

# Turbulence and Intermittent Transport in Magnetized Plasmas

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# Motivation

Cross field transport of particles and heat in magnetically confined plasmas is dominated by anomalous - turbulent transport!

In the edge/scrape-off-layer (SOL) region transport is strongly intermittent and characterized by:

- large-amplitude, radially propagating blob-like structures of particles and heat, generated close to the last closed flux surface (LCFS),
- resulting in asymmetric conditional wave forms, and skewed and flattened PDFs,
- results in localized power loads at PFCs.

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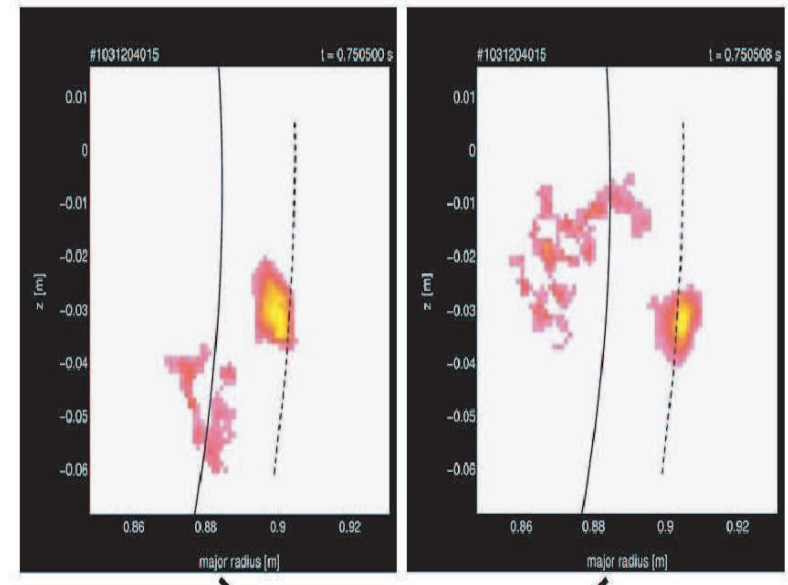
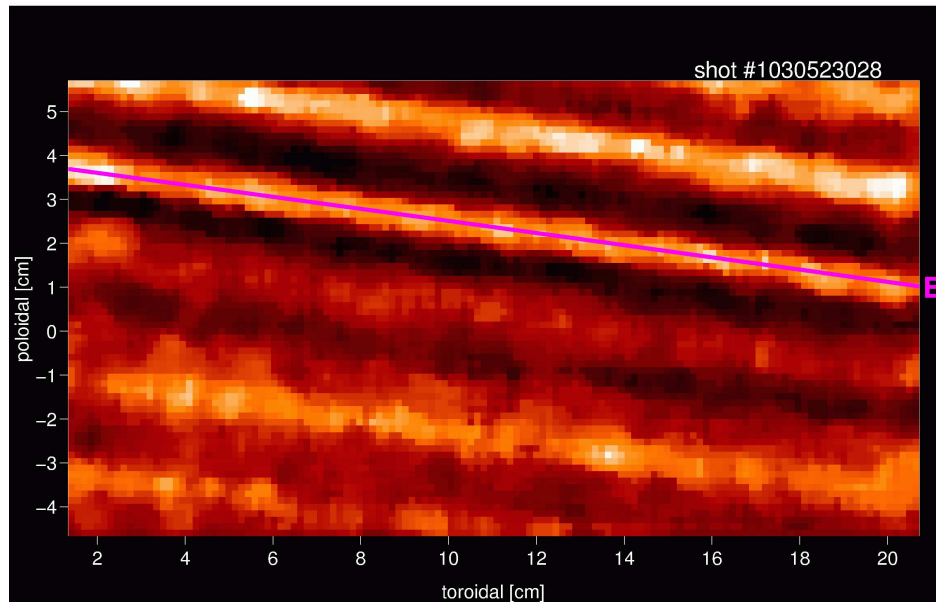
Observed under a variety of conditions:

see, e.g., Zweben *Phys. Fluids* **28** 974 (1985) Antar *et al*, *PoP* **10** 419 (2003); Boedo *et al*, *PoP* **10**, 1670 (2003); Zweben *et al*, *Nucl. Fus.* **44**, 134 (2004); Grulke *et al* *PoP* **13**, 012306 (2006); Garcia *et al* *PPCF* **48**, L1 (2006)

# Density blob observations

Observations of density blobs at the outboard midplane of ALCATOR C-mod ( $D_\alpha$  - light)

O. Grulke *et al.* PoP **13**, 012306 (2006).



Structure along B.

Radial propagation,  $V \approx 0.05c_s$ .

# Density blob theory

Several recent works:

Krasheninnikov *PLA* **283**, 268 (2001)

Curvature drift: charging of a density blob → radial propagation, velocity fraction of  $c_s$ , linear model, no self-consistent blob formation

D'Ippolito *et al*, *PoP* **9**, 222 (2002)

Bian *et al*, *PoP* **10**, 671 (2003)

D'Ippolito *et al*, *CPP* **44**, 205 (2004)

Garcia *et al*, *PoP* **12**, 090701 (2005)

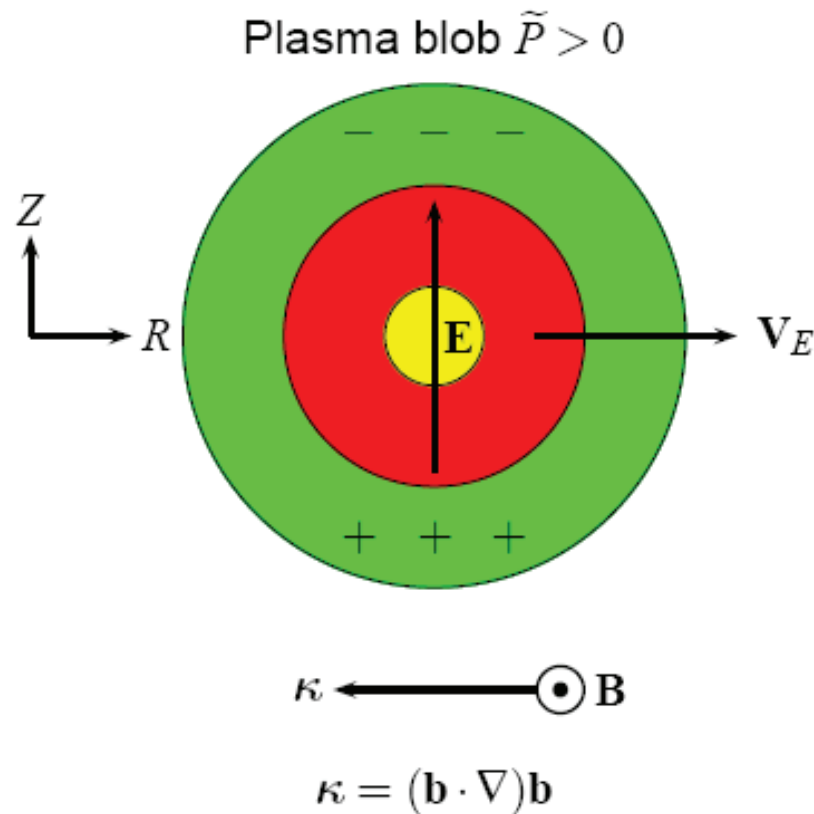
Garcia *et al*, *PoP* **13**, in press (2006)

Similar mechanism for uprising density bubbles in ionospheric E-F layer – inverse stratification in a gravitational field.

e.g. Ott *JGR* **83**, 4369 (1978); Fejer and Kelley *Rev. GSP* **18**, 401 (1980)

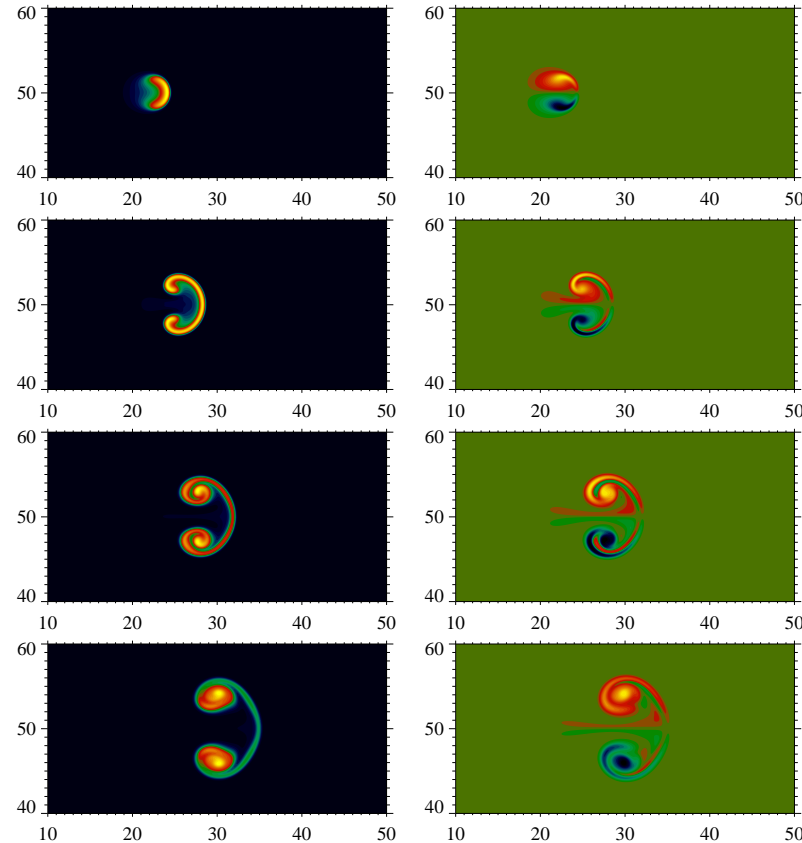
# Blob propagation mechanism

Charge separation by curvature drift:



# Density blob propagation

Garcia *et al*, PoP 12, 090701 (2005),



Density and vorticity (“charge”) evolution. Initial Gaussian density blob.

Radial velocity:  $v/c_s \leq (2\Delta l/R\Delta n/n_0)^{1/2} \approx 0.2$

# Turbulence Model : ESEL

A self-consistent description of fluctuations and intermittent transport in the edge/SOL by employing the RISØ ESEL ( Edge SOL Electrostatic) model for interchange dynamics that:

- include separate plasma production “edge” and loss region “SOL”,
- allow self-consistent flows and profile relaxations,
- profiles and fluctuations are **NOT** separated,
- conserve particles and energy in collective dynamics.

