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SCHOOL and CONFERENCE
on
COMPLEX SYSTEMS
and
NONEXTENSIVE STATISTICAL MECHANICS

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The Olami-Feder-Christensen Model on a Small World topology

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The Olami-Feder-Christensen Model on a *Small World topology*

F. Caruso, A. Pluchino, V. Latora, A. Rapisarda and B. Tadić,
EPJB 50 (2006) 243

F. Caruso, A. Pluchino, V. Latora, S. Vinciguerra A. Rapisarda,
cond-mat/0606118

Outline

- Self-organized Criticality (SOC)
- Earthquake fenomenology
  - Gutenberg-Richter Law
- SOC Model:
  - Olami-Feder-Crinstensen Model (OFC)
- Power Law and Finite-Size Scaling
  - OFC on a small world topology
- Turbulence-like analysis in OFC models

Self-organized criticality

- Self-organized criticality (SOC) can describe emergent complex behavior in physical systems
- SOC is a *out-of-equilibrium mechanism* that drives a system towards a critical state

Self-organized criticality (SOC) …

- is *manifested* by temporal and spatial scale invariance (power laws)
- is *driven* by intermittent evolutions with bursts/avalanches that extend over a wide range of magnitudes

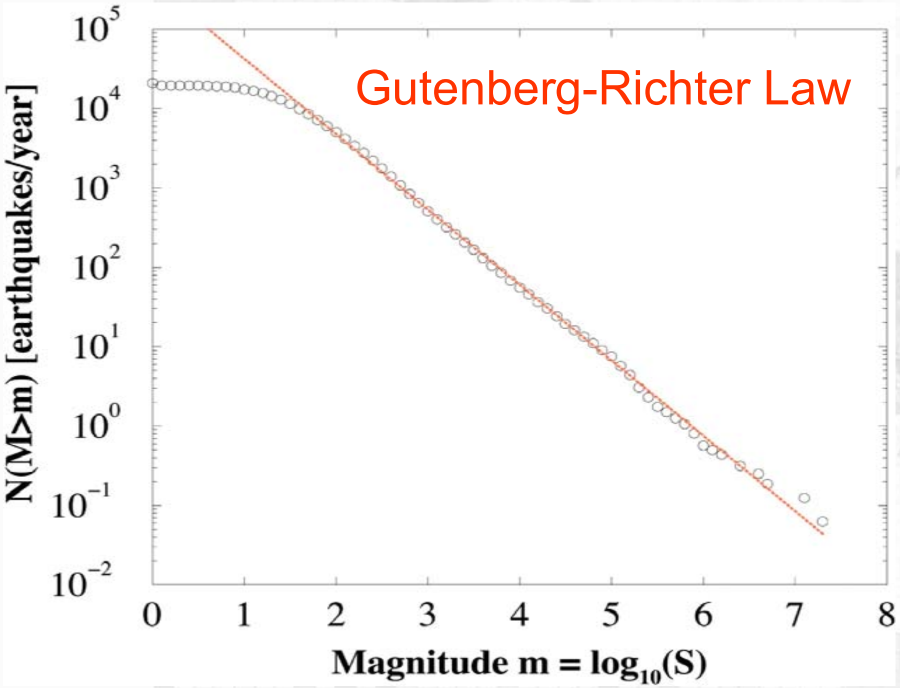
Some examples:

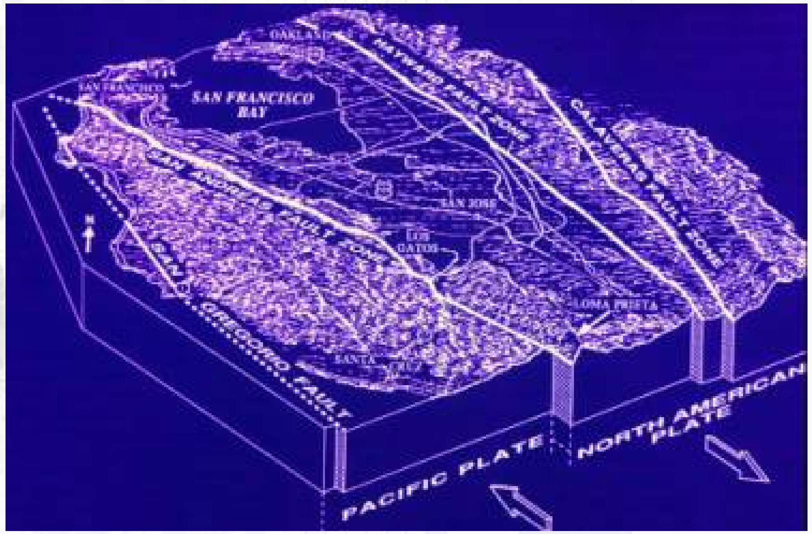
- Earthquakes dynamics
- Sand-pile models (Bak et al. 1987)
- Evolution (Bak and Sneppen 1993)
- Solar flares
- …etc.

Eartquake and Gutenberg-Richter Law

- It is generally believed that earthquakes result from a stick-slip dynamics involving the Earth's crust sliding along faults.
- San-Andreas Fault marks the contact between the Pacific and North American Plates

San-Andreas fault, where the great 1906 San Francisco earthquake occurred



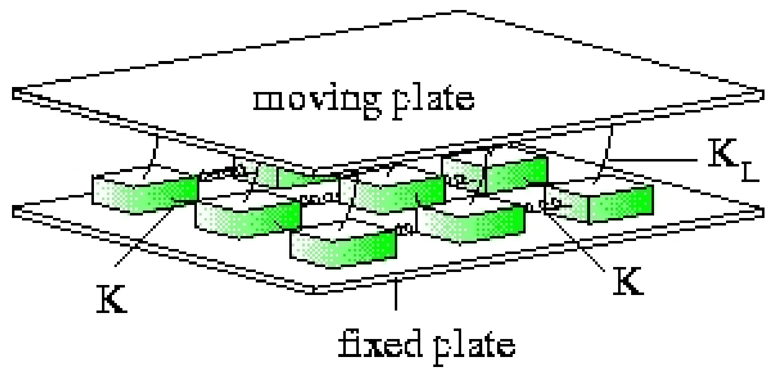


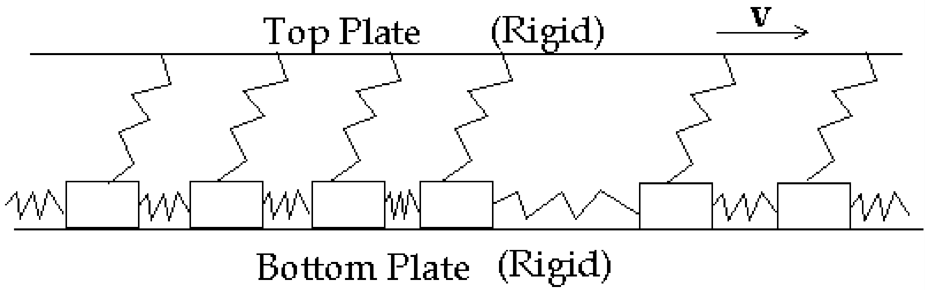
There are more small earthquakes than large ones. But there is no apparent cut-off in the possible size of an earthquake; earthquakes of all sizes are possible.

Spring-Block Model of Earthquakes

A simple computer model of the earthquake fault is composed of one elastic plate and one rigid plate, for simplicity's sake.

