

A FLR-Landau fluid model for the simulation of Alfvén wave cascade and mirror structures in a collisionless plasma

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Turbulent cascade in a magnetized plasma is strongly anisotropic: small-scale fluctuations are generated mostly in quasi-transverse directions, with associated frequencies much lower than the ion gyrofrequency.

In the solar wind or the magnetosheath, the cascade extends beyond the ion Larmor radius: kinetic effects play a significant role \Rightarrow change in the exponent of power-law spectrum of magnetic fluctuations.

From Leamon et al. (1998):

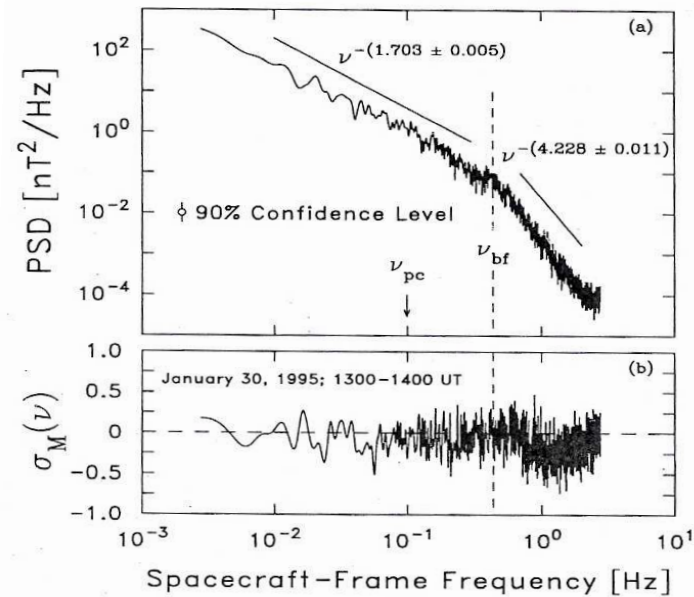
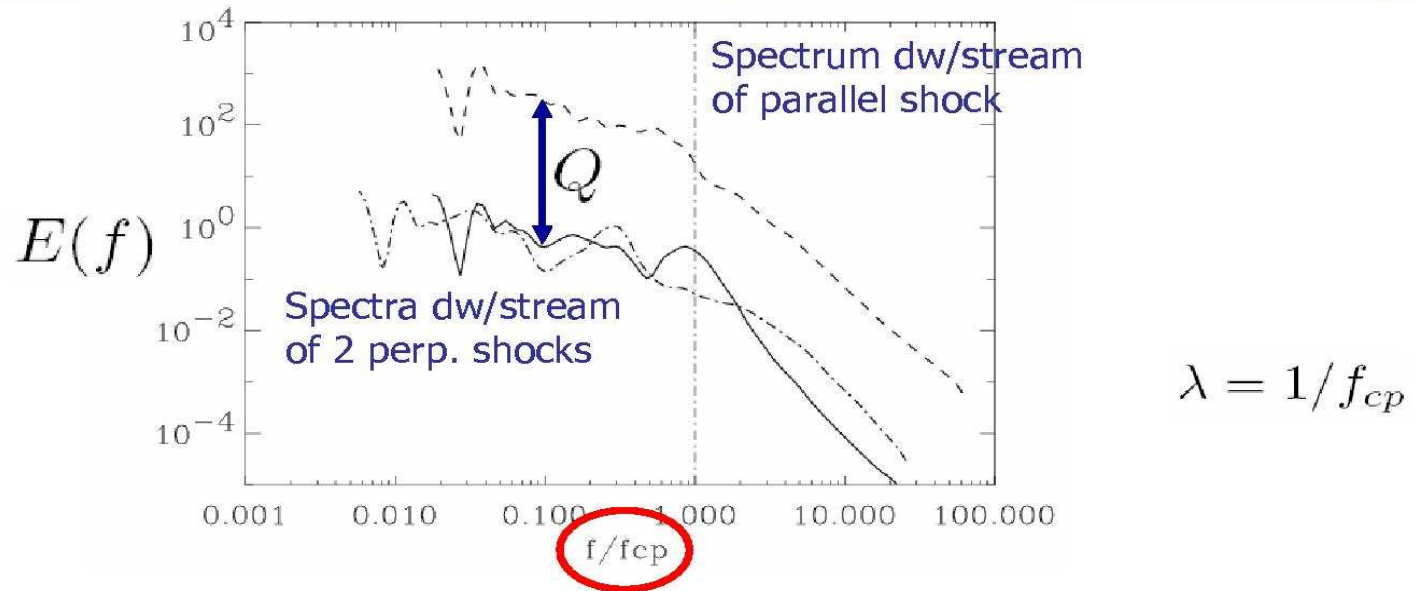


Figure 1. Typical interplanetary power spectrum showing the inertial and dissipation ranges. (a) Trace of the spectral matrix with a break at ~ 0.4 Hz where the dissipation range sets in. (b) The corresponding magnetic helicity spectrum. The date and time of the data used are given.

From Alexandrova et al., EGU 2006:

Turbulent spectra in the magnetosheath downstream of the shock : Cluster observations (up to 12.5 Hz, FGM+STAFF)



Is there any universal spectrum in the MSH?

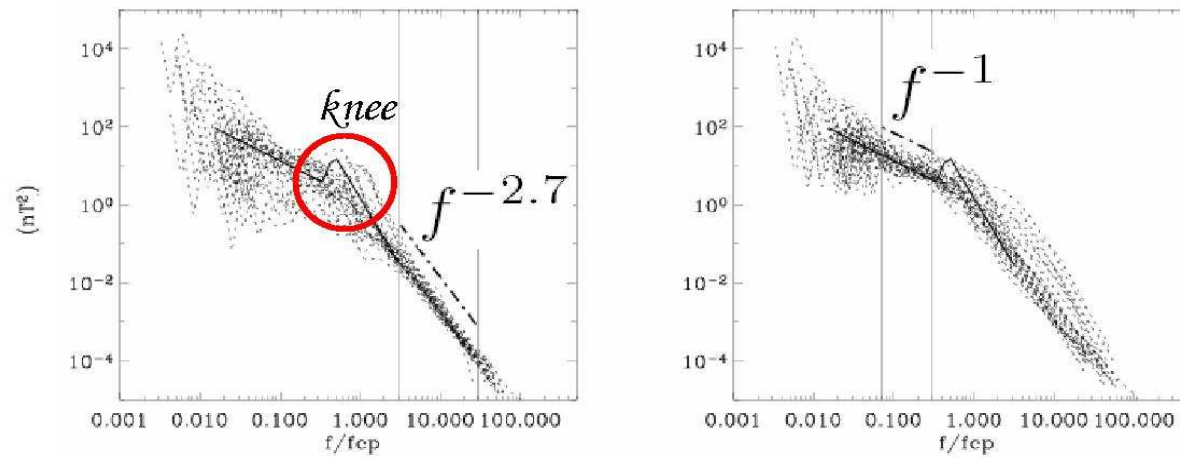
$$E(f) = Qg(\lambda f) \quad g \text{ universal spectrum (if it exist)}$$

Q, λ scale factors (depend on the local properties)

From Alexandrova et al., EGU 2006:

Superposition of 32 magnetic spectra for the magnetosheath parameters $\Theta_{Bn} = [15^\circ - 90^\circ]$ & $\beta = [0.5 - 15]$

Well defined spectral slopes in 2 frequency domains :



"Universal" spectrum :

$$\begin{aligned} g(\lambda f > 1) &\sim f^{-2.7} \\ g(\lambda f < 1) &\sim f^{-1.0} \end{aligned}$$

$$\lambda = 1/f_{cp}$$

Another issue: formation and evolution of mirror structures.

Mirror modes have been reported in various space plasma environments (solar wind, magnetosheath of solar system planets, cometary environments) in regions of high enough β and significant proton temperature anisotropy ($T_{\perp} > T_{\parallel}$).

