



*The Abdus Salam*  
**International Centre for Theoretical Physics**



SMR.1766 - 1

**Miniworkshop on  
New States of Stable and Unstable Quantum Matter  
(14 - 25 August 2006)**

**Electronic Structure of Strongly Correlated Electron  
Materials: A Dynamical Mean Field Perspective**

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These are preliminary lecture notes, intended only for distribution to participants

*Electronic Structure of Strongly  
Correlated Electron Materials:  
A Dynamical Mean Field  
Perspective.*

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Center for Materials Theory

Rutgers University Rutgers University

*Collaborators: G. Kotliar, S. Savrasov, V. Oudovenko*

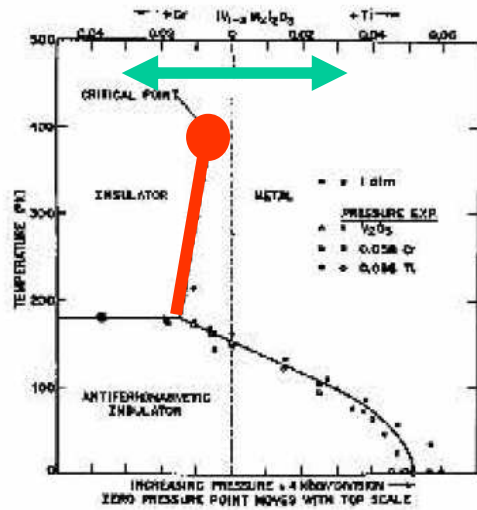
*ICAM Symposium on Frontiers in Correlated Matter, Snowmass 2006*

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# Overview

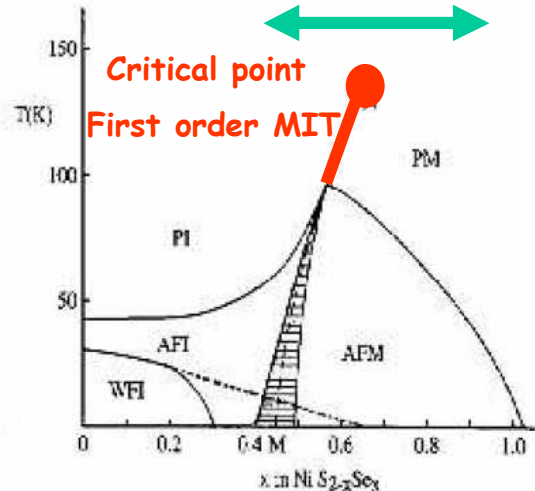
- Application of DMFT to real materials (Spectral density functional approach). Examples:
    - alpha to gamma transition in Ce, optics near the temperature driven Mott transition.
    - Mott transition in Americium under pressure
    - Antiferromagnetic transition in Curium
  - Extensions of DMFT to clusters.  
Examples:
    - Superconducting state in t-J the model
    - Optical conductivity of the t-J model
-

# Universality of the Mott transition

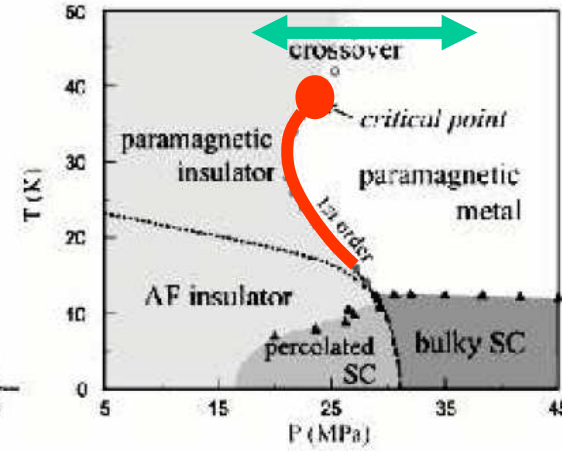


$V_2O_3$

Crossover: bad insulator to bad metal

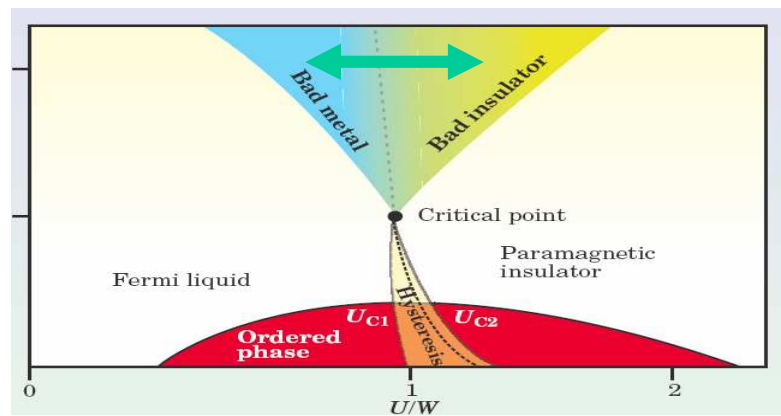


$Ni_{2-x}Se_x$

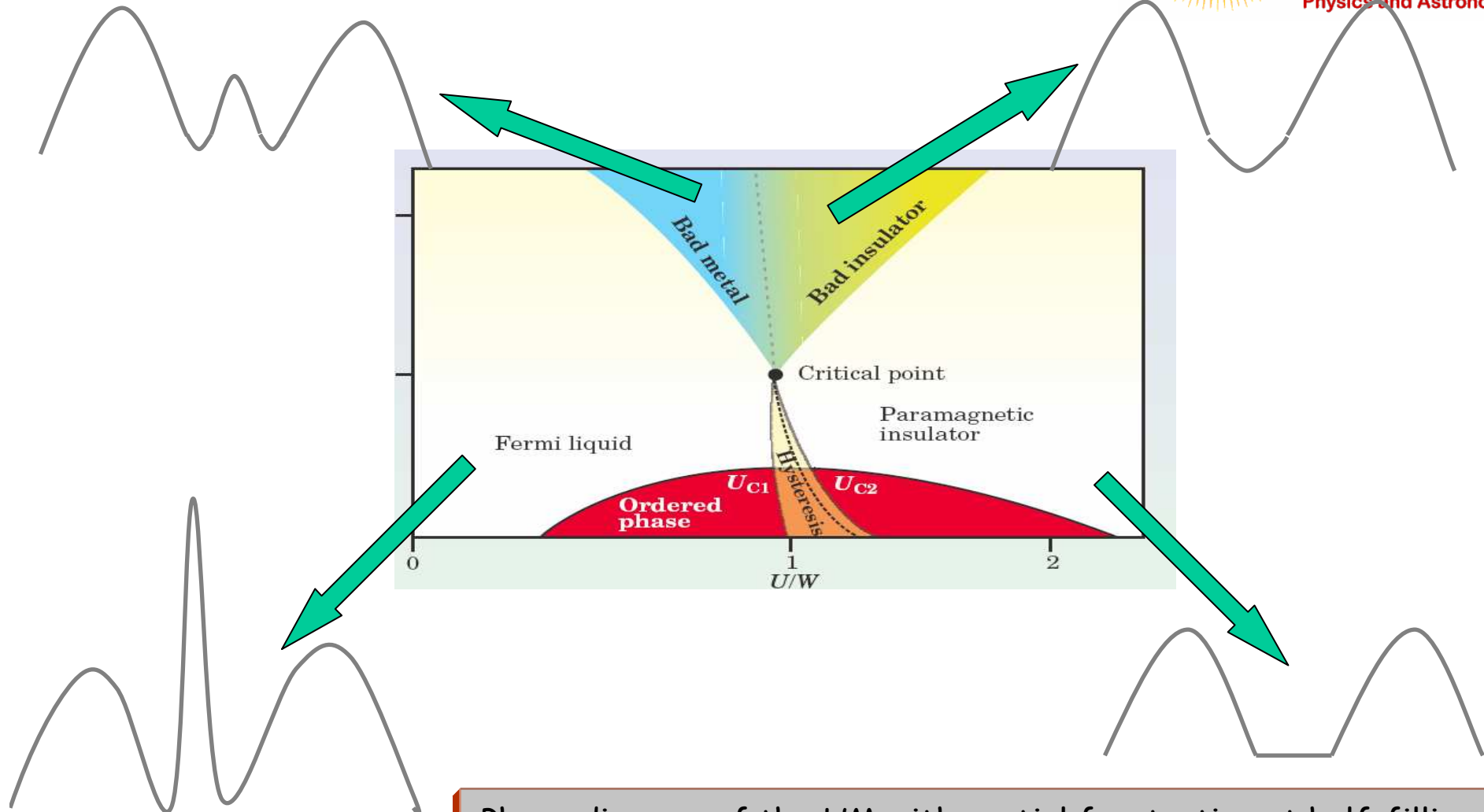


k organics

1B HB model (DMFT):



# Coherence incoherence crossover in the 1B HB model (DMFT)



Phase diagram of the HM with partial frustration at half-filling  
M. Rozenberg et al., Phys. Rev. Lett. **75**, 105 (1995).

# DMFT + electronic structure method

**Basic idea of DMFT:** reduce the *quantum many body problem* to a *one site* or a *cluster* of sites problem, *in a medium* of non interacting electrons obeying a self-consistency condition. (A. Georges et al., RMP 68, 13 (1996)).

