Collective Interactions in Quantum Plasmas

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OUTLINE

INTRODUCTION

- The field of quantum plasmas is quite new.
- Quantum plasmas are ubiquitous in ultrasmall electronic devices and micromechanical systems, in laser and microplasmas, and in dense astrophysical environments.
- Traditionally, quantum effects are important when the de Broglie wavelength of the charge carriers (electrons, holes/positrons) is comparable to the dimension of the system.
- Here quantum mechanical effects (e.g. tunneling) can play an important role at nanoscales.

INTRODUCTION (Continued)

- We are thus dealing with extremely high density and low-temperature plasmas, in contrast to traditional plasmas that have high temperature and low density.

- In quantum plasmas, strong electron correlations exist at quantum scales. They lead to dispersion at quantum scales. We can have new wave modes and nonlinear structures (dark & gray solitons, quantum vortices) in ultracold quantum plasmas.

- the $\hbar$ physics in quantum plasmas is interesting as it shares knowledge with the BEC physics.

PROPERTIES OF QUANTUM PLASMAS

- Quantum effects can be measured by the thermal de Broglie wavelength of the particles composing the plasma

$$\lambda_B = \frac{\hbar}{mV_T}$$

which roughly represents the spatial extension of a particle's wave function due to quantum uncertainty. For classical regimes, the de Broglie wavelength is so small that particles can be considered as pointlike, and therefore there is no overlapping of the wave functions and no quantum interference.

PROPERTIES OF QUANTUM PLASMAS (Continued)

- It is reasonable to postulate that quantum effects start playing a significant role when the de Broglie wavelength is similar to or larger than the average interparticle distance $n^{-1/3}$, i.e. when $n\lambda_B^3 \geq 1$.

- Quantum effects become important when the temperature is lower than the so-called Fermi temperature T_F, defined as

$$k_B T_F \equiv E_F = \frac{\hbar^2}{2m}(3\pi^2)^{2/3} n^{2/3}.$$

- When the temperature approaches T_F, the relevant distribution changes from Maxwell-Boltzmann to Fermi-Dirac.

PROPERTIES OF QUANTUM PLASMAS (Continued)

- It is easy to see that

$$\chi = \frac{T_F}{T} = \frac{1}{2}(3\pi^2)^{2/3}(n\lambda_B^3)^{2/3}.$$

Thus quantum effects become important when $\chi \geq 1$.

- The relevant velocity FOR A FERMI-DIRAC distribution is

$$V_F = (2E_F/m)^{1/2} = \frac{\hbar}{m}(3\pi^2 n)^{1/3}.$$

PROPERTIES OF QUANTUM PLASMAS (Continued)

- The Fermi screening scalelength

$$\lambda_F = \frac{V_F}{\omega_p}$$

is the quantum analogue of the Debye length.

- The quantum coupling parameter

$$G_q = \frac{E_{int}}{E_F} \sim \left(\frac{1}{n\lambda_F^3}\right)^{2/3} \sim \left(\frac{\hbar\omega_p}{E_F}\right)^2,$$

is completely analogous to the classical one when one substitutes $\lambda_F \to \lambda_D$.

Model Equations

- The most fundamental model for the quantum N body problem is the Schroedinger equation for the N-particle wave function $\psi(x_1, x_2, x_N, t)$. We assume that the N-body wave function can be factored into the product of N-one body functions:

$$\psi(x_1, x_2, ...x_N, t) = \psi_1(x_1, t)\psi_2(x_2, t)...\psi_N(x_N, t)$$

  For Fermions a weak form of the Pauli exclusion principle is satisfied if none of the wave functions on the r.h.s. are identical.

- We can then introduce the density matrix formalism-Wigner and Hartee models.

Model Equations (Continued)

- The Wigner-Poisson model for our purposes is then

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \frac{e}{m} \nabla \phi \cdot \nabla_{\mathbf{v}} f \approx \frac{\hbar^2}{24 m^3} \nabla \nabla^2 \phi \nabla_{\mathbf{v}}^3 f$$

$$\nabla^2 \phi = \frac{e}{\epsilon_0} \left(\int f d^3 v - n_0 \right).$$

QUANTUM HYDRODYNAMICAL MODEL

- We have

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$\frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{e}{m}\nabla\phi + \frac{\hbar^2}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) - \frac{1}{mn}\nabla P,$$

where

$$P = \frac{mV_F^2}{3n_0^2}n^3$$

QUANTUM HYDRODYNAMICAL MODEL (Continued)

Introduce the effective wave function

$$\psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}\exp(iS(\mathbf{r},t)/\hbar)$$

where S is defined according to $m\mathbf{u} = \nabla S$ and $n = |\psi|^2$. It is easy to show that the QHD equations are equivalent to

$$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi + e\phi\psi - \frac{mV_F^2}{2n_0^2}|\psi|^{4/D}\psi = 0$$

and

$$\nabla^2\phi = \frac{e}{\epsilon_0}(|\psi^2| - n_0)$$

- Without the ϕ term, the NLSE is similar to that found in BECs. For $D = 1$, we have analytical solutions (PRL **85**, 1146, 2000).