Results agree very well with experimental observations, e.g., TCV, Lausanne (Garcia *et al* PPCF **48**, L1 (2006)) and JET (Naulin et al IAEA-2006)

Garcia, Naulin, Nielsen, Rasmussen, PRL **92** 165003 (2004); PoP **12**, 090701 (2005);  
Physica Scripta **T122**, 89 (2006).

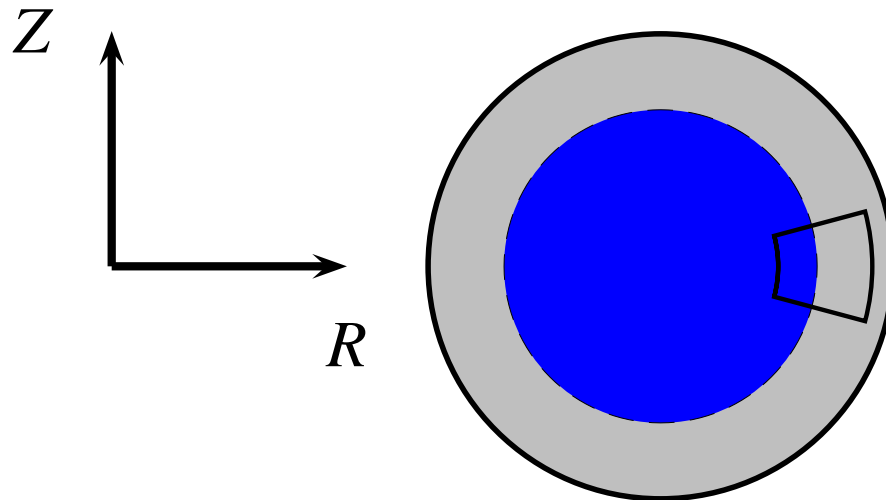


# Geometry and Coordinates

We consider the outboard midplane of a toroidal plasma

The non-uniform magnetic field is  $\mathbf{B} = -(B_0 R_0 / R) \hat{\Theta}$ ,  
described in elementary cylindrical coordinates  $(R, \Theta, Z)$ .

Applying a local slab approximation:  $x = R - R_a$ ,  
 $y = Z$ -poloidal coordinate,  $z = -\Theta$ .



# Model Equations

Fluid model cold ions and quasi-neutrality

$$\frac{dn}{dt} + nC(\phi) - C(nT) = v_n \nabla_{\perp}^2 n - \sigma_n(n-1) + S_n,$$

$$\frac{dT}{dt} + \frac{2T}{3} C(\phi) - \frac{7T}{3} C(T) - \frac{2T^2}{3n} C(n) = v_T \nabla_{\perp}^2 T - \sigma_T(T-1) + S_T,$$

$$\frac{d\Omega}{dt} - C(nT) = v_{\Omega} \nabla_{\perp}^2 \Omega - \sigma_{\Omega} \Omega, \quad \Omega = \nabla_{\perp}^2 \phi.$$

Advective derivative and curvature operators defined by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{B} \hat{\mathbf{z}} \times \nabla \phi \cdot \nabla, \quad C = \nabla \left( \frac{1}{B} \right) \cdot \hat{\mathbf{z}} \times \nabla, \quad B(x) = \frac{1}{1 + \varepsilon + \zeta x}.$$

Conservation of particles and global energy (lowest order in  $\zeta$ )

$$E(t) = \int d\mathbf{x} \left[ \frac{1}{2} (\nabla_{\perp} \phi)^2 + \frac{3}{2} nT \right].$$

Interchange instability:  $N = -B'(p'_0 - \frac{5}{3}B') \leq 0$  instability at low field side.

Naulin et al. PRL **81**, 4148 (1998); PoP **10**, 1075 (2003)

Define the kinetic energy of the fluctuating and poloidal mean motions,

$$v_0(x, t) = \frac{1}{L_y} \int_0^{L_y} v_y(\mathbf{x}, t) dy = \partial\phi_0/\partial x:$$

$$K(t) = \int \frac{1}{2} \left( \nabla_{\perp} \tilde{\phi} \right)^2 d\mathbf{x}, \quad U(t) = \int \frac{1}{2} v_0^2 d\mathbf{x} .$$

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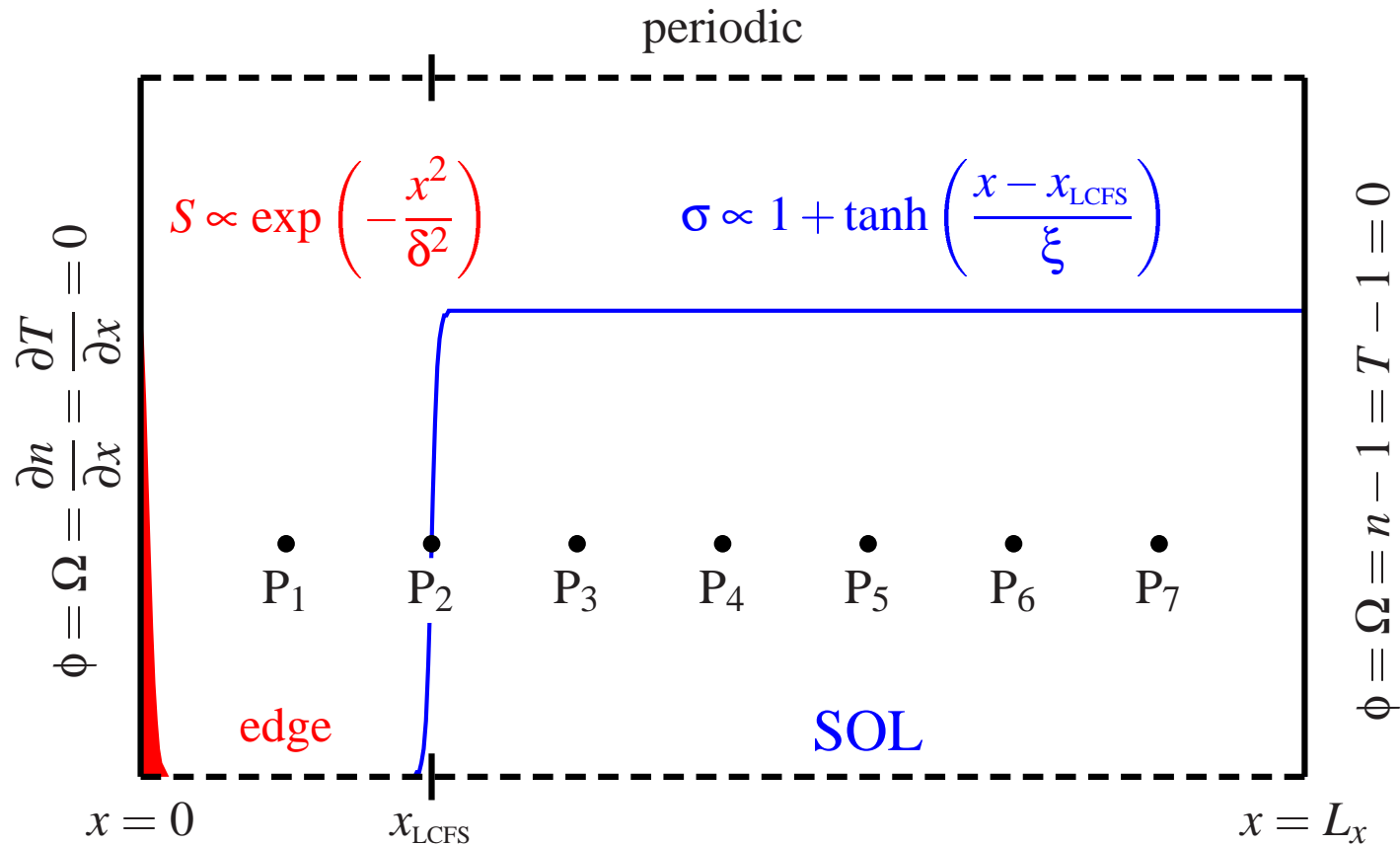
$$K(t) = \int \frac{1}{2} \left( \nabla_{\perp} \tilde{\phi} \right)^2 d\mathbf{x}, \quad U(t) = \int \frac{1}{2} v_0^2 d\mathbf{x} .$$

Energy transfer rates from thermal energy to the fluctuating motions, and from the fluctuating to the poloidal mean flow:

$$F_p(t) = \int p C(\phi) d\mathbf{x}, \quad F_v(t) = \int \tilde{v}_x \tilde{v}_y \frac{\partial v_0}{\partial x} d\mathbf{x}.$$