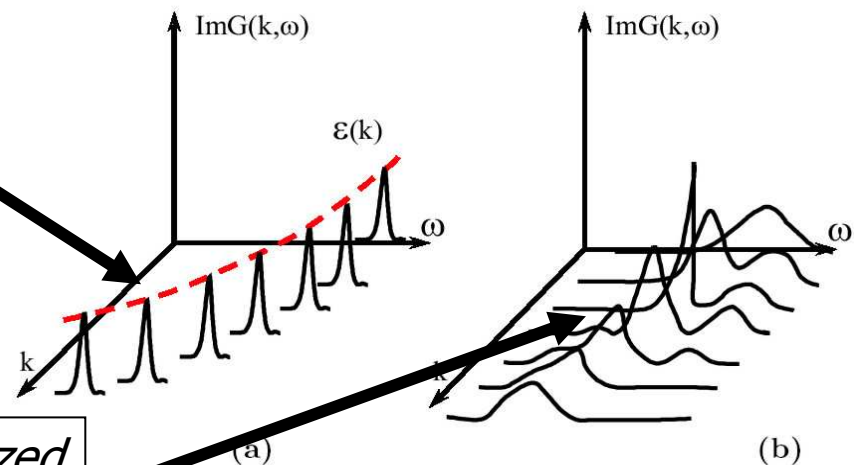
**DMFT in the language of functionals:** DMFT sums up *all local diagrams* in *BK functional*

## Basic idea of DMFT+electronic structure method (LDA or GW):

For less correlated bands (s,p): use LDA or GW

For correlated bands (f or d): *with DMFT add all local diagrams*

Effective (DFT-like) single particle Spectrum consists of delta like peaks



Spectral density usually contains renormalized quasiparticles and Hubbard bands

# How good is single site DMFT for f systems?

H																	He	
Li	Be											B	C	N	O	F	Ne	
Na	Mg											Al	Si	P	S	Cl	Ar	
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	
Cs	Ba		Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	
Fr	Ra		Rf	Db	Sg	Bh	Hs	Mt	Uun	Uuu	Uub							
		La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu		
		Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

Ce

Pu

Cm

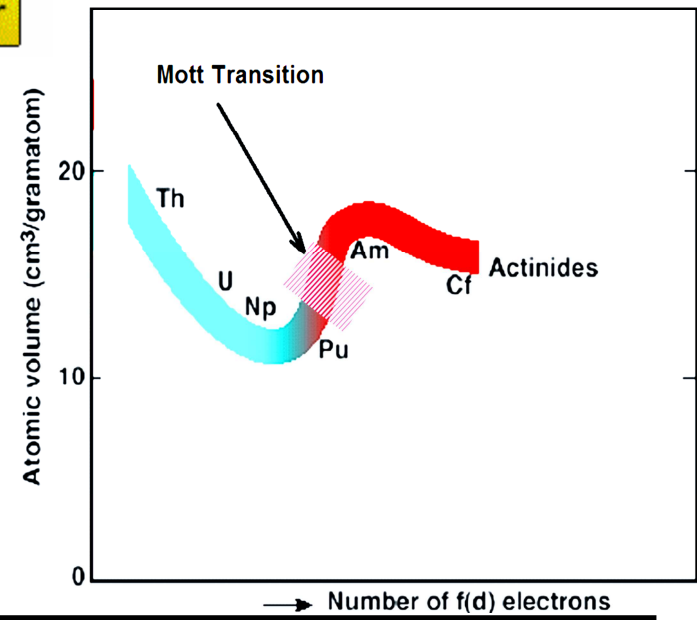
**f5**  
 $L=5, S=5/2, J=5/2$

**f7**  
 $L=0, S=7/2, J=7/2$

**f1**  
 $L=3, S=1/2, J=5/2$

Am

**f6**  
 $L=3, S=3, J=0$



# Cerium





# Ce overview

- isostructural phase transition ends in a critical point at (T=600K, P=2GPa)

→  $\gamma$  (fcc) phase

[ magnetic moment  
(Curie-Wiess law),  
large volume,  
stable high-T, low-p]

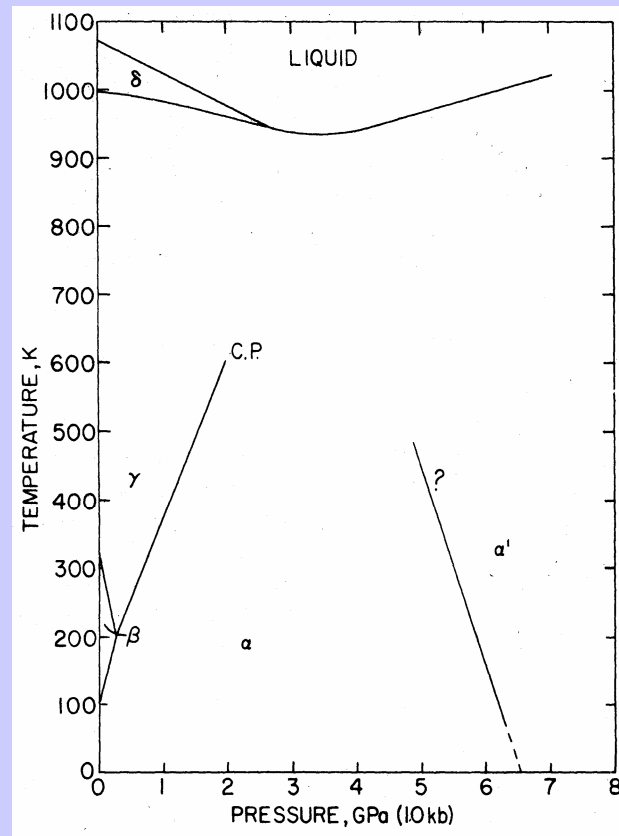
→  $\alpha$  (fcc) phase

[ **loss of magnetic  
moment** (Pauli-para),  
smaller volume,  
stable low-T, high-p]

with large

**volume collapse**

$\Delta v/v \approx 15\%$

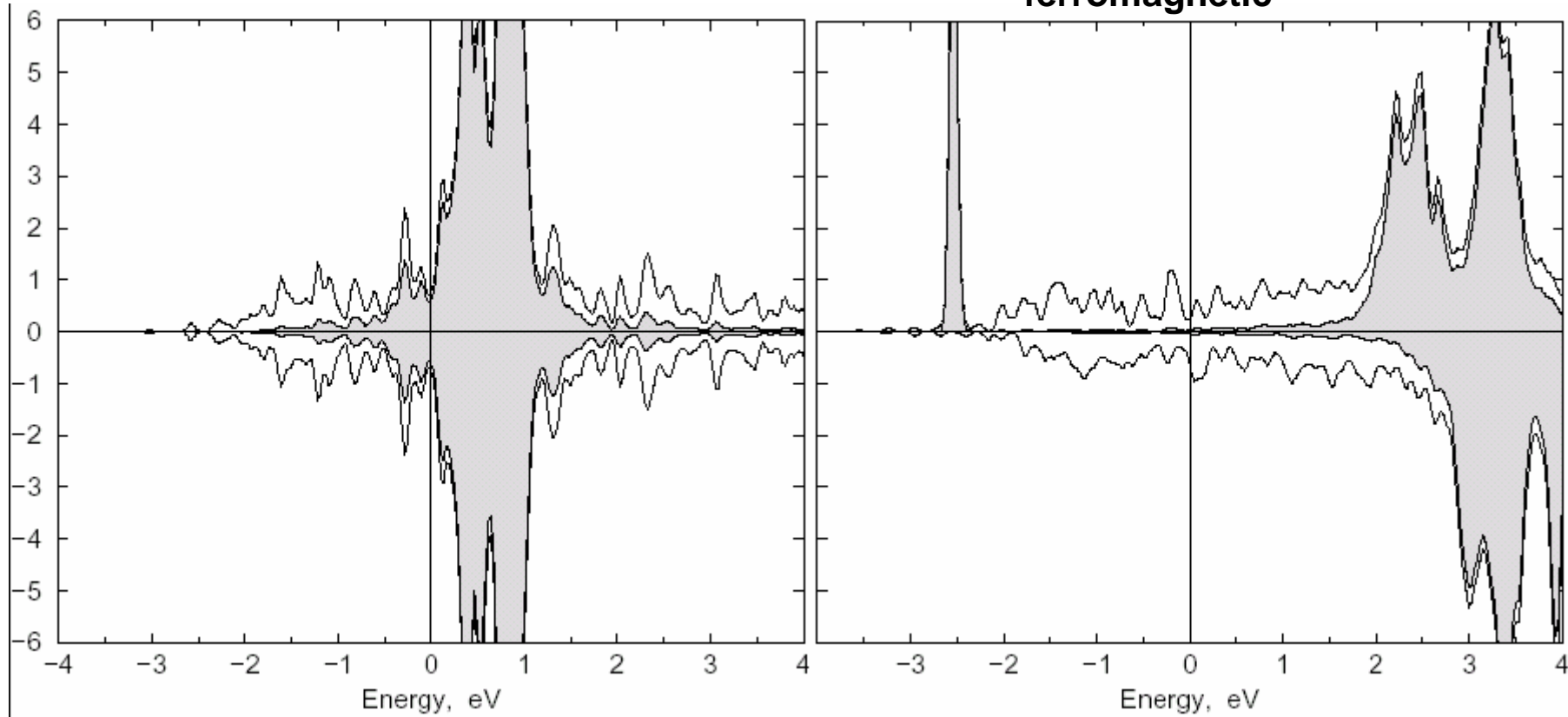


volumes	exp.	LDA	LDA+U
$\alpha$	$28\text{\AA}^3$	$24.7\text{\AA}^3$	
$\gamma$	$34.4\text{\AA}^3$		$35.2\text{\AA}^3$

- *Transition is 1.order*
- *ends with CP*

# LDA and LDA+U

ferromagnetic

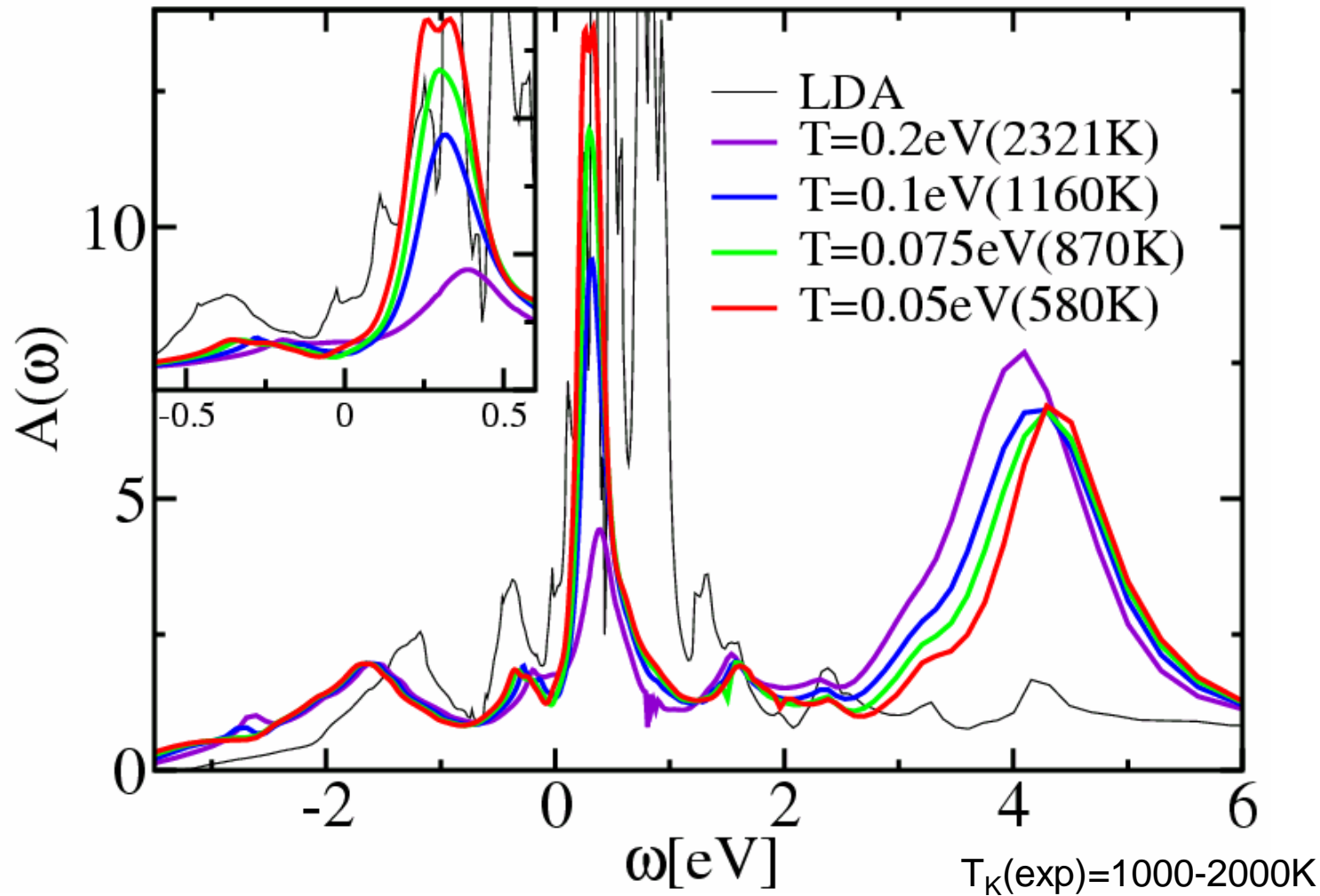


volumes	exp.	LDA	LDA+U
$\alpha$	$28\text{\AA}^3$	$24.7\text{\AA}^3$	
$\gamma$	$34.4\text{\AA}^3$		$35.2\text{\AA}^3$

