Heating of coronal active regions: statistics of dissipation events from reduced MHD simulations

Daniel Gómez [1,2] & Pablo Dmitruk [3]

Group of Astrophysical Plasmas
Buenos Aires - Argentina

Email: dgomez@df.uba.ar Webpage: http://astro.df.uba.ar

(1) Instituto de Astronomia y Fisica del Espacio, CONICET, Argentina
(2) Departamento de Fisica, Universidad de Buenos Aires, Argentina
(3) Bartol Research Institute, University of Delaware, Newark DE 19716, USA

Abstract

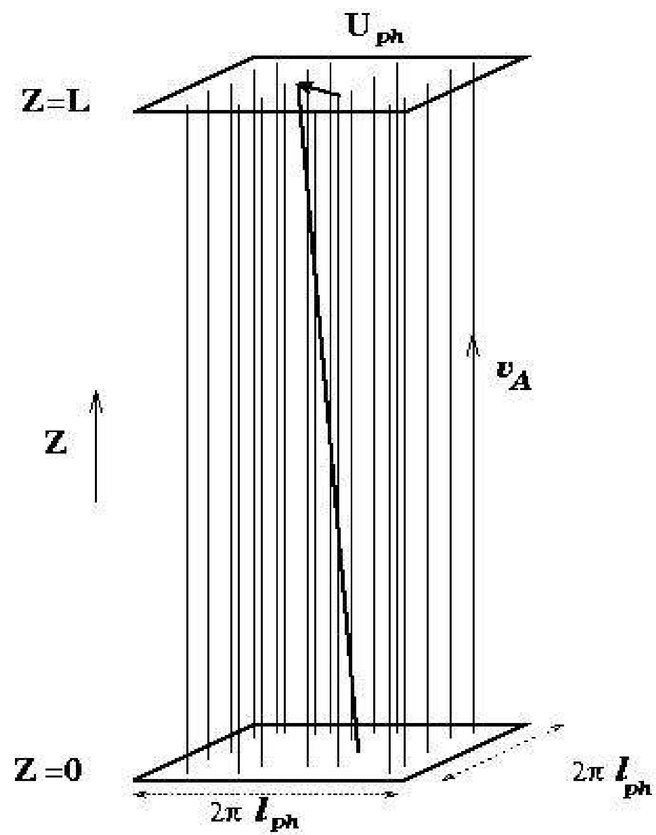
Within the reduced MHD approximation, we numerically simulate the dynamics of a coronal loop driven by a stationary velocity field at the photospheric boundaries.
After several photospheric turnover times, a turbulent stationary regime is reached, characterized by a broadband power spectrum and heating rate levels compatible with the heating requirements of active region loops.

The energy dissipation rate as a function of time displays a complex superposition of impulsive events, which we associate to the so-called nanoflares. A statistical analysis of these events yields a power law distribution as a function of their energies, which is consistent with those obtained for flare energy distributions reported from X-ray observations. We also study the distributions of peak dissipation rates, durations, and waiting times between events.

RMHD Equations

Reduced MHD is a self-consistent approximation of the full MHD equations whenever: (a) one component of the magnetic field is much stronger than the others and, (b) spatial variations are smoother along than accross.

These equations describe the evolution of the velocity (**u**) and magnetic field (**b**) inside the box, with periodic boundary conditions at the sides and given velocity fields ($\mathbf{U}_{ph}$) at the top and bottom plates.



$$\partial_t\, a = v_A \partial_z\, \varphi + [\varphi, a] + \eta \nabla_\perp^2 a$$

$$\partial_t\, \omega = v_A \partial_z\, j + [\varphi, \omega] - [a, j] + \nu \nabla_\perp^2 \omega$$

$$\mathbf{b} = v_A \mathbf{z} + \nabla_\perp \times (a\mathbf{z}) \,, \qquad \mathbf{u} = \nabla_\perp \times (\varphi \mathbf{z})$$

$$\omega = -\nabla_\perp^2 \varphi \,, \qquad j = -\nabla_\perp^2 a$$

RMHD simulations

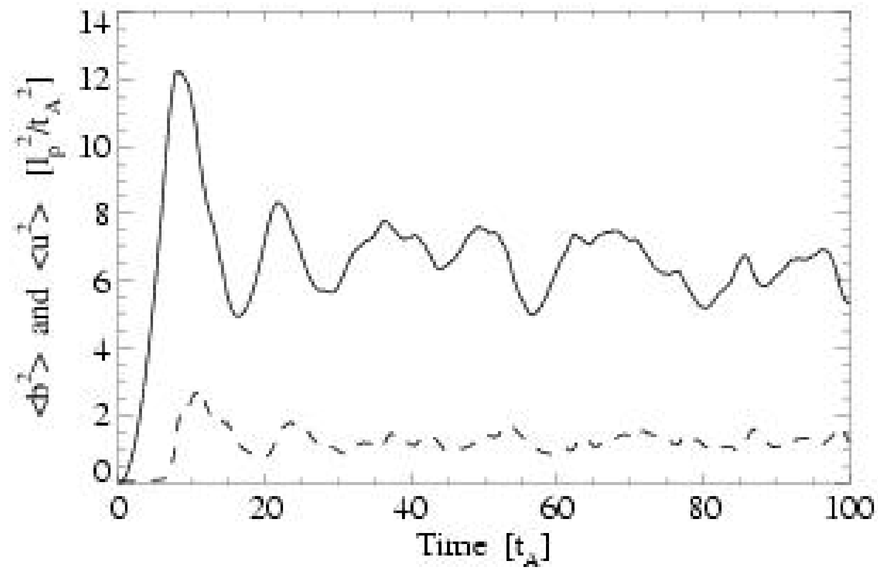
We perform long time integrations of the RMHD equations. Lengths are in units of the photospheric convective motions (ℓ_{ph}) and times are in units of the Alfven time (t_A) along the loop.