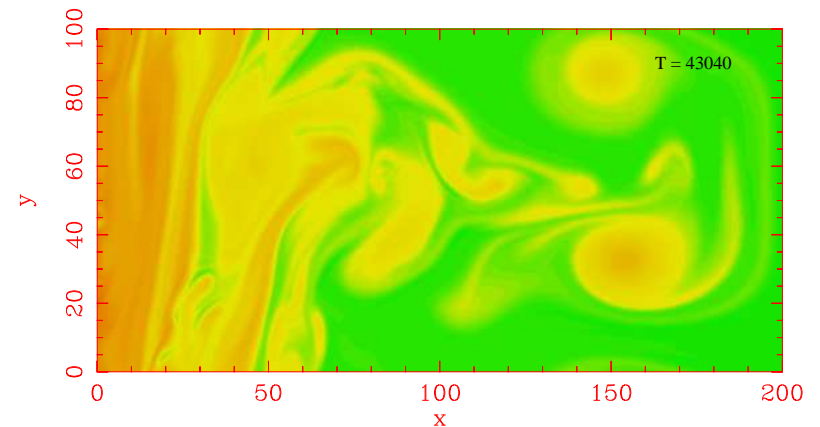
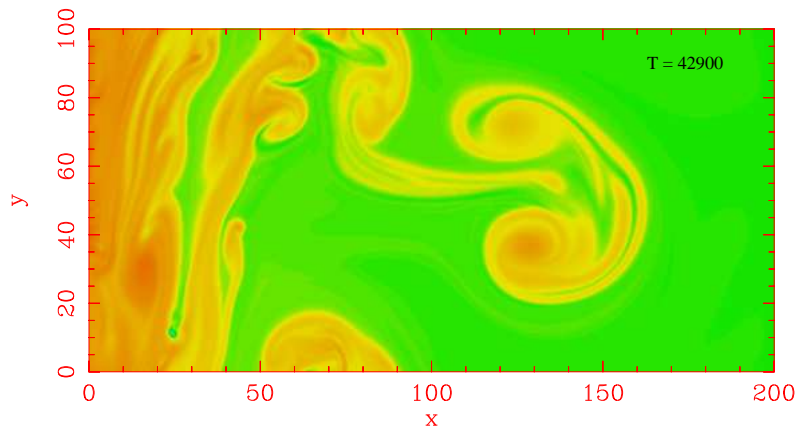
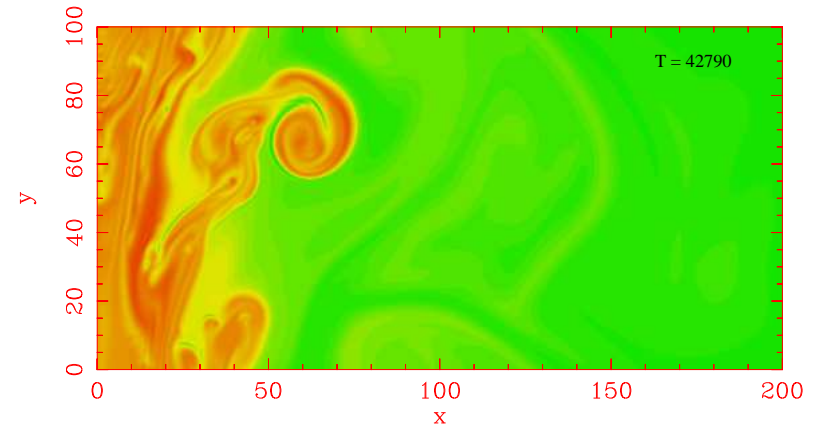
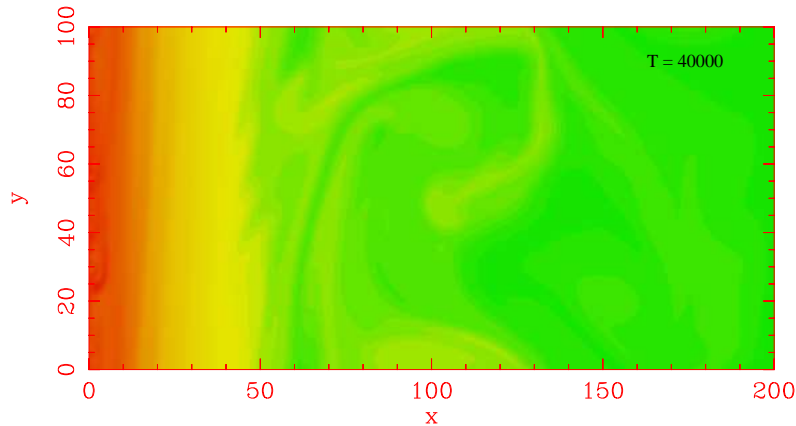
$F_p$  is also a measure of the turbulent energy transport.

# Simulation Geometry



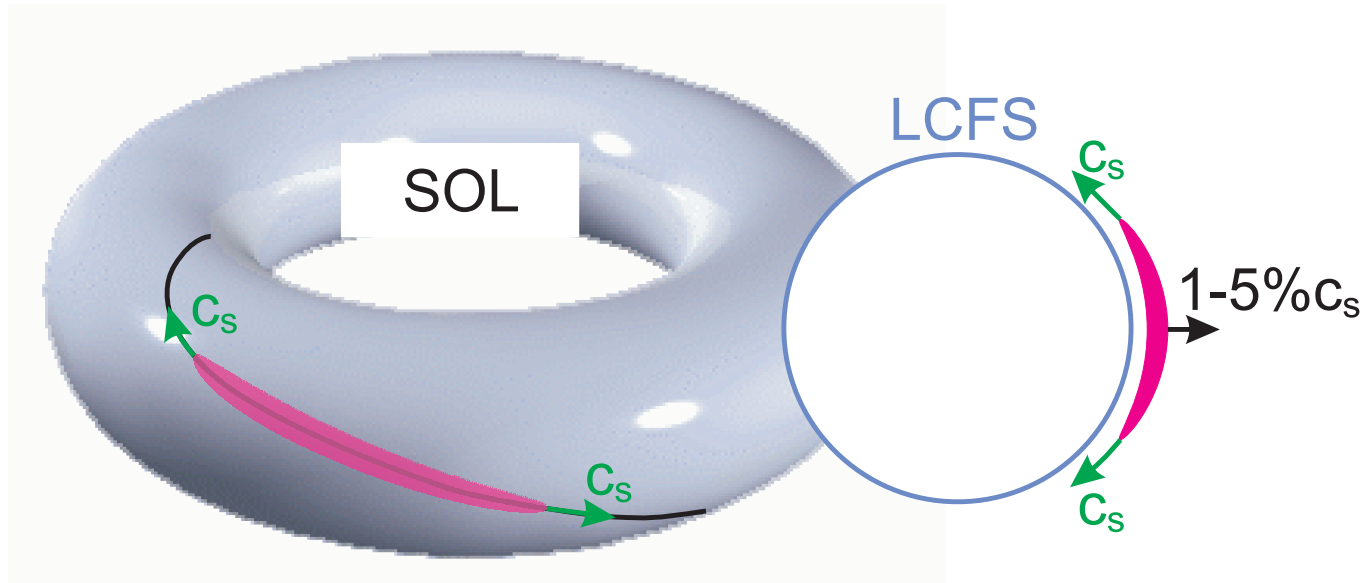
Domain  $L_x = 2L_y = 200$ , resolution  $512 \times 256$ ,  $x_{\text{LCFS}} = 50$ . SOL damping rates  $\sigma_n = \sigma_\Omega = \sigma_T/5 = 3\zeta/2\pi q$  with  $q = 3$ ; magnetic curvature  $\varepsilon = 0.25$ ,  $\zeta = 5 \times 10^{-4}$ ; collisional diffusion  $\nu = 10^{-2}$ ; timespan  $4 \times 10^6$

# Density Evolution



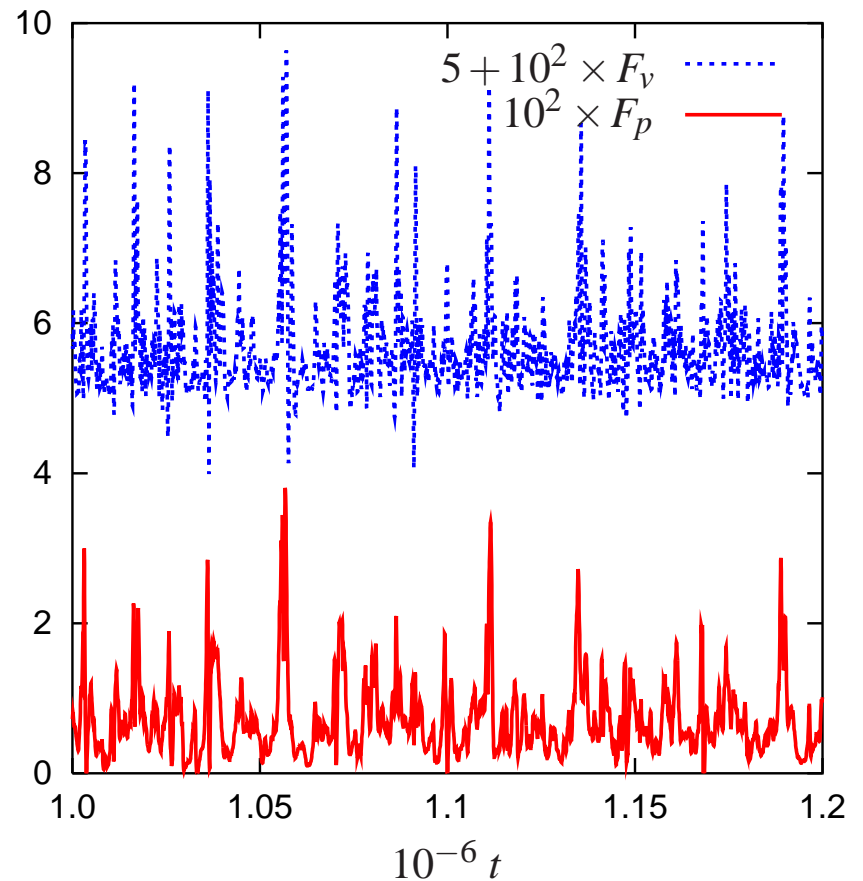
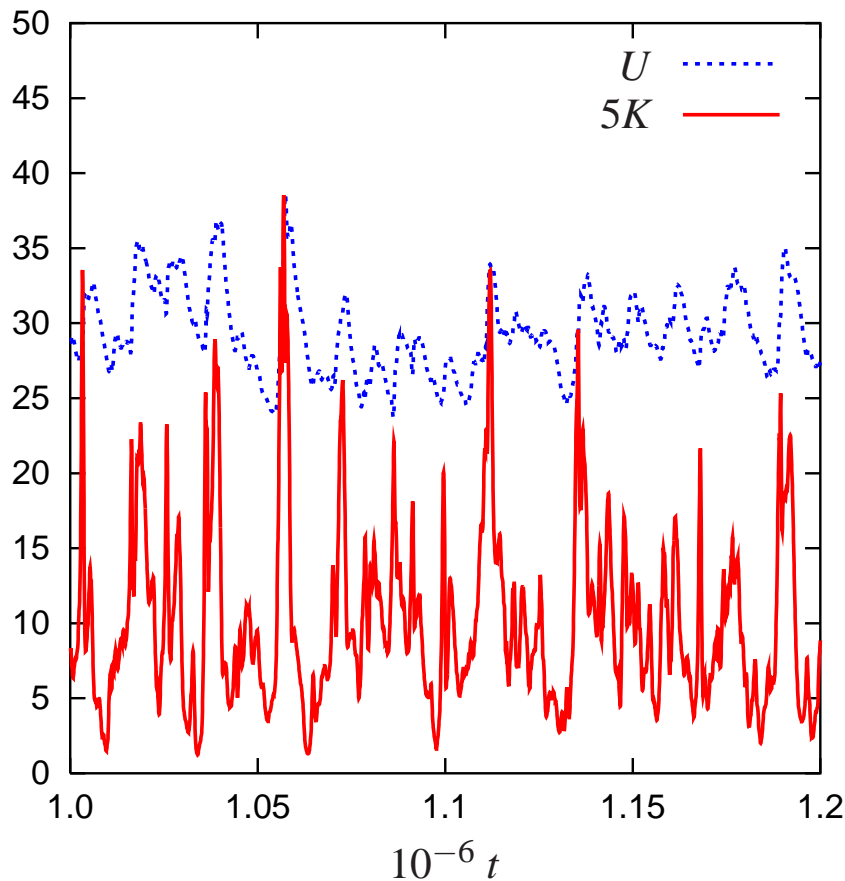
*Density blob propagation: The blob is generated inside LCFS ( $x_{LCFS} = 50$ ). Radial propagation velocity of the structures is estimated to  $\approx 0.05c_s$ ; but with large variance. Envisage the density blob as a filament elongated along the magnetic field with a ballooning structure.*