■ f DOS  
□ total DOS

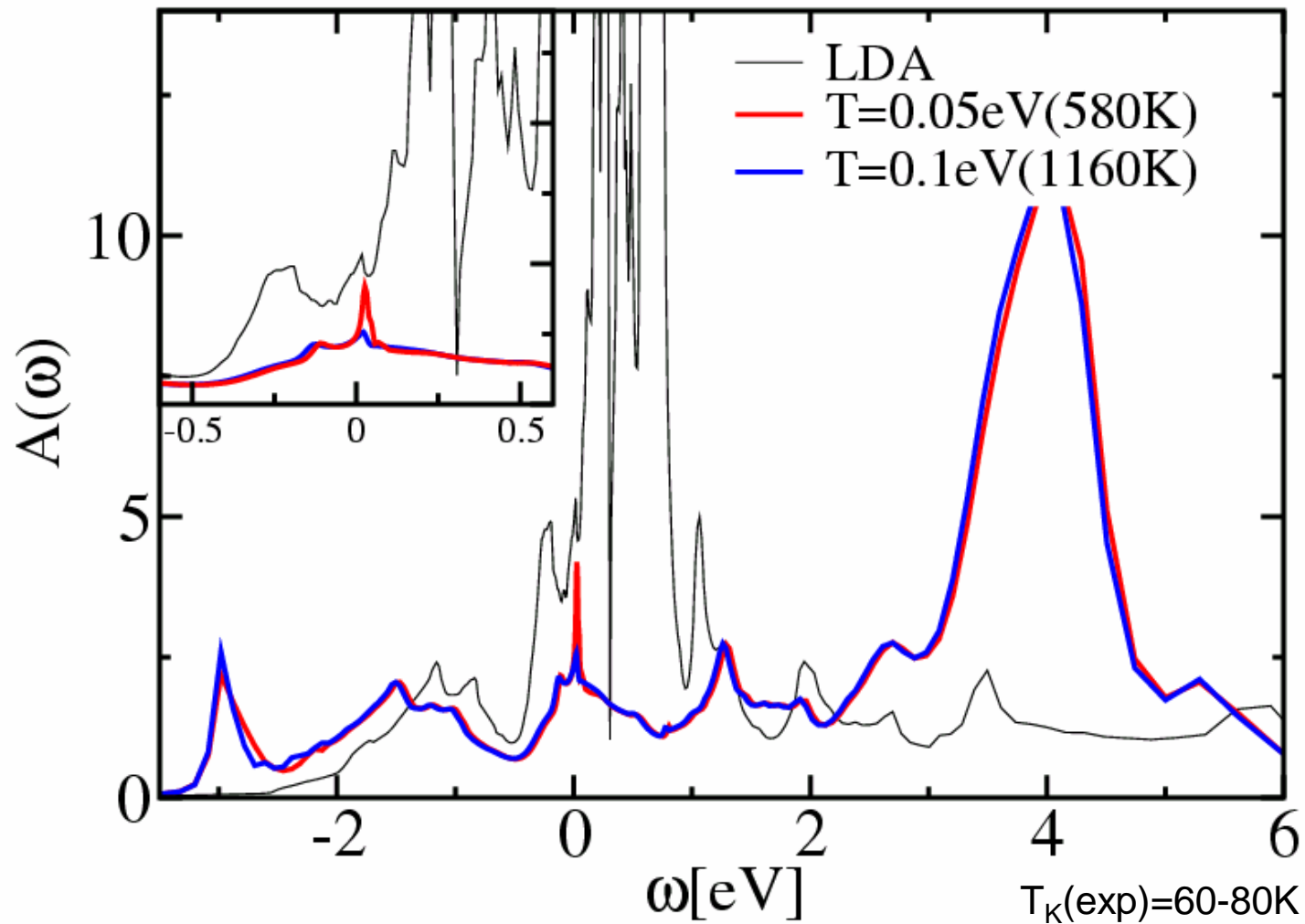
# LDA+DMFT alpha DOS

alpha  $28\text{\AA}^3$   $U=5.5$  eV

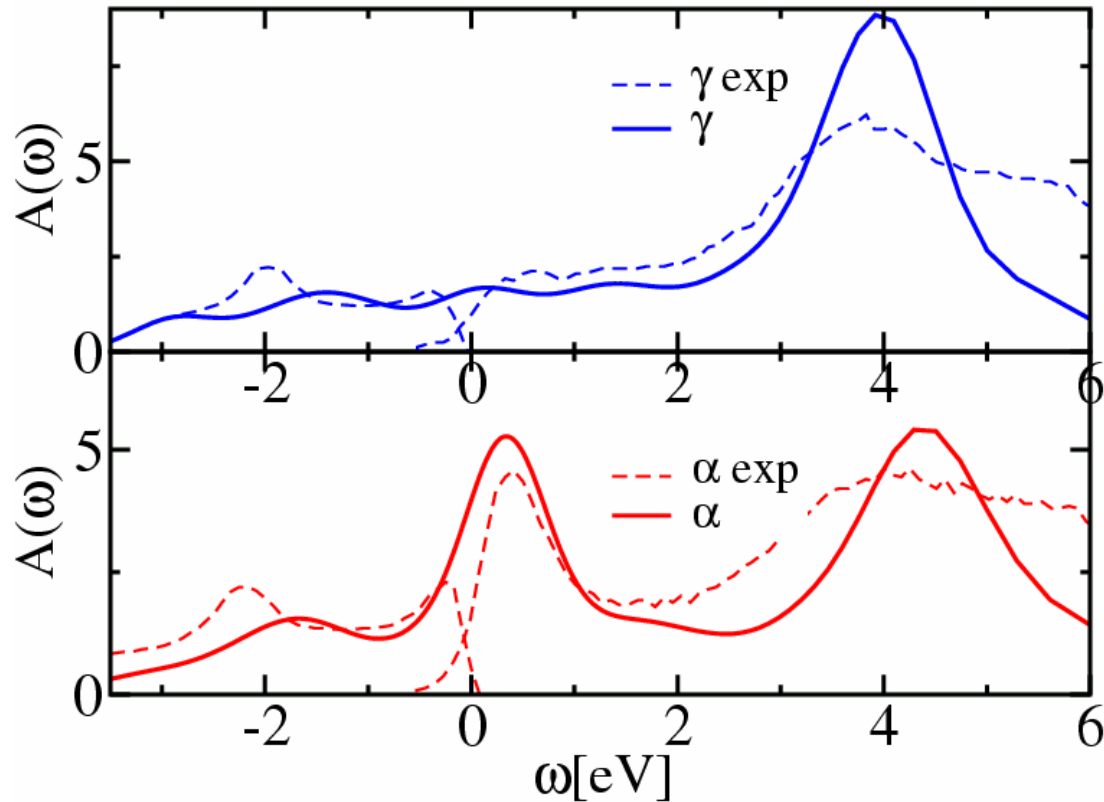


# LDA+DMFT gamma DOS

gamma  $34.4\text{\AA}^3$   $U=6\text{eV}$



# Photoemission & experiment



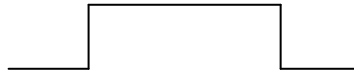
- A. Mc Mahan K Held and R. Scalettar (2002)
- K. Haule V. Udovenko and GK. (2003)

