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**SCHOOL and CONFERENCE
on
COMPLEX SYSTEMS
and
NONEXTENSIVE STATISTICAL MECHANICS**

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Non-additive Dynamics of Soft-condensed Complex Systems

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Non-additive Dynamics of Soft-condensed/ Complex Systems

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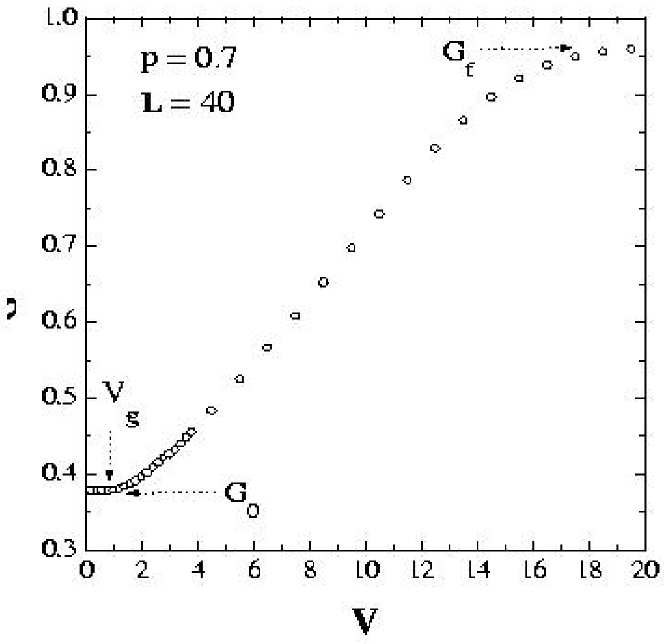
Recapitulation of static G-V response:



V< Vg: Lower Linear Regime (LLR)

V >Vg: Non Linear Regime (NLR)

V >> Vg: Upper Linear Regime (ULR)

Kar Gupta and Sen, PRB 57, 3375 (1998)

Debye relaxation:

- Temporal rate of change in response is proportional to the response itself,

$$\frac{dI}{dt} = -\frac{1}{\tau} I \Longrightarrow I = I_0 \exp\left(-\frac{t}{\tau}\right)$$

- Also called Boltzmann's relaxation time approximation
- Simplest dynamics; possesses a single time-scale τ (Boltzmann-Gibbs-Shannon Statistics)
- Obviously LLR of the RRTN (indeed a RRN) follows this dynamics

Power-law Relaxation/s:

• Belongs to a non-Debye class, as an outcome of an inherent property of the system having multiple/infinite time-constants τ

• An example: a relaxation function $\phi(t)$ with multiple τ's, weighted by a self-similar probability density function (p.d.f.), $w(\kappa t) = t^{-\alpha} w(\kappa)$, where $\kappa = 1/\tau$

$$\phi(t) = \int \exp(-t/\tau) w(\tau) d\tau$$

• With a proper choice of the p.d.f., one finds two power-law relaxations; one at a short-time scale and another at an asymptotically long-time scale

Ref.: Weron and Jurlewicz, J Phys A, **26**, 395 (1993)

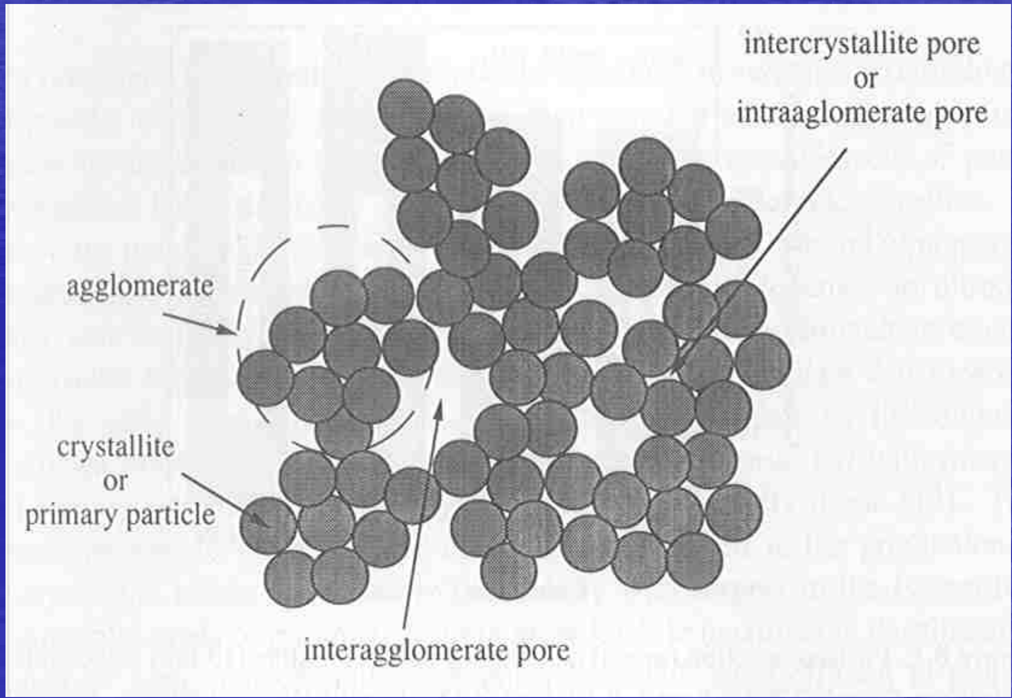
Two initial Power-law Relaxations:

Appear, in general, due to a sequence of

- **Re-distribution of local clusters/fields in time**

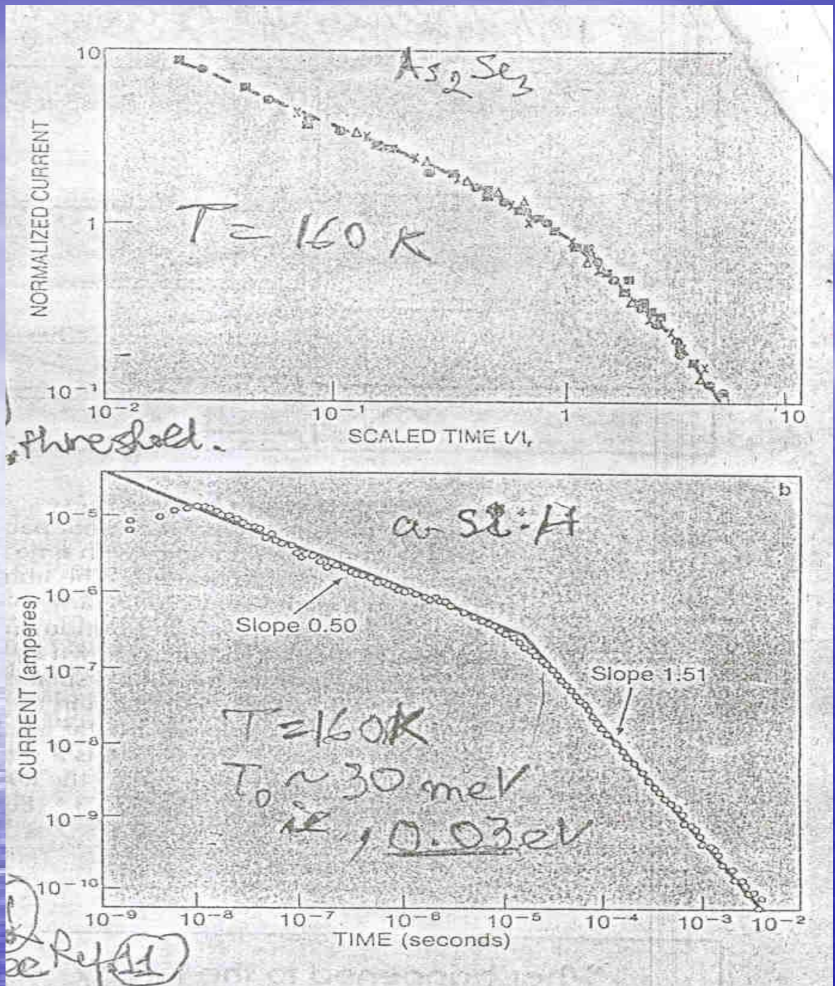
crossing over to a

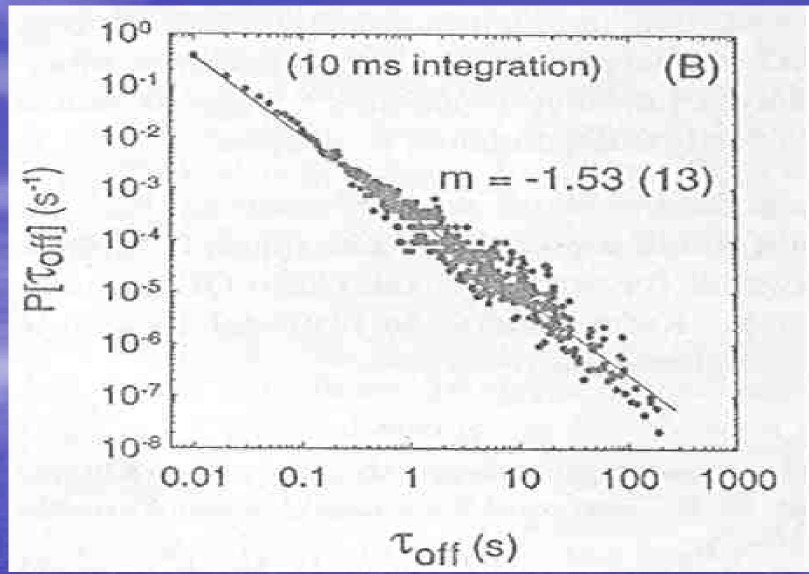
- **re-distribution of global clusters/fields in time**



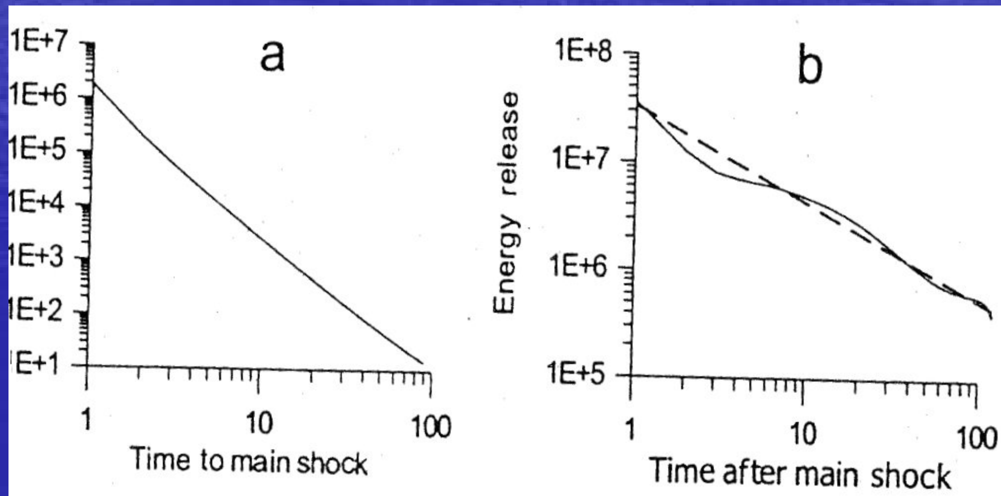
- Eventually,

$\phi(t)$ for $t \rightarrow \infty$,
crosses over to an
exponential dynamics,
towards a steady-state





Blinking kinetics in CdSe Quantum Dots

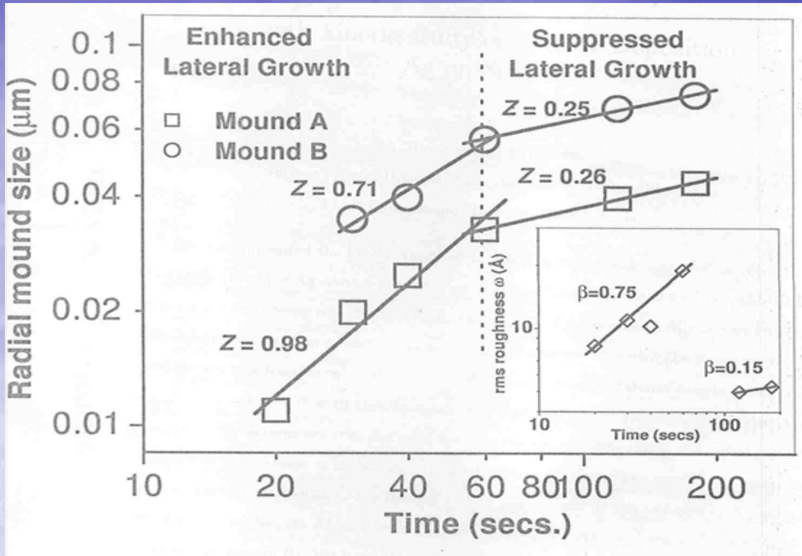


Photocurrent in Amorphous Si:H and

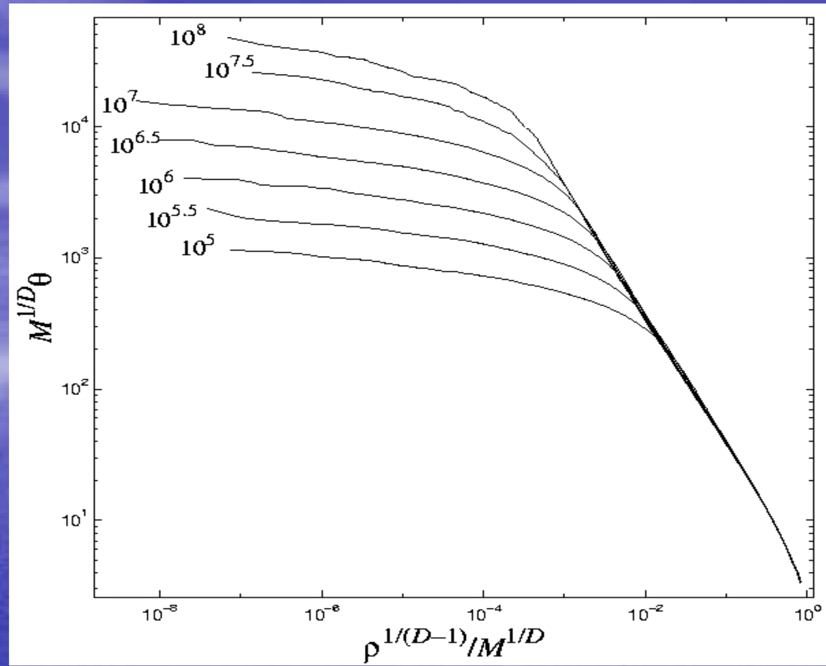
As_2Se_3

Re-association of ligands of Fe in folded heme-Proteins; Parak et al, Physica A**201**, 332 (1993)

