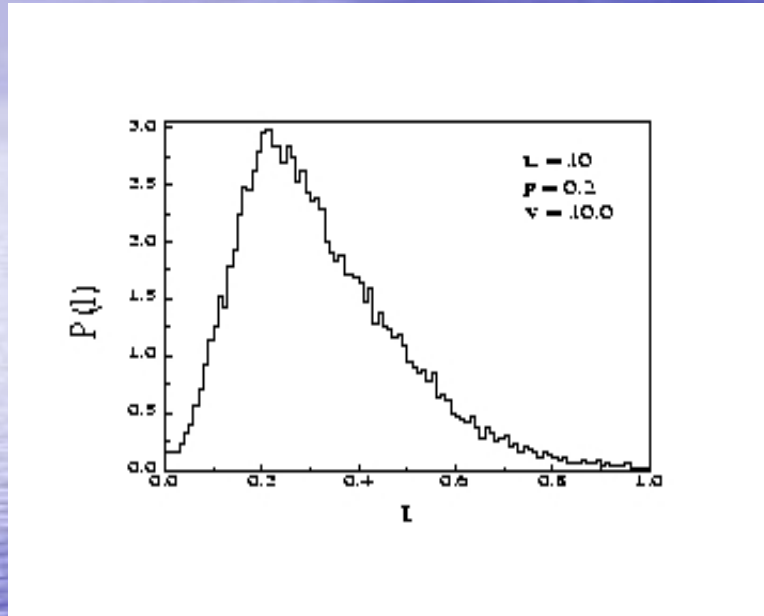
If,  $\kappa(L)$  is **almost constant** with  $L$ , then non-self-averaging

✓ **why electric field rather than voltage ?**



# Histogram $P(I)$ for $p=0.2, L=20$

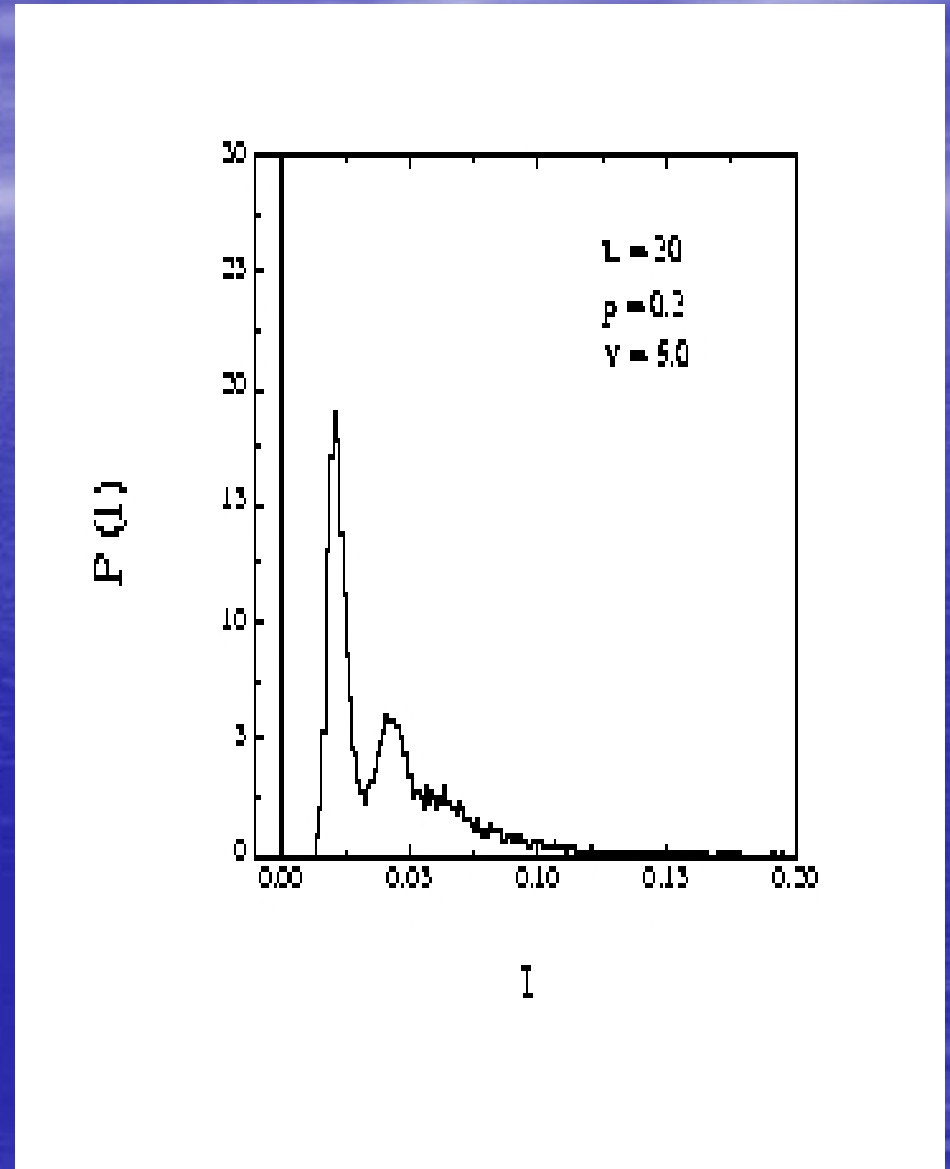
PRB, 57, p3375 (1998)



