Thermodynamical formalism for stationary states of Markov chains

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Carati [1] studied the probability distribution for the number of visits to the cells of a partitioned phase space. He used it as a basis for formulating thermodynamics. The generalization of this approach to arbitrary Markov chains involves the joint probability distribution for the number of transitions between any two states. We prove that this probability distribution always belongs to the exponential family. This result is related to the fluctuation theorem of Gallavotti and Cohen [2], see [3,4]. It makes also clear that in the case of Markov chains the only acceptable definition of dynamical entropy is based on the Boltzmann-Gibbs-Shannon entropy. As an application, we prove linear production of entropy, we study the thermodynamics of a one-parameter model of equilibrium states, and we discuss a model [5,6] of a random walk, used for describing experiments on polymer chains.

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