Earthquake (a) fore- and (b) after-shocks; Case (b) is called Omori law

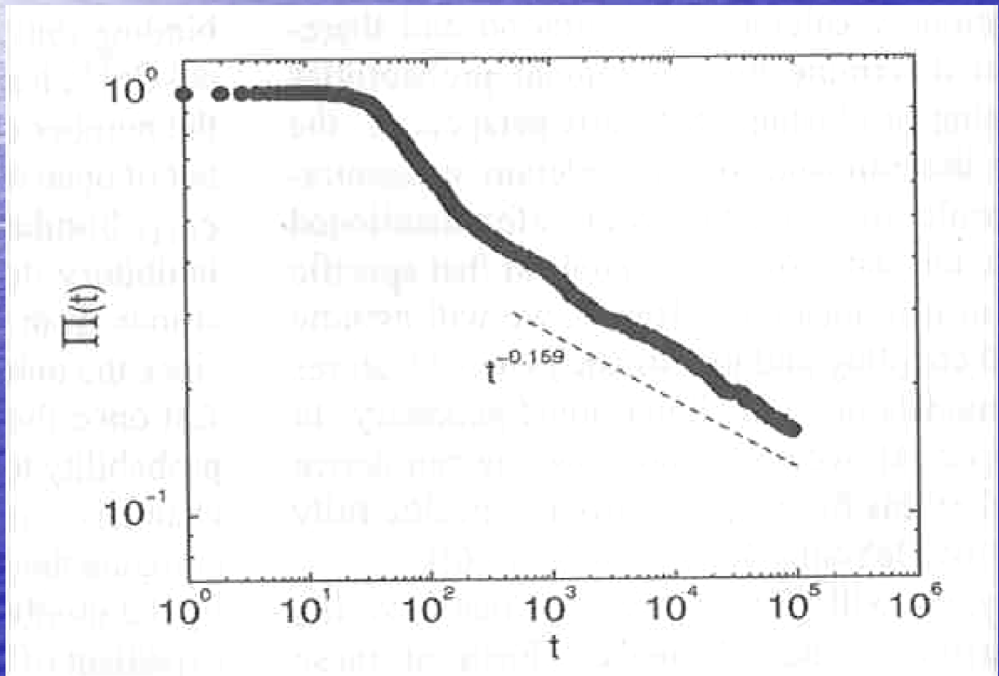


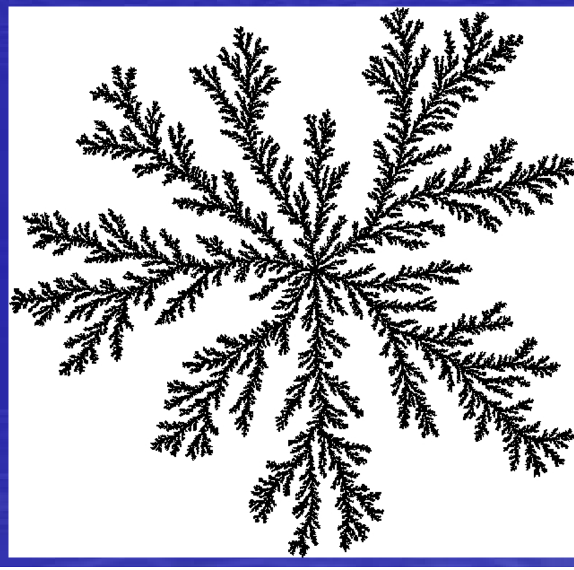
Sputtering of Ag particles on Si(001) surface



Comp-expt On radial DLA growth (N=10^5 - 10^8)

x-axis is time-like; a DLA is shown below.





Ca^{+2} channel dynamics in Living Cells

Relaxation in the RRTN model; some typical parameters:

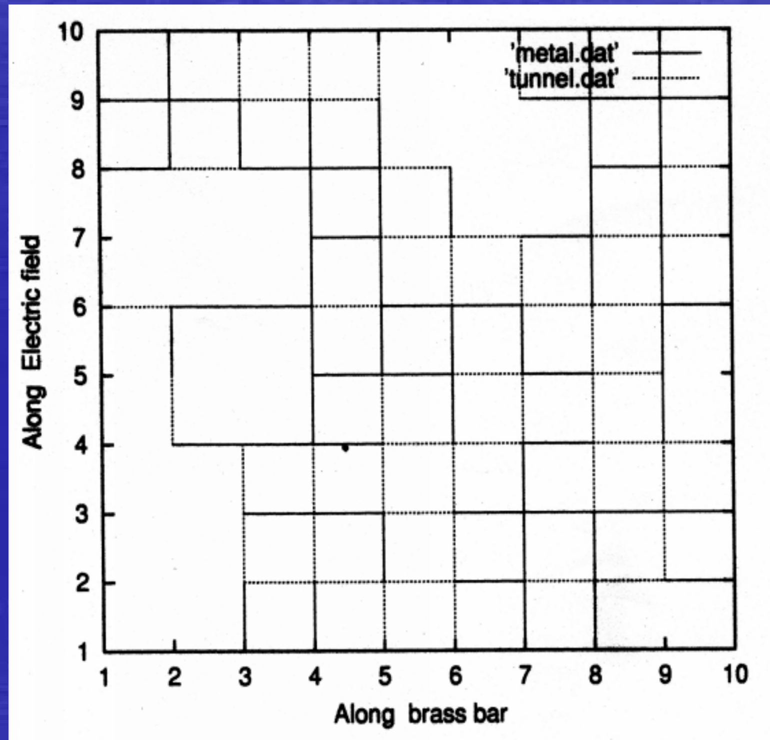
g_o= 1.0, g_t=0.01, c=10^{-5}, displacement current : i_{dis} = c dv/dt for t-bonds with $v < v_g$

- Use a graded random initial voltage configuration v_{ij} at each node

- Update the microscopic voltages at each node using the Continuity Eqn. $\Sigma_{<ij>} I_{ij} = 0$ locally; i.e., a *lattice Kirchhoff's dynamics*:

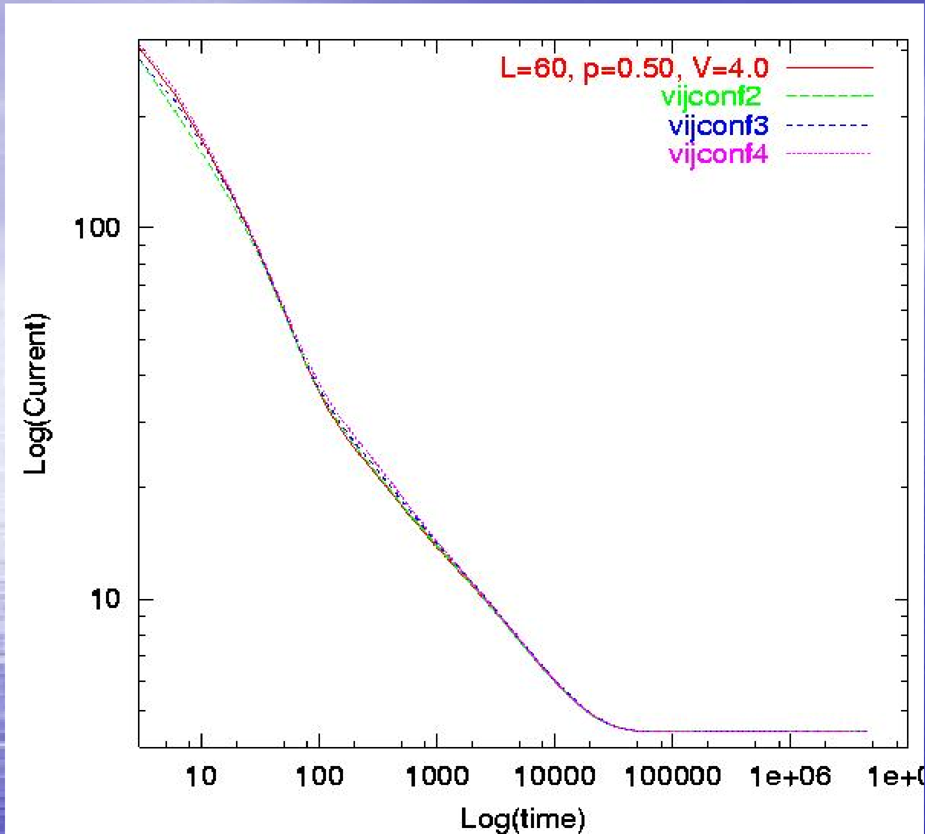
$$v_{ij}(t+1) ☞ v_{ij}(t) + \Sigma_{<ij>} I_{ij} / \Sigma_{<ij>} g_{ij}$$

