The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

SMR.1766 - 4

Miniworkshop on
New States of Stable and Unstable Quantum Matter
(14 - 25 August 2006)

Much Ado about Zeros: Mottness and High T_c

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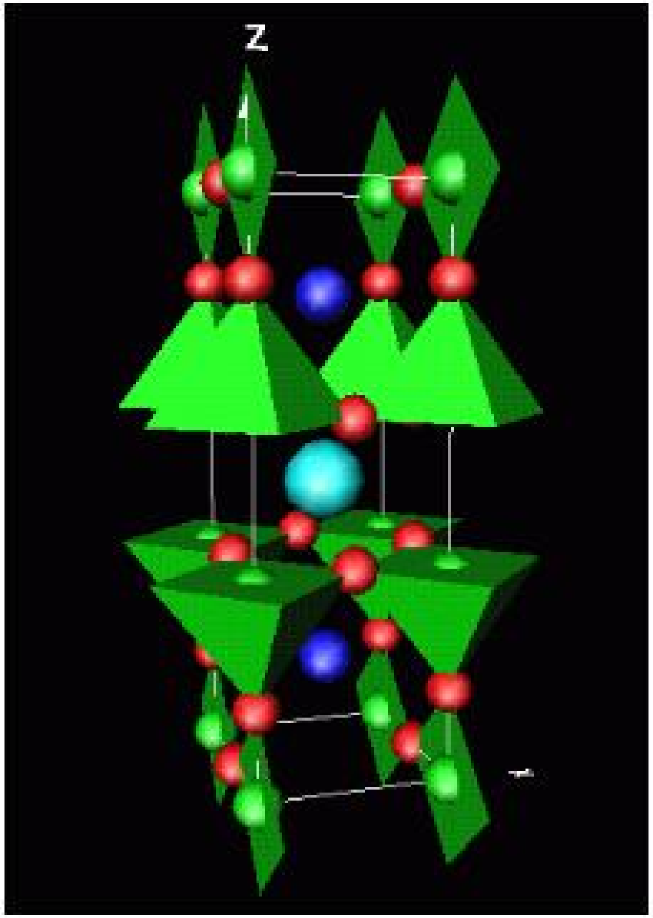
Much Ado about Zeros: Mottness and High T_c

Thanks to:

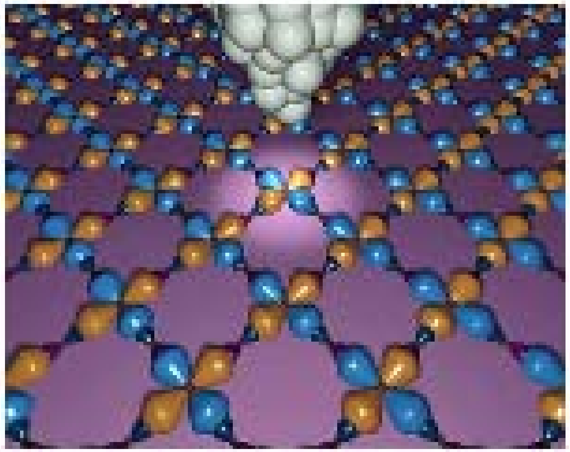
Former student: Tudor Stanescu, Virginia
Present students: D. Galanakis, T.-P. Choy,
Collaborator: C. Chamon, BU



$Y Ba_2Cu_3O_7$

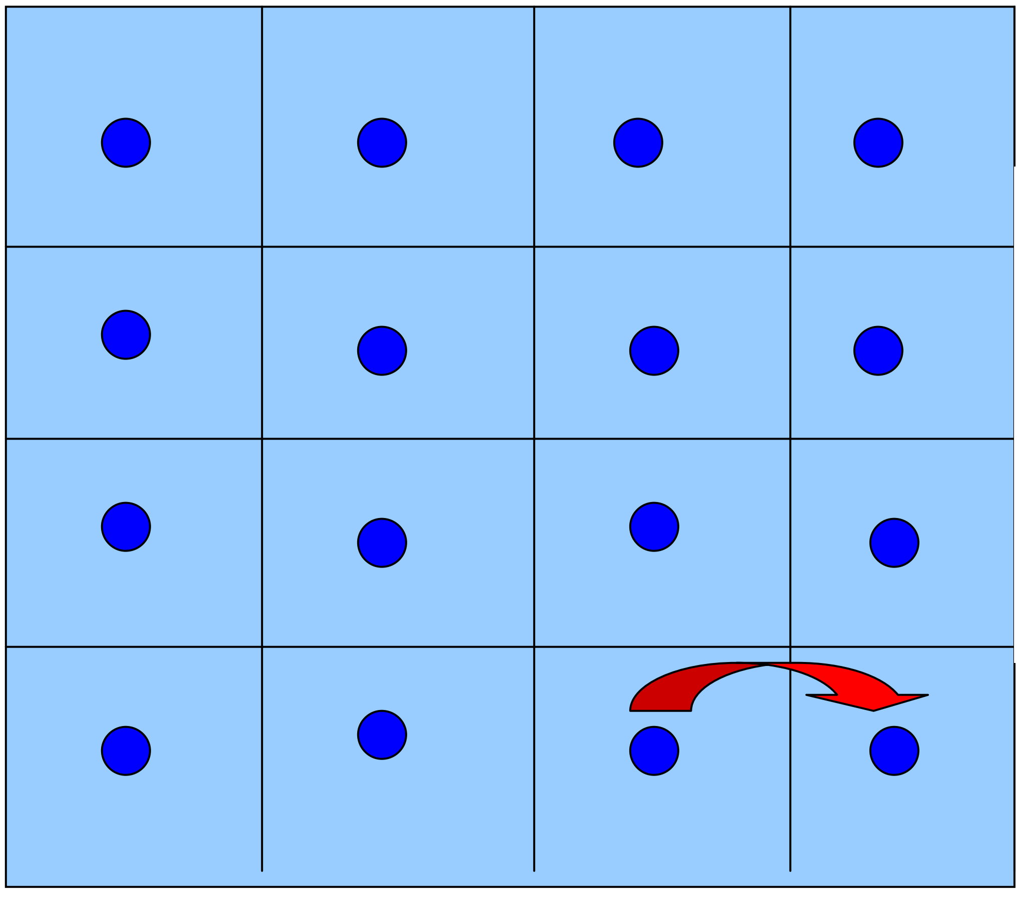
Cuprate Superconductors

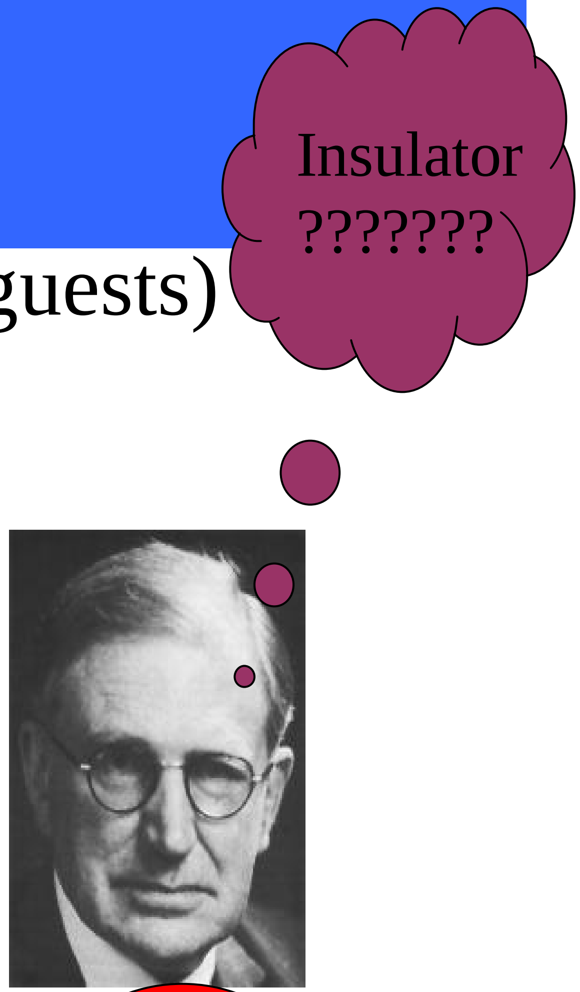
Cu-O plane

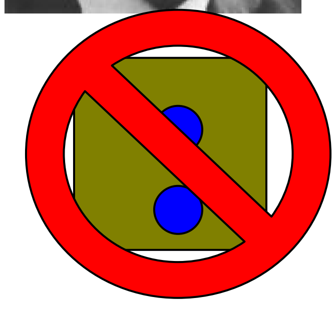


Mott Problem (NiO)

Hubbard Inn floor plan (N rooms N guests)



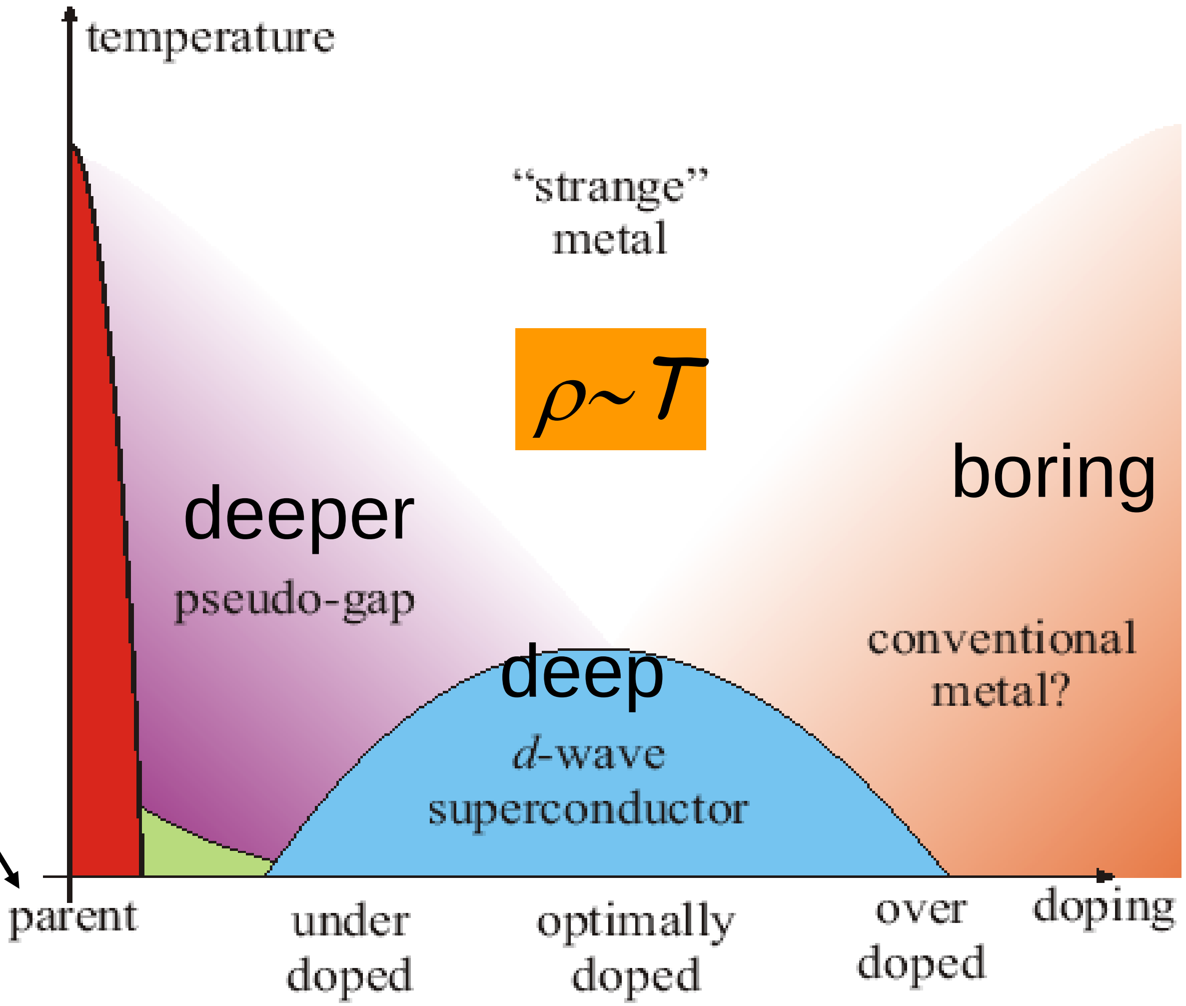




$\Delta E = U \gg K.E.$

Phase diagram

Mott insulator



Why and How?

Organising principle for the Normal State

- Resistivity~T
- Pseudogap
- Absence of quasiparticles
- Spectral-weight transfer

zeros

T-linear Resistivity
Batlogg, et. al., 1990

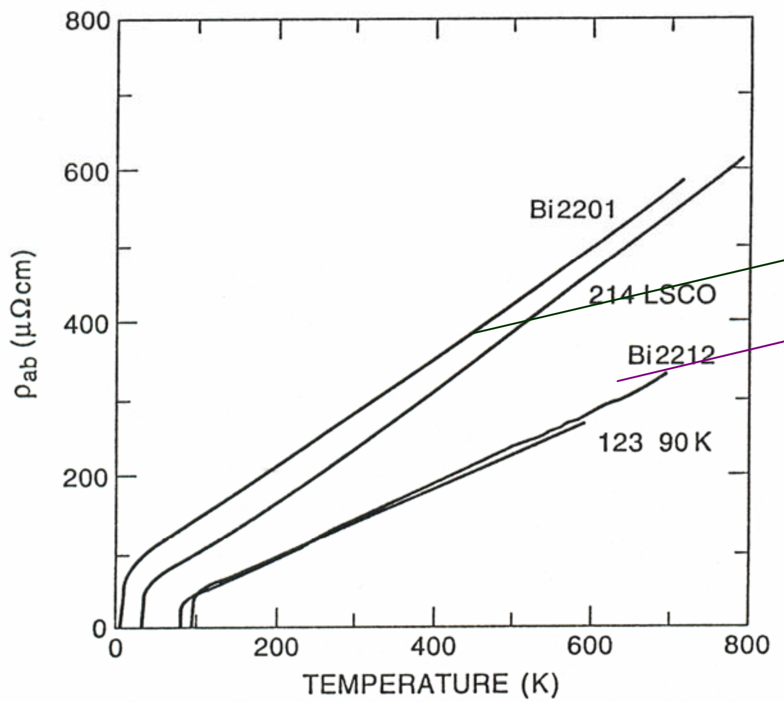
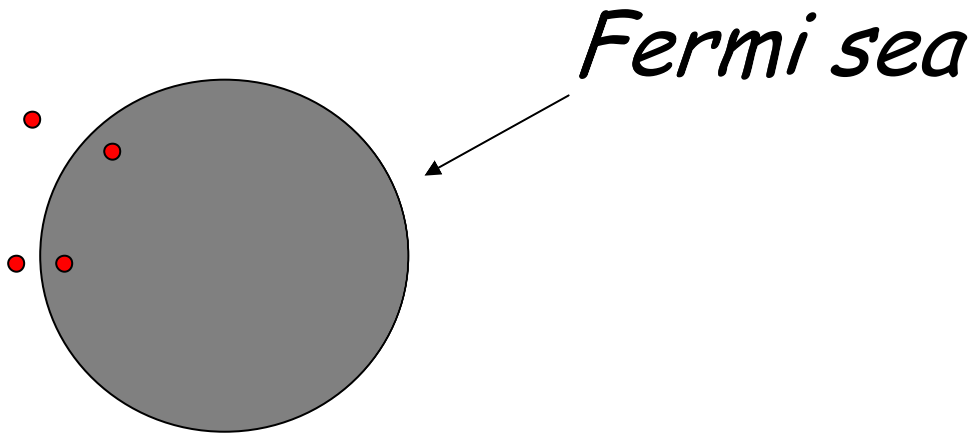


Fig. 1 Temperature dependence of the in-plane resistivity ρ_{ab} measured on crystals of various cuprate superconductors.

Straight line

Metals: $\rho \approx T^2$



Two degrees of freedom

$$\frac{\hbar}{\tau} \approx \frac{\epsilon^2}{\epsilon_F} \propto \frac{T^2}{\epsilon_F}$$

T-linear Resistivity

$$\frac{\hbar}{\tau} \equiv \# k_B T$$

Planckian limit of dissipation
(Zaanen)

Why T-linear Resistivity?

