Theoretical Analysis of Pair-Ion Plasmas in the Light of Recent Experiments

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Abstract

The properties of pair-ion plasmas are briefly discussed. Theoretical analysis of waves in pair plasmas is presented along with a low frequency electrostatic mode which can only propagate in pair plasmas including the electron-positron plasmas. Results of a recent experiment preformed to produce a pure pair-ion fullerene $C_{60}^{\pm}$ plasma are compared with the theoretical calculations. It is pointed out that one of the observed waves in this experiment is the usual ion acoustic wave which itself is an indication of the fact that this plasma is not a pure pair-ion plasma. It is also shown that the frequency of the ion acoustic wave becomes larger as the electron density is reduced in an effort to produce pure pair-ion plasma. Moreover the quasi neutrality can break down in a pair-ion-electron plasma in the perturbed state. Using kinetic model for an unmagnetized case, it is found that in the limit $1 << \lambda_{De}^2 k^2$(where λ_{De} is the electron Debye length) the damping rate is reduced and hence the ion acoustic wave can be easily excited.

I. **Introduction**

The electron-ion (EI) plasmas have been studied extensively in laboratory, space and astrophysical environments [1, 2, 3]. In nature they prevail in planetary ionospheres, magnetospheres, interstellar medium and intergalactic space. The interest to confine (EI) plasmas on laboratory scales is mainly for the purpose of achieving thermonuclear fusion for electricity generation for the present and future needs of mankind. Both the approaches; the magnetic confinement fusion (MCF) [4] and inertial confinement fusion (ICF) [5] are followed to design a fusion reactor for continuous electricity generation. The electron-positron (EP) plasmas have also been investigated in detail because they are believed to be produced by the intense radiation in the presence of large magnetic fields in the magnetospheres of pulsars [6, 7]. They are also believed to be present in the early universe and active galactic nuclei (AGN) [8]. The attempts have been made to collect and confine positrons [10] and anti hydrogen under laboratory conditions [10]. To study the properties of pair plasmas, the (EP) system is not suitable because the annihilation time is very short. The annihilations do not occur in pair-ion plasmas. Therefore efforts are being made to produce pure (PI) plasmas on experimental scales [11, 12] to understand the behavior of equal mass plasmas.

In this experiment a cathode is heated over 2000^0C and thermionically emitted electrons are accelerated by electric field between the cathode and the anode forming a hollow electron beam. The fullerene vapors are introduced into the cylinder. Positive ions C_{60}^+ are produced by the electron impact ionization along with low energy electrons. The negative ions C_{60}^- are produced by the attachments of these low energy electrons. The fullerenes have a good electron attachment cross-section. Then the electrons are separated from the system by magnetic filtering effect. Finally a so called pure (PI) fullerene plasma is collected in a cylindrical chamber of diameter $\sim$ 3cm and about 70cm long. This set up includes an externally applied uniform axial magnetic field $B = 0.3T$,and the density of negative and positive ions in the chamber is almost the same $n_0 \simeq n_0^+ \simeq n_0^- = 10^7 cm^{-3}$. The plasma is assumed to be in thermal equilibrium with temperature $T = T_e^+ = T^- = 0.5eV$. Since the density and temperature are relatively low and the induction current of the ions is very small therefore the electromagnetic modes have been neglected. The electrostatic modes have been excited by temporarily alternating the annular exciter. Further experimental details can be found in Ref [12]. We shall concentrate here on the theoretical analysis of the experimental observations and point out some interesting characteristics of pure (PI) and pair-ion electron (PIE) plasmas.

In Ref. [12], it has been explained that the three kinds of electrostatic waves can propagate in this (PI) plasma along the field lines. These waves are the ion plasma wave (IPW), the ion acoustic wave (IAW) and the third one has been named as the intermediate frequency wave (IFW). Note that the speed of IAW in this experiment has been defined as $v_s = \left(\frac{\gamma_i T_i}{m_i}\right)^{1/2}$ (where

$T_i(m_i)$ denote the ion temperature (mass), respectively) and γ_i is the ratio of the ion specific heats. Note that a general definition of ion acoustic speed in (EI) plasmas is $c_s = (\frac{T_e}{m\,i})$ where T_e is the electron temperature. Since $n_{0e} \simeq 0$ is assumed in this plasma therefore authors have defined the IAW frequency as $\omega_s^{'} = v_s k$.

Let us look at Fig.1 which is Fig.2 of Ref.[12]. The authors of this paper have pointed out that the observed frequency of IAW is larger than the theoretical value given by the linear dispersion relation,

$$\omega_s' = v_s^2 k_z^2 \tag{1.1}$$

where the external magnetic field is along z- axis i.e.$\mathbf{B}_0 = B_0\mathbf{z}$.