Burridge-Knopoff Model (1967)





Olami-Feder-Christensen (OFC) Model of earthquakes

Z. Olami, H. J. S. Feder, and K. Christensen, Phys. Rev. Lett. 68, 1244 (1992)

The blocks interact with the rigid plate through friction.

- The system is driven through slow uniform movement …
- Whenever the spring force exceeds a critical value, the block slides and the spring force is reduced.
- The force lost by the block is transferred to its neighbours; this may causes one or more of its neighbours to slide, and so on …. an earthquake is generated.
- The size of the earthquake in the model is defined as the number of blocks that have slided during the earthquake.

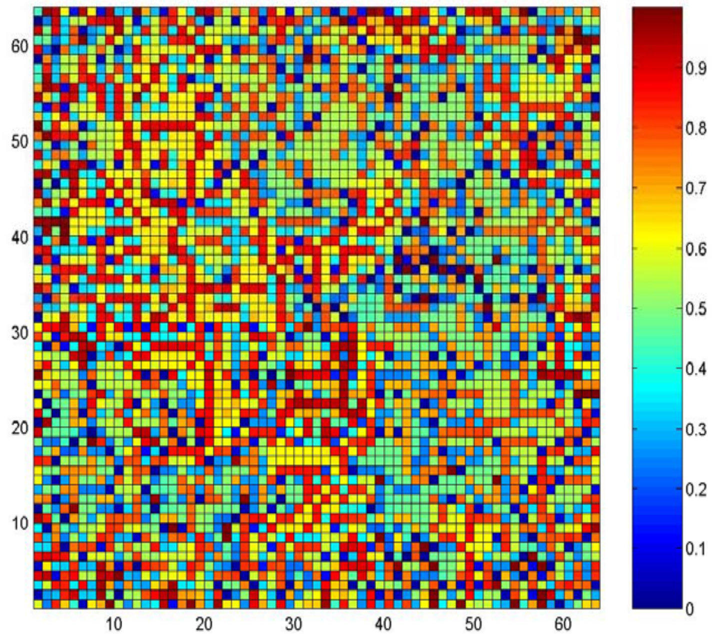
Olami-Feder-Christensen Model

- **System**: discrete system of blocks on a square lattice, each carrying a force, a real variable, F_i in the range (**0, F_{th}**).
- **Slow Driving**: all the forces are increased uniformly until one of them reaches the threshold value $\mathbf{F_{th}}$ and becomes unstable (**Fi > Fth**).
- **Earthquake:** the uniform driving is then stopped and an "earthquake" (or avalanche) starts:

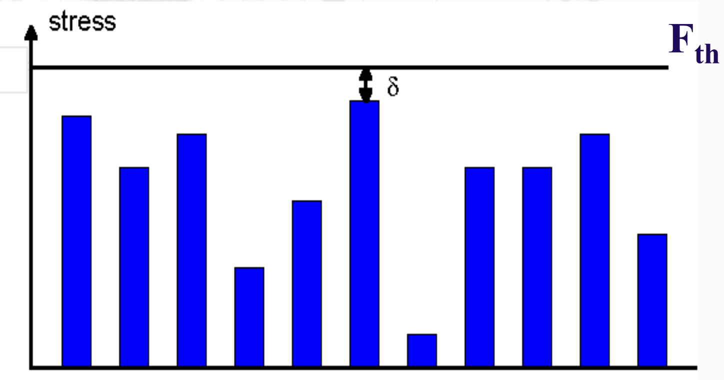
$$F_i \geq F_{th} \Rightarrow \begin{cases} F_i \rightarrow 0 \\ F_{nn} \rightarrow F_{nn} + \alpha F_i \end{cases}$$

where "**nn**" denotes the set of nearest-neighbor sites of **i**.

The earthquake is over when there are no more unstable sites in the system ($\mathbf{F_i < F_{th}}$). The number of topplings during an earthquake defines its size, **s.**

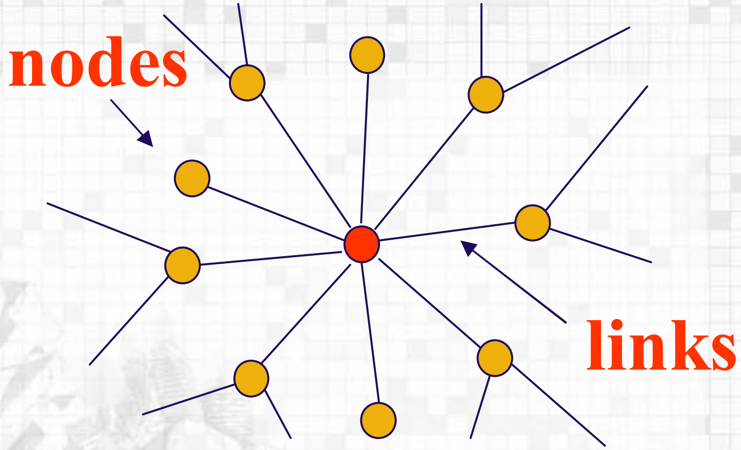


- **Parameters**: **α** controls the level of conservation of the dynamics and, in the case of a graph with fixed connectivity **q**, it takes values between 0 and 1/q (=1/q corresponding to the conservative case).
- **Data**: to collect the earthquake statistics, we need to skip some initial number of earthquakes (transient behaviour → critical state).
- Simulations on a lattice of L=32, 64, 128 (NN OFC model) after 1E+09 avalanches with OPEN boundary conditions.