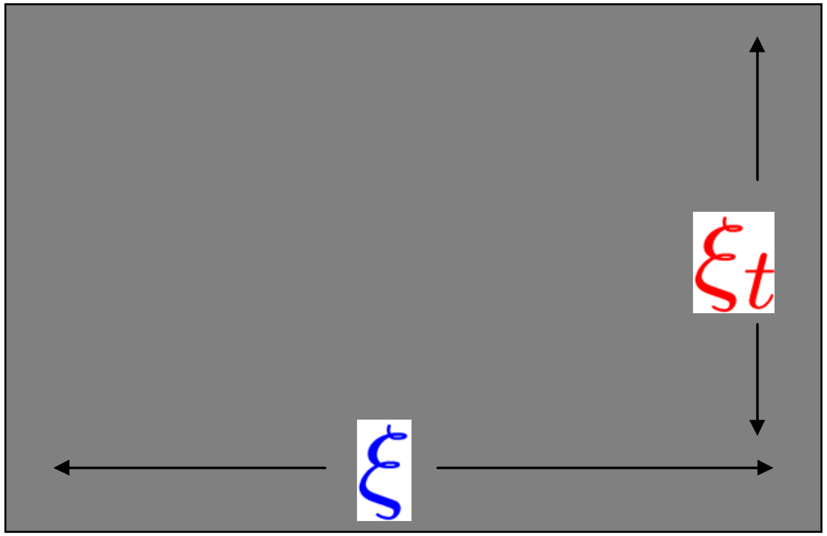
Leading explanation

Quantum criticality

Varma, Anderson, Abrahams, others

Quantum criticality

- Zero-temperature (quantum fluctuations)

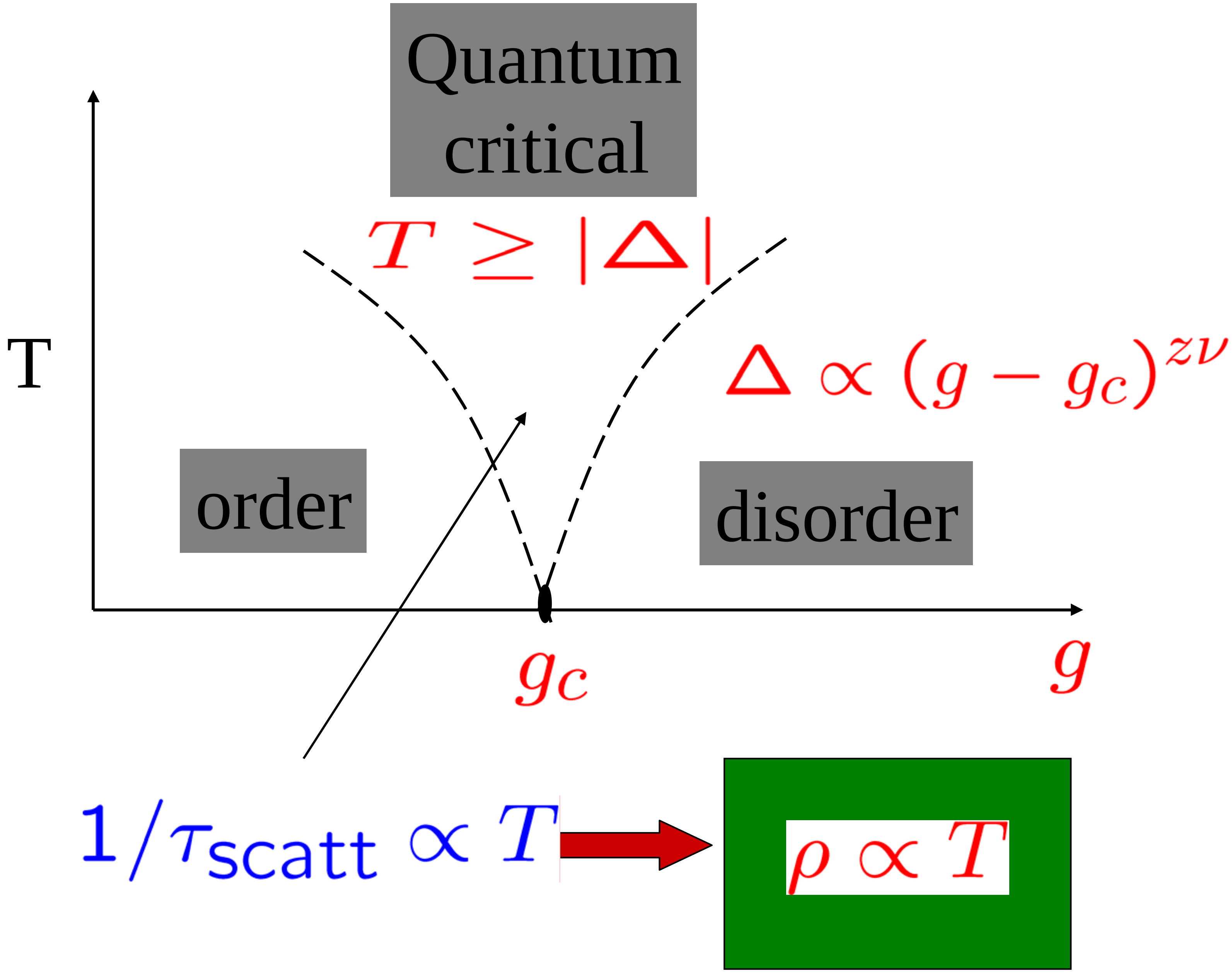


Temporal Correlations Diverge

$\xi_t \propto \xi^z$

dynamical exponent

Spatial correlations diverge

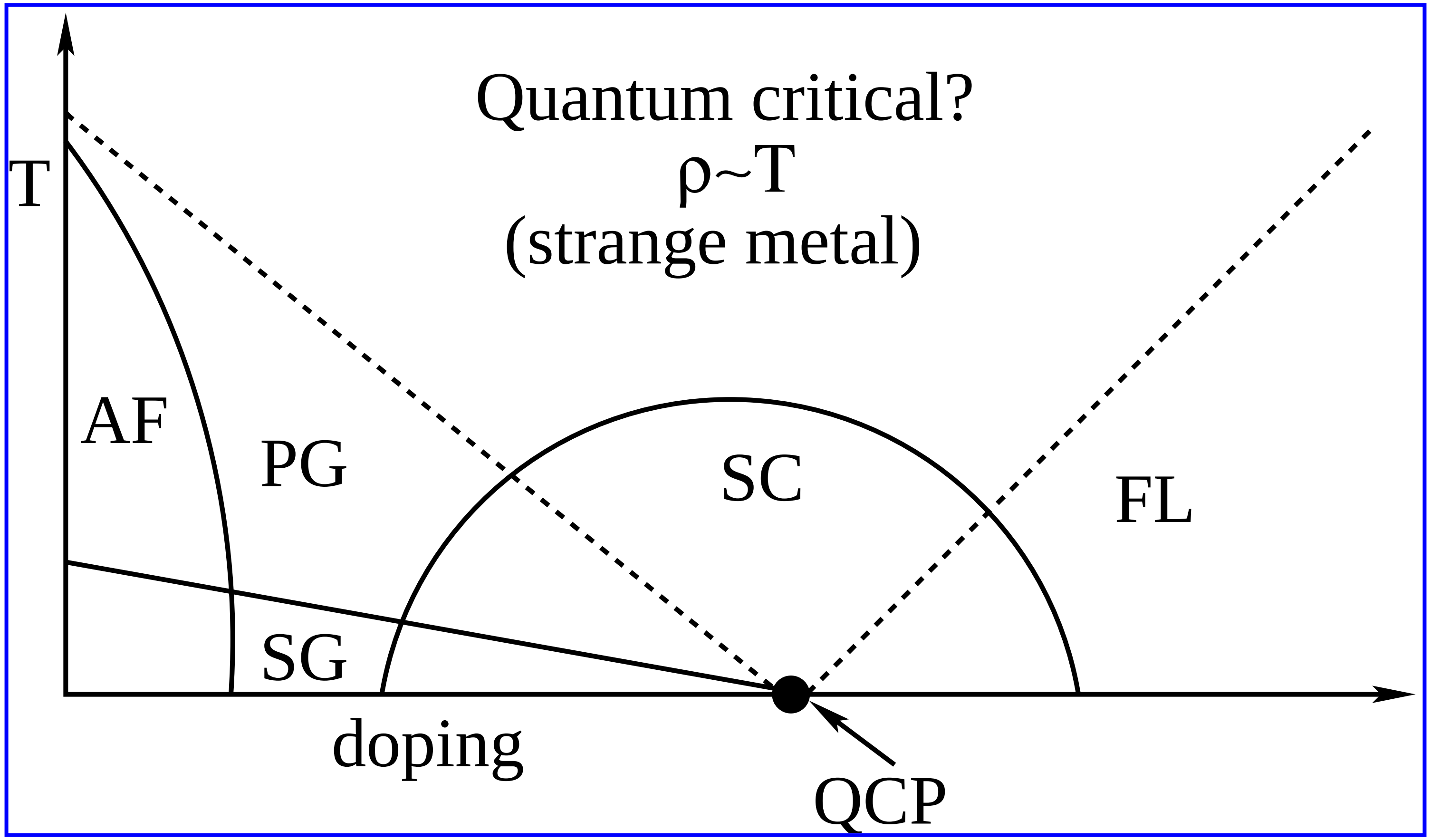

Quantum critical
$T \geq |\Delta|$
$\Delta \propto (g - g_c)^{z\nu}$
T
order
disorder
g_c
g
$1/\tau_{\mathrm{scatt}} \propto T$
$\rho \propto T$

Quantum Critical modes

Carry the current

$$\rho \propto T$$

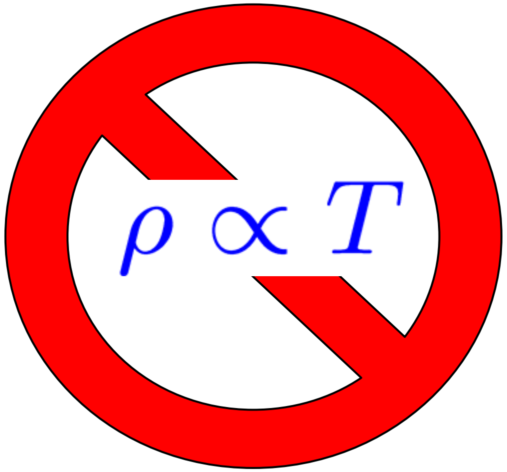
Phase diagram of High-T_c materials



Is this correct?

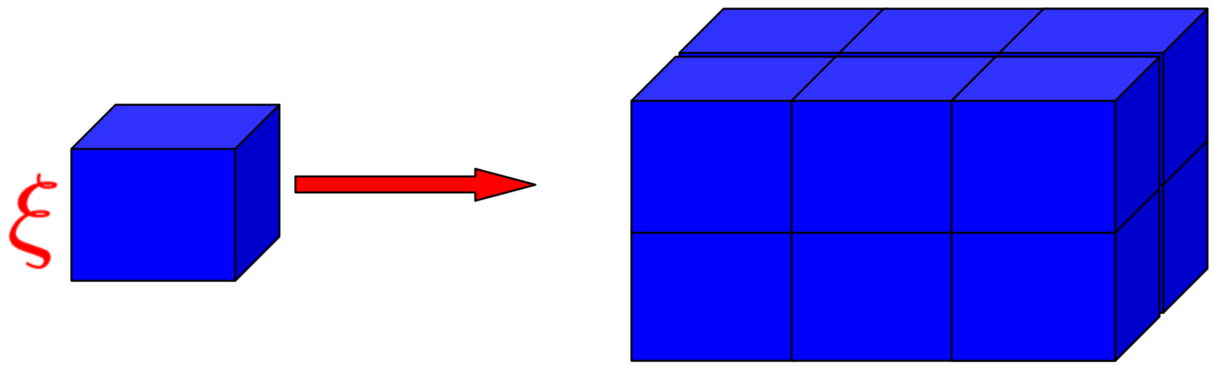
General Result

- One critical length scale
- Charges are critical
- Charges are neither created nor annihilated (charge conservation)



$$\rho \propto T$$

Scaling hypothesis



$L \rightarrow L/\xi$

$\beta \rightarrow \beta/\xi_\tau = \beta/\xi^z$

$$S \to S_{\mathsf{whatever}} + \int d\tau d^d x A^\mu j_\mu$$

Vector potential

current

scaling

$$\ln Z = \frac{L^d \beta}{\xi^d \xi_t} F(\delta \xi^{d_\delta}, \{A^i_\lambda \xi^{d_A}\})$$

$$A^i_\lambda = A^i(\omega = \lambda \xi_t^{-1})$$

conductivity

$$\sigma_{ij} = \frac{1}{L^d \beta} \frac{1}{\omega} \frac{\delta^2 \ln Z}{\delta A^i(-\omega) \delta A^j(\omega)}$$

$$\sigma_{ij} = \frac{Q^2}{\hbar} \xi^{2d_A - d} \Sigma_{ij}(\omega \xi_t)$$

$$\sigma_{ij} = \frac{Q^2}{\hbar}\xi^{2d_A-d}\Sigma_{ij}(\omega\xi_t)$$

Charge conservation

$$d_A \equiv 1$$

$$\xi \propto \xi_t^{1/z} \propto T^{-1/z}$$

$$\sigma(\omega, T) = \frac{Q^2}{\hbar}T^{(d-2)/z}\Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$

General result

$$\sigma(\omega, T) = \frac{Q^2}{\hbar} T^{(d-2)/z} f\left(\frac{\hbar\omega}{k_B T}\right)$$

↓

T-linear resistivity

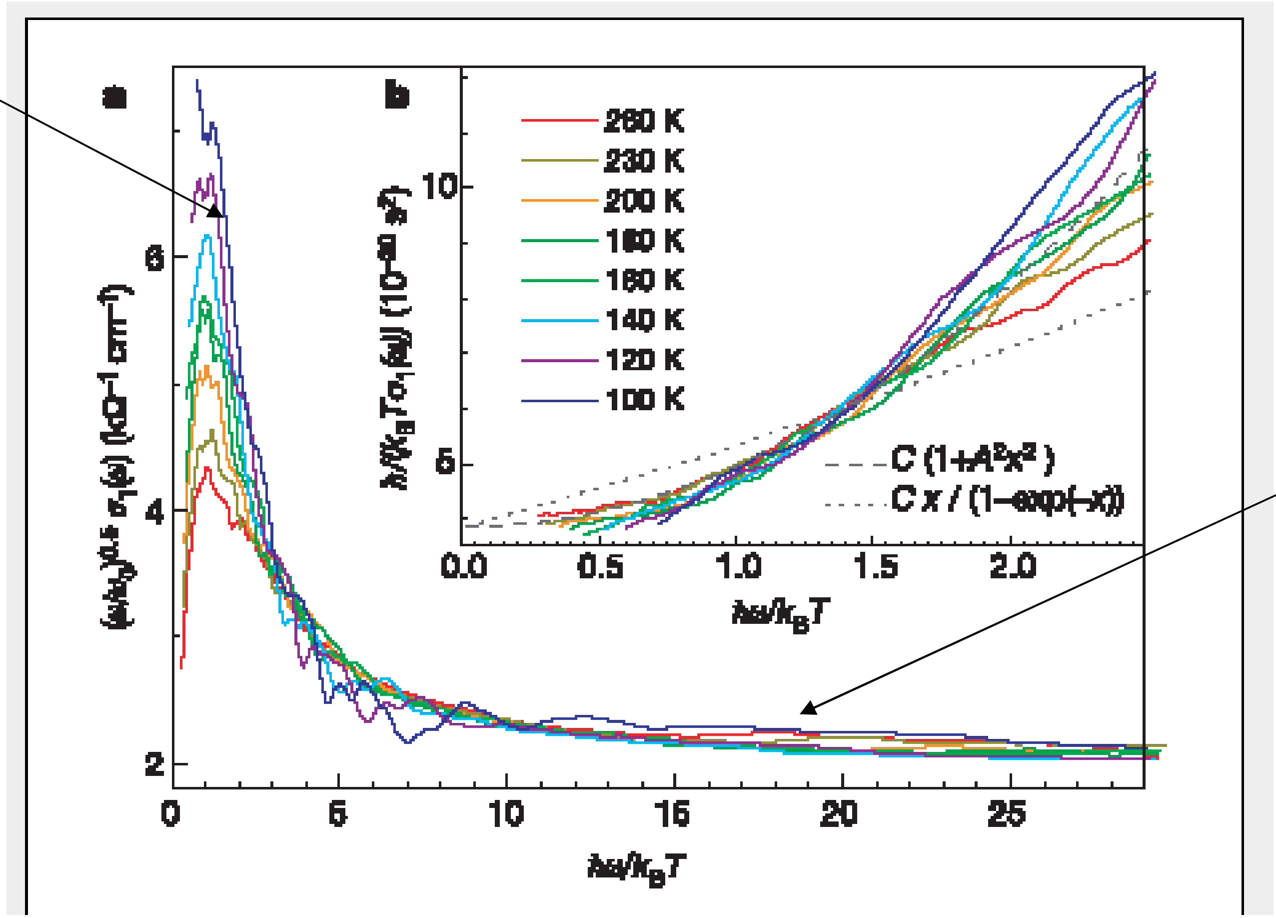
(d=3 for cuprates)

$z = -1!$ ⟶ Impossible?