Dark quantum electron solitons/vortices in D dimensions

Normalized system

$$i\frac{\partial \Psi}{\partial t} + A\nabla^2\Psi + \varphi\Psi - |\Psi|^{4/D}\Psi = 0,$$

$$\nabla^2\varphi = |\Psi|^2 - 1,$$

Conserved quantities

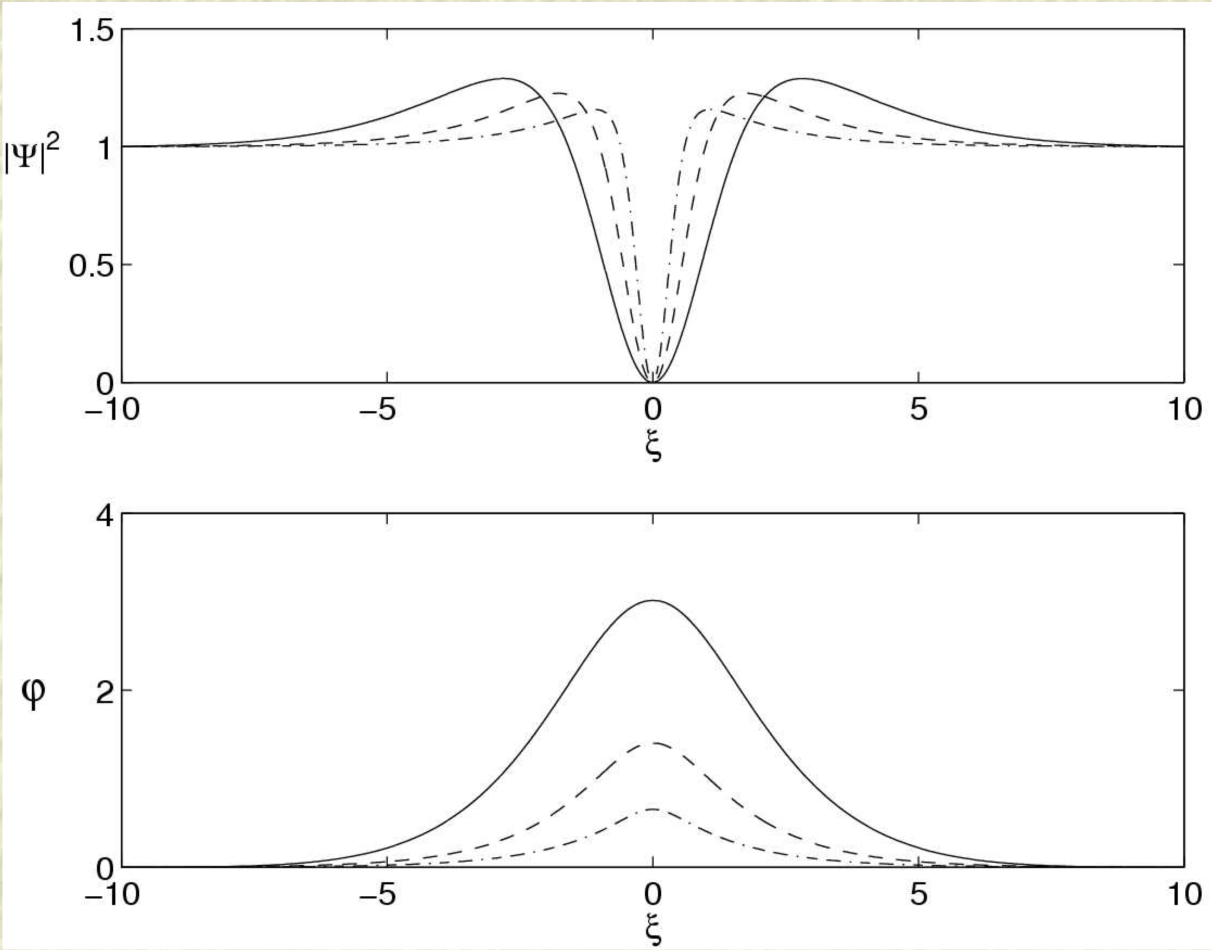
$$N = \int |\Psi|^2 \, d^3x$$

