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SCHOOL and CONFERENCE on COMPLEX SYSTEMS and NONEXTENSIVE STATISTICAL MECHANICS

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Temperature Fluctuations of the Cosmic Microwave Background Radiation: A Case of Nonextensivity?

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Temperature Fluctuations of the Cosmic Microwave Background Radiation: A Case of Nonextensivity?

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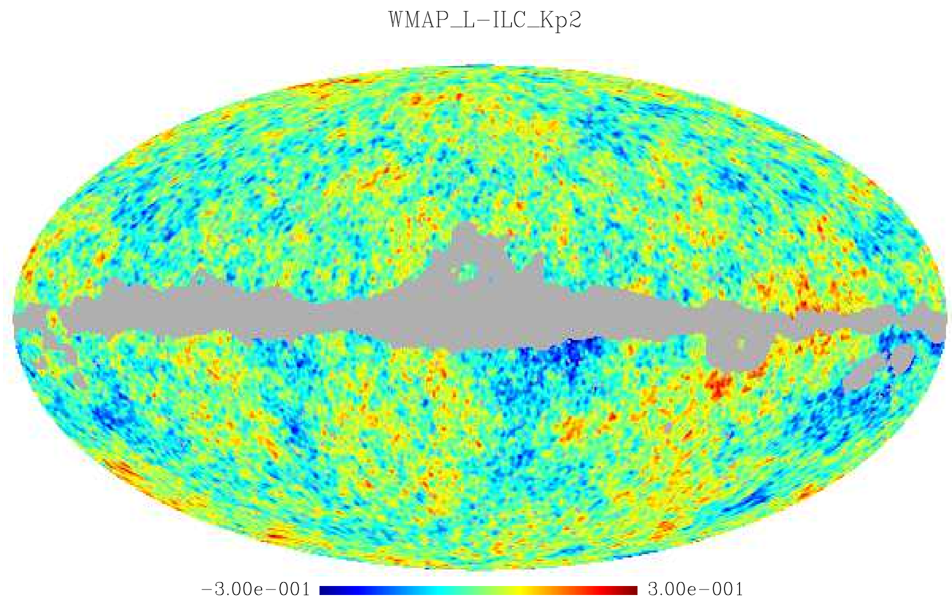
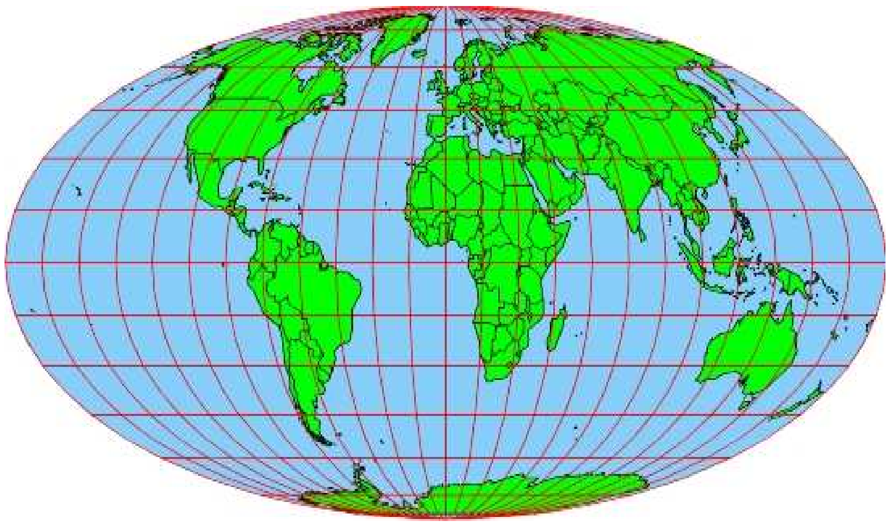
The WMAP Cosmic Microwave Background sky: tiny temperature fluctuations from the early Universe



Sphere S^2

Mollweide Projection on the plane



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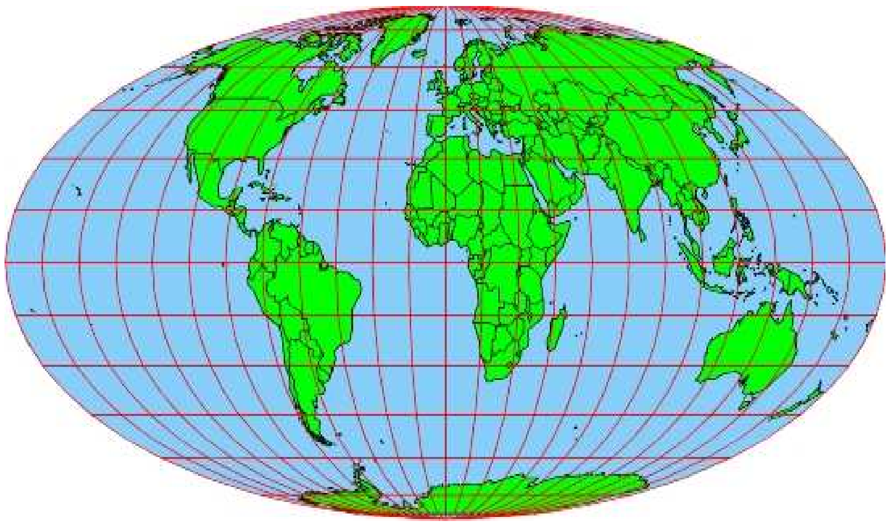
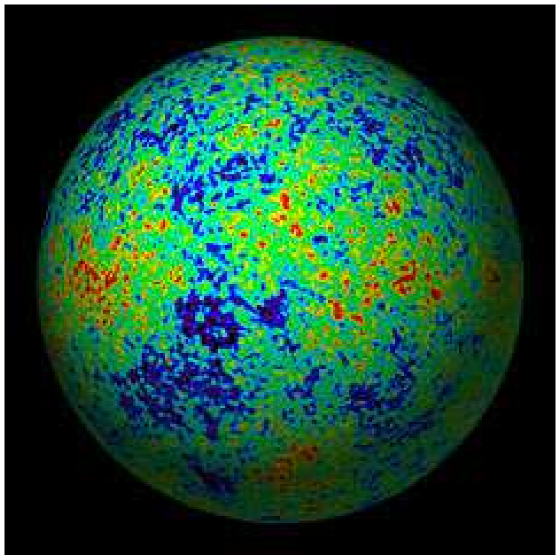
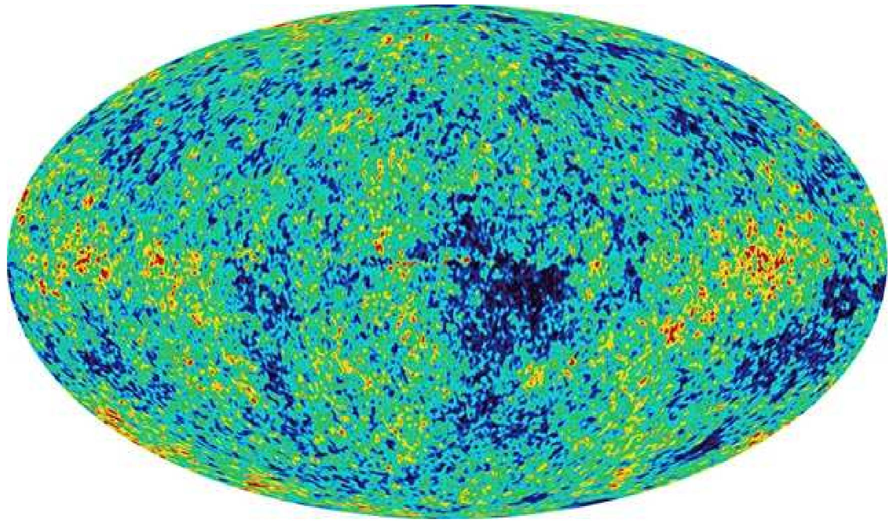
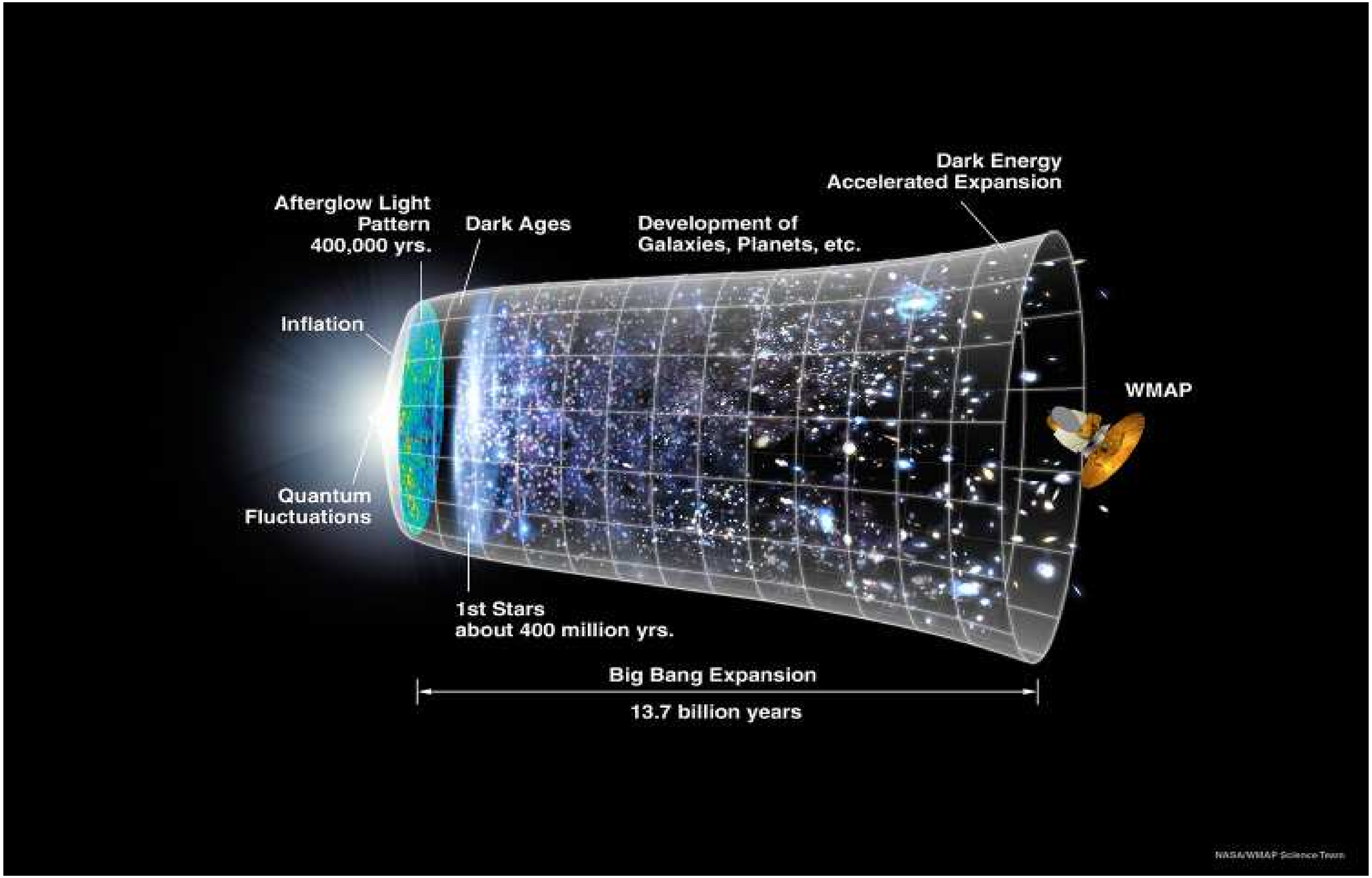