Ultra low frequency waves propagating very slowly in directions making a large angle with the ambient field.

Appear as magnetic holes (moderate β) or magnetic humps (large β), with an anticorrelated density profile.

Size of these structures: a few Larmor radii.

Also observed in conditions for which the plasma is linearly stable.

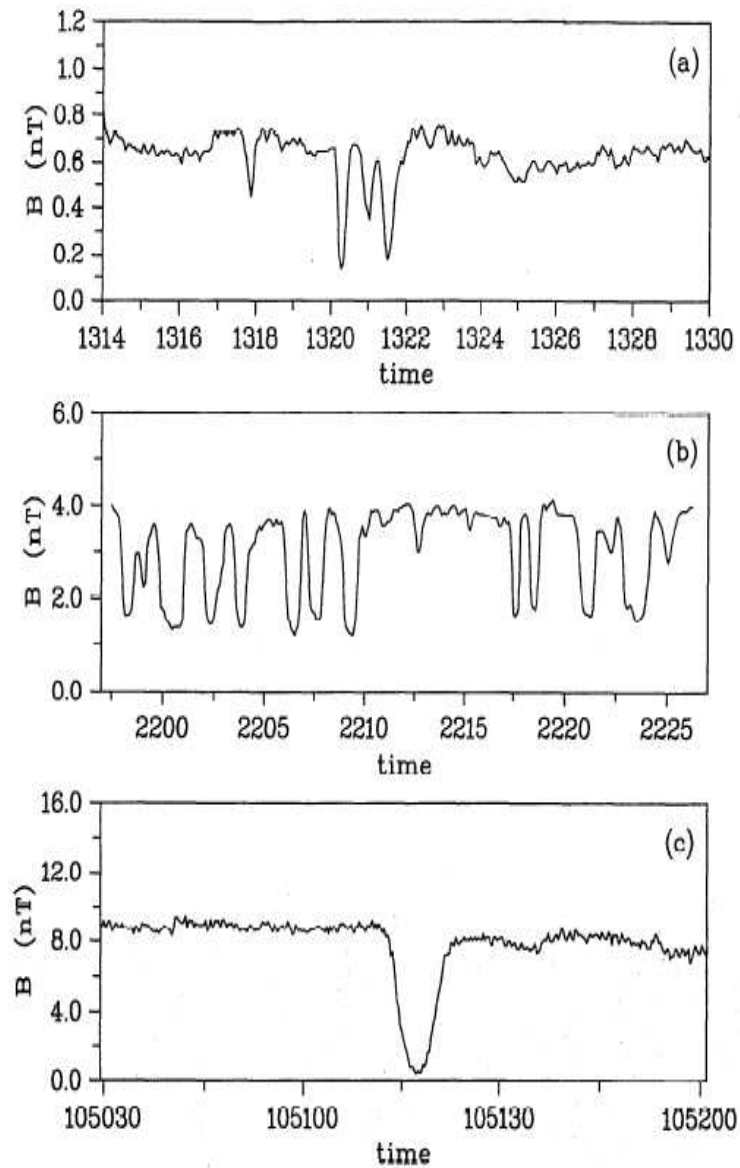


Figure 1. Examples of magnetic holes observed (a) by Ulysses in the free solar wind (taken from Figure 2 of *Winterhalter et al.* [1994]), (b) by Ulysses in the magnetosheath of Jupiter, called mirror mode structures (from Figure 5 of *Erdős and Balogh* [1996]), and (c) by Helios in the free solar wind (data courtesy of K. Sperveslage and F.M. Neubauer, University of Köln, 1999). Shown is the magnetic field magnitude.

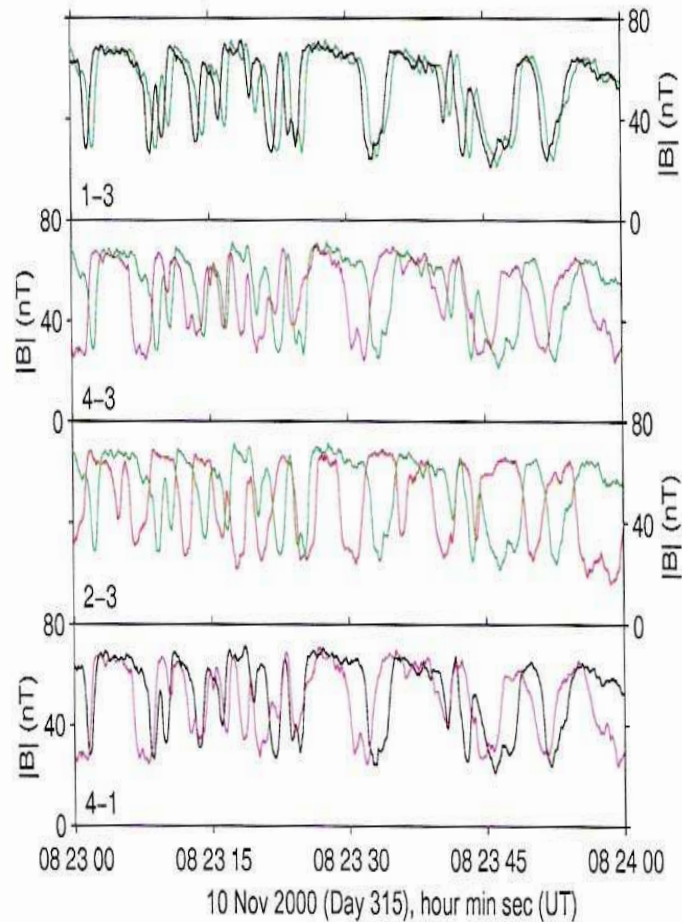


Fig. 5. A comparison of the magnetic field magnitude seen at different spacecraft pairs. The panels show Cluster 1 (black) and 3 (green), Cluster 4 (magenta) and 3 (green), Cluster 2 (red) and 3 (green) and finally Cluster 1 (black) and 4 (magenta).

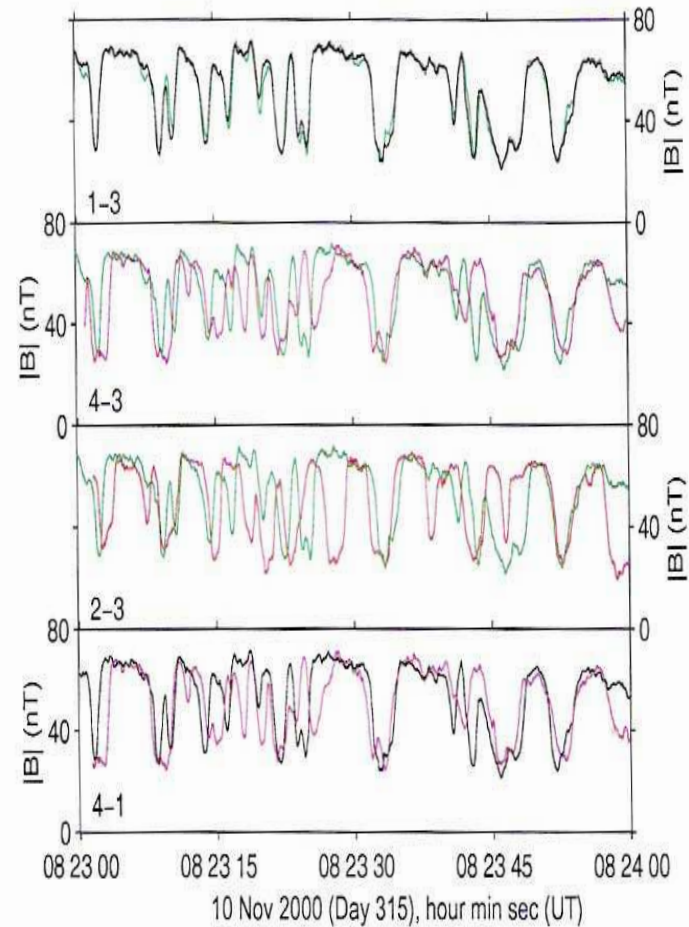


Fig. 6. The same data as in Fig. 5, but one data set is shifted by the time of the peak cross-correlation coefficient. Cluster 3 is used as the reference in the top three panels, and Cluster 1 is used as the reference in the bottom panel.

