First lecture:

The manifestation of q-statistics at the transitions to chaos

Alberto Robledo

Instituto de Fisica, Universidad Nacional Autonoma de Mexico (UNAM),Mexico

Abstract:

In this lecture we present a detailed account of recent understanding on the dynamics at critical attractors of simple one-dimensional nonlinear maps. This dynamics is relevant to a discussion about the applicability of q-statistics to the transition to chaos along its three known routes. The critical attractors considered are those for the familiar pitchfork and tangent bifurcations, the period-doubling, and the quasiperiodic onset of chaos. The fluctuating, nonexponential, sensitivity to initial conditions and its related spectra of q-generalized Lyapunov coefficients are determined with use of the properties of the fixed-point maps under functional renormalization group (RG) transformations. We have found equality between the q-Lyapunov coefficients and the corresponding rates of entropy production at the critical attractors provided the rates are obtained from the q-entropy expression. We establish the links that exist between the thermodynamic approach - originally employed by Mori and colleagues to characterize dynamical phase transitions in attractors of nonlinear iterated maps - and the q-statistical properties of the fluctuating dynamics at the transitions to chaos. We identify the Mori singularities in the Lyapunov coarse-grained function at the onset of chaos with the appearance of special values for the entropic index q. We find that the occurrence of a dynamical phase transition at the onset of chaos is associated to trajectories that connect specific regions in the multifractal attractor. For these trajectories we obtain a q-exponential sensitivity to initial conditions, and the value of the 'thermodynamic field' q at a dynamical transition is the same as that for the index q in the sensitivity. We prove that at such dynamical transition the q-entropy grows linearly with time and with a rate equal to the generalized q-Lyapunov coefficient. The intricate dynamics at these critical attractors consists of an infinite family, with a hierarchical structure, of q-phase transitions where the values of the indexes q are determined by the universal jump discontinuities of the trajectory scaling function.