Exponential tail 

Power-law tail 

Non self-averaging depends on the nature of the tail in  $P(I)$

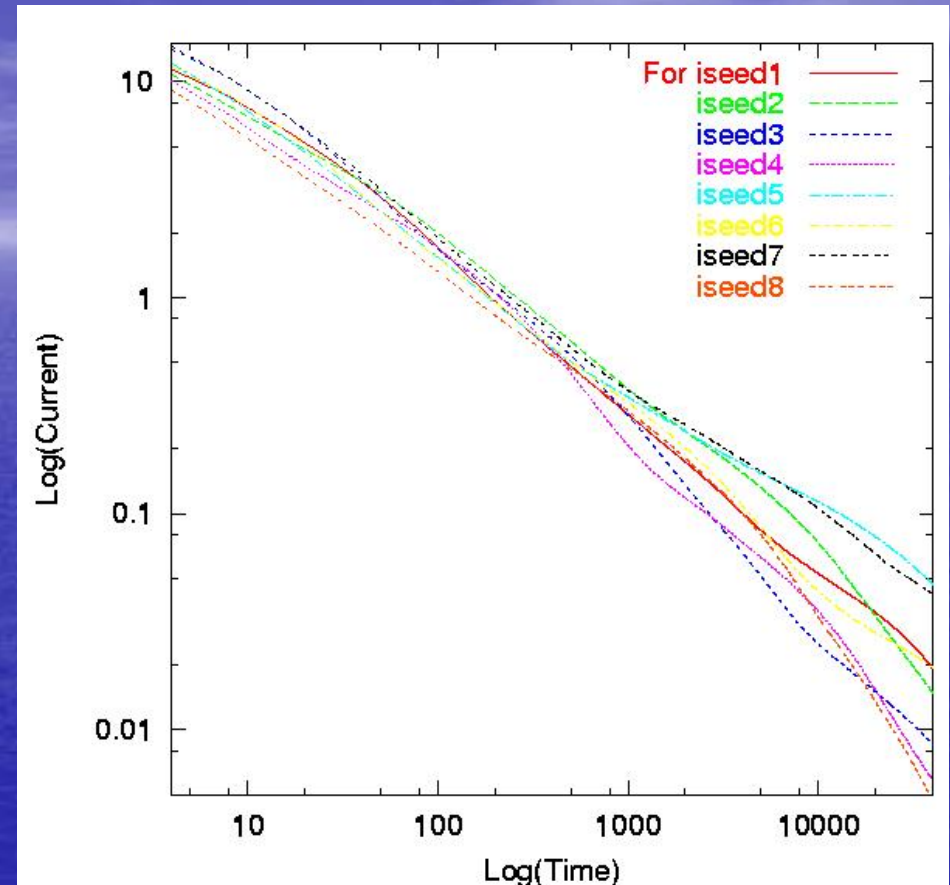
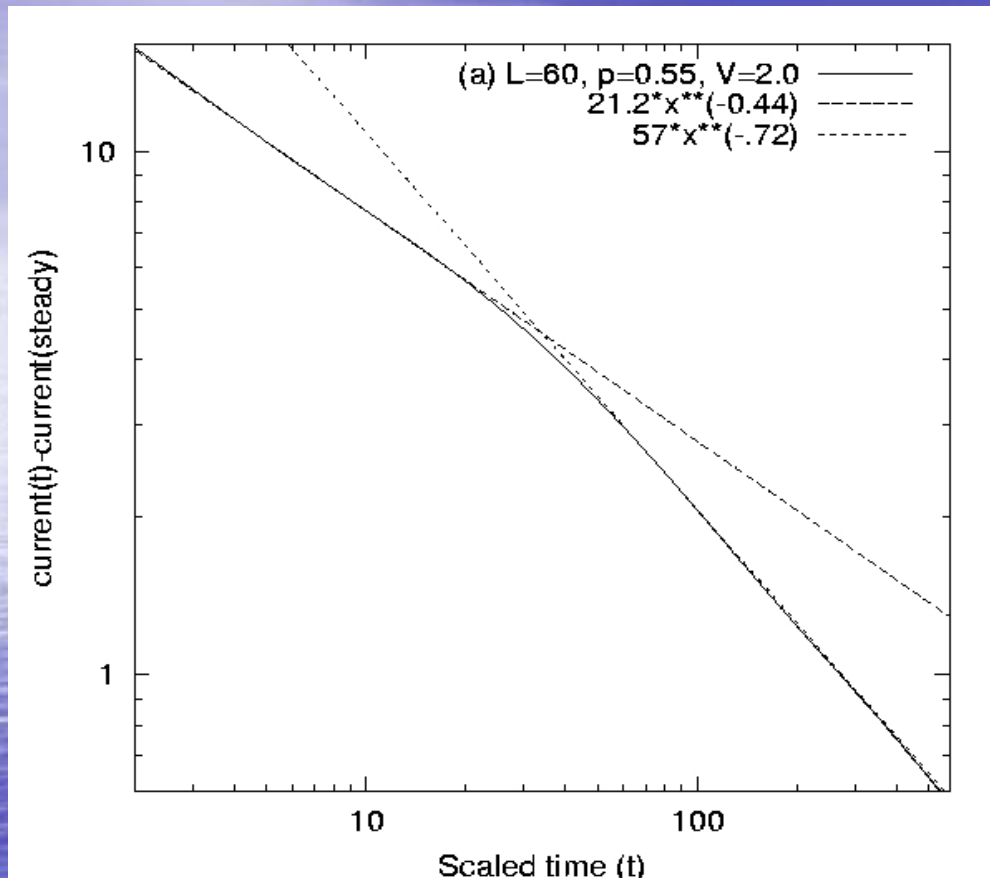