# Density Evolution



*Density blob propagation: The density blob as a filament elongated along the magnetic field with a ballooning structure.*

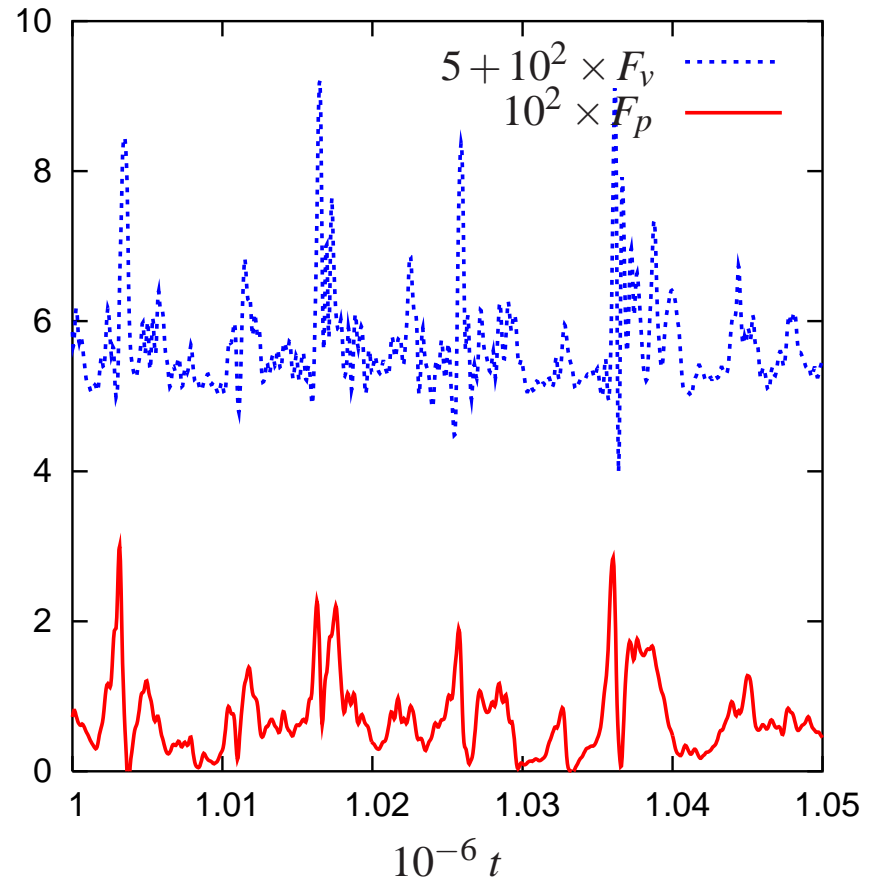
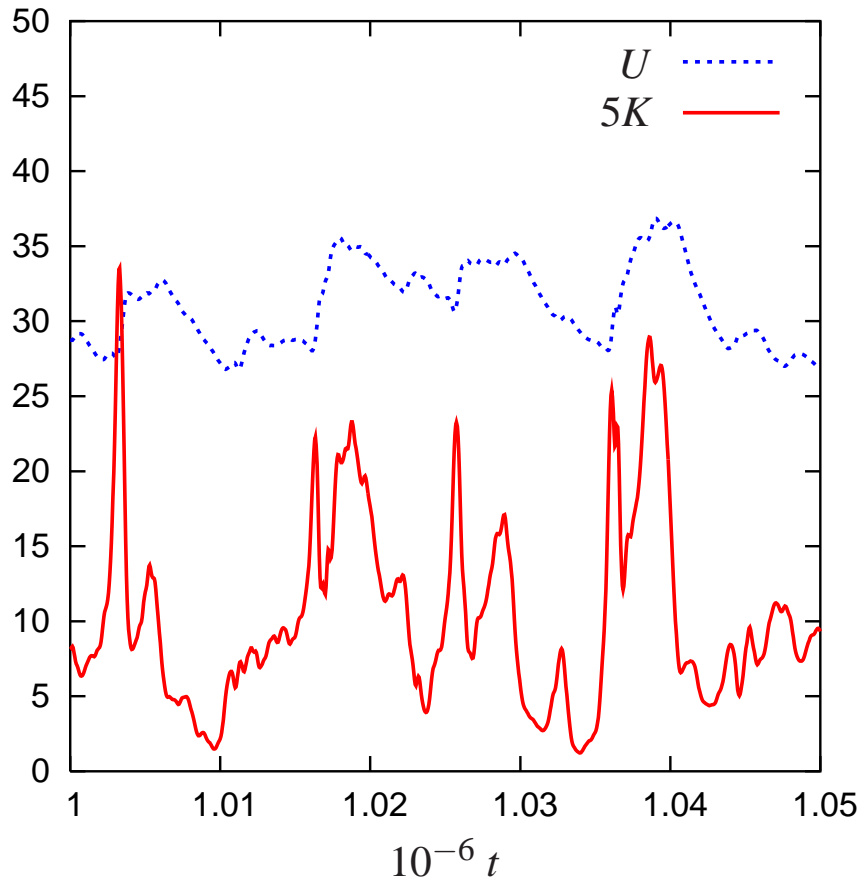
# Energy Transfer



*Bursting : Kinetic energy contained by the mean  $U$  and fluctuating  $K$  motions and the collective energy transfer terms  $F_p$  and  $F_v$  .*

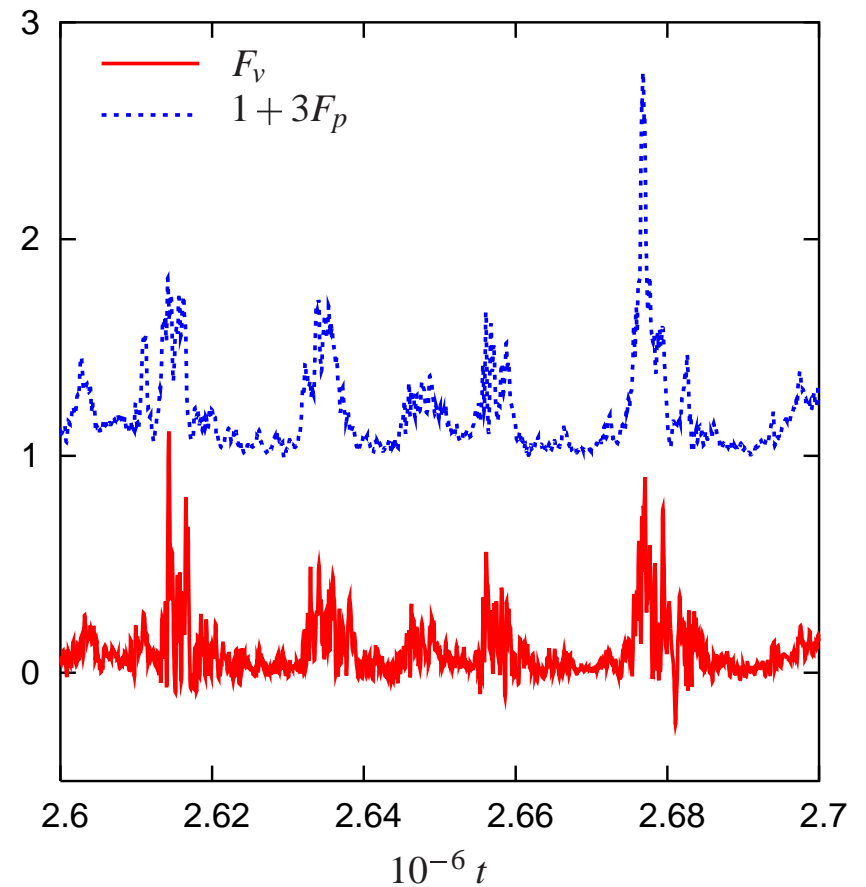
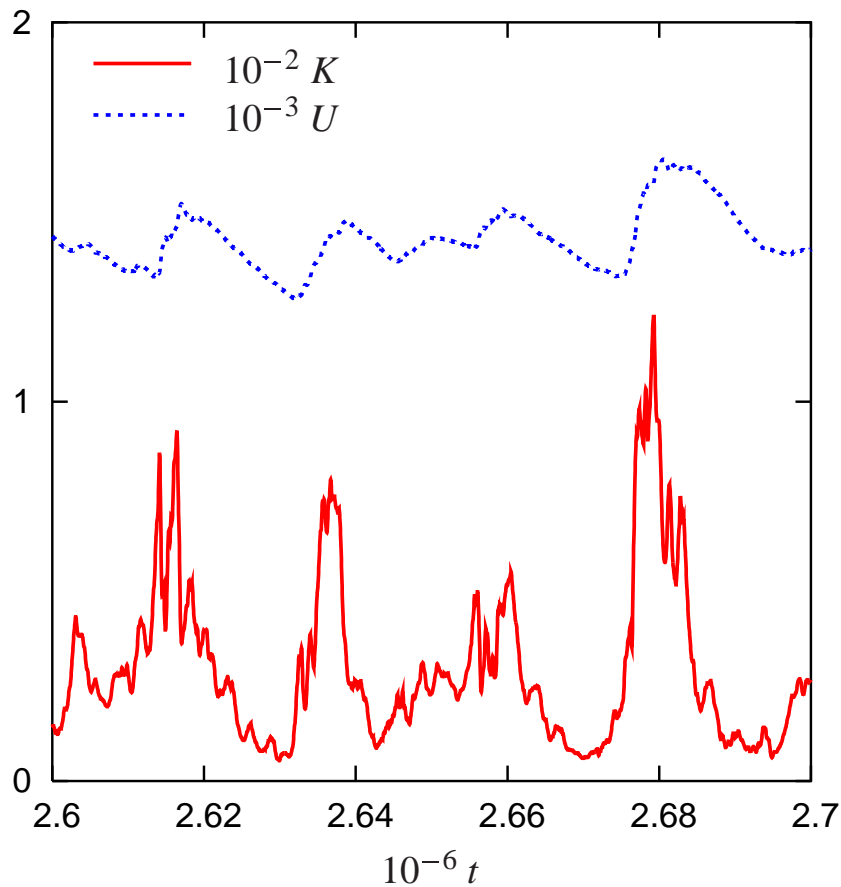