If the plasma is very near to thermal equilibrium and electron density is not zero then we may expect $T_i < T_e$. Even if $T_i \leq T_e$ (which will cause heavy Landau damping and IAW many not be excited in this situation) we shall notice that mathematically the frequency ω_s of IAW turns out to be larger than $c_s k = \left(\frac{T_e}{m\,i}\right)^{1/2} k$ in a pair-ion plasma comprising electrons. Therefore the experimental observation of IAW frequency larger than $\omega_s' = v_s k = (\gamma_i T_i/m_i)^{1/2} k$ is perfectly in agreement with the theory. Only point to note is that the produced fullerene plasma should be treated as a (PIE) plasma. It will also be pointed out that the quasi neutrality may not hold in (PIE) plasmas because λ_{De}(where$\lambda_{De} = (\frac{T_e}{4\pi n_{0e} e^2})^{1/2}$ is the electron Debye length) becomes very large as the equilibrium electron density n_{0e} is decreased in the system in an effort to produce (PI) plasma. Furthermore in the limit $1 << \lambda_{De}^2 k^2$ the damping is reduced and hence acoustic waves can be easily excited. This can be a reason for the large amplitude of the density fluctuations $\frac{n_1}{n_0} \sim 0.1$ associated with the IAW in this experiment. A few authors have presented theoretical analysis of (PI) and (PIE) plasmas using fluid and kinetic models [13, 14].

In the next section we describe the basic fluid equations for the (PIE) plasmas. In section III a few linear modes of (PI) plasmas are discussed in the light of experimental observations using fluid model. The linear dispersion relation of ion acoustic wave in (PIE) plasmas is also obtained using kinetic model in section IV to investigate the wave damping. The nonlinear dynamics of (PI) and (PIE) plasmas are presented in section V. The results are briefly summarized in section VI.

II. **Basic Fluid Equations:**

Let us assume that the positive and negative ion species follow the fluid equations. Let the magnetic field be constant along the $z-$ axis and consider the plasma to be homogeneous. The equation of motion for $\alpha-$species can be written as,

$$m_\alpha n_\alpha \partial_t \mathbf{v}_\alpha = n_\alpha q_\alpha (\mathbf{E} + \frac{1}{c}\mathbf{v}_\alpha \times B_0 \mathbf{z}) - \nabla p_\alpha, \quad (2.1)$$

where the subscript $\alpha = \pm$ denotes positive and negative ions. We also assume $\mathbf{E} = -\nabla\varphi$ and $p_\alpha = n_\alpha T_\alpha$. The above equation yields,

$$(\partial_t^2 + \Omega_\alpha^2)\mathbf{v}_{\alpha\perp} = \frac{q_\alpha}{m_\alpha}(\partial_t \mathbf{E} + \Omega_\alpha \mathbf{E}_\perp \times \mathbf{z}) - \frac{\Omega_\alpha}{m_\alpha}\frac{\nabla_\perp p_\alpha \times \mathbf{z}}{n_\alpha} - \partial_t \left(\frac{\nabla_\perp p_\alpha}{m_\alpha n_\alpha}\right), \quad (2.2)$$

and

$$\partial_t v_{\alpha z} = \frac{q_\alpha}{m_\alpha} E_z - \left(\frac{\partial_z p_\alpha}{m_\alpha n_\alpha}\right), \quad (2.3)$$

where $\Omega_\alpha = \frac{eB_0}{m_\alpha e}$. The continuity equation can be written as,

$$\partial_t n_\alpha + n_{0\alpha}\nabla_\perp.\mathbf{v}_{\alpha\perp} + n_{0\alpha}\partial_z v_{\alpha z} = 0 \quad (2.4)$$

Equations.(2.2 - 2.4) give,

$$\{\omega^2(\omega^2 - \Omega_\alpha^2) - v_{T\alpha}^2 k^2 \omega^2 + v_{T\alpha}^2 k_z^2 \Omega_\alpha^2\} n_\alpha$$

.

$$-\frac{n_{0\alpha} q_\alpha}{m_\alpha} k_\perp^2 \omega^2 \varphi - \frac{n_{0\alpha} q_\alpha}{m_\alpha}(\omega^2 - \Omega_\alpha^2) k_z^2 \varphi = 0 \quad (2.5)$$

Writing Eq. (2.5) for $\alpha = \pm$ and then subtracting one equation from the other, we obtain.

$$[\omega^2(\omega^2 - \Omega_i^2) - v_{Ti}^2 k^2 \omega^2 + v_{Ti}^2 k_z^2 \Omega_i^2](n_+ - n_-)$$

$$-(n_+^0 + n_-^0)\frac{q}{m_i} k_\perp^2 \omega^2 \varphi$$

$$-(n_+^0 + n_-^0)\frac{q}{m_i} k_z^2 (\omega^2 - \Omega_i^2)\varphi = 0, \quad (2.6)$$

Here the superscript naught (0) denotes the equilibrium values.

we have assumed that the magnitude of the charge on both ions is the same i.e. $q_- = q_+ = q$ and they have equal mass m_i.The temperatures of both ions have also been assumed to be equal, i.e.,$T_+ = T_- = T_i$ and hence we define the ion thermal velocity as $v_{Ti} = \left(\frac{\gamma_i T_i}{m_i}\right)^{1/2}$ where, γ_i is the ratio of specific heats for the adiabatic ions. The Poisson equation reads,

$$-\nabla^2\varphi = 4\pi q(n_+ - n_-) \tag{2.7}$$

We are interested to find a criterion to determine the percentage concentration of electrons in the system. For this we have to investigate the ion acousitic wave frequency and, therefore, we shall also need an electron Boltzmann density distribution which is given as,

$$n_e = n_{0e}\exp\left(\frac{e\varphi}{T_e}\right) \tag{2.8}$$

III. **Linear Modes in PI and PIE Plasmas**

Now we discuss a few linear waves in (PI) and (PIE) plasmas.The set of equations (2.6)-(2.8) yield a few simple but interesting results. Let us discuss the limiting cases one by one.