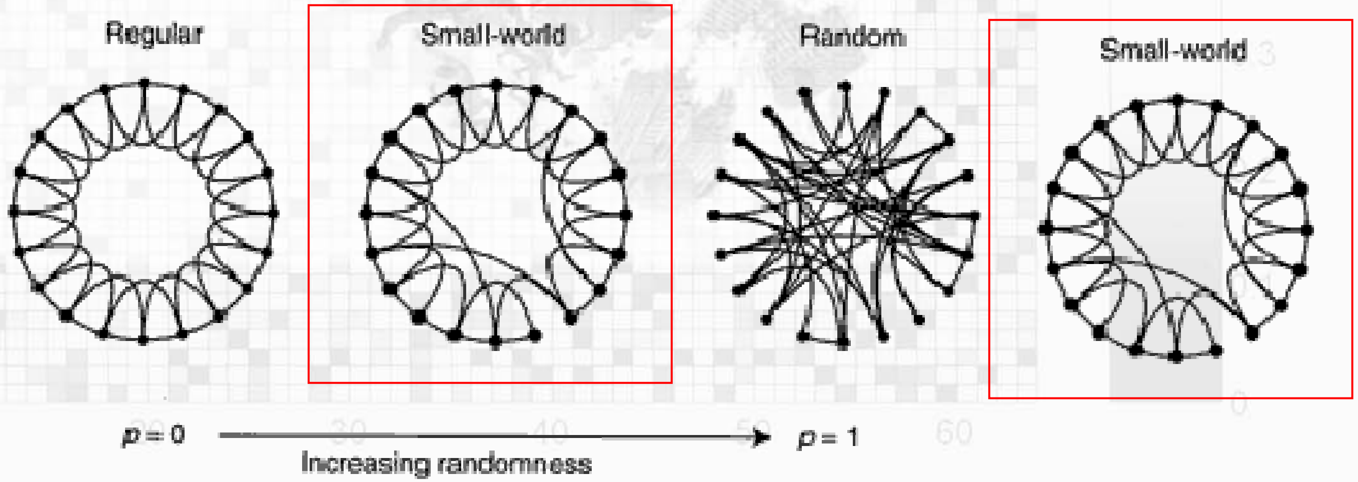


Random Graph, Lattice and Small World

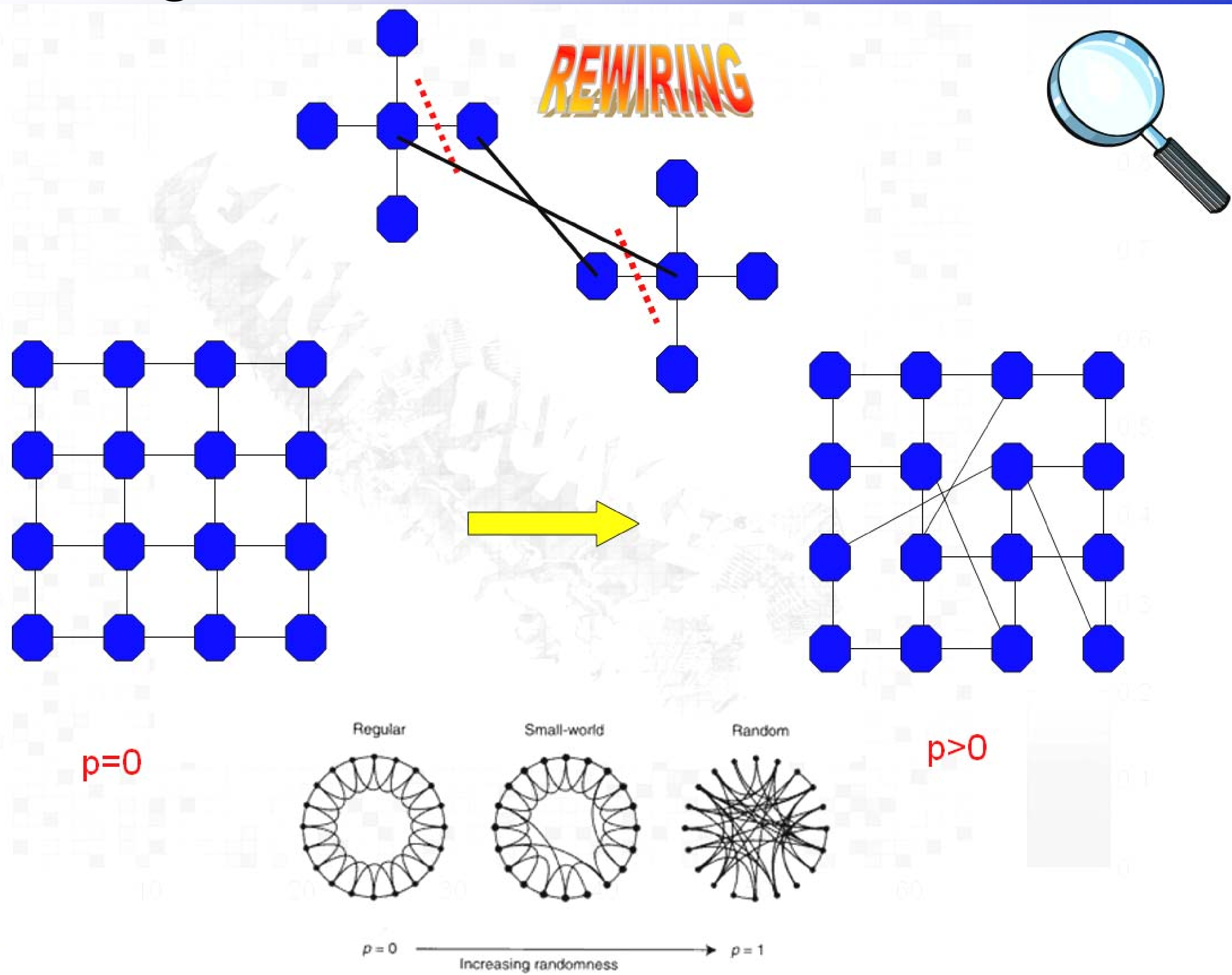
The small-world behavior is characterized by the fact that the distance between any two vertices is of the order of that for a random network and, at the same time, the concept of neighbourhood is preserved, as for regular lattices. For this reason, we will expect to obtain SOC in a small world.



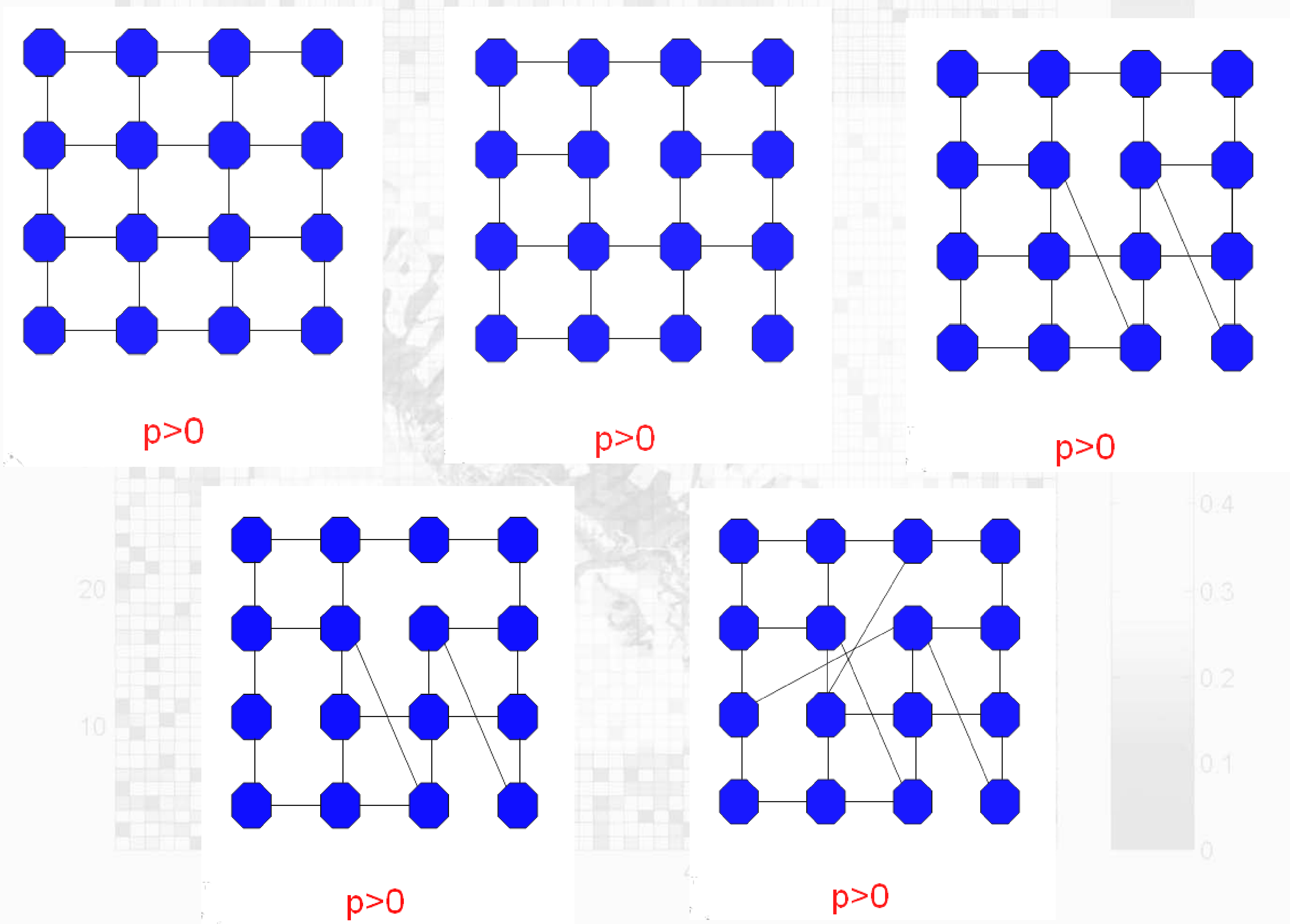
1998 - Watts and Strogatz (USA)