$$\mathbf{P} = -i\int \Psi^* \nabla\Psi \, d^3x$$

$$\mathbf{L} = -i\int \Psi^* \mathbf{r} \times \nabla\Psi \, d^3x$$

$$\mathcal{E} = \int [-\Psi^* A\nabla^2\Psi + |\nabla\varphi|^2/2 + |\Psi|^{2+4/D} D/(2+D)] \, d^3x$$

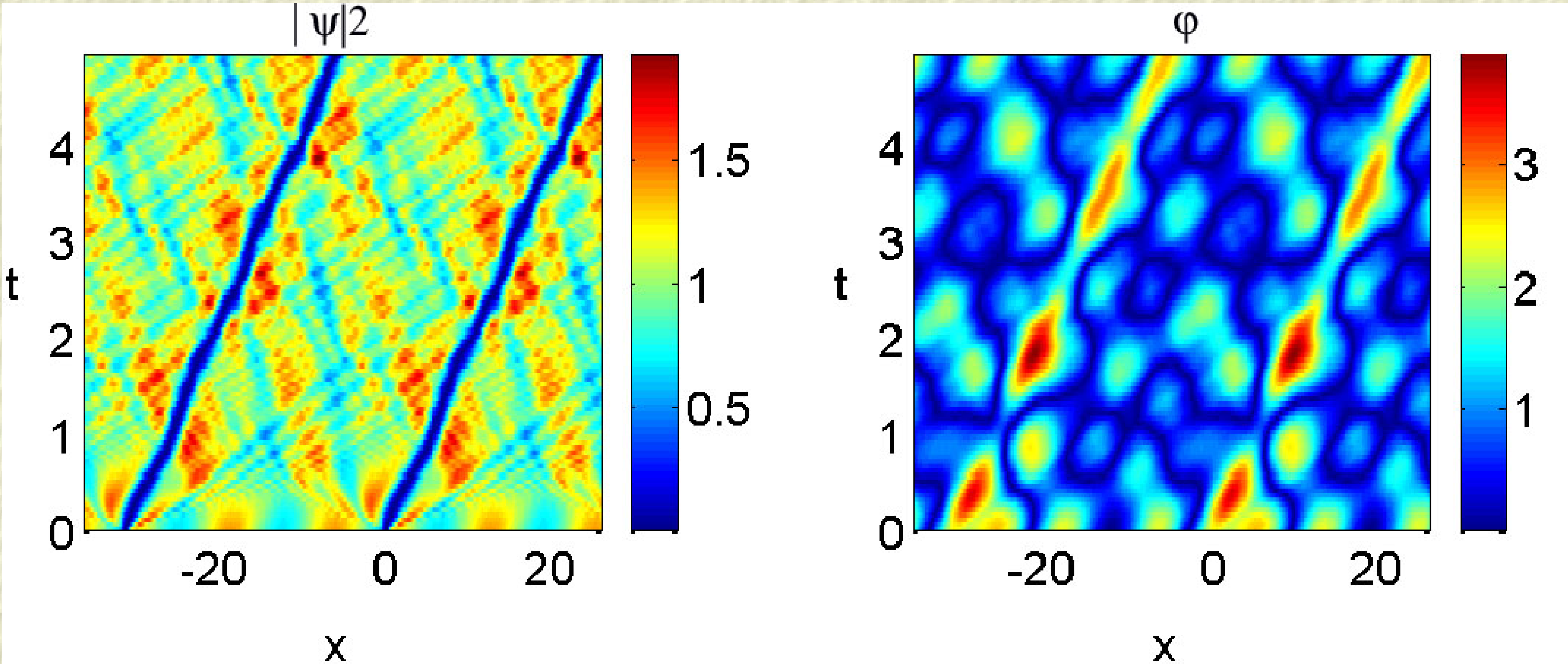
1D DARK SOLITONS



Shukla & Eliasson, PRL **96**, 245001 (2006)