# Energy Transfer



*Bursting : Expanded time scale, ( $\nu = 10^{-2}$ ).*

# Energy Transfer



*Bursting : Half viscosity  $\nu = 5 \times 10^{-3} \rightarrow$  double time span*

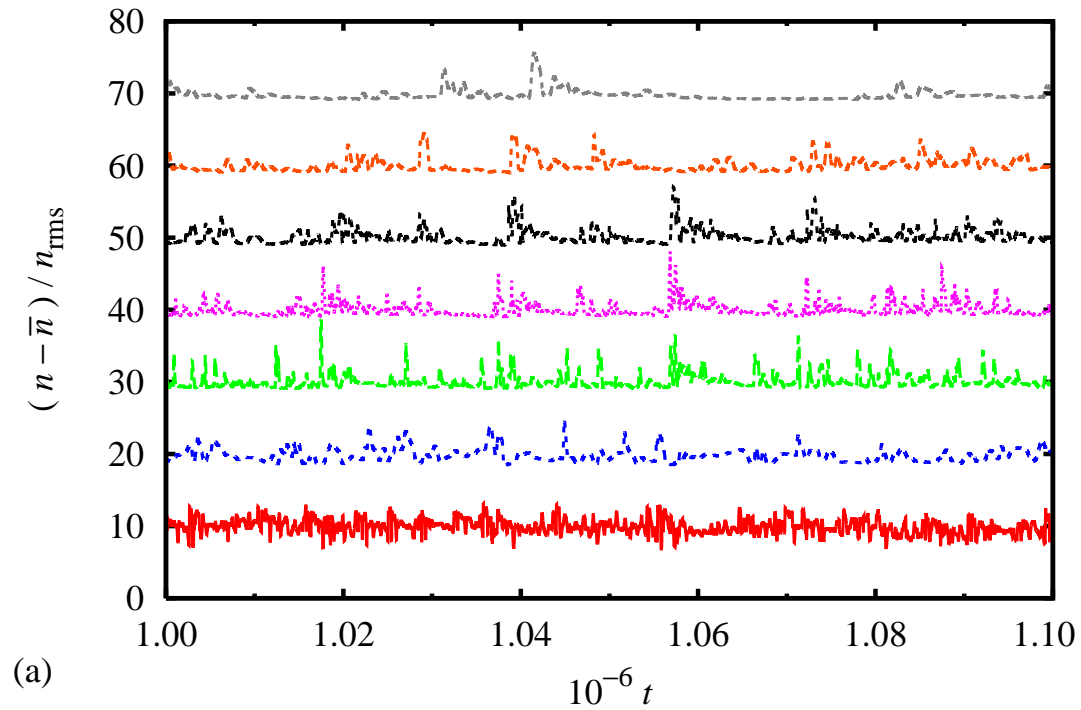
**Robust behavior**

# Energy flow

- Instability drive in the edge
- Turbulence propagating into the SOL
- Saturates and via particle and momentum fluxes
- Profile modification and flow generation; weak transport
- Shear flow stabilizes the instability ( [Benilov \*et al\* Phys. Fluids 14 1674 \(2002\)](#) )
- Profile steepening, flow damping (viscous timescale)
- Instability drive

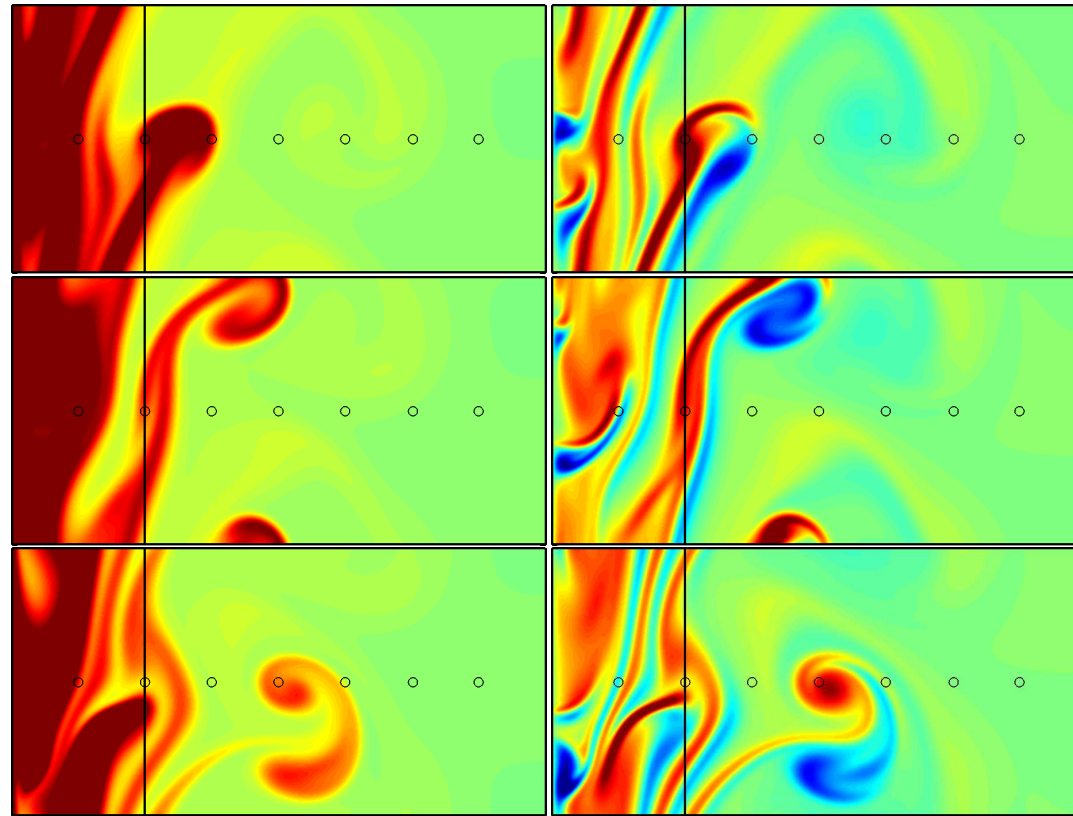
Bursting period related to viscous timescale.

# Density fluctuations



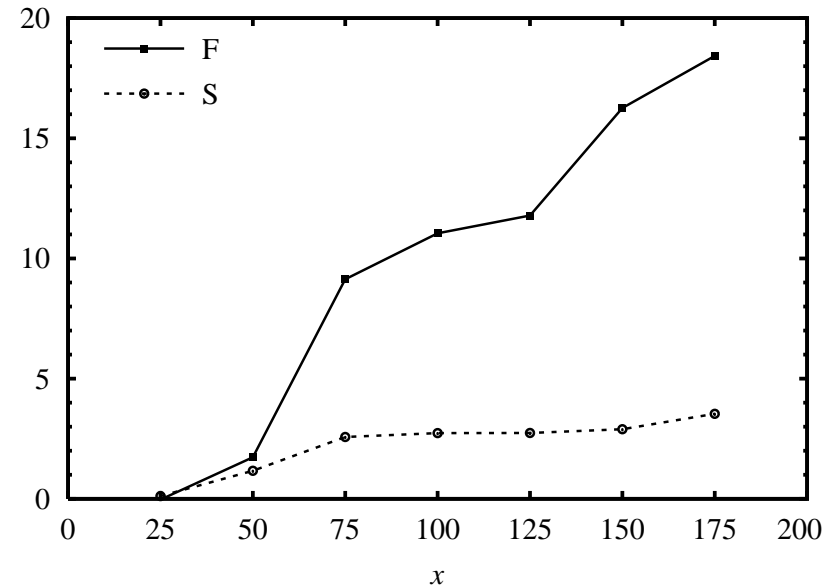
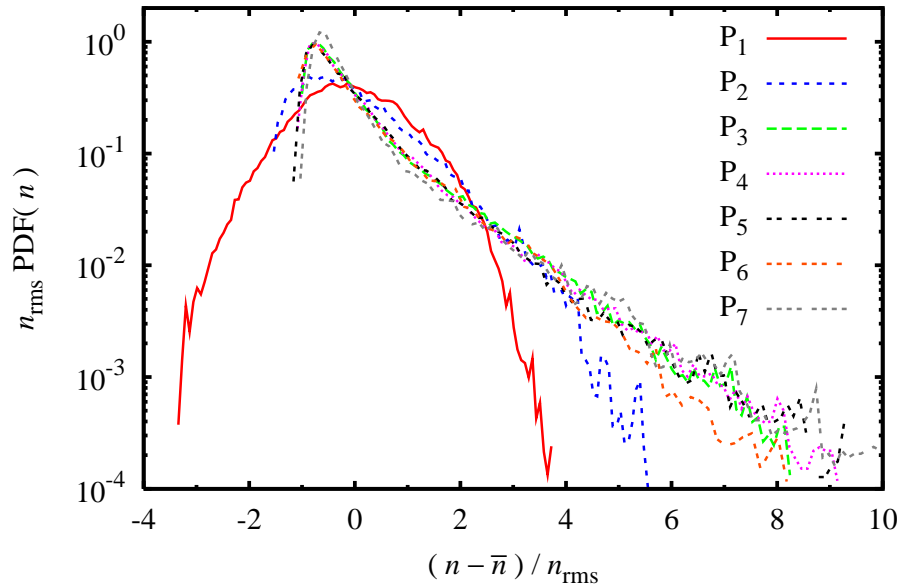
*Temporal evolution of the particle density as recorded by the probes  $P_i$ .*