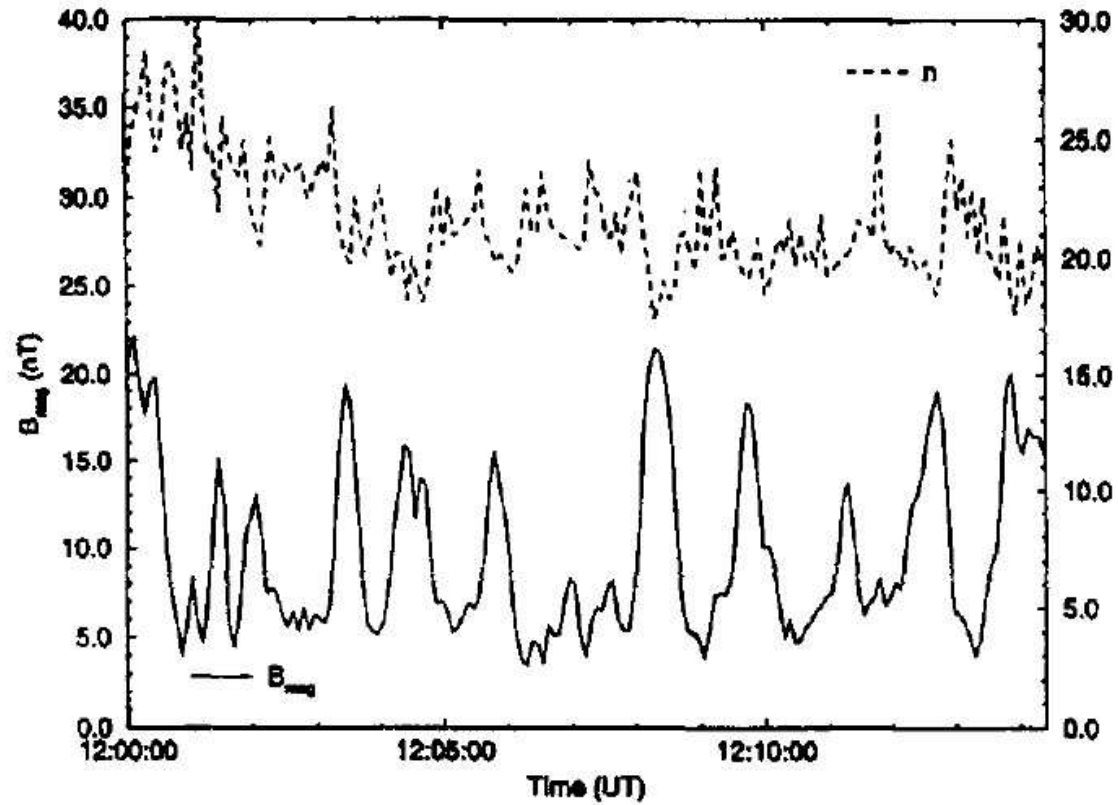


Figure 1: Large beta regime ($\beta \sim 30$): Magnetic humps and plasma density holes [from Leckband et al. 1995].

It was recently suggested to use the [gyrokinetic equation](#) to study turbulent astrophysical plasmas.

[Howes et al. 2006 \(astro-ph/0511812\)](#): evaluation of dispersion and damping of KAW's ($0 < k_{\perp} r_L < 100$). Excellent agreement with hot-plasma kinetic theory, for a broad range of values of beta and electron-ion temperature ratio.

Computational cost of gyrokinetic simulations in turbulent regimes involving a broad range of scales is very high.

Question: Can fluid models provide a cheaper alternative to kinetic calculations, even if they possibly are somewhat less accurate?

From kinetic to fluid approaches

Collisional plasmas: the distribution function relaxes to a Maxwellian before the plasma can change its properties; description in terms of the parameters characterizing this Maxwellian.

Collisionless plasmas : the hierarchy for velocity moments derived from the kinetic equation governing the distribution function, **cannot be rigorously closed**.

Various approaches were suggested.

We here concentrate on the so-called **Landau-fluid models:**

- Introduced by Hammett & Perkins (1990) as a **closure retaining phase mixing and linear Landau damping**.
- Implemented in the context of **large-scale MHD** dynamics by Snyder, Hammett & Dorland (1997) for closing the moment equation hierarchy within the **drift kinetic approximation**.
- Extended to **dispersive MHD by including FLR corrections** computed perturbatively within fluid formalism, starting from Vlasov-Maxwell (Goswami, Passot & Sulem 2005 and references within).

Aim of this talk: **revisit Landau fluid approach to resolve transverse scales much smaller than the ion gyroradius: FLR-Landau fluids.**

Landau-fluids are based on a full description of the hydrodynamic nonlinearities supplemented by a linear (or quasi-linear) description of low-frequency kinetic effects.

Alternative approach: **Gyro-fluids** (Brizard 1992, Dorland & Hammett 1993, Beer & Hammett 1996):

- Obtained by taking velocity moments of the gyrokinetic equation.
- Nonlinear FLR corrections to all order are captured.
- Linear closure of the hierarchy as for Landau fluids.
- Equations are rather complex and not written in the physical coordinates but in the gyrocenter variables. The transformation from one set of variables to the other involves **additional approximations**.
- All fast magnetosonic waves are ordered out, while FLR-Landau fluids retain large-scale fast magnetosonic waves.

Detailed comparison of gyrofluids and FLR-Landau fluids would be most useful.

Landau fluids

For each species, starting from the Vlasov-Maxwell system, write the usual fluid equations governing

- **density** (electron inertia usually neglected)

- **velocity**: involves pressure tensor

$$\overline{\overline{P}}_p = p_{\perp p} \overline{\overline{n}} + p_{\parallel p} \overline{\overline{\tau}} + \overline{\overline{\Pi}} \quad \text{or} \quad \overline{\overline{P}}_e = p_{\perp e} \overline{\overline{n}} + p_{\parallel e} \overline{\overline{\tau}}$$

with $n_{ij} = \delta_{ij} - \hat{b}_i \hat{b}_j$ and $\tau_{ij} = \hat{b}_i \hat{b}_j$. Here, $\hat{b}_i = B_i / B_0$.

- **parallel and perpendicular pressures**: involve heat flux tensors

- **gyrotropic heat fluxes q_{\perp} and q_{\parallel}** : involve gyrotropic fourth rank tensors characterized by 3 scalars

$$r_{\parallel\parallel\parallel} = 3 \frac{p_{\parallel}^2}{\rho} + \tilde{r}_{\parallel\parallel\parallel}, \quad r_{\parallel\perp} = \frac{p_{\perp} p_{\parallel}}{\rho} + \tilde{r}_{\parallel\perp}, \quad r_{\perp\perp} = 2 \frac{p_{\perp}^2}{\rho} + \tilde{r}_{\perp\perp}$$

(Bi-Maxwellian contributions + cumulants)

supplemented with quasi-neutrality ($n_e = n_p$); $j = \frac{c}{4\pi} \nabla \times b$;

induction equation (retaining Hall-effect and electron pressure gradient)

2 main problems:

- (1) Closure relations are needed to express the 4th order cumulants $\tilde{r}_{\parallel\parallel\parallel}$, $\tilde{r}_{\parallel\perp}$, $\tilde{r}_{\perp\perp}$
(closure at lowest order also possible, although usually less accurate)
- (2) FLR corrections to the various moments are to be evaluated

The starting point for addressing these points is the linear kinetic theory in the low-frequency limit $\omega/\Omega \sim \epsilon \ll 1$,

For a unified description of fluid and kinetic scales, FLR-Landau fluids retain contributions of:

- quasi-transverse fluctuations ($k_{\parallel}/k_{\perp} \sim \epsilon$) with $k_{\perp}r_L \sim 1$
- hydrodynamic scales with $k_{\parallel}r_L \sim k_{\perp}r_L \sim \epsilon$.