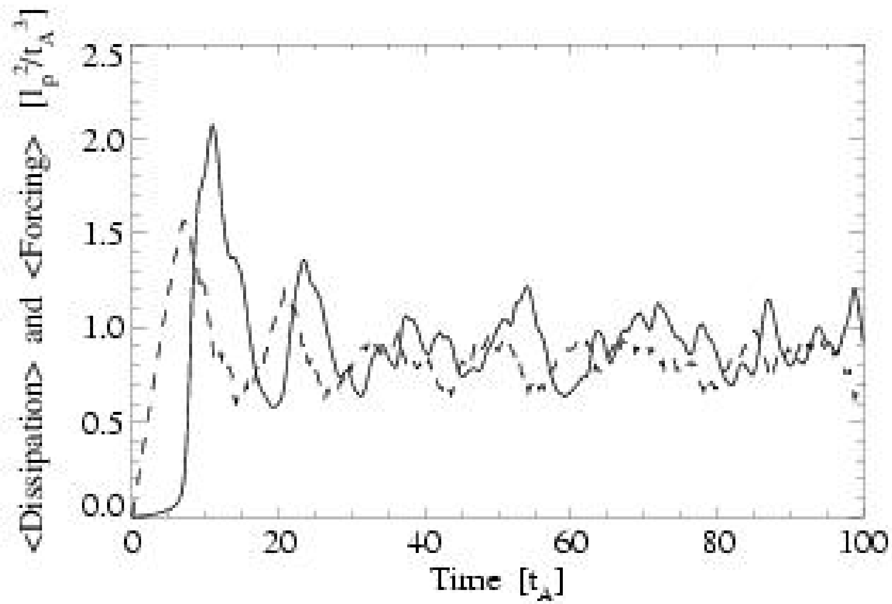
Spatial resolution is 256x256x30 and the integration time is 4000 t_A.

The time averaged dissipation rate scales like (Dmitruk & Gómez 1999)

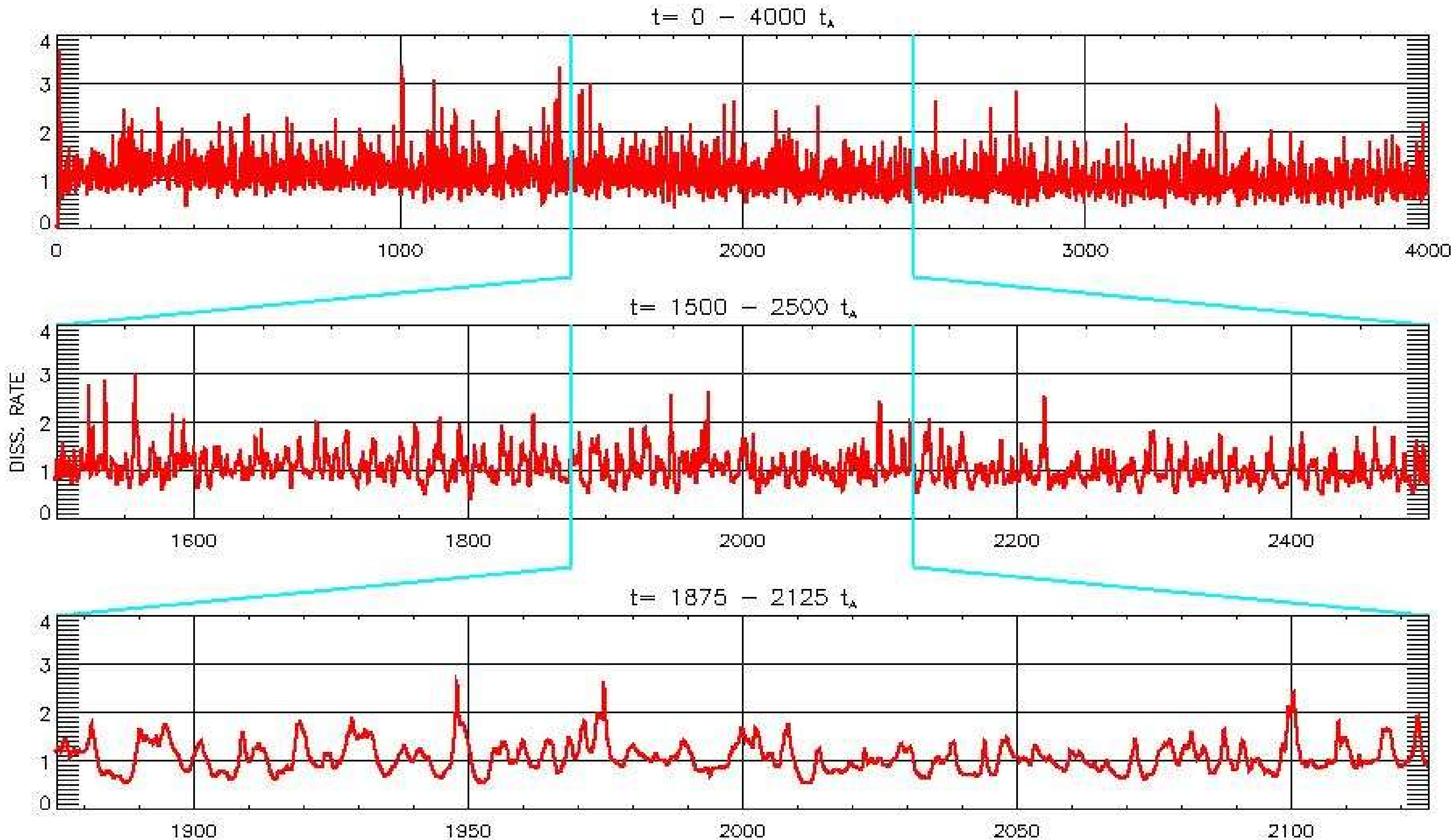
$$\boxed{\varepsilon \approx \frac{\rho\, \ell_{ph}^2}{t_A^3} \left(\frac{t_A}{t_{ph}}\right)^{3/2}}$$

It is esentially independent of the Reynolds number, as expected for stationary turbulence.





