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SCHOOL and CONFERENCE
on
COMPLEX SYSTEMS
and
NONEXTENSIVE STATISTICAL MECHANICS

31 July - 8 August 2006

NonBoltzmann-Gibbsensemblesinhadronicproductionprocesses

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Non Boltzmann-Gibbs ensembles in hadronic production processes

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International Summer School and Workshop on

Complex Systems and Nonextensive Statistical Mechanics

(31 July-8August 2006)

Miramare, Trieste, Italy

or:

Story of
fluctuations, correlations and the nonextensivity
in high energy multiparticle production processes

on examples of:

(*) **G.Wilk and Z.Włodarczyk,** *Fluctuations, correlations and nonextensivity;* ***cond-mat/0603157***

(*) **M.Biyajima, T.Mizoguchi, N.Nakamija, N.Suzuki and G.Wilk,** *Modified Hagedorn formula including temperature fluctuations...;* ***hep-ph/0602120;***

(*) **T.Osada and G.Wilk,** *Nonextensive hydrodynamics for high energy heavy ion collisions;* **in preparation**

Our group: **O.Utyuzh, G.Wilk, Z.Włodarczyk (Poland); F.S.Navarra (Brazil)**
M.Biyajima, M.Kaneyama, T.Mizoguchi, N.Nakamija, N.Suzuki,
T.Osada (Japan)

Content: in the field of hadronic multiparticle production processes, it will be shown that:

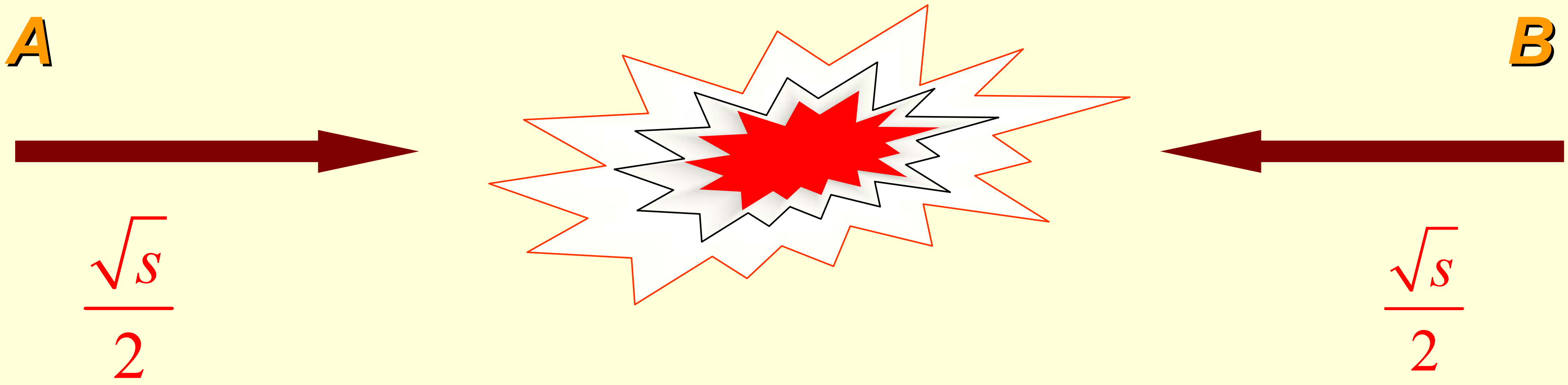
1. **Measured particle distributions are affected by internal fluctuations and correlations presented in the hadronizing system (i.e., in system converting initial energy into observed particles):**
   $f(y) \Rightarrow f_q(y)$
   Poissonian P(N) ➔ Negative Binomial P(N)
   flows…

2. **The best way to account for them in a model independent way is to use some form of non Boltzmann-Gibbs ensembles, for example Tsallis or EGE (or …(?)).**

3. **The more we know on the internal dynamics (for example in the form of mass spectrum of resonances in Hagedorn model) the near we are the BG approach. However, for a time being such knowledge is not model independent, only q-statistics allows for model independent presentation of experimental results.**

Hadronic production processes:

Total energy available is E=√s (~ 100 − 10000 GeV)

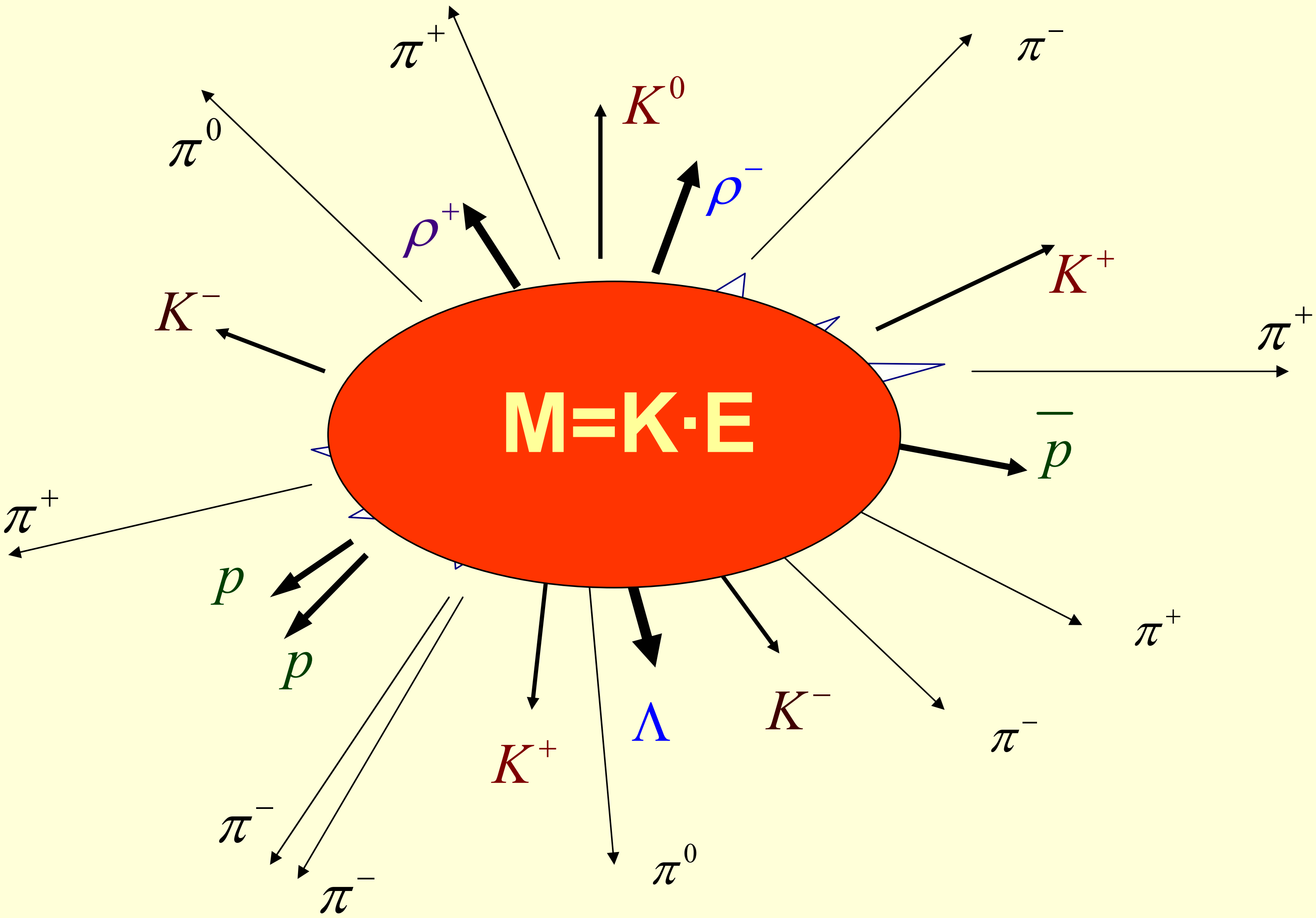


but :

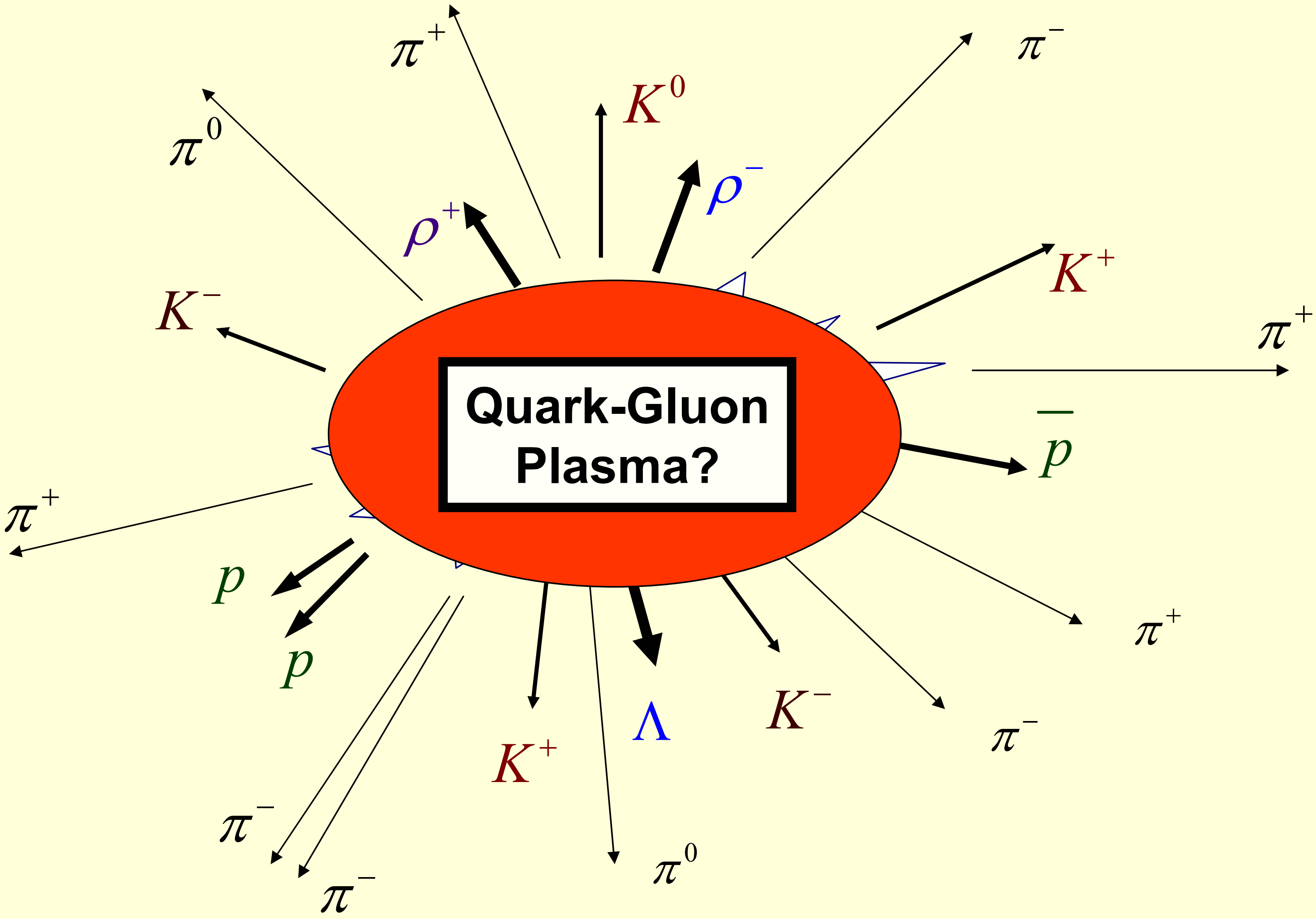
*(*) only fraction M=K- E of initial energy E is used for production of secondaries (**K = inelasticity**)*

*(*) the remaining (1-K)•E is going into remnants of original colliding particles (called **leading particles**)*

Hadronic production processes: **general view**

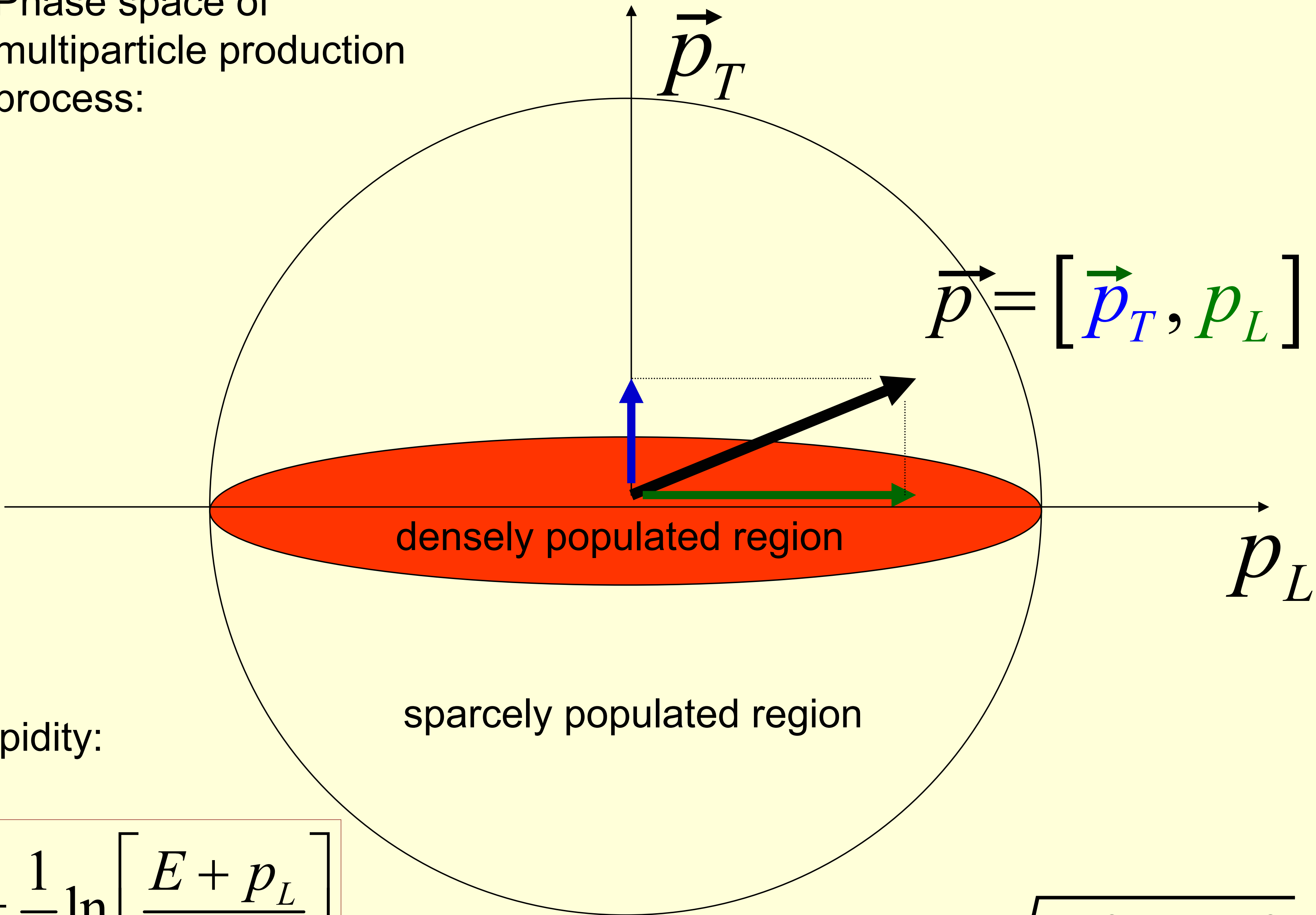


…large number <N> ~ 100 – 1000 secondaries is produced



The most important question is: ***what kind of matter is in this blob? One expects to get answer from heavy ion coolisions and expects that they confirm our expectations that a new kind of matter, the so called Quark-Gluon Plasma is being produced there and hadronizes into observed secondaries.***

Phase space of multiparticle production process:

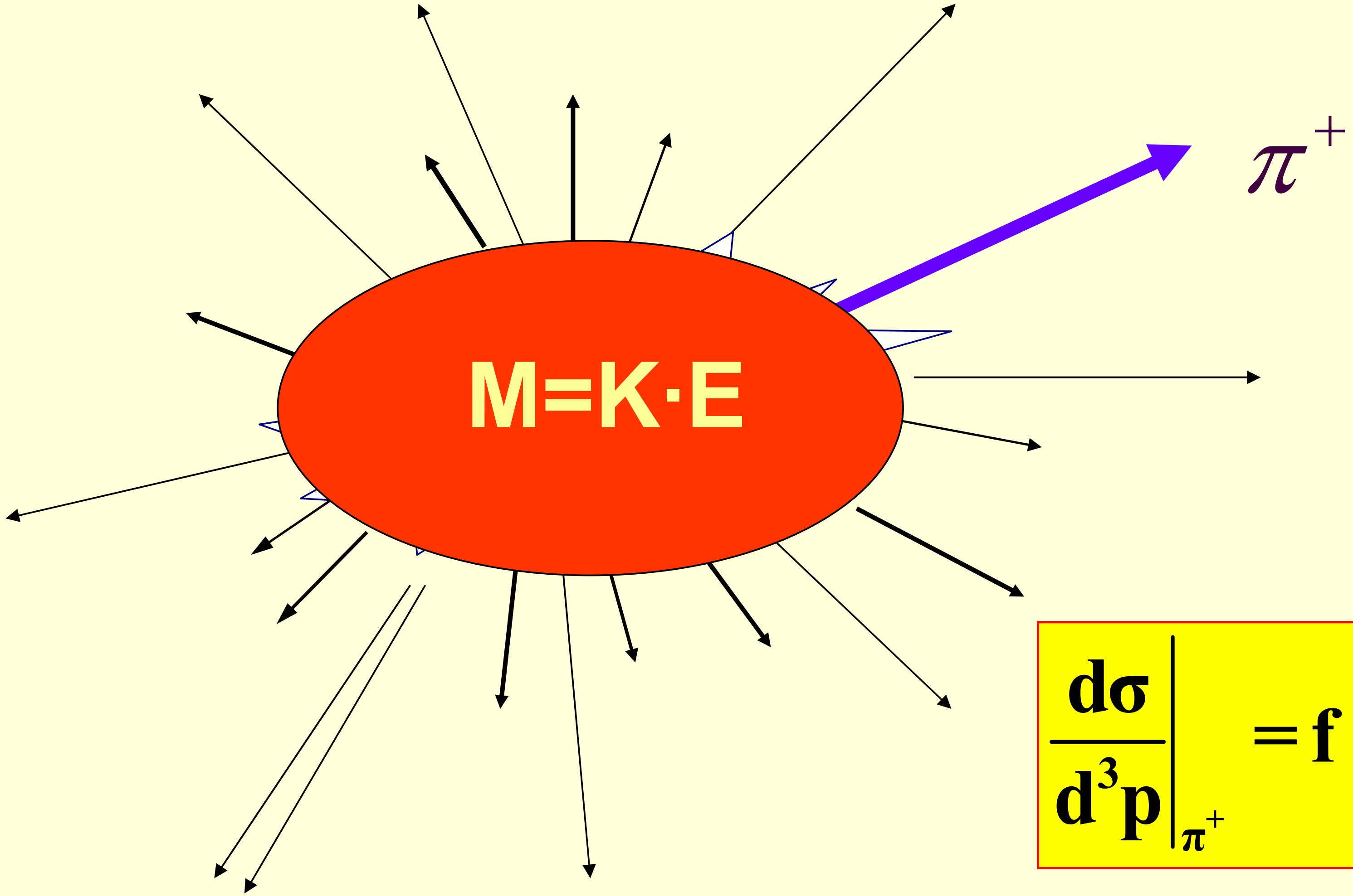


Rapidity:

$$y = \frac{1}{2} \ln \left[\frac{E + p_L}{E - p_L} \right]$$

$$E = \sqrt{m^2 + \vec{p}^2}$$

Hadronic production processes: ***large number secondaries is produced ...***

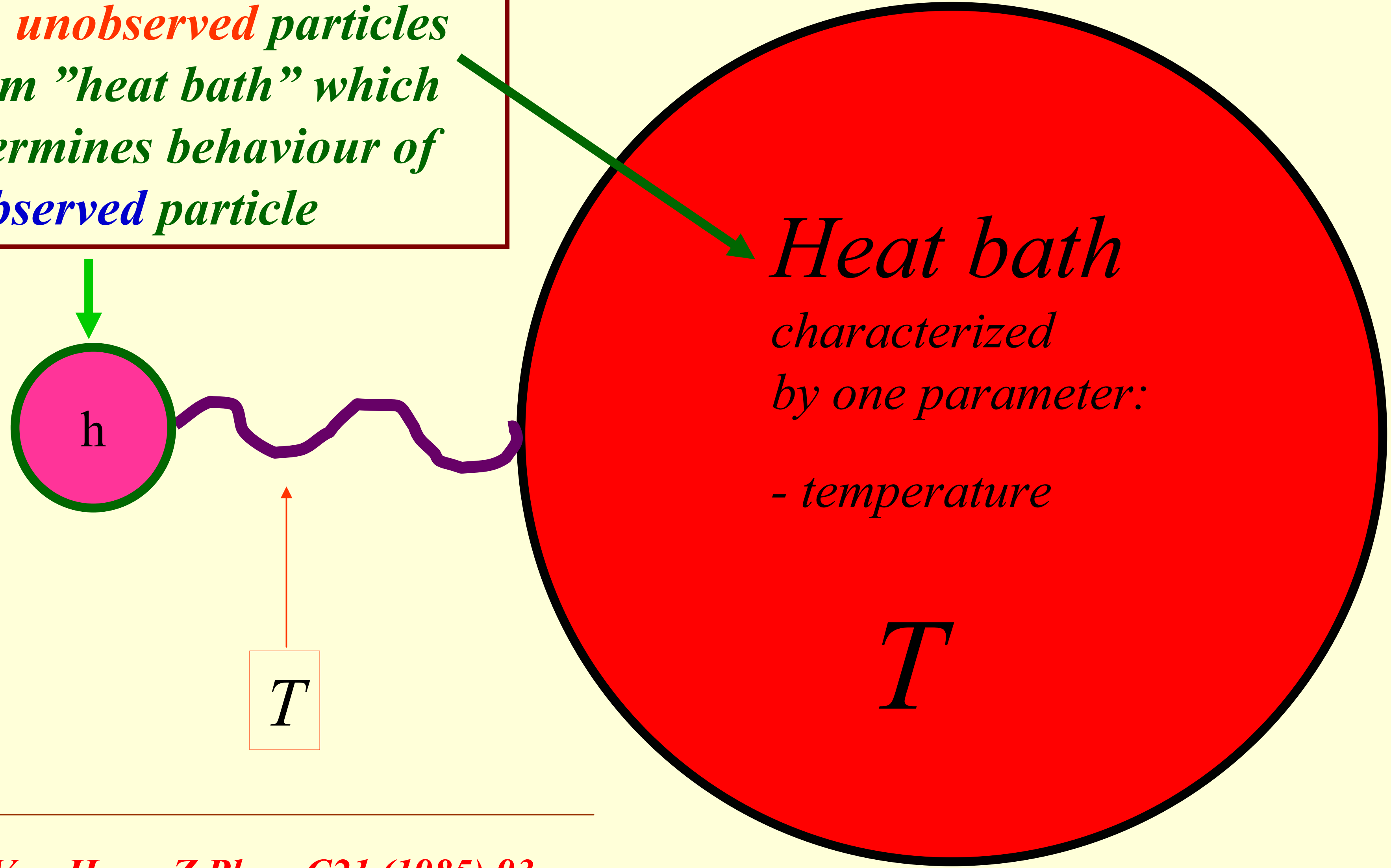


$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f\left(\vec{p}_T, p_L\right)$$

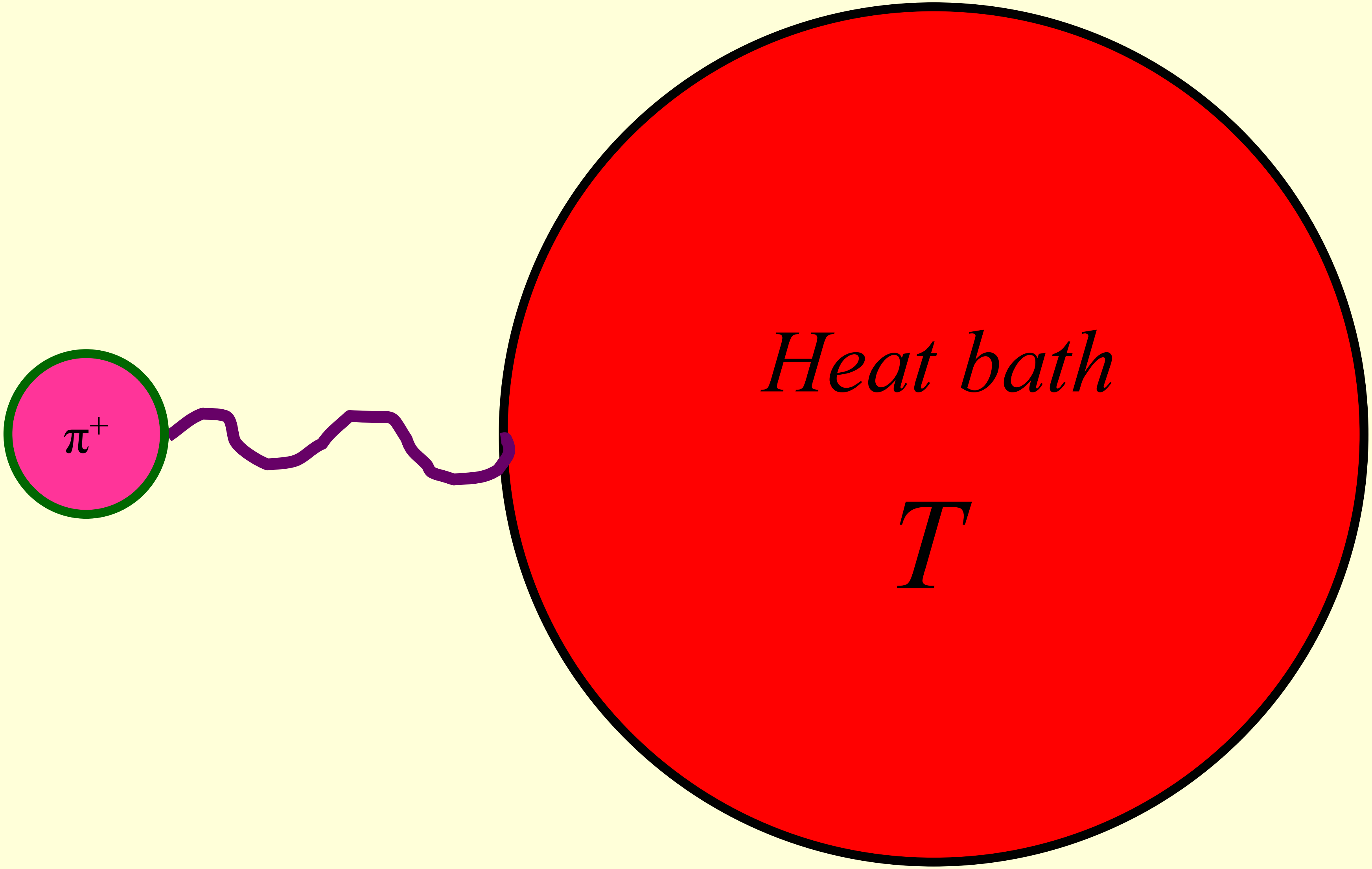
... but usually only one of them is observed and single particle distributions are formed

➨➨ *this calls for statistical (thermodynamical) description of such processes....*

N-particle system ⇒
N-1 unobserved particles
form "heat bath" which
determines behaviour of
1 observed particle



L.Van Hove, Z.Phys. C21 (1985) 93,
Z.Phys. C27 (1985) 135.



$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f\left(\vec{p}_T, p_L\right) = C \cdot \exp\left(-\frac{E}{T}\right) = C \cdot \exp\left(-\frac{\sqrt{\vec{p}_T^{\,2} + p_L^2 + m^2}}{T}\right)$$

BUT: ***in such "thermodynamical" approach one has to***

remember tacit assumptions of infinity and homogenity

made - only then behaviour of the observed particle will

be characterised by single parameter - the "temperature" **T**

In reality: ***this is true only approximately and we are interested***

in the examples when system is

not infinite or/and not homogeneous

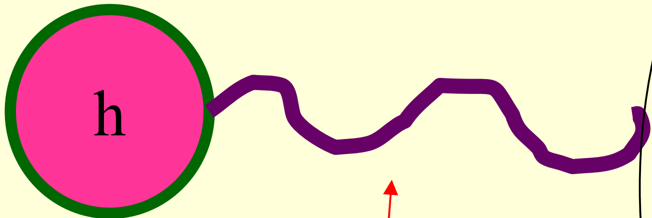
In both cases: ***fluctuations occur and new parameters in***

addition to T are necessary

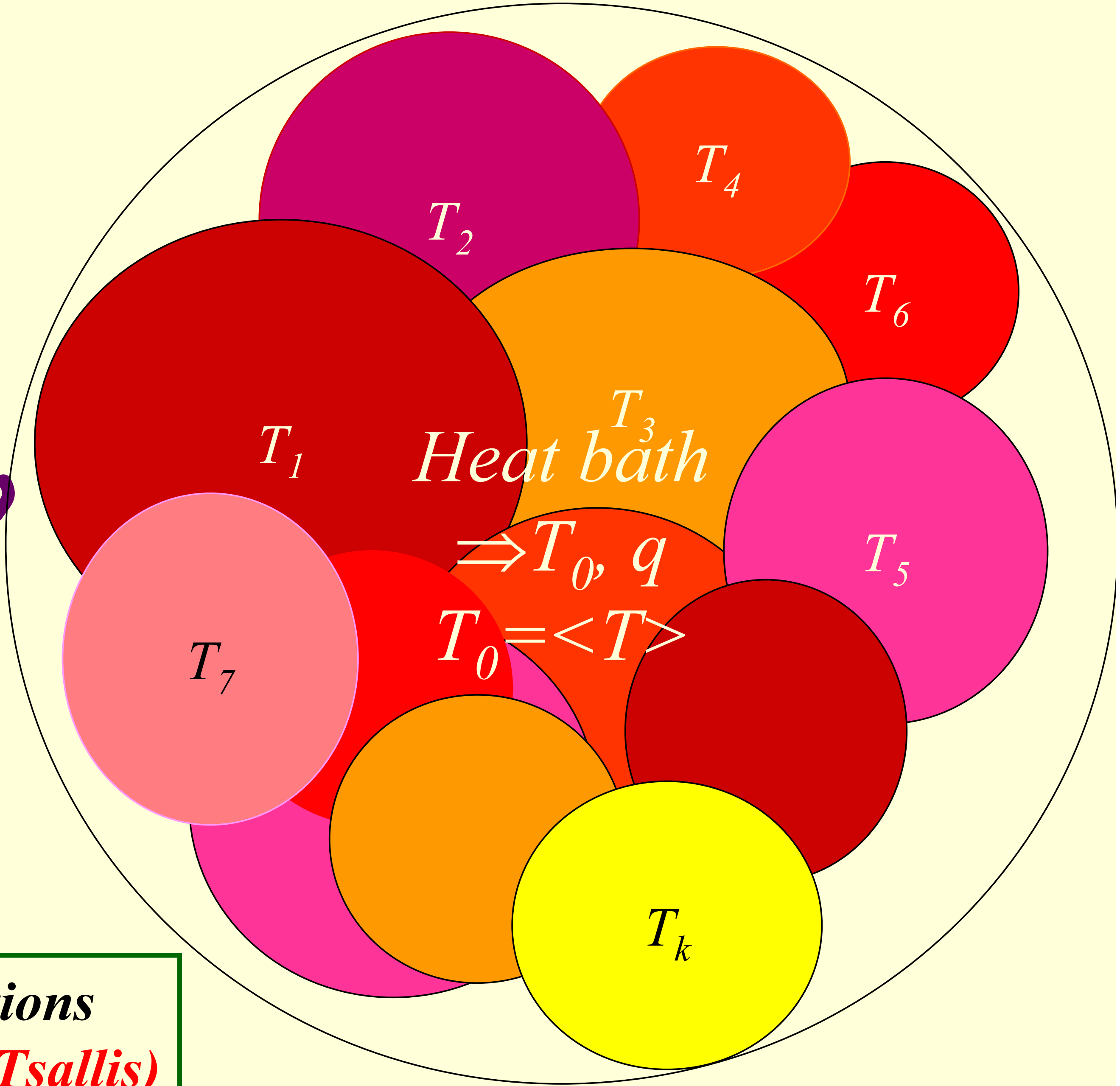
Heath bath is not homogeneous ......

T varies ⇔

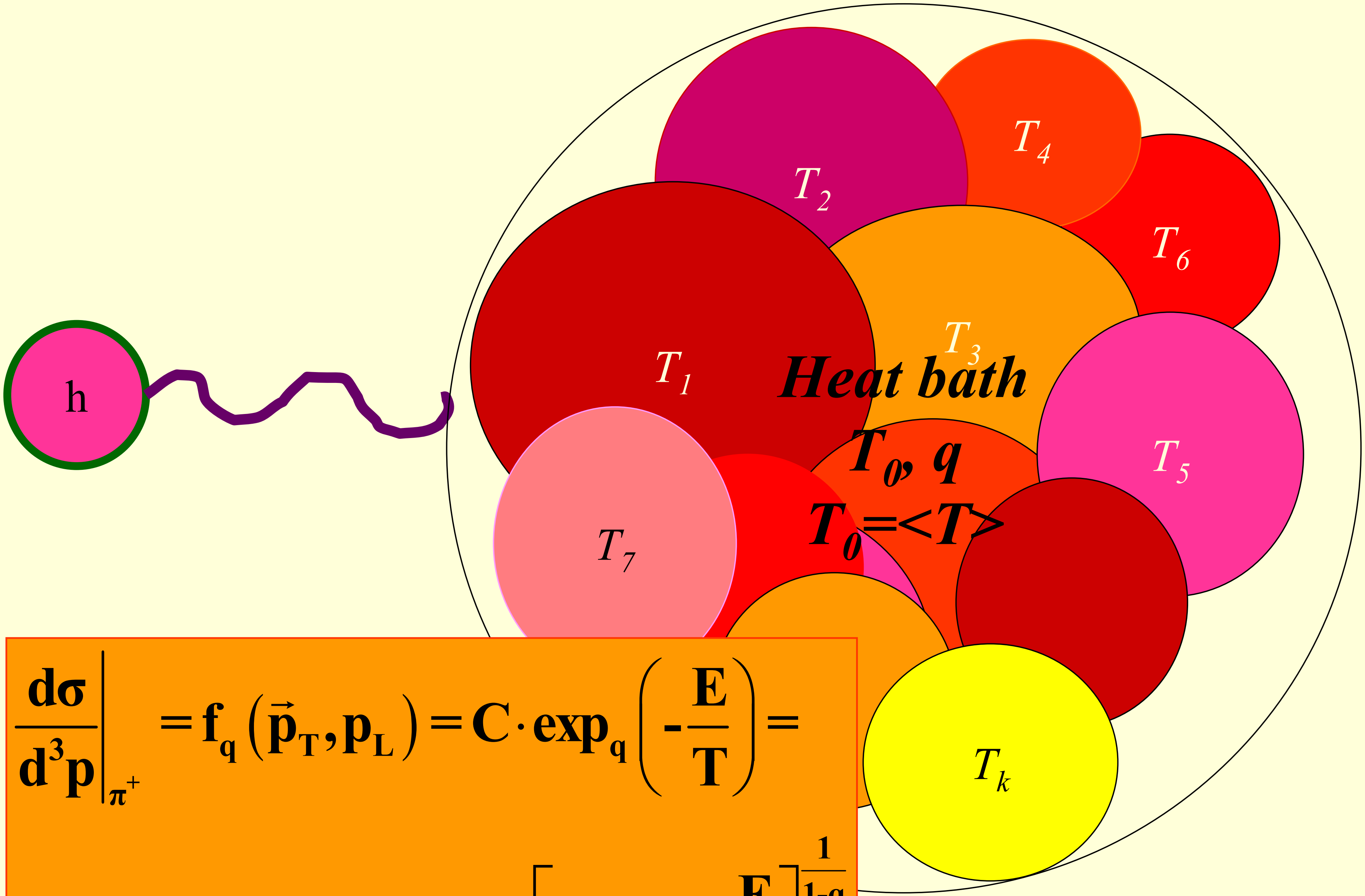
fluctuations...