We observe that a new mode which may be called a finite frequency pair plasma convective cell (PPCC) can exist in such systems in the quasi-neutral approximation with $\omega << \Omega_i$. Let us assume that the concentration of electrons is negligible in the system and the net electron momentum exerted on the collective plasma motion can be ignored. In this situation we obtain from Eq. (2.6) using $n_+ \simeq n_-$,the linear dispersion relation as,

$$\omega^2 = \frac{k_z^2}{k^2}\Omega_i^2 \tag{3.1}$$

It may be noted that this mode requires the condition $k_z << k$ and hence the assumption $\omega << \Omega_i$ remains valid. Otherwise the ion cyclotron and also the ion plasma waves should appear which do not allow us to assume quasi-neutrality in the absence of electrons.

This mode may exist, in principle, in (EP) plasmas as well with $\omega = \frac{k_z}{k}\Omega_e$ where $\Omega_e = \frac{eB_0}{m_e c}$. However, in pulsar magnetospheres since $\omega_{pe} << \Omega_e$ (where ω_{pe} and Ω_e are the electron plasma oscillation and gyro-frequencies, respectively), therefore this mode has not been studied yet (to the best of author's knowledge). The oscillations of electrons and positrons in the parallel electric field will produce the plasma wave which has a much smaller frequency than the gyro-frequency of these light particles in the strong magnetic field of the order of 10^{12} Gauss. Hence, the plasma wave may appear where the quasi-neutrality approximation does not remain valid. In laboratory (EI) plasmas, however, the condition $\Omega_e << \omega_{pe}$ generally holds. Nevertheless, a mode with the dispersion relation $\omega = [n_{0e}m_i/(n_{0i}m_e)]^{1/2}(k_z/k_\perp)\Omega_i$ has been obtained in the dusty plasma environment. A detailed discussion about this mode and the conditions for its existence can be found in Ref [15].

When k_z and T_i are assumed to be zero, and the Poisson equation is used instead of quasi-neutrality in Eq(2.6) one obtains $\omega^2 = \Omega_i^2 + 2\omega_{pi}^2$, which is the analogue of the upper hybrid oscillations in (EI) plasmas. Here $\omega_{pi} = \left(\frac{4\pi n_0 q^2}{m_i}\right)$ is the ion plasma oscillation frequency and n_0 is the background number density of positive and negative ions which are equal in this case.

Now we discuss the purely parallel propagating waves. Assuming quasi-neutrality along with $k_\perp = 0$ and $B_0 = 0$, Eqs. (2.6)and (2.8) yield,

$$\omega^2 = \frac{q}{2} N_0 c_s^2 k_z^2 + \upsilon_{Ti}^2 k_z^2, \tag{3.2}$$

where $N_0 = \frac{1+\epsilon}{1-\epsilon}$ and $\epsilon = n_-^0/n_+^0 \leq 1$. If we have $1 << N_0$, the frequency of the wave may exceed the ion cyclotron frequency that is we may have $\Omega_i < \omega$ even if $c_s k_z < \Omega_i$. Not that we have $\epsilon = 0$ for the case of ordinary (EI) plasmas and $\epsilon = 1$ for the pure (PI) plasmas. It may be noticed that for $\epsilon = 1$, the Eq. (3.2) is trivially satisfied if we multiply it with $(1-\epsilon)$ since $c_s \to 0$ as $\epsilon \to 1$.The dispersion relation (3.2) shows that the frequency of the (IAW) will be greater than the ion thermal speed even if $T_e = T_i$ and $q = e$ due to the first term on the right-hand side which can have $1 << N_0$. This seems to be the reason that the observed IAW frequency line is above the ion thermal wave plot in Fig.2 of Ref.[12]. Thus the observation of the IAW frequency can be a test to determine the percentage concentration of electrons in the system. For IAW we must have $T_i << T_e$.

On the other hand, if we assume $n_{0e} = 0$ and $B_0 = 0$, and we use Poisson equation along with $k_\perp = 0$ in Eq. (2.6), we obtain the (IPW) dispersion relation $\omega^2 = 2\ \omega_{pi}^2 + \upsilon_{Ti}^2 k_z^2$. Now we show that the ion cyclotron wave dispersion relation will be modified in plasma with the same positive and negative ions. Let us assume that the plasma is quasi-neutral in the presence of Boltzmann electrons. For simplicity we ignore the ion temperature effects. Equations (2.5) and (2.7) then yield,

$$\omega^4 - \left(\Omega_i^2 + \frac{q}{e} N_0 c_s^2 k^2\right)\omega^2 + \frac{q}{e} N_0 c_s^2 k_z^2 \Omega_i^2 = 0 \tag{3.3}$$