The complete time series of energy dissipation rate is displayed below. It shows a mean value (consistent with the scaling law given above) plus a rather spiky structure.



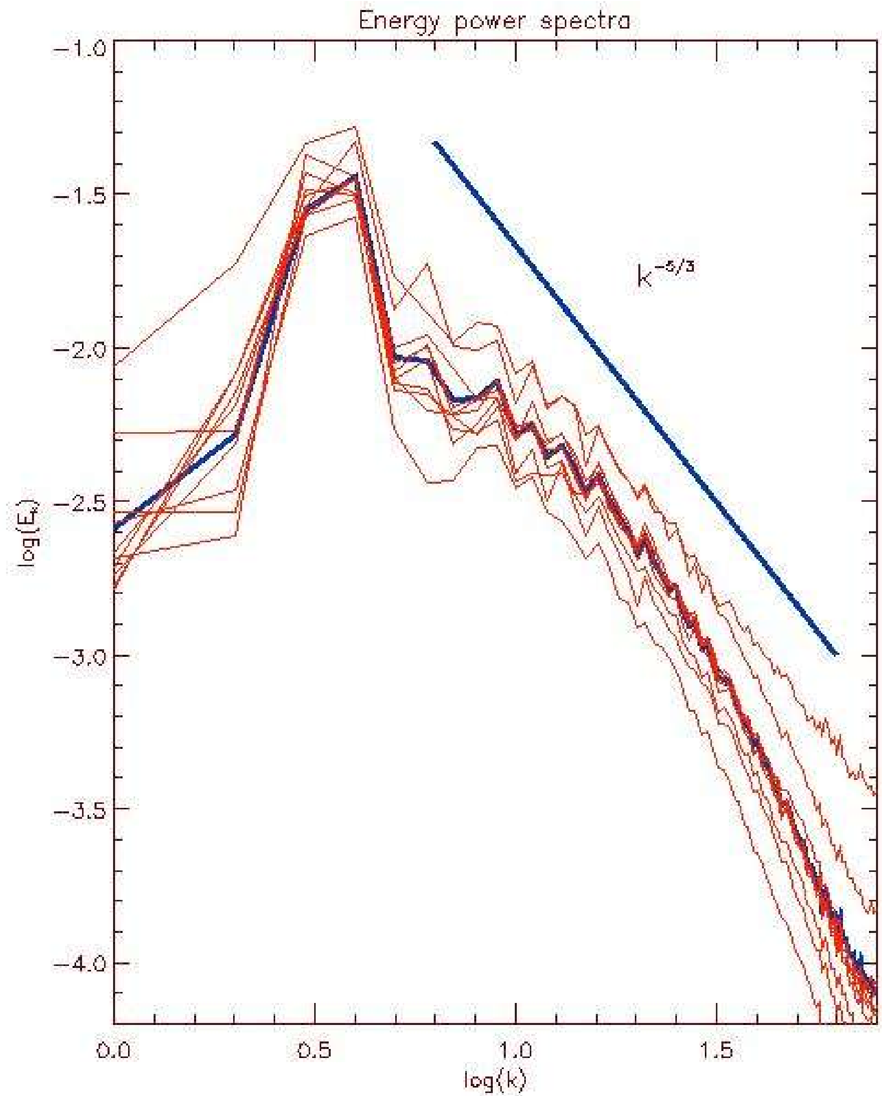
Energy power spectra

The energy spectra are shown here. The red lines correspond to ten spectra taken at different times (separated by 10 t_A). The blue trace is the time averaged version.

The Kolmogorov slope is displayed for reference, but the moderate spatial resolution of these runs is insufficient for a serious spectral analysis.

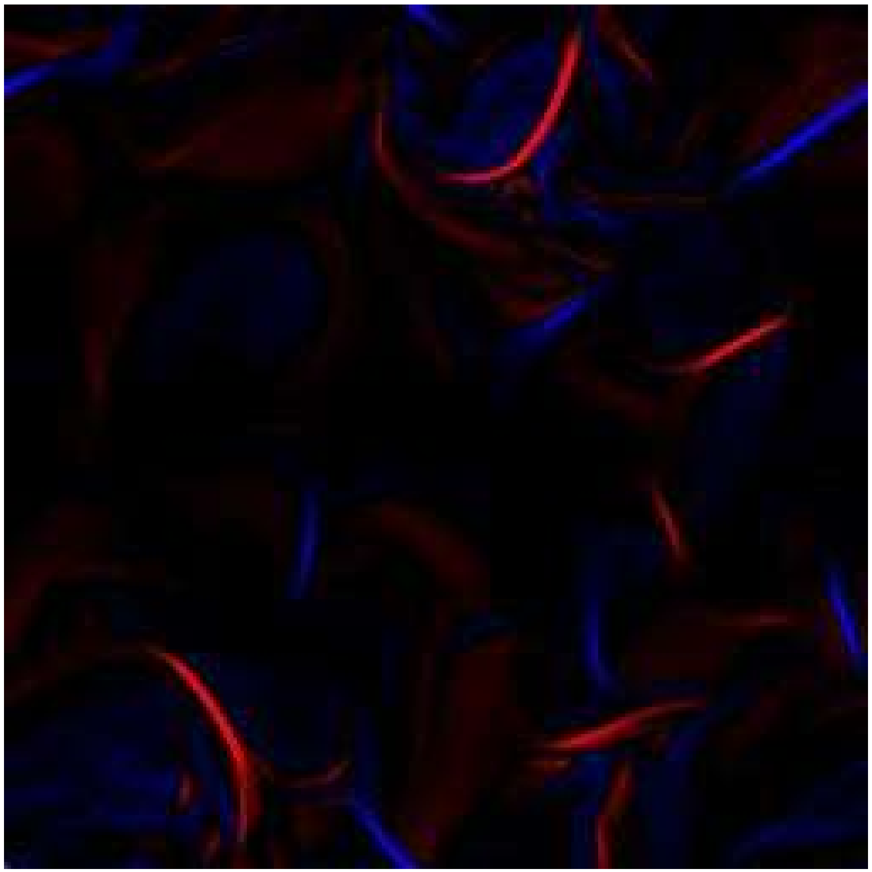
Viscosity and resistivity are large enough to spatially resolve the dissipative structures properly.