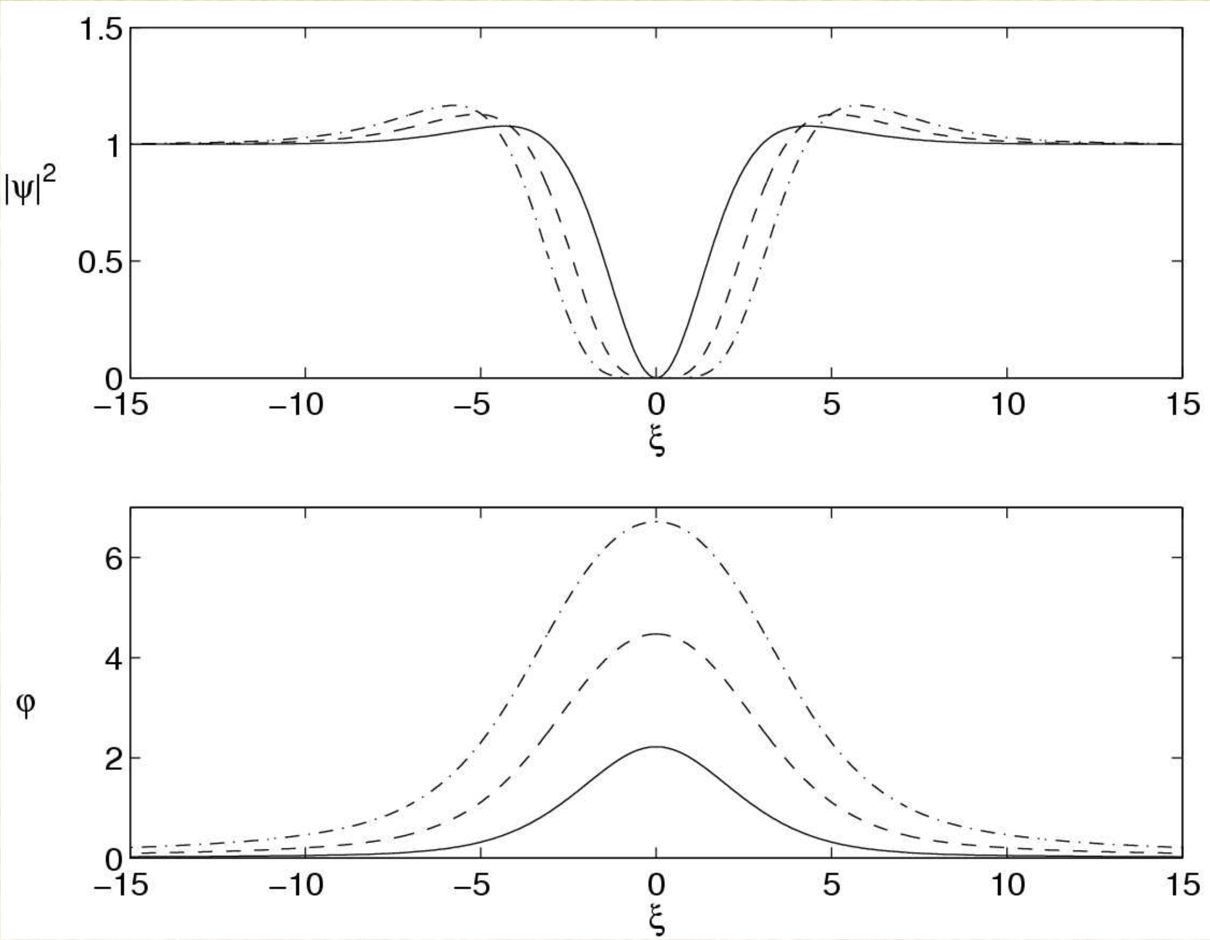
DYNAMICS OF 1D DARK SOLITONS

Electron density (left) & electrostatic potential (right)



Shukla & Eliasson, PRL **96**, 245001 (2006)

