Scaling property and the generalized entropy uniquely determined by a fundamental nonlinear differential equation

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Abstract

We derive a scaling property from a fundamental nonlinear differential equation whose solution is the so-called q-exponential function. A scaling property has been believed to be given by a power function only, but actually more general expression for the scaling property is found to be a solution of the above fundamental nonlinear differential equation. In fact, any power function is obtained by restricting the domain of the q-exponential function appropriately. As similarly as the correspondence between the exponential function and Shannon entropy, an appropriate generalization of Shannon entropy is expected for the scaling property. Although the q-exponential function is often appeared in the optimal distributions of some one-parameter generalized entropies such as Rényi entropy, only Tsallis entropy is uniquely derived from the algebra of the q-exponential function, whose uniqueness is shown in the two ways in this talk.

1. H. Suyari, Mathematical structures derived from the q-multinomial coefficient in Tsallis statistics, Physica A, **368**, pp.63-82 (2006).

2. H. Suyari, Generalization of Shannon-Khinchin axioms to nonextensive systems and the uniqueness theorem for the nonextensive entropy, IEEE Trans. Inform. Theory **50**, 1783-1787 (2004).

3. H. Suyari and M. Tsukada, Law of error in Tsallis statistics, IEEE Trans. Inform. Theory **51**, 753-757 (2005).

4. T. Wada and A.M. Scarfone, A non self-referential expression of Tsallis' probability distribution function, Eur.Phys.J.B **47** 557-562 (2005).

5. H. Suyari, The unique non self-referential q-canonical distribution and the physical temperature derived from the maximum entropy principle in Tsallis statistics, Prog. Theor. Phys. Suppl., **162**, pp.79-86 (2006). (See the author's website (http://www.ne.jp/asahi/hiroki/suyari/publications.htm) for the detail derivation.)

6. H. Suyari and T. Wada, Scaling property and the generalized entropy uniquely determined by a fundamental nonlinear differential equation, to appear in *cond-mat* soon.