In the (EI) plasma case $N_0 = 1$ and for the ion cyclotron wave we have $k_z << k_\perp$, therefore Eq.(3.3) yields the well-known dispersion relation $\omega^2 =$

$\Omega_i^2 + c_s^2 k_z^2$

In the present situation, $1 << N_0$ is possible along with $\omega^2 < N_0 c_s^2 k_z^2$ therefore we retain the last term in Eq. (3.3). It gives for $q = e$,

$$\omega^2 = \frac{1}{2}[(\Omega_i^2 + N_0 c_s^2 k^2)$$
$$\pm((\Omega_i^2 + N_0 c_s^2 k^2)^2 - 4N_0 c_s^2 k_z^2 \Omega_i^2)^{1/2}]. \tag{3.4}$$

This is the modified ion cyclotron wave dispersion relation. In the limit $\omega << \Omega_i$ it reduces to the ion acoustic wave,

$$\omega^2 = \frac{qN_0 c_s^2 k_z^2/e}{1 + qN_0 \rho_s^2 k^2/e}. \tag{3.5}$$

It may also be noticed that low frequency electromagnetic Alfven waves are not dispersive in a pair-ion plasma. If we assume $n_{e0} = 0$ and use quasi-neutrality because of the low frequency limit along with $\mathbf{E} = -\nabla\phi - (\partial A_z/\partial t)\mathbf{z}$ then we obtain the following Alfven wave dispersion relation

$$\omega^2 = \upsilon_A^2 k_z^2/2, \tag{3.6}$$

where $\upsilon_A^2 = (B_0^2/4\pi n_0 m_i)$

IV. **Kinetic Theory of IAW in PIE Plasmas**

Since $n_{0e} << n_0^+$ is generally the case in (PIE) plasmas, therefore one must not use quasi-neutrality because the inequality $1 << \lambda_{De}^2 k^2$can be easily satisfied in such systems. The charge separation plays a crucial role in increasing and decreasing the Landau damping of the IAW in (PIE) plasmas in certain limits.

It will be shown through analytical calculations that the IAW can not be easily excited in the system if $n_{0e} << n_0^+$ holds which seems quite reasonable. The observation of acoustic wave frequency ω significantly larger than $c_s k$ in the experiment [12] indicates that $T_e \neq 0$ and the electron density is smaller than positive ion density and hence $1 < N_0$.

However, we can not predict the electron concentration n_{0e} in this experiment because T_e has not been measured and hence c_s is unknown.

Let us consider the linear dispersion relation of low frequency electrostatic waves propagating parallel to the external magnetic field ($\mathbf{k} \,||\, \mathbf{B}_0$) in a hot plasma as follows,

$$1 + \sum_j \frac{1}{k^2 \lambda_{Dj}^2}\{1 + i\sqrt{\pi} Z_j W_{(Z_j)}\} = 0 \tag{4.1}$$

where $Z_j = \frac{\omega}{\sqrt{2}k\upsilon_{Tj}}, \lambda_{Dj} = \frac{\upsilon_{Tj}}{\omega_{pj}}, \omega_{pj} = (\frac{4\pi n_{0j} q_j^2}{m_j})^{1/2}, \upsilon_{Tj} = (T_j/m_j)^{1/2}$ and $q_j(T_j)$ are the charge (temperature) of the j-th species, respectively. The plasma dispersion fuction $J(Z_j) = \frac{1}{\sqrt{\pi}} \int_c \frac{e^{-y2}}{z-y} dy$ is defined as $J(Z_j) = i\sqrt{\pi} W(Z_j)$.

For $j = e, +, -$, the Eq. (4.1) can be expressed as,

$$1 + \frac{1}{\lambda_{De}^2 k^2}\{1 + \iota\sqrt{\pi} Z_e W_{(Z_e)}\} + \frac{1}{\lambda_{D+}^2 k^2}\{1 + \iota\sqrt{\pi} Z_+ W_{(Z_+)}\}$$

$$+\frac{1}{\lambda_{D-}^2 k^2}\{1 + \iota\sqrt{\pi} Z_e W_{(Z_-)}\} = 0 \qquad (4.3)$$

The IAW exists in an (EI) plasma in the limit $\upsilon_{Ti}k \ll \omega \ll \upsilon_{t_e}k$. To investigate the linear dynamics of IAW in (PIE) plasmas, we use the same approximation i.e. $\upsilon_{Ti}k \ll \omega \ll \upsilon_{T_e}k$. Following (EI) plasma case, we use the limit $|Z_e| << 1$ and $1 << |Z_\pm|$ to obtain,

$$\left\{(1 + \lambda_{De}^2 k^2) - \frac{\lambda_{De}^2}{\lambda_{D+}^2}\left(\frac{1}{2Z_+^2} + \frac{3}{4Z_+^4}\right) - \frac{\lambda_{De}^2}{\lambda_{D-}^2}\left(\frac{1}{2Z_-^2} + \frac{3}{4Z_-^4}\right)\right\}$$

$$+\iota\sqrt{\pi}\left\{Z_e + \frac{\lambda_{De}^2}{\lambda_{D+}^2} Z_+ e^{-z_+^2} + + \frac{\lambda_{De}^2}{\lambda_{D-}^2} Z_- e^{-z_-^2}\right\} = 0 \qquad (4.4)$$