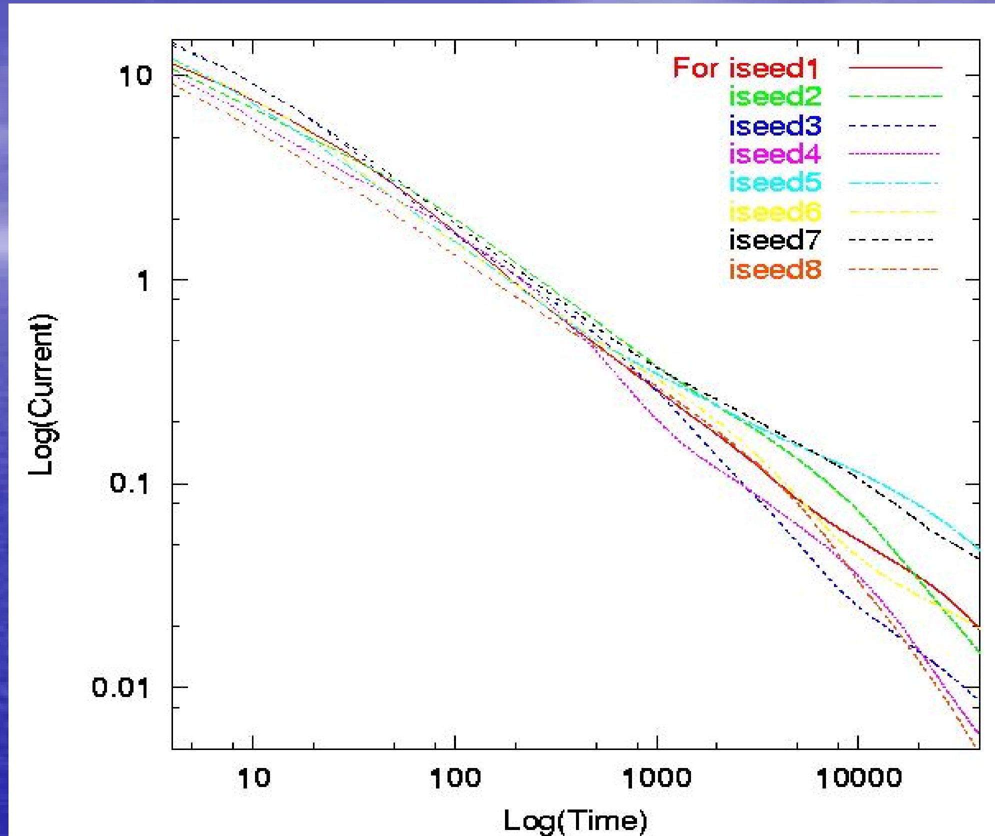
- Check the global continuity to ascertain the final steady state, i.e., stop iteration when $| I(1^{st} \text{ layer}) - I(N^{th} \text{ layer}) | \leq \varepsilon$, a pre-assigned small +ve number (for controlling precision)



Bhattacharya and Sen, Europhys. Lett. **71**, 797 (2005)

Two early-stage power-law dynamics for a p=0.50 RRTN sample



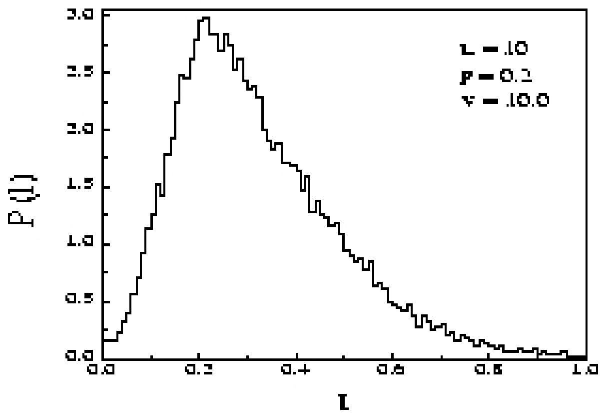


A typical two power-law dynamics ($t^{-\alpha_1}$ and $t^{-\alpha_2}$) and final exponential dynamics to an unique steady current (strong memory). The α_1 and α_2 are app. robust; for some separate classes of $v_{ij}(t=0)$; thus on the edge of chaos

The steady current is subtracted out to treat all cases under the same footing; also the final exponential dynamics is not shown further. For different samples (iseeds), α_1 and α_2 vary widely; non-self averaging

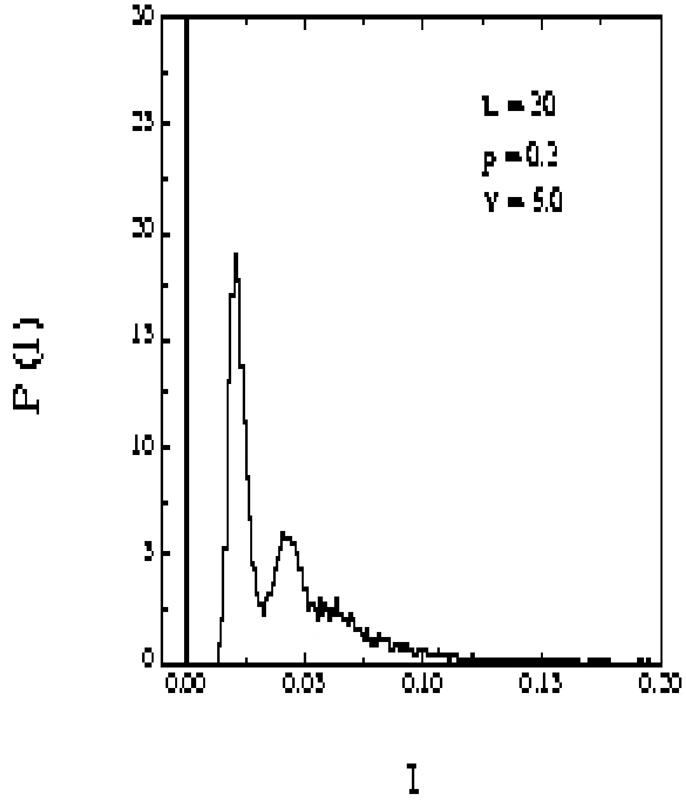
Histogram of P(I) for p=0.2, L=20; Kar Gupta and Sen, PRB**57**, 3375(1998)

