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Possible FFLO inhomogeneous superconducting state in CeCoIn₅

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These are preliminary lecture notes, intended only for distribution to participants
Possible FFLO inhomogeneous superconducting state in CeCoIn$_5$

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"The human understanding is of its own nature prone to suppose the existence of more order and regularity in the world than it finds."
F. Bacon
Outline

1. Introduction to FFLO state,
2. The H-T phase diagram in CeCoIn$_5$
3. Experimental results suggesting an FFLO state in CeCoIn$_5$,
4. Perspective & Conclusion
Part 1:  
Introduction to FFLO state

- What is FFLO state?  
- Why is it inhomogeneous?
The superconducting state becomes inhomogeneous at high magnetic fields.

Cooper pair $|k, \uparrow\rangle \bar{\epsilon} \cdots \bar{\epsilon} |{-k, \downarrow}\rangle$

Pair breaking mechanisms at high magnetic fields:

**“Orbital”**

$H_{\text{orb}} = \Phi_0 / 2 \pi \xi^2$

Vortex state (Abrikosov, 1957)

**“Spin”**

$H_p = \sqrt{2\Delta/g\mu_B}$

A. Clogston, *PRL* 9, 266 (1962)

Superconductivity survives if:

- spin triplet
- FFLO
An inhomogeneous state is predicted in a spin-singlet superconductor when $g\mu_B H \geq \Delta$

A.I. Larkin & Y.N. Ovchinnikov, *JETP*. 47, 1136 (1964)

Ginzburg-Landau free energy:

$$F = \alpha |\Delta|^2 + \beta |\Delta|^4 + \kappa |\nabla \Delta|^2 + \ldots + O(|\Delta|^6)$$

$\rightarrow$ non-uniform solution when $\kappa < 0$

For $s$-wave, in clean limit: 1st order transition

Maki Parameter: a useful criteria for the FFLO state

\[ \alpha = \sqrt{2 \frac{H_{\text{orb}}}{H_p}} \]


FFLO state can exist if \( \alpha > 1.8 \)

Gruenberg & Gunther, *PRL* 16, 996 (1966)

**Estimated Maki Parameter for CeCoIn\(_5\):**

\[ \alpha = 4.5 \quad H \perp c \]

\[ 3.6 \quad H \parallel c \]

Theoretical fit to \( H_{c2} \) shows that Pauli limiting is important for \( H \perp c \) only

H.Won *et al.* *PRB* 69, 180504 (2004)
Origin of inhomogeneity in the FFLO state

The spin degeneracy is lifted at high fields.

Cooper pairs acquire a finite momentum $q \approx g \mu_B H / \hbar v_F$

$\Delta(r) = \Delta_0 e^{i q \cdot r}$ (FF)
$\Delta(r) = \Delta_0 \cos(qr)$ (LO)

Nodal planes!
Structure of the vortex+FFLO state?

Take into account orbital effects:
\[ \nabla \psi \rightarrow (\nabla - 2 ieA/c)\psi \]

\[ \Rightarrow \text{minimize } F \text{ with Landau level (LL) wavefunctions} \]

Kinetic energy of the \( n^{th} \) LL: \(|(2n+1)/\ell^2 - q^2|\)

"modulated" vortex lattice in 3D (\( n=0, q \neq 0 \))

\[ \Delta(r) \]

multiquanta vortices in 2D (\( n>0, q \neq 0 \))

Search for FFLO in unconventional superconductors

Pauli limit is rarely reached in conventional superconductors!