# Spatial Structure



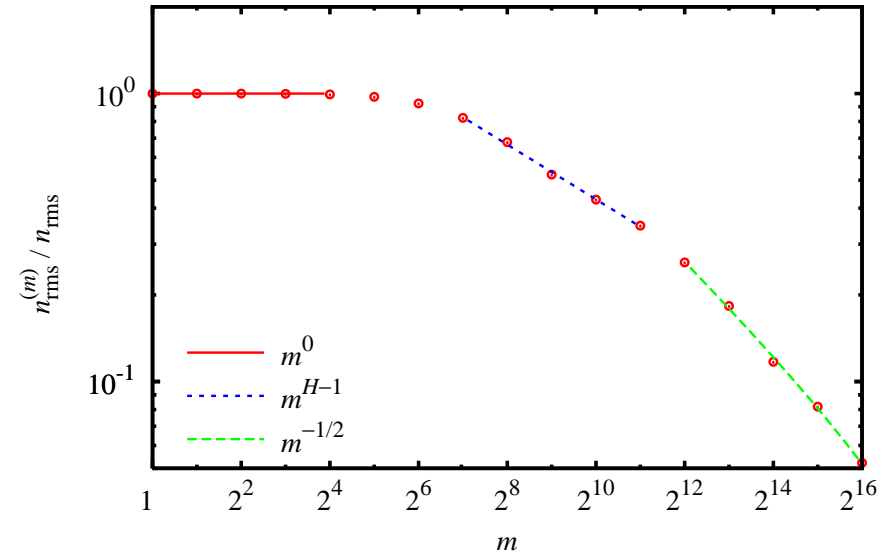
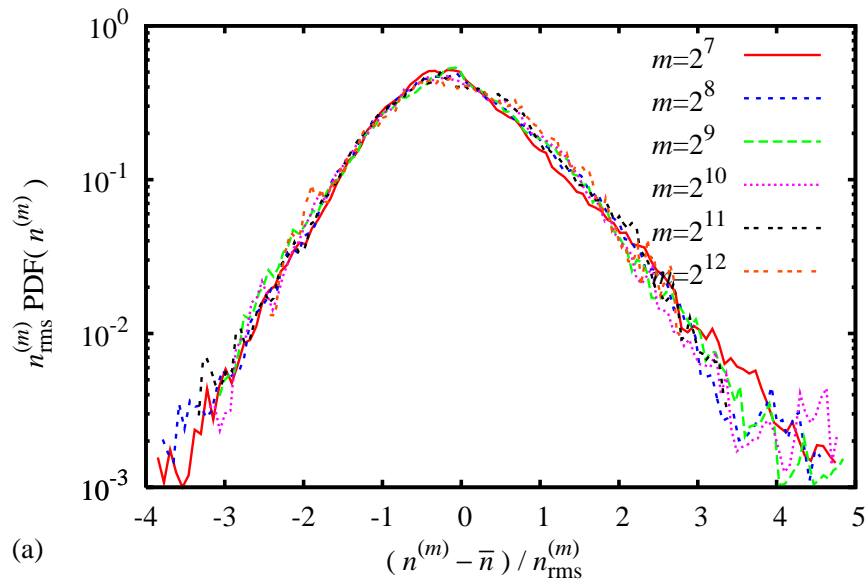
Particle density (left) and vorticity (right) during a burst  
( $\Delta t = 500$ )

# Single-Point PDFs



*Scaled probability density distribution functions, PDF, of density fluctuations at  $P_i$ .  $i > 2$  exponential tails, indicating strong blob structures. Skewness,  $s$  and flatness,  $F$ , factors for the density fluctuations*

# Single-Point PDFs

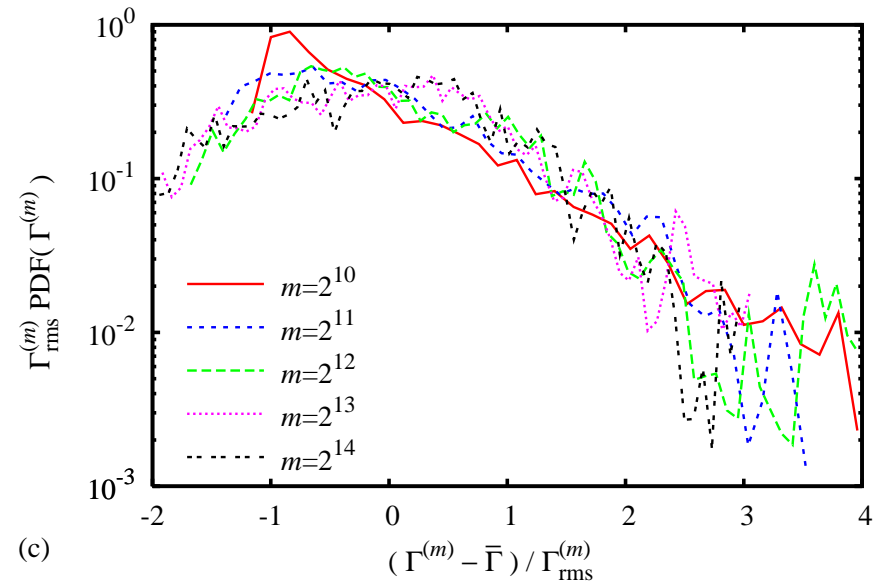
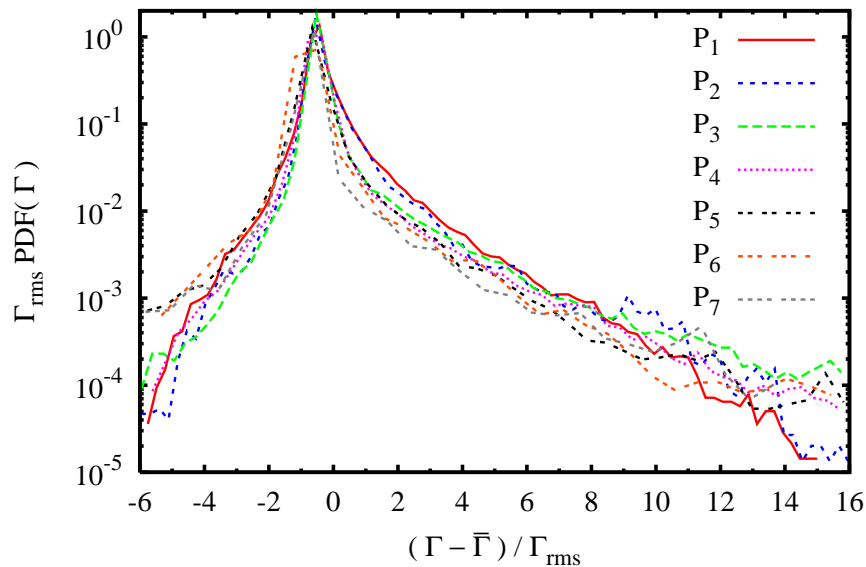


Coarse grained PDF for density fluctuations at  $P_3$ .  $m$  number of points in coarse graining. Self-similarity for scales up to  $m = 2^{12}$  ( $\tau \approx 10^4$ ).

Scaling with  $m$ , for mesoscales:  $H = 0.68$  revealing long range correlations.

For long scales  $m \geq 2^{12}$  ( $\tau > 10^4$ )  $H = 0.5$  uncorrelated random noise.

# Particle Density Flux

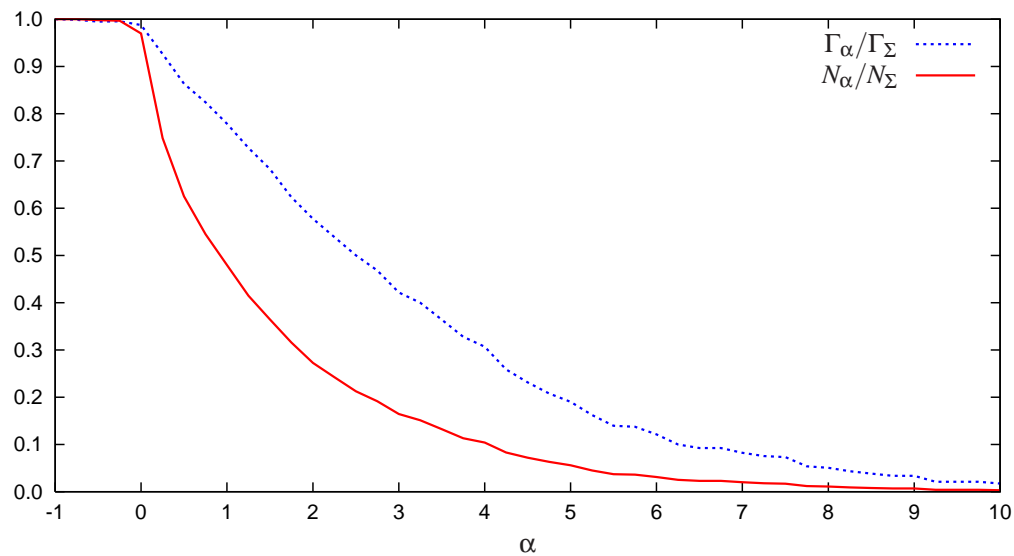


*Re-scaled PDF of particle density flux,  $\Gamma = (n - \bar{n})v_x$ , measured at the probes,  $P_i$ . Exponential tails: flux dominated by strong bursts.*

*Coarse grained PDF at  $P_3$ : Self-similarity for mesoscales with  $H = 0.5$ .*



# Particle Density Flux



*Conditional particle flux  $\Gamma_\alpha = \langle \Gamma_0 | \Gamma_0 - \bar{\Gamma}_0 > \alpha \Gamma_{0rms} \rangle$  relative to total flux  $\Gamma_\Sigma$  at  $P_3$ .*

*Relative count number  $N_\alpha$  of sub-records.*

*Few events contain most of the flux. Burst rate  $\propto$  viscosity.*

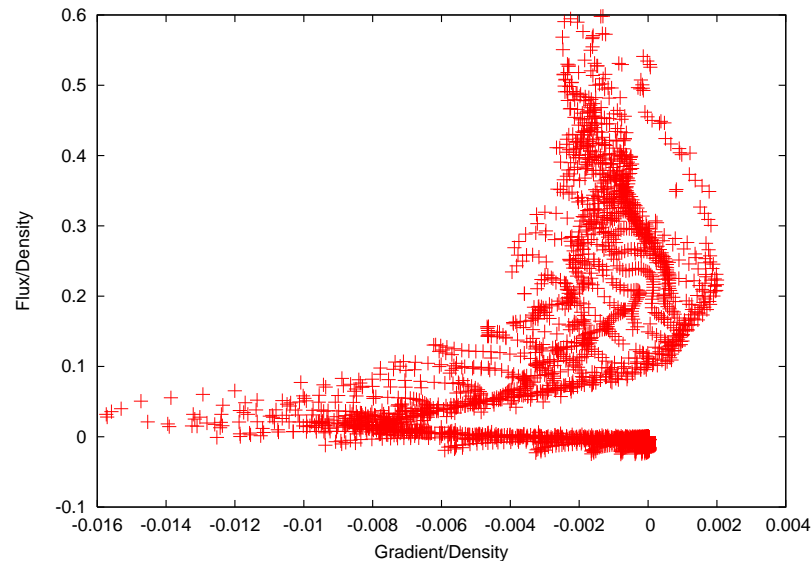
Transport characterized by the Flux PDF; not diffusive: find the “unique PDF”

Prediction of loads to divertor plates and PFC.