Origin of non self-averaging property of the dc response in the RRTN



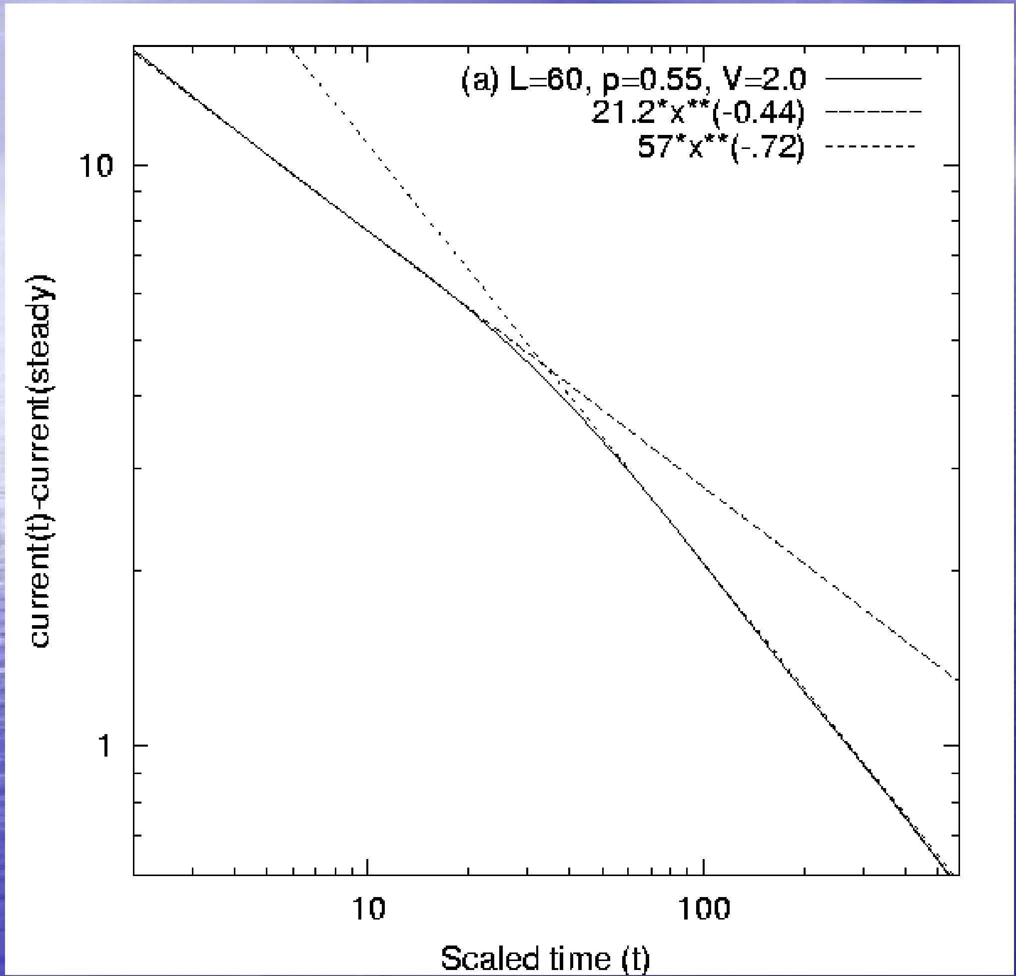
Self-averaging w/ Exponential tail

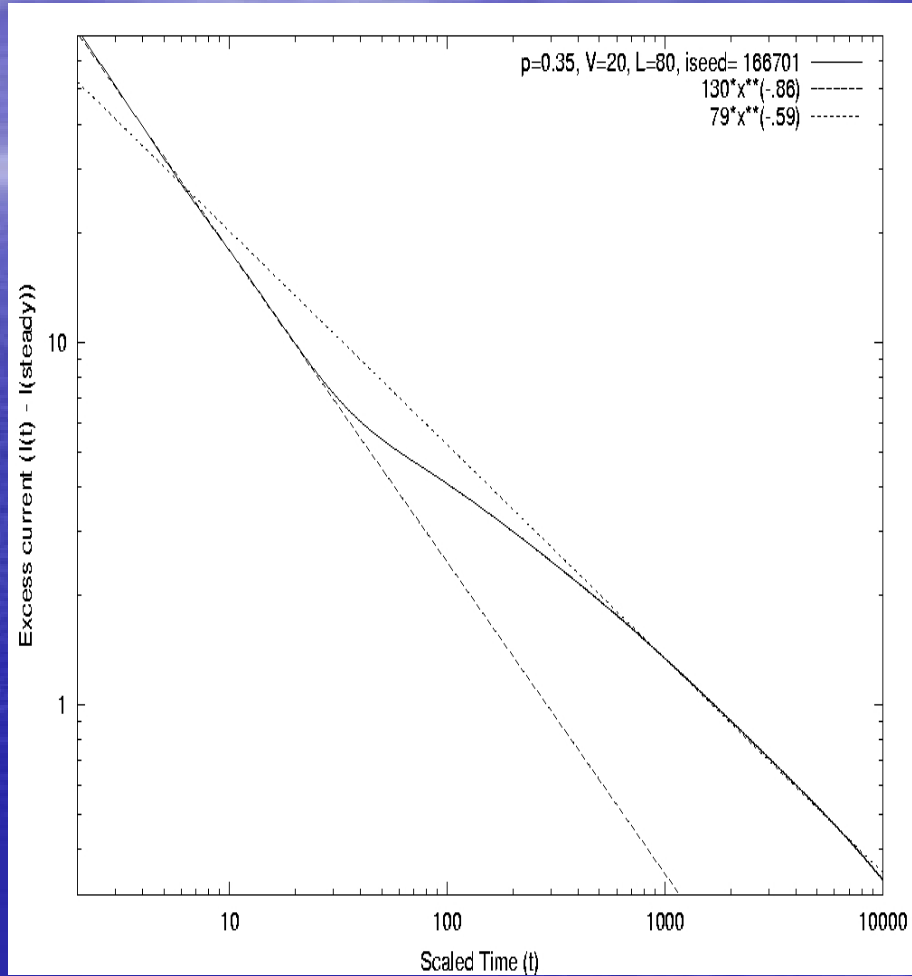
Power-law tail



Non self-averaging depends on the nature of the tail in P(I)

Two early-stage power-laws in RRTN current dynamics





$\alpha_1 < \alpha_2$ (appear in most expts.)

$\alpha_1 > \alpha_2$ (somewhat rare in expts.)

S. Bhattacharya and A.K. Sen, Europhys. Lett. **71**, 797 (2005)

Thus, the out-of-equilibrium dynamics of various systems of nature as well as the same in the RRTN model, suggests that:

- ✓The 1st. order D.E. for the relaxation is strongly non-Debye type, and
- ✓In particular, it should have the empirical form,

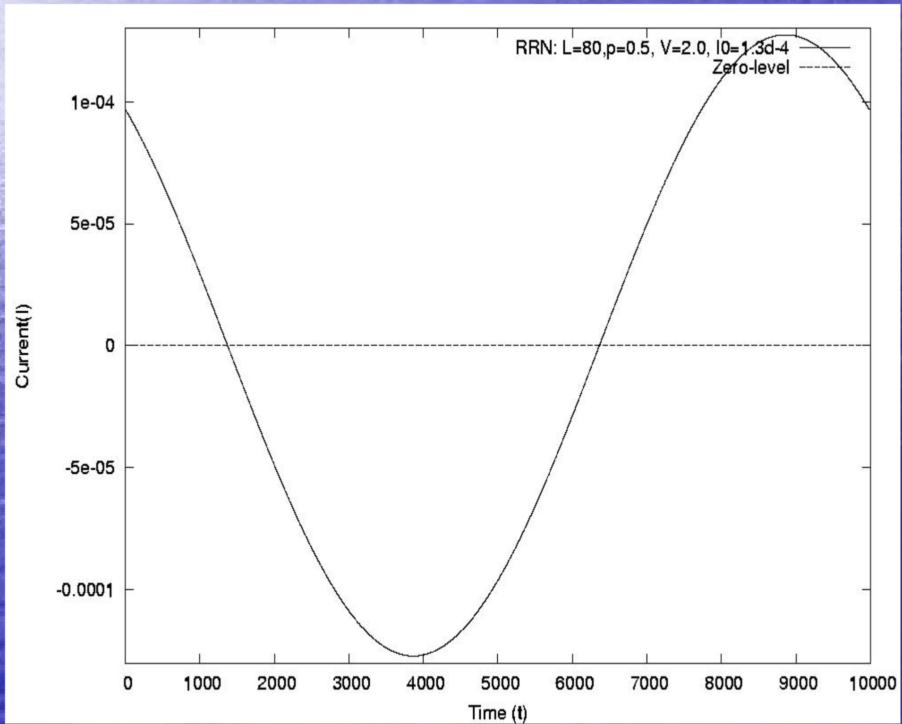
$$dI/dt = -\lambda_q I^q - \lambda_r I^r - (1/\tau) I \; ; \quad \text{with } q, r > 1$$

