Generalizing Boltzmann factors

Stefan Thurner and Rudolf Hanel

University of Vienna, Austria

We show by a simple argument that the most general Boltzmann factor allowed within the canonical ensemble is a q-exponential rather than the classical exponential.
Based on this finding we ask what happens if one relaxes the requirement of a Boltzmann factor to be an exponential and to allow for a broader range of experimentally measurable distribution functions.
In order to describe the correct thermodynamics of systems leading to non-Boltzmann distributions and to - at the same time - keep the maximum entropy principle with standard (!) constraints - this has consequences for the definition of entropy. We discuss these consequences and suggest an idea of how an 'entropy' can self-consistently be 'constructed' on the basis of various classes of (measurable) distribution functions. We show how Boltzmann-Gibbs and Tsallis entropies evolve as natural special cases.