The real part of this equation yields,

$$\omega^2 = \omega_s^2\left(1 + \frac{3\upsilon_{T+}^2 k^2}{\omega^2}\frac{L_0^\pm}{N_0^\pm}\right)$$

where $N_0^\pm = \left(\frac{n_+^0}{n_{0e}} + \frac{n_-^0}{n_{0e}}\frac{T_-}{T_+}\frac{m_+}{m_-}\right), L_0^\pm = \left(\frac{n_+^0}{n_{0e}} + \frac{n_-^0}{n_{0e}}\frac{T_e T_-}{T_+^2}\left(\frac{m_+}{m_-}\right)^2\right)$

and

$$\omega_s^2 = N_0^\pm \frac{c_{s+}^2 k^2}{1 + \lambda_{De}^2 k^2} \qquad (4.5)$$

Here $c_{s+} = \left(\frac{T_e}{m_+}\right)^{\frac{1}{2}}$ is the ion acoustic speed corresponding to positive ions. Using Eq. (4.4) the real frequency can be approximated as,

$$\omega_r \simeq \omega_s\left(1 + \frac{3}{2}\frac{\upsilon_{T+}^2 k^2}{\omega_s^2}\frac{L_0^\pm}{N_0^\pm}\right) \qquad (4.6)$$

for $\frac{3\upsilon_{T+}^2 k^2}{\omega_s^2}\frac{L_0^\pm}{N_0^\pm} < 1$.

Assuming $\omega = \omega_r - i\gamma$ along with $\gamma << \omega_r$ the damping rate turns out to be,

$$\gamma \simeq \sqrt{\frac{\pi}{8}} \frac{N_0^{\pm} c_{s+} k}{(1+\lambda_{De}^2 k^2)} \left(\frac{T_e}{T_+}\right)^{1/2} \left\{ \left(\frac{m_e}{m_+}\right)^{\frac{1}{2}} \left(\frac{T_+}{T_e}\right)^{\frac{1}{2}} \right.$$

$$\left. + \left(\frac{T_e}{T_+}\frac{n_+^0}{T_+ n_{0e}}\right) e^{-\frac{\omega_r^2}{2k^2 v_{T+}^2}} + \left(\frac{T_e}{T_-}\frac{n_-^0}{n_{0e}}\right) e^{-\frac{\omega_r^2}{2k^2 v_{T-}^2}} \right\} \tag{4.7}$$

It is obvious from this equation that the damping of IAW depends upon concentration ratios and temperatures ratios of different species. The magnitude of γ will be different in the limits $T_- < T_+$ and $T_+ < T_-$. But this difference in magnitude will not be qualitative as long as $T_\pm << T_e$ remains valid.

If the temperatures of the ions are not much smaller than the electron temperature, i.e if the limit $T_\pm \leq T_e$ holds then we can not use the assumption $1 << |Z_\pm|$and $|Z_e| << 1$. The investigation of this situation is out of scope of the present work. Apart from the particular experiment [12], our aim is to point out how the damping rate and real frequency of IAW is affected because of $n_{0e} \neq 0$ in (PIE) plasmas.

In deriving above relations, we have assumed $q_+ = q_- = 1$. Otherwise ω_r and γ will become functions of these parameters as well. Such effects are similar to the case of other multi component plasmas like the dusty plasmas and (EPI) plasmas and can be found in the existing literature.

In the present work, we are mainly interested in discusing the role of Debye shielding in the damping rate γ of IAW due to the fact that we have $n_{0e} << n_0^+$ and $n_{0e} \neq 0$ in (PIE) plasmas. It is also to be pointed out that IAW can be easily excited in PIE plasmas in the limit $1 << \lambda_{De}^2 k^2$. Now to focus our attention on this particular point, we assume $T_+ = T_- = T_i$ and $m_+ = m_- = m$ similar to the approximations used in theoretical calculations of Ref.[12]. But we also assume $T_e \neq 0, n_{0e} \neq 0$ and $T_\pm << T_e$ for an analysis of IAW dynamics in (PIE) plasmas. Then Eq. (4.3) can be written as,

$$(1+\lambda_{De}^2 k^2) + \frac{T_e}{T_i} N_0 \left\{ -\left(\frac{1}{2Z_i^2} + \frac{3}{4Z_i^4}\right) \right\} + i\sqrt{\pi}$$

$$\left(Z_e + \frac{T_e}{T_i} N_0 Z_i e^{-z_i^2}\right) = 0 \tag{4.8}$$

The real part of this equation yields the real frequency as,

$$\omega_r(k) \simeq \omega_s \left(1 + \frac{3}{2}\frac{\upsilon_{Ti}^2 k^2}{\omega_s^2}\right) \tag{4.9}$$

here $\omega_s^2 = N_0 \frac{c_s^2 k^2}{1+\lambda_{De}^2 k^2}$. In the limit $\lambda_{De}^2 k^2 << 1$, Eq. (4.9) reduces to $\omega^2 = N_0 c_s^2 k + 3\upsilon_{Ti}^2 k^2$. In the limit $1 << \lambda_{De}^2 k^2$ for $n_{e0} << n_0^+$ and assuming $\omega = \omega_r - \iota\gamma$, one obtains

$$\omega_r(k) \simeq (1+\epsilon)\omega_{pi}^+ \left[(1 - \frac{1}{2\lambda_{De}^2 k^2})\times \right.$$

$$\left. (1 + \frac{3}{2}\frac{n_+^0}{n_{0e}}\frac{\lambda_{D+}^2 k^2}{N_0})\right] \tag{4.10}$$

where $\omega_{pi}^+ = \left(\frac{4\pi n_{0i}^+ e^2}{m_i}\right)^{\frac{1}{2}}$.