CLOSURE RELATIONS are based on linear kinetic theory (near bi-Maxwellian equilibrium) in the low-frequency limit.

For example, for each species, (assuming the ambient magnetic field along the z direction),

$$\tilde{r}_{\parallel\perp} = \frac{p_{\perp}^{(0)2}}{\rho^{(0)}} \left[1 - R(\zeta) + 2\zeta^2 R(\zeta) \right] \left[[2b\Gamma_0(b) - \Gamma_0(b) - 2b\Gamma_1(b)] \frac{b_z}{B_0} + b[\Gamma_0(b) - \Gamma_1(b)] \frac{e\Psi}{T_{\perp}^{(0)}} \right]$$

$$\Gamma_n(b) = e^{-b} I_n(b), \quad b = (k_{\perp}^2 T_{\perp}^{(0)}) / (\Omega^2 m), \quad I_n(b) \text{ modified Bessel function, } E_z = -\partial_z \Psi$$

R is the plasma response function, $\zeta = \frac{\omega}{|k_{\parallel}| v_{th}}$. (For electrons, $b \approx 0$, $\Gamma_0 \approx 1$, $\Gamma_1 \approx 0$)

It turns out that $\tilde{r}_{\parallel\perp}$ can be expressed in terms of perpendicular gyrotropic heat flux q_{\perp} and the parallel current j_z . One has

$$\tilde{r}_{\parallel\perp} = \sqrt{\frac{2T_{\parallel}^{(0)}}{m} \frac{1 - R(\zeta) + 2\zeta^2 R(\zeta)}{2\zeta R(\zeta)}} \left[q_{\perp} + [\Gamma_0(b) - \Gamma_1(b)] \frac{p_{\perp}^{(0)} p_{\parallel}^{(0)}}{\rho^{(0)} v_A^2} \left(\frac{T_{\perp}^{(0)}}{T_{\parallel}^{(0)}} - 1 \right) \frac{j_z}{en^{(0)}} \right].$$

The **approximation** consists in replacing the plasma response function R by its three pole Padé approximant $R_3(\zeta) = \frac{2 - i\sqrt{\pi}\zeta}{2 - 3i\sqrt{\pi}\zeta - 4\zeta^2 + 2i\sqrt{\pi}\zeta^3}$.

This leads to the approximation $\frac{1 - R(\zeta) + 2\zeta^2 R(\zeta)}{2\zeta R(\zeta)} \approx \frac{i\sqrt{\pi}}{-2 + i\sqrt{\pi}\zeta}$.

(A lower order approximant would overestimate the Landau damping in the large ζ limit).

This leads to a closure relation in the form of the evolution equation (for each species)

$$\left[\frac{d}{dt} - \frac{2}{\sqrt{\pi}} \sqrt{\frac{2T_{\parallel}^{(0)}}{m}} \mathcal{H}_z \partial_z \right] \tilde{r}_{\parallel\perp} + \frac{2T_{\parallel}^{(0)}}{m} \partial_z [q_{\perp} + [\Gamma_0(b) - \Gamma_1(b)] \frac{p_{\perp}^{(0)} p_{\parallel}^{(0)}}{\rho^{(0)} v_A^2} \left(\frac{T_{\perp}^{(0)}}{T_{\parallel}^{(0)}} - 1 \right) \frac{j_z}{en^{(0)}}] = 0,$$

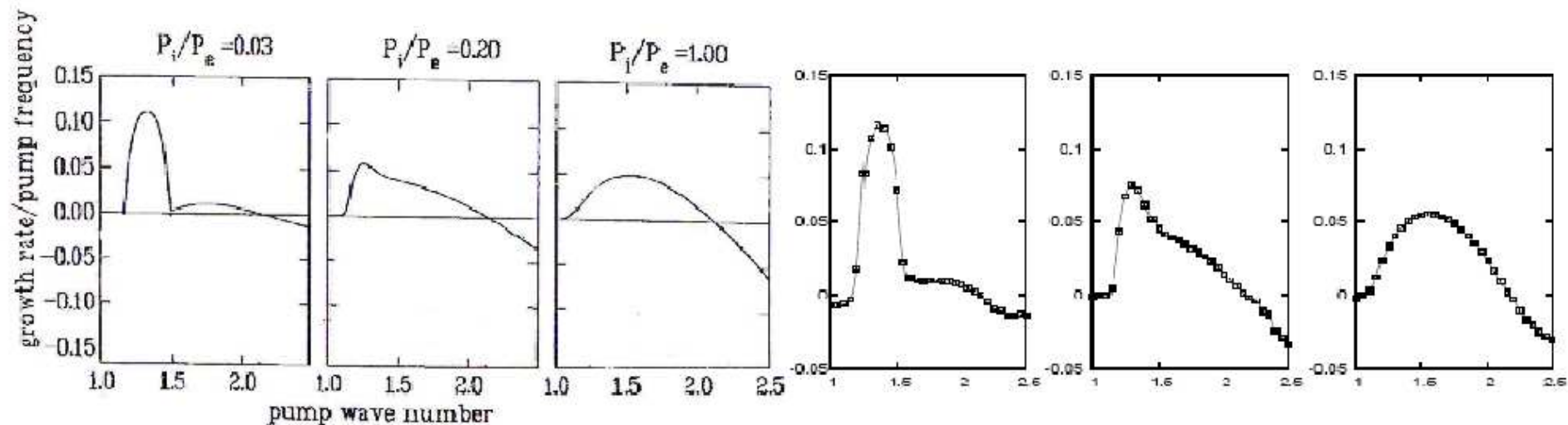
In Fourier space, Hilbert transform \mathcal{H}_z reduces to the multiplication by $i \operatorname{sgn} k_z$.

Improvement: Retain the evolution of the equilibrium state by replacing the (initial) equilibrium pressures and temperatures by the instantaneous fields averaged on space.

In the large-scale limit, $\Gamma_0(0) = 1$ and $\Gamma_1(0) = 0$.

An alternative description of the large-scale dynamics based on a Chapman-Enskog-like approach, that directly provides a linear closure relation in terms of lower-order moments (typically velocity and temperature) [Chang and Callen 1995].

Decay instability of parallel Alfvén waves in the long-wavelength limit (no FLR corrections)



Drift-kinetic analysis (from Inhester 1990)

Landau fluid simulation

Figure 2: Maximum growth rates of the density modes versus wavenumber (normalized by the pump wavenumber) resulting from the **decay instability of a non dispersive Alfvén wave** of amplitude $b_0 = 0.447$ in a plasma with $\beta_{\parallel p} = 0.3$ and isotropic temperatures such that $T_e^{(0)}/T_p^{(0)} = 33$ (left), $T_e^{(0)}/T_p^{(0)} = 5$ (middle) and $T_e^{(0)}/T_p^{(0)} = 1$ (right).

Reducing electron temperature tends to broaden the spectral range and to reduce the growth rate of the instability.