PP, CC, PRL, vol. 95, 107002 (2005)

$$\sigma \neq T^{-z} f(\omega/T)$$

$z = 1.0$



$z = 0.5$

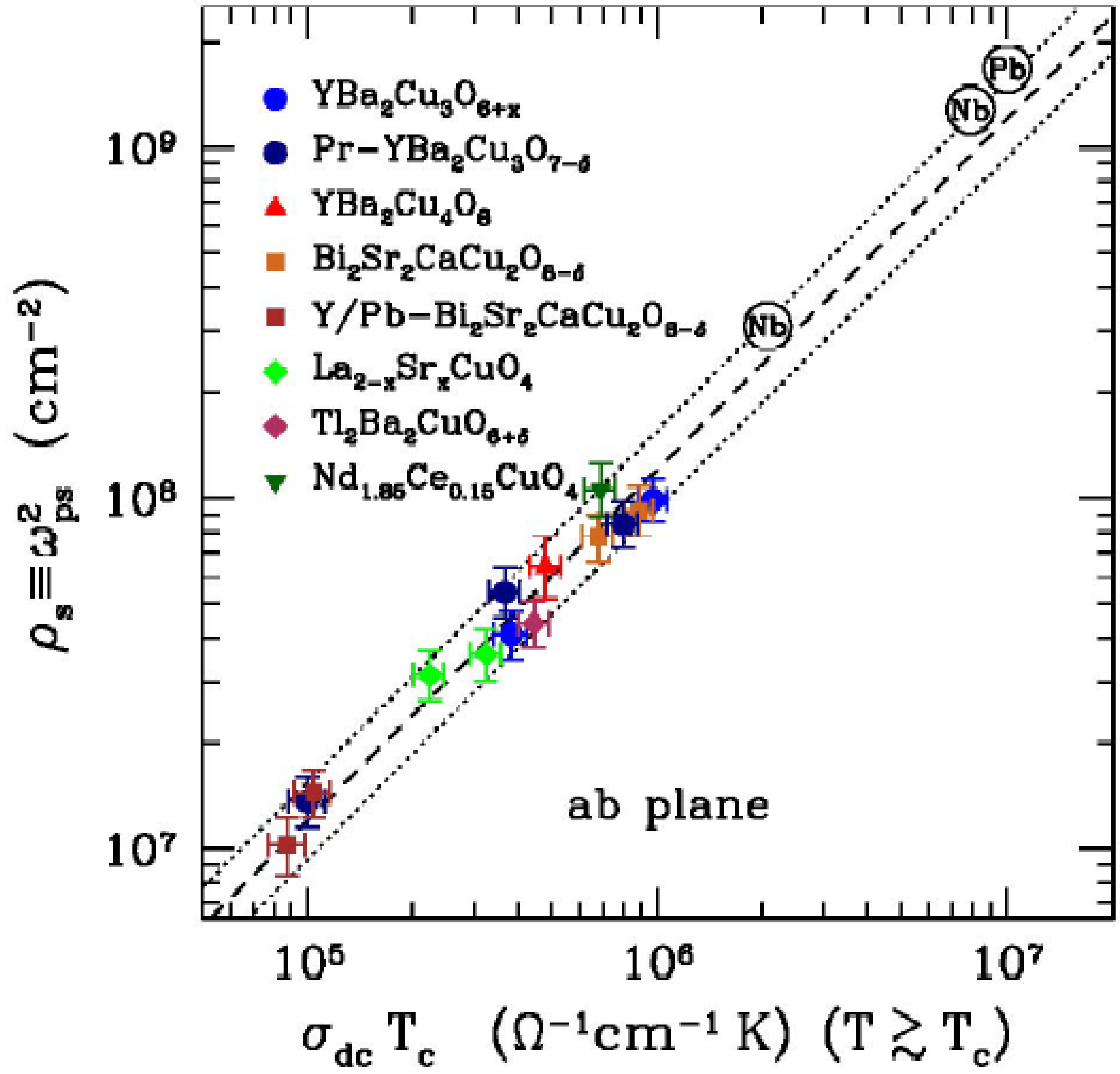
Van der Marel, et. al. Nature 2004

Drude Conductivity

$$\sigma(\omega, T) = \frac{1}{4\pi} \frac{\omega_{pl}^2 \tau_{tr}}{1 + \omega^2 \tau_{tr}^2}$$

new
energy
scale

Not of scaling form:
Except for z=-1



YBa2Cu3O6+x
Pr−YBa2Cu3O7−δ
YBa2Cu4O8
Bi2Sr2CaCu2O8−δ
Y/Pb−Bi2Sr2CaCu2O8−δ
La2−xSrxCuO4
Tl2Ba2CuO6+δ
Nd1.85Ce0.15CuO4
Pb
Nb
Nb
ab plane
10^9
10^8
10^7
10^5
10^6
10^7
ρs ≡ ω²ps (cm⁻²)
σdc Tc (Ω⁻¹cm⁻¹ K) (T ≳ Tc)

What does Homes' Law Mean?

$\rho_s = A\sigma_{dc}(T_c)T_c$

Express in units of 1/time^2 (Zaanen)

$\Omega^2_{p,s} = 4\pi\, n_s\, e^2/m_e$

$\Omega^2_{p,N}\tau(T\_c)/8\pi^2$

(Drude)

$2\pi\, k_B\, T_c/\hbar$

$\tau(T_c) = \hbar/(2\pi\, k_B T_c)$

Planckian Dissipation (QC)

Is this true?

$\rho_s = A\sigma(T_c)T_c$

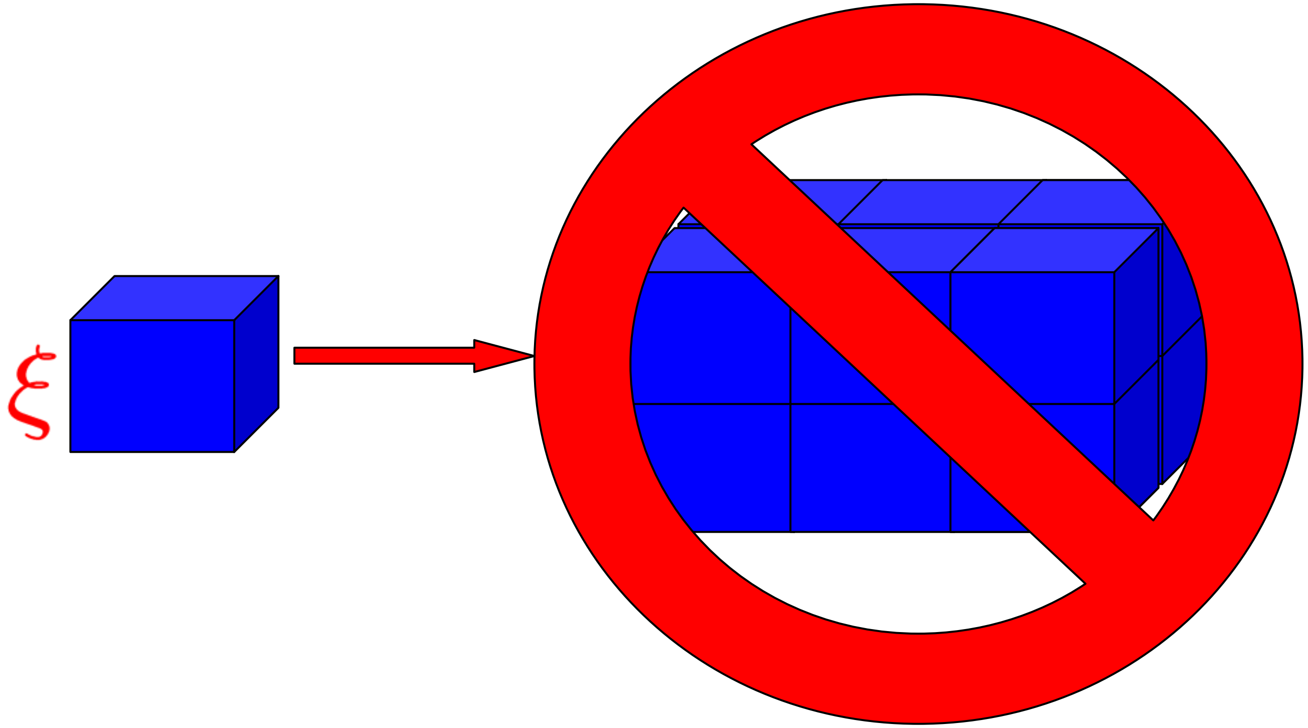
$\downarrow$ $T^{(d-2)/z} f(0)$ (QC)

ρ_s / T^{α}_c

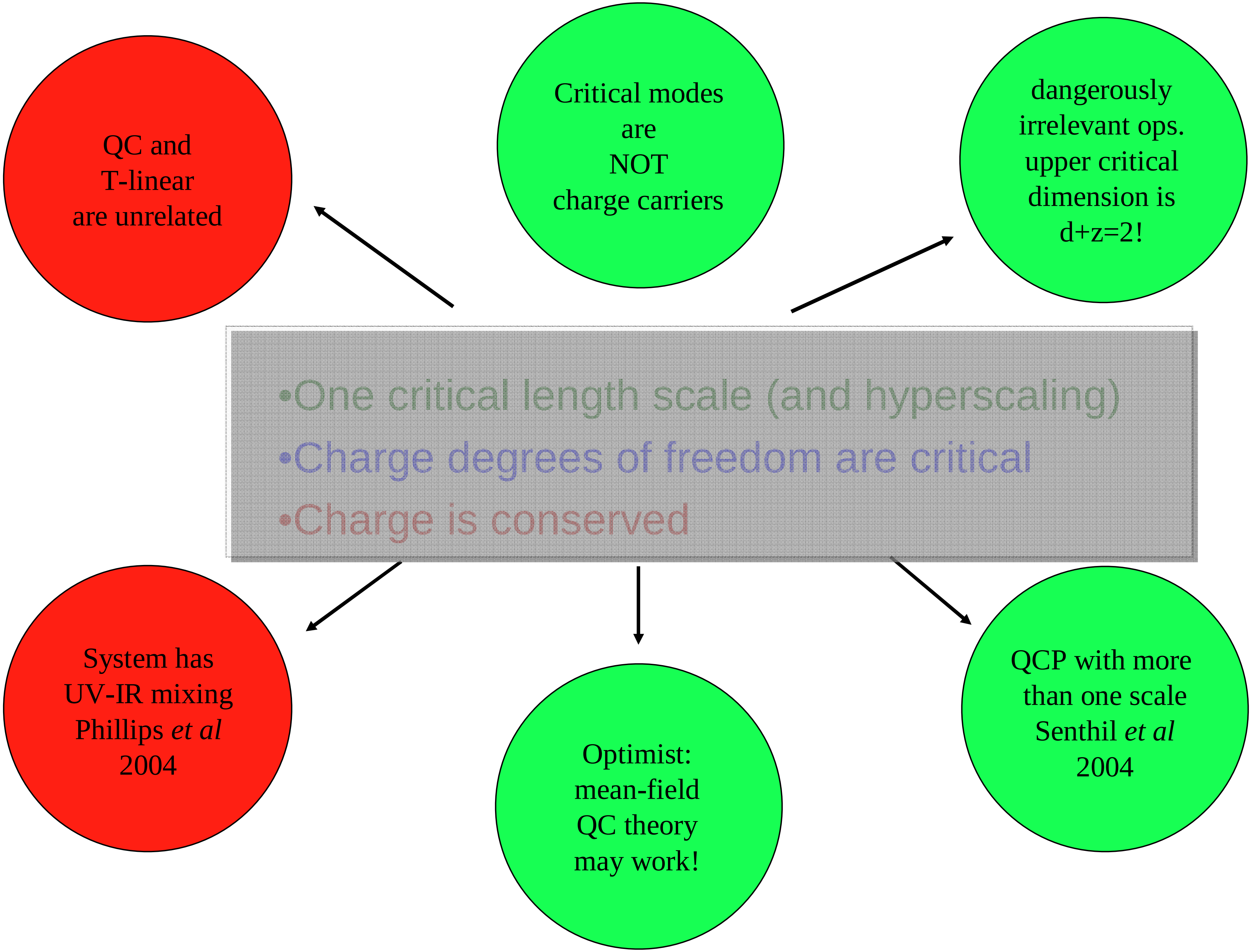
$\uparrow$

Uemura Relationship $\neq$ Homes Law

Scaling hypothesis



One-parameter Scaling breaks down


•One critical length scale (and hyperscaling)
•Charge degrees of freedom are critical
•Charge is conserved
QC and T-linear are unrelated
Critical modes are NOT charge carriers
dangerously irrelevant ops. upper critical dimension is d+z=2!
System has UV-IR mixing Phillips *et al* 2004
Optimist: mean-field QC theory may work!
QCP with more than one scale Senthil *et al* 2004

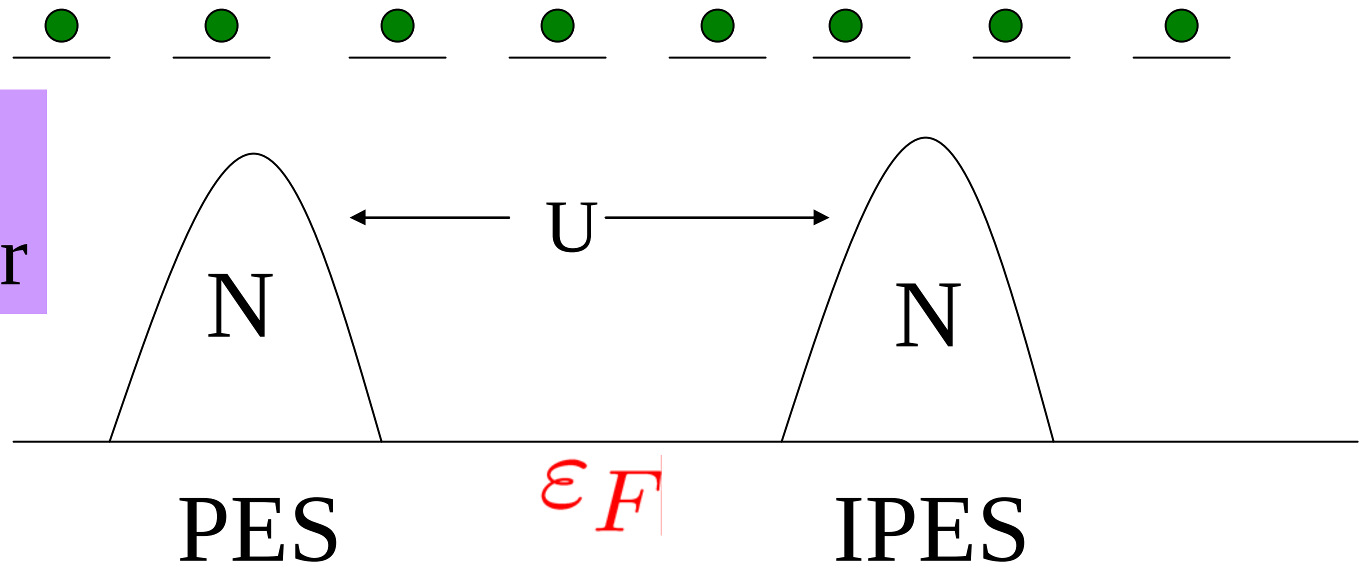
New direction

(UV-IR mixing)