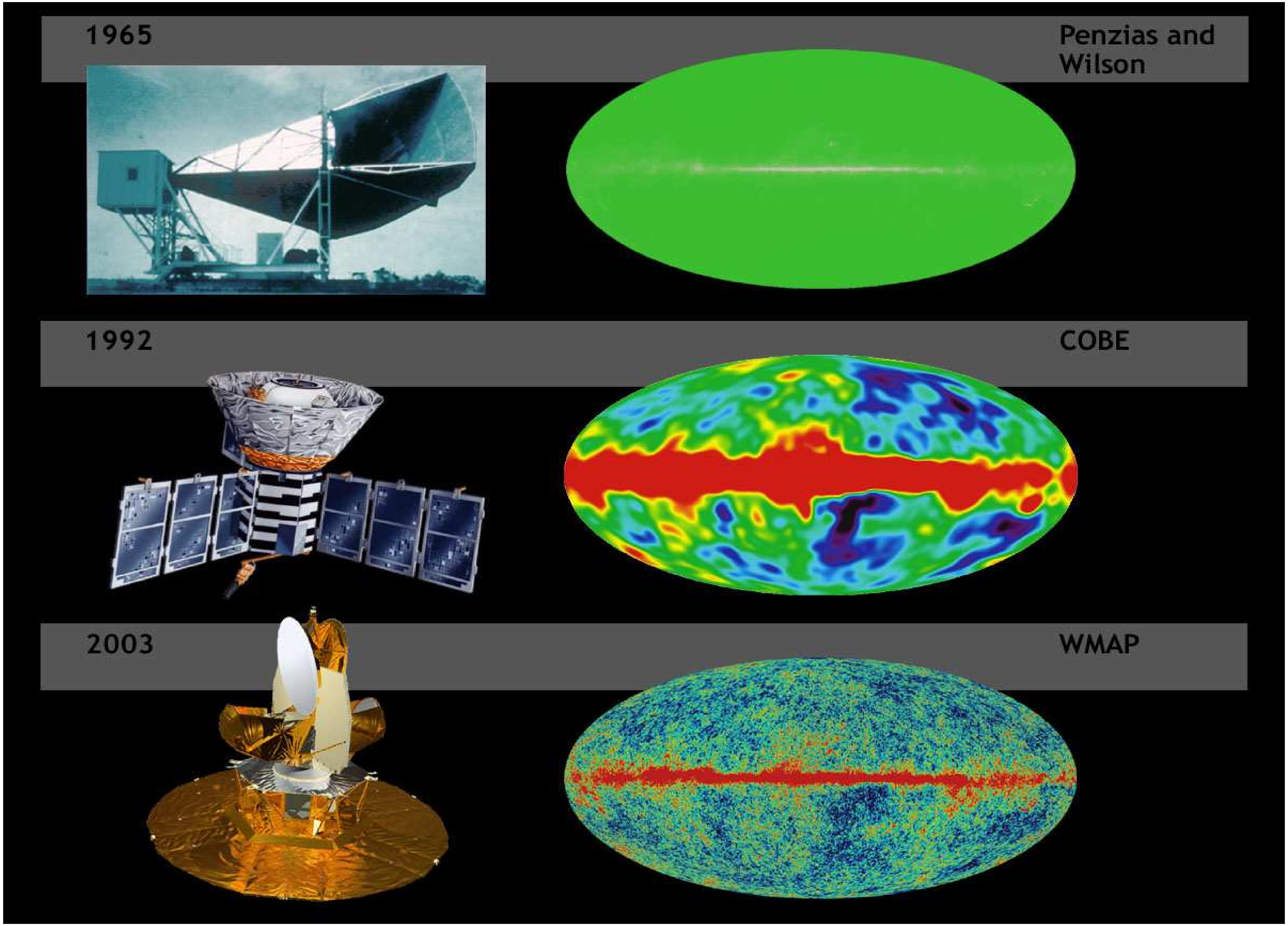




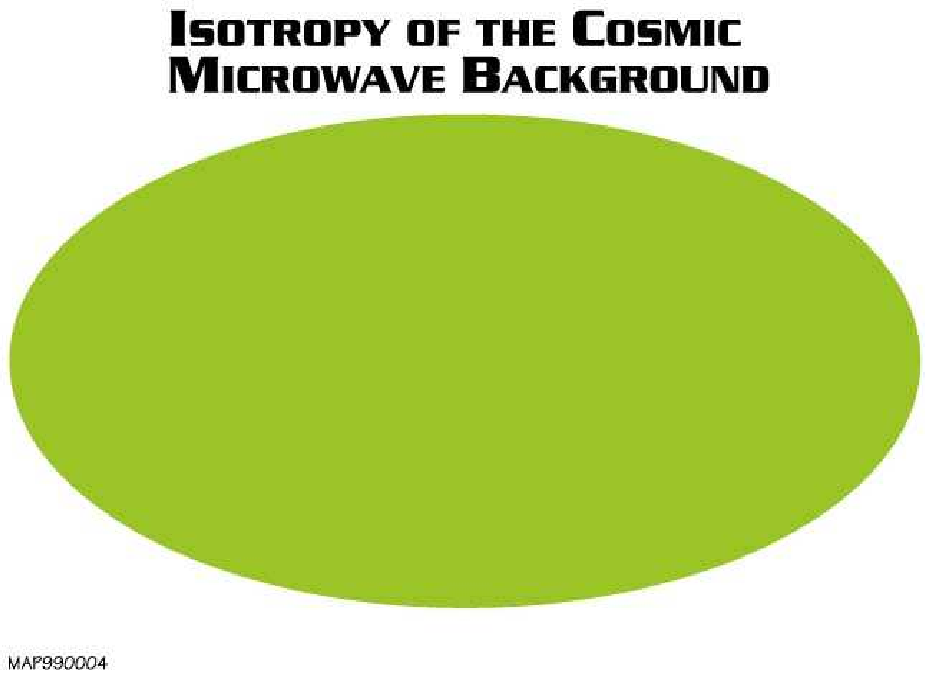
A brief history of Cosmic Background Radiation CMB

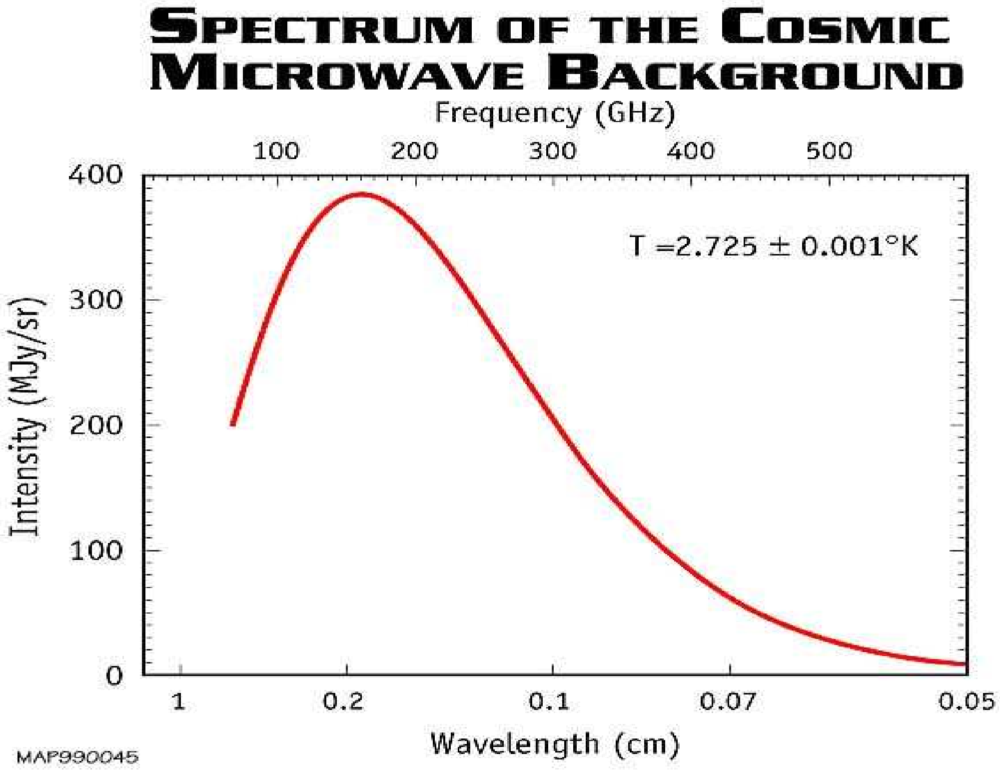


A brief history of CMB detection



Note that in the last two CMB maps was extracted the black body temperature $T_{Black Body}$=2.725K