Difficulty: In plasmas with anisotropic ion temperature ($T_{\perp p} > T_{\parallel p}$) and β exceeding a few units, mirror instability can occur. (At $\beta \sim 1$, ion cyclotron instability that is not captured by low-frequency asymptotics dominates). **The growth rate of mirror instability in the quasi-hydrodynamic limit is accurately captured by Landau fluids (in contrast with anisotropic MHD models).**

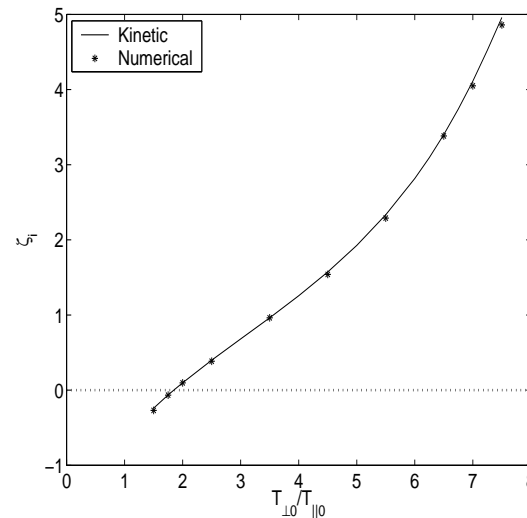


Figure 3: Mirror mode growth rate $\zeta_p = \text{Im}(\omega)/(k_z v_{th})$ in a quasi-transverse direction ($\cot \theta = 0.01$), predicted by the kinetic theory and given by time integration of the Landau fluid model, versus equilibrium temperature anisotropy for a plasma with $\beta = (2\beta_{\perp} + \beta_{\parallel})/3 = 1$ and equal temperatures for electrons and protons.

Mirror instability extends to TRANSVERSE SCALES COMPARABLE TO ION LARMOR RADIUS. Such scales cannot thus be ignored.

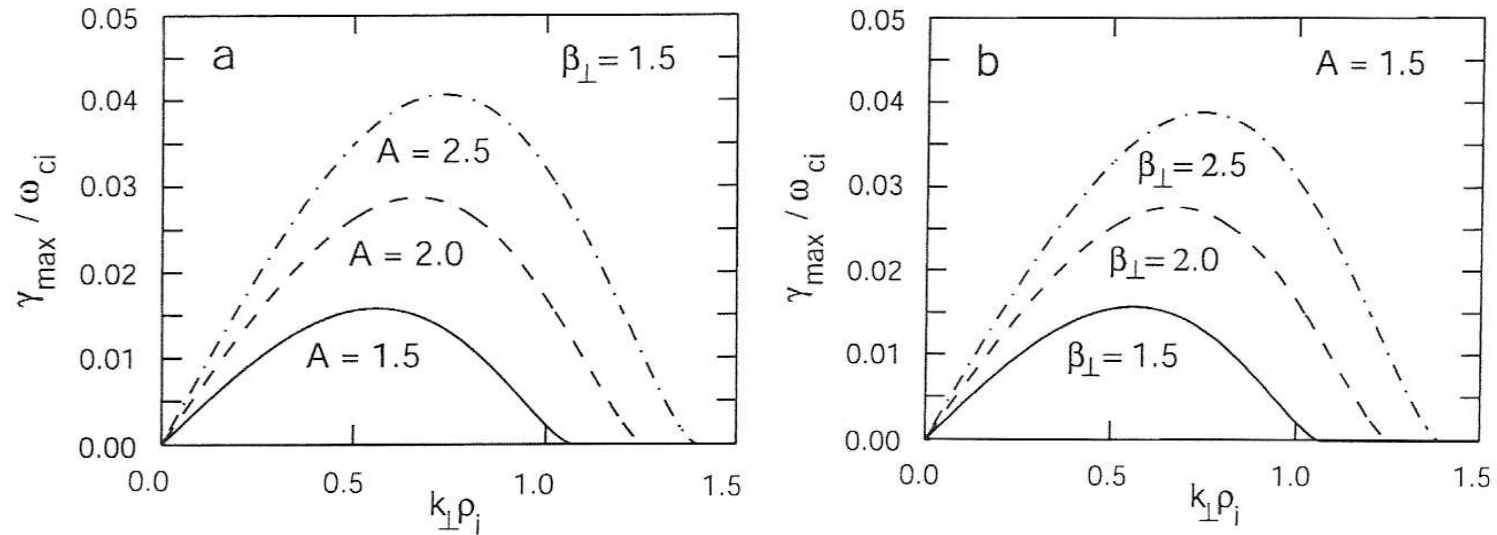


Figure 4: Instability growth rate versus the transverse wavenumber for various $\beta_{\perp p}$ and anisotropy factor $A = T_{\perp p}^{(0)} / T_{\parallel p}^{(0)} - 1$ [From Pokhotelov et al. (2004) JGR **109**, A09213].

In the quasi-hydrodynamic approach, maximum growth rate proportional to $k_{\perp} r_L$, whereas kinetic theory predicts a quenching of the instability for perpendicular scales of the order of the ion Larmor radius.

Retaining only the large scales leads to an ill-posed problem (the smallest retained scales are the most unstable).

FLR effects at SMALL transverse scales should be retained to reproduce the quenching of the mirror instability.

Gyroviscous tensor: $\overline{\overline{\Pi}} = \overline{\overline{\Pi}}_{\perp} + \Pi_z \otimes \hat{b} + \hat{b} \otimes \Pi_z$

It is convenient to write $\frac{1}{p_{\perp p}^{(0)}} \nabla_{\perp} \cdot \overline{\overline{\Pi}}_{\perp} = -\nabla_{\perp} \mathcal{A} + \nabla_{\perp} \times (\mathcal{B} \hat{z})$.

By combining expressions of the various fields provided by the kinetic theory in order to eliminate the plasma response function, one gets

$$\mathcal{A} = \left[1 - \frac{\Gamma_1(b)}{b[\Gamma_0(b) - \Gamma_1(b)]} + \frac{\Gamma_1(b)}{\Gamma_0(b)} \right] \frac{1}{\Omega} (ik_{\perp} \times u_{\perp p}) \cdot \hat{z} - \frac{\Gamma_1(b)}{\Gamma_0(b)} \frac{T_{\perp p}^{(1)}}{T_{\perp p}^{(0)}}$$

$$\mathcal{B} = - \left[\frac{\Gamma_0(b) - 1 - \Gamma_1(b)}{b} + 2(\Gamma_0(b) - \Gamma_1(b)) + \frac{\Gamma_0(b) - \Gamma_1(b)}{1 - \Gamma_0(b)} (\Gamma_0(b) - \Gamma_1(b) - \frac{1 - \Gamma_0(b)}{b}) \right]$$

$$\times \frac{c}{\Omega B_0} (ik_{\perp} \times E_{\perp}) \cdot \hat{z} + \frac{1}{1 - \Gamma_0(b)} \left[\Gamma_0(b) - \Gamma_1(b) - \frac{1 - \Gamma_0(b)}{b} \right] \frac{1}{\Omega} (ik_{\perp} \cdot u_{\perp p}).$$

In the large scale limit $b = \frac{k_{\perp}^2 T_{\perp p}^{(0)}}{\Omega^2 m_p} \rightarrow 0$, the usual fluid estimates are recovered:

$$\mathcal{A}_{fluid} = \frac{1}{2\Omega} (\nabla_{\perp} \times u_{\perp}) \cdot \hat{z}, \quad \mathcal{B}_{fluid} = \frac{1}{2\Omega} (\nabla_{\perp} \cdot u_{\perp})$$

In order to reproduce the leading-order nonlinear fluid theory, replace $p_{\perp p}^{(0)}$ by $p_{\perp p}$

Similar description of $\Pi_{\parallel} = -\nabla\mathcal{C} + \nabla \times (\mathcal{D}\hat{z})$ and of the relevant (transverse) non-gyrotropic heat fluxes S_{\perp}^{\perp} and S_{\perp}^{\parallel} .

Specific difficulty:
$$\mathcal{D} = \frac{\alpha(b)}{\Omega}(q_{\perp} + p_{\perp}^{(0)}u_z) + \dots$$

The fluid hierarchy provides a dynamical equation for the heat flux q_{\perp} . Using this determination leads to a **spurious instability** for a quasi-transverse non-propagating mode, at a scale of the order of the ion gyroradius.