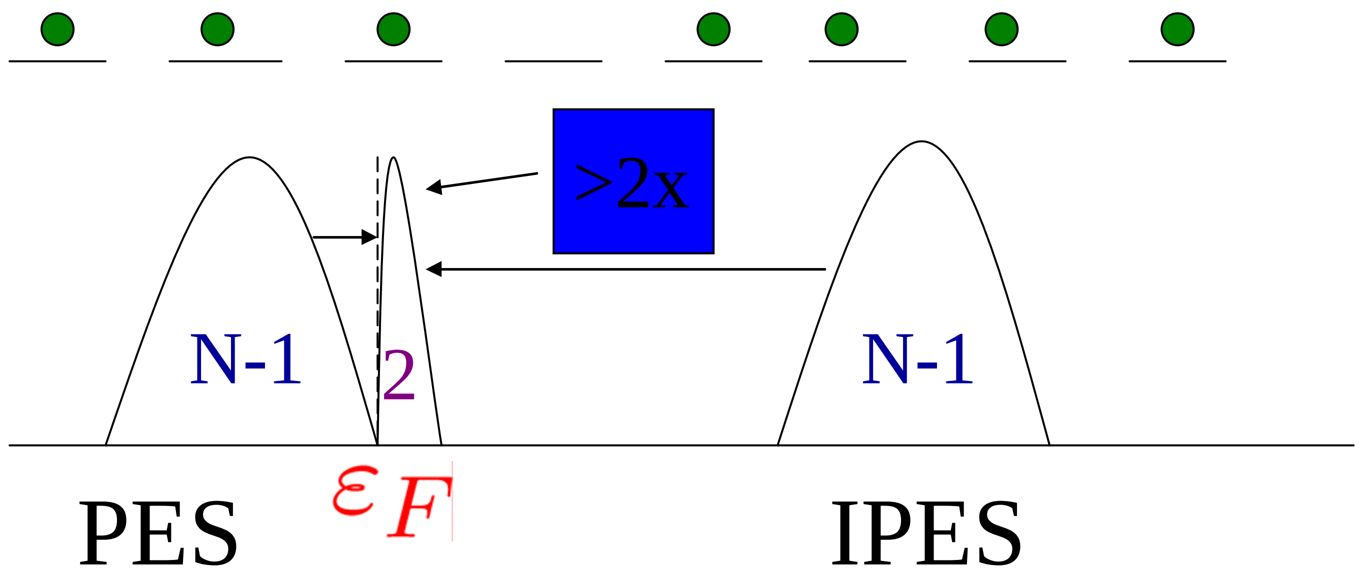
↓ all energy scales are mixed

Asymptotic slavery

Mott insulator



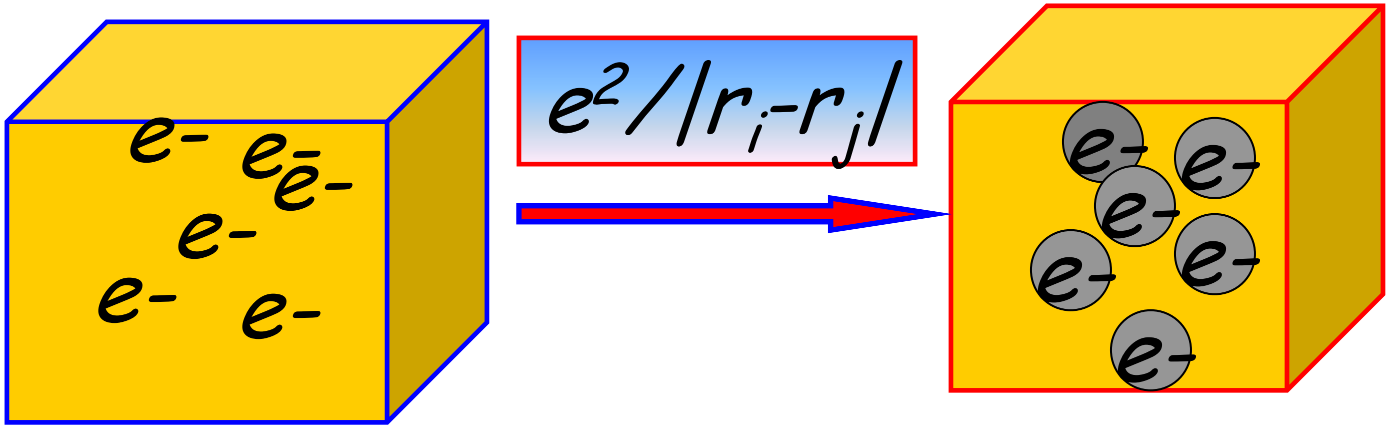
UV-IR Mixing



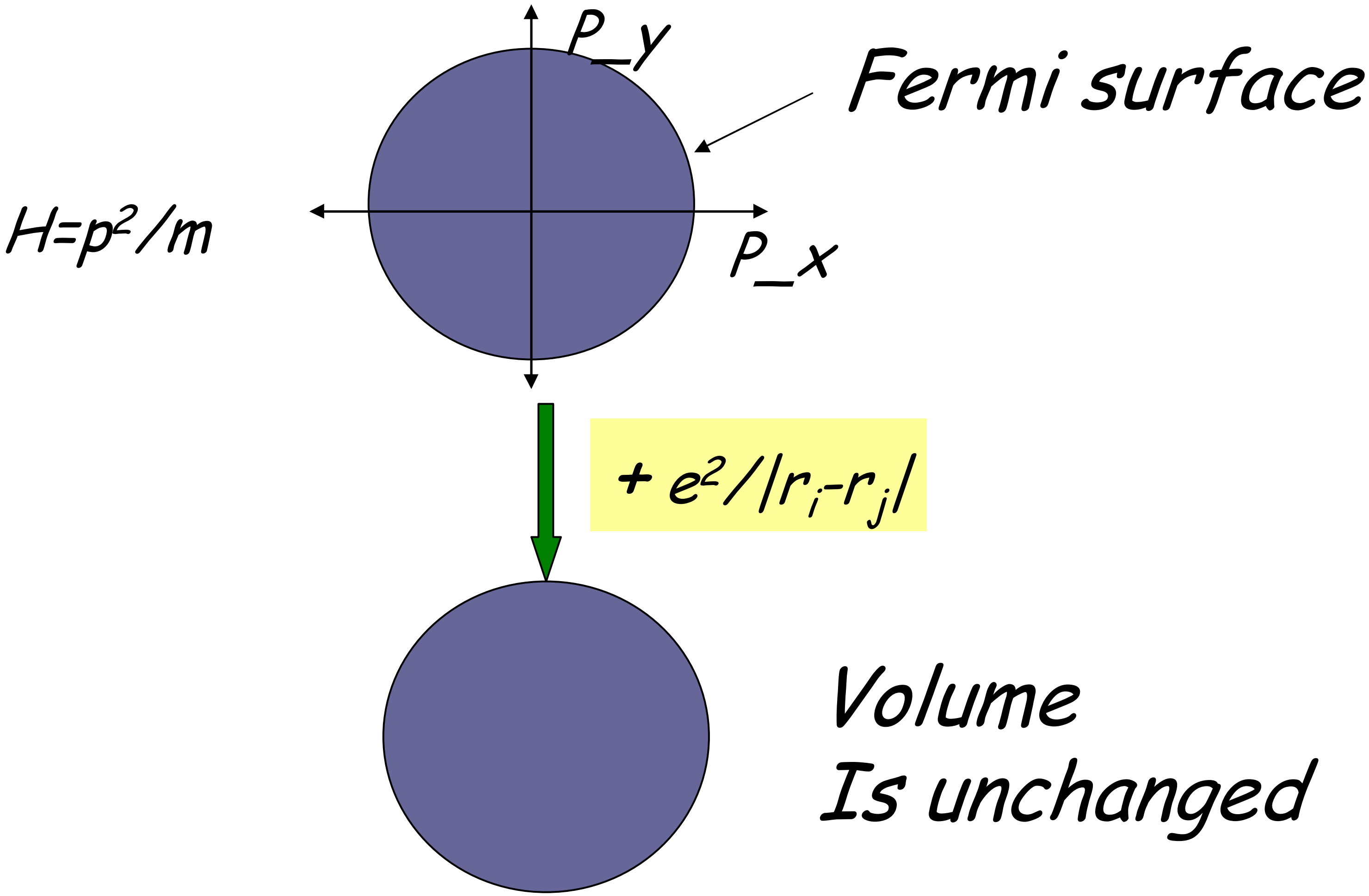
Sawatzky, et. al.

What is lost when
high energy scale
is removed?


Fermi
Liquid?


e-
$e^2/|r_i-r_j|$

Luttinger's theorem



Luttinger's Theorem

$$\frac{N}{V} = 2\int_{G(0,p)>0} \frac{d^d p}{(2\pi)^d}$$

$Im\Sigma=0$

Poles:
Quasiparticles
Fermi surface

Zeros: Gaps
Luttinger surface
(Dzyaloshinskii)

Fermi liquid:
$G(\varepsilon,p)=Z/(\varepsilon-\varepsilon(p))$

(Mott Insulator):
$G(\varepsilon,p)\sim(\varepsilon-\varepsilon(p))$

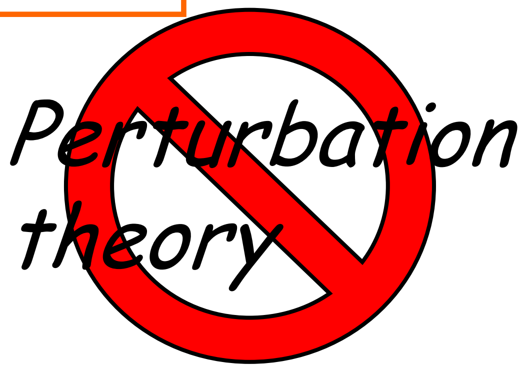
Closer look at zeros

$$G = \frac{1}{\omega - \epsilon(p) - \Sigma(p,\omega)}$$

$$\Sigma = G^{-1} - G_0^{-1}$$

If G=0

$\Sigma = 1$



Is the surface of
Zeros the Mott analogue
Of the Fermi surface?

Does its volume=
particle density?

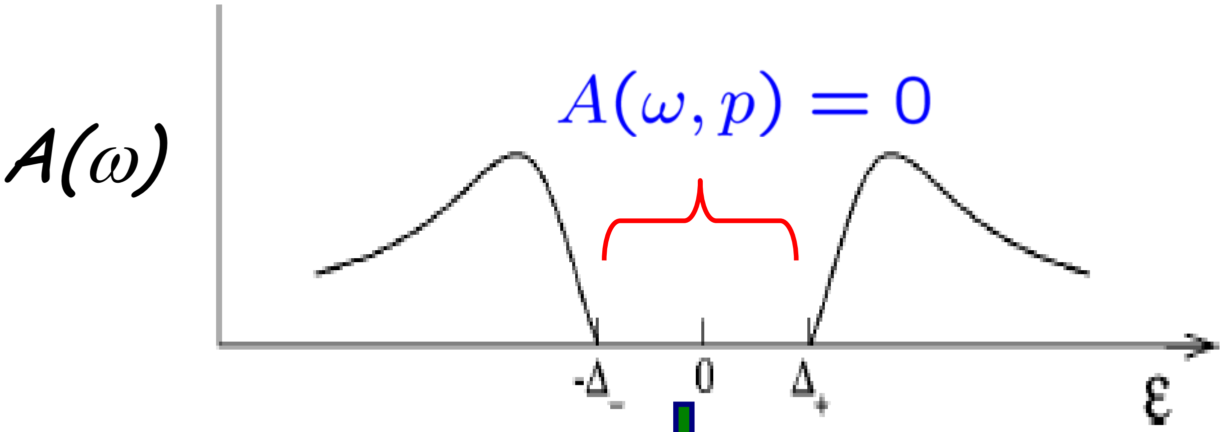
$$-\frac{1}{\pi} ImG$$

$$G(\omega, p) = \int_{-\infty}^{\infty} \frac{A(\omega', p)\, d\omega'}{\omega - \omega' + i\eta}$$

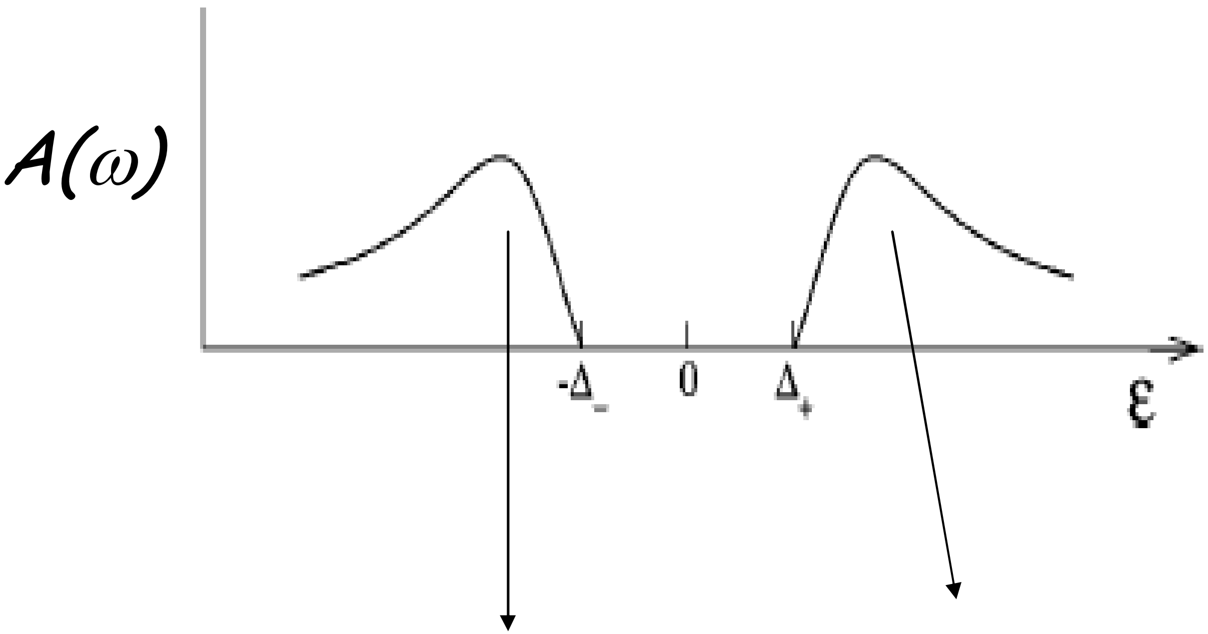
Lehman

Mott gap

$$A(\omega, p) = 0$$



$$ReG(0, p) = \text{below gap} + \text{above gap}$$

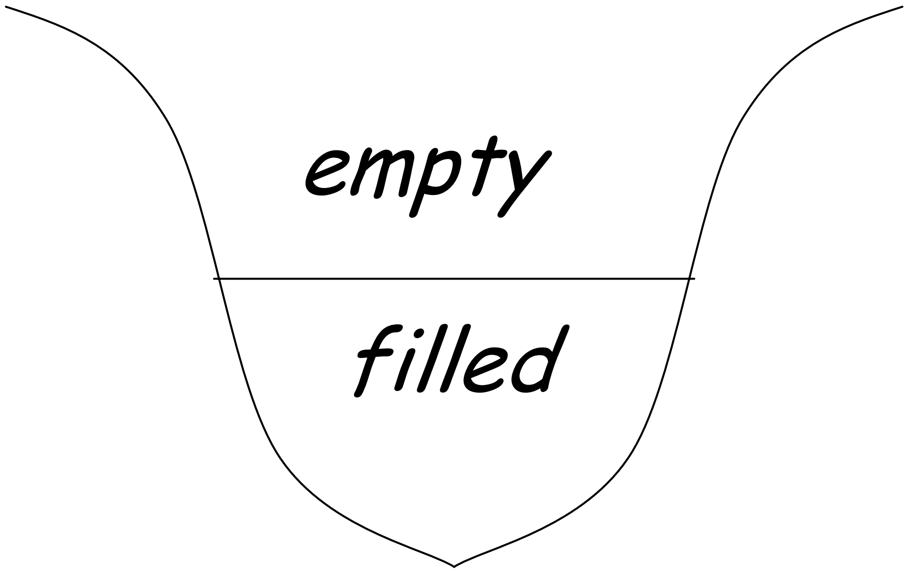


$$ReG(0,p) = \int_{-\infty}^{-\Delta_-} \frac{A(\omega',p)d\omega'}{-\omega'} + \int_{\Delta_+}^{\infty} \frac{A(\omega',p)d\omega'}{-\omega'}$$

Zero condition: Re G(0,p)=0

A(ω,p) is an even function of ω

Symmetry of half-filling



$$n=1-n=1/2$$

Particle-hole symmetry

$$c_{i\sigma} \to e^{iQ\cdot R(i)}\, c_{i\sigma}^{\dagger}$$

$$A(\omega,p)=A(-\omega,-p-Q+2n\pi)$$

$$P= Q/2+n\pi$$

Interchange x,y

$$p_x=-p_y-q+2n\pi$$

$$p_x=p_y-q+2n\pi$$

$p_x = -p_y - q + 2n\pi$
$p_x = p_y - q + 2n\pi$

$Q = (\pi, \pi)$

$$p_x \pm p_y = -\pi + 2n\pi$$

Fermi surface
Of non-interacting
Electrons on a Square lattice

Generalisation

Asymmetric
Band structure

$$M_n^{\sigma}(k) \equiv \int \frac{d\omega}{2\pi} \omega^n d\omega G_{\sigma}^{\text{ret}}(k,\omega)$$