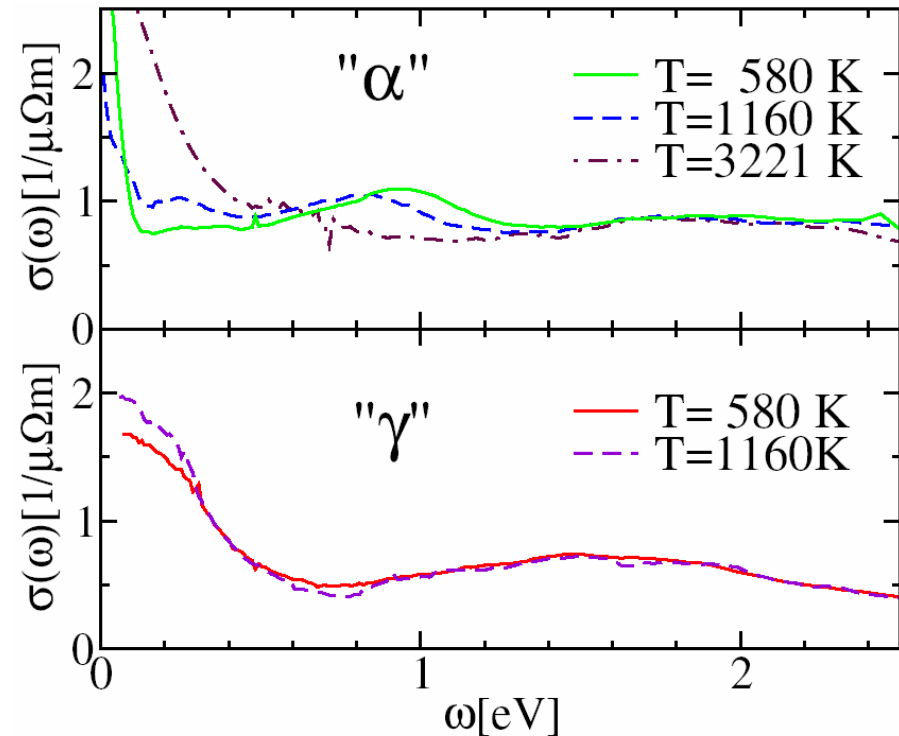
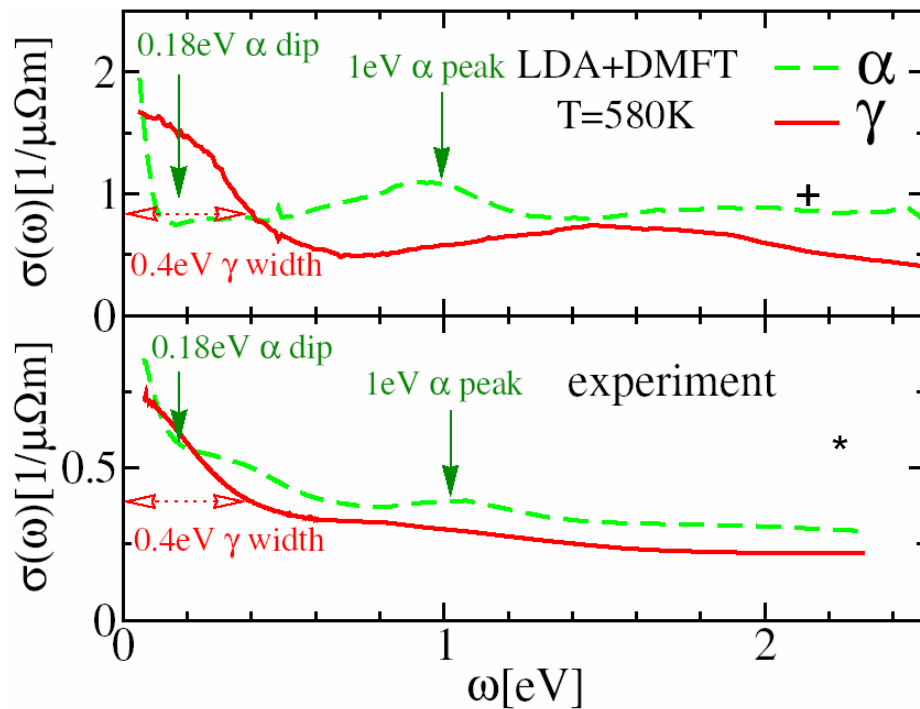
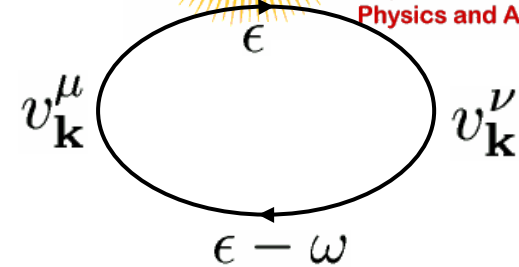
Phenomenological approach  
describes well the transition

Kondo volume collapse (J.W. Allen, R.M. Martin, 1982)

# Optical conductivity

$$\sigma^{\mu\nu}(\omega) = \int w(\epsilon, \omega) \sum_{\mathbf{k}} \text{Tr} (v_{\mathbf{k}}^{\mu} \rho_{\mathbf{k}}(\epsilon) v_{\mathbf{k}}^{\nu} \rho_{\mathbf{k}}(\epsilon - \omega))$$

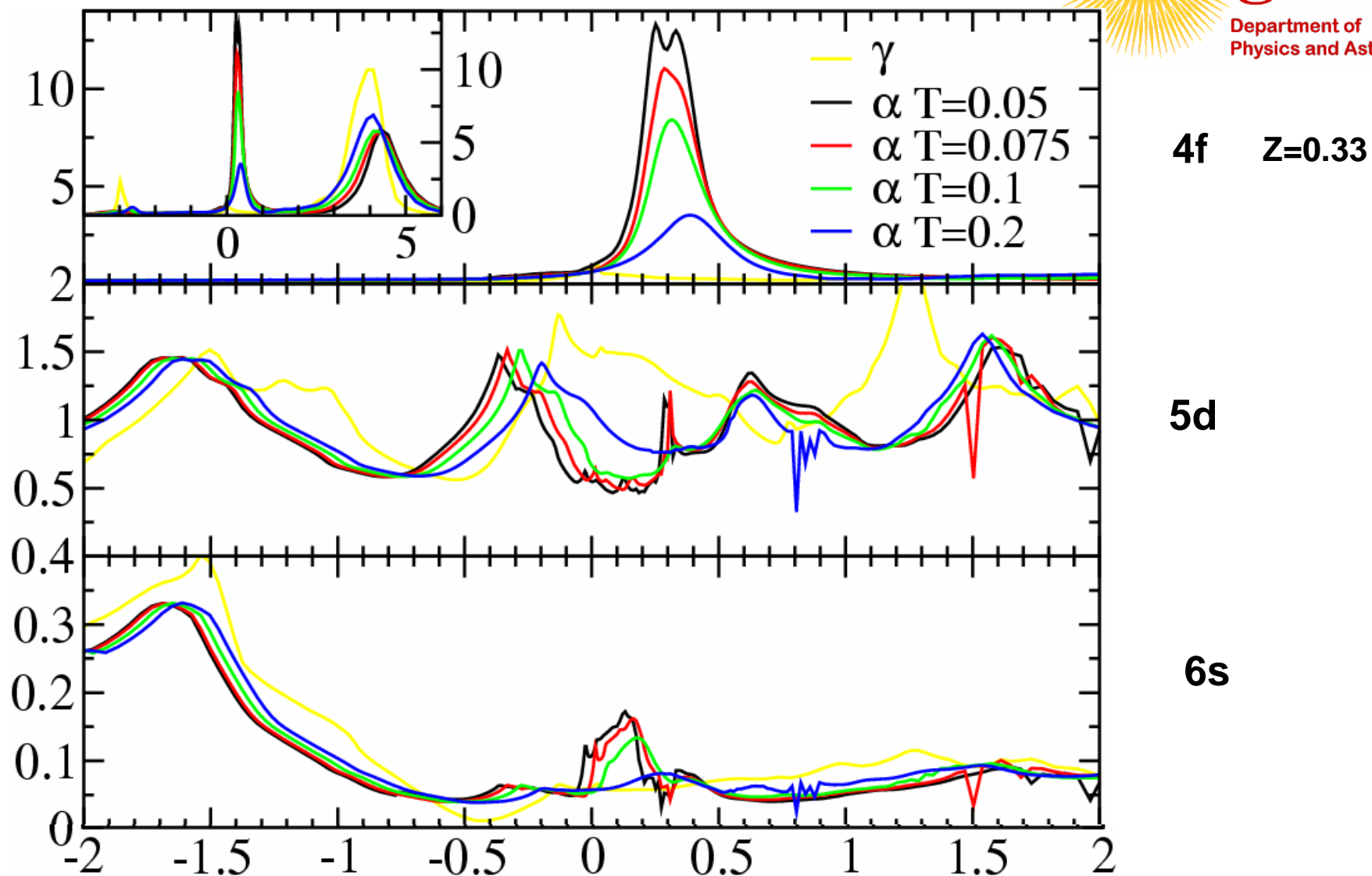
$$w(\epsilon, \omega) = \frac{f(\epsilon - \omega) - f(\epsilon)}{\omega}$$




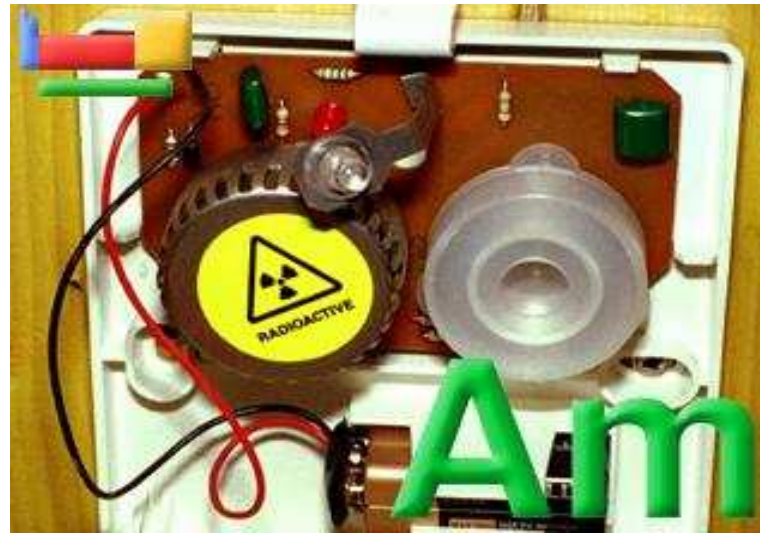
+ K. Haule, et.al.,  
 Phys. Rev. Lett. **94**, 036401 (2005)

\* J.W. van der Eb, A.B. Ku'zmenko, and D. van der Marel,  
 Phys. Rev. Lett. **86**, 3407 (2001)