$T_0 = \langle \mathbf{T} \rangle$, q

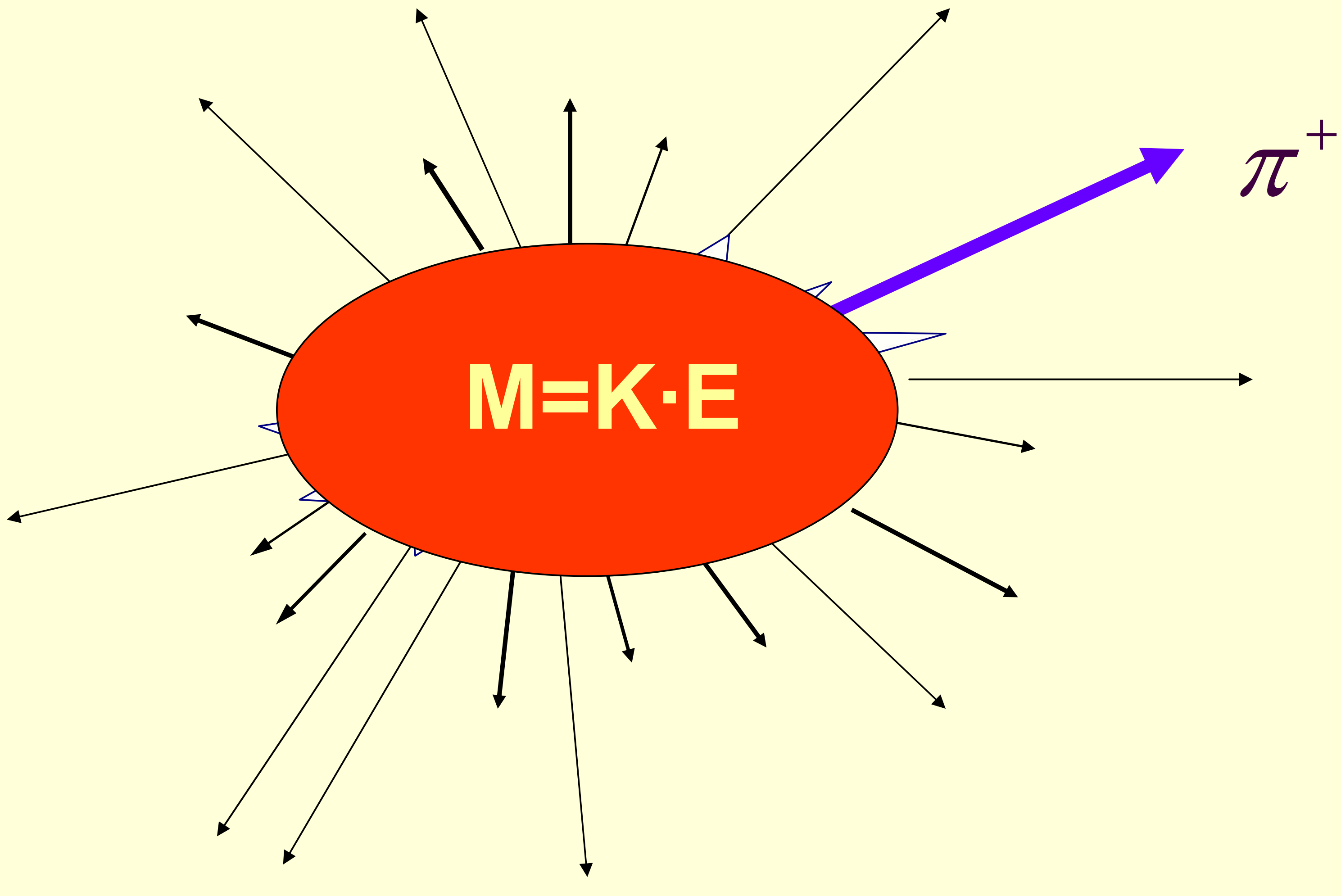


q - measure of fluctuations of T ➡ ***q-statistics (Tsallis)***



$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f_q\left(\vec{p}_T, p_L\right) = C\cdot \exp_q\left(-\frac{E}{T}\right) = C\cdot\left[1-(1-q)\frac{E}{T}\right]^{\frac{1}{1-q}}$$

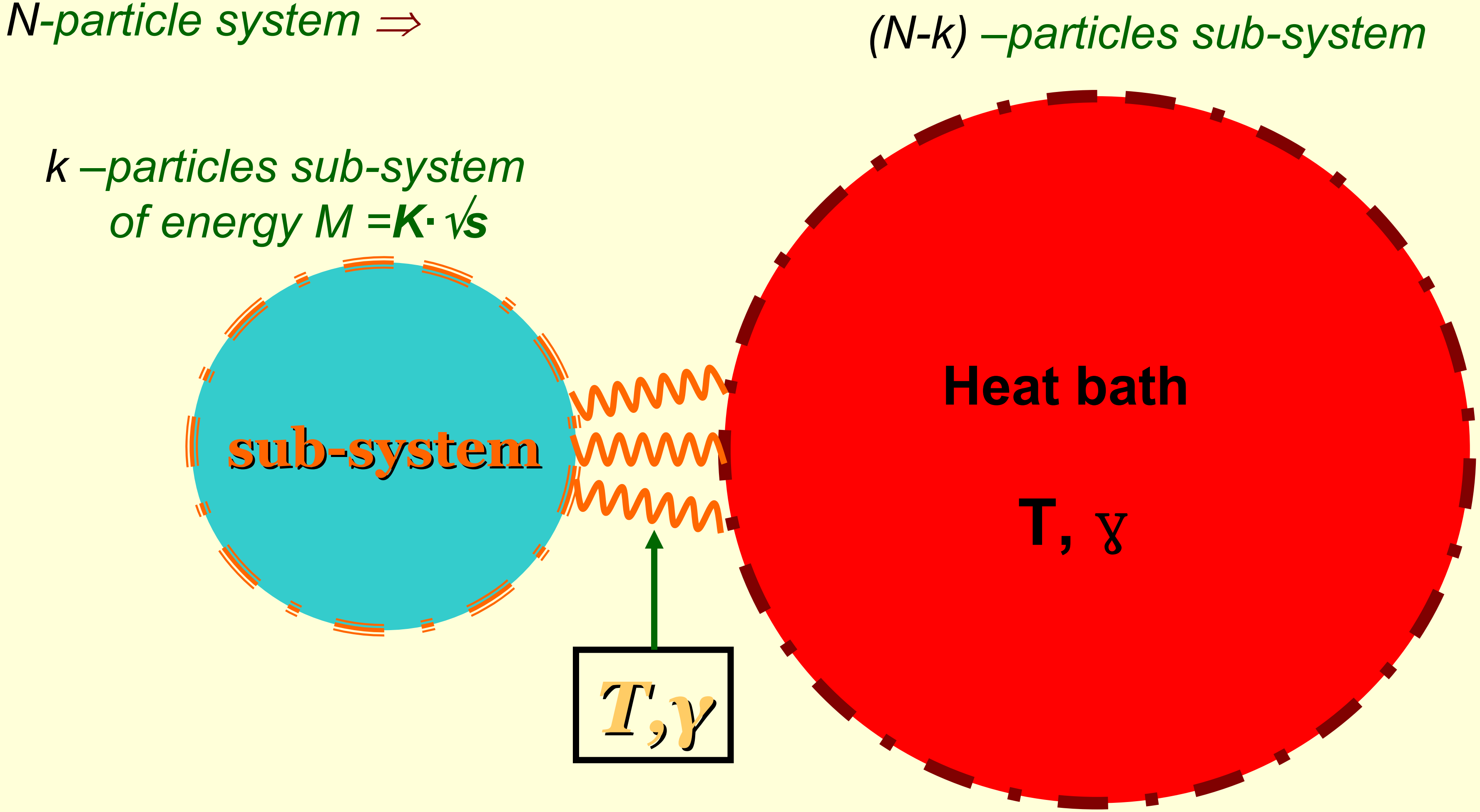
Hadronic production processes: ***... only one of them (here π⁺) is observed***



…. but amount of energy used for production of particles, M=K•E, can fluctuate

Heath bath is "small"......

N-particle system ⇒

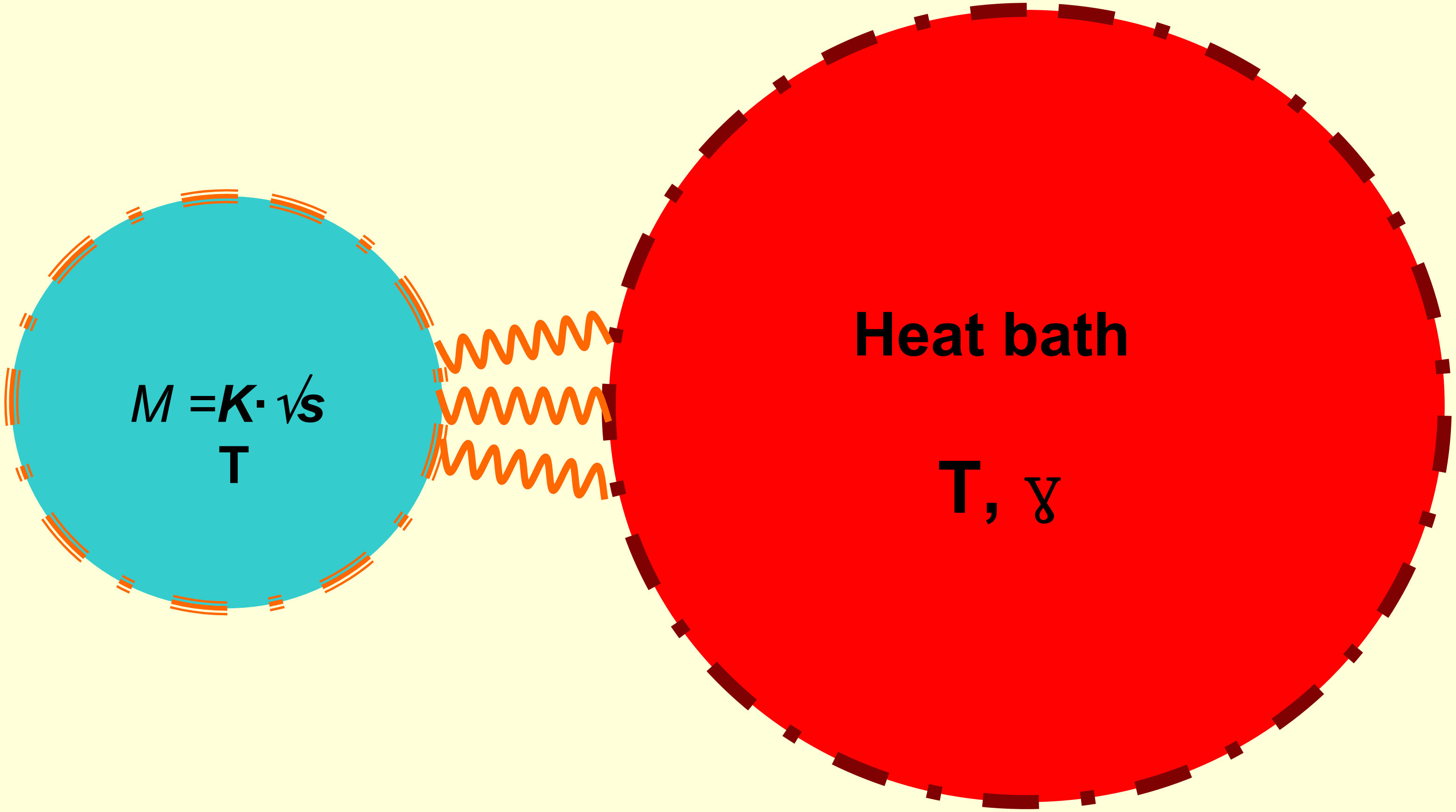


ɣ *- measure of fluctuations of (inelasticity) K*
➡ *in* ***Extended Gaussian ensemble***

T.Osada,O.Utyuzh, G.Wilk, Z.Włodarczyk
Eur. Phys. J. B 50, 7 (2006)

k –particles sub-system
of energy ***M =K· √s***

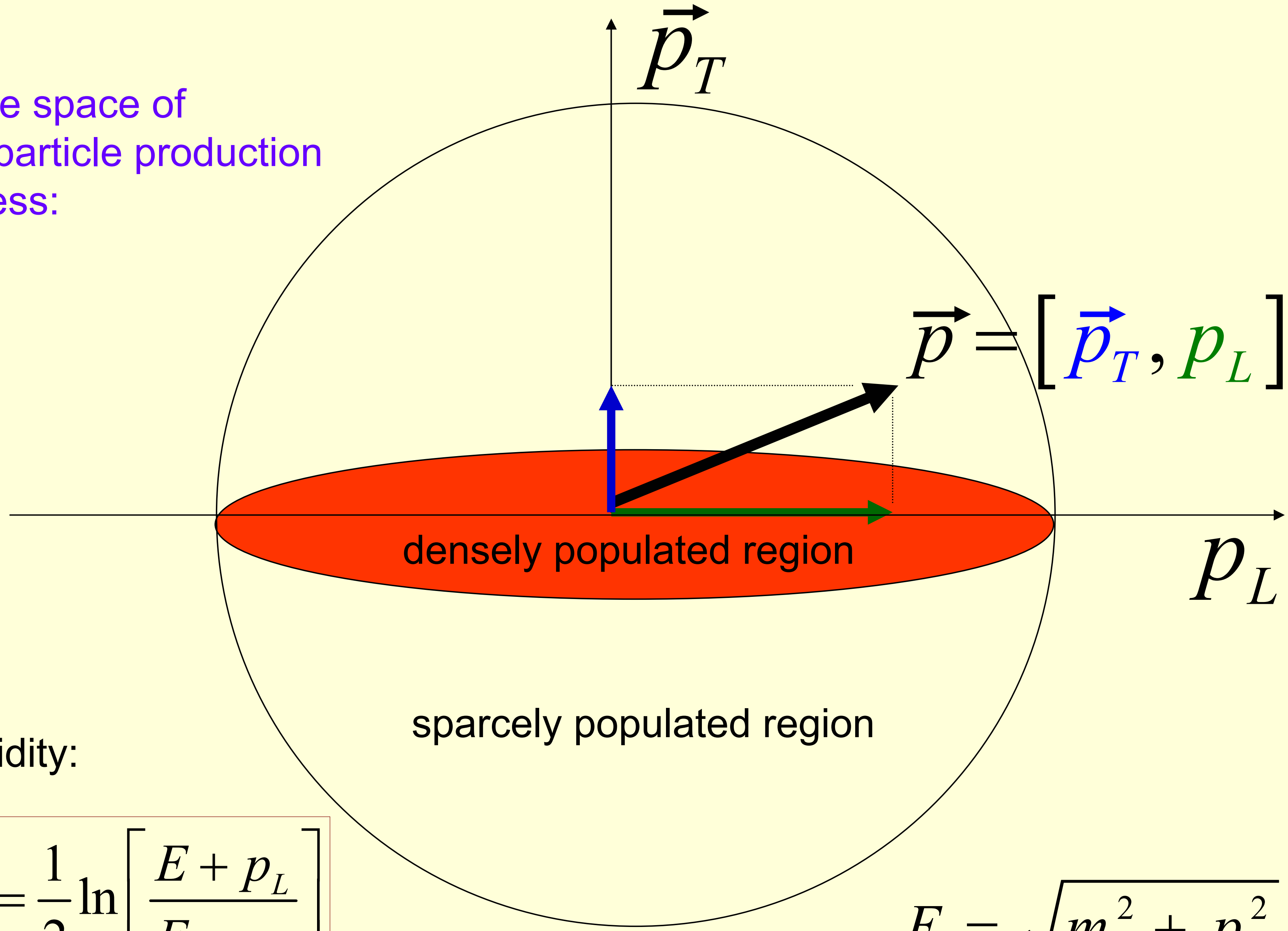
(N-k) –particles sub-system



$$f(M) = C \cdot \exp\left[-\gamma \left(M - \langle M \rangle \right)^2 \right]$$