Since $\epsilon \neq 0$ if $n_{0e} \neq 0$, therefore ω_r becomes larger than the case of (EI) plasmas corresponding to the same density of n_0^+. Note that $\lambda_{D+}^2 = \lambda_{De}^2$ in (EI) plasmas because $n_{0e} = n_0^+$ and Eq.(4.10) in this case becomes the same as Eq. (4.2.4.4)of Ref.[16]. However, in (PIE) plasmas the result is different from the case of (EI) plasmas due to $\epsilon \neq 0$ and $1 < \frac{n_+^0}{n_{0e}}$. Therefore in (PIE) plasmas we may have $\omega_{pi} < \omega_r$. This seems to be the case in Fig. 1.

To write imaginary part of the frequency as $\gamma = \gamma_e^\pm + \gamma_i^\pm$ where superscripts ($\pm$)denote the case of (PIE) plasmas, we obtain,

$$\gamma_e^\pm = N_0 \gamma_e \tag{4.11}$$

$$\gamma_i^\pm = N_0^2 \gamma_i \tag{4.12}$$

where

$$\gamma_e(k) = (\frac{\pi}{8})^{1/2}(\frac{m_e}{m_i})^{1/2}\frac{c_s k}{(1+\lambda_{De}^2 k^2)^2} \tag{4.13}$$

$$\gamma_i(k) = (\frac{\pi}{8})^{1/2}(\frac{T_e}{T_i})^{3/2}\frac{c_s k}{(1+\lambda_{De}^2 k^2)^2}\exp(-\frac{\omega_r^2}{2k^2\upsilon_{Ti}^2}) \tag{4.14}$$

Equations (4.12) and (4.13) are the same as Eq. (4.2.4.6) of Ref [16] which represent the damping rate of IAW in (EI) plasmas where $N_0 = 1$.

Note that $\lambda_{De}^2 = \epsilon_1 \lambda_{D+}^2$ in (PIE) plasmas where $\epsilon_1 = \frac{n_+^0}{n_{0e}}$. Corresponding to a wavelength $\frac{1}{k}$ for which $\lambda_{De}^2 k^2 = \lambda_{D+}^2 k^2 < 1$ in EI plasmas, we may have $1 << \lambda_{De}^2 k^2$ in (PIE) plasma because the inequality $1 << \epsilon_1$ may hold due to $n_{0e} << n_+^0$.

Therefore quasi-neutrality approximation must not be used in (PIE) plasmas in general. Furthermore,the use of Poisson equation has also shown that in (PIE) plasmas we may have $\omega_{pi} < \omega_r$ because of the factor $1 + \epsilon$. For the case of longer wavelengths i.e. $\lambda_{De}^2 k^2 < 1$, the damping rate of IAW can be very large in (PIE) plasmas compared to (EI) plasmas because we may have $1 << N_0$ and hence $\gamma_i << \gamma_i^{\pm}$ and $\gamma_e << \gamma_e^{\pm}$. In this case, one can not excite IWA easily.

On the other hand, if $1 << \lambda_{De}^2 k^2$ holds which can be the case when $1 << \epsilon_1$ (even if $\lambda_{D+}^2 k^2 \leq 1$), then the damping rate of (IAW) can reduce significantly in (PIE) plasmas because $1 << N_0 << \lambda_{De}^2 k^2$ can be generally valid. In this situation one may find $\gamma_i^{\pm} << \gamma_e$ and $\gamma_e^{\pm} << \gamma_e$. Therefore the IAW can be excited easily in PIE plasmas if the condition $1 << N_0 << \lambda_{De}^2 k^2$ is satisfied

In the limit $1 << \lambda_{De}^2 k^2$, one may express the above relations as,

$$\gamma_{e(k)}^{\pm} \simeq \left[\sqrt{\frac{\pi}{8}} \sqrt{\frac{m_e}{m_i}} \frac{1 - \epsilon^2}{(\lambda_{D+}^2 k^2)^2} \right] c_s k \tag{4.15}$$

and

$$\gamma_{i(k)}^{\pm} \simeq \left[\sqrt{\frac{\pi}{8}} \left(\frac{T_e}{T_i} \right)^{\frac{3}{2}} \frac{1 - \epsilon^2}{(\lambda_{D+}^2 k^2)^2} e^{\frac{\omega_r^2}{2k^2 v_{Ti}^2}} \right] c_s k \tag{4.16}$$