# Partial DOS



# Americium



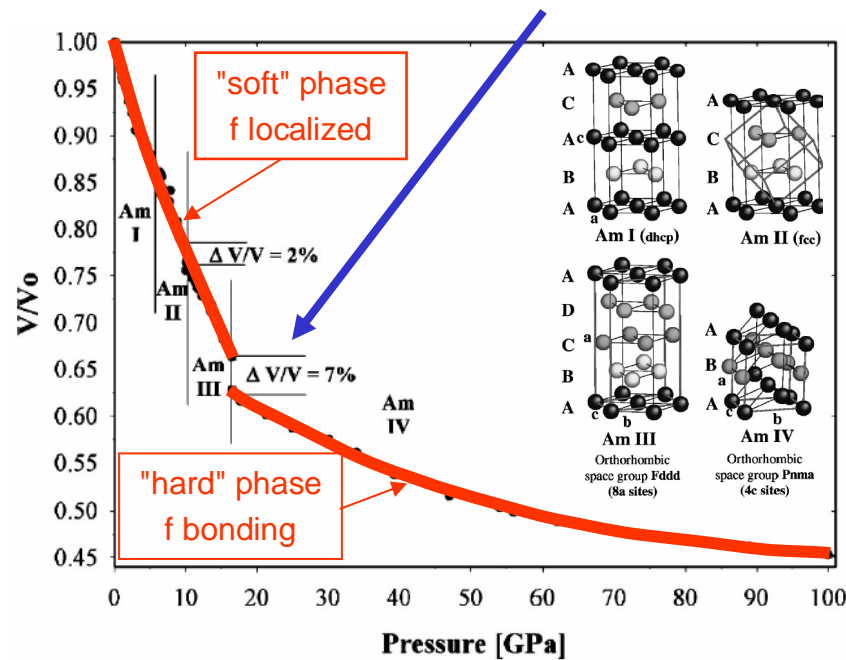


# Americium

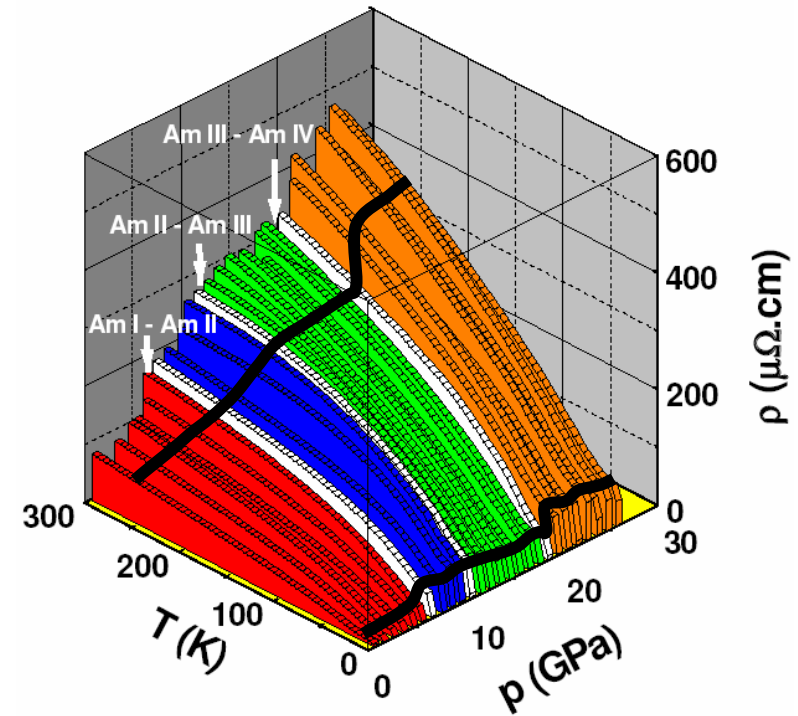
lanthanons	58	59	60	61	62	63	64	65	66	67	68	69	70	71
	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
actinons	90	91	92	93	94	95	96	97	98	99	100	101	102	103
	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

$$f^6 \rightarrow L=3, S=3, J=0$$

## Mott Transition?



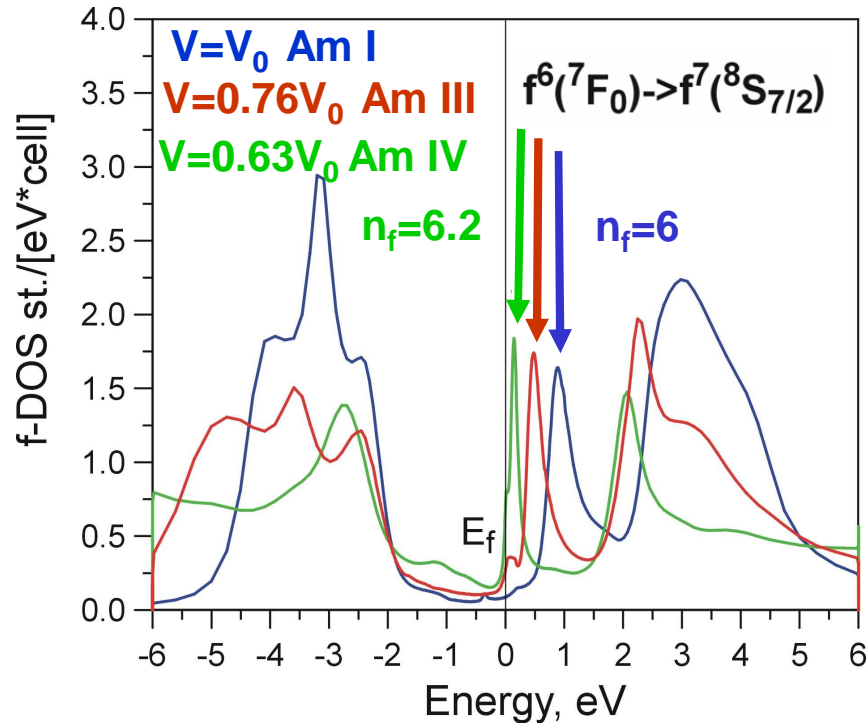
A.Lindbaum, S. Heathman, K. Litfin, and Y. Méresse,  
 Phys. Rev. B **63**, 214101 (2001)



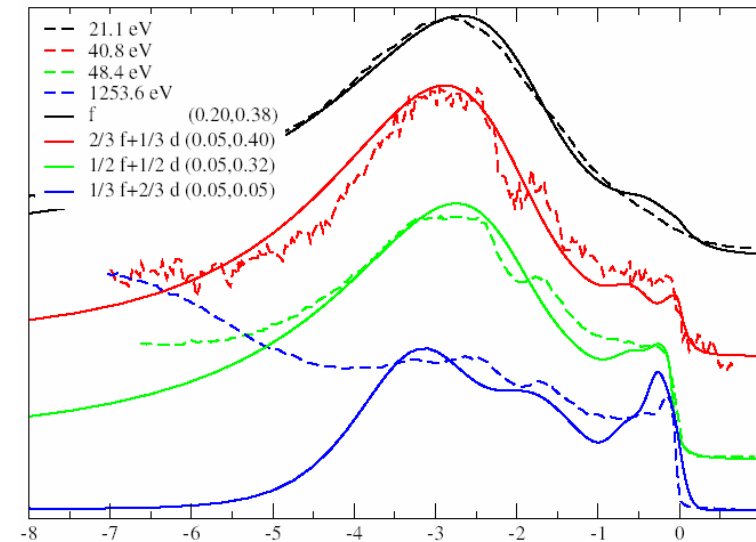
J.-C. Griveau, J. Rebizant, G. H. Lander, and G. Kotliar  
 Phys. Rev. Lett. **94**, 097002 (2005)

# Am within LDA+DMFT

from  $J=0$  to  $J=7/2$



Comparison with experiment



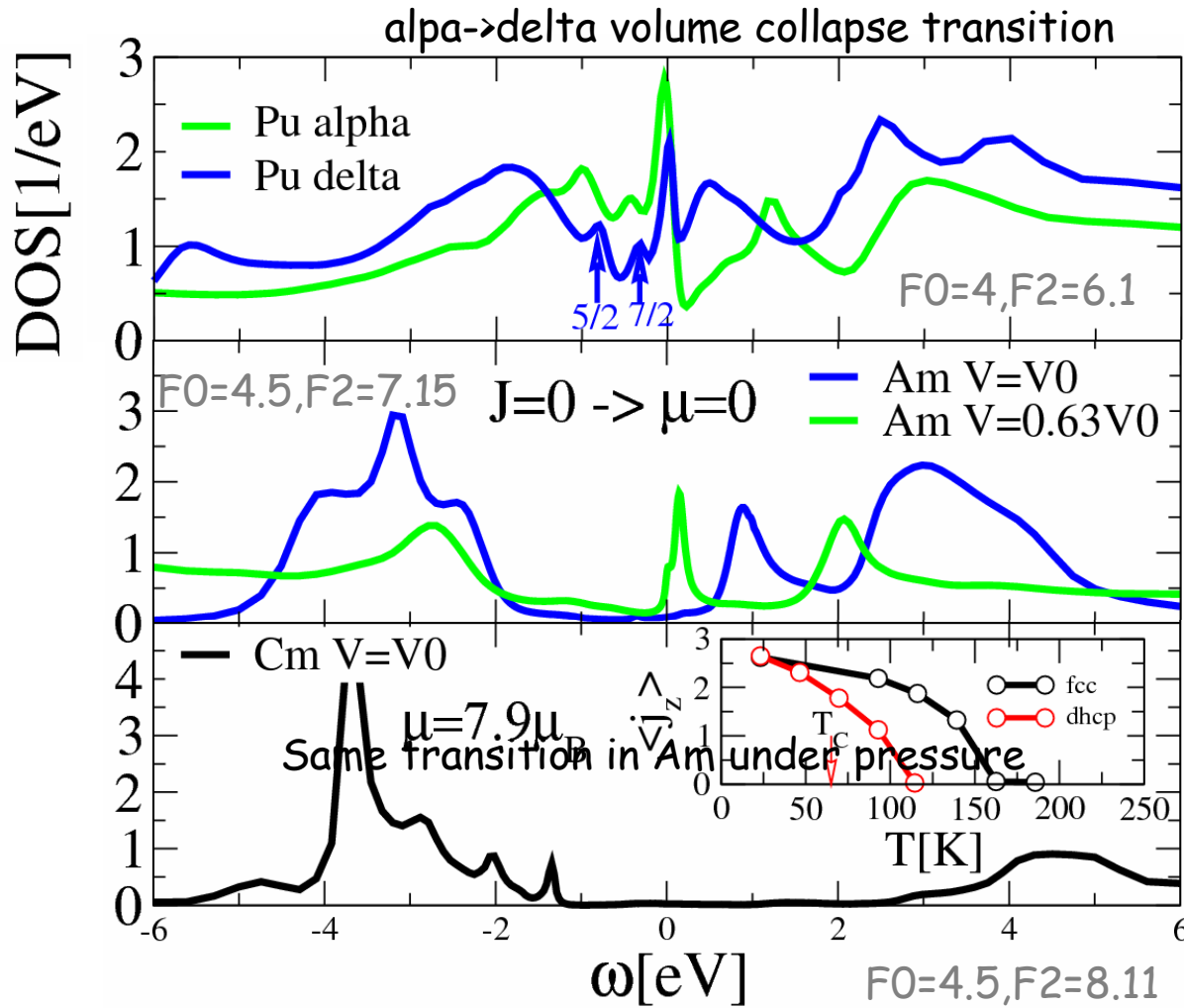
- "Soft" phase very different from  $\gamma$  Ce  
not in local moment regime since  $J=0$  (no entropy)
- "Hard" phase similar to  $\alpha$  Ce,  
Kondo physics due to hybridization, however,  
nf still far from Kondo regime

Exp: J. R. Naegele, L. Manes, J. C. Spirlet, and W. Müller  
Phys. Rev. Lett. **52**, 1834-1837 (1984)

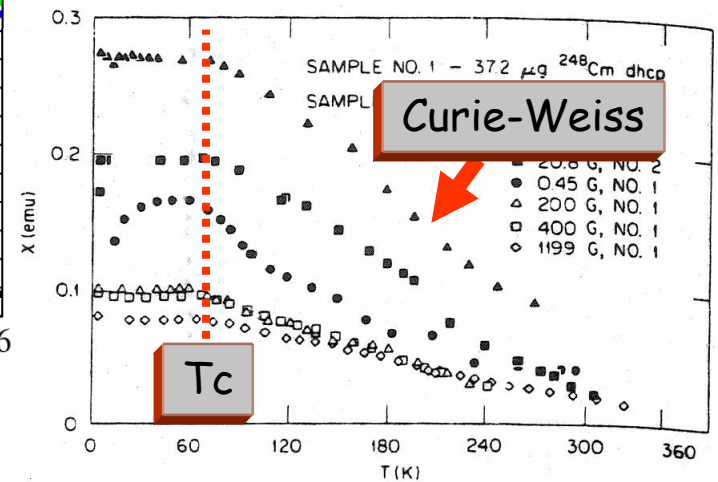
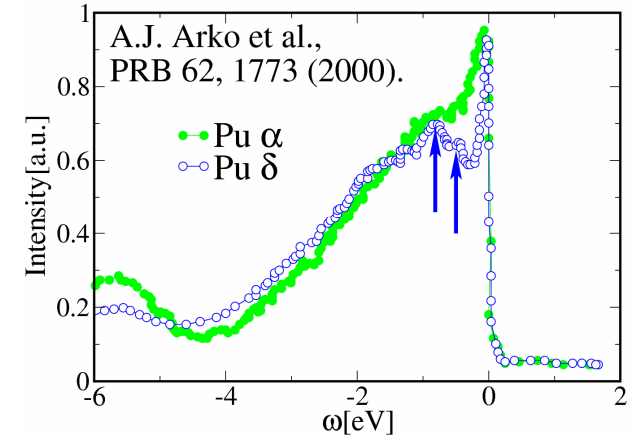
Theory: S. Y. Savrasov, K. Haule, and G. Kotliar  
Phys. Rev. Lett. **96**, 036404 (2006)

Different from Sm!