## Main qualitative results on averaging:

The steady state response is non-self-averaging

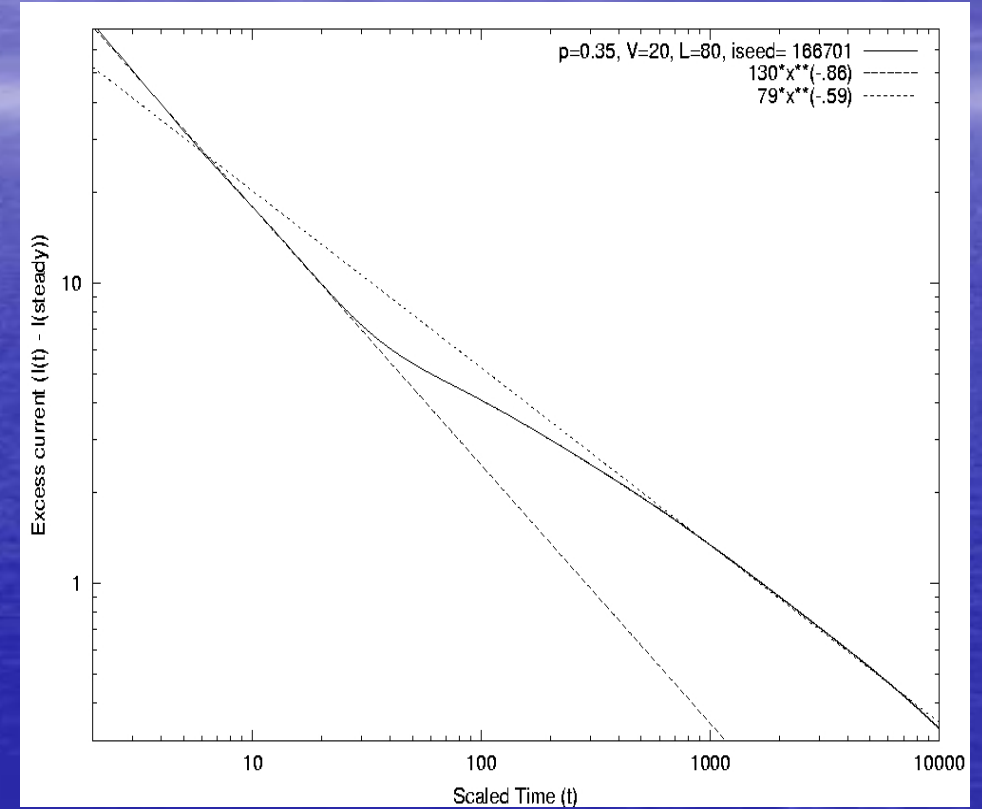
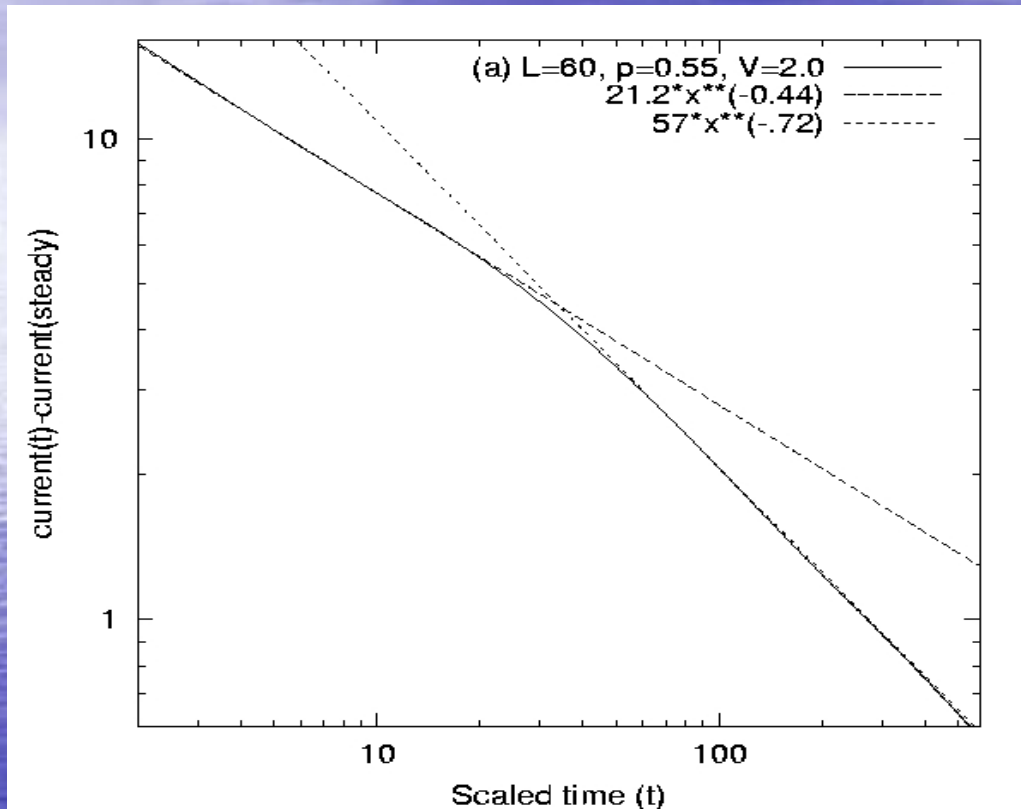
- ✓ at **low electric field** such that  $E \leq 0.25$   
and at  $p \cong p_{ct} (=0.18)$  ; we checked the result for  **$p=0.2$**
- ✓ Temperature can not change the situation, unless  $p$  and  $E$  values determine the phase

S. Bhattacharya and A. K. Sen, *Europhys. Lett.* 71, 797 (2005)



**Too much fluctuation** among the ensemble members even **during dynamics**

# Early-stage power-laws in RRTN current dynamics



$$\alpha_1 < \alpha_2$$

$$\alpha_1 > \alpha_2$$

S. Bhattacharya and A. K. Sen, **Europhys. Lett. 71, 797 (2005)**



**Thank you all !!!**

for your kind  
attention