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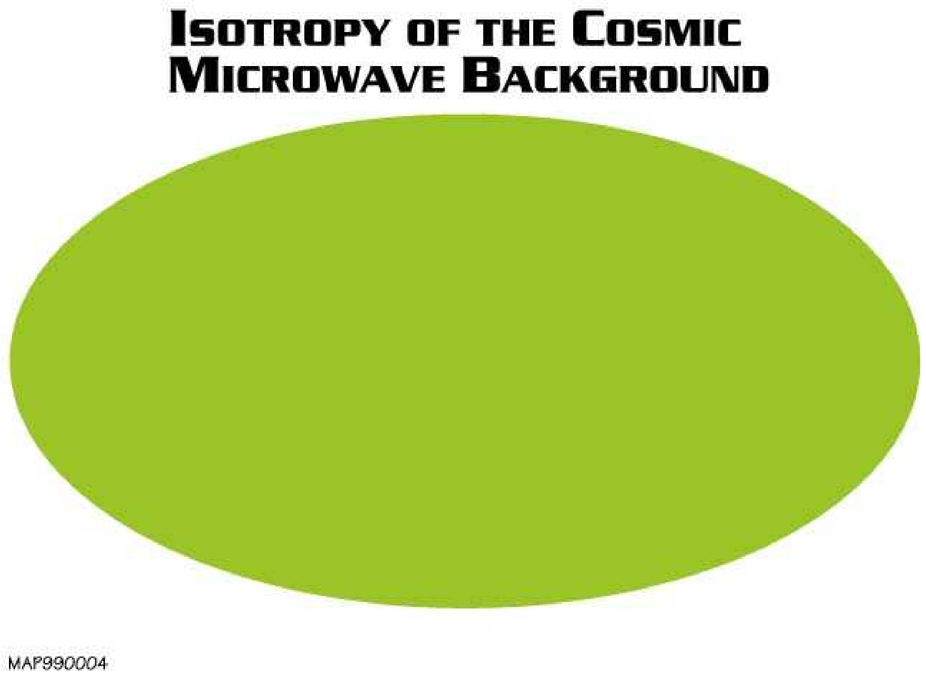
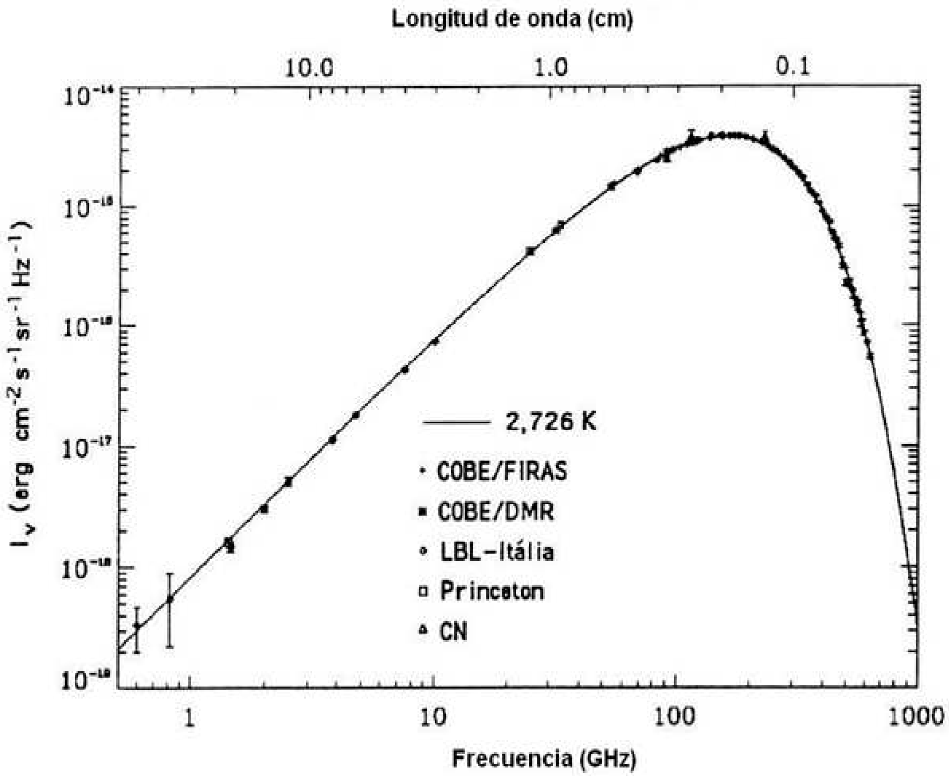
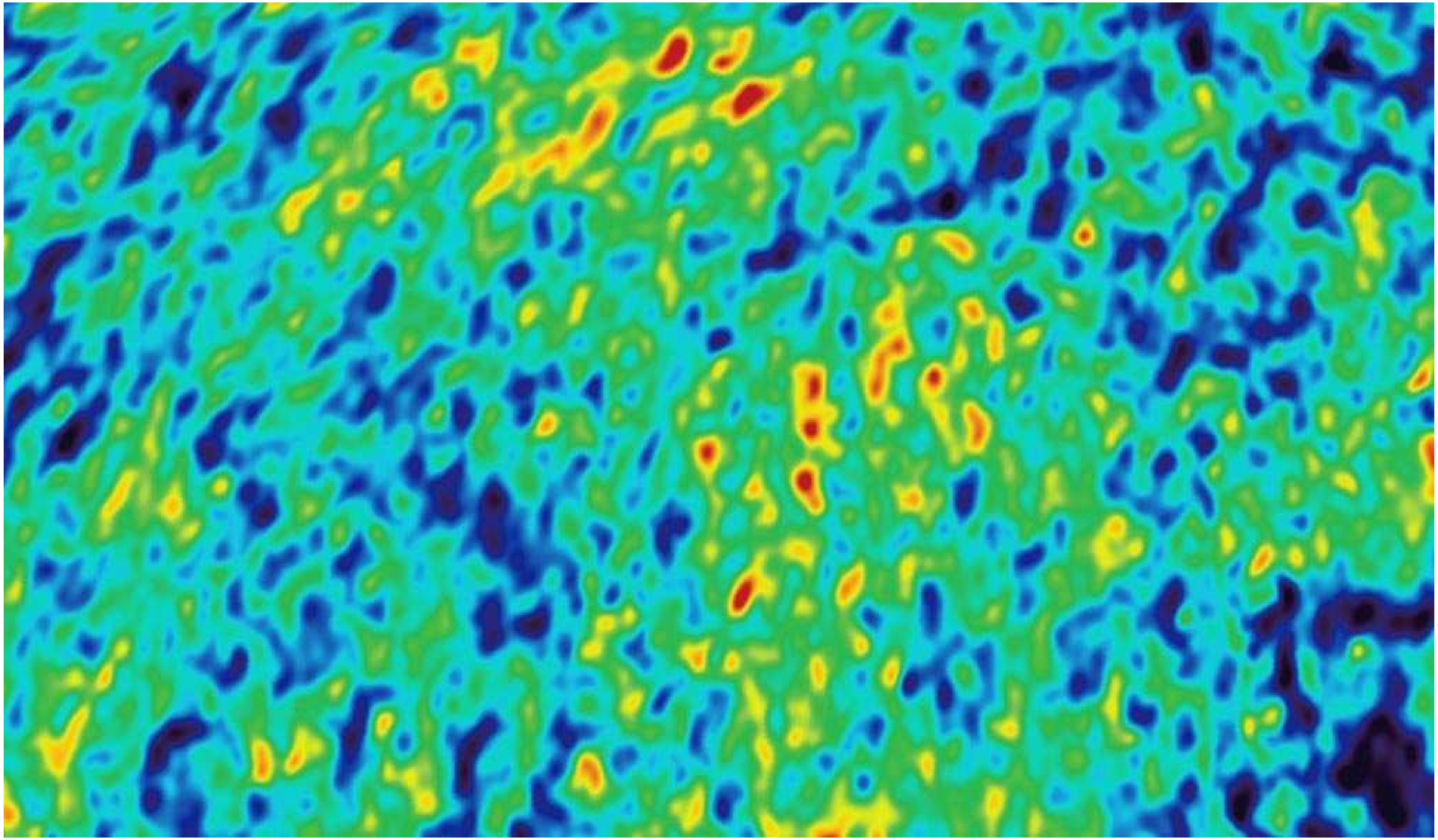




… remaining the so-called

- and + **CMB temperature fluctuations** ($\sim 10^{-5}$K around $\mathbf{T}_{BB}$)



What is the NATURE of these TEMPERATURE FLUCTUATIONS?