Constructing the network …

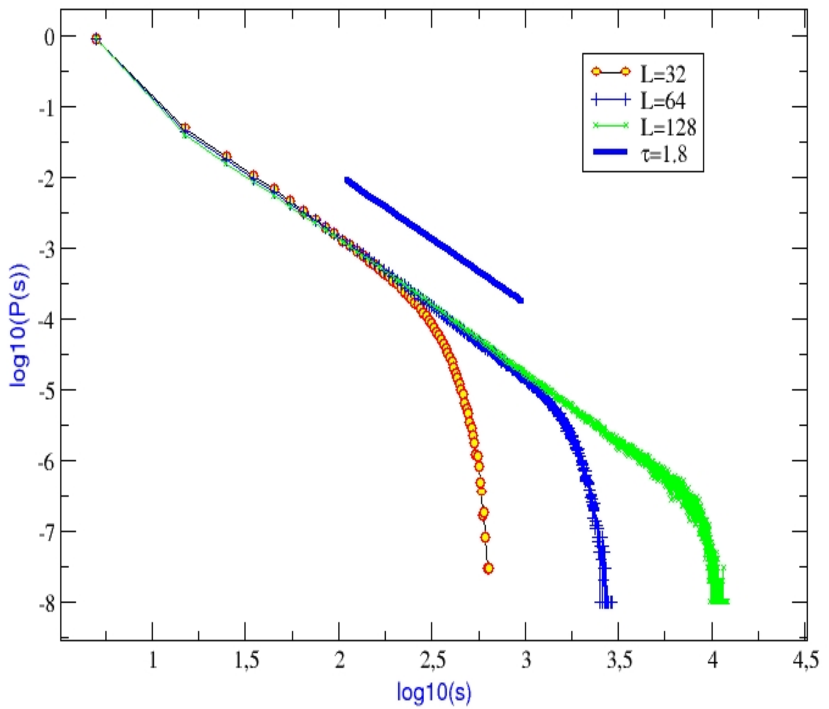


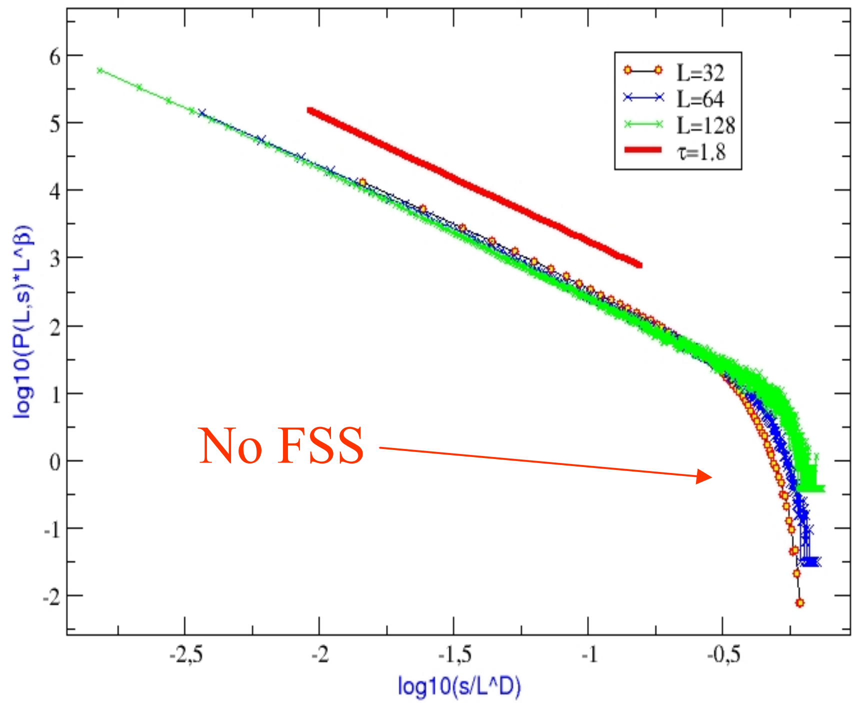
Constructing the network …



To date … for dissipative NN OFC model

Dissipative Nearest Neighbor OFC Model (α=0.21)





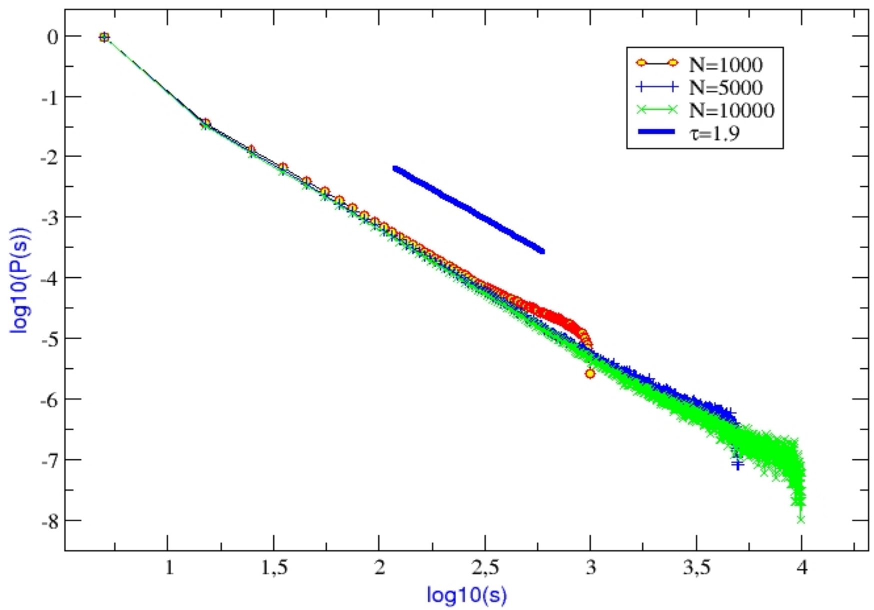
In order to characterize the critical behavior of the model, a finite size scaling (**FSS**) ansatz is used:

$$P_N(s) \simeq N^{-\beta} \cdot f(s/N^D)$$

where f is a suitable scaling function and **β** and **D** are critical exponents describing the scaling of the distribution function.