Examples from hadronic production:

Phase space of multiparticle production process:



Rapidity:

$$y = \frac{1}{2} \ln \left[\frac{E + p_L}{E - p_L} \right]$$

$$E = \sqrt{m^2 + \vec{p}^{\,2}}$$

In multiparticle production processes one usually measures distributions of:

(*) either $\mathbf{p_L}$ (and averages over p_T) ⇨ $\mathbf{T=T_L}$ (**"partition" temperature**)

(*) or $\mathbf{p_T}$ (for fixed, usually small, p_L) ⇨ $\mathbf{T=T_T}$ (**"usual" temperature**)

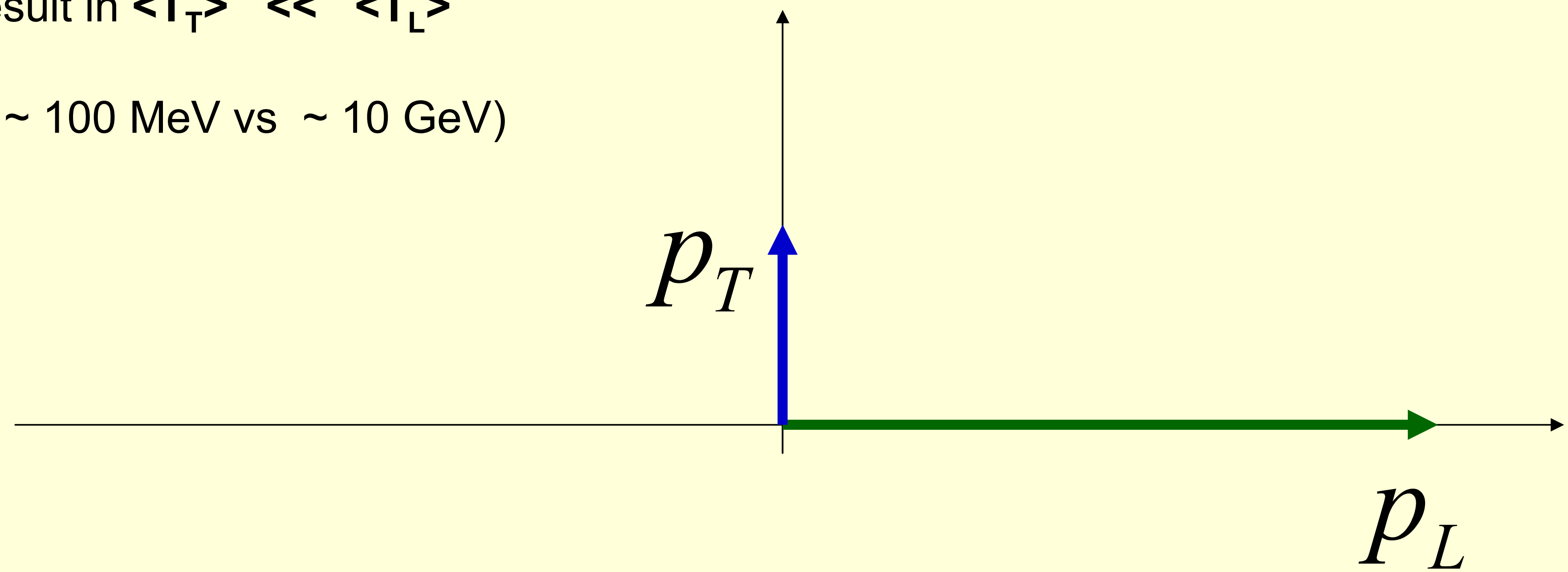
Data show that $\langle p_T \rangle \ll \langle p_L \rangle$ and analyzed by means of formula:

$$F(p) \sim \exp[-p/T] \quad \Rightarrow \quad F_q(p) \sim \exp_q[-p/T]$$

$$q = (q_L, q_T)$$

result in $\langle T_T \rangle \ll \langle T_L \rangle$

(~ 100 MeV vs ~ 10 GeV)



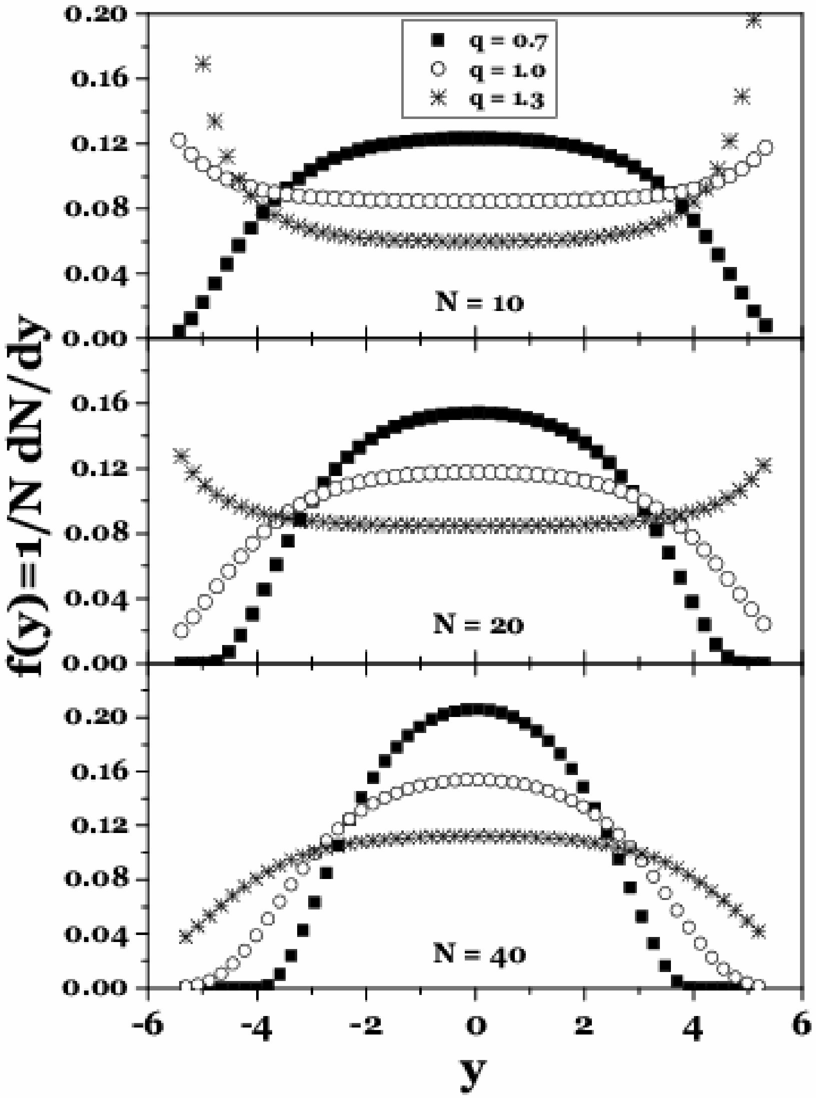


Fig. 1. The examples of the most probable rapidity distributions obtained by q-statistical approach. The object (fireball, string,...) of mass $M = 100$ GeV decays into N secondaries of (transverse) mass $m_T = 0.4$ GeV each. Figs. (a) and (b) show results for N leading to *Feynman scaling* ($\beta = 0$) or *Feynman q-scaling* ($\beta_{q=1.3} = 0$). Fig. (c) shows example of such N that all $\beta > 0$.

examples of:

$$p(y) = \frac{1}{Z}\exp\left[-\beta\mu_T \cdot \cosh y\right]$$

$$(q = 1)$$

and

$$p_q = \frac{1}{Z_q}\left[1-(1-q)\beta_q\mu_T \cdot \cosh y\right]^{\frac{1}{1-q}}$$

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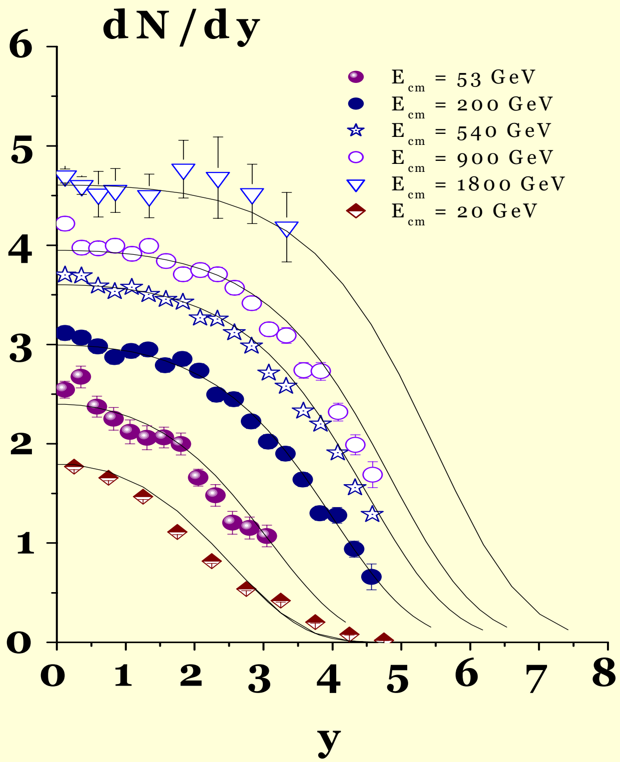
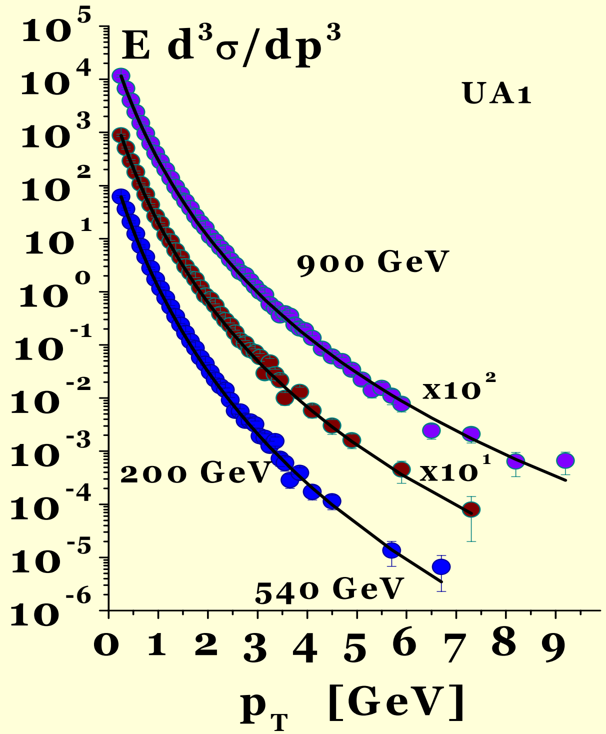


$$y = \frac{1}{2} \ln \left[\frac{E+p_L}{E-p_L} \right]$$



$S^{1/2}$	q_L	$T_L=1/\beta_L$
200	1.203	12.12
546	1.262	22.38
900	1.291	29.47

$$q = \frac{q_L T_L^2 + q_T T_T^2}{T^2} - \frac{T_L^2 + T_T^2}{T^2} + 1$$

$$\Rightarrow \quad q \approx q_L \;\; because \;\; T = \sqrt{T_L^2 + T_T^2} \approx T_L$$

$S^{1/2}$	q_T	$T_T=1/\beta_T$
200	1.095	0.134
546	1.105	0.135
900	1.110	0.140

For meaning of $\boldsymbol{q_T}$ ↔ *see GW and ZW, PRL 84 (2000) 2770; Physica A305 (2002) 227*

GW and ZW, PRL 84 (2000) 2770; Physica A305 (2002) 227 (and many others, like C.Beck ....):

➡➡➡

q results from intrinsic fluctuations in the hadronizing system, here (in p_T distributions) from fluctuations of the temperature at which hadronization process takes place.

$$\left[1-(1-q)\beta_0 E\right]^{\frac{1}{1-q}} = \int_0^{\infty} d\beta\, f(\beta)\exp(-\beta E)$$

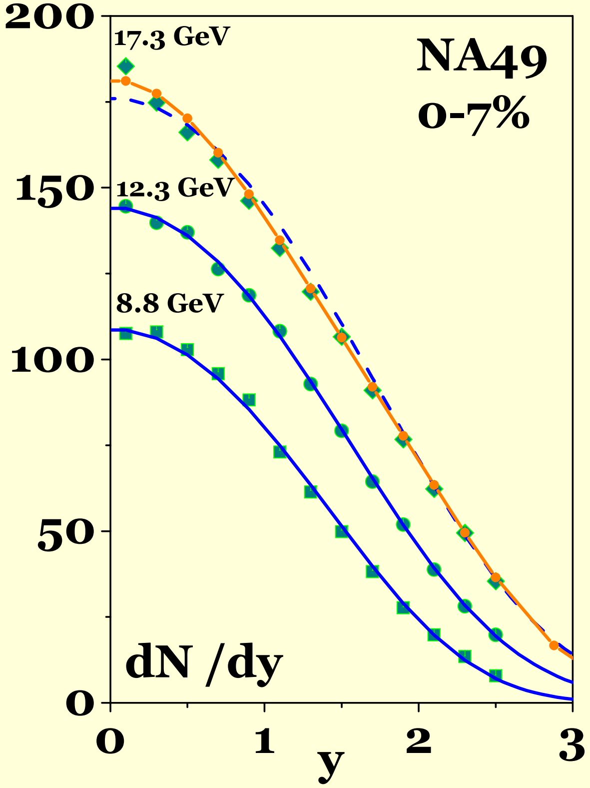
$$f(\beta) = \frac{1}{\Gamma(\alpha)}\left(\frac{\alpha}{\beta_0}\right)^{\alpha}\beta^{\alpha-1}\exp\left(-\frac{\alpha}{\beta_0}\beta\right); \quad \alpha = \frac{1}{q-1}; \quad \beta_0 = \langle\beta\rangle$$

$$q = 1 + \frac{\langle\beta^2\rangle - \langle\beta\rangle^2}{\langle\beta\rangle^2}$$

$$\frac{1}{C_h} = q - 1$$

C_h is the total heat capacity of the hadronizing system ⇨ expectation: because C_h ~V therefore q should decrease vith volume V: q(e^+e^-)>q(pp)>q(AA). Fits to the corresponding data confirm this.

Applications: AA



Example of use of MaxEnt method applied to some NA49 data for π^- production in PbPb collisions (centrality 0-7%):

- **(blue lines)**

$S^{1/2}$	q	K_q
8.8	1.040	0.22
12.3	1.164	0.30
17.3	1.200	0.33

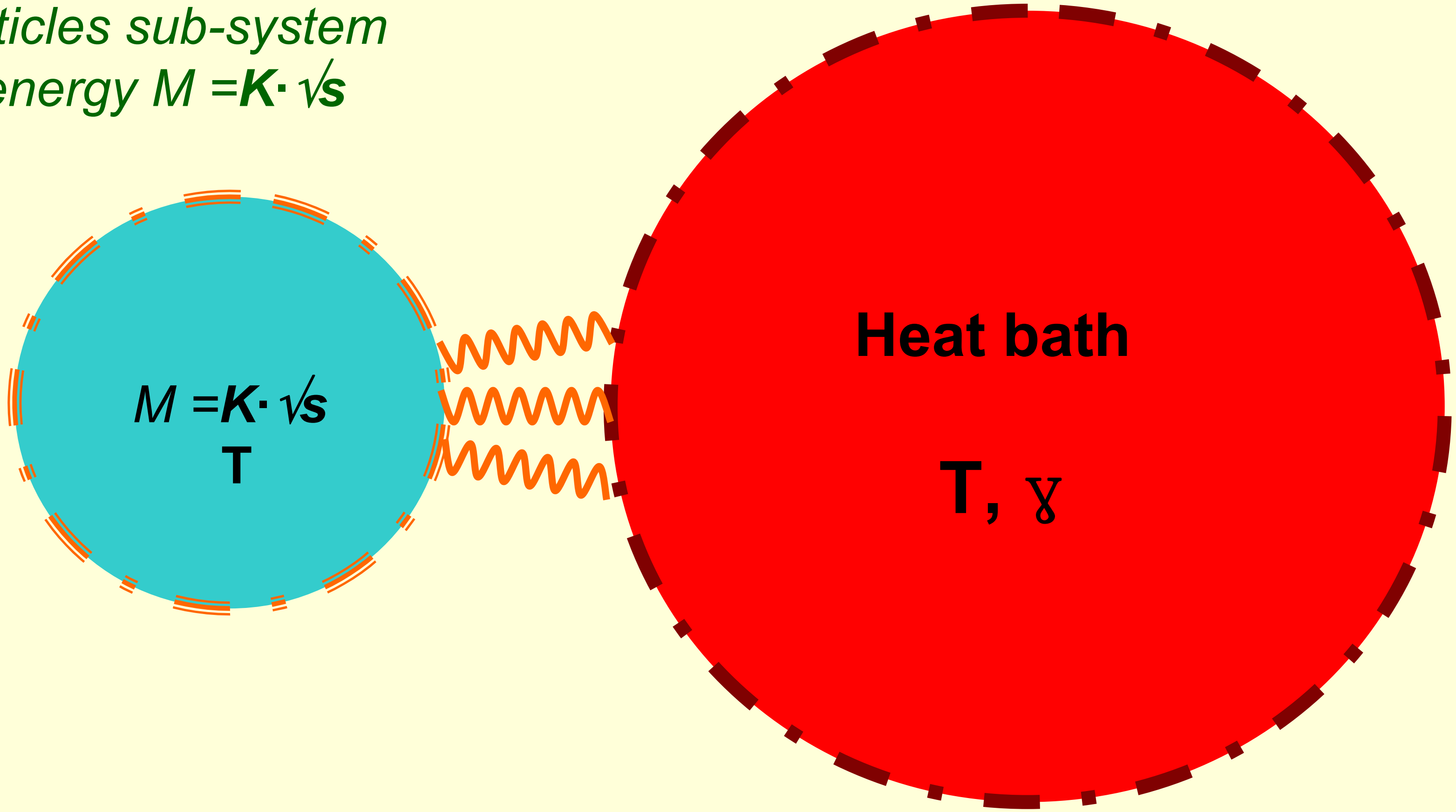
- **(orange line)** **q=1, two sources of mass M=6.34 GeV located at |y|=0.83**

this is example of adding new dynamical assumption

Returning to small heat baths:

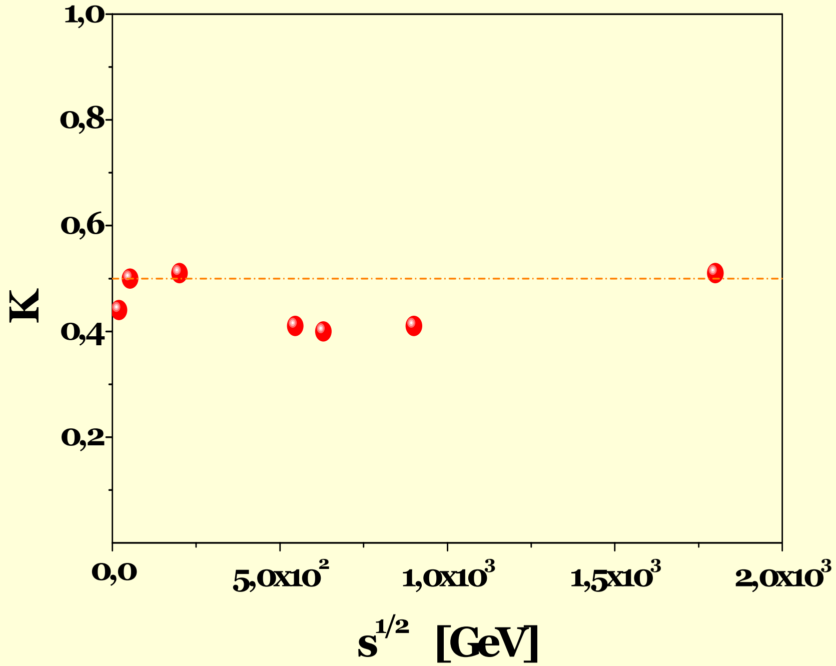
(N-k) –particles sub-system

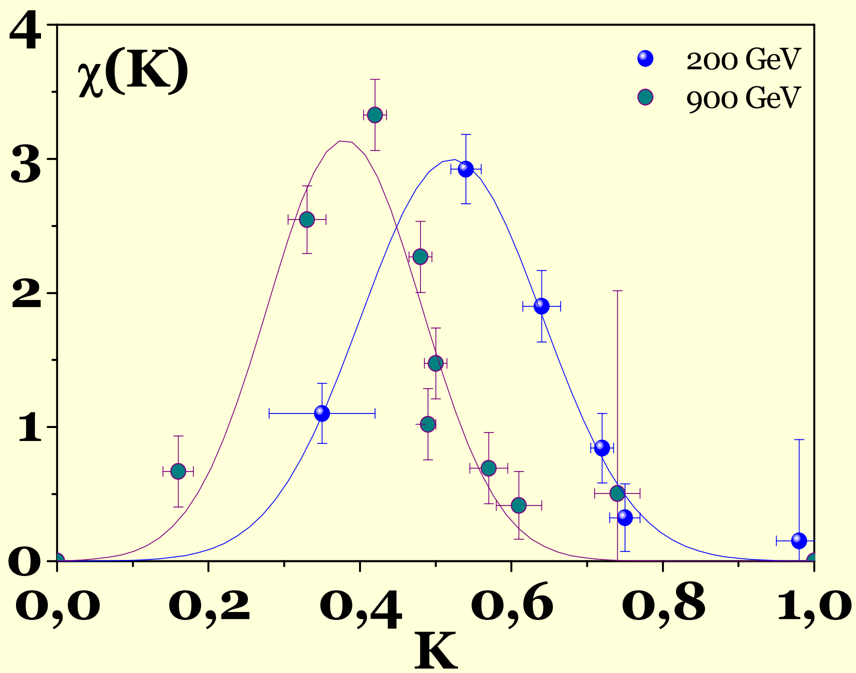
k –particles sub-system
*of energy M =**K·√s***



$$\mathbf{f(M) = C \cdot exp\left[-\gamma \left(M - \langle M \rangle \right)^2 \right]}$$

For small heat baths we have results for inelasticity K:
(*) its energy dependence K(√s) and
(*) its distribution χ(K)





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Derivation of inelasticity distribution $\chi(K)$ from EGE

(*) The whole energy available for reaction, $E = \sqrt{s}$, is divided into two parts:

one part equal to $E_1 = K \cdot \sqrt{s}$ is going into system producing observed secondaries

second part $E_2 = E - E_1$, acts as a "heat bath" for first one

(*) $$p_1(E_1) = \frac{\Omega_1(E_1)\Omega_2(E_2)}{\Omega_{1+2}(E)}$$ **where Ω denotes the corresponding number of states**

Defining entropy as $S_i(E_i) = \ln \Omega_i(E_i)$ one gets

$$p_1(E_1) = \frac{1}{\Omega_{1+2}(E)} \cdot \exp\left[S_1(E_1) + S_2(E_2)\right]$$

Derivation of inelasticity distribution $\chi(K)$ from EGE (*cont.*)

(*) Expanding entropy around $E_1 = U$**,**
keeping only linear and quadratic terms,

assuming that $\beta = \frac{1}{T_0} = \left[\frac{\partial \ln \Omega}{\partial E_1}\right]_{E_1=U}$ **and** $\gamma = -\left[\frac{\partial^2 \ln \Omega}{\partial E_1^2}\right]_{E_1=U}$

are the same for both parts of the system

one gets

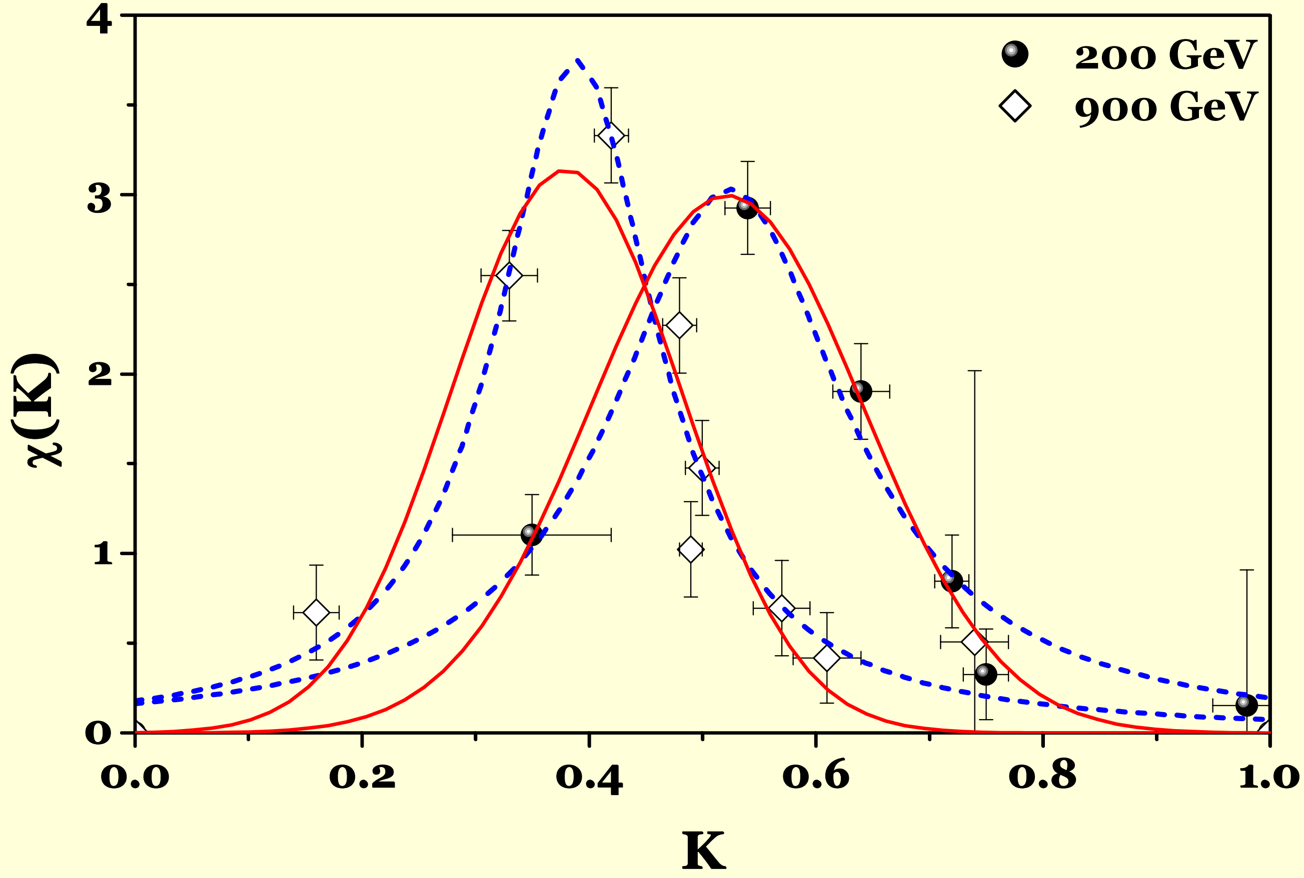
$$p_1(E_1) = \frac{1}{Z_G}\exp\left[-\gamma(E_1 - U)^2\right]$$

$$\chi(K) = \chi_0 \exp\left[-\frac{1}{2\sigma^2}(K - \langle K \rangle)^2\right]$$

(see figure)

Notice, however, that:

apparently better fit is obtained for Lorentzian distribution which can be regarded as q_gaussian with q=2….



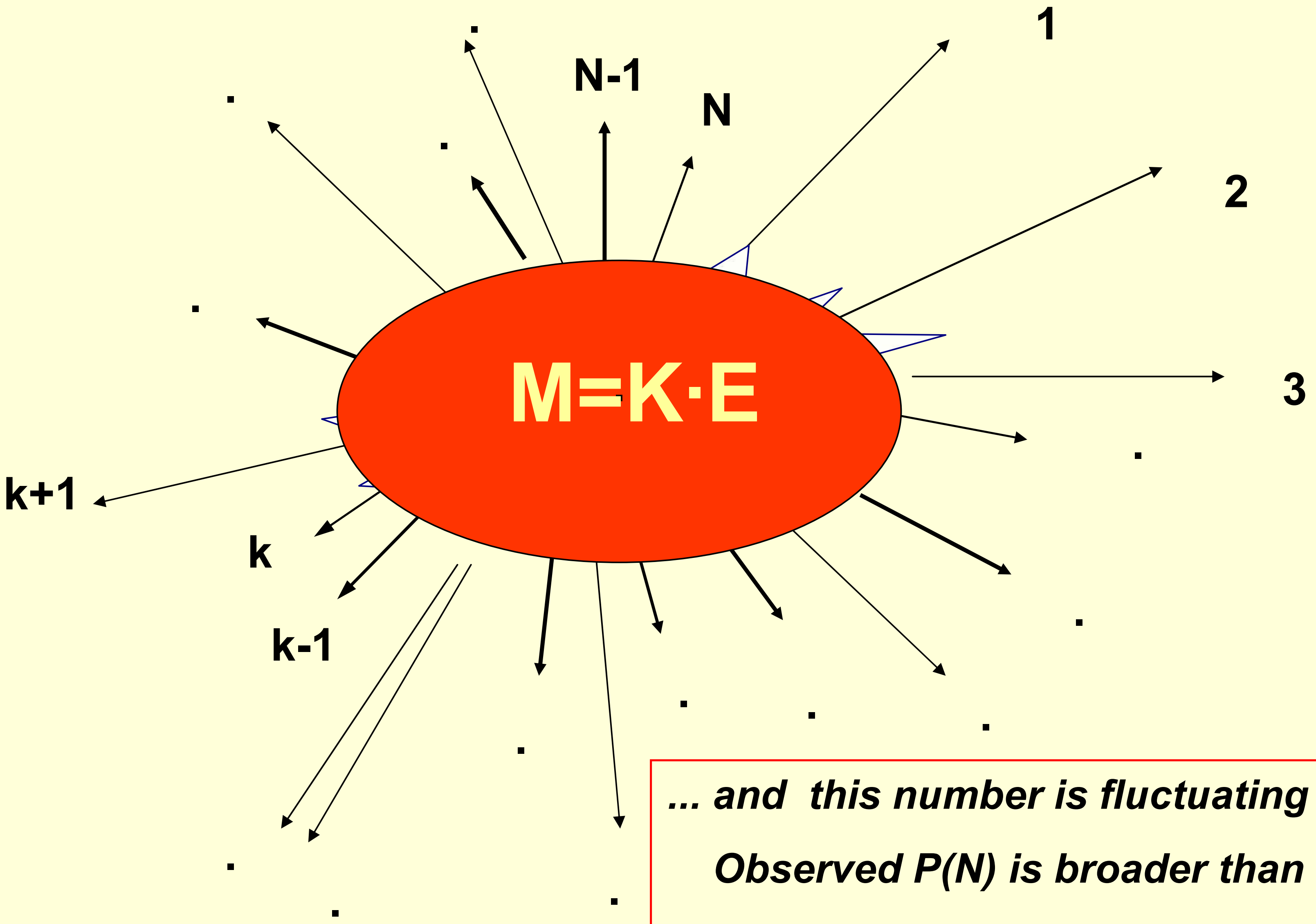
… ➡.. q-EGE ? …

Gaussian distribution

Lorentzian distribution

$\chi_{1<q<2}(\mathbf{K}) \leftrightarrow$ ***Student distribution***

Hadronic production processes: ***large number secondaries is produced ...***



... and this number is fluctuating ⇨ P(N).

Observed P(N) is broader than Poissonian,

usually Negative Binomial type

These are the fluctuations leading to $f_q(y)$ rather than $f(y)$ ⇨⇨⇨

Possible meaning of parameter q in rapidity distributions

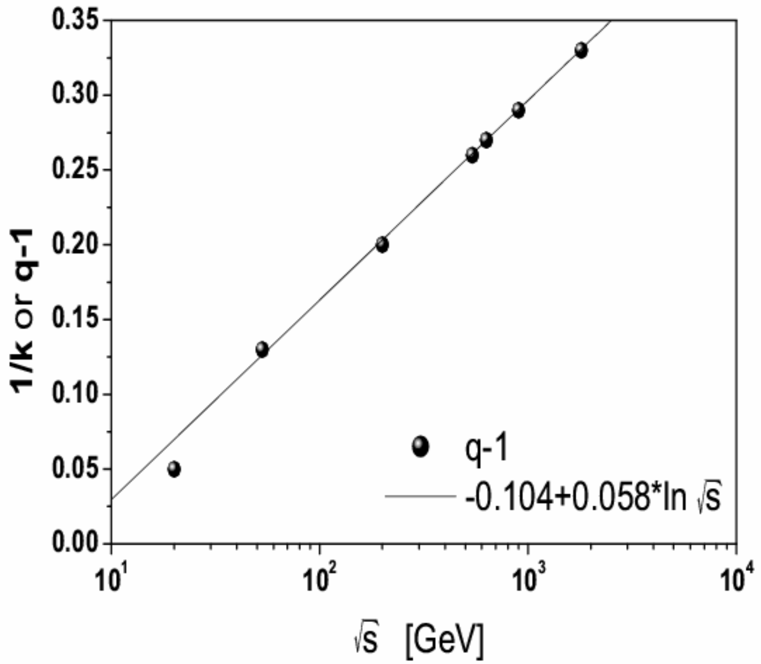


Figure 6:

The values of the nonextensivity parameter q obtained in fits shown here and listed in Table I compared with the values of the parameter k of Negative Binomial distribution fit to the corresponding multiplicity distributions (as given by C.Geich-Gimbel, *Int. J. Mod. Phys.* A4 (1989) 1527).

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(*) From fits to rapidity distribution data one gets systematically q>1 with some energy dependence

(*) What is now behind this q?

(*) y-distributions ⇔ 'partition temperature'
$T \approx K \cdot \sqrt{s}/N$ (N=multiplicity)

(*) q ⇔ fluctuating T ⇔ fluctuating N

(*) Conjecture: q-1 should measure amount of fluctuation in P(N)

(*) It does so, indeed, see Fig. where data on q obtained from fits are superimposed with fit to data on parameter k in Negative Binomial Distribution!