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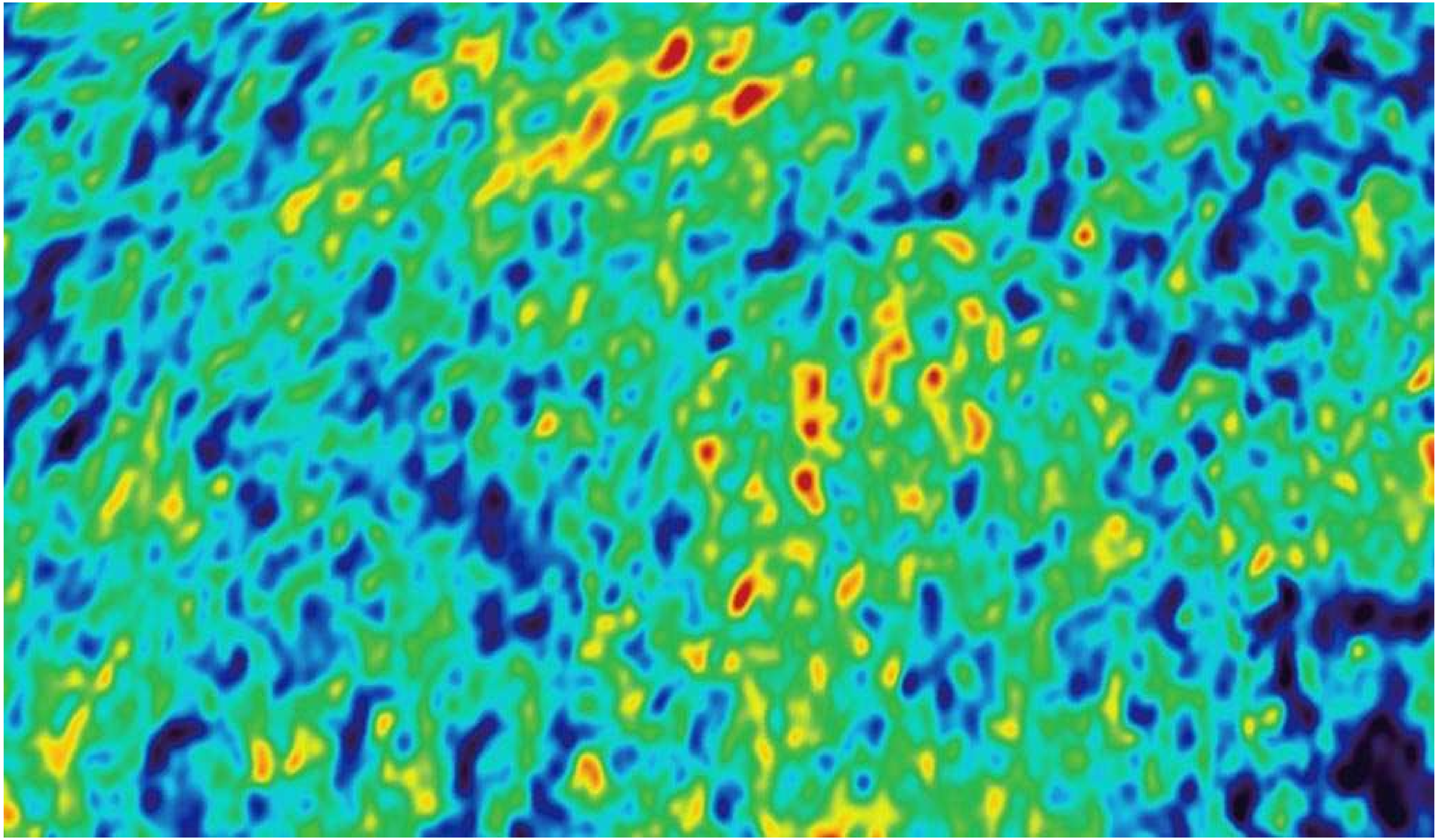
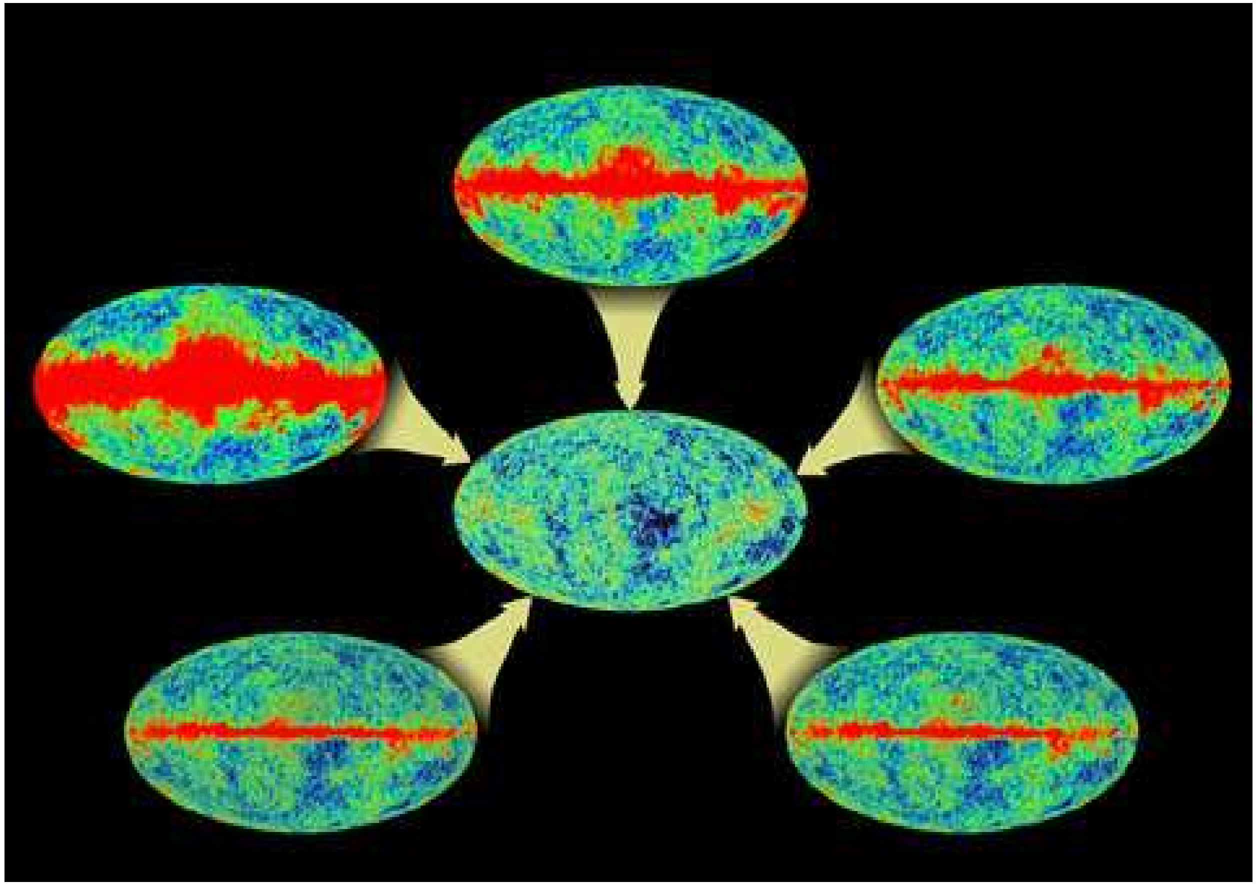
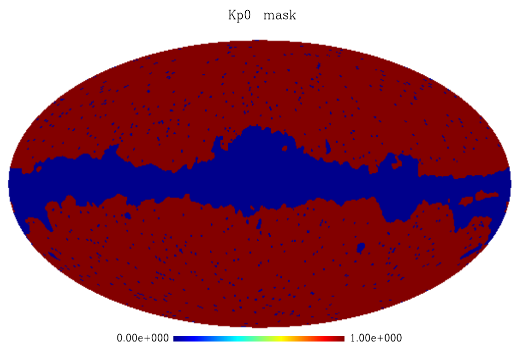


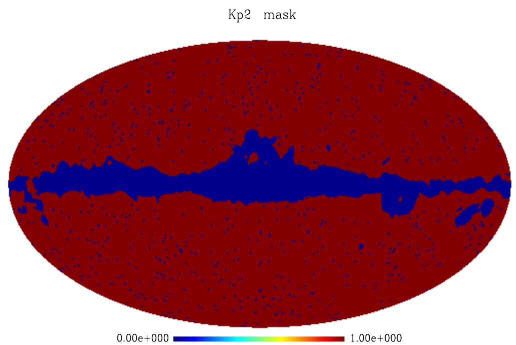
Are they Gaussian TEMPERATURE FLUCTUATIONS?

Cosmic Microwave Background analyses: WMAP data





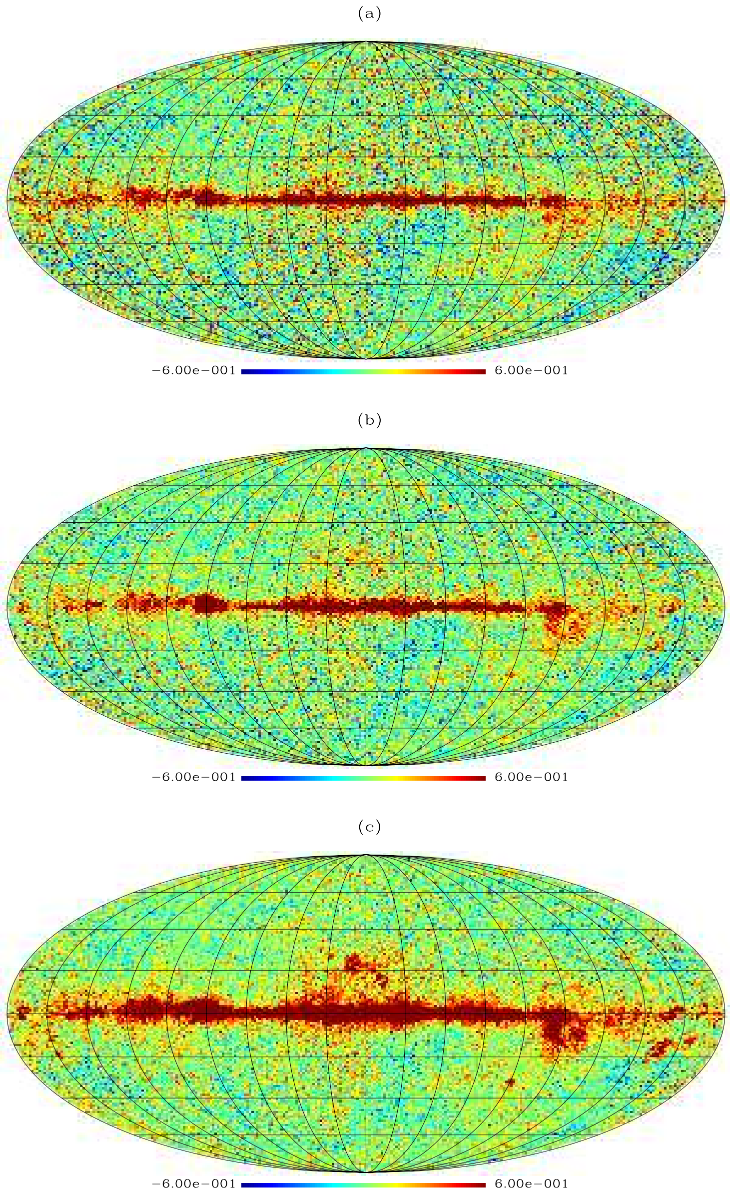
Kp0



Kp2

We acknowledge CMB maps from WMAP team

WMAP first-year data: W-, V-, and Q- bands



The data contained in each CMB maps are the temperature of each pixel, given by

$$T_{\text{pixel}} = T_{\text{CMB}} + T_{\text{instrum.noise}} + T_{\text{foregrounds}}$$