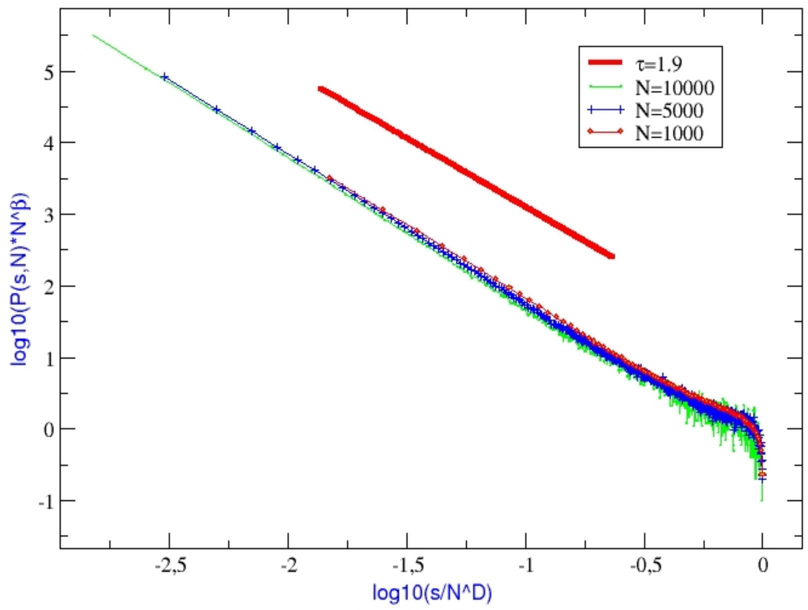
… and for random graph

Dissipative OFC Model on a Random Graph with $\alpha=0.15$



Finite-size scaling for OFC on a Random Graph with $\alpha=0.15$

The critical exponents are D=1 and $\beta=1.65$

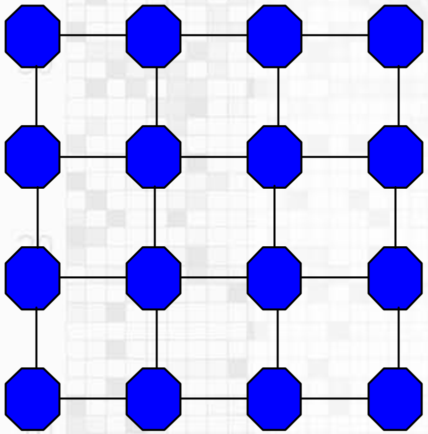


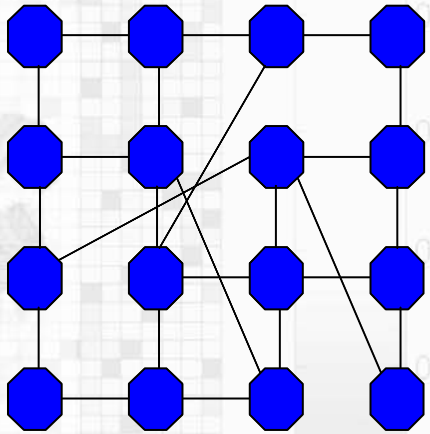
In a random regular graph all sites have exactly the same number of nearest neighbours **q**.
In this case, it's verified that (both for q=4 and q=6) the system organizes into a subcritical state.
In order to observe scaling in the avalanche distribution, one has to introduce some inhomogeneities. For the OFC model on a (quenched) random graph, it's found that it suffices to consider just two sites in the system with coordination q-1.

S. Lise and M. Paczuski, Phys. Rev. Lett. 88, 228301 (2002)

… SW topology?

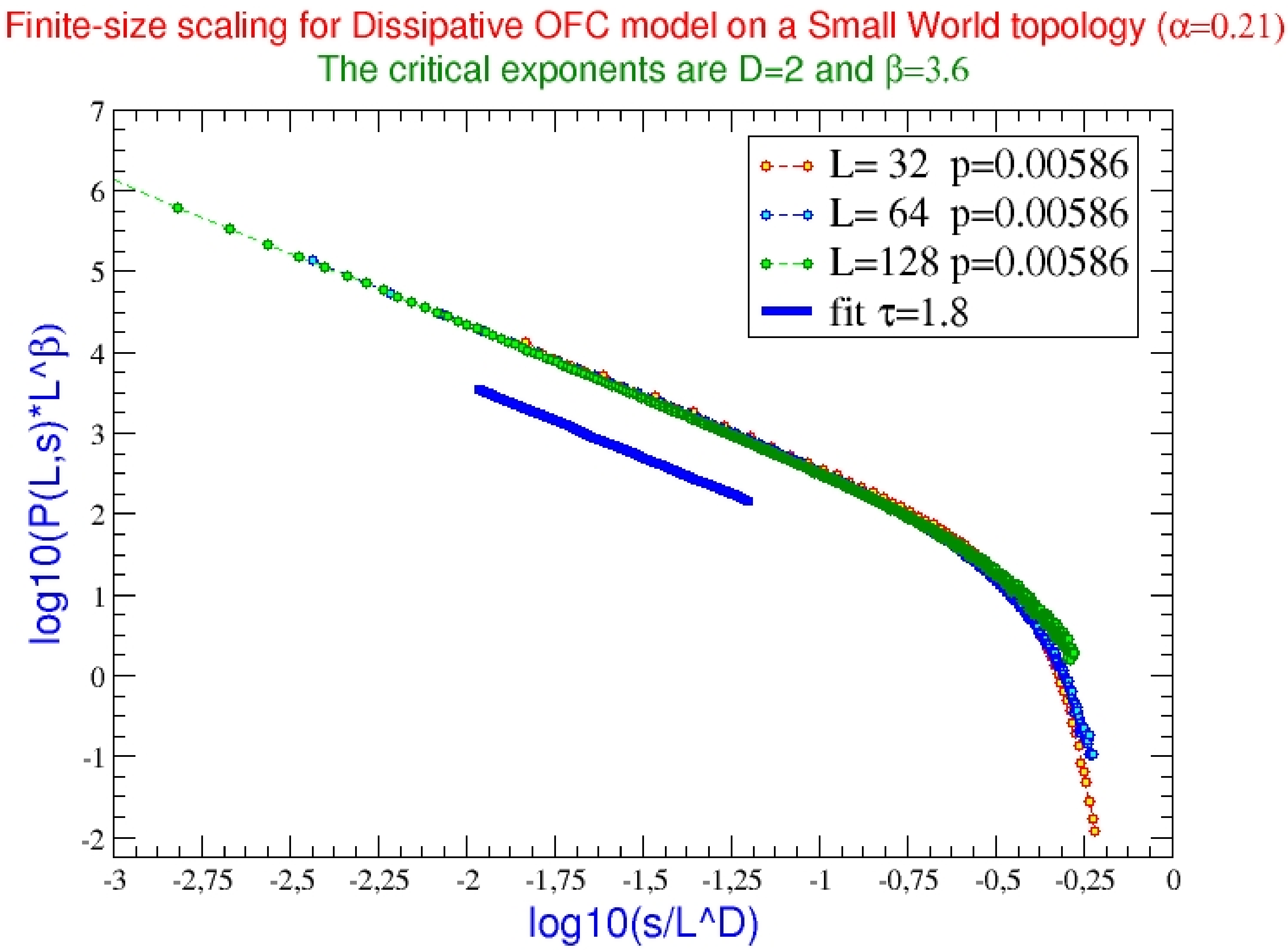
What happens for a small world topology?