This phenomenon is related to the **difference between fast** ($\frac{\omega}{k} \sim v_{\text{th}}$) **and slow** ($\frac{\omega}{k} \ll v_{\text{th}}$) **dynamics**.

For slow dynamics, q_{\perp} IS PRESCRIBED BY THE EQUATION FOR THE PERPENDICULAR PRESSURE, OR EQUIVALENTLY THE PERPENDICULAR TEMPERATURE (where $\partial_t \frac{T_{\perp}}{T_{\perp}^{(0)}}$ is subdominant)

$$\partial_t \frac{T_{\perp}}{T_{\perp}^{(0)}} + \nabla_{\perp} \cdot u_{\perp} + \frac{1}{p_{\perp}^{(0)}} (\partial_z q_{\perp} + \nabla_{\perp} \cdot S_{\perp}^{\perp}) = 0.$$

Kinetic expression gives

$$\frac{T_{\perp}^{(1)}}{T_{\perp}^{(0)}} = \frac{1}{\Omega} (ik_{\perp} \times u_{\perp}) \cdot \hat{z} + \Gamma_0(b) \frac{b_z}{B_0} - \Gamma_0(b) \frac{T_{\perp}^{(0)}}{T_{\parallel}^{(0)}} R(\zeta) \frac{b_z}{B_0}.$$

Approximate R by the one-pole approximant $R_1 = 1/(1 - i\sqrt{\pi}\zeta)$ or, in physical space, $R_1 = (1 + \frac{\sqrt{\pi}}{v_{th}} \mathcal{H}_z \partial_z^{-1} \partial_t)^{-1}$ (sufficient for slow dynamics).

Write the resulting equation in the form of a partial differential equation and replace the temperature time-derivative by the expression given by the fluid equation.

The resulting (closed) description avoids spurious instabilities.

The resulting model accurately reproduces kinetic Alfvén wave dynamics

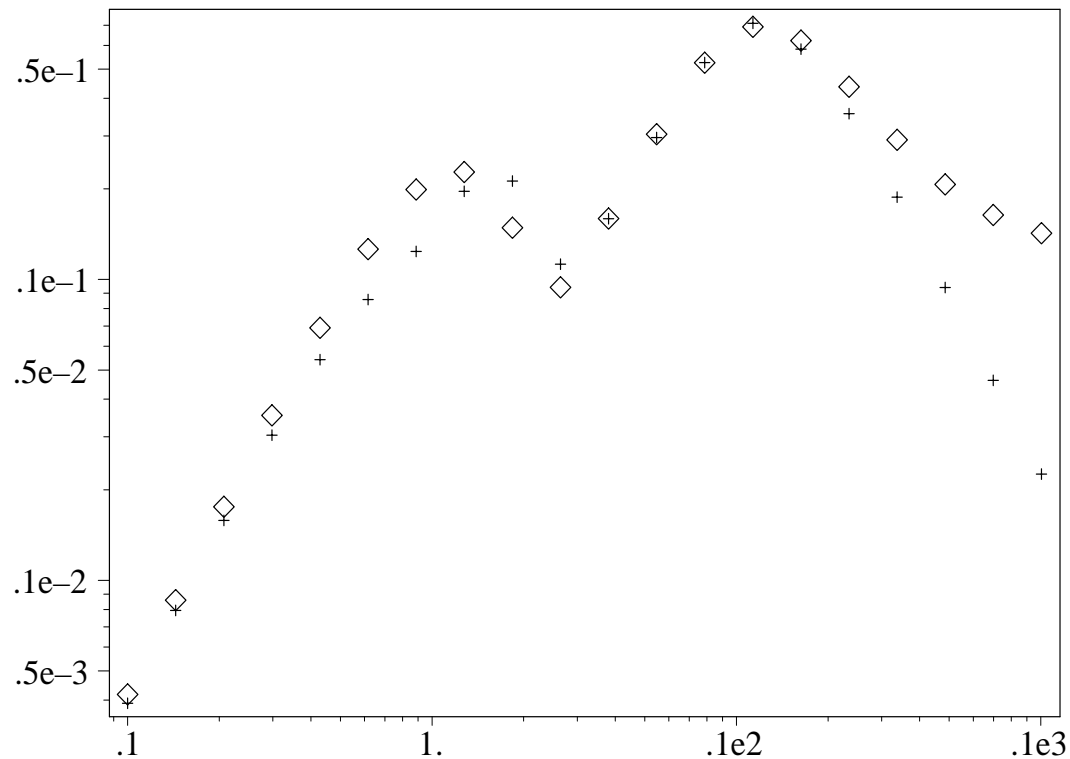


Figure 5: KAW normalized damping rate $-\gamma/(k_{\parallel}v_A)$ vs $k_{\perp}r_L$ for $T_e^{(0)}/T_p^{(0)} = 0.01$ and $\beta_p = 1$. **Diamonds:** FLR-Landau fluid model; **Crosses:** low-frequency kinetic theory.

- Small-scale inaccuracy (at about $r_L/30$) originates from neglected electron inertia.
- Agreement with gyrokinetic simulations and hot plasma stability analysis (Howes et al. 2006).

Mirror instability:

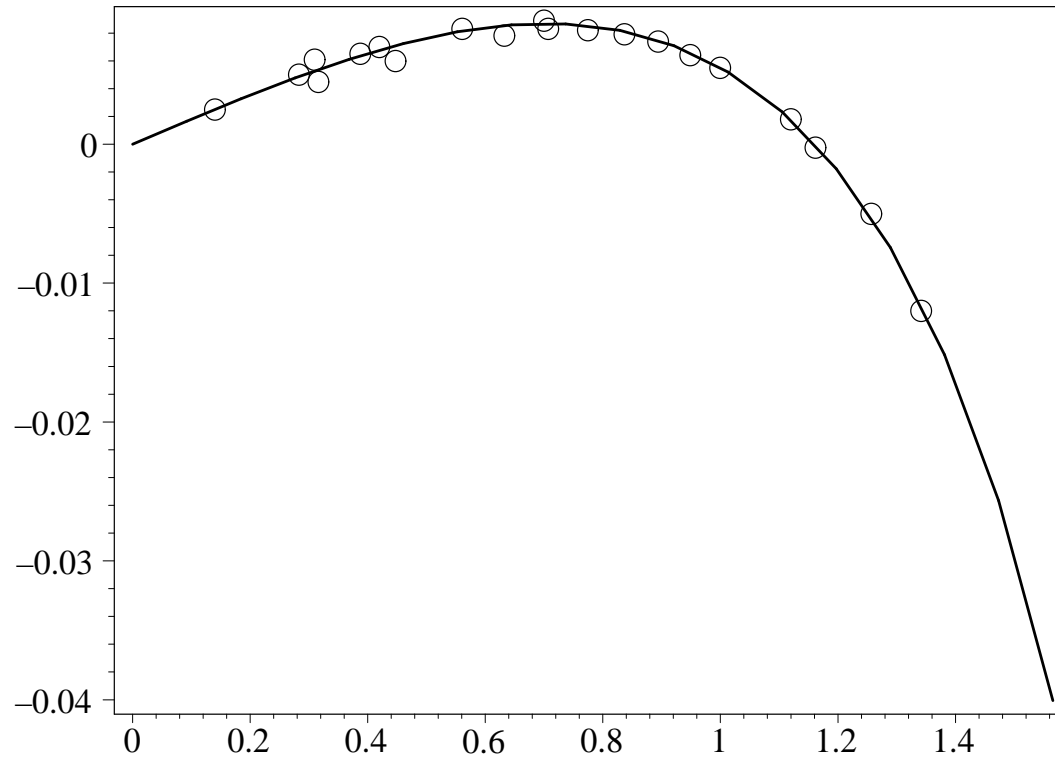


Figure 6: Normalized mirror instability growth rate $\gamma / (k_{\parallel} v_A)$ versus $k_{\perp} r_L$ for $\beta_{\parallel p} = 5$, $T_{\perp p} / T_{\parallel p} = 1.4$, $T_{\parallel e} / T_{\parallel p} = 1$, $T_{\parallel e} = T_{\perp e}$, $\cos \theta = 0.2$. Circles: FLR-Landau fluid simulations, solid line: low-frequency kinetic theory.