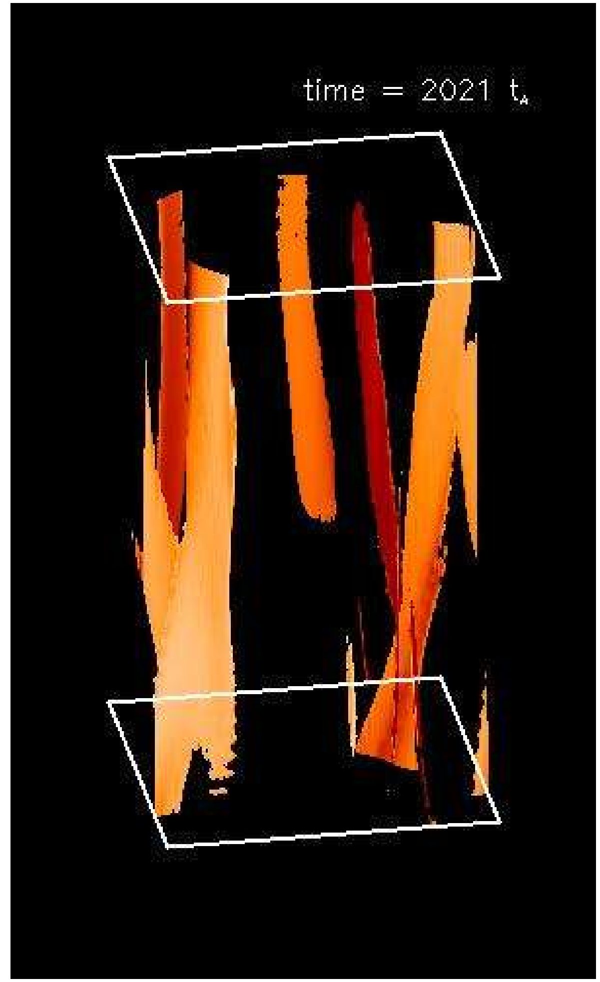
The spectra of kinetic energy (not shown) remain much smaller than magnetic energy, although they tend to equipartition at the largest wavenumbers.



Dissipative structures: current sheets

Most of the energy dissipation takes place in current sheets. Below we display the current density along the loop in a transverse cut by the middle of the loop. The different colors correspond to different flow directions.





3D distribution of the upward flowing current density, showing the regions where the current exceeds 10% of its maximum.

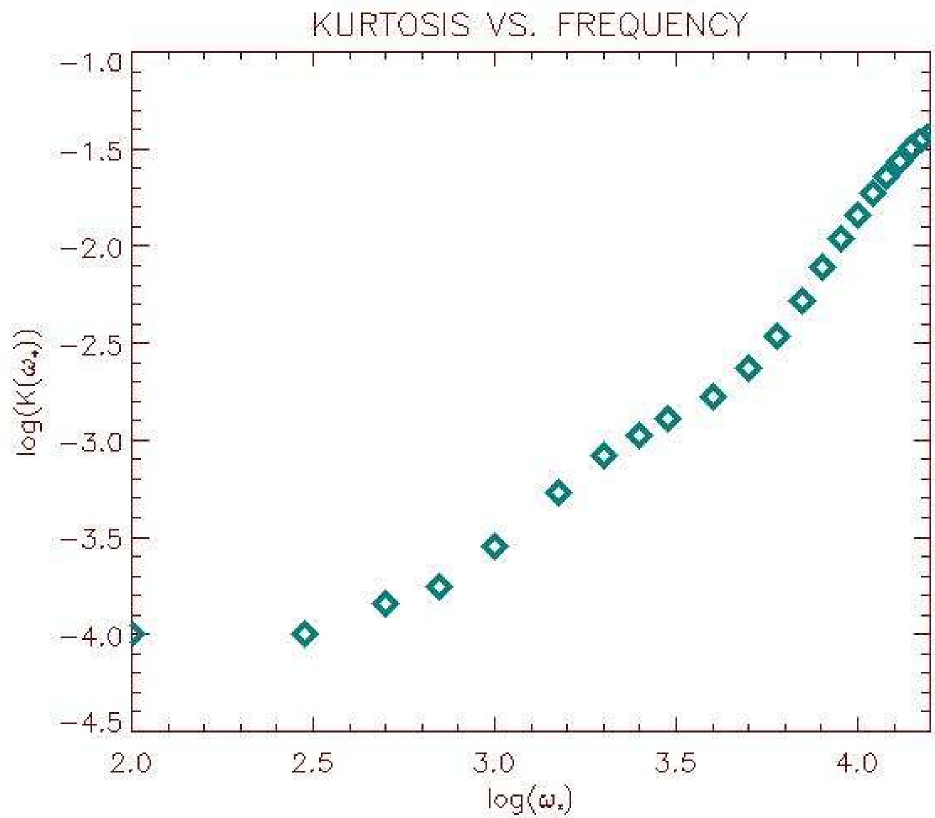
Intermittency

The intermittency of a time series can be artificially emphasized by performing high-pass filtering, to remove slow trends.