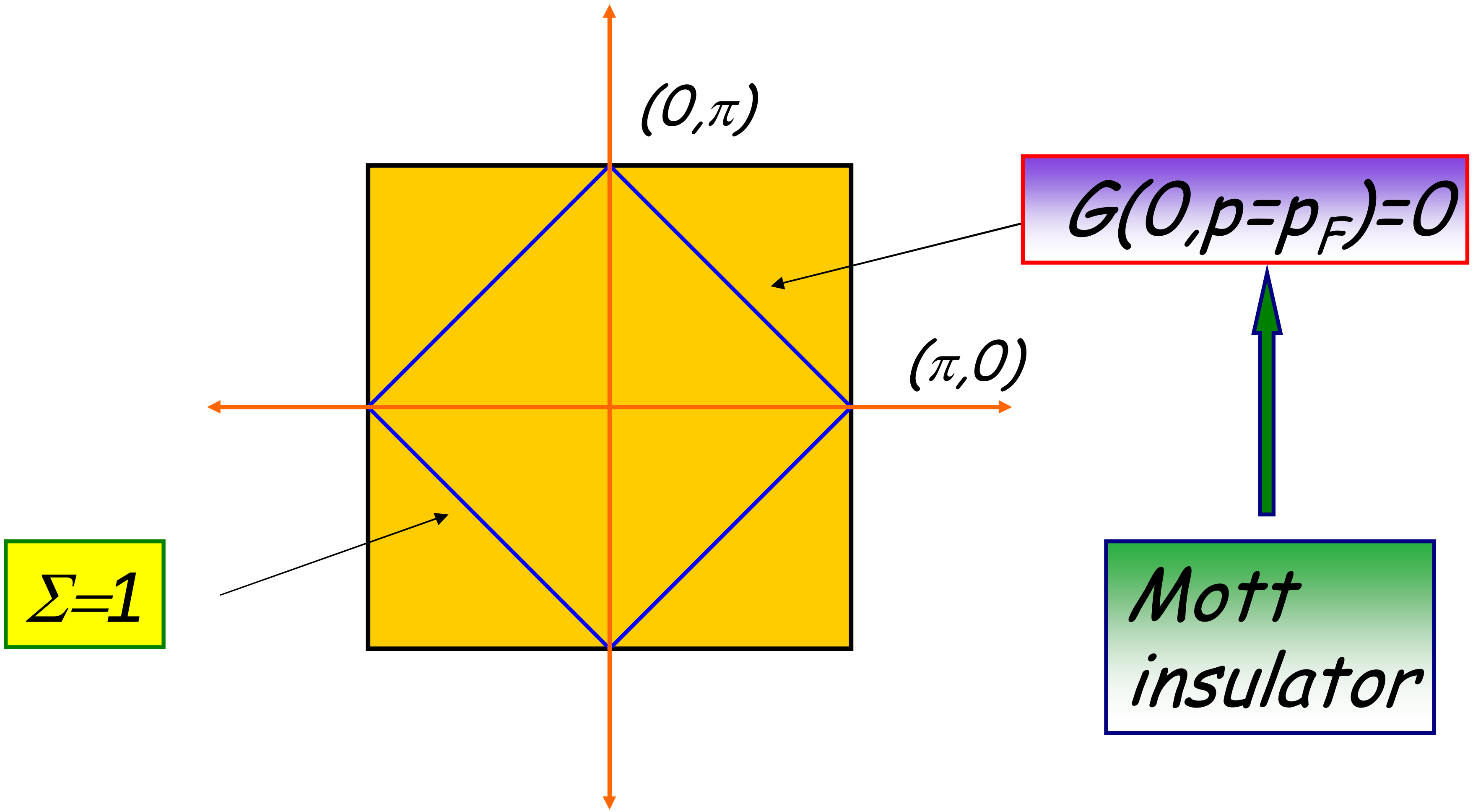
Only even moments
Survive when
$\varepsilon(k)=0$

Particle-hole symmetry

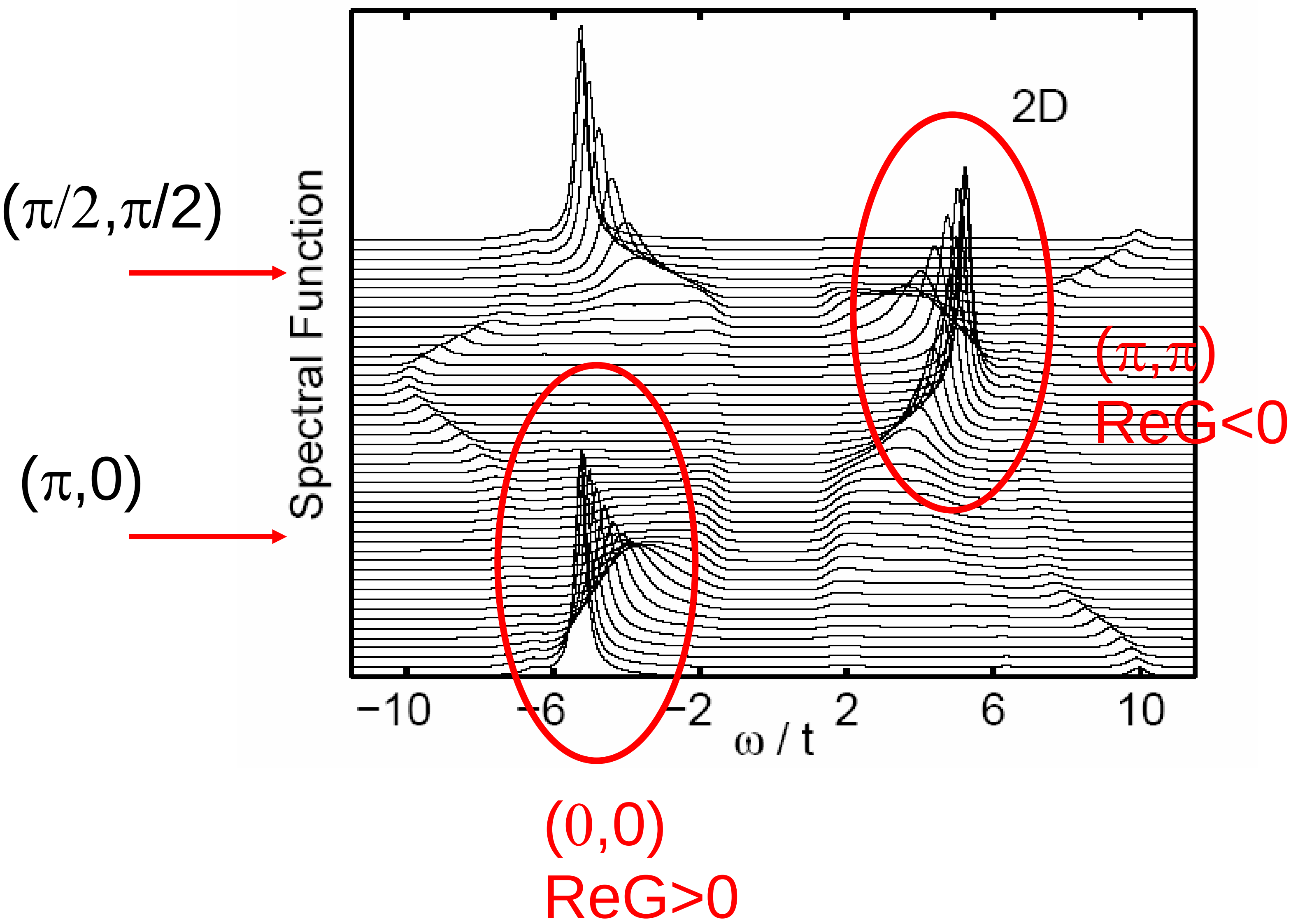
Mott insulator

$G(0, p=p_F)=0$

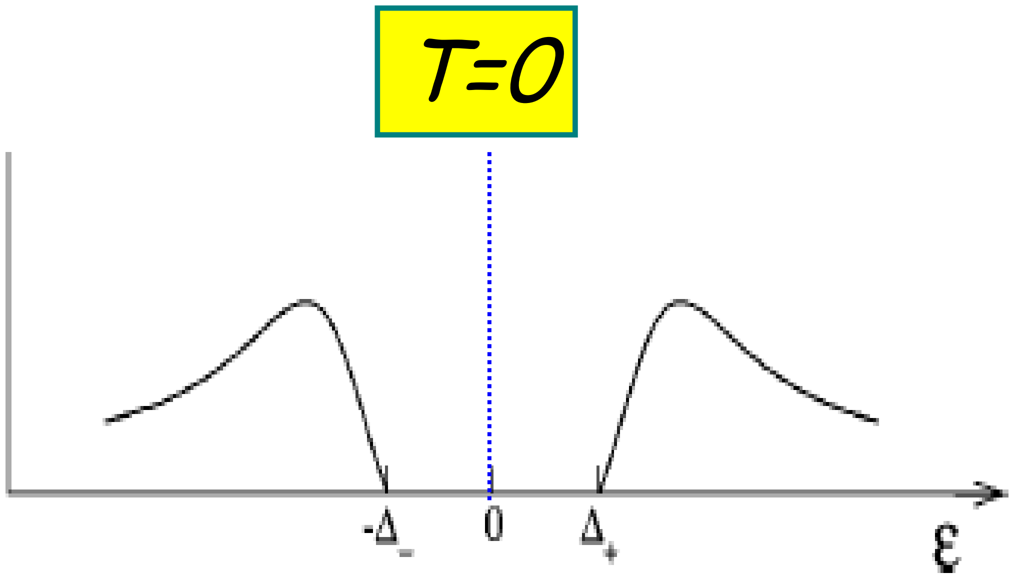
Surface of zeros



Spectral function



Robustness of zeros



What happens to the zeros?

$$ReG(0,p) = \int_{-\infty}^{-\Delta_-} \frac{A(\omega',p)d\omega'}{-\omega'} + \int_{\Delta_+}^{\infty} \frac{A(\omega',p)d\omega'}{-\omega' - E}$$

If E!1

Zeros vanish

Mott Insulator

Zeros along a momentum surface for some range of energies within the gap

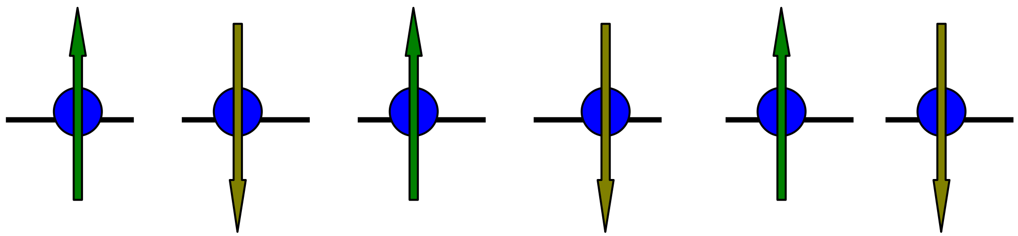
Mottness

Mott insulators

Strong electron correlations

Not considered by Mott

Ordering

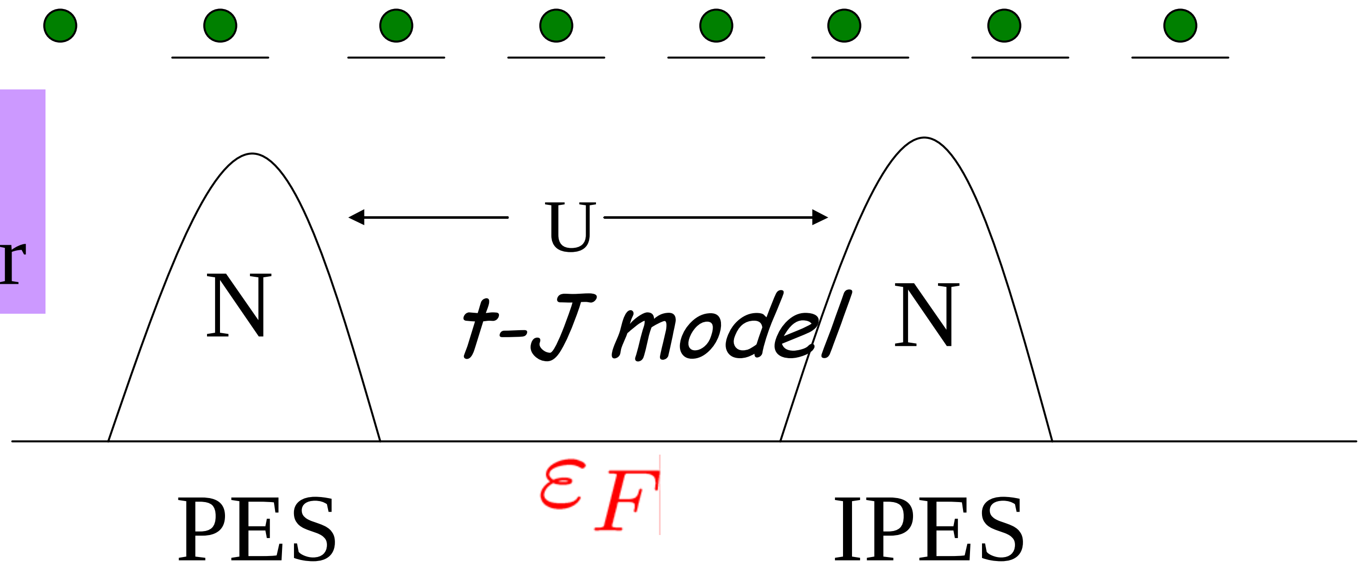


Anti-
ferromagnet

Mottness=Mott insulator-ordering

In what low-energy reductions, do zeros survive

Mott insulator



What changes?

$$ReG(0,p) = \int_{-\infty}^{-\Delta_-} \frac{A(\omega',p)d\omega'}{-\omega'} + \int_{\Delta_+}^{\infty} \frac{A(\omega',p)d\omega'}{-\omega'}$$

Ze | No Zeros: $ReG(0,p) \neq 0$ | p)=0

Zeros are an
Indication that
The UV and IR physics
Cannot be disentangled.

low-energy reductions
Fail (t-J model at n=1)

Volume of zero surface?

$$\frac{N}{V} = \sum_{p,\omega} G(p,\omega) 1$$

trick

$$1 = \frac{\partial}{\partial \omega}(G^{-1} + \Sigma)$$

$$\frac{N}{V} = n_{\mathrm{Lutt}} + \sum_{p,\omega} G \frac{\partial}{\partial \omega}\Sigma$$

I_2

Fermi liquid

Particle-hole symmetry

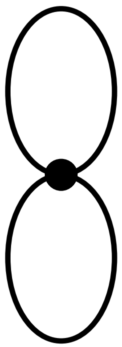
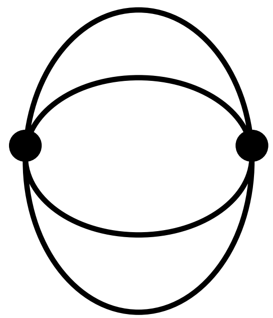
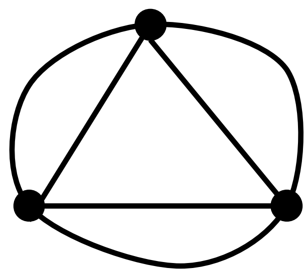
$n=n_{Lutt}$

$n=n_{Lutt}$

Luttinger-Ward Functional, Φ[G]

$$\delta\, \Phi[G] = \sum_k \text{s}\; d\omega\; \Sigma(k,\omega)\; \delta G(k,\omega)$$

Perturbation expansion in U

$\Phi[G] = 2[\,\frac{1}{1}$  $+ \frac{1}{2}$  $+ \frac{1}{3}$  $+ \ldots]$

$$\Rightarrow \frac{\partial \Phi[G]}{\partial \omega} = \sum_k \int d\omega \Sigma(k,\omega) \frac{\partial G(k,\omega)}{\partial \omega} = 0$$

$\Rightarrow$ **Luttinger Theorem**

Hubbard Model (t=0)

$$\Sigma_{loc}(k,\omega) = \frac{(U/2)^2}{\omega + \mu} \neq \Sigma_{\rm pert} = U$$

$$G(\Sigma - U/2)^2 + (\Sigma - U/2) - (U/2)^2 G = 0$$

$$\Sigma[G] = \frac{U}{2} + \frac{-1 \pm \sqrt{1+U^2G^2}}{2G}$$

Functional Integration

Frequency expansion: $$\Phi[G] = \int \frac{d\omega}{2\pi} Y(i\omega)$$

$$\begin{aligned} Y(i\omega) &= \frac{1}{2}\left(UG(i\omega) \pm \sqrt{1+U^2G^2(i\omega)} \mp \log\left[1+\sqrt{1+U^2G^2(i\omega)}\right]\right) \\ &= \frac{1}{2}\left[-\log G(i\omega) \pm \sqrt{1+U^2G^2(i\omega)} \pm \frac{1}{2}\log\left(\frac{\sqrt{1+U^2G^2(i\omega)}-1}{\sqrt{1+U^2G^2(i\omega)}+1}\right)\right] \\ &= Y_{reg}(i\omega) + Y_{sing}(i\omega) \end{aligned}$$

Singular part

$$Y_{\text{sing}}(i\omega) = \frac{1}{2}\log\frac{G_0(i\omega)}{G(i\omega)}$$

$$\Rightarrow \frac{\partial \Phi(i\omega)}{\partial \omega} = \int \frac{d\omega}{2\pi}\frac{\partial Y_{\text{sing}}(i\omega)}{\partial \omega} \neq 0$$

- Singularity in self-energy
- Perturbation breaks down
- Self-energy can not be obtained pertubatively!!

$$\Sigma_{loc}(k,\omega) = \frac{(U/2)^2}{\omega + \mu}$$

Diverges!!

$$I_2 = -Tr\sum_k \int \frac{d\omega}{2\pi} G(k,\omega)\frac{\partial}{\partial i\omega}\Sigma(k,i\omega)$$

$$= \Theta(\mu - U/2) + \Theta(\mu + U/2) - 2\Theta(\mu) \neq 0$$

Agrees with Rosch

Luttinger Theorem is not applicable in Mott insulator!!