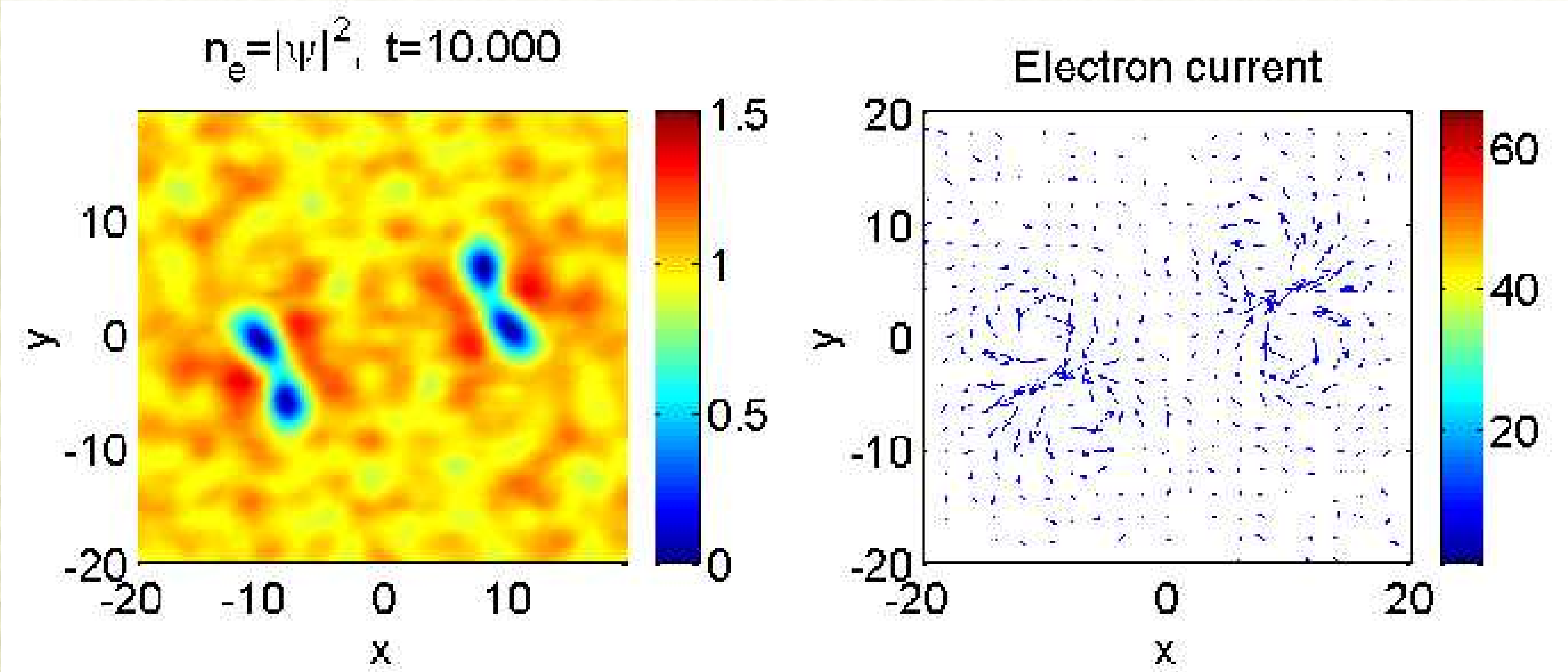
2D QUANTUM ELECTRON VORTICES



Shukla & Eliasson, PRL **96**, 245001 (2006)

INTERACTING 2D QUANTUM VORTICES

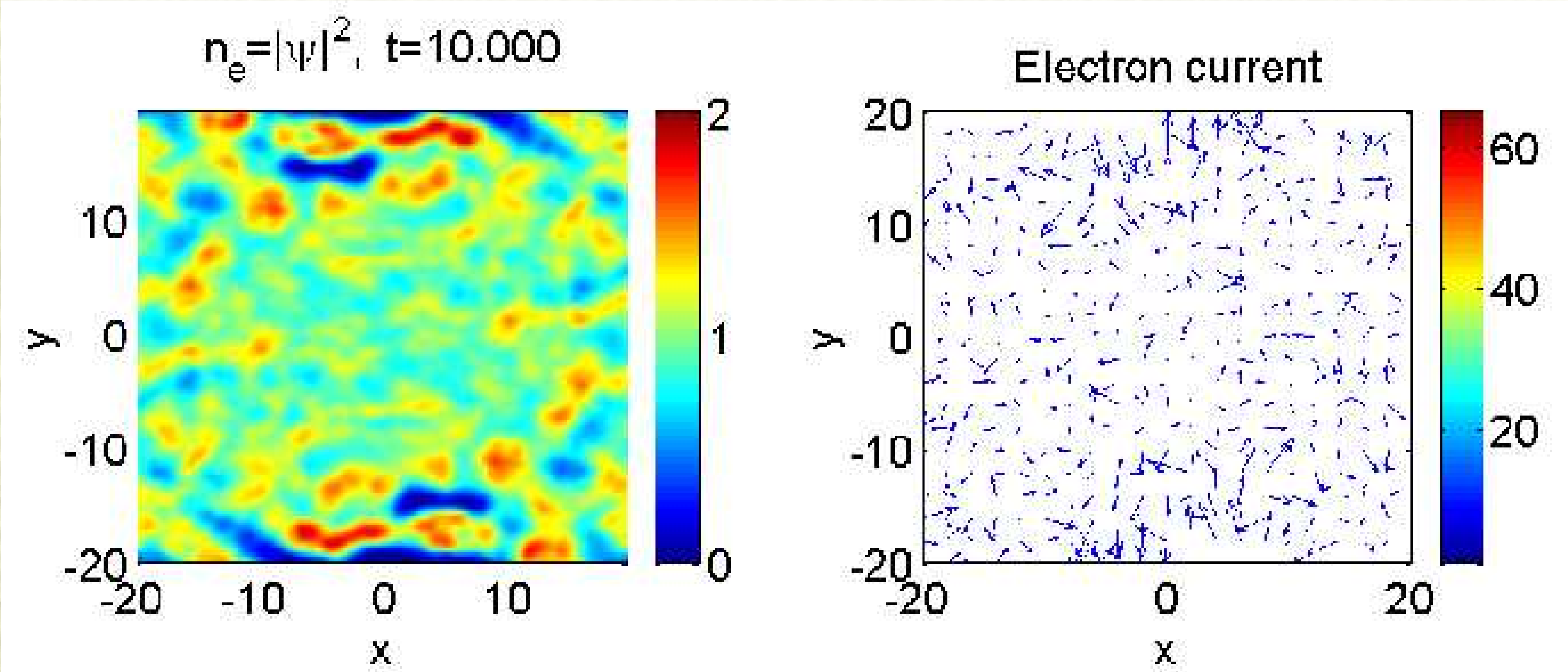
SINGLE CHARGE STATES ($n = 1$)