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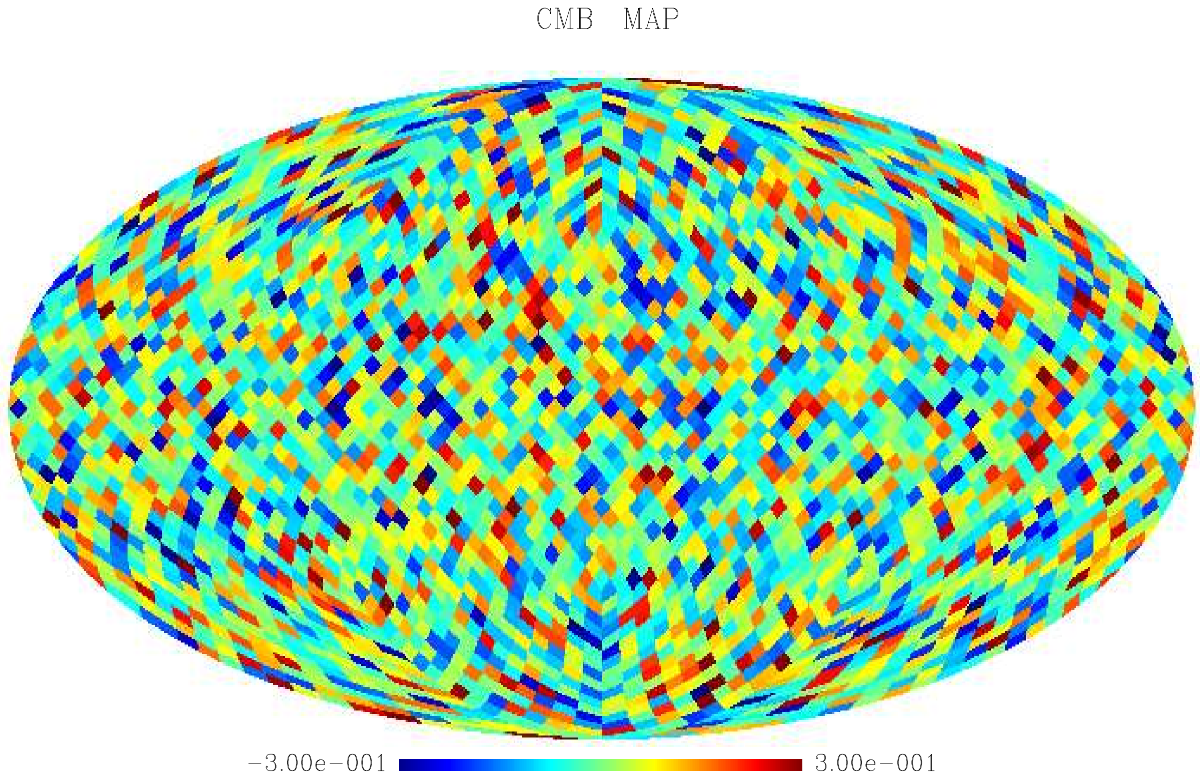
But, what is a pixel?

$\Rightarrow$ any equal-sized region in which the CMB sky has been divided:



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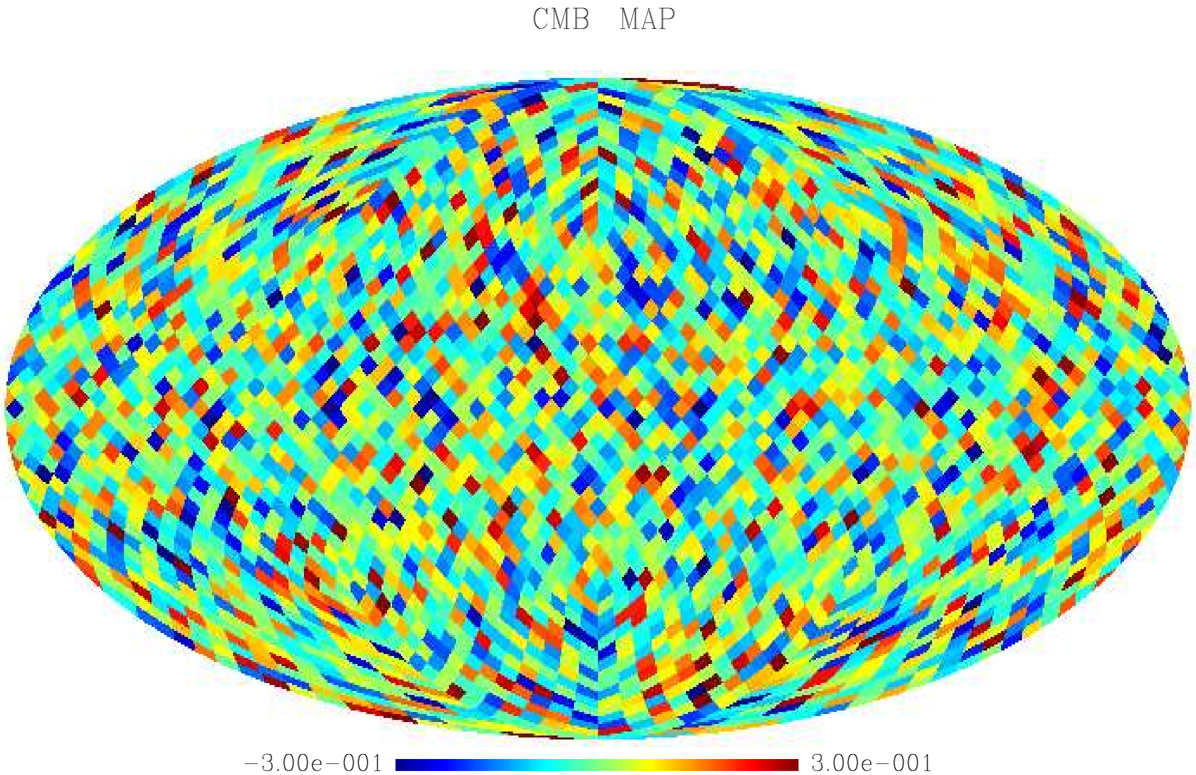

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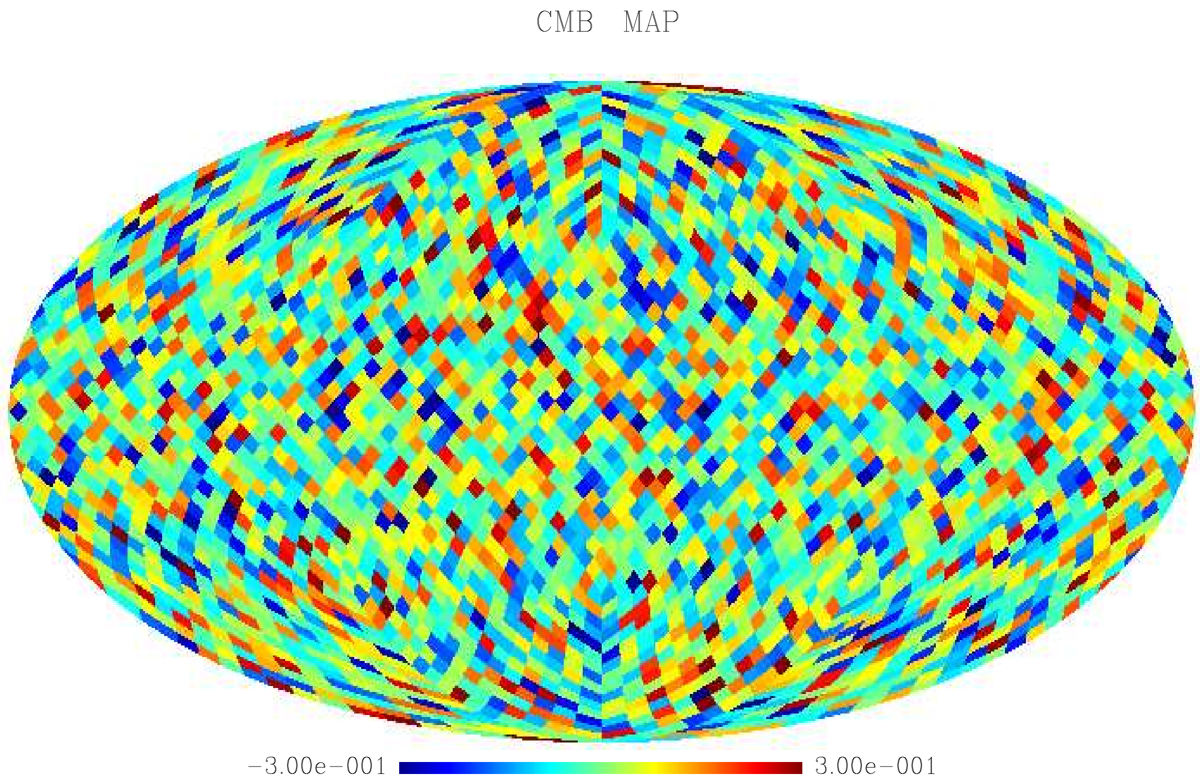