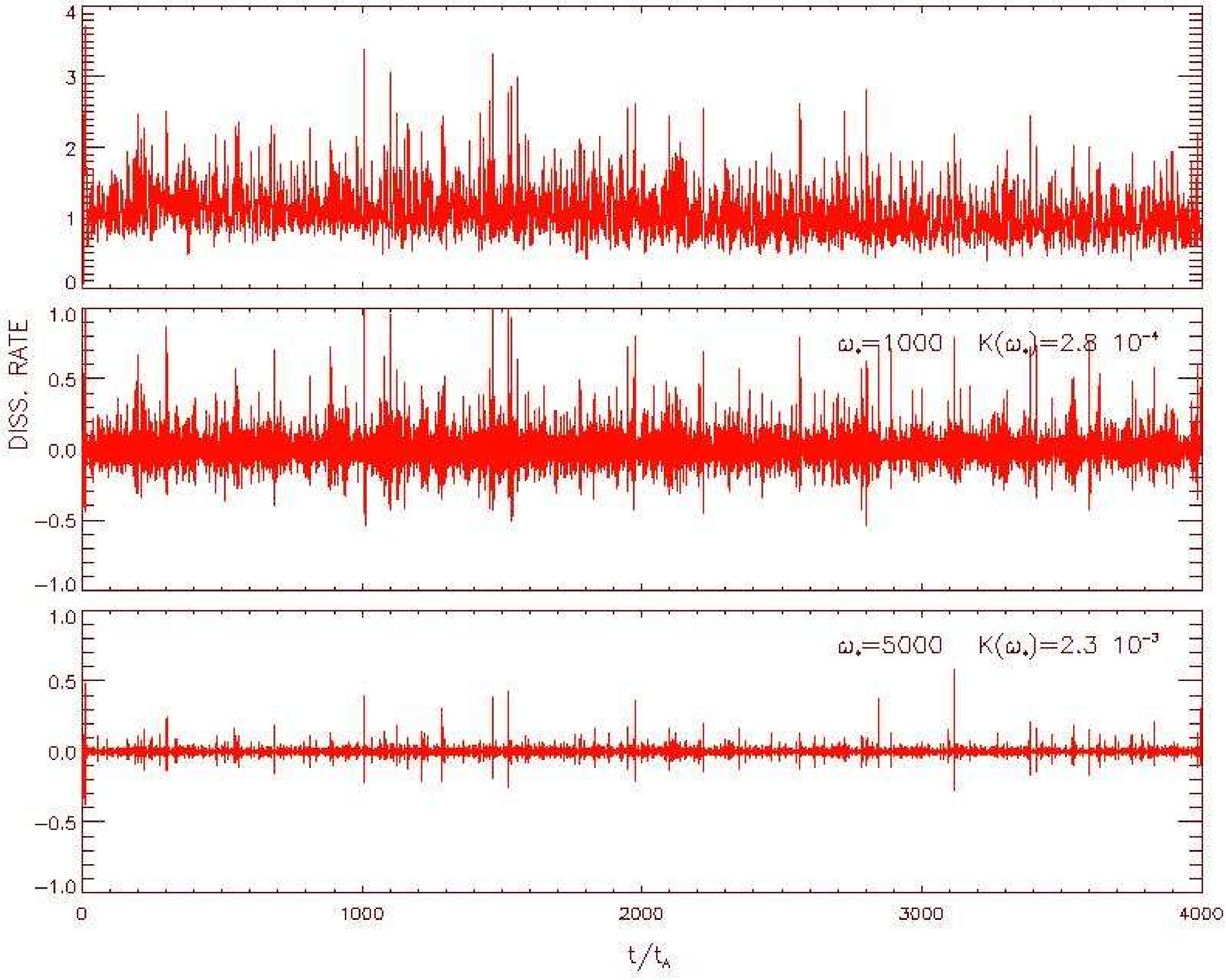
Below we display the total dissipation rate vs. time as well as the result of high-pass filtering for two cut-off frequencies.

Even if we move the cut-off frequency to higher values, intermittent signals always show bursts of activity.

This activity can be quantified by the flatness of the filtered signal. The flatness is the fourth order moment of the filtered signal, divided by the second order moment squared.



The monotonic increase of the flatness as the cut-off frequency of the high-pass filter is raised, has been proposed as a definition of intermittency (Frisch 1995).



