Dynamics and nonextensivity of elementary 1D cellular automata with long range memory

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We numerically study the dynamics of elementary 1D cellular automata (CA), where the binary state $\sigma_i(t) \in \{0,1\}$ of a cell i does not only depend on the states in its local neighborhood at time $t-1$, but also on the memory of its own past states $\sigma_i(t-2), \sigma_i(t-3), ...$ We assume that the weight of this memory decays proportionally to $(t-\tau)^{-\alpha}$, with $\alpha \geq 0$ ($\alpha \to \infty$ corresponds to the usual CA, where the states at time t are determined solely by the neighbor states at time $t-1$). Since the memory function is summable for $\alpha > 1$ and nonsummable for $0 \leq \alpha \leq 1$, we expect pronounced qualitative and quantitative changes of the dynamical behavior at $\alpha = 1$. This is precisely what our simulations exhibit, particularly for the time evolution of the Hamming distance H of initially close trajectories. More specifically, we typically expect the asymptotic behavior $H(t) \propto t^{1/(1-q)}$, where q is the entropic index associated with nonextensive statistical mechanics. In all cases, the function $q(\alpha)$ exhibits a sensible change at $\alpha = 1$. Focusing on the "regular" (Wolfram class II) rules 61 and 111, we obtain that, for rule 61, $q = 0$ for $0 \leq \alpha \leq 1$, and $q < 0$ for $\alpha > 1$, whereas the opposite behavior is found for rule 111. This fact points at a very rich dynamics intimately linked to the interplay of local lookup rules and long range memory. Finite size scaling proportional to the number N of cells indicates that the range of the power-law regime for $H(t)$ indeed diverges with system size N. Similar studies have been carried out for other rules, e.g., the famous "universal computer" rule 110.