Parameter q measures dynamical fluctuations in P(N)

(*) Experiment: P(N) is adequately described by NBD depending on <N> and k (k≥1) affecting its width:

$$\frac{1}{k} = \frac{\sigma^2(N)}{<N>^2} - \frac{1}{<N>}$$

(*) If 1/k is understood as measure of fluctuations of <N> then

$$P(N) = \int_0^\infty d\bar{n} \frac{\bar{n}^n \exp(-\bar{n})}{n!} \cdot \frac{\gamma^k \bar{n}^{k-1} \exp(-\gamma\bar{n})}{\Gamma(k)}$$

$$= \frac{\Gamma(k+n)}{\Gamma(1+n)\Gamma(k)} \cdot \frac{\gamma^k}{(\gamma+1)^{k+n}}$$

(P.Carruthers,C.C.Shih, Int.J.Phys. A4 (1989)5587)

with $\gamma = \frac{k}{<\bar{n}>}$

$$\frac{1}{k} = D(\bar{n}) = \frac{\sigma^2(\bar{n})}{<\bar{n}>^2} = q-1$$

(*) → one expects: q=1+1/k what indeed is observed

Another derivation of Negative Binomial multiplicity distribution P(N):

Boltzmann exp(- E/T) ⇦⇨ Poisson P(N)
Tsallis $\exp_q$(- E/T) ⇦⇨ Negative Binomial P(N)

(*) If one has procees in which N particles with energies $\{E_{1,\dots,N}\}$ are produced in the independent manner and in which energies are distributed according to Boltzmann distribution

$$f(E_i) = \frac{1}{\lambda}\cdot\exp\left(-\frac{E_i}{\lambda}\right); \qquad \lambda = \langle E\rangle$$

then, the corresponding multiplicity distribution has known Poissonian form (with <N>=E/λ):

$$\mathbf{P(N) = G_{N+1}(E) - G_N(E) = \frac{\langle N\rangle^N}{N!}\cdot\exp\left(-\langle N\rangle\right)}$$

where distribuant $G_N(E)$ is given by

$$G_N(E) = 1 - \sum_{i=1}^{N-1}\frac{1}{(i-1)!}\cdot\left(\frac{E}{\lambda}\right)^{i-1}\cdot\exp\left(-\frac{E}{\lambda}\right)$$

G.Wilk and Z.Włodarczyk, Fluctuations, correlations and nonextensivity;cond-mat/0603157

(*) If one has procees with N indpendent particles with energies $\{E_{1,\dots,N}\}$ but this time distributed according to Tsallis distribution:

$$h(\{E_{1,\dots,N}\}) = C_N \left[1-(1-q)\frac{\sum_{i=1}^{N} E_i}{\lambda}\right]^{\frac{1}{1-q}+1-N}$$

then, the corresponding multiplicity distribution has form of Negative Binomial distribution (NBD) widely encountered in all analyzes of high energy multiparticle production of all kinds

$$P(N) = H_{N+1}(E) - H_N(E) = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)} \cdot \frac{\left(\frac{<N>}{k}\right)^N}{\left(1+\frac{<N>}{k}\right)^{N+k}}; k = \frac{1}{q-1}$$

where: $<N>=E/\lambda$, $Var(N)=<N>+(q-1)<N>^2$ and distribuant $H_N(E)$ is given by

$$H_N(E) = 1 - \sum_{j=1}^{N-1}\left\{\frac{E^{i-1}\prod_{i=1}^{j}\left[(i-1)q-(i-3)\right]}{(j-1)!\lambda^j}\left[1-(1-q)\frac{E}{\lambda}\right]^{\frac{1}{1-q}+1-j}\right\}$$

Notice that: **NBD becomes**

Poisson distribution

$$P(N) = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)} \cdot \frac{\left(\dfrac{<N>}{k}\right)^{N}}{\left(1+\dfrac{<N>}{k}\right)^{N+k}} \xrightarrow[(k\to\infty)]{q\to 1} \frac{\langle N\rangle^{N}}{N!} \cdot \exp\left(-\langle \mathrm{N}\rangle\right)$$

$$\xrightarrow[(k\to 1)]{q\to 2} \frac{<N>^{N}}{\left(1+<N>\right)^{N+1}}$$

Geometrical distribution

and that **parameter k in NBD can be expressed by the correlation coefficient ρ for the two-particle energy correlations:**

$$k = \frac{\rho+1}{\rho}$$

$$\rho = \frac{Cov(E_1, E_2)}{\sqrt{Var(E\ )}\sqrt{Var(E_2)}} = \frac{q-1}{2-q}$$

Some remarks concerning distribution:

$$h(\{E_{1,\ldots,N}\}) = C_N \left[1 - (1-q)\frac{\sum_{i=1}^{N} E_i}{\lambda} \right]^{\frac{1}{1-q}+1-N}$$

➡ Negative Binomial form of multiparticle distributions P(N)

Fluctuating in $\mathbf{f(x) = \frac{1}{\lambda} \cdot \exp\left(-\frac{x}{\lambda}\right)}$ parameter $\lambda = \lambda' = \lambda'(\varepsilon) = \frac{\lambda}{\varepsilon}$

where: $$g(\varepsilon) = \frac{1}{(q-1)\Gamma\left(\frac{1}{q-1}-1\right)}\left[\frac{\varepsilon}{q-1}\right]^{-2+\frac{1}{q-1}} \cdot \exp\left(-\frac{\varepsilon}{q-1}\right)$$

one gets Tsallis distribution:

$f(x;\lambda \to \lambda/\varepsilon)$

$$\mathbf{h(x)} = \int_0^{+\infty} d\varepsilon\, g(\varepsilon)\left(\frac{\varepsilon}{\lambda}\right)\exp\left(-\frac{\varepsilon}{\lambda}\cdot x\right) = C_1\left[1-(1-q)\frac{x}{\lambda}\right]^{\frac{1}{1-q}}$$

where $C_1 = \frac{2-q}{\lambda}$; $\langle\varepsilon\rangle = 2-q$ and $\frac{\langle\varepsilon^2\rangle}{\langle\varepsilon\rangle^2} - 1 = \frac{q-1}{2-q}$

Tsallis distributions: fluctuations *vs* correlations :

One needs at least two particles to discuss correlations ⇨

(*) ***Consider two independent random variables (x,y) each following its own independent distribution and let***

$$f(x,y) = f(x) \cdot f(y) = \frac{1}{\lambda^2} \cdot \exp\left[-\frac{(x+y)}{\lambda}\right]$$

There are two ways to obtain from it two-particle Tsallis distribution:

- fluctuating λ for each variable separately ⇨
joint Tsallis distribution for uncorrelated random variables

- fluctuating λ for both variables ⇨
joint Tsallis distribution for correlated random variables

Uncorrelated random variables

(*) Fluctuating λ for each variable separately leads to joint Tsallis distribution for **uncorrelated** random variables:

$$\mathbf{h(x,y) = h(x) \cdot h(y) = C^2 \left[1 - (1-q)\frac{(x+y)}{\lambda} + (1-q)^2 \frac{xy}{\lambda^2} \right]^{\frac{1}{1-q}}}$$

(*) **However:** The same **h(x,y)** results starting from exponential distribution of **correlated** variables **(x,y):**

$$f(x,y) = \left\{ \frac{\exp[1/(1\text{-}Q)]}{(Q\text{-}1)\Gamma(0,1/(q\text{-}1))} \right\} \cdot \exp\left[-\frac{x+y}{\lambda'} + (1-Q)\frac{xy}{\lambda\lambda'} \right]$$

and **fluctuating** λ' using the same prescription as before when getting h(x) with q=Q. The resulting **positive** corelations cancel the negative correlations introduced above and one gets joint distribution of uncorrelated variables.

This example shows that effects of correlations and fluctuations can cancel each other.

Correlated random variables

Proceeding in the same way as for single variable case, i.e., fluctuating **λ→λ'=λ/ε**, one gets:

$$\mathbf{h(x,y)} = \int_0^{+\infty} d\varepsilon \left(\frac{\varepsilon}{\lambda}\right)^2 \exp\left[-\frac{(x+y)\varepsilon}{\lambda}\right] g(\varepsilon) = \mathbf{C_2}\left[\mathbf{1-(1-q)\frac{(x+y)}{\lambda}}\right]^{\frac{q}{1-q}};\ C_2 = \frac{2-q}{\lambda^2}$$

The corresponding marginal probability has Tsallis form (notice: **1/(1-q)=q/(1-q)+1):**

$$h(x) = \int dy\, h(x,y) = C_1\left[1-(1-q)\frac{x}{\lambda}\right]^{\frac{1}{1-q}}$$

$$< x > = < y > = \frac{\lambda}{3-2q};\quad Var(x) = Var(y) = \frac{\lambda^2(2-q)}{(3-2q)^2(4-3q)};$$

$$Cov(x,y) = \frac{\lambda^2(q-1)}{(3-2q)^2(4-3q)};\quad \rho = \frac{Cov(x,y)}{\sqrt{Var(x)}\sqrt{Var(y)}} = \frac{q-1}{2-q} = \frac{1}{2-q} - 1$$

It means that **correlation coefficient ρ is entirely given by the parameter q** or **q=1+ρ/(1+ρ)**

Finally, in general:

$$h(\{x_{1,\dots,N}\}) = \int_0^{+\infty} d\varepsilon \left(\frac{\varepsilon}{\lambda}\right)^N \exp\left[-\frac{\varepsilon}{\lambda}\cdot\sum_{i=1}^N x_i\right] g(\varepsilon) = C_N \left[1-(1-q)\frac{\sum_{i=1}^N x_i}{\lambda}\right]^{\frac{1}{1-q}+1-N}$$

where:

$$C_N = \frac{1}{\lambda^N}\prod_{i=1}^N [(i-2)q-(i-3)] = \frac{(q-1)^N}{\lambda^N}\cdot\frac{\Gamma\left(N+\frac{2-q}{q-1}\right)}{\Gamma\left(\frac{2-q}{q-1}\right)}$$

Effective q_N

$$h(\{x_{1,\dots,N}\}) = C_N\left[1-\frac{1-q_N}{1+(N-1)(1-q_N)}\cdot\frac{\sum_{i=1}^N x_i}{\lambda}\right]^{\frac{1}{1-q_N}}$$

$$\frac{1}{1-q_N}-\frac{1}{1-q_1} = 1-N \quad \text{or} \quad q_N = 1+\frac{q_1-1}{1+(q_1-1)(N-1)} \xrightarrow{N\to\infty} 1$$

Notice that:

(i) Fluctuations lead always to Tsallis distribution, irrespectively of the presence or absence of correlations.

(ii) For simple prescription:

$$f(x,y) \to h(x,y) \cong \left[1-(1-q)\frac{(x+y)}{\lambda}\right]^{\frac{1}{1-q}}$$

the marginal probabilities are not reproduced:

$$h(x) = \int dy\, h(x,y) \neq C_1\left[1-\left(1-q\right)\frac{x}{\lambda}\right]^{\frac{1}{1-q}}$$

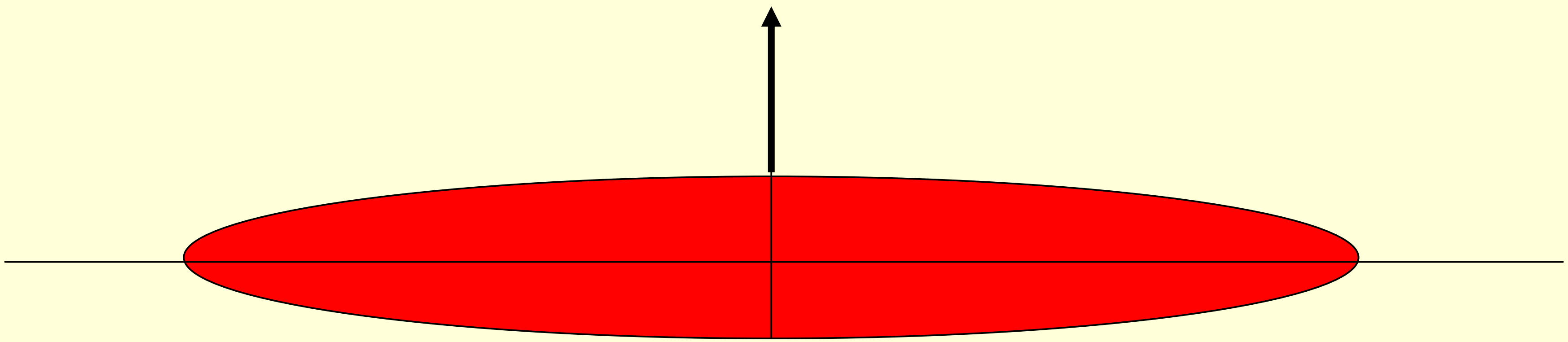
To summarize this part let us repeat again that:

Measured particle distributions are affected by internal fluctuations and correlations presented in the hadronizing system (i.e., in system converting initial energy into observed particles):

$$f(y) \Rightarrow f_q(y) \quad \text{and} \quad \text{Poissonian } P(N) \Rightarrow \text{Negative Binomial } P(N)$$

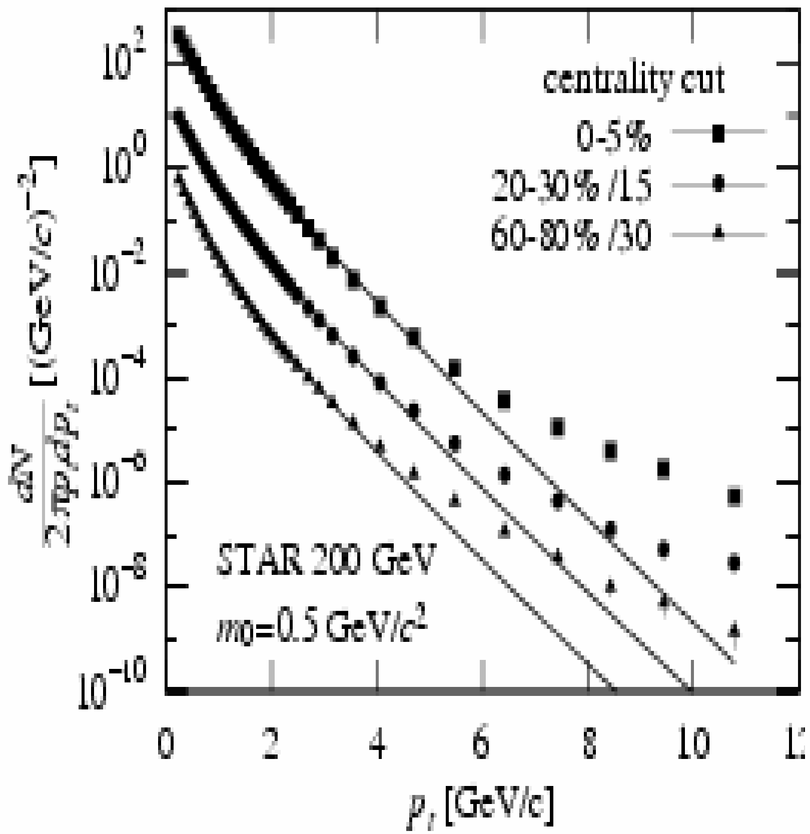
Let us now proceede to the problem of description of transverse momentum spectra, which are important because they provide us with information on thermal properties of the hadronizing system

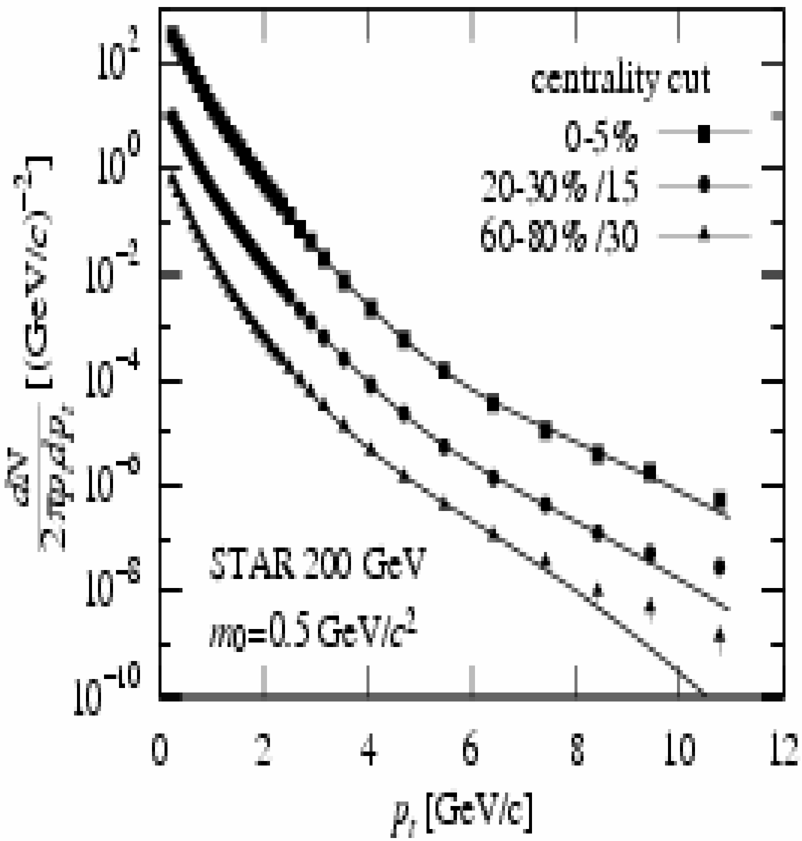
$$p_T \leftrightarrow T_T = T$$



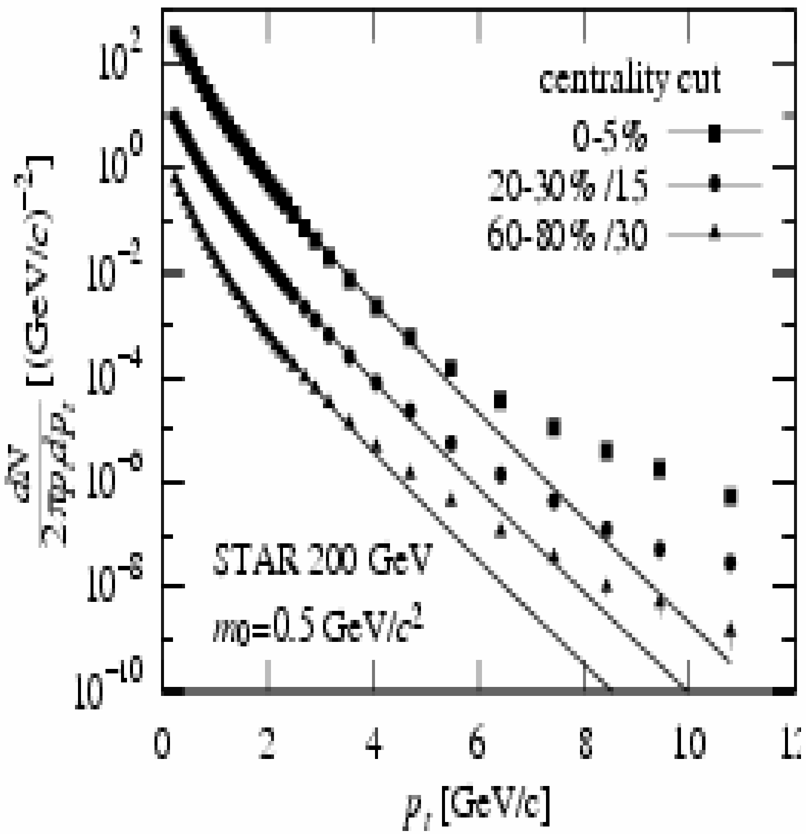
M.Biyajima, T.Mizoguchi, N.Nakamija, N.Suzuki and G.Wilk, *Modified Hagedorn formula including temperature fluctuation;* hep-ph/0602120