Small finite hopping, t

- Hubbard II approximation

$$I_2 = \sum_k \left[\Theta(-\mu + \frac{\sqrt{U^2 + \varepsilon^2(k)}}{2}) + \Theta(-\mu - \frac{\sqrt{U^2 + \varepsilon^2(k)}}{2}) - 2\Theta(\mu) \right] \neq 0$$

Finite temperature, T

$$I_2(T > 0) \quad = f(-\mu + \frac{\sqrt{U^2 + \varepsilon^2(k)}}{2}) + f(-\mu - \frac{\sqrt{U^2 + \varepsilon^2(k)}}{2}) - 2 f(-\mu)$$

$$\xrightarrow{T \to 0} I_2(T = 0)$$

Smoothly connected to T>0

- No T=0 Transition: See ACM, EPL 41, 401 (1998).

Dzyaloshinskii (and Rosch) are wrong

Modified Luttinger Theorem

$$n = \int_{G(k,0)>0} \frac{d^2k}{(2\pi)^2} + \int_{G_0(k,0)>0} \frac{d^2k}{(2\pi)^2}$$

If Σ does
Not have a perturbative
Expansion from the non-interacting
limit

↓ Mott insulator

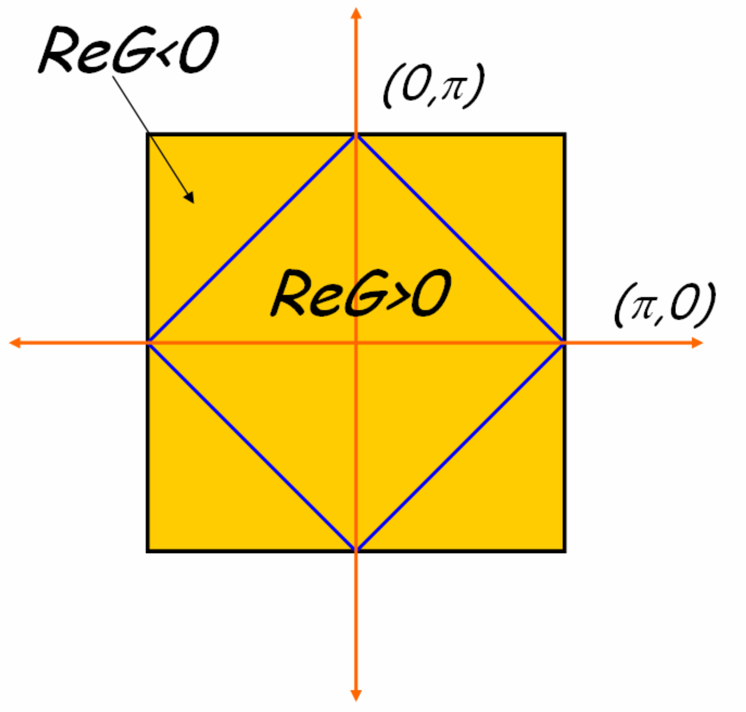
No sum-rule on surface of
zeros (just get over it!)

Have zeros been Seen experimentally?

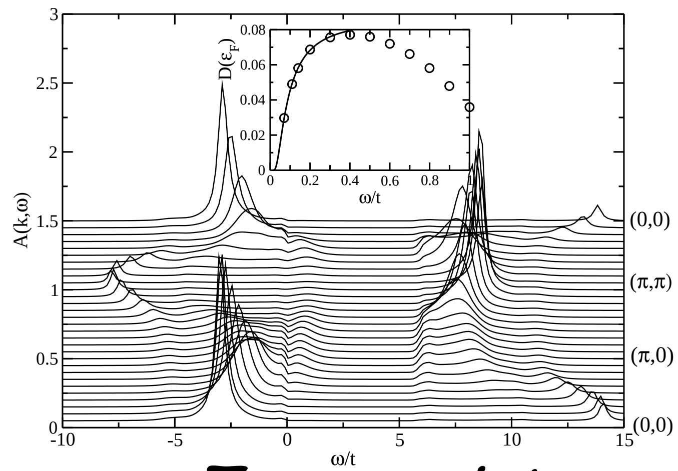
Are they important?

yes

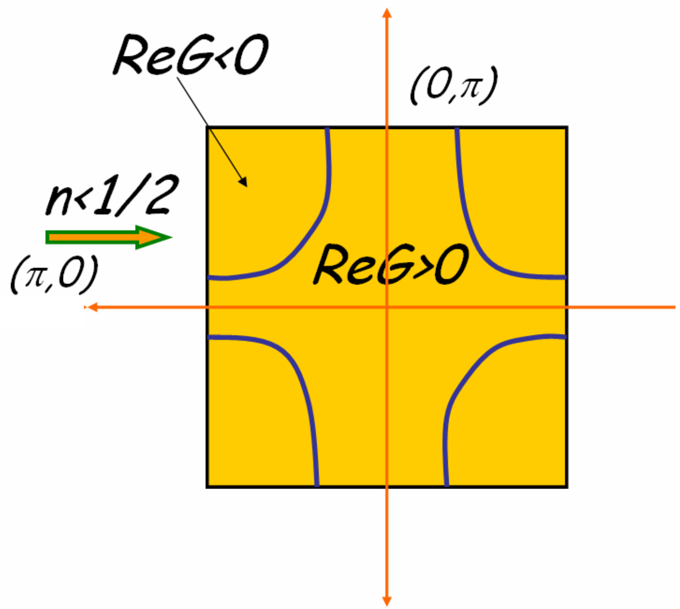
Away from half-filling



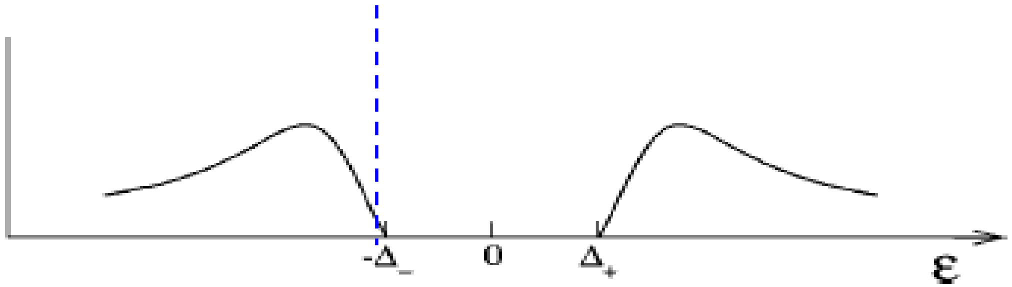
Mott insulator



Zeros exist



Surface changes

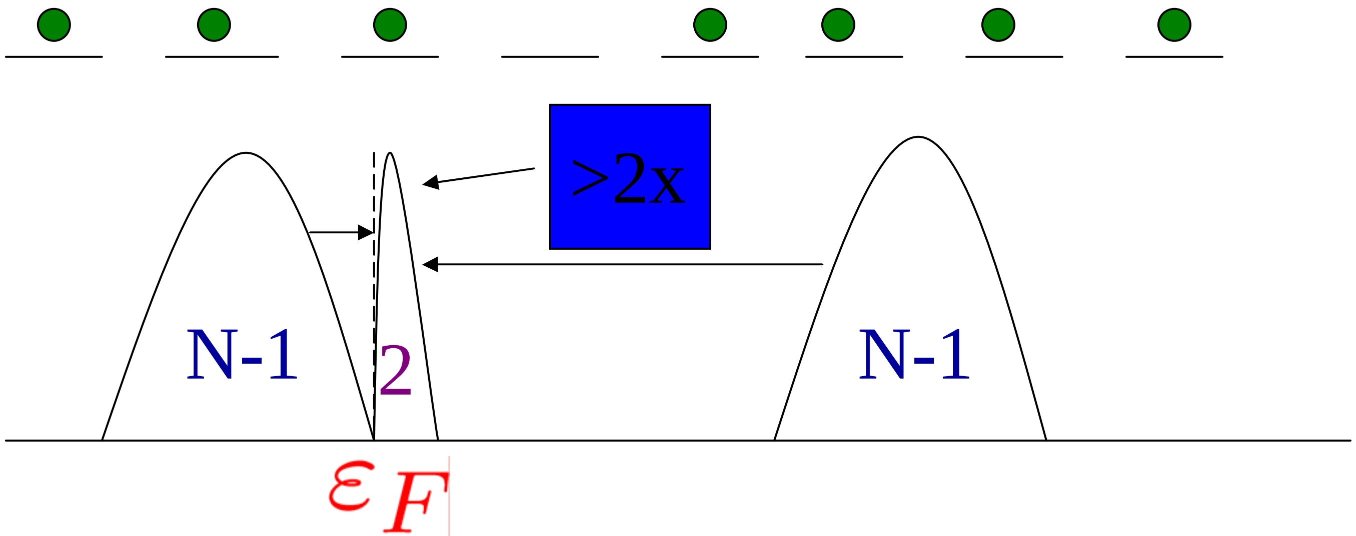


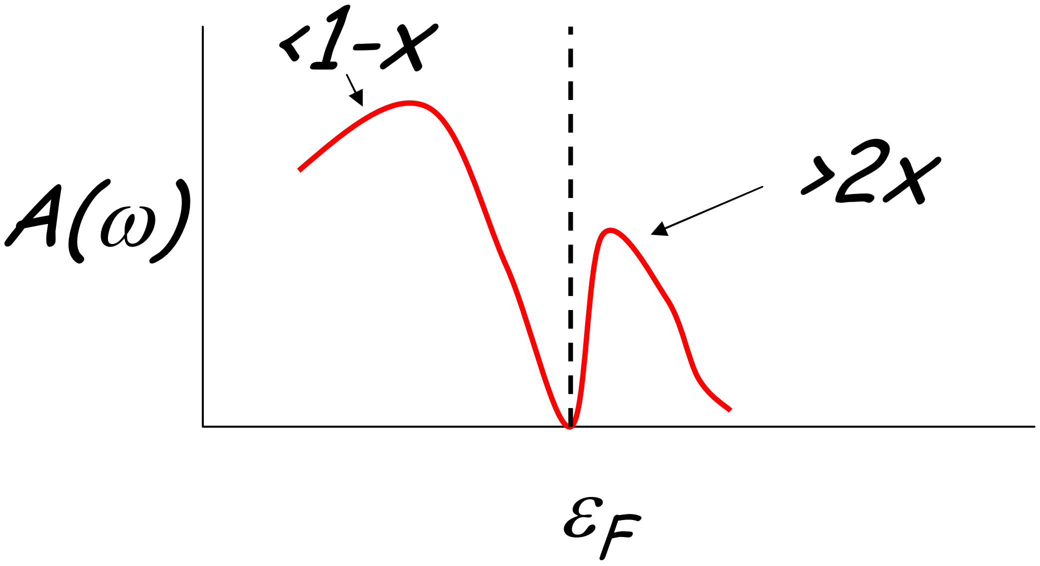
$$ReG(0,p) = \int_{-\infty}^{-\Delta_-} \frac{A(\omega',p)d\omega'}{-\omega'} + \int_{\Delta_+}^{\infty} \frac{A(\omega',p)d\omega'}{-\omega'}$$

If ReG=0

Spectral weight transfer

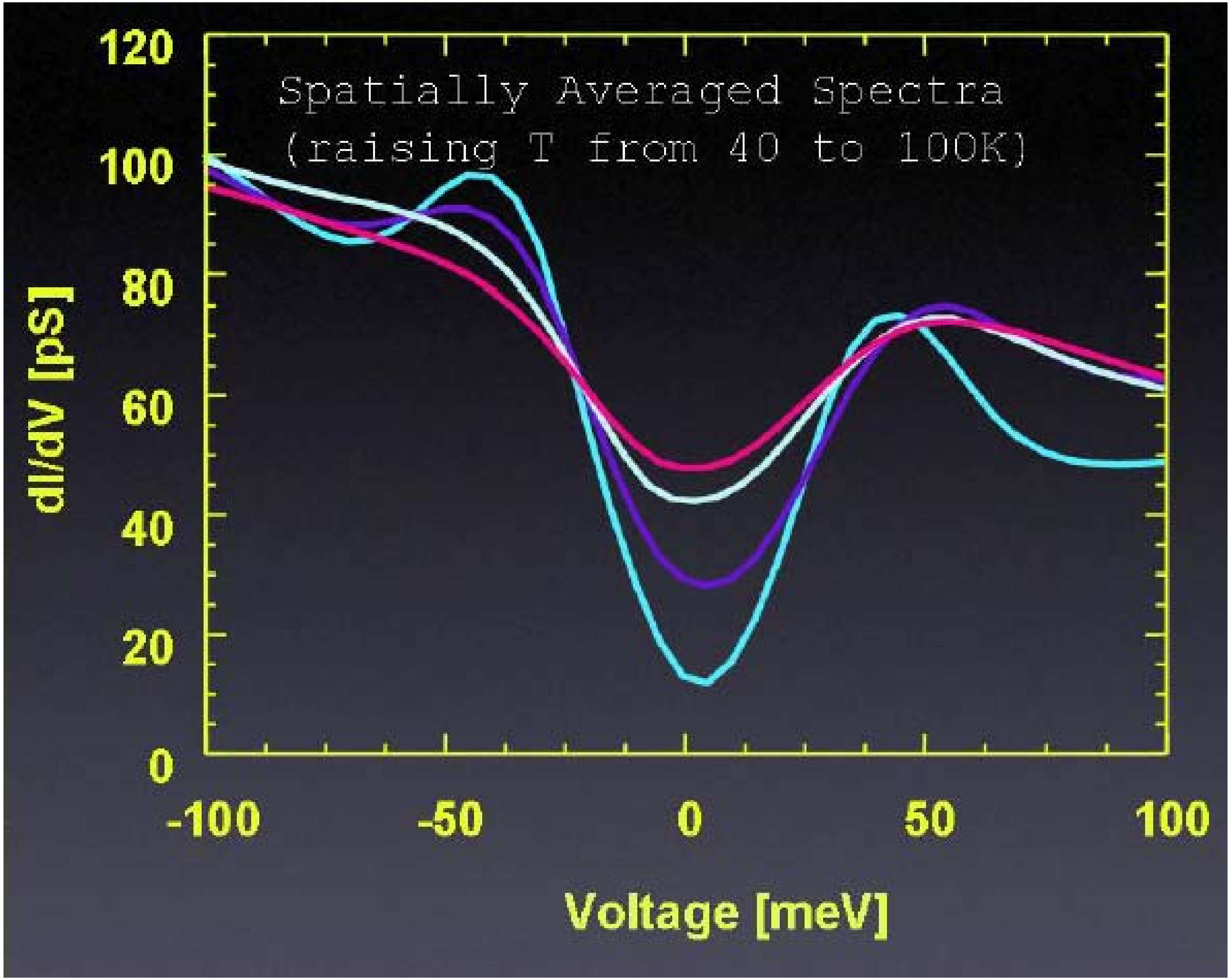
UV-IR Mixing





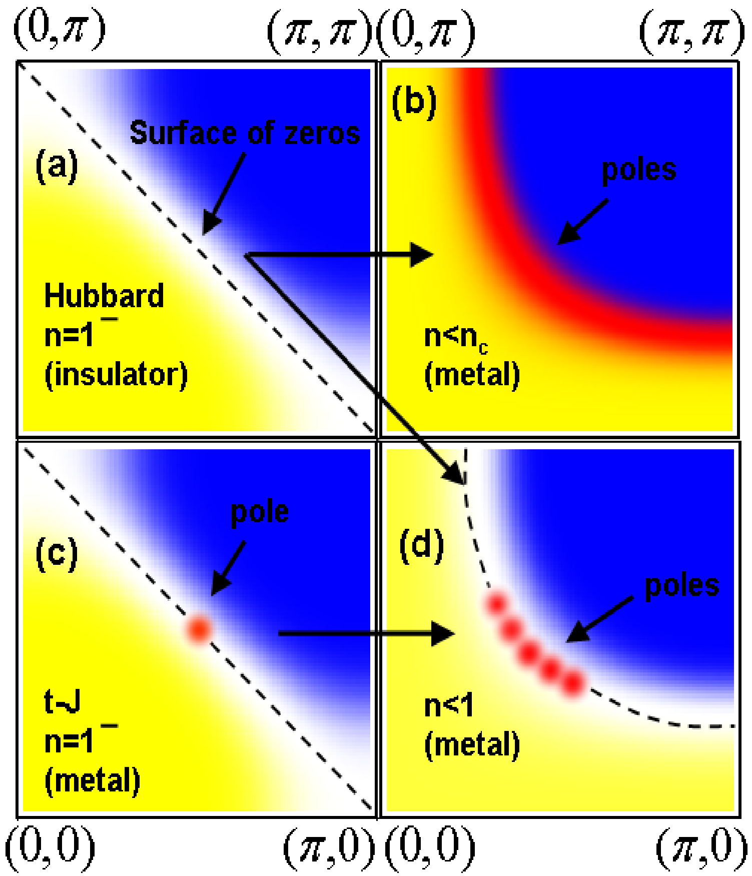
Pseudogap:
Not symmetrical

Spectral weight transfer



Yazdani, et. al.

