Nonlinear quantum electrodynamics in magnetized plasmas

Mattias Marklund
Umeå University, Sweden

With G. Brodin, P. K. Shukla, B. Eliasson, L. Stenflo

Overview

- Nonlinear QED.
- Pair plasmas and pulsar magnetospheres.
- Magnetars and ultra-strong magnetic fields.
- Plasma-QED effects
- Conclusions

Nonlinear quantum electrodynamics

- Even before the event of QED, Weisskopf, Heisenberg, Euler etc. derived nonlinear effects based on vacuum fluctuations.
- Schwinger presented derivation of vacuum polarization.
- Nonlinear self-interaction of photons via virtual pairs gives rise to (Marklund & Shukla, RMP (2006))
  - Delbrück scattering in strong static inhomogeneous external fields,
  - photon splitting in strong external static fields, and
  - elastic scattering among real photons.
- Pair production: $\begin{cases} \gamma + \gamma \rightarrow e^- + e^+ \\ \gamma + B_0 \rightarrow e^- + e^+ + B_0 \end{cases}$

Pair plasmas and magnetospheres

- Rotating strongly magnetized stars ⟹ Pair plasma creation from photons produced by accelerated charges.
- Goldreich-Julian density: $n_{\mathrm{GJ}} \sim 10^{11} - 10^{12}\,\mathrm{cm}^{-3}$ (from screening) close to neutron star surface ($B_0 \sim 10^{12}\,\mathrm{G}$).
- Density can be increased depending on multiplicity (e.g. secondary pair production in the neutron star double layer).
- Magnetars: Even more extreme objects, $B_0 \sim 10^{15}\,\mathrm{G}$, QED effects may have significant effects on magnetosphere.

Strong field effects

- The Heisenberg-Euler Lagrangian (Schwinger, PR (1951)):

$$\mathcal{L} = -\tfrac{1}{2}\epsilon_0(E^2 - c^2B^2) + \kappa\epsilon_0^2\left[(E^2 - c^2B^2)^2 + 7c^2(\boldsymbol{E}\cdot\boldsymbol{B})^2\right]$$

where $\kappa = \alpha/90\pi\epsilon_0 E_{\mathrm{crit}}^2$ and $E_{\mathrm{crit}} = m_e^2c^3/e\hbar \sim 10^{16}$ V/cm.

- Nontrivial dispersion relation for photons with four-momentum $(\omega, \boldsymbol{k})$ in external field $(\boldsymbol{E}, \boldsymbol{B})$:

$$\omega = c|\boldsymbol{k}|\left(1 - \tfrac{1}{2}\lambda\epsilon_0|\hat{\boldsymbol{k}}\times\boldsymbol{E} + c\hat{\boldsymbol{k}}\times(\hat{\boldsymbol{k}}\times\boldsymbol{B})|^2\right)$$

where $\lambda = 8\kappa, 14\kappa$ depending on photon polarization (Bialynicka-Birula & Bialynicki-Birula, PRD (1970)).

Plasma-QED dispersion

- The combined effect of ultrarelativistic magnetized plasmas and nonlinear QED gives rise to new low-frequency EM modes:

$$\frac{k^2c^2}{\omega^2} = \frac{4\alpha}{45\pi}\left[\left(\frac{E}{E_{\rm crit}}\right)^2\frac{k^2c^2}{\omega^2} + \left(\frac{cB_0}{E_{\rm crit}}\right)^2\right]\frac{k^2c^2}{\omega^2} \mp \frac{\omega_p^2}{\omega\omega_e}\frac{E_{\rm crit}}{E}$$

where $\omega_e = m_e c^2/\hbar$ (Marklund *et al.*, PPCF (2005)). Also for dusty plasmas (Marklund *et al.*, PoP (2005)).

- Shock wave and soliton formation in magnetized dusty QED plasmas (Marklund *et al.*, EPL (2005)).