The low dimensionality + small coherence length in unconventional SC makes them potential candidates for FFLO

\( \kappa-(BETS)_2GaCl_4 \)


\( \kappa-(BEDT-TTF)_2Cu(NCS)_2 \)

Part 2:
H-T phase diagram in CeCoIn$_5$

- Unconventional heavy fermion superconductivity in CeCoIn$_5$
- 1$^{st}$ order transition and the novel superconducting state at high fields
- Field-tuned quantum critical point near $H_{c2}$
CeCoIn$_5$: a heavy fermion superconductor

- **Strong coupling** ($\Delta C/C \approx 4.5 \ k_B T_c$)
- **Heavy fermion** ($\gamma_0 \approx 0.3 \ JK^{-2} \ mol^{-1}$)
- **Clean limit** ($l \geq 1 \mu m$, $\rho_0 \leq 0.3 \mu \Omega cm$)
- **Quasi-2D Fermi surface sheets**
- **$d_{x^2-y^2}$ symmetry**

Step-like change at the superconducting transition

1. Magnetocaloric effect:

A. Bianchi et al. PRL 89, 137002 (2002)

2. Thermal conductivity:

C. Capan et al. PRB 70, 134513 (2004)

K. Izawa et al. PRL 87, 057002 (2001)
First order superconducting transition in CeCoIn$_5$

- Observed for both $H \parallel [100]$ ($T < T_0 \approx 1\text{K}$) and $H \parallel [001]$ ($T < T_0 \approx 0.7\text{K}$)
- 1$^{\text{st}}$ order transition persists and $T_0$ increases under pressure

T. Tayama et al. PRB 65, 180405 (2002)
T. Tayama et al. JPSJ 74, 1115 (2005)
See also T.P. Murphy et al. PRB 65, 100514 (2002)
Novel superconducting state at high fields

Additional transition line within the superconducting state:
- Second anomaly in specific heat
- Kink in thermal conductivity $\kappa(H)$

A. Bianchi et al. PRL 91, 187004 (2003)
C. Capan et al. PRB 70, 134513 (2004)
The second transition is also observed with various other techniques

Kink in the penetration depth $\lambda$ (=inverse superfluid density)

C. Martin et al. PRB 71, 020503 (2005)

Kink in the ultrasound attenuation (transverse sound velocity)

T. Watanabe et al. PRB 70, 020506 (2004)
H-T phase diagram of CeCoIn$_5$

Consistent with a mean field phase diagram obtained in a GL model of a 2D d-wave superconductor


C.F.Miclea et al. PRL 96, 117001 (2006)
Field tuned quantum critical point in CeCoIn$_5$

Strong deviation from Fermi Liquid behavior in resistivity and specific heat close to $H_{c2}$

$\gamma(\Delta, T) = (J/mol-K^2)$

$H_{c2}$

$\rho = \rho_0 + AT^2$

$A(\Delta) \propto (\Delta - 5.1)^{-1.37}$


QCP coincides with $H_{c2}$ in CeCo(In,Sn)$_5$.

The quantum critical behavior is suppressed with pressure

With increasing pressure, $H_{QCP}$ is suppressed much faster than $H_{c2}$.

⇒ The origin of QCP in CeCoIn$_5$ is not superconductivity

F. Ronning et al. PRB 73, 064519 (2006)
Is there a connection between the QCP and the novel high field state?

- The origin of QCP is unlikely related to the FFLO state! (see pressure + Sn-doping results)

- Effect of strong AF spin fluctuations:
  - On the 1st order transition?
  - On vortex cores?
  - On nodal planes?

  T.Tayama et al. JPSJ 74, 1115 (2005)

- Enhanced $X_{\text{spin}}$ leads to a reduced phase space available for FFLO

  A.B.Vorontsov, M.J.Graf cond-mat/0606572

→ consistent with the observed pressure dependence of $T_{\text{FFLO}}$
Part 3:
Existing evidence for FFLO in CeCoIn$_5$

- Enhanced QP density in the high field state
- Probing the nodal planes with the anisotropy of thermal conductivity
Enhanced QP density in the high field state:

✓ Superfluid density decreases faster
C.Martin et al. PRB 71, 020503 (2005)

✓ Thermal conductivity is enhanced
C.Capan et al. PRB 71, 020503 (2005)

✓ $X_{\text{spin}}$ is enhanced
V.Mitrovic et al. cond-mat/0605305
Probing the nodal planes with thermal conductivity.