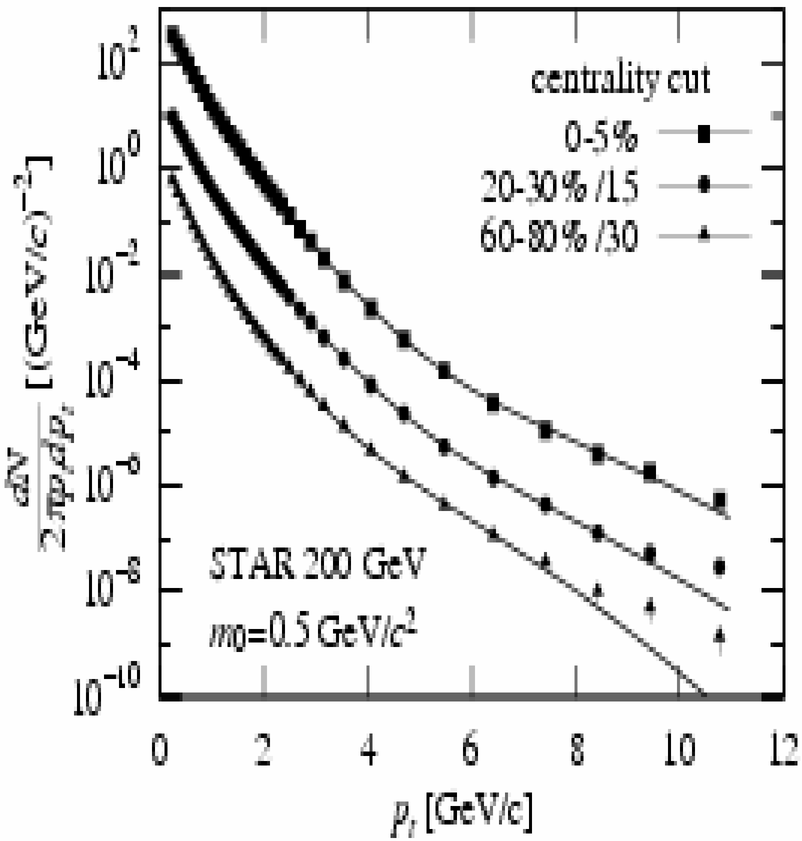
Experiments on Au+Au → π + X collisions at energy 200 GeV per nucleon (in CMS system) performed at RHIC accelerator in Brookhaven NL (USA) show nonexponential, power-like tail for large transverse momenta. These experiments are important because from such distributions one expects to deduce the temperature T of the hadronizing system and to learn about possible phase transition from the hypothetical Quark-Gluon Plasma to hadronic matter.





There are many attempts to explain these results using some kind of nonequilibrium approach like, for example, flow of prehadronic and hadronic matter and/or decay of hadronic resonances produced in such collisions. Rather than excluding them we have investigated the possibility that observed nonexponential spectra could result from some kind of equilibrium characteristic of nonextensive thermodynamics.One can then successfully account for the whole range of the observed transverse momenta.





M.Biyajima, T.Mizoguchi, N.Nakamija, N.Suzuki and G.Wilk, ***Modified Hagedorn formula including temperature fluctuation;*** **hep-ph/0602120**

Hagedorn bootstrap model: *gas of noninteracting resonances which consist of resonances etc. with charecteristic exponential mass spectrum ρ(m)* ➡➡➡

(*) limiting temperature: hadronic phase can live only at $T<T_H = 1/\beta_H$

(*) energy distributions are governed by temperature $T_0=1/\beta_0$

$$\frac{d^3\sigma}{dp^3} = \int_0^\infty dm\,\rho(m)\exp\left(-\sqrt{\mathrm{p}_\mathrm{T}^2 + p_L^2 + m^2}\cdot\beta_0\right)$$

➡

$$f(p_T) = \frac{d^2\sigma}{2\pi p_T dp_L} = C\int_{m_0}^\infty dm\,\rho(m) m_T K_1(m_T\beta_0)$$

$$\rho(m) = \frac{\exp(m\beta_H)}{\left(m^2+m_0^2\right)^{5/4}};\quad m_T = \sqrt{p_T^2+m^2}$$

However: **the above formula can describe data only in limited range of transverse momenta (cf. previous Fig.) ....**

First attempts to fit the whole range of p_T are from 1977 (C.Michael):

$$f(p_T) = C\left(1 + \frac{p_T}{p_0}\right)^{-n} \rightarrow \begin{cases} \exp\left(-\frac{n}{p_0} p_T\right) & \text{for} \quad p_T \rightarrow 0 \\ \left(\frac{p_0}{p_T}\right)^{n} & \text{for} \quad p_T \rightarrow \infty. \end{cases}$$

"soft" (nonperturbative) physics

"hard" (perturbative) physics

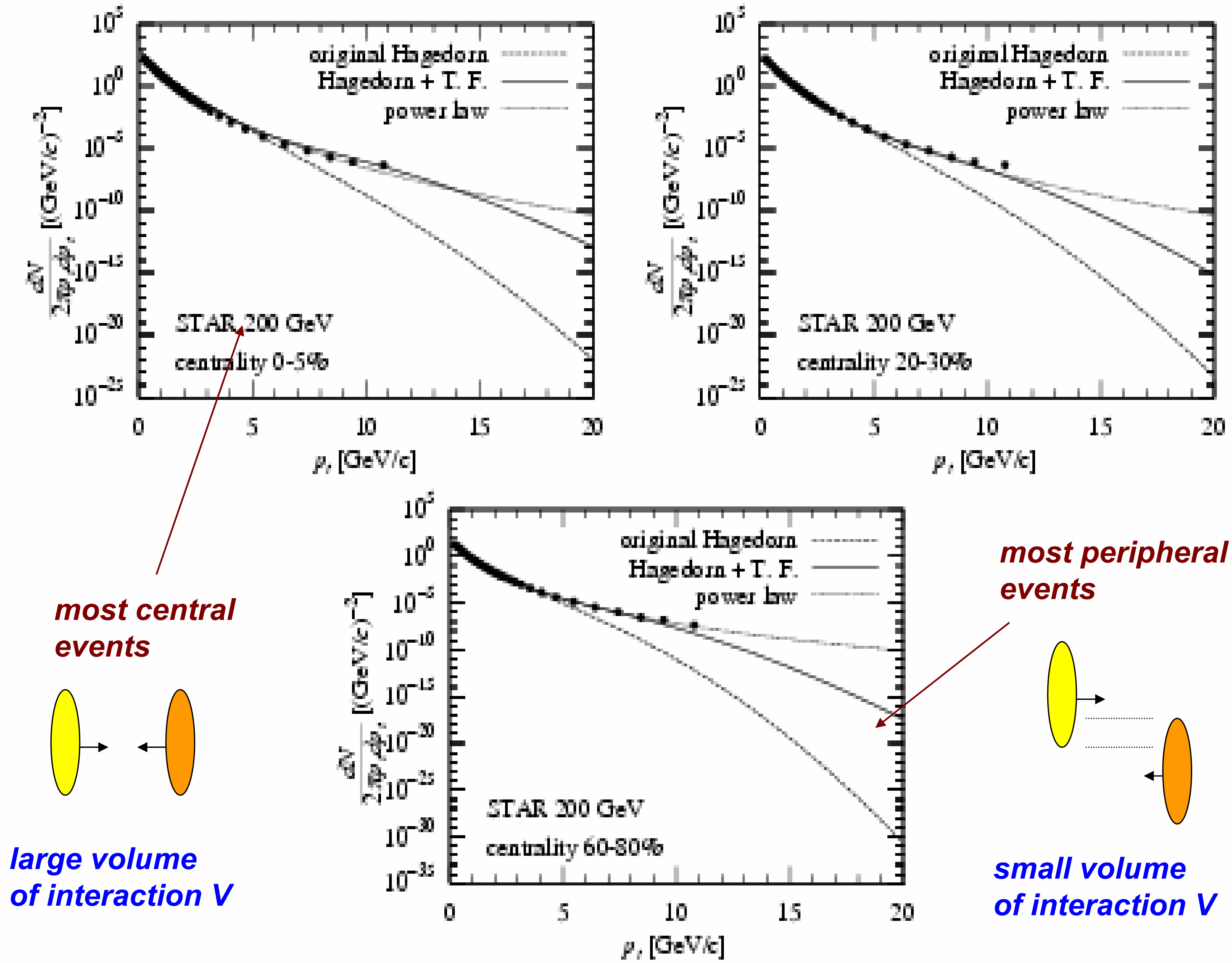
Insted these two separate mechanisms we have investigated the possibility that observed nonexponential spectra could result from some kind of equilibrium characteristic of nonextensive thermodynamics. Two formulas were considered:

$$f(p_T) = C\int_0^{\infty} dp_L \left[1 - (1-q)\frac{\sqrt{p_L^2 + m_T}}{T_0}\right]^{\frac{1}{1-q}}$$

(T) Simple nonextensive formula

$$f(p_T) = C\int dy \cosh y \int dm\rho(m) m_T \left[1 - (1-q)\frac{m_T \cosh y}{T_0}\right]^{\frac{1}{1-q}}$$

(H) Modified Hagedorn formula


original Hagedorn
Hagedorn + T. F.
power law
STAR 200 GeV
centrality 0-5%
STAR 200 GeV
centrality 20-30%
STAR 200 GeV
centrality 60-80%
p_t [GeV/c]
$\frac{dN}{2\pi p_t dp_t}$ [(GeV/c)$^{-2}$]
most central events
large volume of interaction V
most peripheral events
small volume of interaction V

Notice: fitting data by using some form of Tsallis distribution must be supplemented by attempt to understand what is the possible meaning of The parameter q in given circumstances.

What is observed here:

(*) Parameter q is increasing when going from most central to most peripheral events.

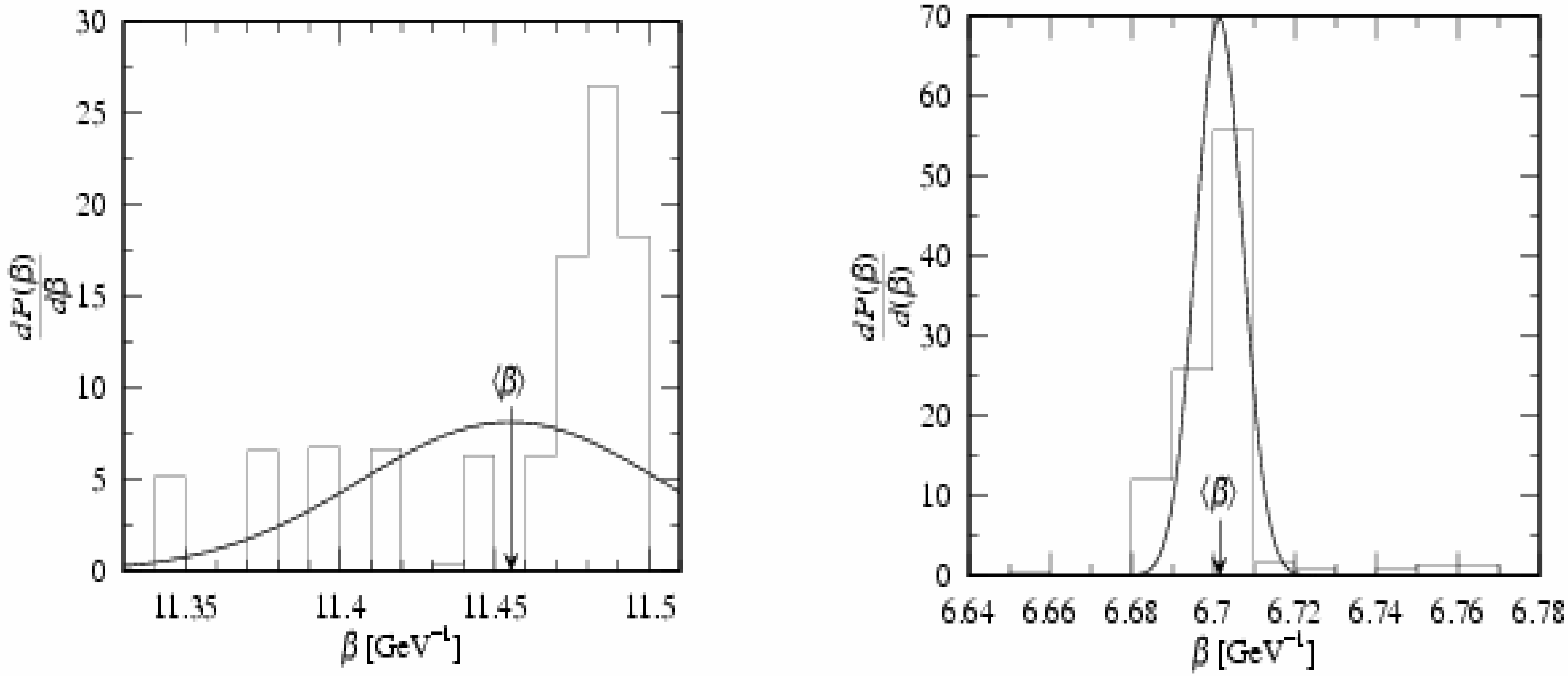
This observation fits nicely the idea that q when representing fluctuations in the system depends (via heat capacity C) inversely on the volume of the hadronizing system, which decreases from central to peripheral collisions.

(*) Both T an q estimated by using simple nonextensive formula (T) are systematically higher than those estimated by using modified Hagedorn model (H).

This observation fits nicely the idea that q represents a joint action of intrinsic fluctuations in the system. The difference between model (T) and model (H) lies in the additional fluctuations present in (H) due to the assumed spectrum mass of resonances (described by Hagedorn temperature T_H). Therefore part of fluctuations described in model (T) by parameter q is in model (H) accounted by parameter T_H .

*(*) **This is best seen below.*** ***The centrality cut 0-5% region of STAR data was fitted (point by point, separately) using model (H) with q=1 (left panel) and with q=1.00014 (right panel). Fit was performed by fixing all parameters except of β_0. It is assumed then the reciprocal of the each error bar calculated by fitting program MINUIT is proportional to the corresponding probability of this value of β. In this way a whole probability distribution for β is obtained in form of histogram – see Figure. The histogram in each panel is fitted to gamma distribution (colid curves).***

NOTICE: good fit can be obtained only with q>1 – in this case distribution of temperature is very narrow.



To summarize this part let us repeat again what we have shown here is:

(*) The more we know on the internal dynamics (for example in the form of mass spectrum of resonances in the Hagedorn model) the near we are the true BG approach. However, for a time being such knowledge is not model free. One is therefore using q-statistics to allow for the better model independent presentation of experimental results.

(*) As long as in fits to data one gets |q-1| ≠1, it means that some dynamics is still there to be uncovered

(dynamics = everything outside the true BG approach).

Nonextensive hydrodynamics for high energy heavy ion collisions?
(T.Osada and G.Wilk, preliminary)

Reasoning:

(*) high energy collisions producing many particles ➔ non-equilibrium problem

(*) tools for it ➔ linear response theory (LRT)
hydrodynamics (H)
kinetic theory (KT)

(*) only LTR and H do not rely on perturbation approach used in KT

(*) H with transport coefficients from LTR – all we need is assumption on near-equlibrium state (or local equilibrium)

(*) the q-versions of KT are already known, the q-H was so far not yet investigated

(*) Following q-KT (A.Lavagno, PL A301 (2002) 13) with nonextensive Boltzmann equation (f(x,p) is phase space distribution function and $C_q(x,p)$ is q-collision term):

$$p^{\mu}\partial_{\mu} f^{q}(x,p) = C_q(x,p)$$

(*) one gets that:

☺

$$\partial_{\mu} T_q^{\mu\nu} = 0$$

$$\partial_{\mu} S_q^{\mu\nu} = 0$$

where

$$T_q^{\mu\nu}(x) = \frac{g}{(2\pi)^3}\cdot\frac{1}{Z_q}\int\frac{d^3p}{p^0}p^{\mu}p^{\nu}f^{q}(x,p)$$

$$S_q^{\mu}(x) = \frac{g}{(2\pi)^3}\cdot\frac{1}{Z_q}\int\frac{d^3p}{p^0}p^{\mu}\left[f^{q}\ln_q f - 1\right]$$

➔ *In this way we obtain f(x,p)*

(*) However, we are interested rather in $dN/[d^2p_T dp_L]$ resulting from eqs. ☺ *supplemented by:*

- *some choice of initial conditions*
- *some choice of relation between pressure and energy density (equation of state).*

So far only first preliminary results available....