DISS, RATE
$\omega_* = 1000$ $K(\omega_*) = 2.8 \ 10^{-4}$
$\omega_* = 5000$ $K(\omega_*) = 2.3 \ 10^{-3}$
t/t_A

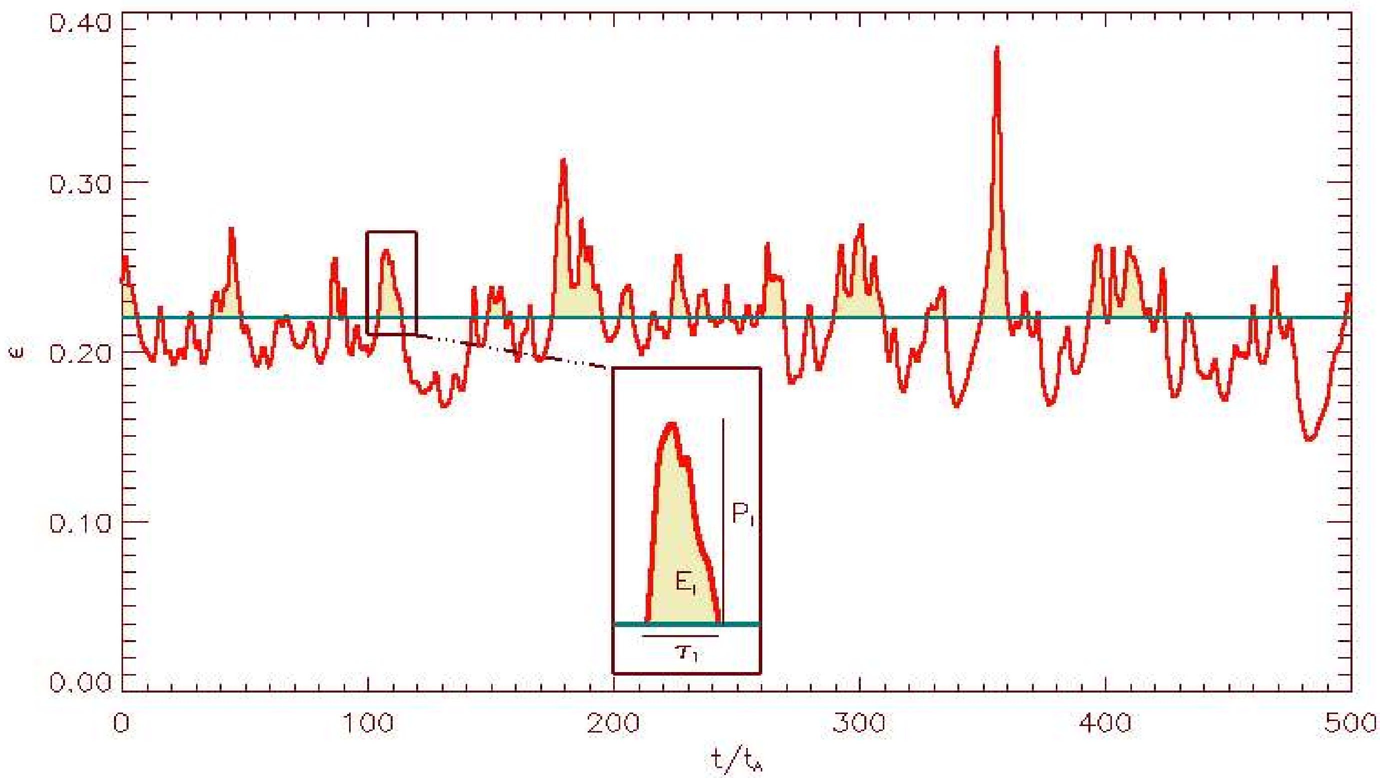
Dissipation events

We associate the intermittent bursts of dissipation with Parker´s nanoflares.

To carry out our statistical analysis, we draw a threshold and count as events everything that pops up above that line.

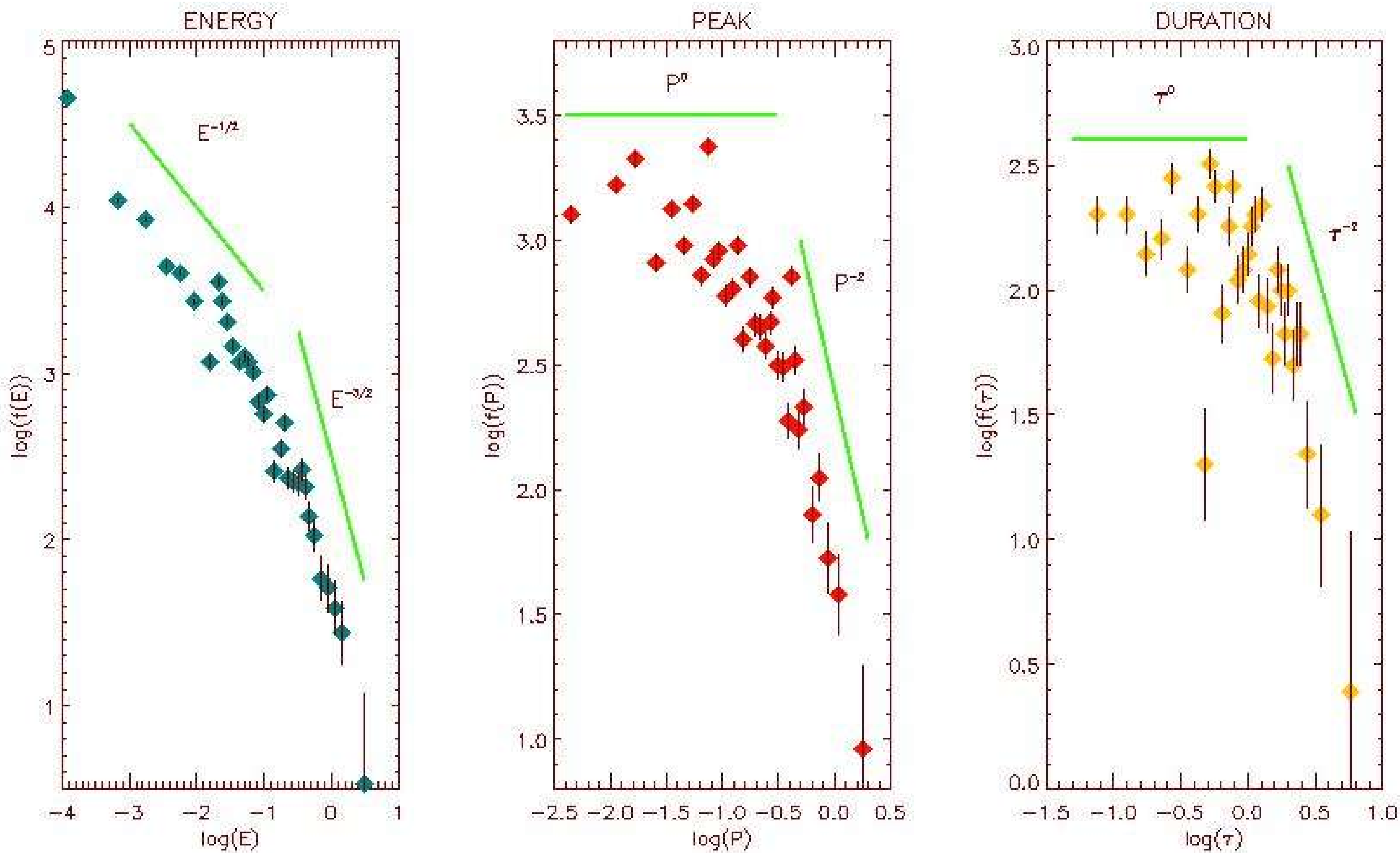
For each event, we compute its peak dissipation (P_i), its total dissipated energy (E_i), and its duration (T_i).