# Effective Diffusion?

Transport modelling: linear combination of effective convection and diffusion

$$\Gamma = nV_{\text{eff}} - D_{\text{eff}} \frac{\partial n}{\partial r} \Rightarrow \frac{\Gamma}{n} = V_{\text{eff}} - D_{\text{eff}} \frac{1}{n} \frac{\partial n}{\partial r}.$$



*Scatter plot for the flux–gradient relation.*

Transport **cannot** be parameterized by an effective particle diffusivity and a convective velocity

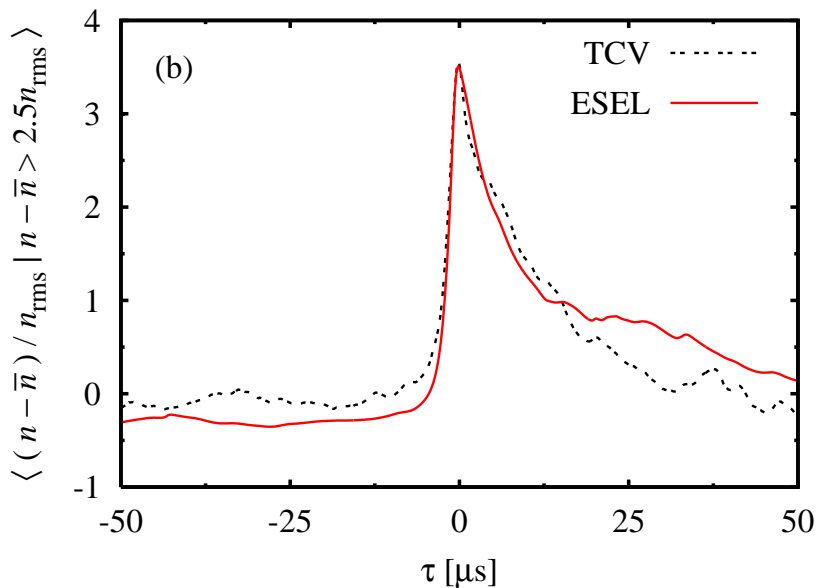
# Conclusions ...

The non-linear dynamics of interchange turbulence (2-D ESEL-code) yields very good agreement with experimental measurements:

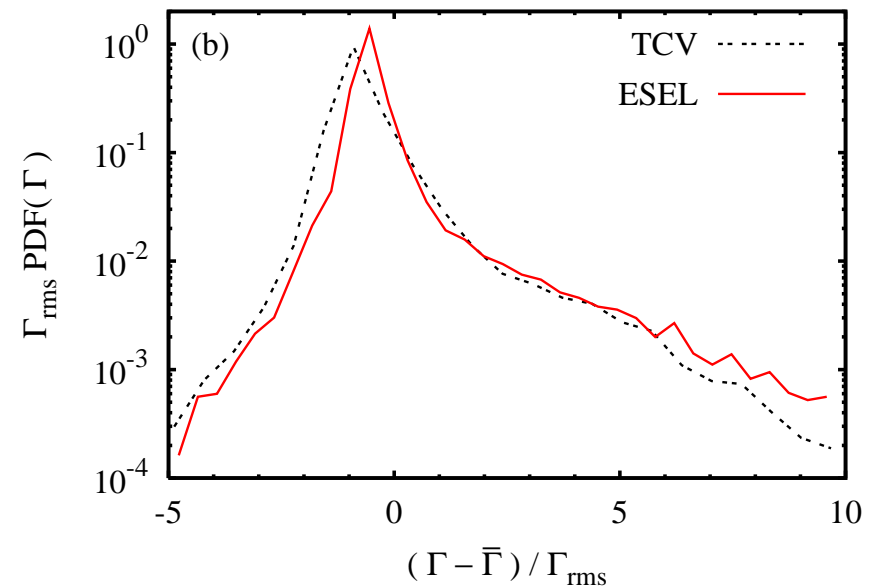
- the formation of blobs due to profile relaxations,
- radial propagation velocities around 0.1 acoustic speed,
- asymmetric wave forms; skew and flat PDFs,
- intermittent transport,
- cannot be parameterized by an effective diffusion coefficient and an effective convection velocity,
- lead to localized in space and time power loads on PFCs.

# Experiment Comparison

Experimental results from TCV-Tokamak, Lausanne: excellent quantitative agreement:  
 Garcia *et al* PPCF 48, L1 (2006)



*Conditional averaged density blob*



*PDF of the particle density flux*

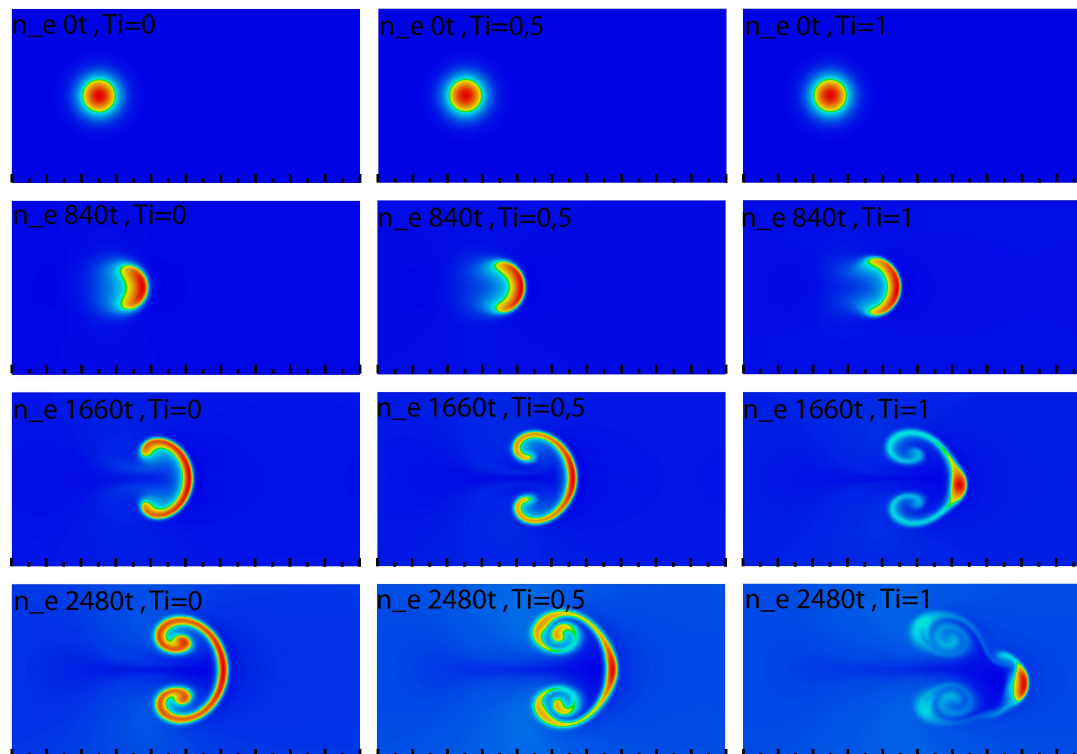
# ... and Outlook

More complete modelling of edge and SOL turbulence

- should be 3-D, non-local, and energy-conserving,
- with geometry effects and sheath boundary conditions,
- finite ion temperature: GESEL,
- address the relation to ELMs.

# Blobs in Hot Plasmas

**Finite ion temperature:** Gyrofluid model GESEL;  
(J. Madsen, J. Staerk *et al.*, EPS-2006, Rome, Italy)



# Impurity Dynamics

Impurity dynamics are modeled by tracing passive particles convected by the turbulent field.

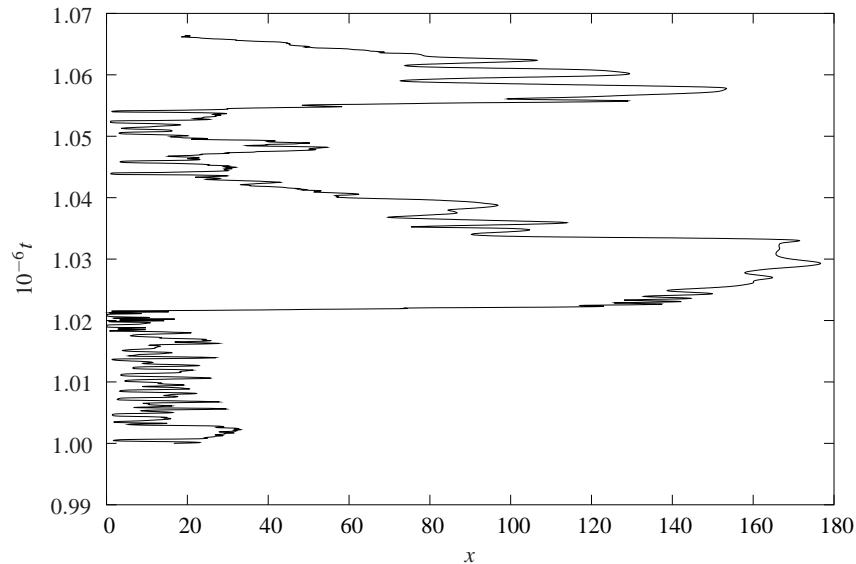
Assumptions: Impurity density low, fully ionized, cold.

Impurity convection:  $\frac{d\vec{x}}{dt} = \vec{v}_{part} = \frac{1}{B}\hat{z} \times \nabla\phi$   $\nabla \cdot \vec{v}_{part} \neq 0$  due to curvature.

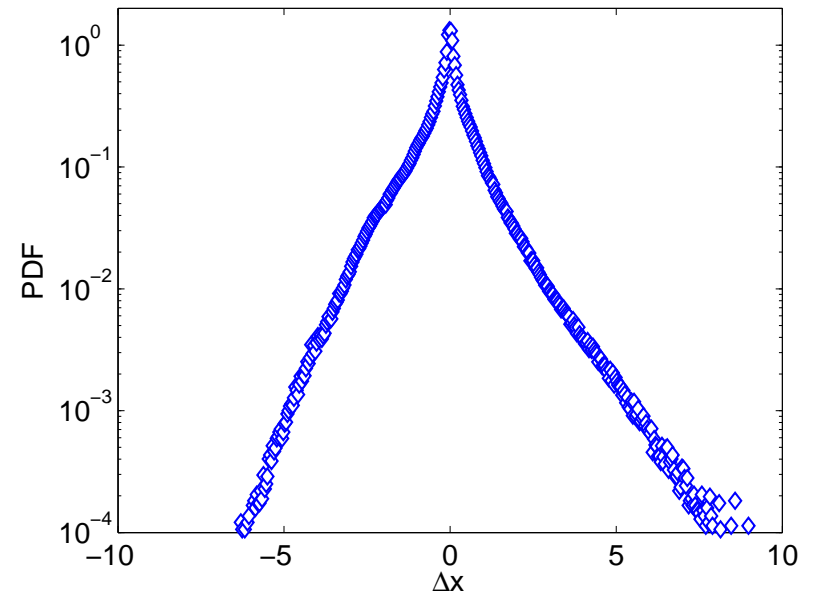
Neglecting inertia effects  $\propto M_{imp}/Zm_i$ : only lighter impurities.