Since $e^{-\frac{\omega_r^2}{2k^2 v_{Ti}^2}} << 1$, $\epsilon \leq 1$ or $\epsilon \ll 1$, therefore $\gamma_i^{\pm} \leq \gamma_i$ can be the case. That is corresponding to the same value of $\lambda_{D+}^2 k^2$ we have $\gamma_i^{\pm} \leq \gamma_i$ because in (PIE) plasmas $\omega_r^{ei} < \omega_r^{\pm}$. But $\gamma_e^{\pm} < \gamma_e$ always holds due to $(1 - \epsilon^2) < 1$. Therefore we conclude that in the limit $1 << \lambda_{De}^2 k^2$, the damping rate of IAW decreases if the electron concentration decreases as ϵ becomes nearer to 1. Since ω_r depends upon T_e which is unknown in the observations of Ref [12], therefore we can not estimate the value N_0 from the Fig.1 by using the value ω_s and k. Furthermore to find the value of $\lambda_{D+}^2 k^2$, we again need the value of T_e. Therefore in the present work we can not predict any estimate of ϵ or N_0 in this experiment which can give the value of the ratio $\frac{n_{0e}}{n_+^0}$.

For example if $n_-^0 = 0.9 n_+^0, N_0 = 19$, then $\lambda_{De}^2 = 10 \lambda_{D+}^2$. If $\lambda_{D+}^2 k^2 = 10$

is assumed (which is equivalent to $\lambda_{De}^2 k^2 = 10$ in case of (EI) plasmas), then in (PIE) plasmas we find $\epsilon_1 = 10$ and hence $\lambda_{De}^2 k^2 = 10^2$.

Important points of this section are the following. First it has been pointed out that the quasi-neutrality is not a good approximation in the case of (PIE) plasmas where the condition $1 \leq \lambda_{De}^2 k^2$ can hold in general due to $n_{0e} << n_+^0$ even if $\lambda_{D+}^2 k^2 < 1$. Second the real frequency of the wave can be larger than ion plasma oscillation frequency i.e. $\omega_{pi} << \omega_r$ because $1 < \epsilon$ in Eq. (4.10) provided that $1 << \lambda_{De}^2 k^2$ is satisfied. Third, the IAW has lesser value of the damping factor in (PIE) plasmas compared to (EI) plasmas corresponding to the same value of $\lambda_{D+}^2 k^2$ if $1 << \lambda_{De}^2 k^2$ holds. Therefore this wave can be easily excited in (PIE) plasmas in the limit $1 << \lambda_{De}^2 k^2$. Fourth, for $\lambda_{De}^2 k^2 << 1$, the damping of (IAW) is larger in (PIE) plasmas compared to (EI) plasmas corresponding to the same values of $\lambda_{D+}^2 k^2$ and hence this wave can not be easily excited.

V. **Nonlinear Dynamics of PI Plasmas.**

For the nonlinear study of PPCC mode, we ignore the ion pressure term assuming T_i to be small.

Then under drift approximation $|\partial_t| << \Omega_i$,Eq. (2.1) gives

$$\mathbf{v}_{\alpha\perp} = \frac{c}{B_0}(\mathbf{E}_\perp \times \mathbf{z}) - \frac{1}{\Omega_\alpha}(\partial_t + \mathbf{v}_\alpha.\nabla)\mathbf{v}_\alpha \times \mathbf{z} = \mathbf{v}_E + \mathbf{v}_{\alpha p} \tag{5.1}$$

and

$$(\partial_t + \mathbf{v}_\alpha.\nabla)v_{\alpha z} = \frac{q_\alpha}{m_\alpha}E_z, \tag{5.2}$$

where $\mathbf{v}_E$ is the electric drift and $\mathbf{v}_{\alpha p}$ is the polarization drift for $\alpha = \pm$ ions. Note that

$$\nabla.(\mathbf{v}_{+p} - \mathbf{v}_{-p})$$
$$= -\frac{2c}{B_0\Omega_i}\left[\partial_t + \frac{c}{B_0}z \times \nabla_\perp\varphi.\nabla_\perp\right]\nabla_\perp^2\varphi$$

Therefore, the parallel Eq. (5.2) yields,

$$\left[\frac{\partial}{\partial t} + \frac{c}{B_0}\mathbf{z} \times \nabla_\perp(\varphi).\nabla_\perp\right](v_{+z} - v_{-z}) = -b\frac{\partial\varphi}{\partial z}, \tag{5.3}$$

and the continuity equation can be written as,

$$\left[\frac{\partial}{\partial t} + \frac{c}{B_0}\mathbf{z} \times \nabla_\perp\varphi.\nabla_\perp\right]a\nabla_\perp^2\varphi = -\frac{\partial}{\partial z}(v_{+z} - v_{-z}) \tag{5.4}$$

where $a = -2c/(B_0\Omega_i)$ and $b = 2q/m_i$.
The factor 2 in constants a and b is the effect of pair-ion plasma.
Equations (5.3) and (5.4) can be written, respectively , as,

$$(\partial_t + D_i \mathbf{z} \times \nabla_\perp \Phi.\nabla)V = 2v_{Ti}^2 \partial_z \Phi \tag{5.5}$$

and

$$(\partial_t + D_i \mathbf{z} \times \nabla_\perp \Phi.\nabla)D_T^2 \Phi = -\frac{1}{2\rho_i^2}\partial_z V \tag{5.6}$$

where $\Phi = \frac{e\varphi}{T_i}, D_i = \frac{cT_i}{eB_0}, \rho_i = \frac{v_{Ti}}{\Omega_i}$ and $V = (v_{z-} - v_{+z})$.