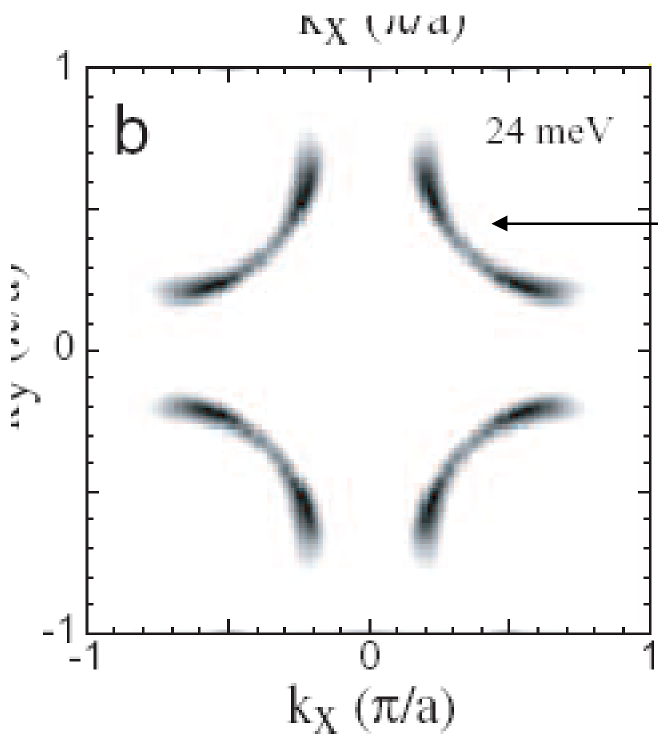
Evolution of surface of zeros



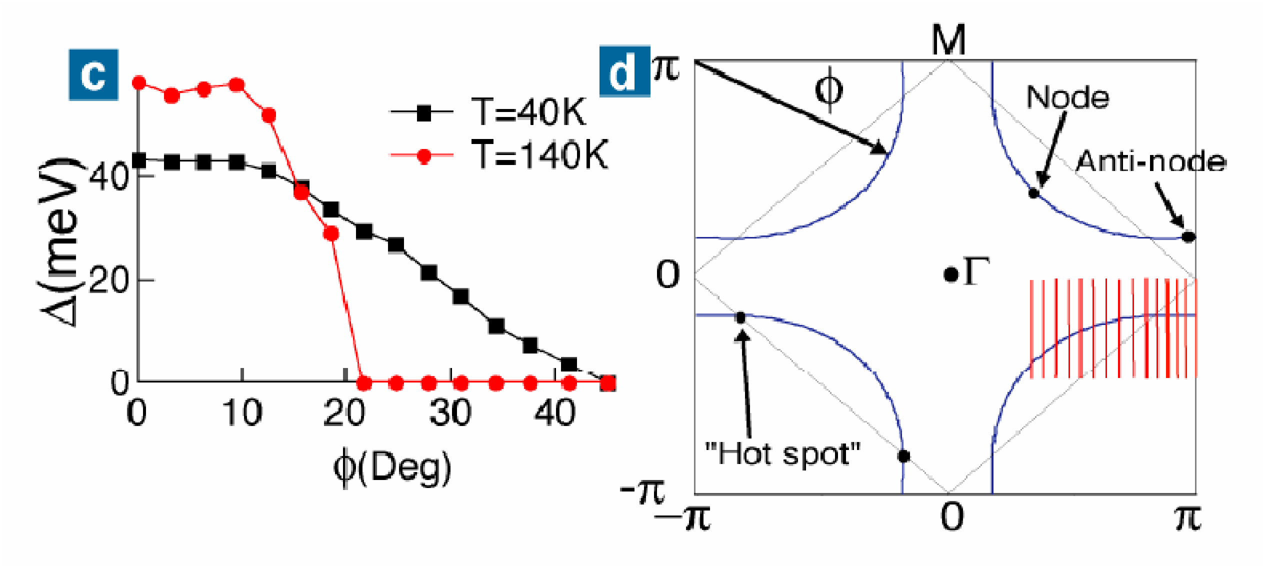
Perturbative schemes

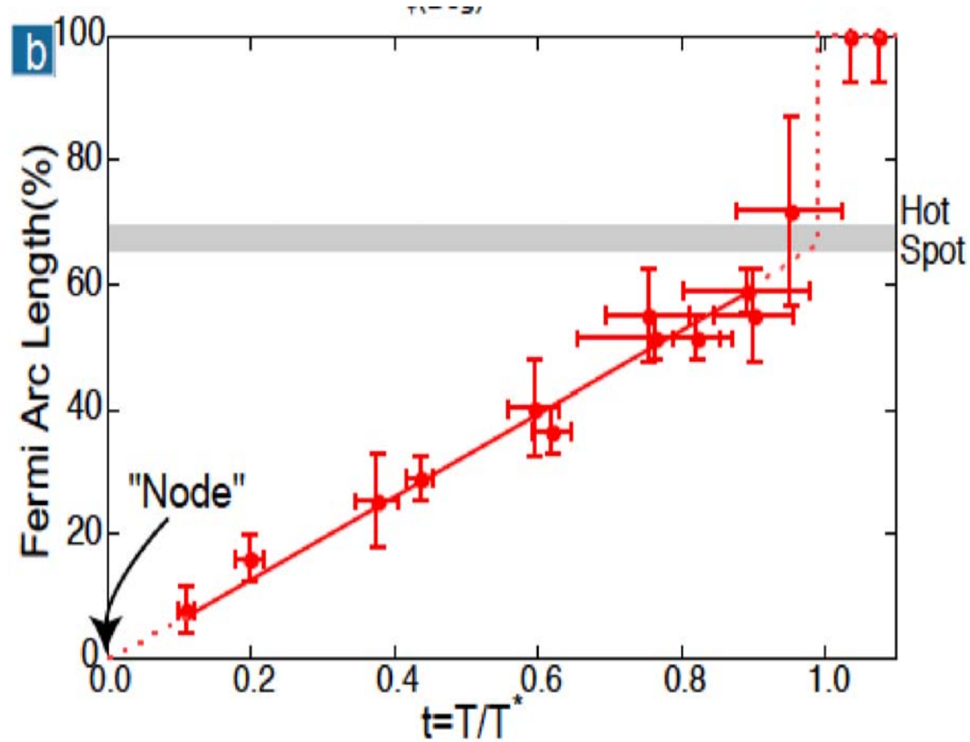
$ReG \neq 0$

Fermi Arcs



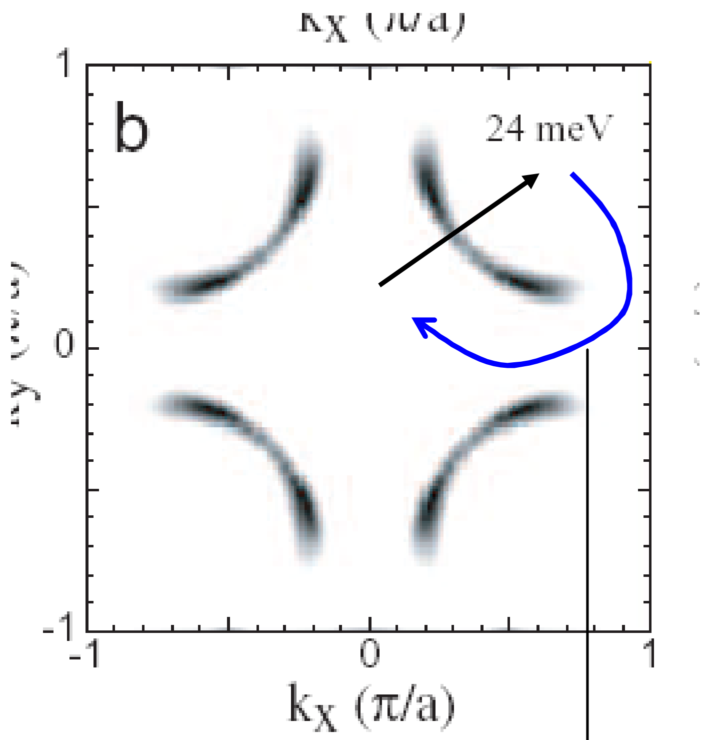
Are these Quasiparticles?





Campuzano, Et. Al. NP, 2006.

Fermi Arcs



Re G
Changes
Sign across
An arc

Must cross
A zero line!!!

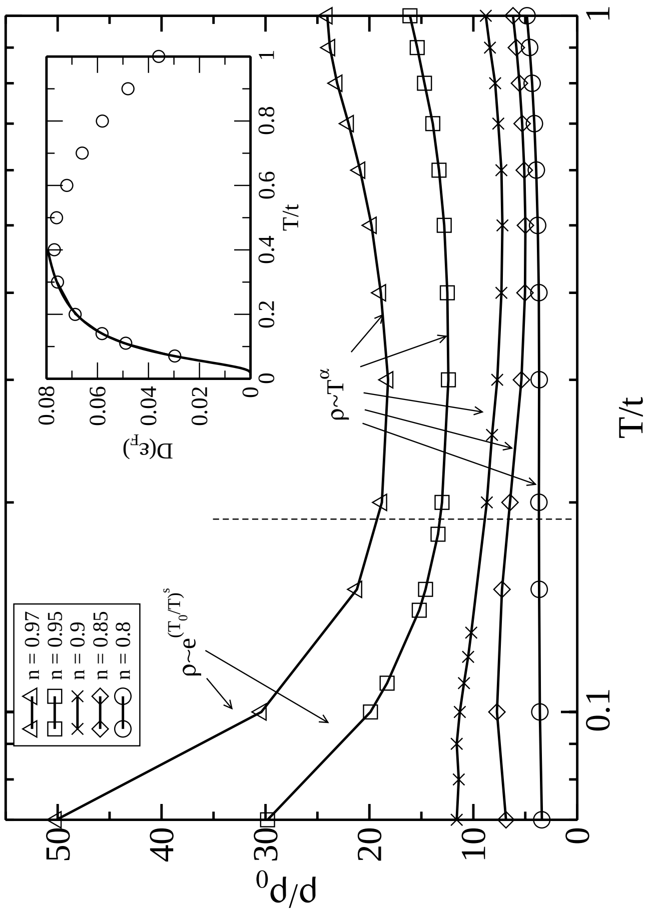
Fermi arcs and zeros are related

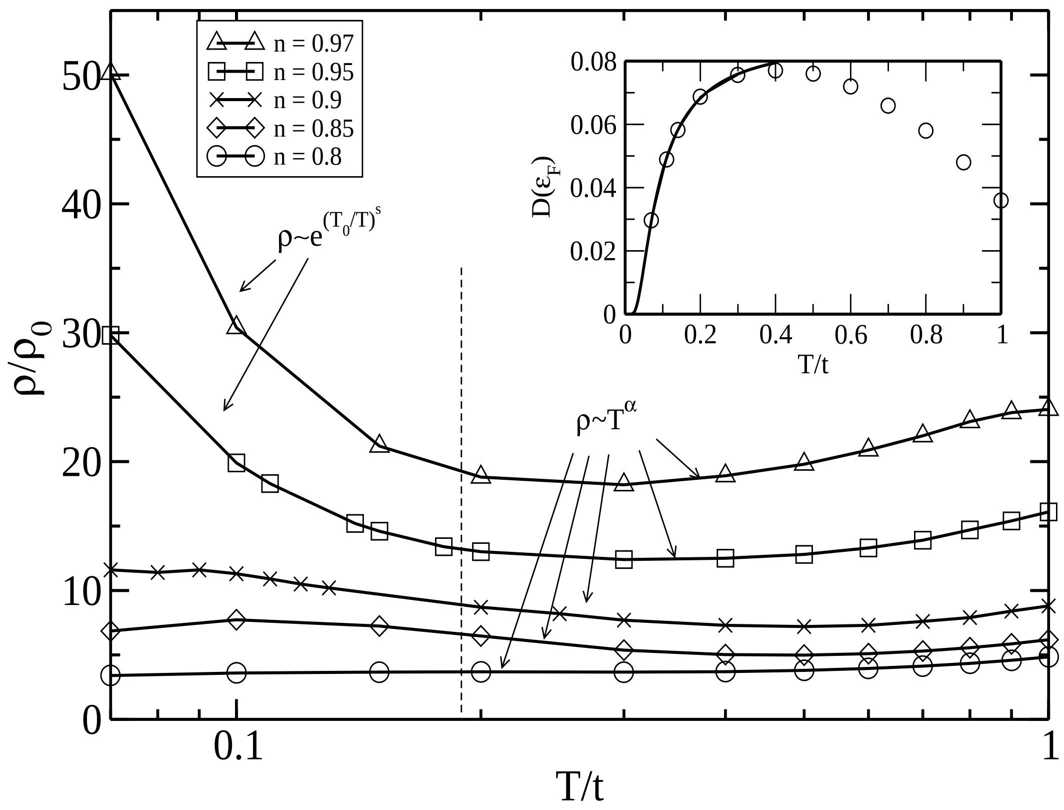
Zeros define the pseudogap

↓

Pseudogap is insulating

Doped Mott insulators are insulators





TPC, PP, PRL, 95, 196405 (2005)

Ando, et. al.