Nonlinear development of the mirror instability: numerical integration of the model

At early times, longitudinal magnetic fluctuations grow in anti-correlation with density and temperatures \Rightarrow quasi-monochromatic waves with wavelength close to that of the most unstable mode.

At the time of nonlinear saturation, strong variations of the parallel and perpendicular temperatures before they stabilize.

Non trivial transient dynamics resulting in different structures, depending on β .

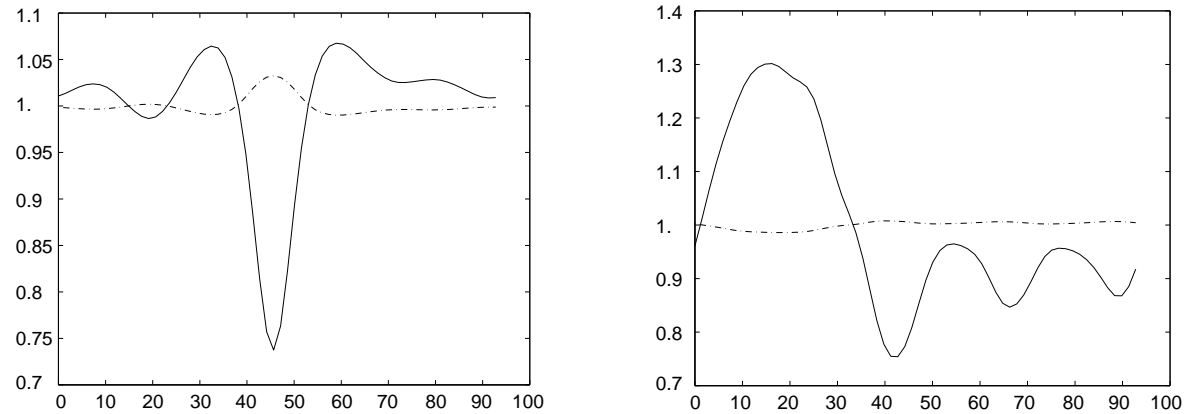


Figure 7: Magnetic field amplitude (solid) and density (dotted) as functions of space in units of inertial length, for $\beta_{\parallel p} = 5$ with $T_{\perp p}/T_{\parallel p} = 1.5$ (left) at $t = 2000 \Omega_p^{-1}$ and for $\beta_{\parallel p} = 20$ with $T_{\perp p}/T_{\parallel p} = 1.4$ (right) at $t = 850 \Omega_p^{-1}$, when $T_{\parallel e}/T_{\parallel p} = 1$, $T_{\perp e}/T_{\perp p} = 0.05$, $\cos \theta = 0.2$. Width of the structures: about 6 Larmor radii.

For $\beta_{\parallel p} = 5$: formation of magnetic holes

For $\beta_{\parallel p} = 20$: formation of magnetic peaks.

At very long times ($t > 3000 \Omega_p^{-1}$), the peak amplitude is observed to decrease and a stable hole forms. This effect could be due to the absence of Landau damping saturation.

The existence of such quasi-static mirror structures can be interpreted, using a simple variational argument.

A simple QUASI-STATIC model at scale large enough for kinetic effects to be negligible (with V. Ruban).

When closing the stationary moment hierarchy by assuming bi-Maxwellian distribution functions, one gets the equations of state $(A = \frac{T_{\perp}^{(0)}}{T_{\parallel}^{(0)}} - 1)$

$$\frac{T_{\parallel}}{T_{\parallel}^{(0)}} = 1 \quad ; \quad \frac{T_{\perp}}{T_{\perp}^{(0)}} = \frac{B/B_0}{(A+1)B/B_0 - A},$$

These expressions can also be obtained by solving the stationary Vlasov-Maxwell system with bi-Maxwellian distributions as boundary conditions.

The problem of stationary structures is then amenable to a variational formulation (assume cold electrons for simplicity).

On stationary configurations, the functional

$$\mathcal{H} = \int \left(\frac{\bar{B}^2}{\beta_{\parallel}} + n \left[F(\bar{B}) + \ln n - 1 \right] + 1 \right) dr,$$

(where $F(\bar{B}) = \ln[(A+1)\bar{B} - A] - \ln \bar{B}$. and $\bar{B} = B/B^{(0)}$)
should achieve a minimal value.

One-dimensional solutions: the only solutions are piecewise constant functions.
Reproduce typical large-scale features of mirror structures.

Periodic piecewise constant solutions in $0 < x < 1$:

$$\begin{aligned}\mathbf{B}_1 &= (\cos \theta, \sin \theta - a), & B_1 &= |\mathbf{B}_1|, & n_1 &= 1 + \delta/b & \text{if } 0 < x < \lambda, \\ \mathbf{B}_2 &= (\cos \theta, \sin \theta + b), & B_2 &= |\mathbf{B}_2|, & n_2 &= 1 - \delta/a & \text{if } \lambda < x < 1,\end{aligned}$$

displays discontinuities at $x = 0$ (or 1) and at $x = \lambda = b/(a + b)$.

Expressions of λ , n_1 and n_2 result from the constraint that the magnetic flux and the number of particles should be the same for the nonlinear state and for the trivial equilibrium.

The solutions are characterized by three parameters δ , a , and b to be determined.

The system admits a **trivial solution**. In addition, a **nontrivial solution** is possible.

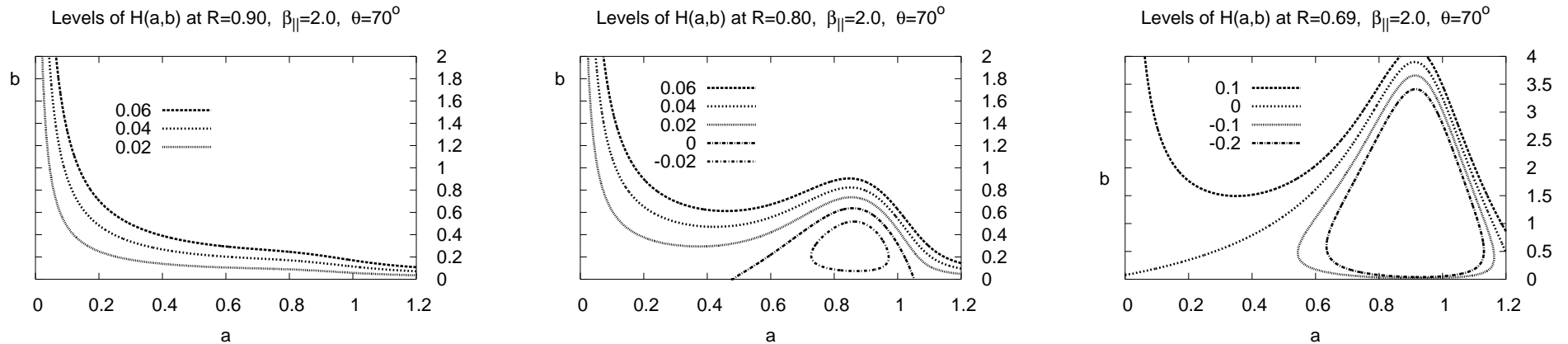


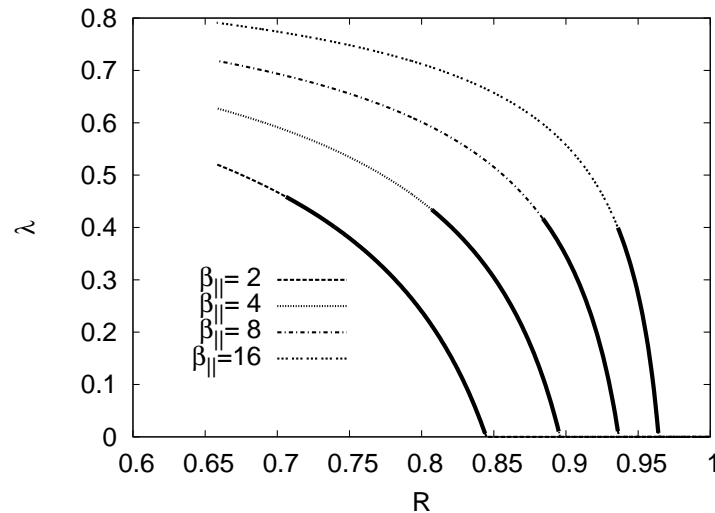
Figure 8: Contours $H(a, b) = \text{const}$, with $\beta_{\parallel} = 2.0$, $\theta = 70^{\circ}$.