One would expect an enhanced QP density for $H \perp J \Rightarrow K_{H \perp J} > K_{H \parallel J}$.
The anisotropy of $\kappa$ in the normal state is well understood:

The anisotropy of thermal conductivity in the normal state is due to the Lorentz force on electrons.
The anisotropy of $\kappa$ in the mixed state is opposite to what we expected for nodal planes:

**We expected:**

$\kappa_{H \perp J} > \kappa_{H \parallel J}$

**We observe:**

$\kappa_{H \perp J} < \kappa_{H \parallel J}$

Close to $H_{c2}$, the anisotropy of $\kappa$ should be small since vortices overlap strongly.

In CeCoIn$_5$, we found a large anisotropy at low temperatures!
The anisotropy of $\kappa$ in the mixed state increases upon cooling:

- $\kappa/T$ is reminiscent of $C/T$ divergence close to $H_{c2}$.
- Sign of anisotropy consistent with previous report at lower fields.
Expected anisotropy of $\kappa$ in the mixed state:

- **At low temperature:** Larger density of quasiparticles excited for $H \perp v$ (Volovik effect)
  \[ \kappa_{H \perp J} > \kappa_{H \parallel J} \]
  - Vortices scatter quasiparticles with $H \perp v$ preferentially
  \[ \kappa_{H \perp J} < \kappa_{H \parallel J} \]

- **s-wave**: reversal of the 0-90° anisotropy of $\kappa$ upon cooling

- **d-wave**: similar result + nodal quasiparticles
Possible reasons for the observed anisotropy

\[ \kappa(H \parallel J) > \kappa(H \perp J) \]

- Vortex contribution dominates over the nodal planes?
  - \rightarrow QP density in vortex core > in nodal plane
  - \rightarrow Scattering by vortices > by nodal planes

- The spatial structure of FFLO state in presence of vortices may not involve nodal planes!
Vortex vs FFLO contribution to the anisotropy of $\kappa$
Interplay between FFLO and vortex structures

- The real space profile of $\Delta$ is not universal
- Near $T_0$ the profile of $\Delta$ is determined by $n$ (LL index) and the vortex lattice structure
Conclusion

- Experimental H-T Phase diagram of CeCoIn$_5$ with H in-plane:
  - 1$^{\text{st}}$ order superconducting transition below $T^*=1K$
  - 2$^{\text{nd}}$ order transition between vortex state-FFLO below $T_{\text{FFLO}}=0.3K$
  - Field tuned QCP near $H_{c2}$ for both $H||[001]$ and [100] at ambient pressure

- The anisotropy of thermal conductivity can be used to probe nodal planes in the high field superconducting state: we found an anisotropy opposite in sign to what is expected from nodal planes.

Need for more theoretical + experimental work!
Controversies about FFLO in CeCoIn$_5$:

- Is there a second transition line for H||[001]?
- Are there nodal planes in the high field state?
- Are the magnetization steps evidence for multiquanta vortices?

K. Kakuyanagi et al., *PRL* 94, 047602 (2005)

See also R. Movshovich *Nature* 427, 802 (2004)
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Thank you!
Supplemental slide: H-T phase diagram from resistivity

- H // ab
- Tc
- T_{FFLO}
- TF
- T*

- H // c
- , TF
- , T*

H / H_{c2} vs T (K)
Supplemental slide: Resistivity under pressure for \( H \parallel [001] \)

Graphs showing resistivity \( \rho \) in \( \mu \Omega \cdot \text{cm} \) as a function of temperature squared \( T^2 \) (in \( K^2 \)) for different pressures: 0 GPa, 0.05 GPa, 0.6 GPa, and 1.3 GPa.

For \( H = 6T \):
- \( 0 \text{ GPa} \)
- \( 0.05 \text{ GPa} \)
- \( 0.6 \text{ GPa} \)
- \( 1.3 \text{ GPa} \)

For \( H = 9T \):
- \( 0 \text{ GPa} \)
- \( 0.05 \text{ GPa} \)
- \( 0.6 \text{ GPa} \)
- \( 1.3 \text{ GPa} \)