Is it sufficient to consider a small world graph, obtained by randomizing a fraction p of the links of the regular nearest neighbour lattice, in order to obtain FSS?

Finite-size scaling …



Turbulence-like analysis in OFC models

The possibility of establishing a close analogy between 2D BTW sandpile dynamics and fully developed turbulent scaling has just been showed by Stella and De Menech in 2001.

A. L. Stella, M. De Menech, Physica A 295, 101-107 (2001)

Is it possible to find a similar connection in OFC models on a small world topology?

Turbulence-like analysis

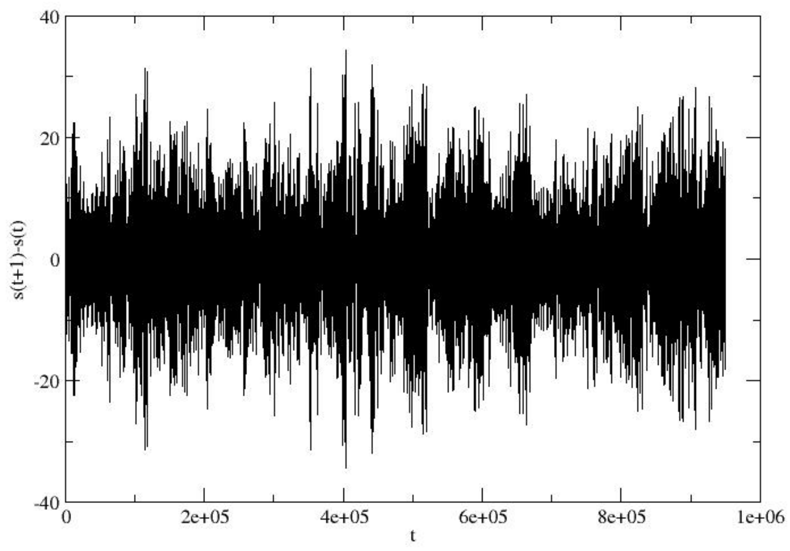
We consider the difference between

two avalanches, i.e. $x = S(t+\delta) - S(t)$

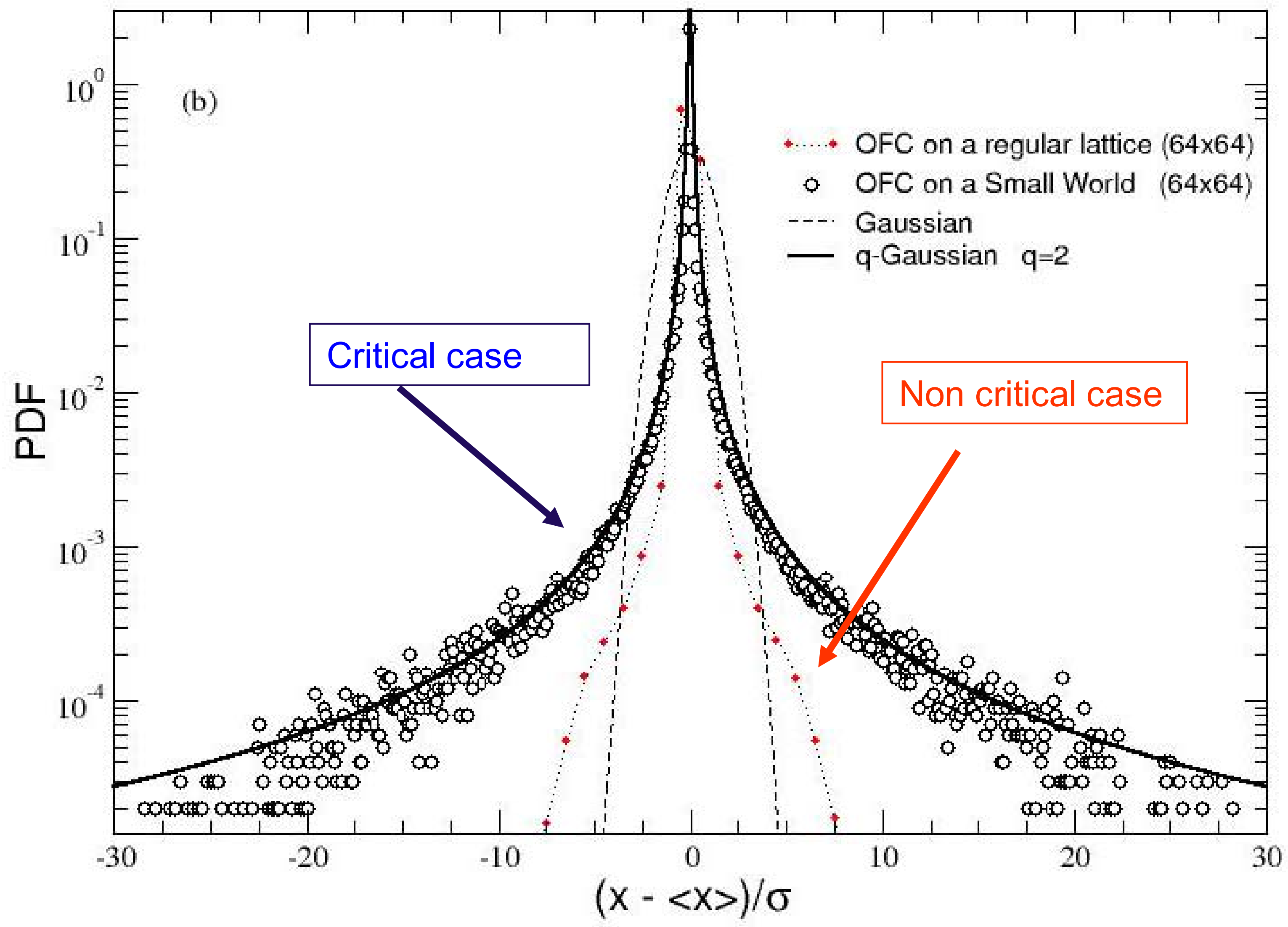
being S(t+δ) and S(t) two successive avalanches with a time difference δ

The time series of x is very intermittent and the pdfs are non Gaussian at criticality, i.e. when long range correlation and finite size scaling exist