# Trends in Actinides



Curium has large magnetic moment and orders antif.



# What is captured by single site DMFT?



- Captures volume collapse transition (first order Mott-like transition)
  - Predicts well photoemission spectra, optics spectra, total energy at the Mott boundary
  - Antiferromagnetic ordering of magnetic moments, magnetism at finite temperature
  - Qualitative explanation of mysterious phenomena, such as the anomalous raise in resistivity as one applies pressure in  $\text{Am}_2\text{S}_7$ ,...
-

# Beyond single site DMFT

## What is missing in DMFT?

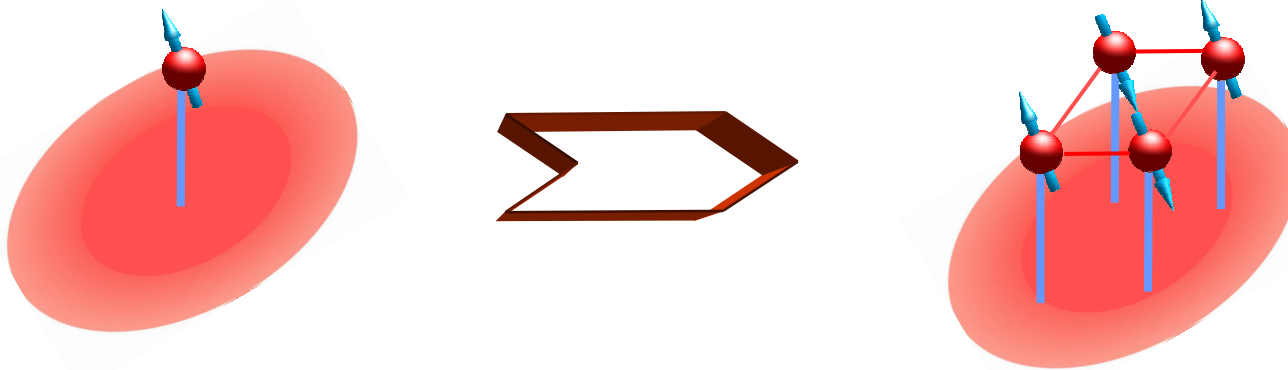
- Momentum dependence of the self-energy  $m^*/m=1/Z$
- Various orders: d-wave SC, ...
- Variation of  $Z$ ,  $m^*$ ,  $\tau$  on the Fermi surface
- Non trivial insulator (frustrated magnets)
- Non-local interactions (spin-spin, long range Columb, correlated hopping..)

### Present in DMFT:

- Quantum time fluctuations

### Present in cluster DMFT:

- Quantum time fluctuations
- Spatially short range quantum fluctuations



# The simplest model of high $T_c$ 's

$$H = - \sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \frac{1}{2} \sum_{ij} J_{ij} \mathbf{S}_i \mathbf{S}_j \quad \text{t-J, PW Anderson}$$

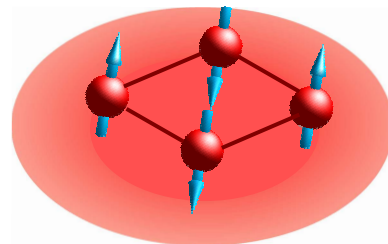
Hubbard-Stratonovich  $\rightarrow$  (to keep some out-of-cluster quantum fluctuations)

$$S = \int_0^\beta d\tau \left\{ \sum_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}^\dagger(\tau) \left( \frac{\partial}{\partial \tau} - \mu + \epsilon_{\mathbf{k}} \right) c_{\mathbf{k}\sigma}(\tau) + \sum_i U n_{i\uparrow}(\tau) n_{i\downarrow}(\tau) + \sum_{\mathbf{q}} \left[ \Phi_{\mathbf{q}}^\dagger(\tau) \frac{2g^2}{J_{\mathbf{q}}} \Phi_{\mathbf{q}}(\tau) + ig \mathbf{S}_{\mathbf{q}} (\Phi_{\mathbf{q}}^\dagger(\tau) + \Phi_{-\mathbf{q}}(\tau)) \right] \right\}$$

$$\Gamma[\mathcal{G}, \mathcal{D}] = -\text{Tr} \log(G_0^{-1} - \Sigma) - \text{Tr}[\mathcal{G}\Sigma] + \frac{1}{2} \text{Tr} \log(\mathcal{D}_0^{-1} - \Pi) + \frac{1}{2} \text{Tr}[\mathcal{D}\Pi] + \Phi[\mathcal{G}, \mathcal{D}] \quad \text{BK Functional, Exact}$$

cluster in  $k$  space

$$\mathcal{G}_{\mathbf{K}}(\omega) \rightarrow \sum_{\mathbf{k} \in \mathbf{K}} \mathcal{G}_{\mathbf{k}}$$

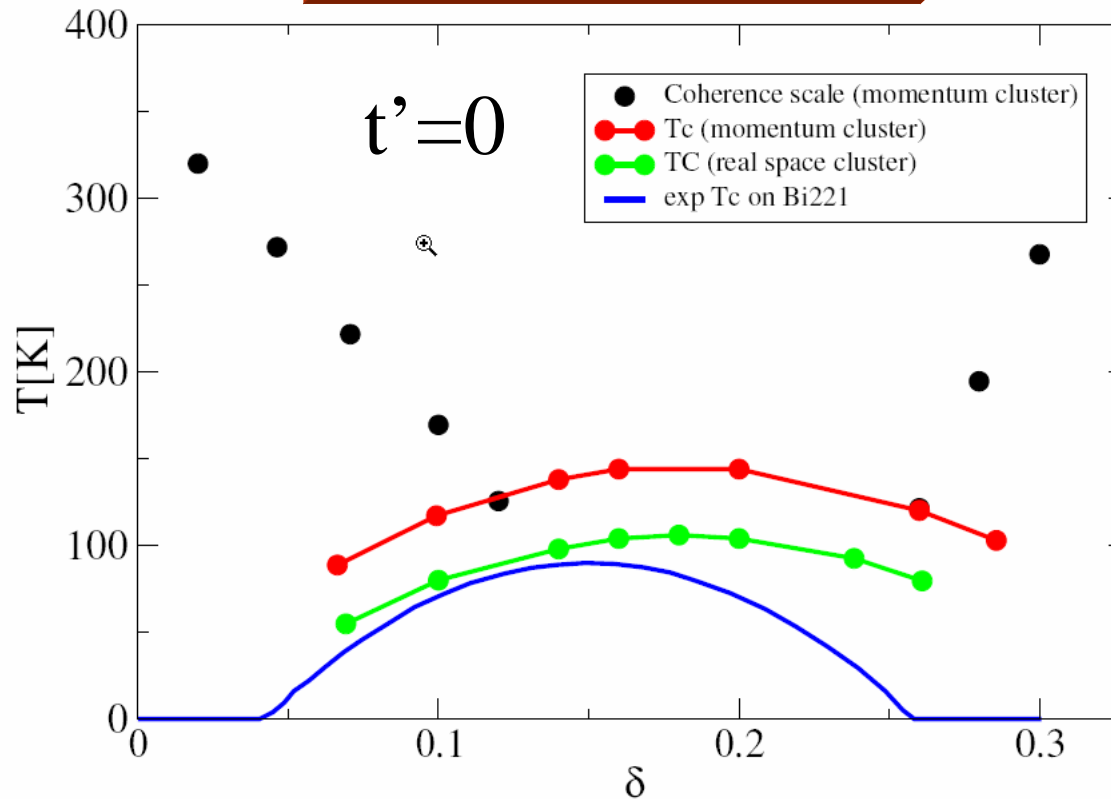


cluster in real space

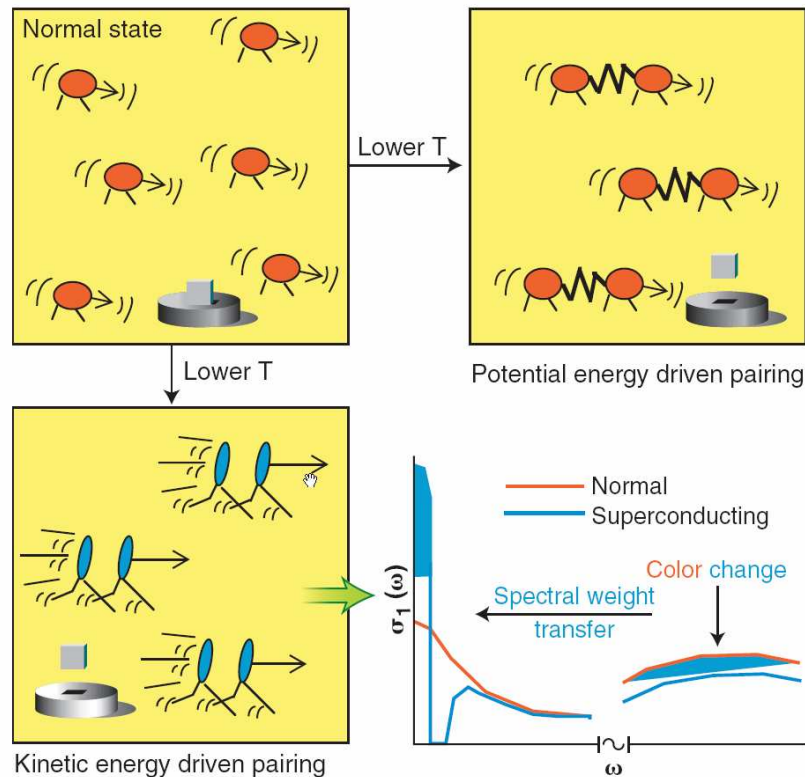
$$\mathcal{G}_{\mathbf{R}_i - \mathbf{R}_j}(\omega) \rightarrow \begin{cases} \mathcal{G}_{\mathbf{R}_i - \mathbf{R}_j}(\omega) & \mathbf{R}_i - \mathbf{R}_j \text{ inside the cluster} \\ 0 & \text{otherwise} \end{cases}$$