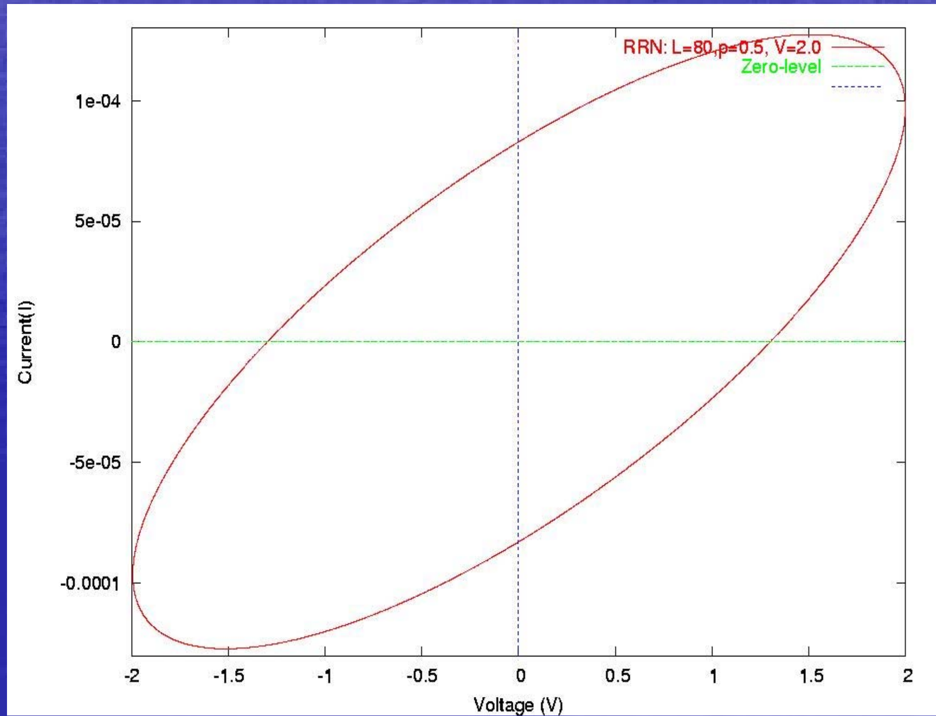
[following Tsallis, Bemski and Mendes; Phys Lett A**257**, 93 (1999), and adding a q=1, τ >> 1 (Boltzmann-Gibbs-Shannon) term explicitly].

- ✓As expected it gives rise to two early power-law relaxations with the following exponents $\alpha_1 = 1/(q-1)$ and $\alpha_2 = 1/(r-1)$ for $\tau >> 1$
- ✓Eventually for t >> τ, RRTN's dc-response is in the Upper Linear Regime (ULR), and there is expnl. relxn. (consistent with B-G-S)
- ✓Very strong memory of the steady state (ULR regime); due to perfect correlation in placing the t-bonds; to be studied by ac field next

Strong Memory and statistically perfect correlation; ac-voltage Hysteresis in a sinusoidal voltage-driven RRTN (LLR regime):

- Driving voltage $V(t) = V_0 \cos(\omega t)$ s.t. $V_0 < V_g$ on an RRTN with $p>p_c$, the stable current response $I(t)=I_1 \cos(\omega t + \phi)$ lags behind by an angle ϕ



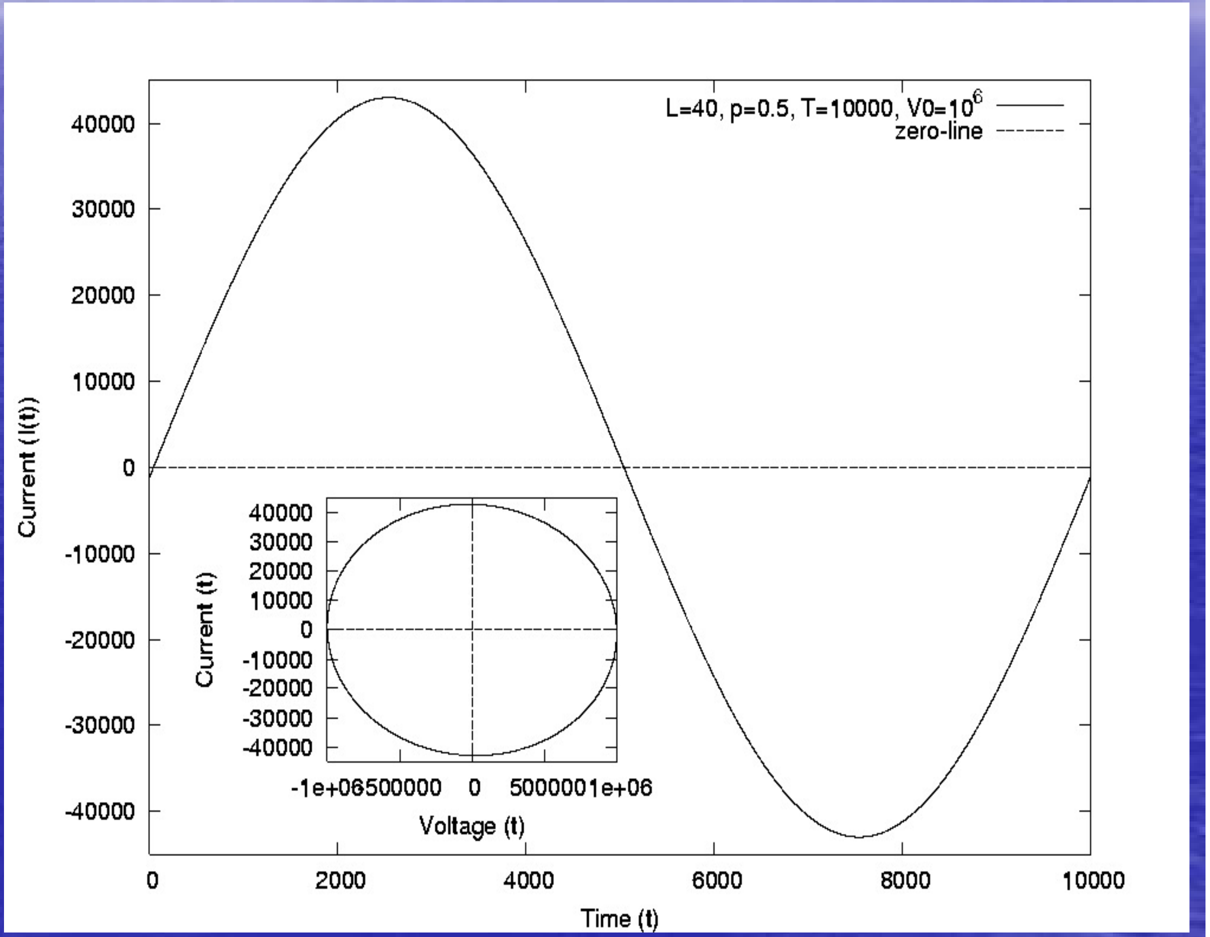


Linear response, same ω and a lag

Hysteresis loop; Lissajous figure

Memory and statistical correlation; Hysteresis in a sinusoidal voltage-driven RRTN (at ULR):

- **$p>p_{ct}$, $V_0>>V_g$, all t-bonds are active (ULR of maximal RRTN)**
- **Again, linear response; with a phase-lag 90^0**
- **Hysteresis/ Lissajous loop, simple ellipse; Area, function of driving time-period (T)**

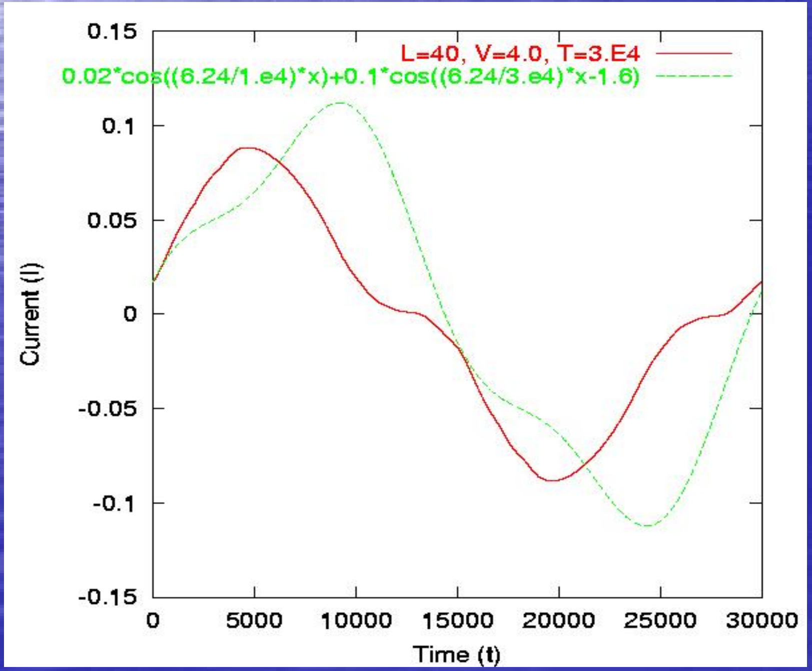


Unconventional Hysteresis loops in the sigmoidal, Non-Linear Regime (NLR) of the RRTN's dc-response:

For $V_0 > V_g$,

- Nonlinearity manifests through generation of higher harmonics of ω (n odd for resistive o-bonds and even for capacitive t-bonds (for $v < v_g$):

$I(t) = I_1 \cos(\omega t - \phi_1) + I_2 \cos(2\omega t - \phi_2) + I_3 \cos(3\omega t - \phi_3) + \ldots\ldots$



Fourier co-efficients:

$I_1 = 7.15.10^{-2}$, $\phi_1 = 1.05$

$I_2 = 4.3\ 10^{-6}$, $\phi_2 = 0.65$

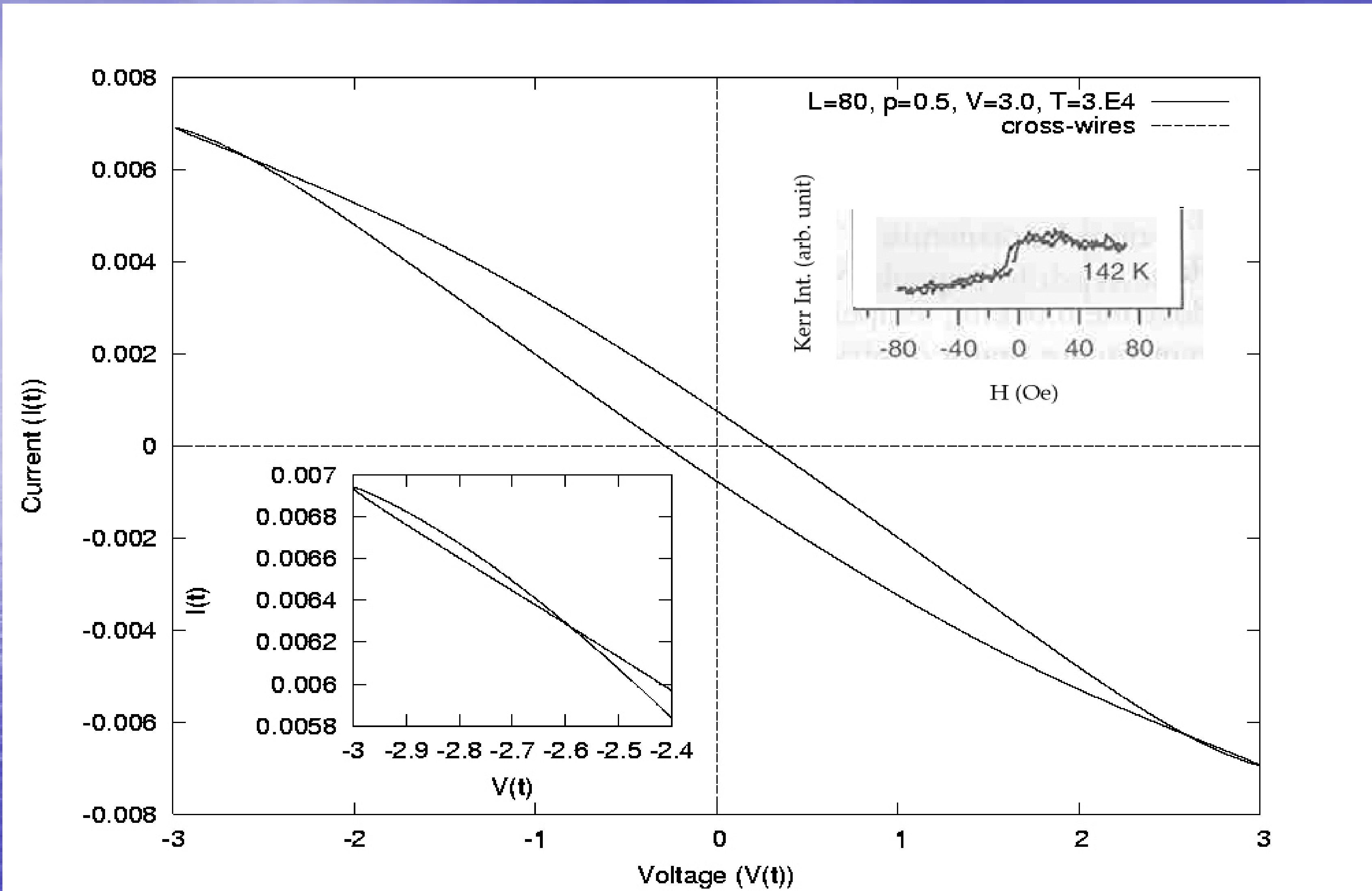
$I_3 = 1.5.\ 10^{-2}$, $\phi_3 = -0.07$

$I_4 = 9.4.\ 10^{-7}$, $\phi_4 = -1.34$

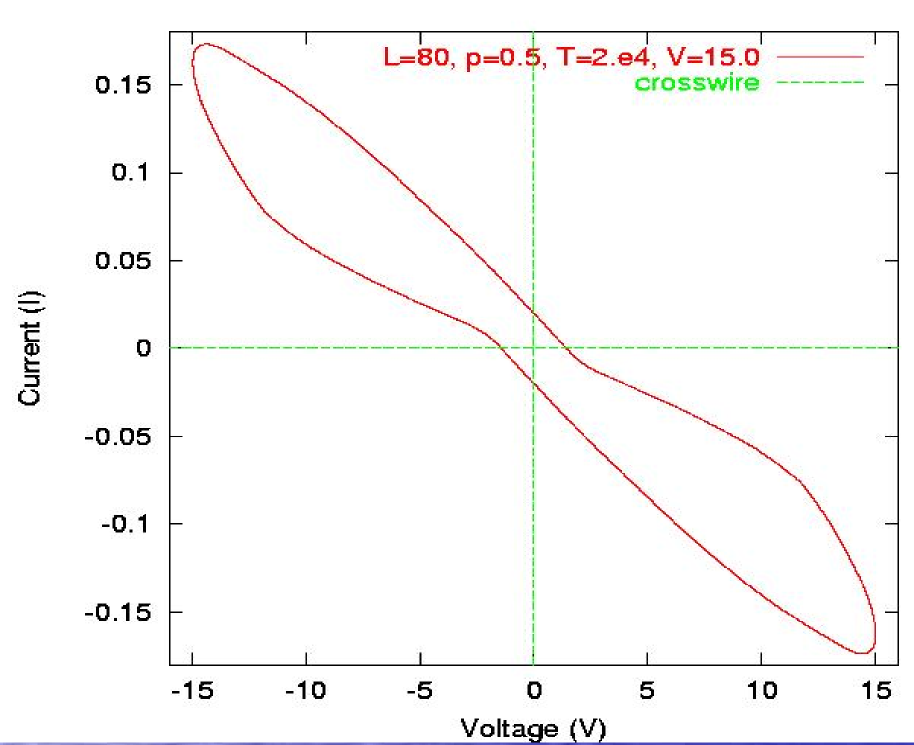
$I_5 = -2.8\ 10^{-3}$, $\phi_5 = -0.9$ .....