We try to find a stationary solution of the above coupled nonlinear partial differential equations. Let us define $\eta = y + \mu z - ut$ coordinate moving with speed u in $yz-$ plane, where $\mu = \frac{u_y}{v_z}$ and $u = (u_y^2 + u_z^2)^{1/2}$. In the moving frame (η, x), the coupled equations (5.5) and (5.6) can be expresed as ,

$$C_1 \partial_\eta \nabla_\perp^2 \phi + v_0 \partial_\eta \phi + \{\nabla_\perp^2 \phi, \phi\} = 0 \tag{5.7}$$

where

$$C_1 = -\frac{u}{D_i}, v_0 = \mu \frac{L_0}{2\rho_i^2 D_i}, L_0 = (C_0 - 2\mu \frac{v_{Ti}^2}{u})$$

and C_0 is an arbitrary constant. The equation (5.7) can have dipolar vortex solutions which can be obtained by dividing the (η, x) plane into two regions, inner region $r < R_0$ and the outer region $R_0 < r$ where R_0 is radius of a circle, with $r = \sqrt{\eta^2 + x^2}$ and $\theta = \tan^{-1}\frac{x}{\eta}$.

VI Summary

Theoretical analysis of pair-ion (PI) and pair-ion-electron plasmas has been presented in the light of a recent experiment [12]. Several linear modes of PI plasmas have been discussed assuming the plasma behavior as a fluid. Here the quasineutrality has been used. It is found that the frequency of ion acoustic wave is larger in the PIE plasmas compared to electron-ion (EI) plasmas corresponding to the same electron temperature T_e and wave vector k because of the inequality $1 < N_0$.

Then it has been shown that the quasi-neutrality is not a good approximation for investigating (PIE) plasmas because the condition $1 \ll \lambda_{De}^2 k^2$ holds in such systems in general. In this case the damping of ion acoustic wave (IAW) is reduced. This seems to be the reason for large density fluctuations $\frac{n_1}{n_0} \sim 0.1$ associated with IAW in the experiment [12]. On the other hand, it has been found that the (IAW) damping increases in the limit $\lambda_{De}^2 k^2 \ll 1$.

The nonlinear dynamics of pure (PI) plasmas has also been discussed. A new linear wave is found to be a normal mode of pair plasmas. It has the frequency $\omega = \frac{k_z}{k}\Omega_i$ in pair-ion (PI) plasmas and $\omega = \frac{k_z}{k}\Omega_e$ in electron-positron (EP) plasmas. In the nonlinear limit this electrostatic wave is described by two coupled equations. These equations can reduce to a single equation in the moving frame (η, x) which has the form similar to Hasegawa-Mima (HM) equation. It admits dipolar and tripolar solutions. It may be interesting to note that this equation looks similar to HM-equation in (η, x) frame but it contains a different physics. The drift waves can not exist in (PI) plasmas. This equation describes the nonlinear dynamics of pair ion convective cell (PPCC) mode.

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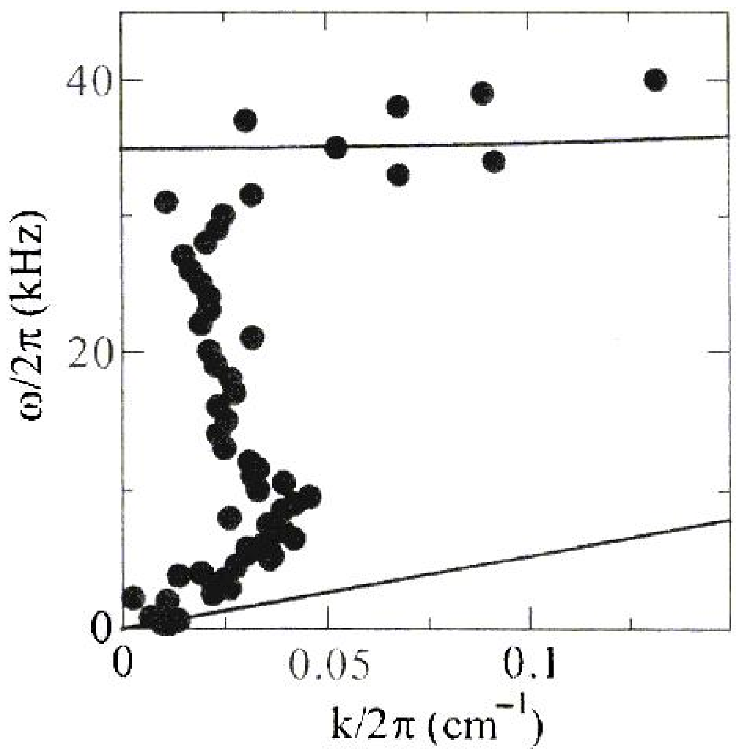

FIG. 1. Dispersion relations for electrostatic waves propagating along *B*-field lines. Solid lines and dots denote results calculated from two-fluid theory and measured experimentally, respectively.