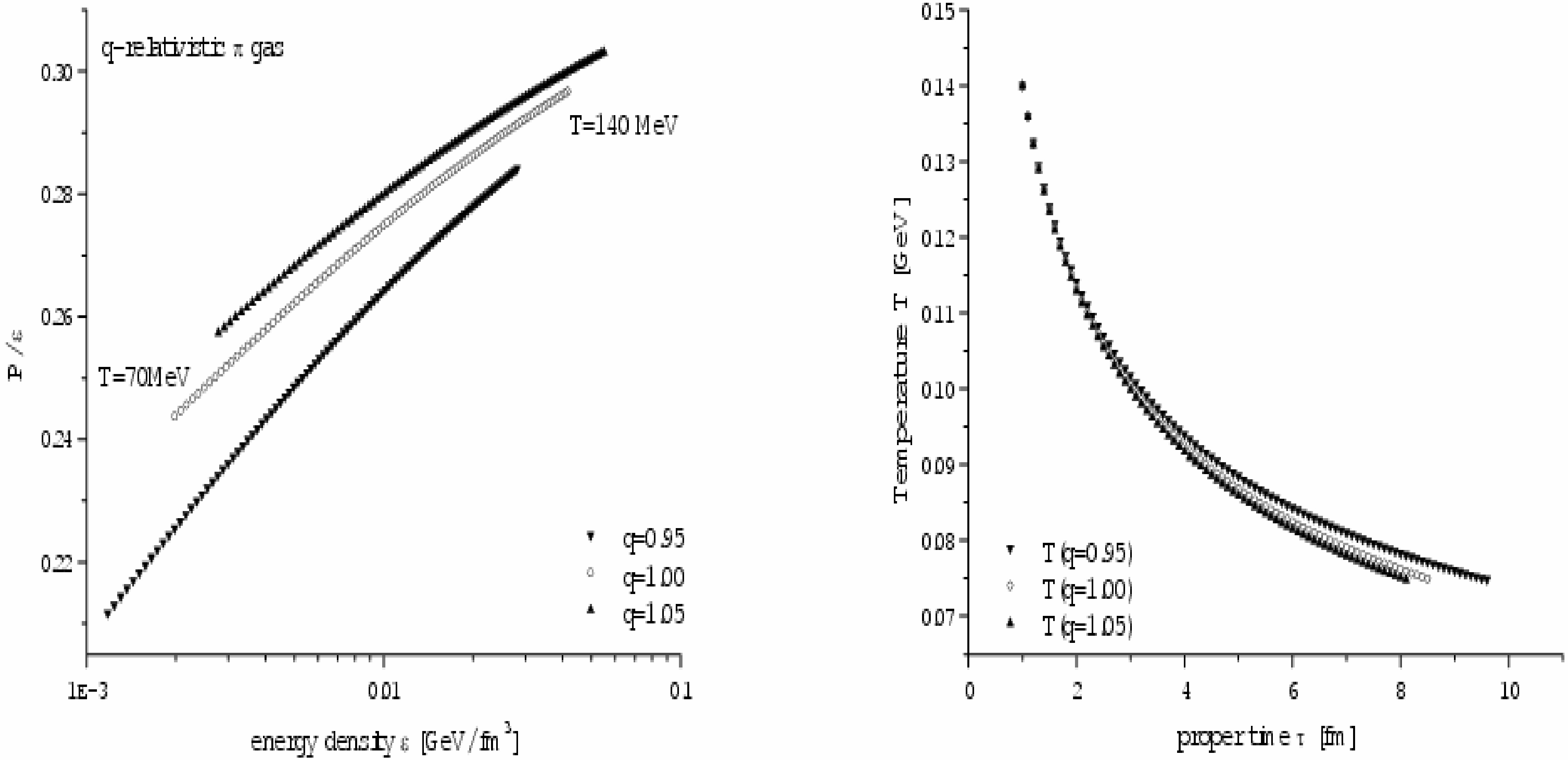


Figure 1:

Left panel; Non-extensive equation of state in $(1+3)$ dimension for relativistic π gas.

Right panel: The (1+1) hydrodynamical evolution of temperature T with (1+3) Eos.

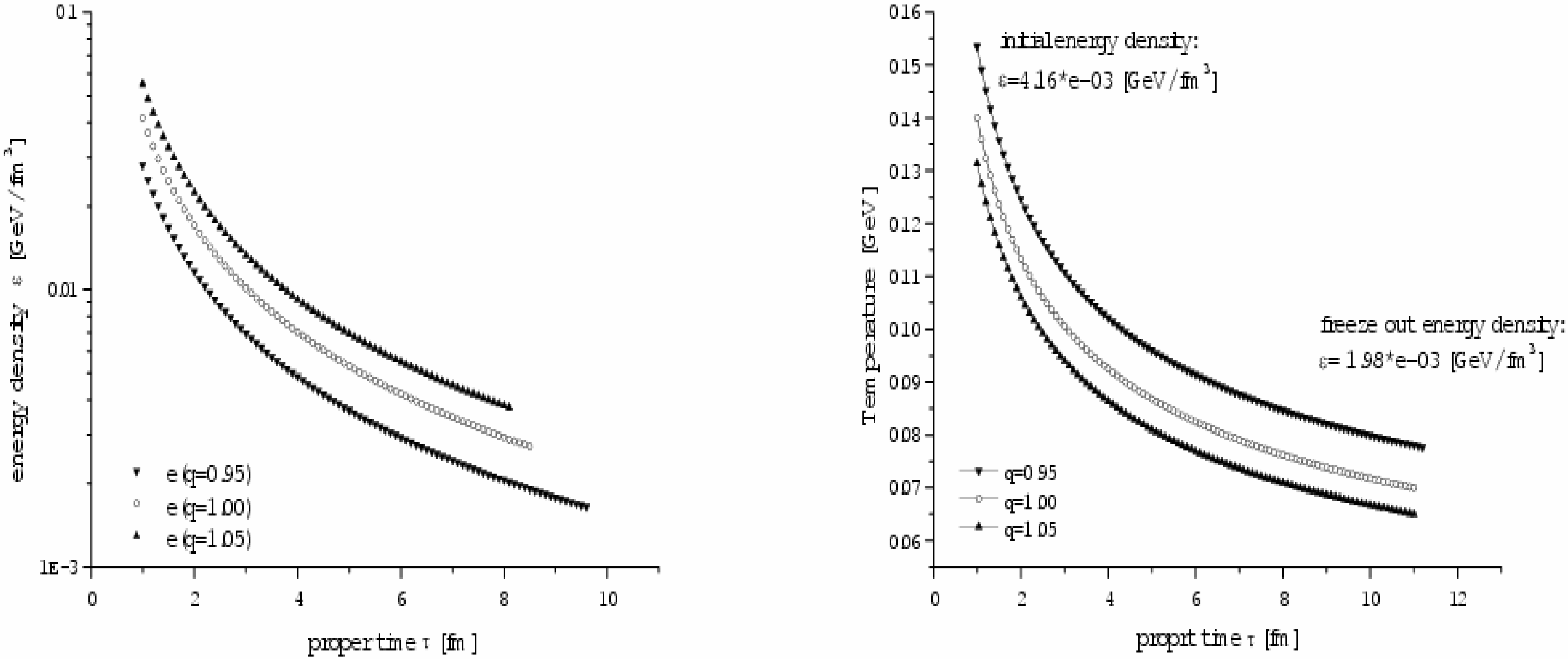


Figure 2:

Left panel: The (1+1) hydrodynamical evolution of temperature T with (1+3) Eos.

Right panel: The q-hydrodynamical evolution in (1+1) dimension starting from energy density ε_0=4.17$\times10^{-2}$GeV/fm^{-2} with relativistic π gas EoS. The freeze out condition is determined by the constant energy density $\varepsilon_{\rm f}$ =1.97$\times10^{-3}$ GeV/fm^{-3}.
The freeze out temperature depends on value of parameter q, which is
$T_{\rm f}$= 65.0, 70.0 and 77.7 MeV for q=1.05, 1.00 and 0.95 respectively.

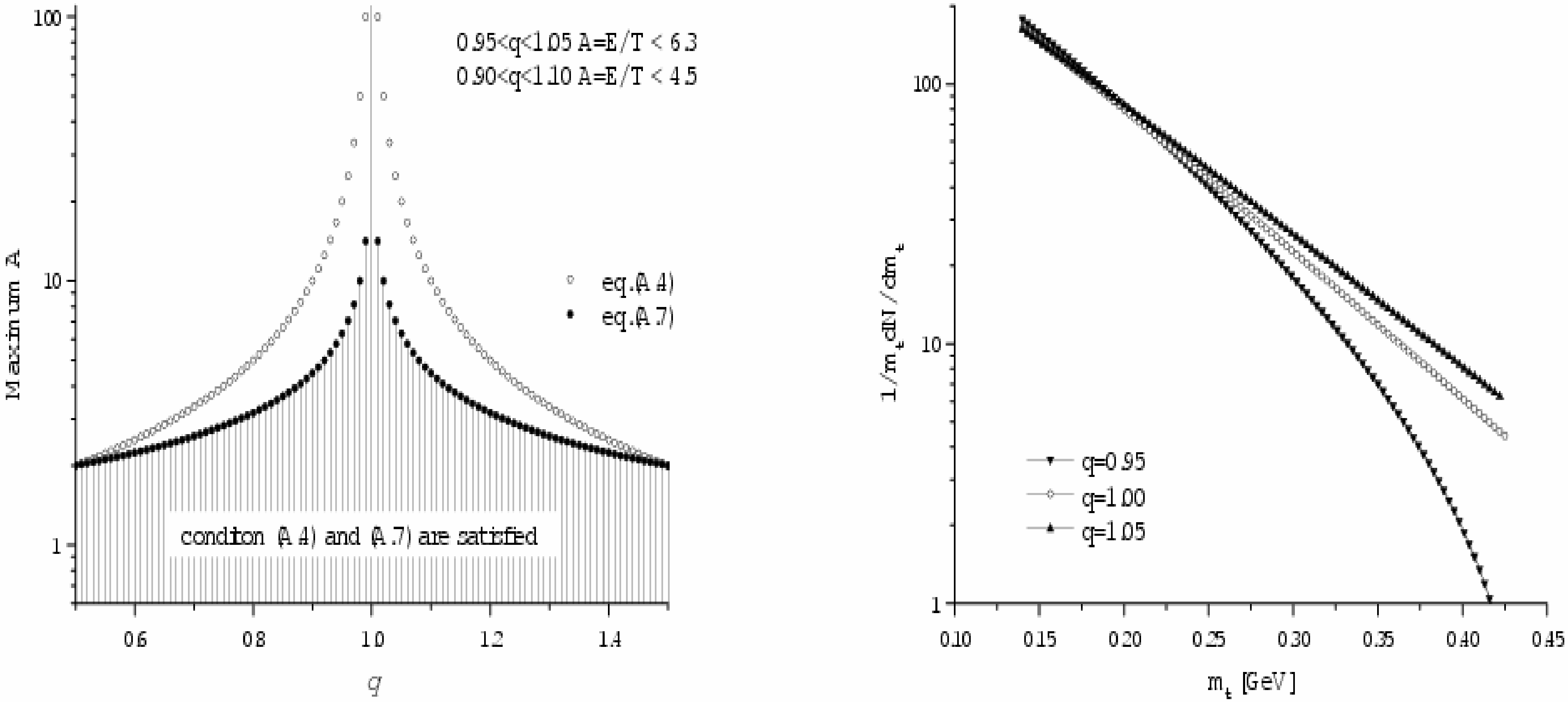


Figure 3:

Left panel: Conditions of linearization for the distribution function in $(q-1)$.

Right panel: The transverse momentum distribution obtained by non-extensive hydrodynamics.

The initial condition and freeze out condition is given by constant energy density ε_0 and ε_{f}, respectively.

Summary:

(*) Even single particle distributions are affected by internal fluctuations and correlations presented in the hadronizing system:

$f(y)$ ➔ $f_q(y)$

Poissonian P(N) ➔ Negative Binomial P(N)

flows ……………………….

(*) The best way to account for them in a model independent way is to use some form of non Boltzmann-Gibbs ensembles, for example Tsallis or EGE (or …(?)).

(*) The more we know on the internal dynamics (for example in the form of mass spectrum of resonances in the Hagedorn model) the near we are the true BG approach. However, for a time being such knowledge is not model free. One is therefore using q-statistics to allow for the better model independent presentation of experimental results.

(*) As long as in fits to data one gets $|q-1| \neq 1$, it means that some dynamics is still there to be uncovered (dynamics = everything outside the true BG approach).

Other posible interpretation of q-parameter: a measure of incomplete occupation of the phase space (limitation or the fractal-like structure of heath bath ⇔ correlations)

(*) Proposition of yet another dynamical origin of power-laws (Kodama et al., Europ.Lett. 70 (2005) 439):

supercorrelated systems ⇒ q-clusters

(*) Motivated by observation (Berges et al., PRL 93 (2001)142002) that dynamics of quantum scalar fields exhibits a prethermalization behaviour : thermodynamical relations become valid long before the real thermal equilibrium is attained.

(*) Possible realisation: strong correlations among some variables (leading to clustration):

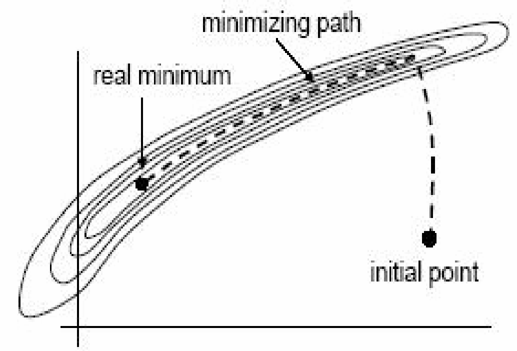


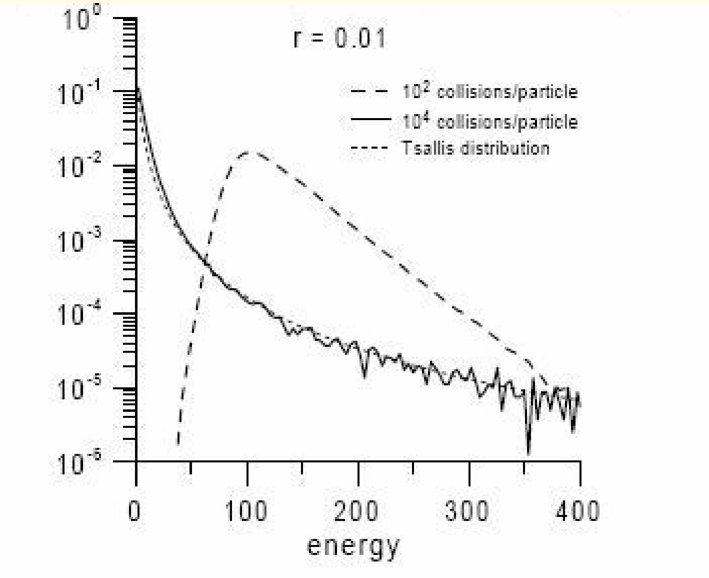
FIG. 1: Schematic illustration of the effect of a strong correlation among two variables for minimizing a function.

***Example:** system composed of N particles with strong correlation among any q of them and with dynamical evolution such that, for a given number of q-clusters, the configuration of the system tries to minimize the energy of the correlated subsystem (i.e., the system first generates correlations among particles minimizing the energy in the clusters) ⇒ power law distribution discussed before with the same q-parameter .*

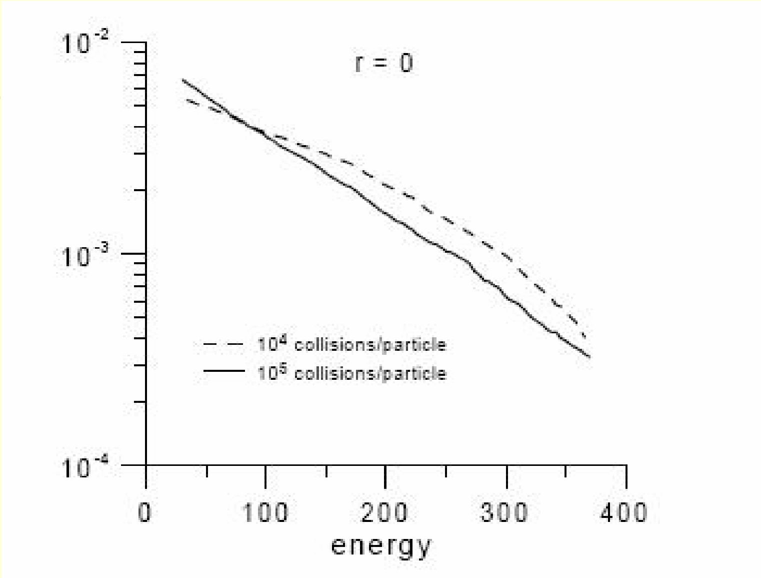
Kodama et al.., Europ.Lett. 70 (2005) 439):

Example how the existence of dynamical correlations leads to a preequilibrium state of the system:

(a)



(b)



Energy spectrum after a given number of collisions per particle, starting from a distribution peaked at E=125 GeV:

(a) correlated system $\Rightarrow$ non-Boltzmann distribution fitted by Tsallis distribution with β=0.39 GeV^{-1} and q=1.42

(b) uncorrelated system $\Rightarrow$ Boltzmann distribution (equilibration needs 10 times more steps now!)