We compute histograms for these quantities and look for correlations between them.



Histograms of events

Histograms of energies, peak dissipation rates and durations display a power law behavior. We estimated the slopes using standard fitting techniques.



The approximate slopes for these distributions are

The slope –3/2 obtained for the energy distribution is consistent with the one derived by Dmitruk et al. (1998) for MHD simulations in 2D.

More importantly, it is also consistent with the energy distributions measured for nanoflares in the quiet Sun, for transient brightenings in active regions, and for hard X-ray flares (Aschwanden, 2004).

For the smaller events (i.e. small energy, peak and duration) the slopes change respectively to

$$\boxed{f(E) \approx E^{-0.5} \quad , \quad f(P) \approx P^{0} \quad , \quad f(T) \approx T^{0}}$$

This abrupt change of slopes arises because we are only considering the part of each event that rises above the threshold. If the total number of events per unit time is large, events will pile-up as shown by our time series of energy dissipation rate. For events such that the fraction of energy rising above the threshold is small, the distribution asymptotically becomes $f(E) \approx E^{-0.5}$

Correlations

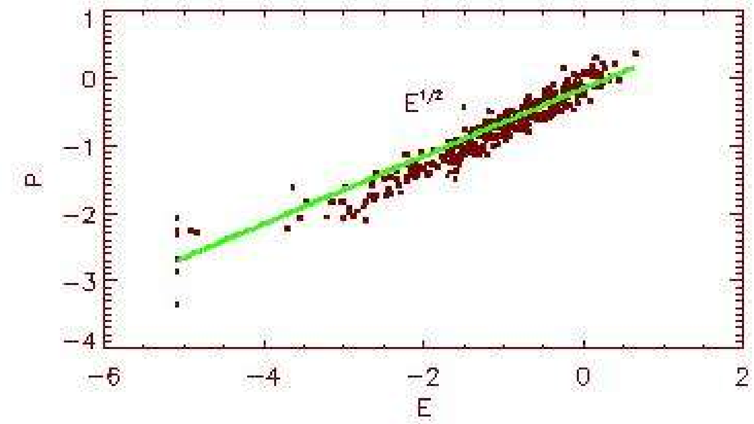
We also performed scatter plots to look for correlations between E, P and T.

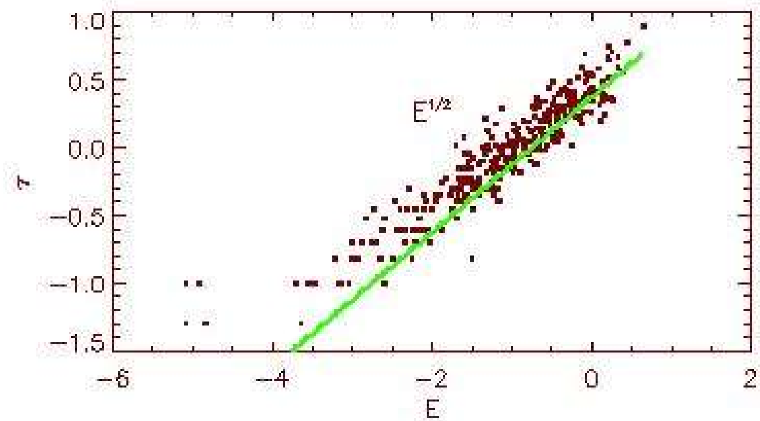
We obtain that both the peak dissipation rate and duration of events can be approximated by power laws of their energies:

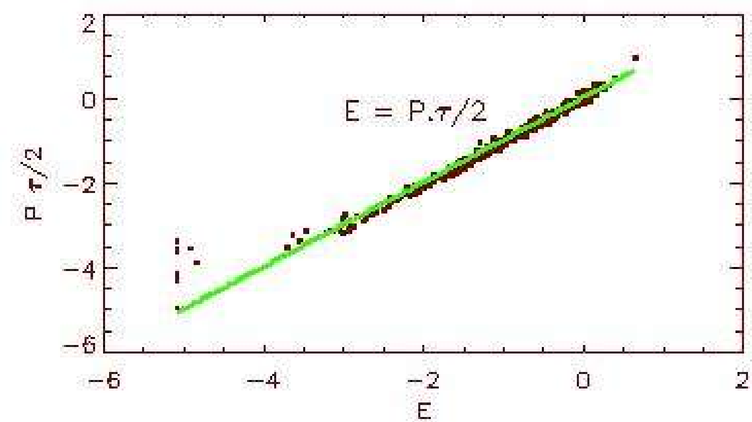
$$\boxed{P \approx E^{0.5} \qquad , \qquad T \approx E^{0.5}}$$

We also performed a scatter plot of E vs. P.T/2. We obtained that this scatter plot can be accurately fitted by a straight line of slope equal to one. This indicates that events can be regarded as displaying triangular profiles, at least for the sake of statistics.