Impurity density:  $D(n_{imp}/B)/Dt = 0$  : Total mixing  $:: n_{imp} \propto B$   
 $::$  Curvature pinch (Naulin, PRE 71, 015402 (2005))

# Impurity Dynamics



*Trajectory of a test particle released inside LCFS*

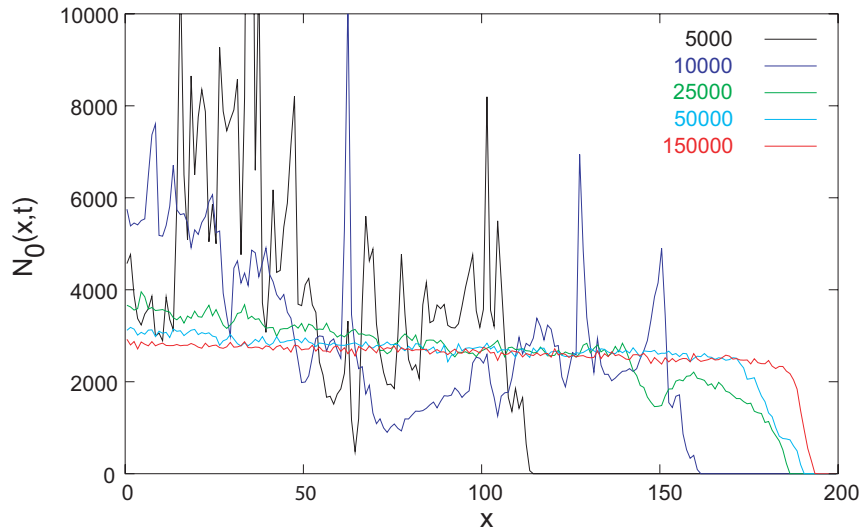


*PDF of the radial displacement over  $\Delta t = 50$  Fat exponential tail!*

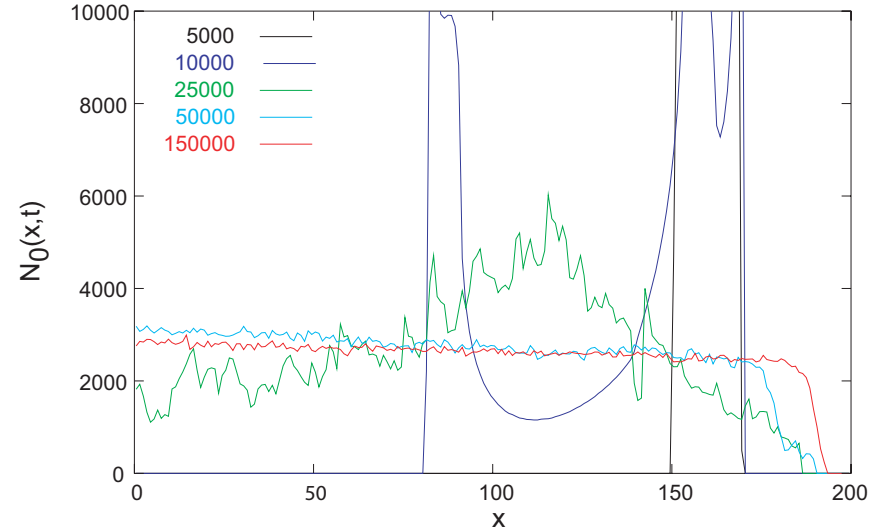


# Impurity Dynamics

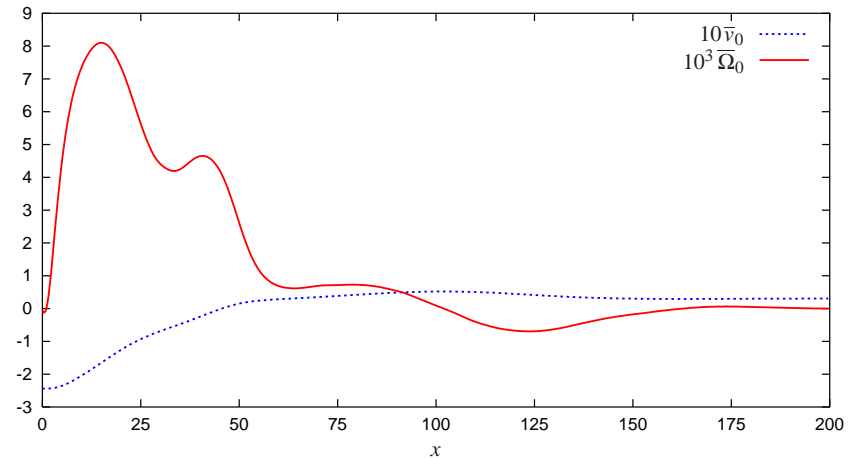
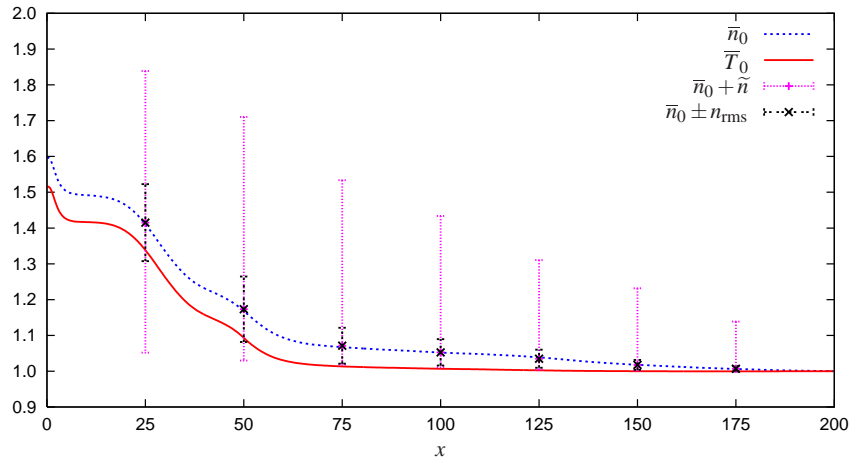
Impurity density:  $D(n_{imp}/B)/Dt = 0$  : Total mixing  $:: n_{imp} \propto B$   
 $::$  Curvature pinch (Naulin, PRE 71, 015402 (2005))



*Distribution of impurities, that are initially released at  $x = 40$ , inside lcfs: Turbulent mixing.*



*Distribution of impurities, that are initially released at  $x = 160$ : Turbulent mixing. Inward (curvature) pinch*



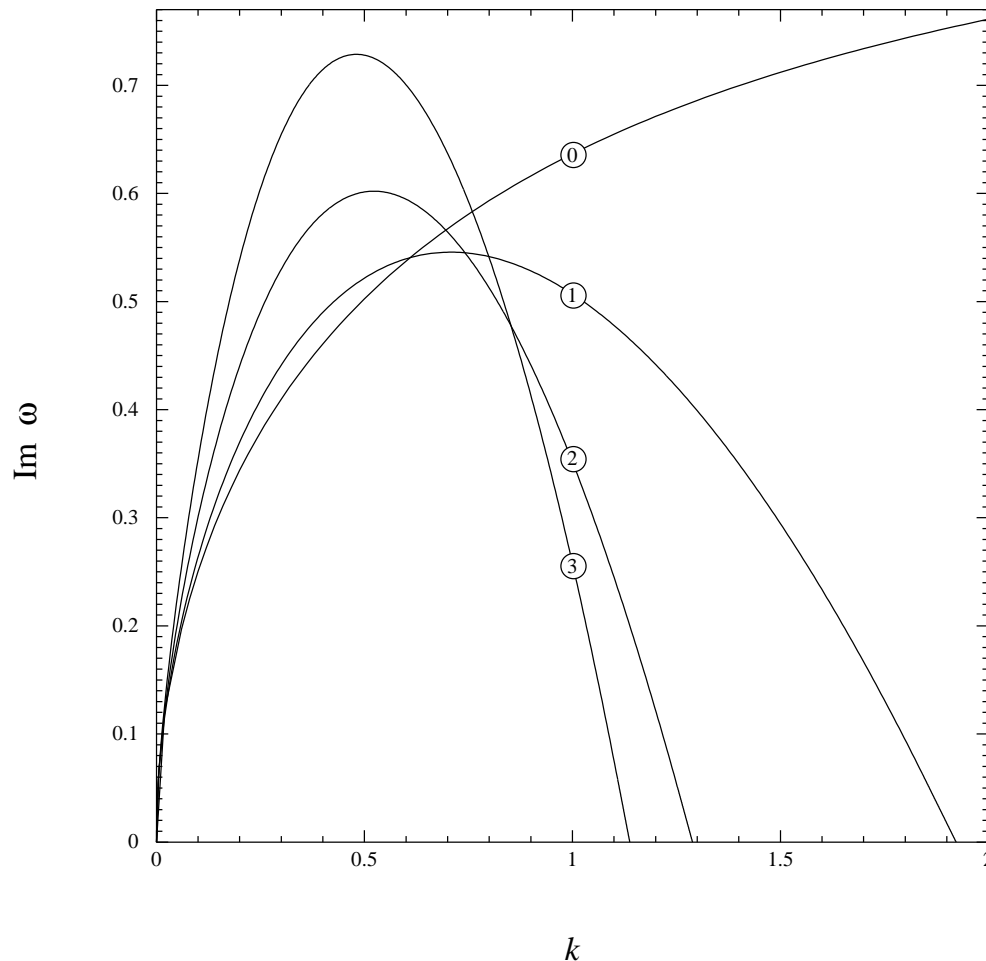
*Time averaged profile of density,  $\bar{n}_0$  and temperature,  $\bar{T}_0$ :  
Strong gradients in the edge region ( $x < x_{LCFS} = 50$ ) and flat profiles in the SOL.*

*Time averaged profile of the poloidal flow,  $\bar{v}_0$ , and vorticity,  $\bar{\Omega}_0$  ( $v = \times 10^{-2}$ )*

# Shear flow stabilization?

Influence of a background shear flow  $V(x)\hat{y}$  on the classical interchange instability

Benilov *et al* Phys. Fluids 14 1674 (2002)



Numerical solution  
of linear dispersion  
relation:  $V(x) = V_0 \tanh x$ ,  
 $V_0 = 0, 0.5, 1.0, 2.0$

NOTE: Stability for  
 $2\pi/L_y > k_c$