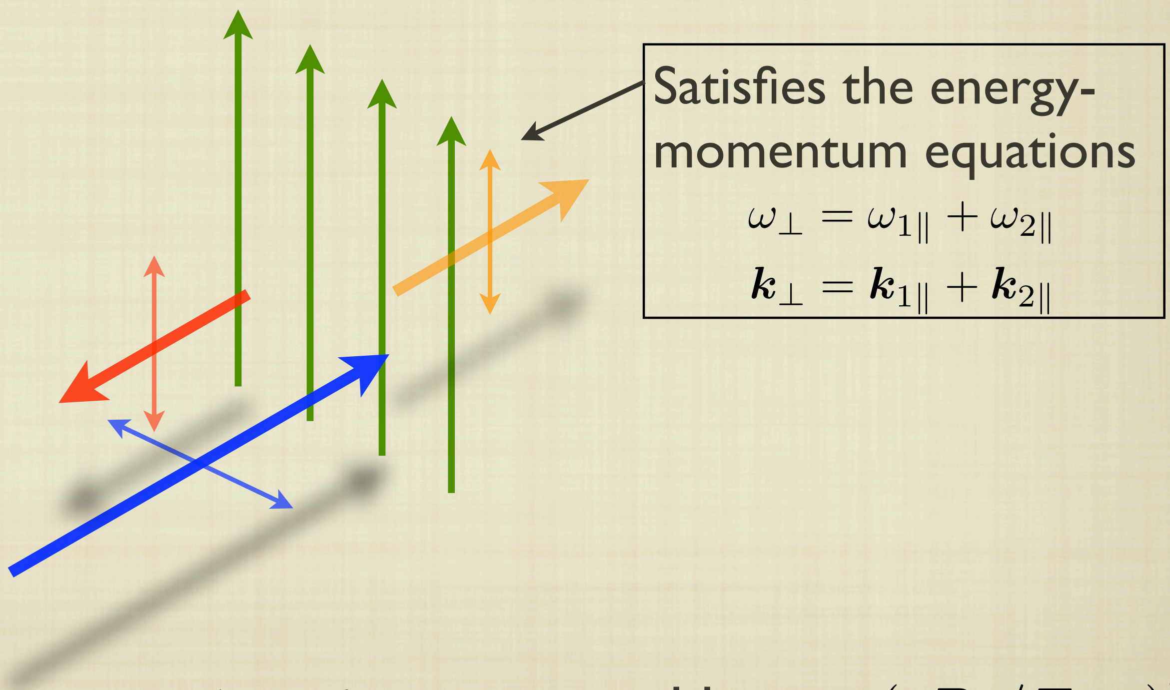
$\sim c\alpha^{1/2}/E_{\rm crit}$

a) Dominant nonlinearity: $\partial_t E \pm a|E|\partial_z E = 0$;

b) weakly nonlinear: $i\partial_t E - \frac{1}{2}v_g'\partial_z^2 E - a^2|E|^2 E = 0$.