Caruso et al. cond-mat/0606118

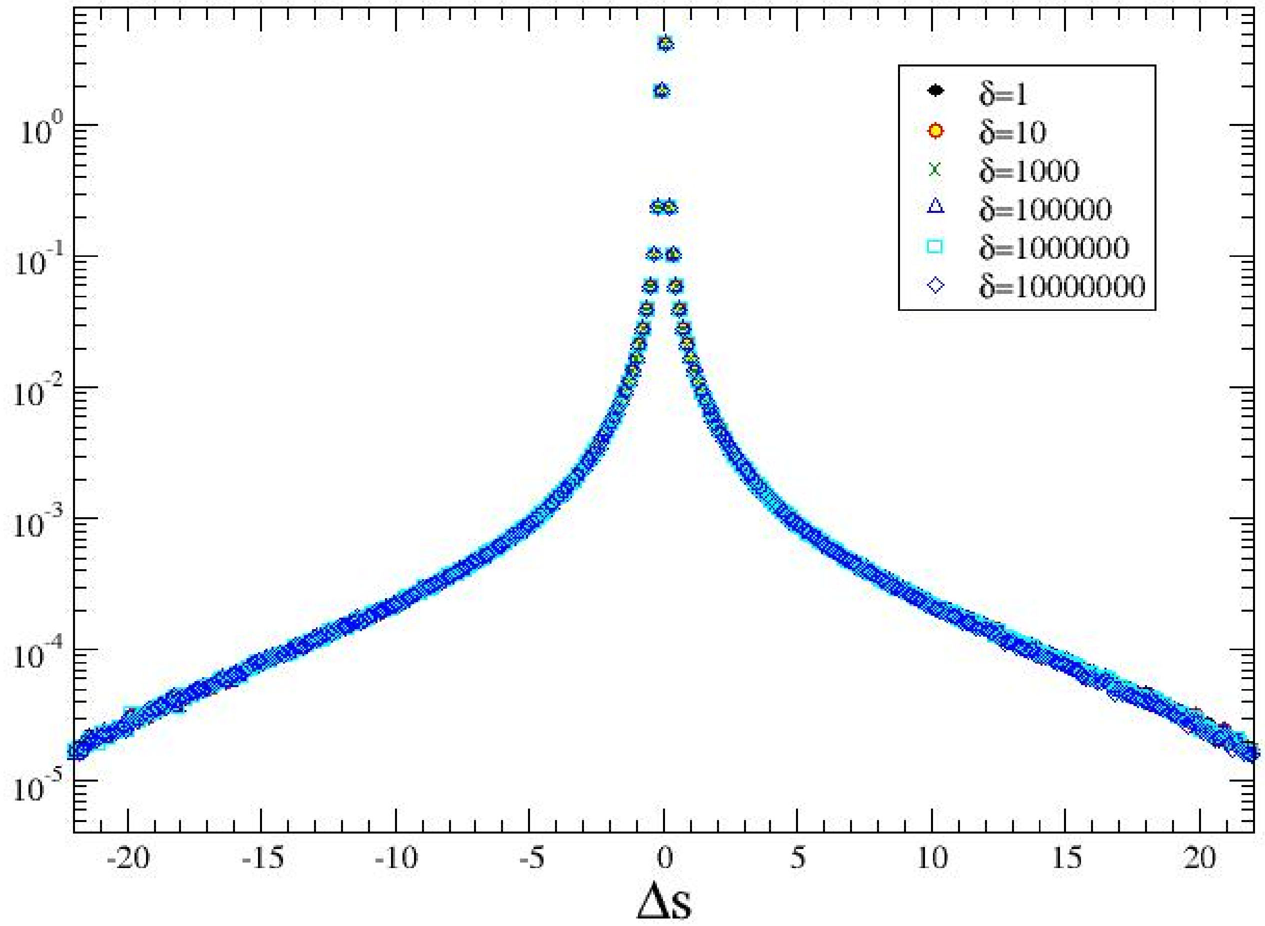


Fat q-gaussian tails at criticality …



Fat q-gaussian tails at criticality …

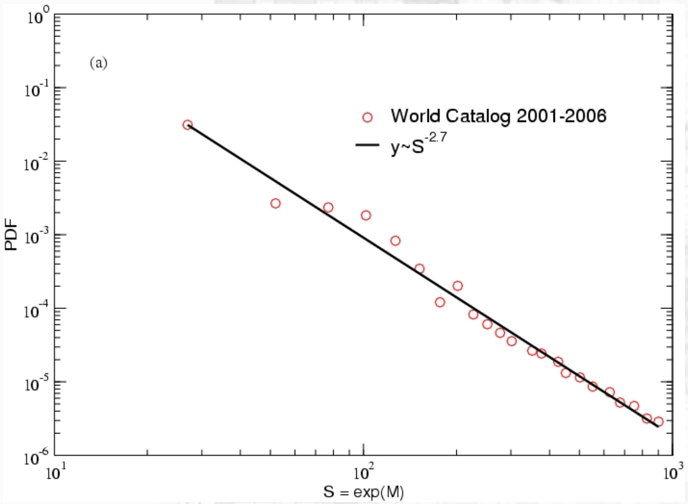
No dependence on the time interval

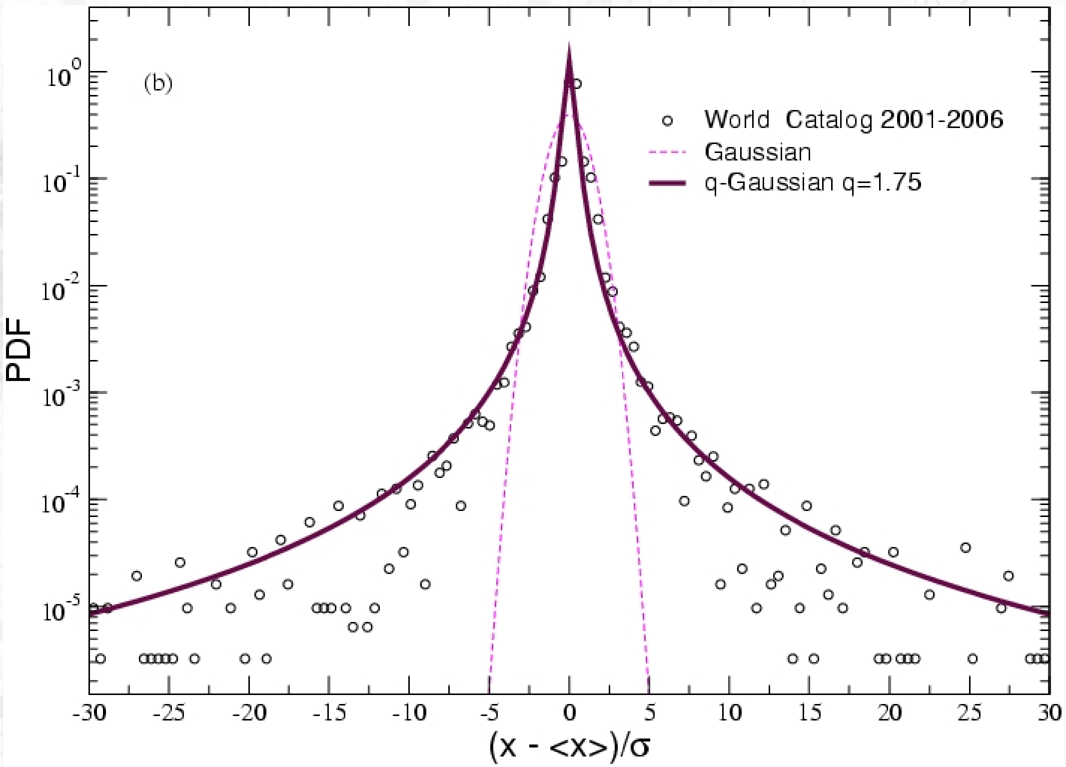


Analysis of real data: World Catalog

We considered S~exp(M), M being the earthquake magnitude

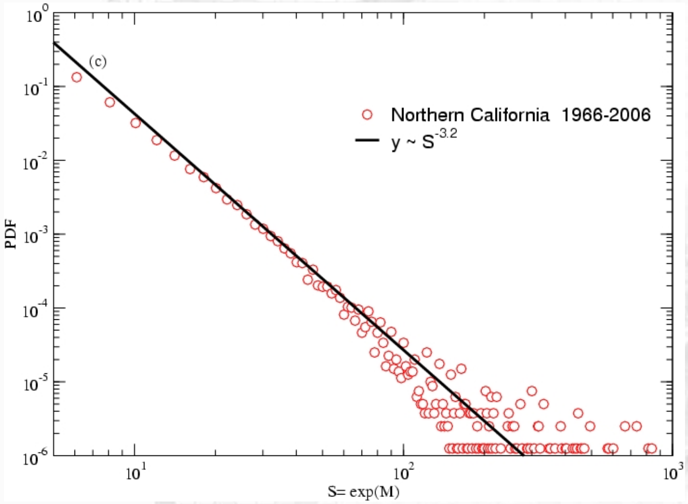
World Catalog: 689000 events in the period 2001-2006