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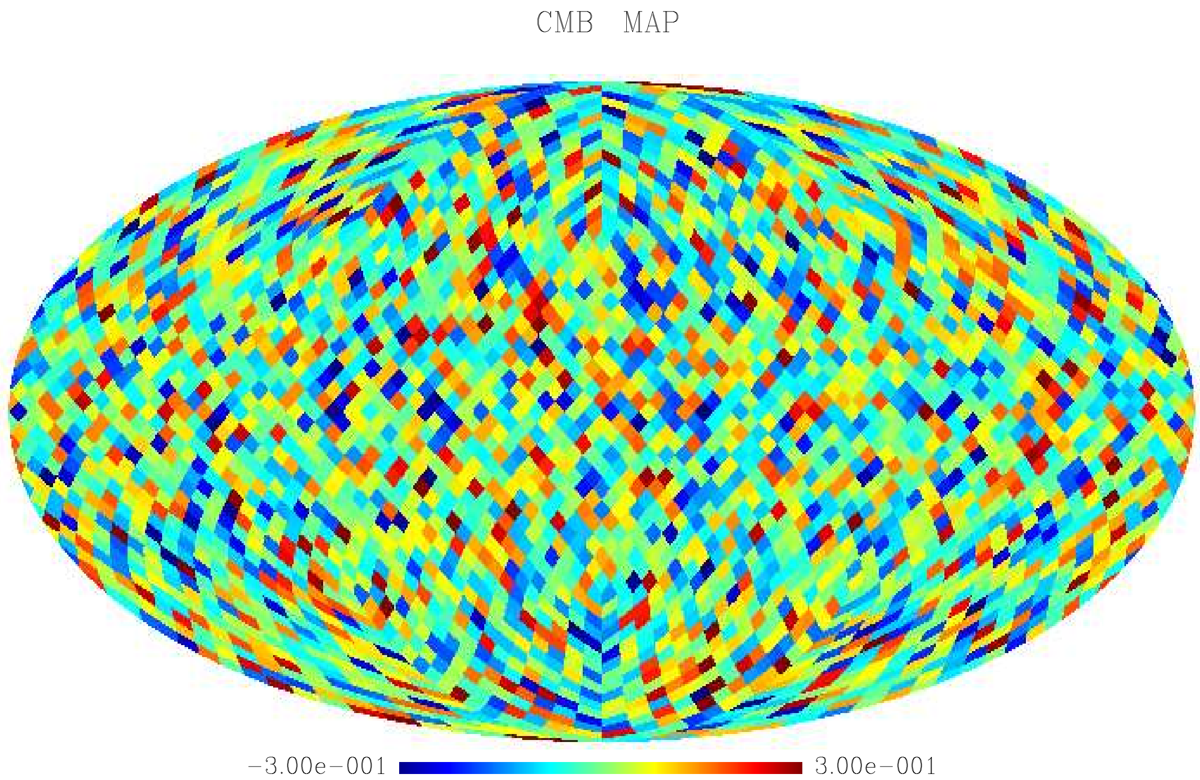

Our statistical analyses of CMB maps consist in calculating their **temperature distribution functions**.

Besides the obvious Galactic Foregrounds (GF) in CMB maps, which are strongly eliminated with the application of a suitable WMAP cut-sky mask, data could still contain contaminations, most probably: unsubtracted GF, detector's noise, systematics, etc.

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How to avoid (minimize?) these effects in CMB data?

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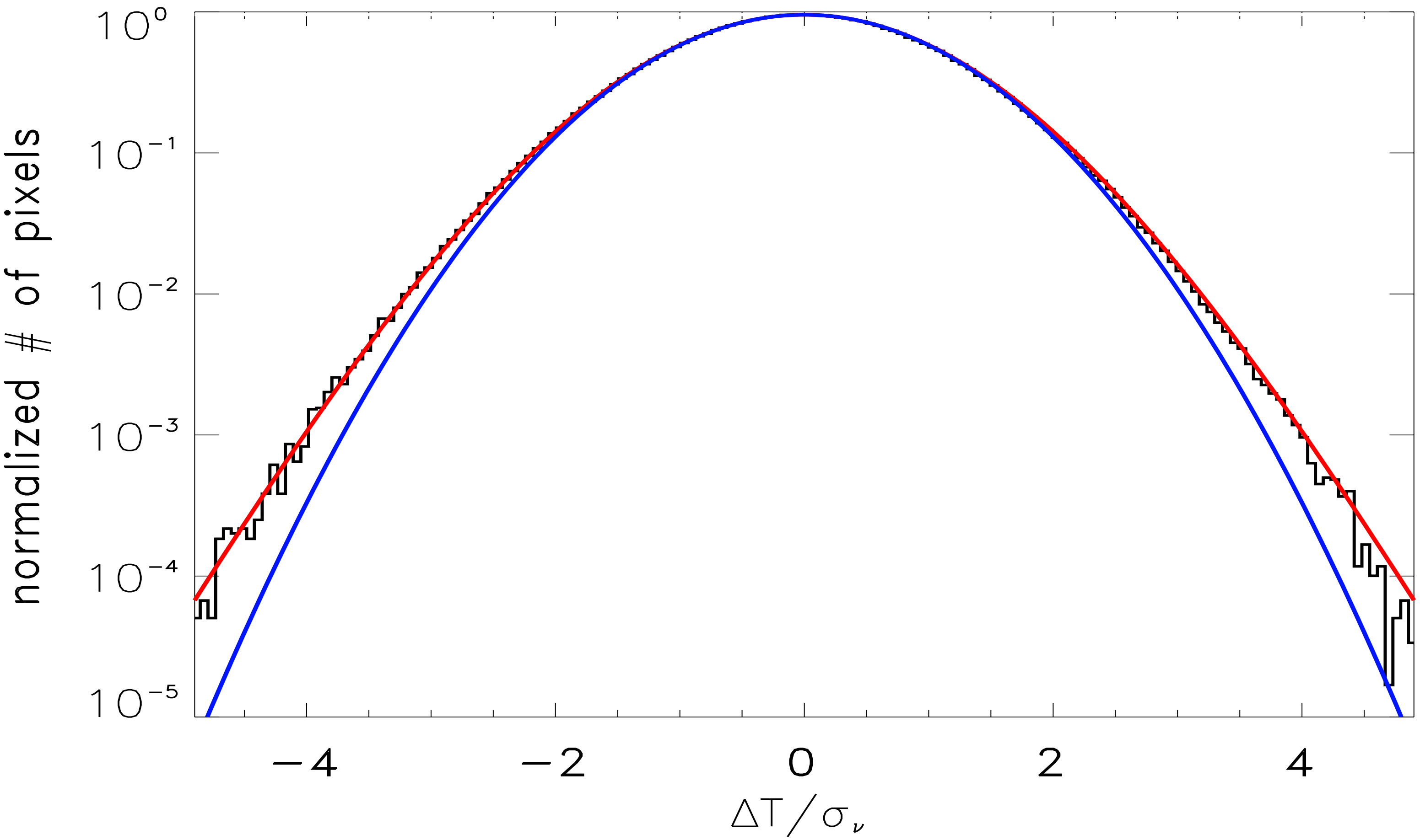
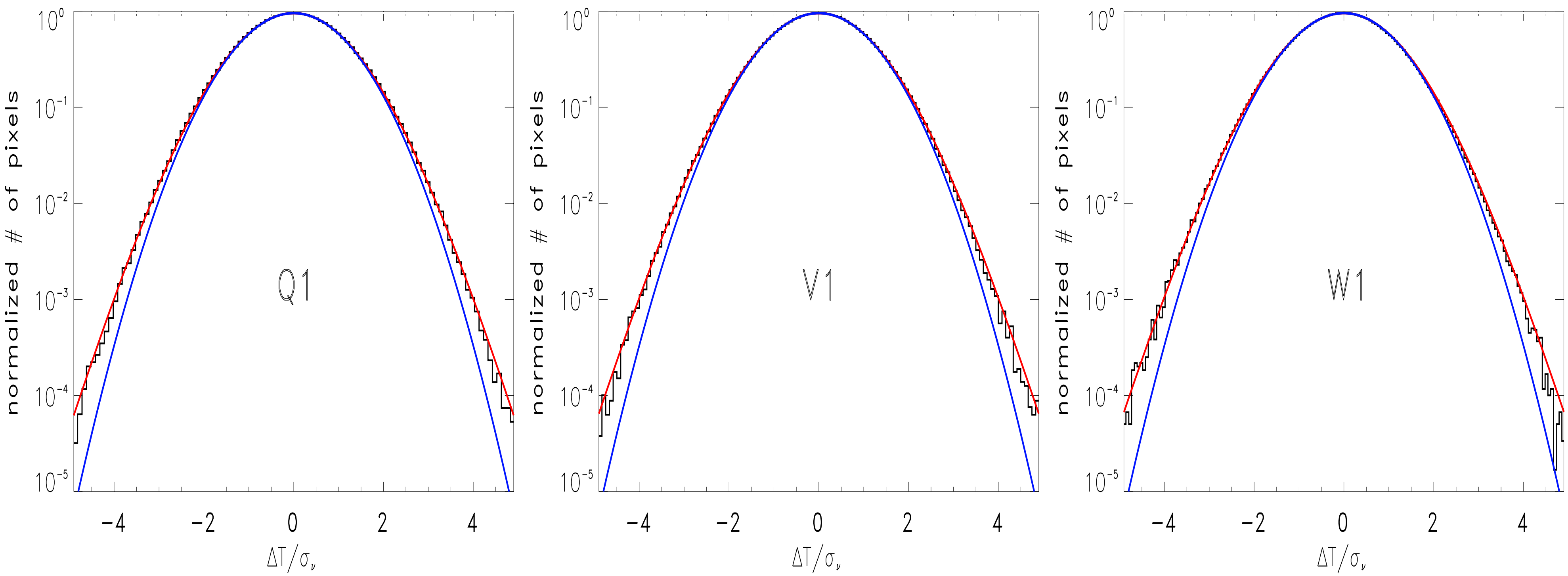
- We investigate just negative temperature fluctuations, since known GF contribute as positive ones.
- We analise 8 WMAP DIFFERENTIAL ASSEMBLIES in 3 high-frequency bands (remember that GF are frequency dependent): Q1, Q2, V1, V2, W1, W2, W3, and W4.
- We use two WMAP masks: Kp0 and Kp2 (23% and 15% cut-sky, respectively), to test the robustness of our results against GF.
- We generate 1000 Monte Carlo sky maps containing CMB signal (with zero mean) plus instrument noise signal (with non-zero mean). The analysis of these maps leds to estimate the effect of detector's noise as being responsible for Non-Gaussianities in WMAP data.