# What can we learn from "small" Cluster-DMFT?

## Phase diagram



# Insights into superconducting state (BCS/non-BCS)?



**BCS:** upon pairing potential energy of electrons decreases, kinetic energy increases

(cooper pairs propagate slower)  
Condensation energy is the difference

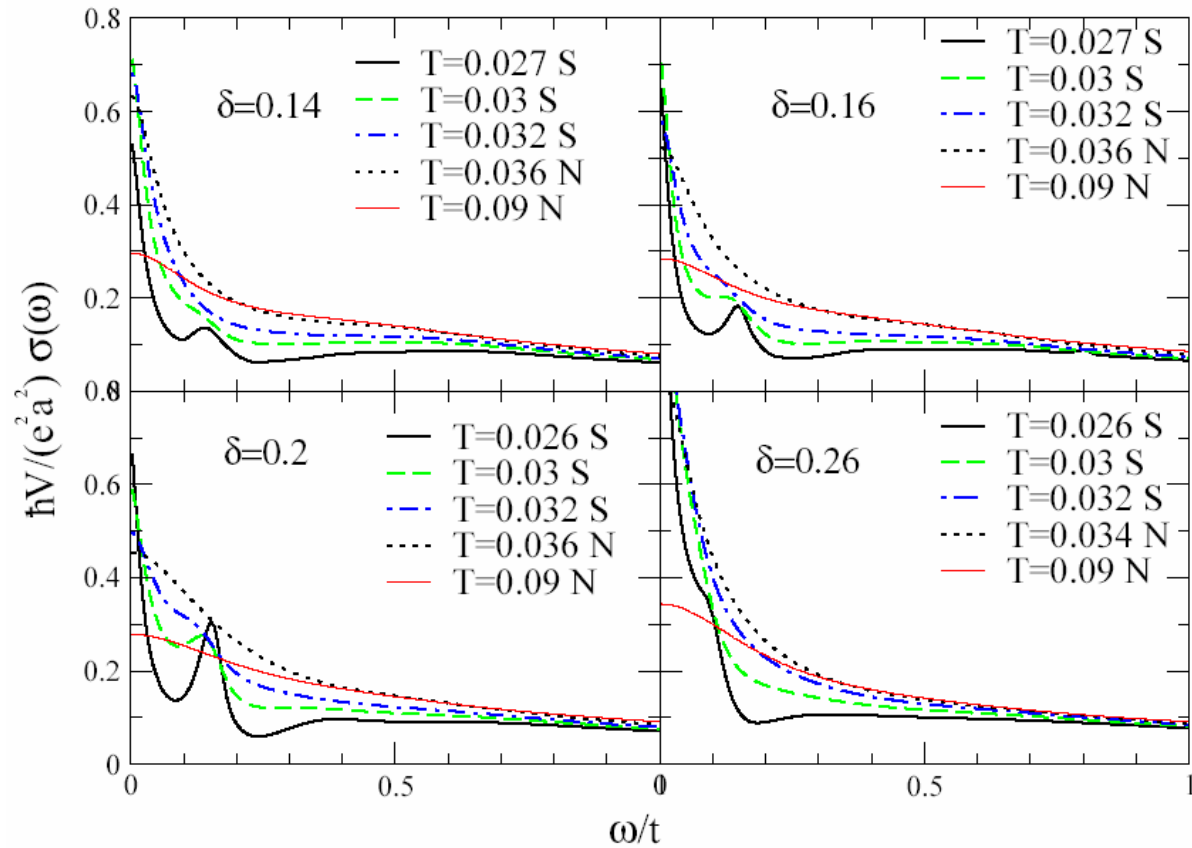
**non-BCS:** kinetic energy decreases upon pairing

(holes propagate easier in superconductor)

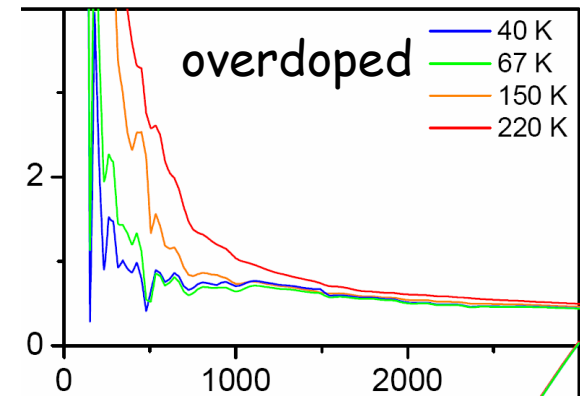
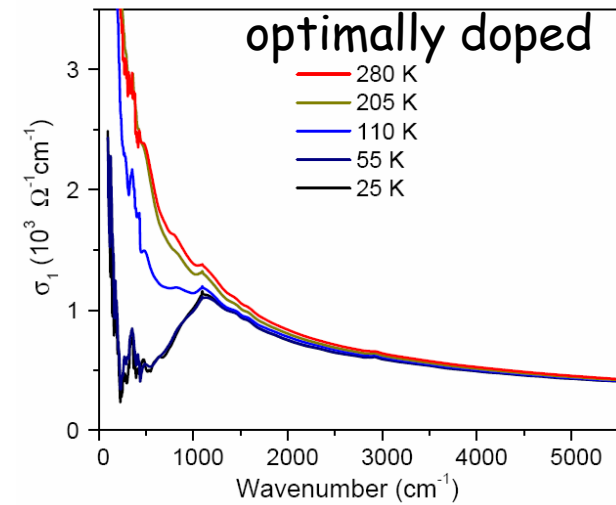
J. E. Hirsch, *Science*, **295**, 5563 (2226)



# Optical conductivity



cond-mat/0601478



D van der Marel, Nature 425, 271-274 (2003)

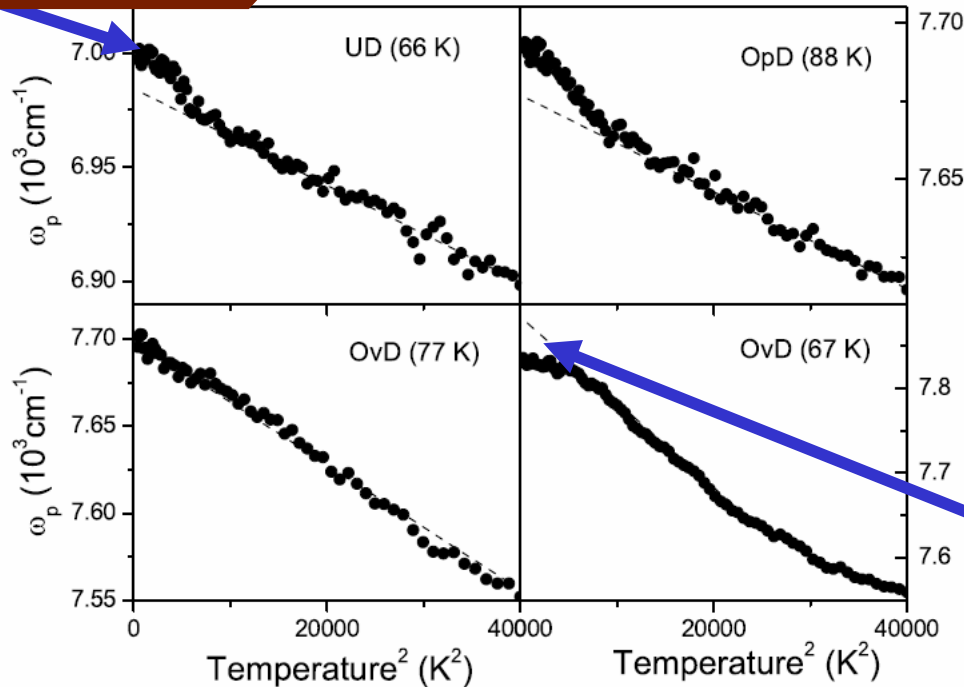
# Optical weight, plasma frequency

$$W(\Omega_c, T) \equiv \int_0^{\Omega_c} \sigma_1(\omega, T) d\omega = \frac{\pi e^2 a^2}{2\hbar^2 V} \langle -\hat{K} \rangle$$

$$\Omega_c \sim 1\text{eV}$$

Weight bigger in SC,  
 K decreases (**non-BCS**)

Bi2212



Weight smaller in SC,  
 K increases (**BCS-like**)

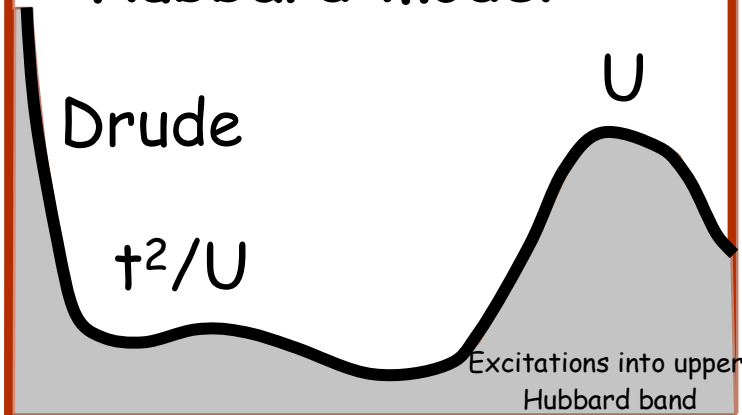
D. van der Marel et.al., in preparation

# Hubbard versus t-J model

## Kinetic energy in Hubbard model:

- Moving of holes
- Excitations between Hubbard bands

### Hubbard model

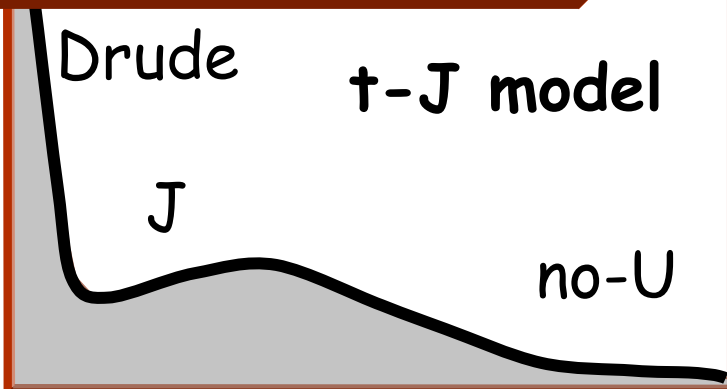


$$W(\Omega_c, T) \equiv \int_0^{\Omega_c} \sigma_1(\omega, T) d\omega = \frac{\pi e^2 a^2}{2\hbar^2 V} \langle -\hat{K} \rangle$$