Photon splitting in magnetic fields

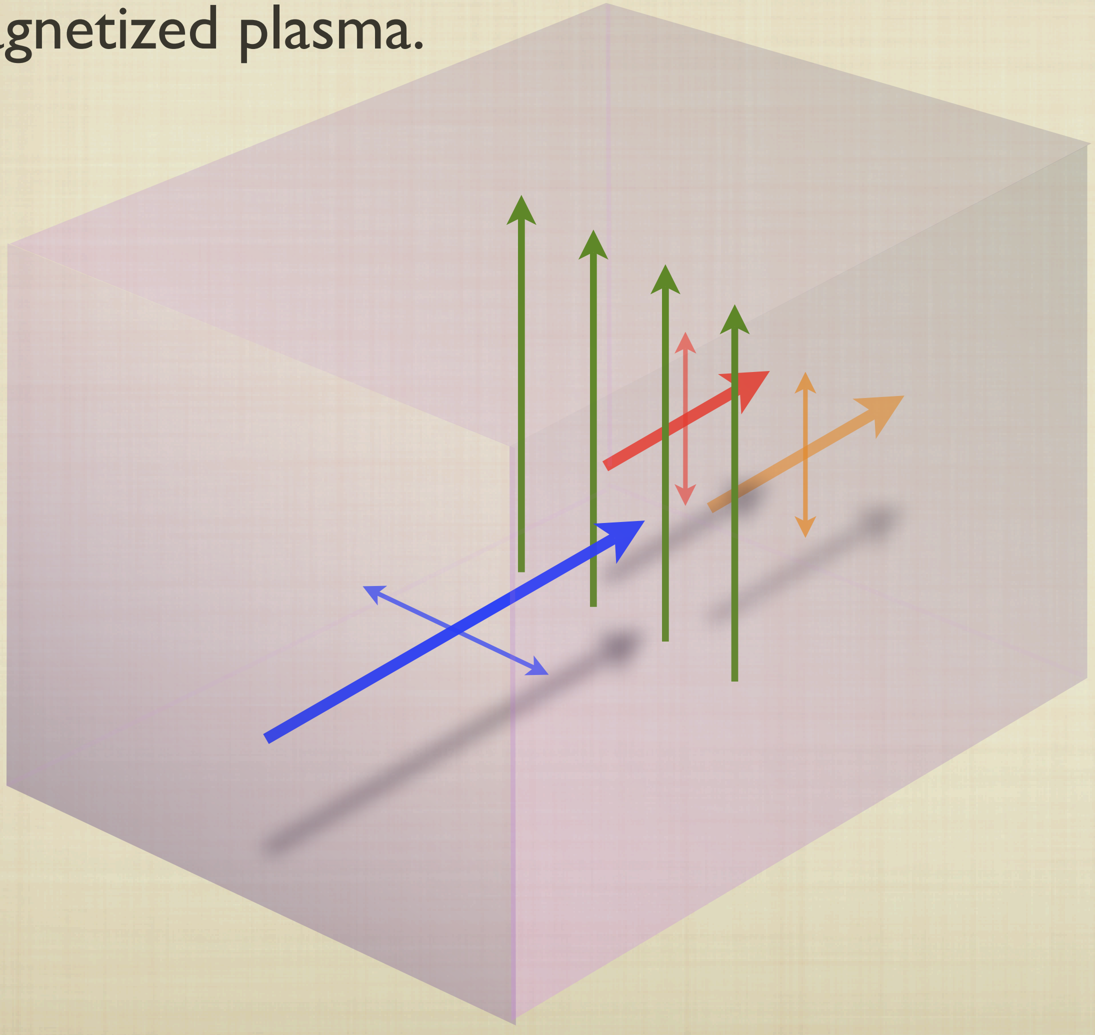
- Photons may be down-converted in external magnetic fields: $\gamma_\perp \to \gamma_\parallel + \gamma_\parallel$ (Adler, AP (1971)).



- The process $\gamma_\perp \to \gamma_\perp + \gamma_\parallel$ is suppressed by $\sim \alpha(cB_0/E_{\rm crit})^2$ (Berestetskii, Lifshitz, & Pitaevskii, 1982).

Photon splitting in magnetized plasma

- In magnetized plasma.



Plasma-QED photon splitting

- Linear EM wave propagation in astrophysical QED plasmas have been analyzed by, e.g., Bakshi *et al.*, PRD (1976); Cover *et al.*, PRD (1979); Péres Rojas & Shabad, AP (1979)).

- May lead to gamma photon capture and pair plasma suppression in pulsar magnetospheres (Shabad & Usov, Nature (1982)).

- Analysis of fully nonlinear photon splitting in a pair plasma has never been done.

- Of interest in highly relativistic environments (e.g. pulsars).

Photon decay in pair plasmas

- Taking into account the nonlinear plasma response, we obtain a new and novel set of coupled mode equations:

$$\partial_t E_\perp + v_{g\perp}\partial_x E_\perp = \omega_\perp C E_{1\|} E_{2\|}/E_{\rm crit}$$

$$\partial_t E_{1\|} + v_{g1}\partial_x E_{1\|} = \omega_{1\|} C E_{2\|}^* E_\perp/E_{\rm crit}$$

$$\partial_t E_{2\|} + v_{g2}\partial_x E_{2\|} = \omega_{2\|} C E_{1\|}^* E_\perp/E_{\rm crit}$$