Here it is: **temperature distribution function of a CMB map**



Temperature distribution function, for several CMB maps



Non-extensive Statistical Mechanics

From the q-entropy[§]

$$S_q \equiv k\{1 - \int dx\,[P_q(x)]^q\}/(q-1)\,,$$

one obtains the non-extensive probability distribution

$$P_q(\Delta T) = A_q\, e_q^{-B_q \Delta T^2}\,,$$

A_q is the normalization constant, obtained from $\int dx\, P_q(x) = 1$.
The q-exponential is defined by $e_q^z \equiv [1+(1-q)z]^{1/(1-q)}$, for $[1+(1-q)z] \geq 0$, while $e_q^z = 0$ otherwise.
Note that, in the limit $q \to 1$, we recover the Gaussian distribution

$$\lim_{q\to 1} P_q(\Delta T) = P^{\mathrm{Gauss}}(\Delta T) = A\, e^{-B(\nu)\Delta T^2}\,,$$

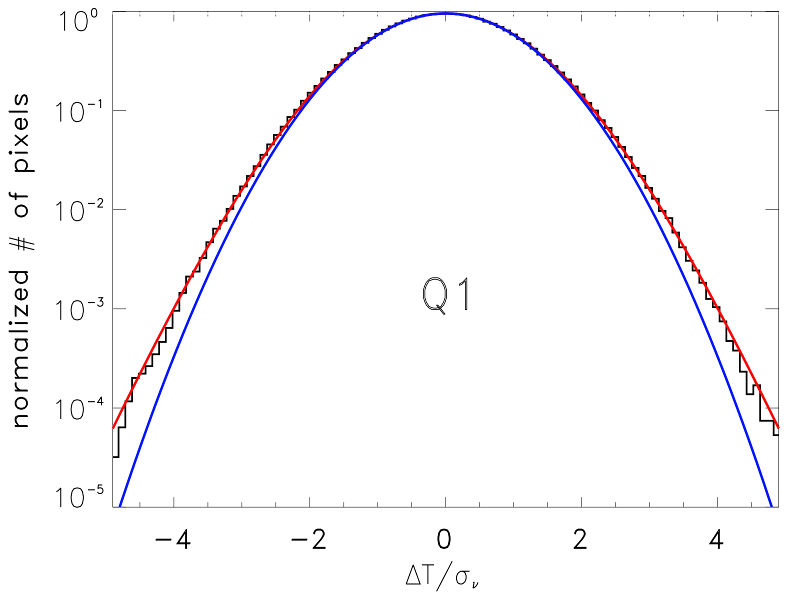
where $A \equiv 1/(\sigma_\nu \sqrt{2\pi})$, $B(\nu) \equiv 1/(2\,\sigma_\nu^2)$, and σ_ν^2 is the variance of the Gaussian distribution.

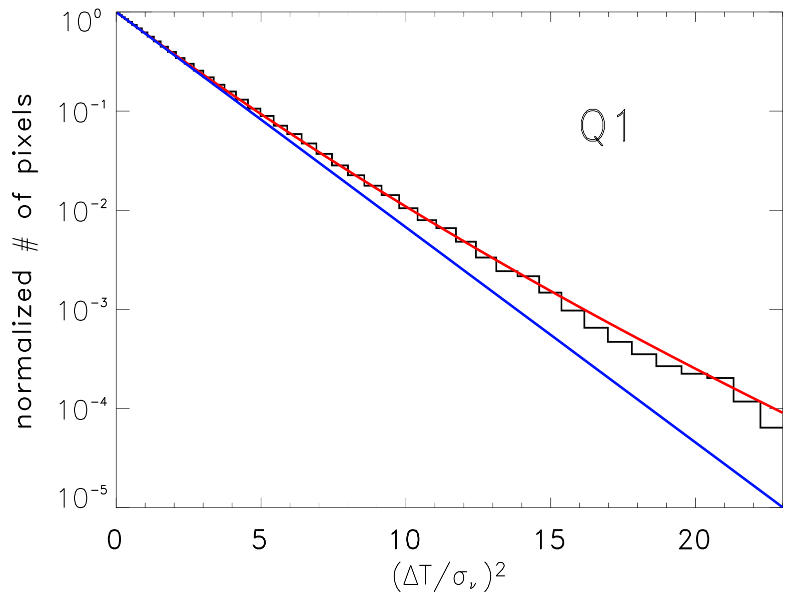
[§] AB, C. Tsallis, and T. Villela, PLA (2006) **356**, 426-430 (astro-ph/0512267)

Assuming $A_q = A = 1$ and $Bq = B(\nu)$ one obtains that both Gaussian and non-extensive distributions have the same amplitude and the same slope at the initial point $\Delta T = 0$ in the following plots:

# of pixels (with temp. fluct. ΔT) *versus* $\Delta T/\sigma_\nu$ (or simply ΔT), and

# of pixels (with temp. fluct. ΔT) *versus* $(\Delta T/\sigma_\nu)^2$





First-year WMAP data $q = 1.045 \pm 0.05$ §

$\mathbf{Q}$ = coadded[Q1+Q2]; $\mathbf{V}$ = coadded[V1+V2]; $\mathbf{W}$ = coadded[W1+W2+W3+W4];

where {Q1,Q2,V1,V2,W1,W2,W3,W4} are the 8 WMAP Differential Assemblies

REMEMBER:

$$P_q(\Delta T) = e_q^{-B\Delta T^2}$$

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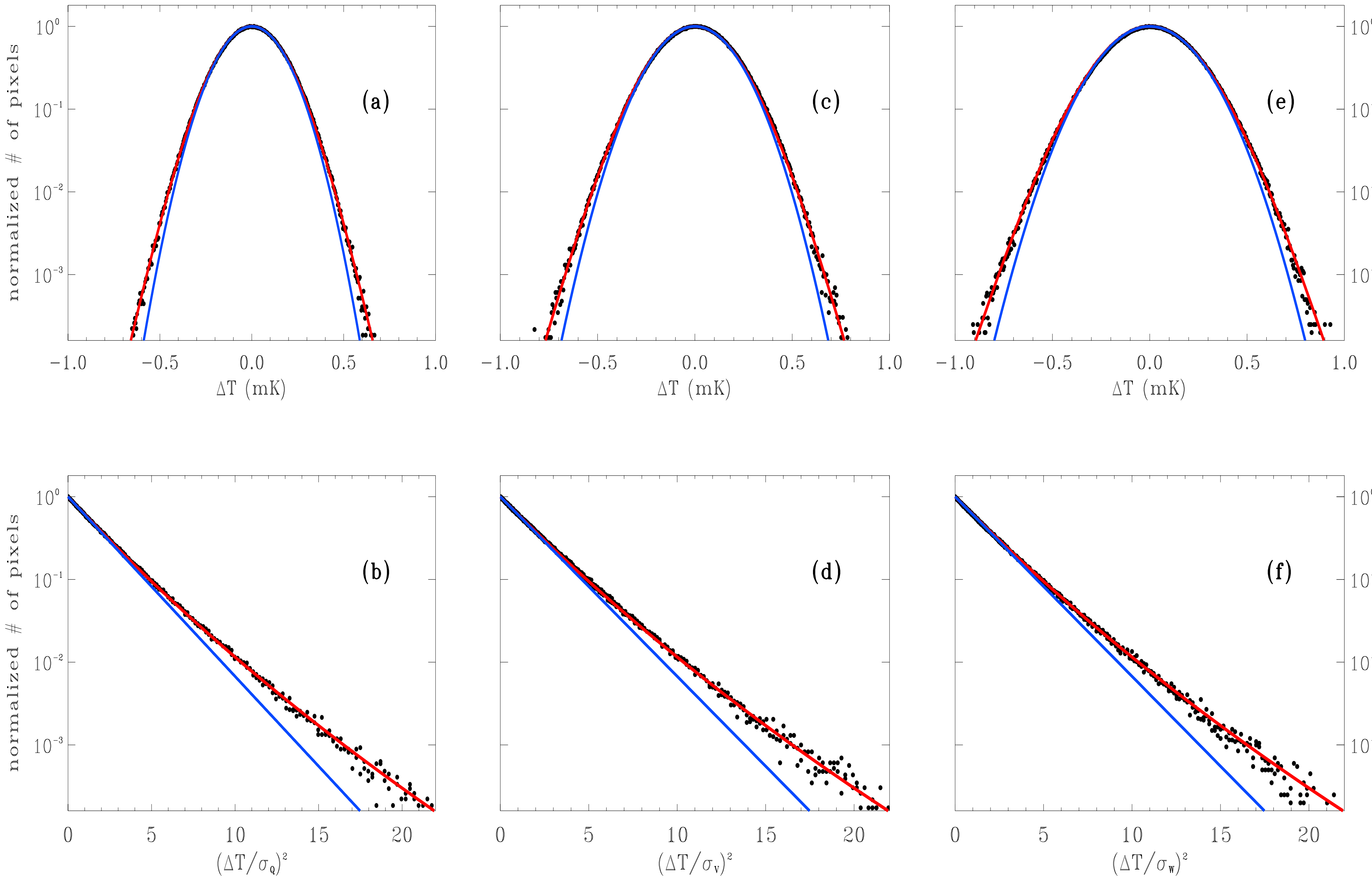
§ AB, C. Tsallis, T. Villela, PLA (2006) **356**, 426-430



normalized # of pixels
(a)
(c)
(e)
(b)
(d)
(f)
ΔT (mK)
$(\Delta T/\sigma_Q)^2$
$(\Delta T/\sigma_V)^2$
$(\Delta T/\sigma_W)^2$

Three-year WMAP data $q = 1.045 \pm 0.05$ §

Analyses of the 8 WMAP Differential Assemblies {Q1,Q2,V1,V2,W1,W2,W3,W4}.