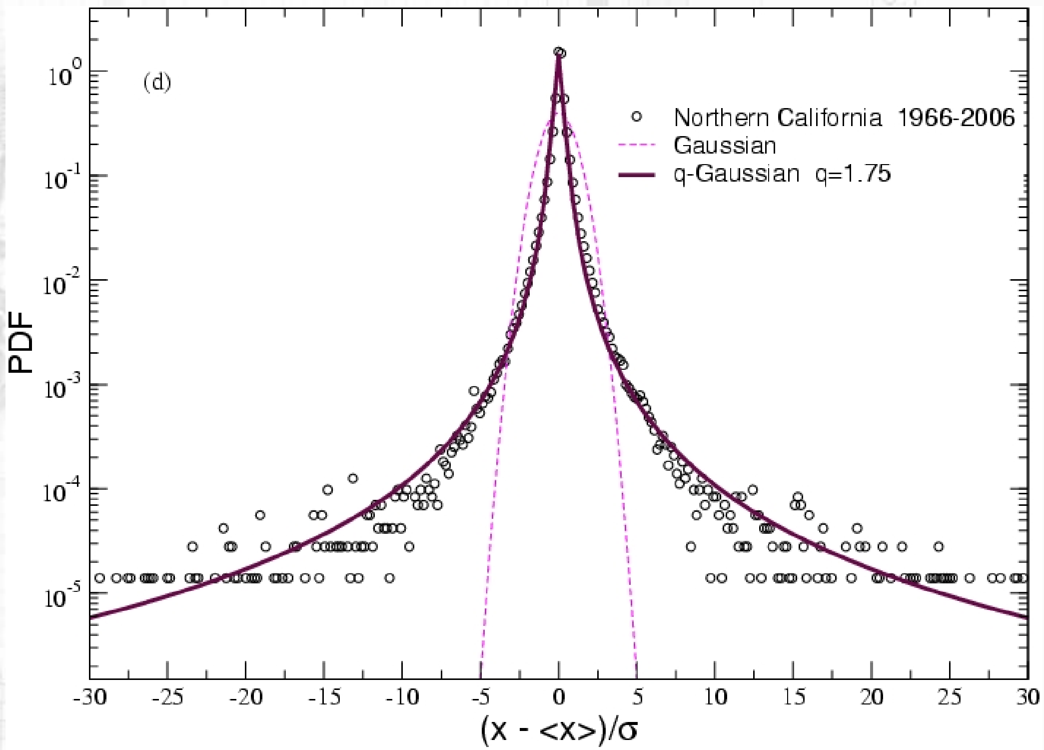




Analysis of real data: Northern California

Northern California: 400000 events in the period 1966-2006





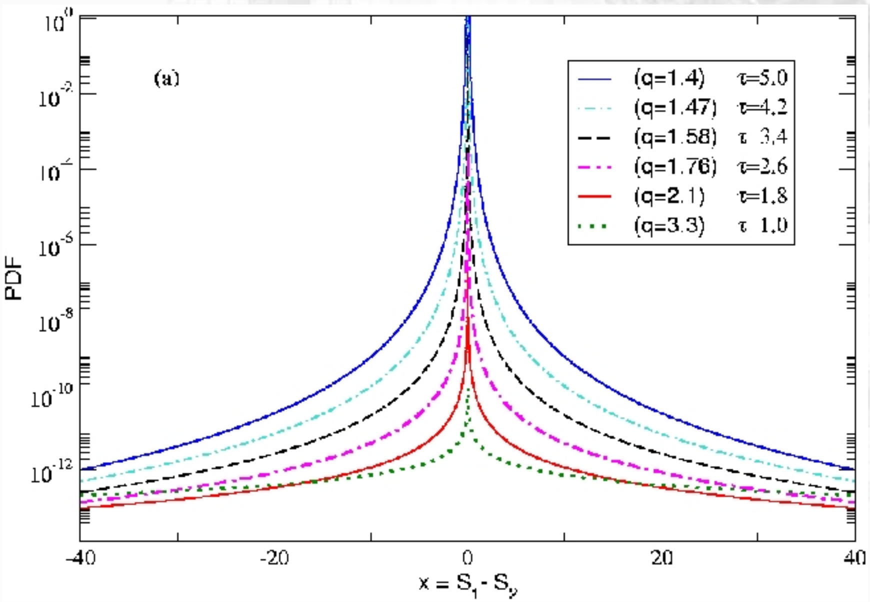
A simple model

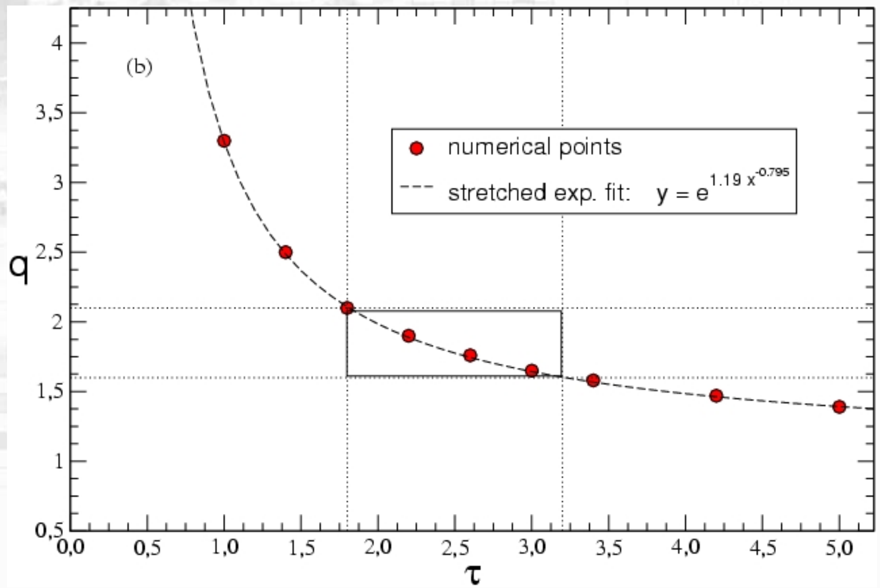
Assuming no correlation in two stochastic variables with a power law distribution and taking their difference, we get for the pdf of the difference x=S1(t+Δ)-S2(t) the formula

$$P(x) = \int_0^\infty dS_1 \int_x^\infty dS_2 (S_1 S_2)^{-\tau} \delta(S_1 - S_2 - x) =$$

$$= K\, {}_2F_1\left(\tau, 2\tau - 1; 2\tau; -\frac{|x|}{\varepsilon}\right)$$

This P(x), which can be approximated by a q-Gaussian, is able to reproduce both the numerical and the experimental data once τ is given





Thanks for the attention