Shukla & Eliasson, PRL **96**, 245001 (2006)

INTERACTING 2D QUANTUM VORTICES

DOUBLE CHARGE STATES ($n = 2$)



Shukla & Eliasson, PRL **96**, 245001 (2006)

QUANTUM WAVE MODES

- The Langmuir wave dispersion relation is

$$\omega^2 = \omega_p^2 + k^2 V_F^2 + \frac{\hbar^2 k^4}{4m^2}$$

- Quantum ion-acoustic waves

$$\omega^2 = \frac{k^2 C_s^2 (1 + H^2 k^2/4)}{1 + k^2 (1 + H^2 k^2/4)}$$

where

$$C_s = \left(\frac{2 k_B T_{Fe}}{m_i} \right)^{1/2}$$

$$H = \frac{\hbar \omega_p}{2 k_B T_{Fe}}$$

QUANTUM WAVE MODES (Continued)

❑ In ultracold quantum plasmas, we have

$$\omega = \left(\omega_p^2 + \hbar^2 k^4/4m^2\right)^{1/2} \equiv \Omega_R$$

for the Langmuir waves, and

$$\omega = \hbar k^2/2\sqrt{m_e m_i} \equiv \Omega_B$$

for the ion oscillations.

EXCITATION OF ES QUANTUM MODES BY LIGHT

- Both Langmuir waves and ion oscillations couple with light nonlinearly. The governing equations are

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \omega_p^2\right)\mathbf{A} + \omega_p^2\frac{n_{e1}}{n_0}\mathbf{A} = 0$$

for the light

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2 + \frac{\hbar^2}{4m_e^2}\nabla^4\right)n_{e1} = \frac{n_0 e^2}{2m_e^2 c^2}\nabla^2|\mathbf{A}|^2$$

for the driven Langmuir waves, and

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\hbar^2}{4m_e m_i}\nabla^4\right)n_{e1} = \frac{n_0 e^2}{2m_e m_i c^2}\nabla^2|\mathbf{A}|^2$$

for the driven ion oscillations in ultracold quantum plasmas.

Parametric Instability Growth Rates

- The SRS and SBS growth rates are

$$\gamma_R = \frac{\omega_p e K |\mathbf{A}_0|}{2\sqrt{2}\sqrt{\omega_0 \Omega_R} m_e c}$$

$$\gamma_B = \frac{\omega_p e K |\mathbf{A}_0|}{2\sqrt{2}\sqrt{\omega_0 \Omega_B} m_e m_i c}$$

Shukla & Stenflo, PoP **13** (2006).

SUMMARY

- We have discussed the properties of quantum plasmas.
- Provided the appropriate models
- Localized nonlinear states can be represented as dark and gray envelope solitons in a quantum electron gas.
- 2D Quantum electron vortices can exist. They are robust identities.
- Discussed possible linear ES quantum modes.
- ES quantum modes can be parametrically excited by light.