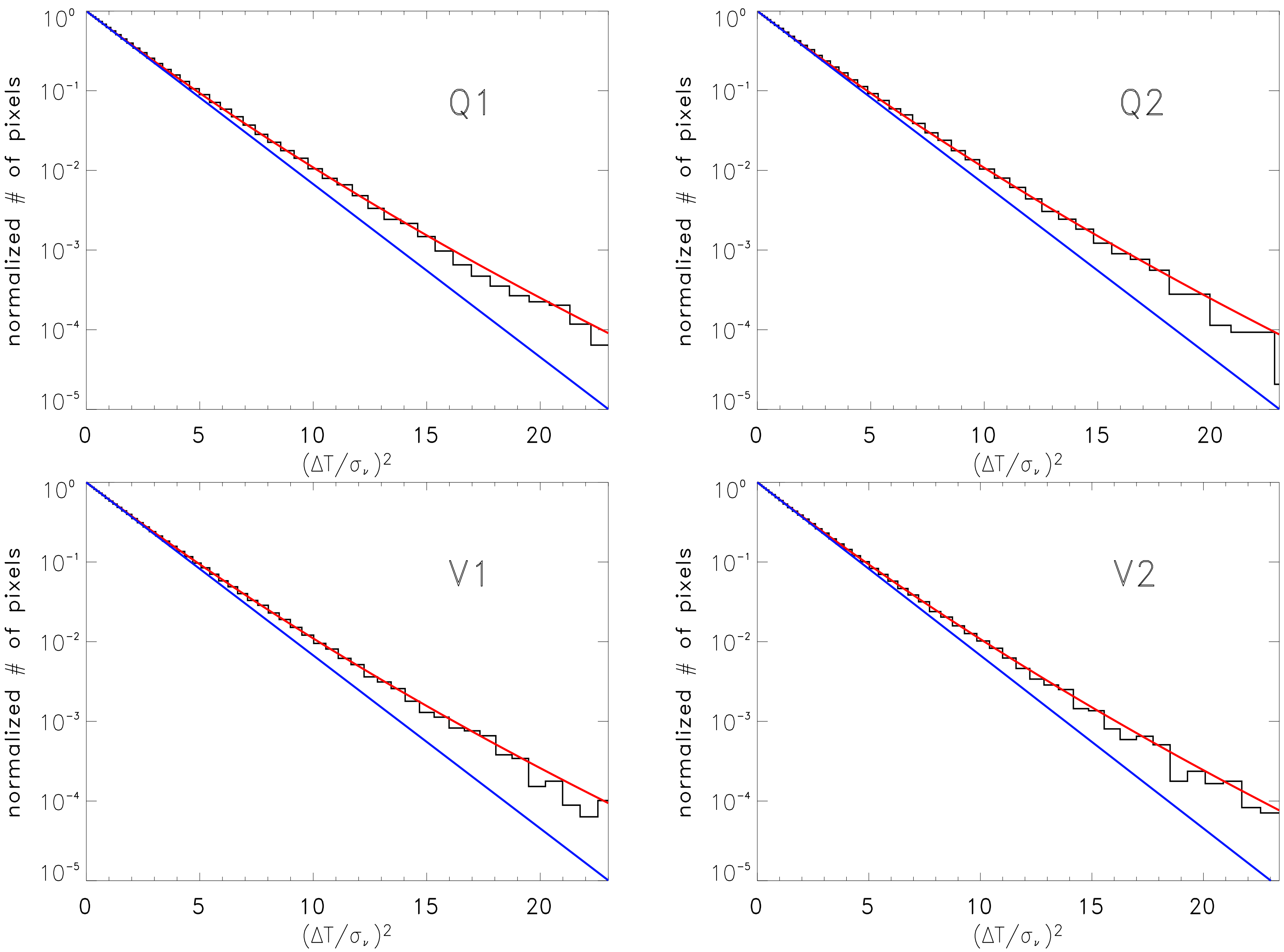
REMEMBER:

$$P_q(\Delta T) = e_q^{-B\Delta T^2}$$

$$P(\Delta T) = e^{-B\Delta T^2}$$

§AB, C. Tsallis, T. Villela, in preparation

Temperature distribution of three-year WMAP data: red curves are e_q with $q = 1.045$



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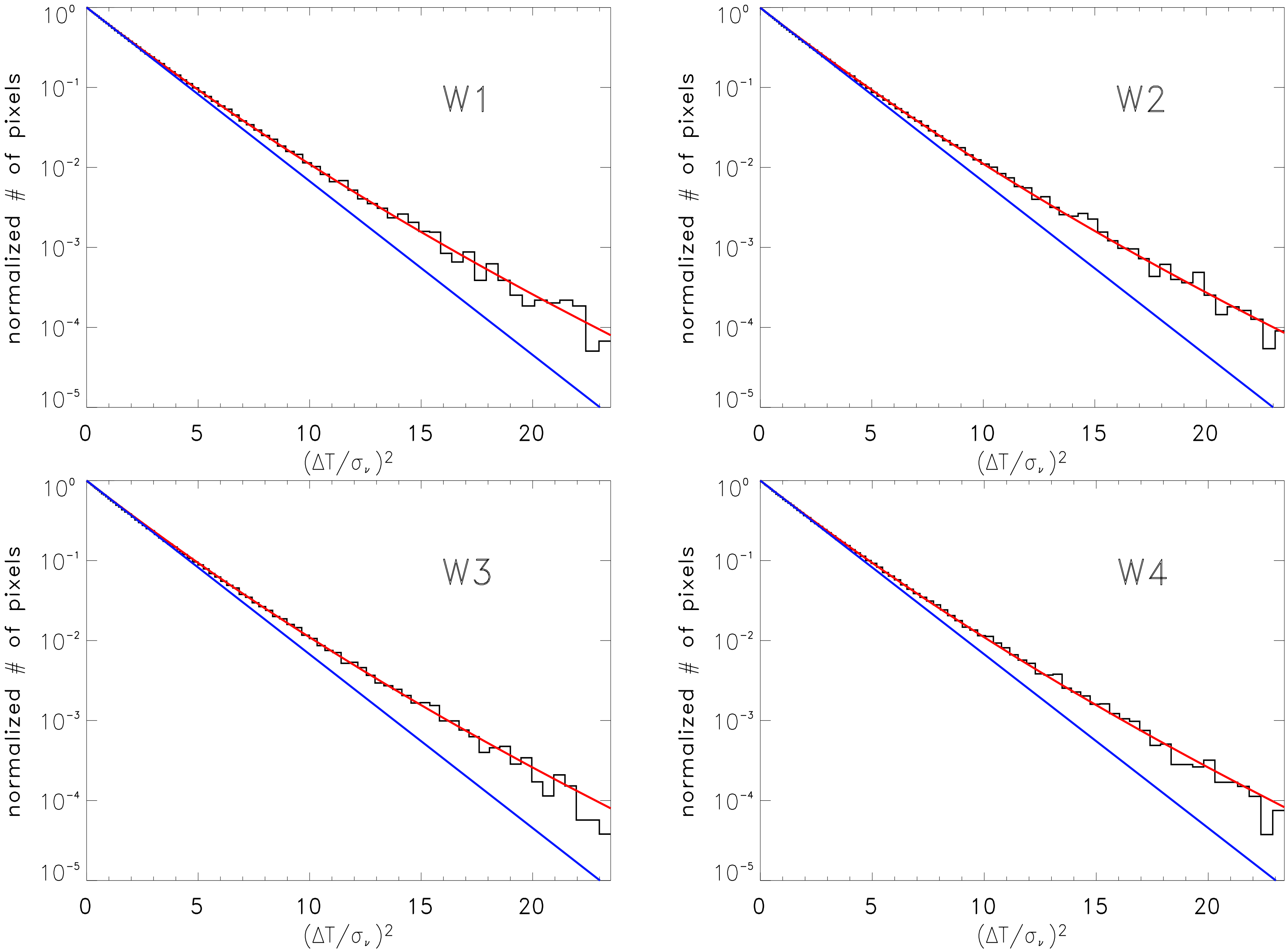

Conclusions

- Small deviations from Gaussianity detected in WMAP CMB maps are well described by a probability distribution emerging from the non-extensive statistical mechanics.

  Since the $\mathrm{T}_{\text{instrument noise}}$ **varies as** $1/\sqrt{\mathrm{N}_{\text{observations}}}$**, the comparison between the first-year data with the three-year data, all data well described with the same parameter** $q = 1.045 \pm 0.005$**, shows that the non-Gaussianities found are intrinsic to the CMB.**