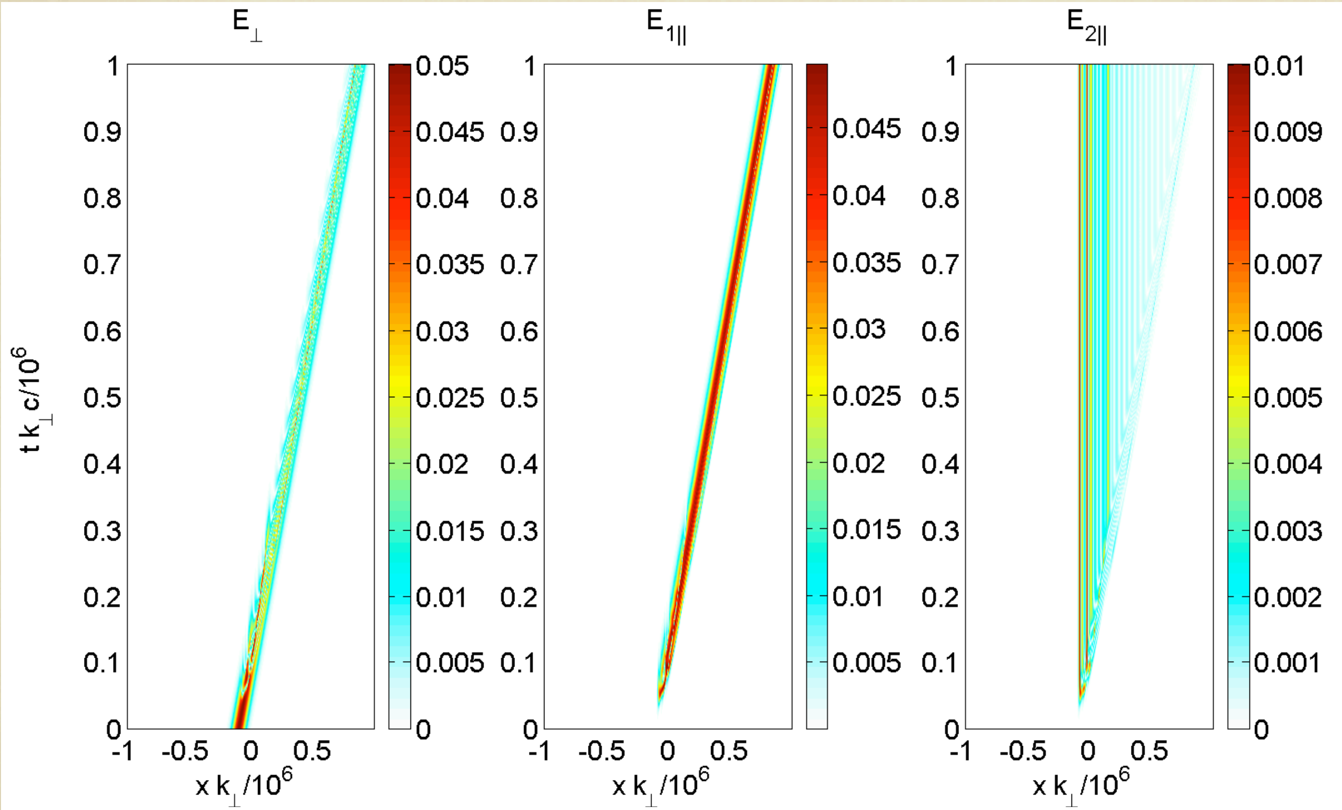
where $C = C_{\rm pl} + C_{\rm QED}$ and

$$C_{\rm pl} = {\rm i}\left(\frac{\alpha}{90\pi\xi}\right)^{1/2} \frac{k_\perp c}{\omega_\perp}\frac{\omega_p^2}{\omega_{1\|}\omega_{2\|}}$$

$$C_{\rm QED} = 2{\rm i}\left(\frac{\alpha\xi}{90\pi}\right)^{1/2}\left[10\frac{k_\perp c}{\omega_\perp} + 7\left(\frac{k_{1\|}c}{\omega_{1\|}} + \frac{k_{2\|}c}{\omega_{2\|}}\right)\right]$$

Note that as $\hbar \to 0$, $C_{\rm pl}/E_{\rm crit}$ remains finite.

Nonlinear decay dynamics



Conclusions

- Relativistic plasma effects crucial in many astrophysical environments, e.g. supernovae, pulsar magnetospheres.
- Strong magnetic fields and intense EM waves often co-exist with plasmas.
- Plasma-QED dynamics in magnetized plasmas could give new insight into emission spectra etc.
- EM down-conversion takes place due to QED effect.
- New EM decay channels, new signatures can occur?