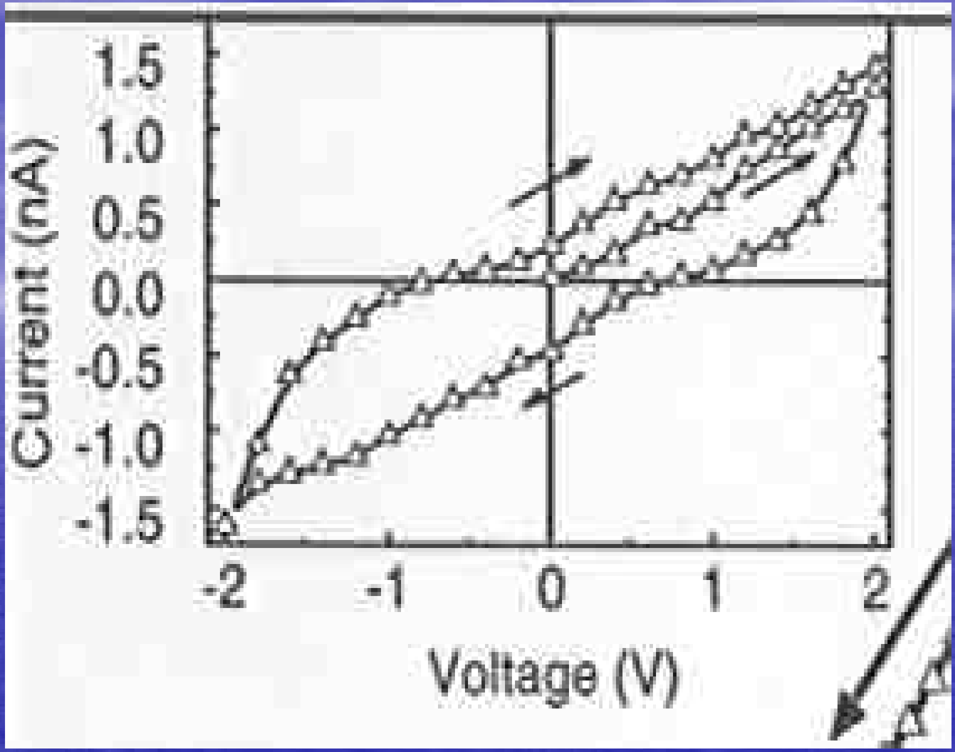
Symmetries in a stable hysteretic response:

- (a) The stable (non-changing) loop-shapes/areas imply neither an equilibrium nor a steady state. For a resistive symmetry, $I(-V) = -I(V)$ leads to $I(t+T/2) = -I(t)$; this allows only odd-n harmonics
- (b) The semi-quantum tunneling/capacitive symmetry of t-bonds ($v<v_g$) *has no loss*; thus zero-crossing with $I(t)=-I(t+T/4)$ and even-n harm.
- For $v>v_g$, the t-bonds become conducting and falls in the class (a) above.
- No percolation with only capacitive t-bonds; also $i_{dis}<<i_{ohm}$; so at low p and $(V-V_g)$, even-harmonics may appear as a small pinching effect near the origin



Odd n, tiny edge loops in the RRTN. Exptl. loop (top R inset) shows Magnetic hyst. in Co nanodot arrays on Cu(001) surface at 142 K; Komori et al, J Phys cond mat 14, 8177





Even harmonic pinching effect in the RRTN

Memory applications and even harmonic effect in a thiophene-based conjugated polymer, sandwiched between InSn-oxide layers; Mazumder et al, J Phys D **36**, 211 (2003)

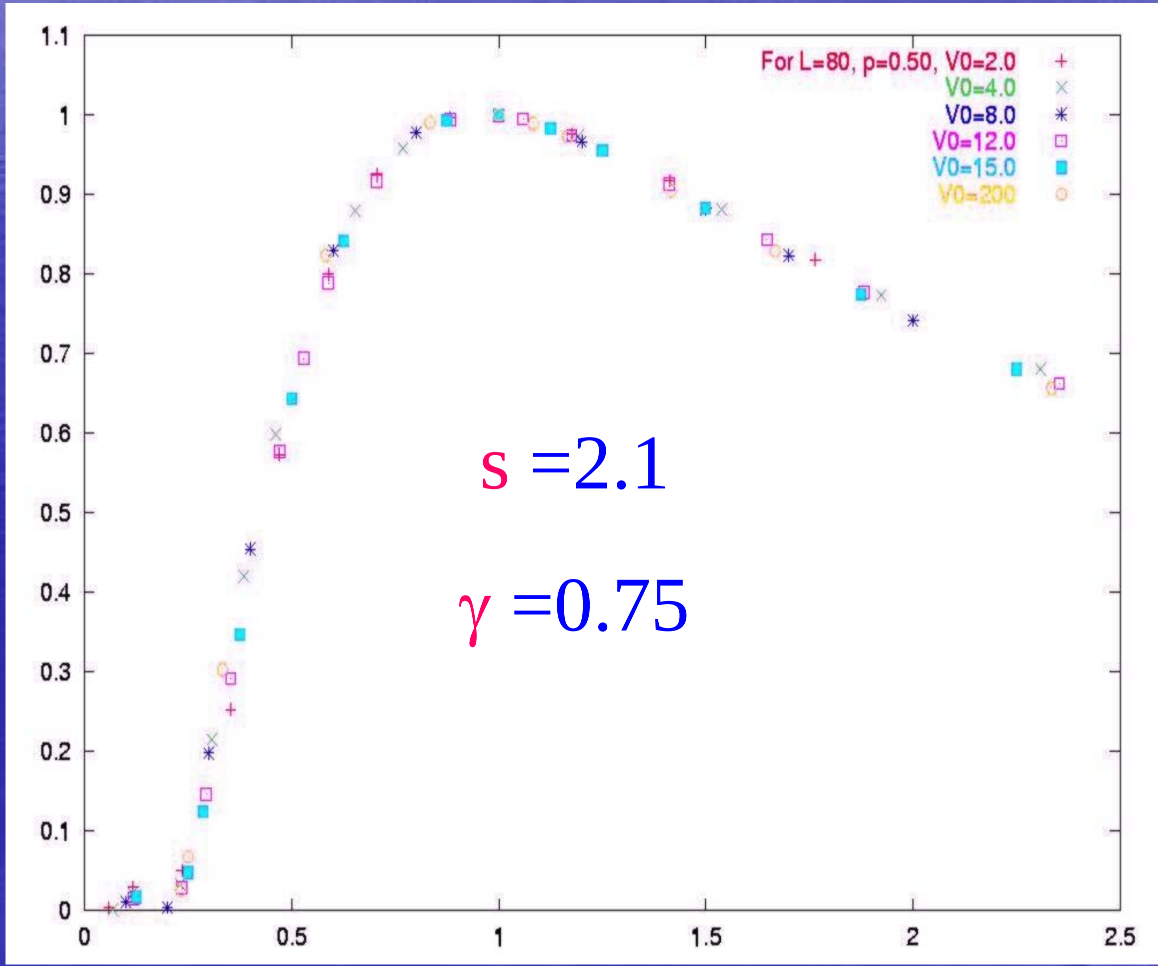
Hysteresis loop-area A *vs* ω (or, T=2π/ω) is found empirically to follow the scaling function:

$$A(\omega)=A_0(\omega/\omega_0)^s \exp[-(\omega_0/\omega)^\gamma],$$

where A_0, ω_0, s and γ depend on V_0

Scaling in A/A_m *vs* T/T_m

The excellent data collapse bears tell-tale testimony of its accuracy.



Conclusion for dynamical study:

- Dynamics in the RRTN in the presence of disorder and nonlinearity/interaction due to its basic and simple ideas on the bond arrangements and microscopic voltage threshold v_g in tunneling bonds, shows intriguing non-Debye relaxation.
- This opens a scope to apply non-extensive thermo-statistics for the explanation of power-law relaxations. Whereas, the steady state/current remains unique, independent of initial voltage configurations, the exponents do not (on the edge of chaos!!).
- Existence of initial power-laws in time-dynamics, far from the critical points, assures that their origin is not related to any self-organized criticality, but due to the nonlinearity and the perfect statistical correlation, in the placement of the t-bonds, in-built in the model.
- Two early power-law dynamics seem to be due to the correlated random fluctuations in the microscopic voltage distribution in different iterations (or, time evolution); and the bulk system still picks up a time-scale (τ) while going through these correlated pathways.
- No chaoticity in reaching the final steady state from any initial voltage configuration leads to a robust pattern-recognition property (i.e., a biased statistics). This property is comparable to the memory of the natured state in a protein-folding problem, and the Leventhal's paradox.

THANK YOU