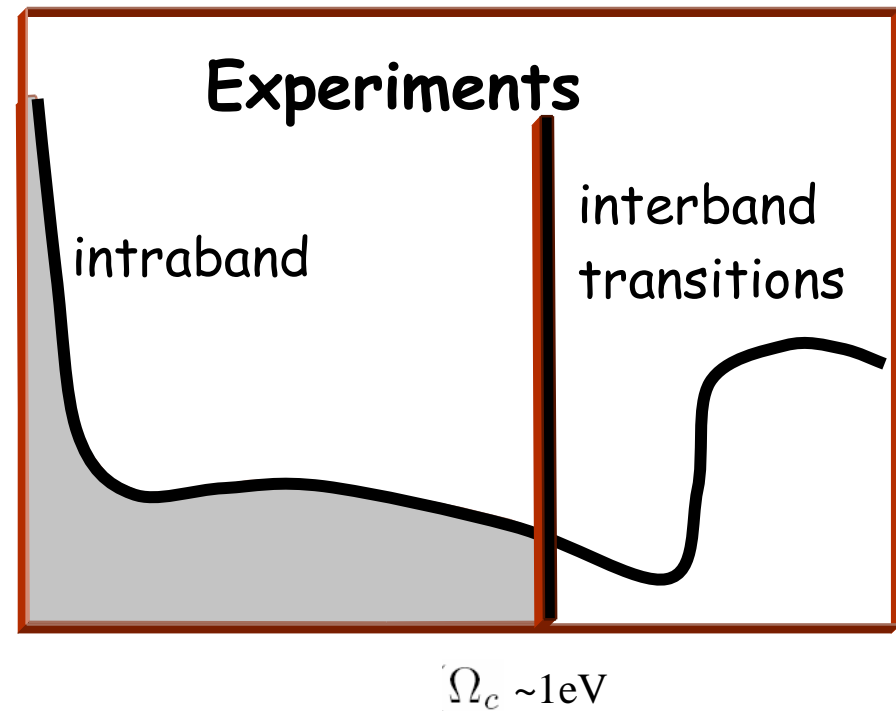
## Kinetic energy in t-J model

- Only moving of holes

### t-J model



### Experiments

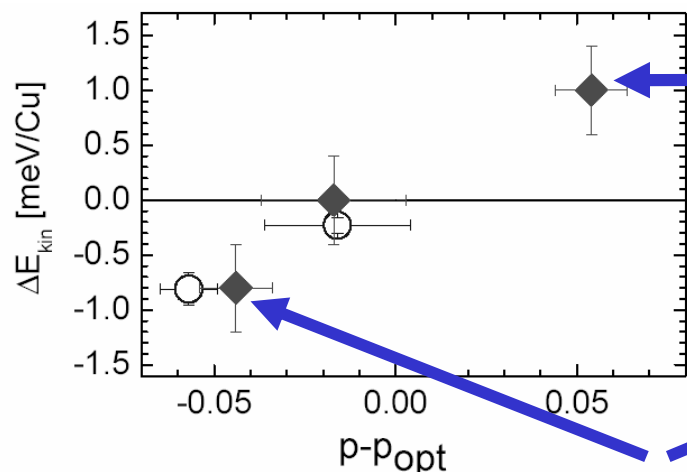


# Kinetic energy change



The State University of New Jersey

Department of Physics and Astronomy

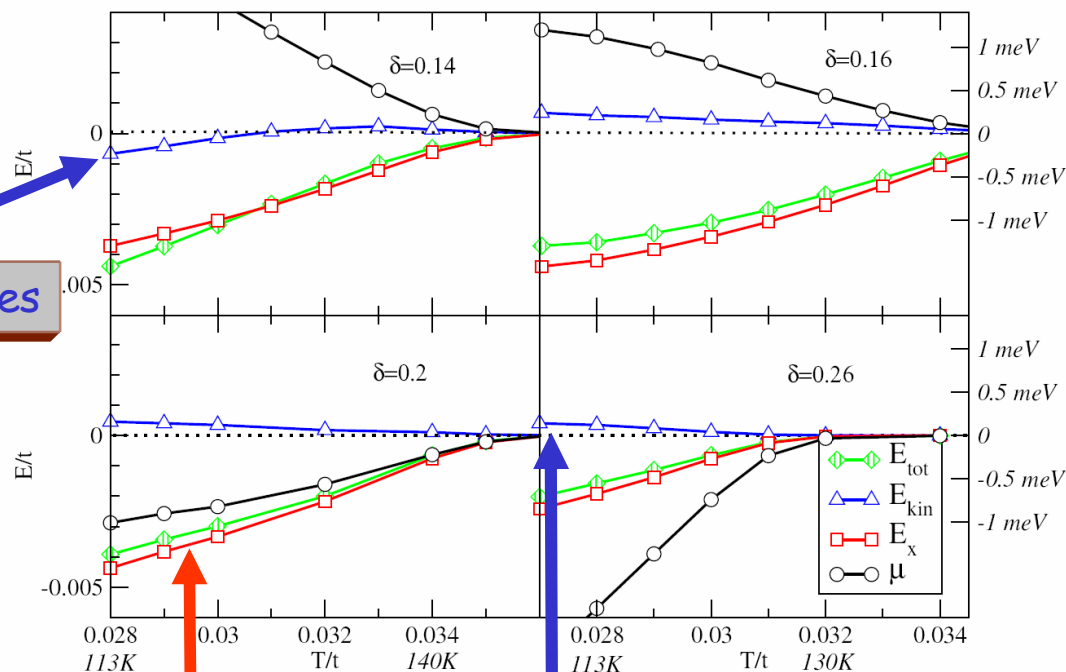


Kinetic energy increases

cluster-DMFT, cond-mat/0601478

Kinetic energy decreases

FIG. 2: Change  $\Delta E_{kin}$  of the kinetic energy, in meV per copper site, calculated from equations (1) and (3), versus the charge  $p$  per copper with respect to  $p_{opt}$  (Eq. 6). Full diamonds: data from Ref. [3], high frequency cut-off 1 eV. Open circles: data from Ref. [1], high frequency cut-off 1.25 eV. Error bars: vertical, uncertainties due to the extrapolation of the temperature dependence of the normal state spectral weight down to zero temperature; horizontal, uncertainties resulting from  $T_c/T_{c,max}$  through Eq. 6 (see text). We have taken  $T_{c,max} = (83 \pm 2)$  K for films and  $(91 \pm 2)$  K for crystals.



Kinetic energy increases

Guy Deutscher<sup>1</sup>, Andrés Felipe Santander-Syro<sup>2</sup>  
and Nicole Bontemps<sup>3</sup>

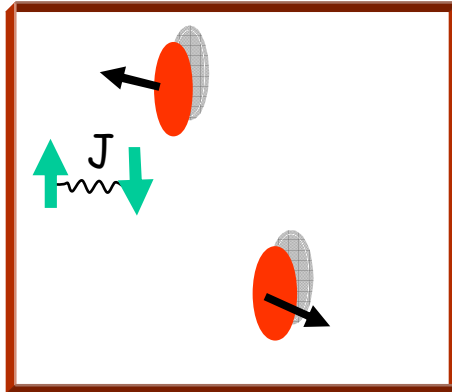
cond-mat/0503073

Phys Rev. B **72**, 092504 (2005)

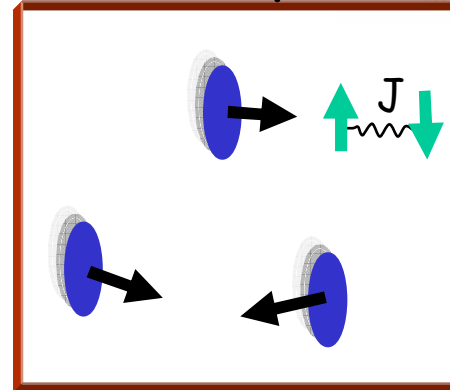
Exchange energy decreases and gives  
largest contribution to condensation energy

# Kinetic energy upon condensation

underdoped

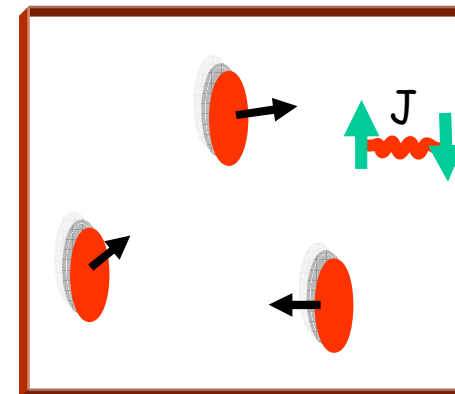
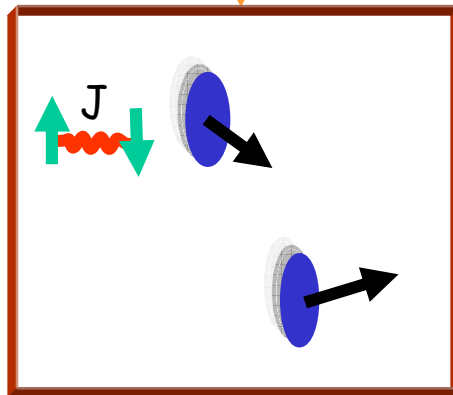


overdoped



electrons gain energy due to exchange energy  
holes gain kinetic energy (move faster)

electrons gain energy due to exchange energy  
hole loose kinetic energy (move slower)



same as RVB (see P.W. Anderson Physica C, 341, 9 (2000),  
or slave boson mean field (P. Lee, Physica C, 317, 194 (1999))

BCS like

## Conclusions



- LDA+DMFT can describe interplay of lattice and electronic structure near Mott transition. Gives physical connection between spectra, lattice structure, optics,....
    - Allows to study the Mott transition in open and closed shell cases.
    - In both Ce and Am single site LDA+DMFT gives the zeroth order picture
    - Am: Rich physics, mixed valence under pressure.
    - Describes magnetism of Curium
  - 2D models of high-T<sub>c</sub> require cluster of sites. Some aspects of optimally doped, overdoped and slightly underdoped regime can be described with cluster DMFT on plaquette:
    - Evolution from kinetic energy saving to BCS kinetic energy cost mechanism
-