Using the correlation $T \approx E^{0.5}$ and a power law energy distribution $f(E) \approx E^{-\alpha}$, we can re-obtain the distribution for peaks and durations shown in the previous page.







Waiting time distributions

We also computed the waiting times between consecutive events.

For a time series of uncorrelated events, the predicted distribution is the Poisson law

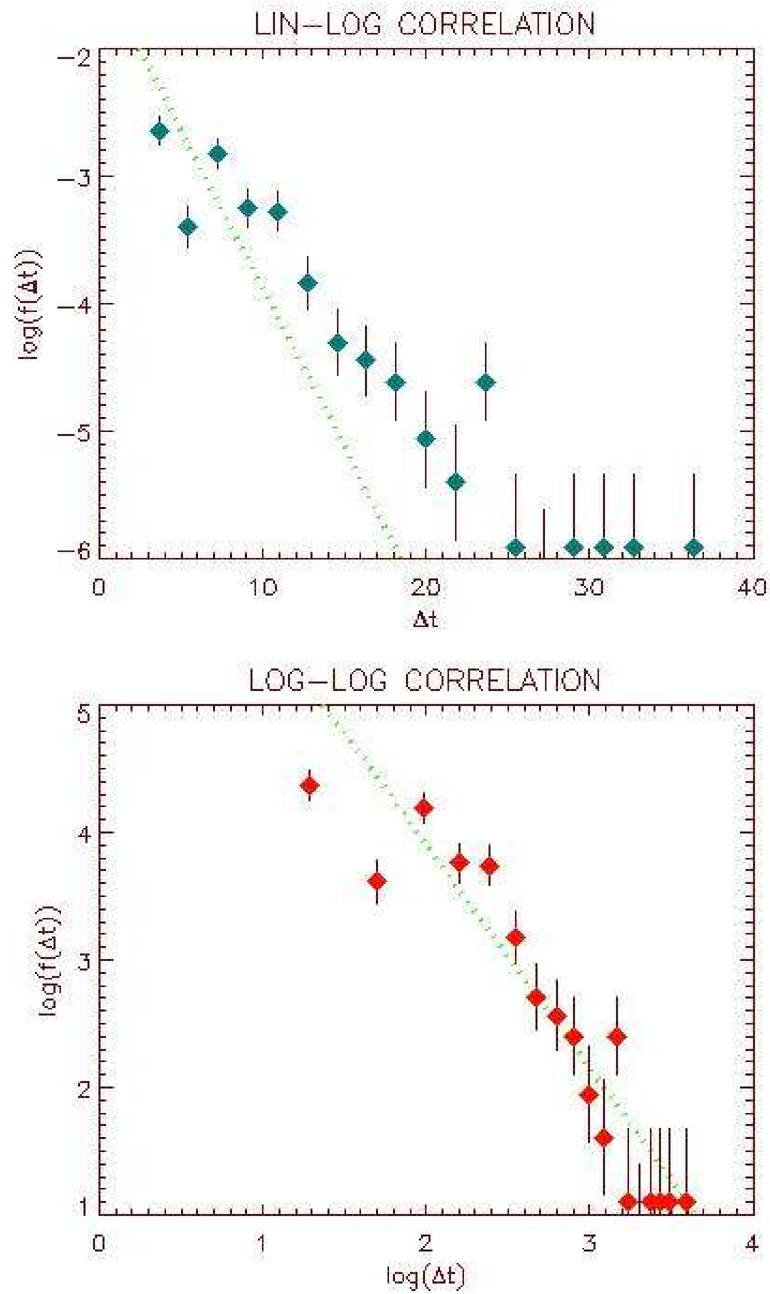
$$f(\Delta t) = \frac{1}{<\Delta t>} e^{-\frac{\Delta t}{<\Delta t>}}$$

In the upper panel, we compare our histogram with the Poisson distribution, and obtain a quality for this fit of $\frac{1}{N}\chi^2 \cong 6.74$

We also fitted a power law

$$f(\Delta t) \approx (\Delta t)^{-1.75}$$

and the quality of the fit is $\frac{1}{N}\chi^2 \cong 0.37$
i.e. Much better than the Poisson fit.



Conclusions

We performed the statistical analysis of dissipation events arising from numerical simulations of the RMHD equations. We associate the intermittent bursts of the energy dissipation rate, which are ubiquitous in turbulent systems, to Parker´s nanoflares.

The energy, peak dissipation rate and duration of these events, display power law distributions, with slopes consistent to those obtained from observations (see Aschwander 2004 for a recent overview).

The statistics of events obtained from our RMHD simulations, are quite comparable to previous results derived from MHD 2D simulations (Einaudi et al. 1996; Dmitruk & Gómez 1997), which are meant to mimic the dynamics of transverse slices of coronal loops.

A statistical study of waiting time distributions shows that a power law fits the results better than a Poisson law, thus indicating that consecutive events are to some extent correlated in time.