$k_F\ell \approx 10$

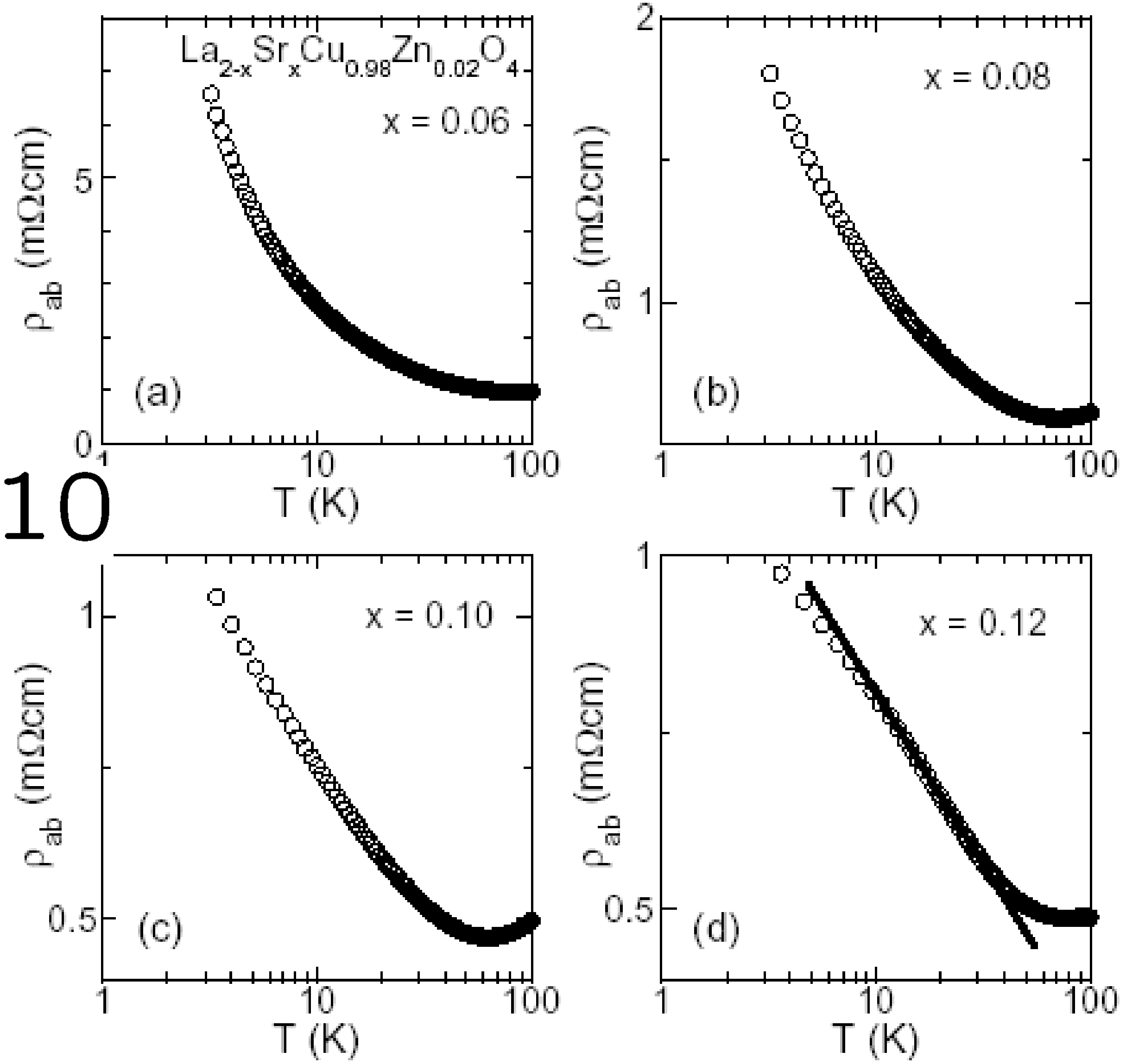
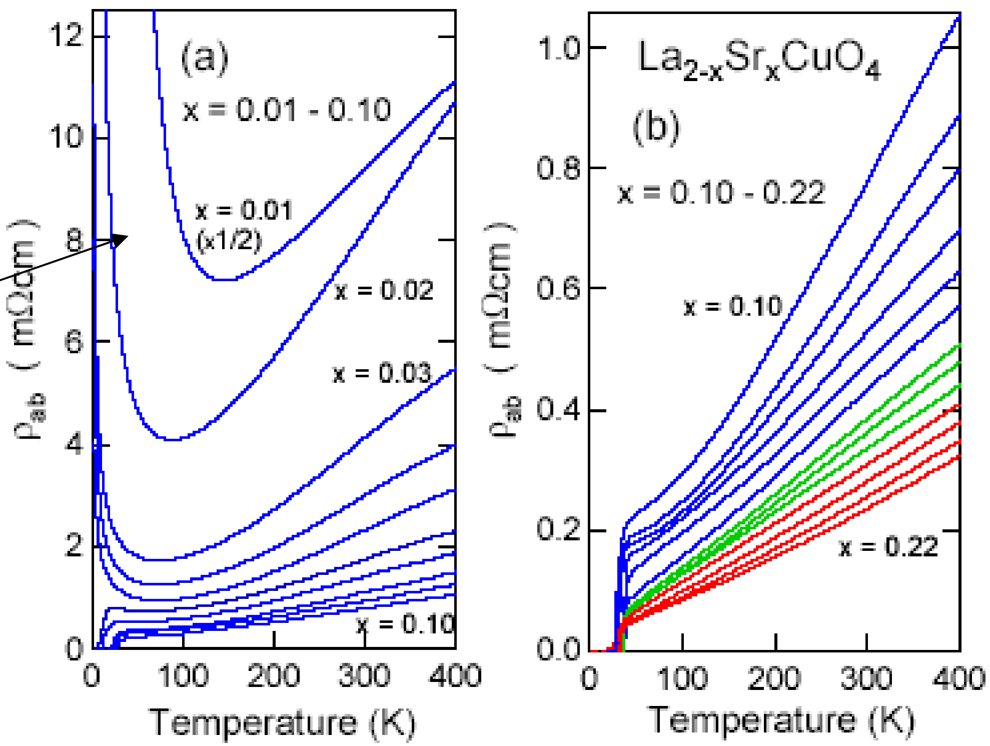


FIG. 4: $\log(T)$ plots of $\rho_{ab}(T)$ for Zn-doped LSCO with $x \leq 0.12$.

insulator



(a)
x = 0.01 - 0.10
x = 0.01
(x1/2)
x = 0.02
x = 0.03
x = 0.10
ρ_{ab} (mΩcm)
Temperature (K)
$La_{2-x}Sr_xCuO_4$
(b)
x = 0.10 - 0.22
x = 0.10
x = 0.22
ρ_{ab} (mΩcm)
Temperature (K)

t-J model

Rosch, Wolfle, 2004

Metallic Not insulator

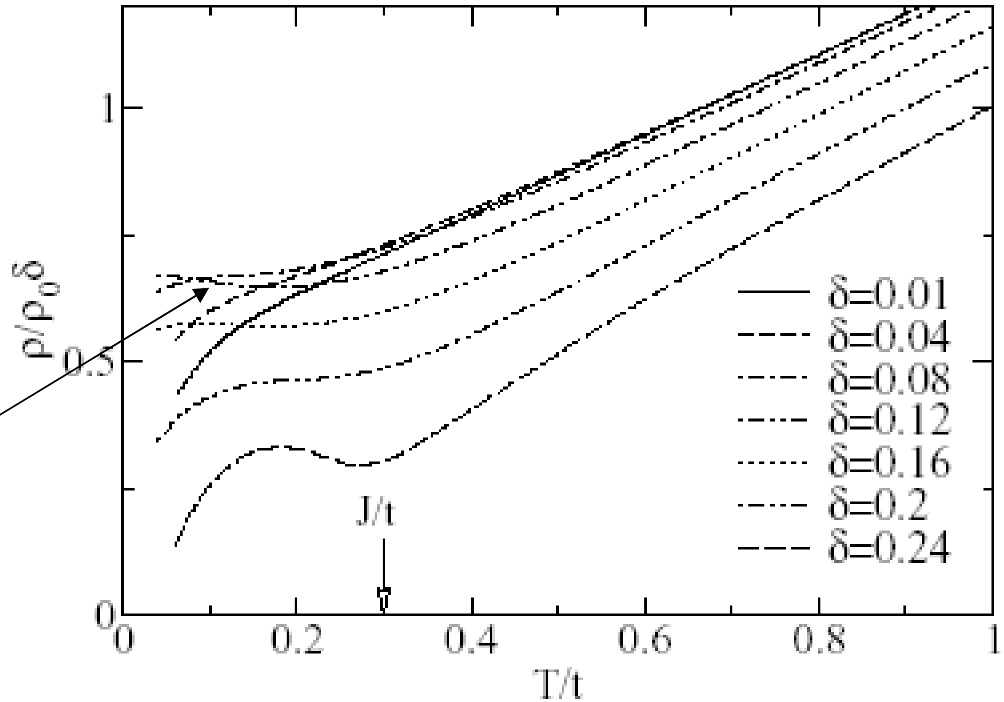
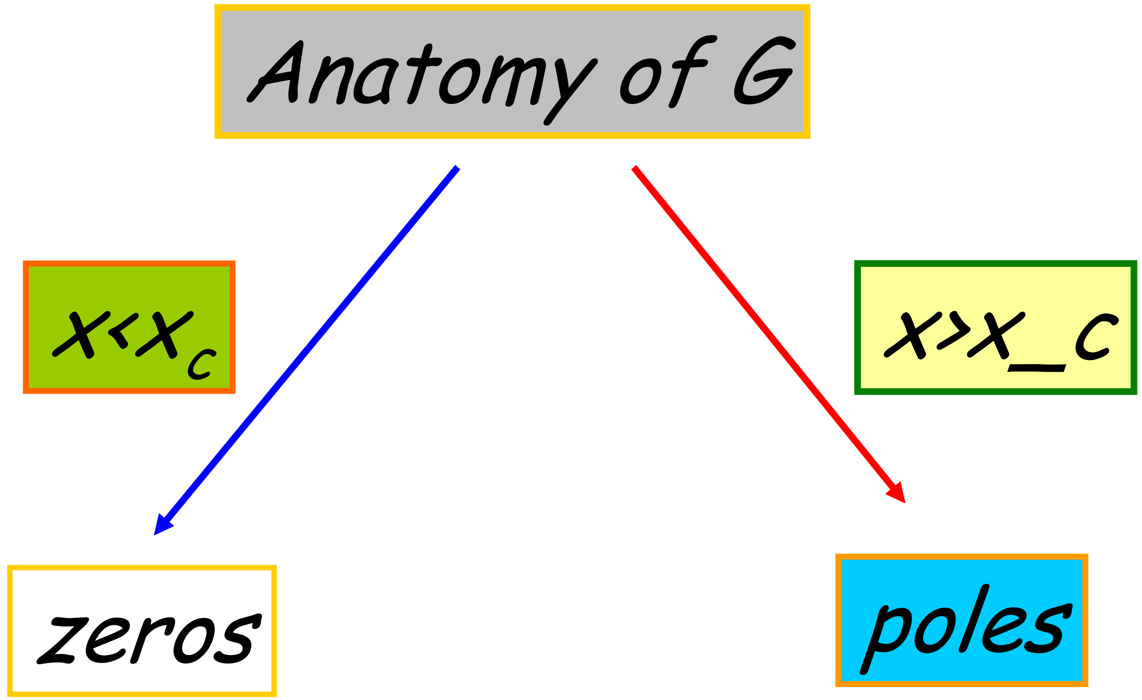
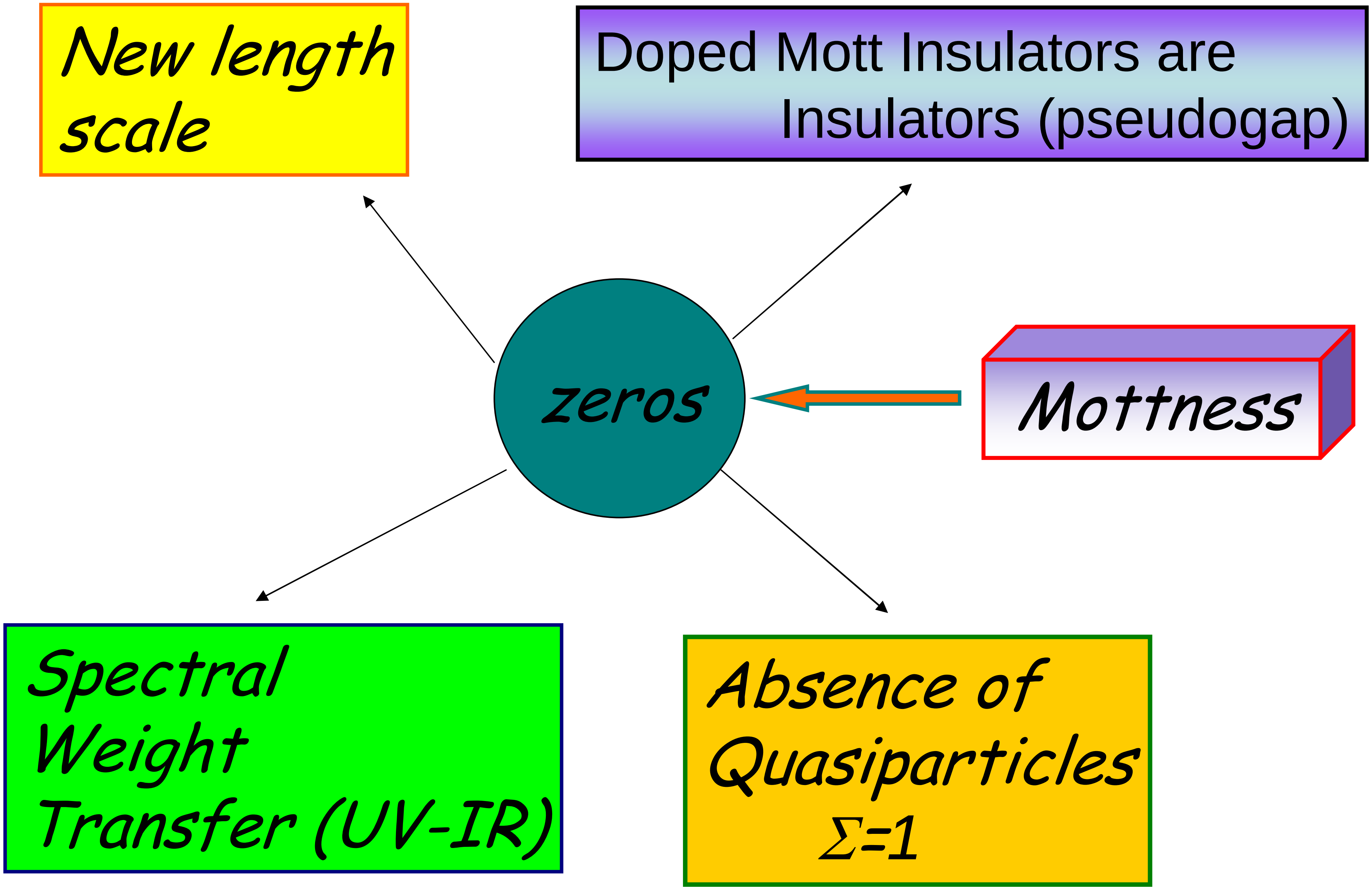


FIG. 19: T-dependence of the resistivity multiplied by doping δ. The linear T behavior for high T flattens for $\delta > 0.1$ at a temperature of the order of J. For $\delta < 0.1$ the resistivity drops in the regime where a pseudogap opens.

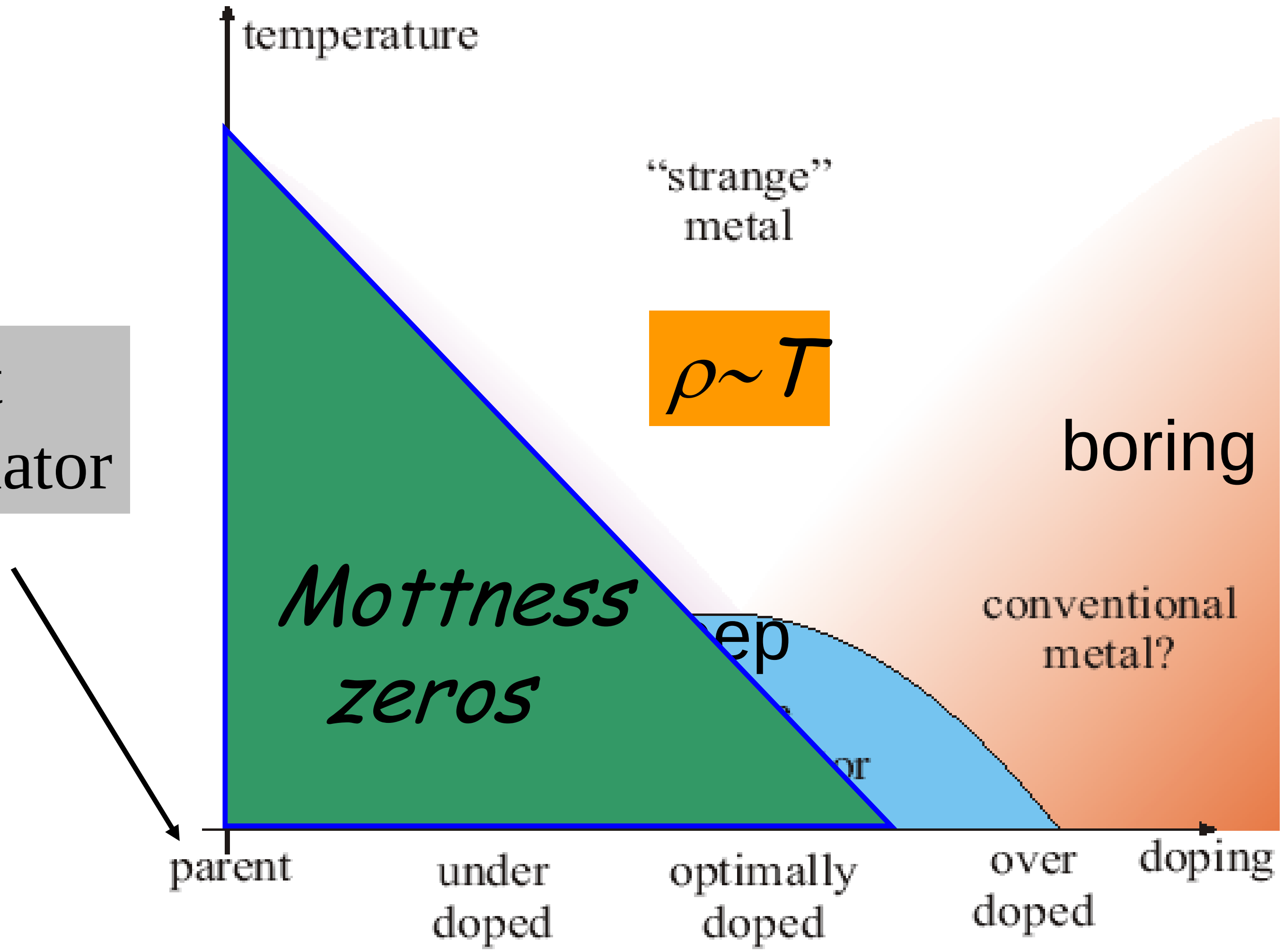
Kotliar, prelovsek


Anatomy of G
$x<x_c$
x>x_c
zeros
poles


New length scale
Doped Mott Insulators are Insulators (pseudogap)
zeros
Mottness
Spectral Weight Transfer (UV-IR)
Absence of Quasiparticles
Σ=1

Phase diagram

Mott insulator



Why and How?

Thanks to:

T. Stanescu, TP Choy,
D. Galanakis, C. Chamon

NSF-DMR 0305864