At $R = T_{\parallel}^{(0)} / T_{\perp}^{(0)} = 0.90$ (left), the only stationary point is the trivial point, and it is stable. At $R = 0.80$ (middle), there is ALSO a nontrivial stable point originating from a bifurcation that occurs at $R_* \approx 0.84$. At $R = 0.69$ (right), the trivial point is unstable and only the nontrivial point is stable.

Stable nontrivial solutions were reported in regimes where the trivial solution is linearly stable (bistability):

- in anisotropic MHD simulations with spectral filtering to quench the mirror instability at small scale (Baumgärtel 2001),
- in observations of the Jovian magnetosheath (Erdős and Balogh 1996).

Influence of β_{\parallel} :



Near threshold, for moderate values of β_{\parallel} , $\lambda = b/(a+b)$ takes values smaller than $1/2$. The periodic pattern thus displays a structure of magnetic holes. In contrast, for large β_{\parallel} , λ is larger than $1/2$. Such a configuration is associated with magnetic humps.

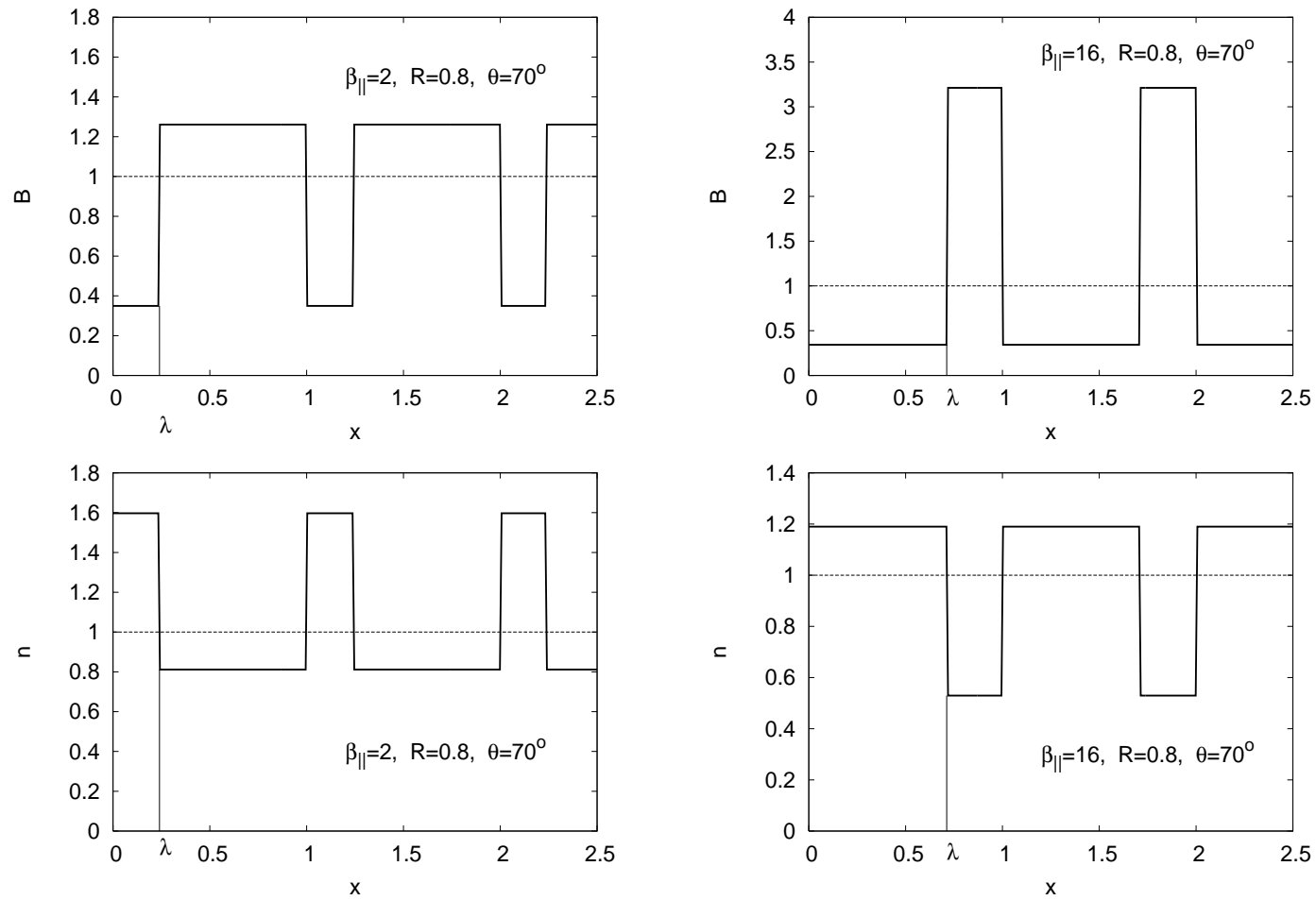


Figure 9: Magnetic field intensity (top) and density (bottom) profiles for $\beta_{\parallel} = 2$ (left), displaying magnetic holes and $\beta_{\parallel} = 16$ (right), displaying magnetic humps, in the case $R = 0.8$.

SUMMARY

FLR-Landau fluid model

- extends the Landau-fluid model developed by Snyder et al. [*Phys. Plasma* (1997) 4, 3974] for (non-dispersive) MHD scales to the much smaller (quasi-transverse) kinetic scales that are excited by the turbulence cascade.
- retains all the hydrodynamic nonlinearities, but kinetic effects (Landau damping and FLR corrections) are treated quasi-linearly.
- accurately reproduces the dispersion and damping of kinetic Alfvén waves up to scales $\sim (1/30) \times$ ion inertial length. Discrepancy at smaller scales could be due to electron inertia that is not retained.
- accurately reproduces the mirror instability (including its quenching at small scales), First simulations of its nonlinear development were presented.
- retains large-scale fast magnetosonic waves (in contrast with gyrokinetic approach).

FORTHCOMING DEVELOPMENTS

- Further benchmarks of the model by comparison with Vlasov-Maxwell and hybrid simulations: **evaluation of the importance of (neglected) nonlinear kinetic effects**, depending on the plasma parameters.
- Simulations of Alfvén turbulence and of the coherent structures (magnetic holes, magnetic filaments, shocklets) observed in the solar wind and the magnetosheath.
- Detailed analysis of the nonlinear development of the mirror instability.
 - FLR-Landau fluids predict that the development of the mirror instability leads to magnetic holes for moderate β and magnetic humps for larger β , indicating that these structures are then stable, as suggested by satellite observations.
 - **However**, hybrid simulations (Baumgärtel et al. 2003) lead to magnetic humps even at moderate β .

Vlasov or gyrokinetic simulations of mirror mode nonlinear dynamics are needed.

- **Challenge: Modeling of particle trapping within a fluid formalism** (Mattor 1999).