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Miniworkshop on
New States of Stable and Unstable Quantum Matter
(14 - 25 August 2006)

Coulomb interacting Dirac fermions in disordered graphene

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Coulomb interacting Dirac fermions in disordered graphene

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Outline of the talk

- Introduction: Dirac fermions in graphene
- Nodal vs conventional Fermi liquids
- Pairing between nodal fermions
- Excitonic instability in HOPG and graphene
- Other effects of Coulomb interactions in graphene
- Quantum Hall Effect in graphene
- Conclusions

References:

cond-mat/0607174, 0604180

PRL 96, 027004 (2006)

PRB 73, 115104 (2006) (with W. Shively)

NPB 642, 515 (2004) (with H. Leal)

PRL 87, 246802 (2001)

PRL 87, 206401 (2001)

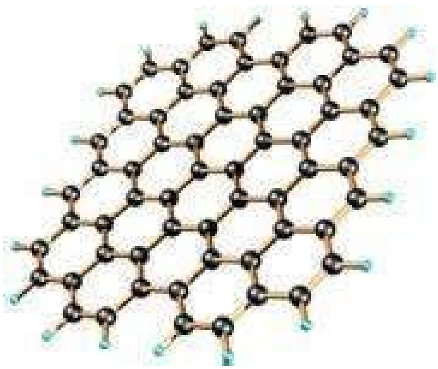
Novel carbon-based materials

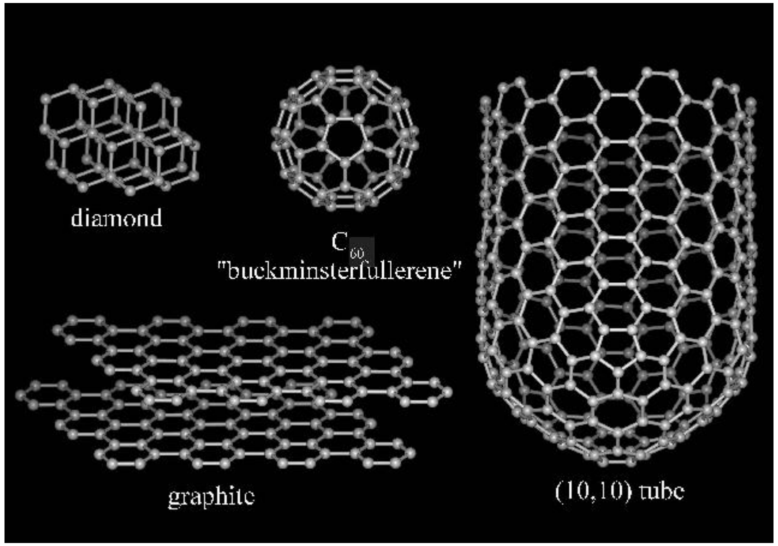
•**Buckyballs** (0D): superconductivity, paramagnetism in crystallized, intercalated state;

•**Nanotubes** (1D): a wealth of interesting mechanical and electrical properties, Luttinger liquid;

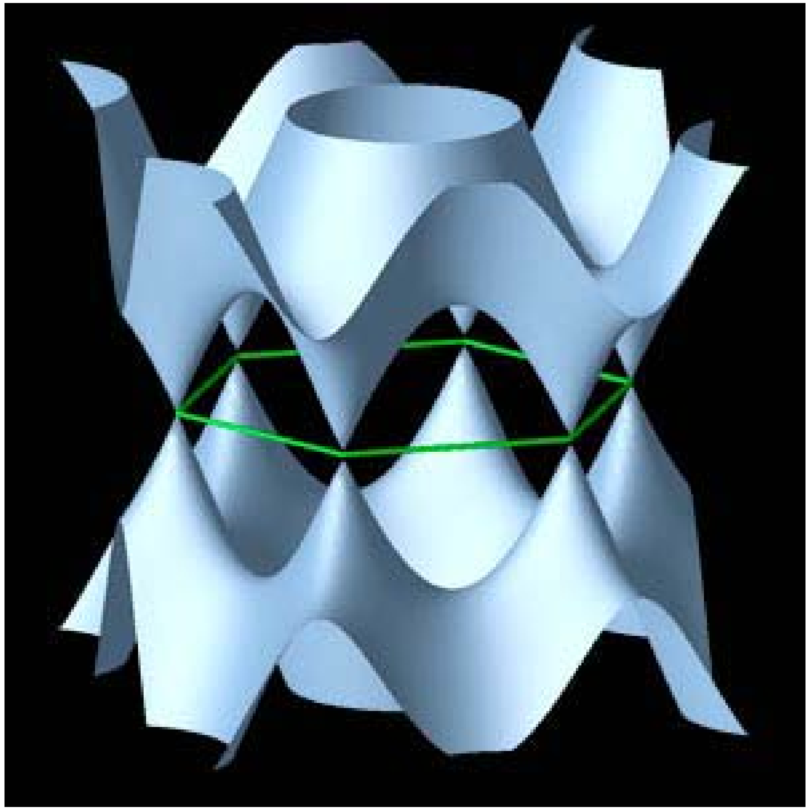
•**HOPG** (quasi-2D): weak ferromagnetism, magnetic field-induced metal-insulator transition, linear qp damping;

•**Graphene** (2D): Berry phase, field-induced Dirac mass (?), anomalous WL behavior,…





Nodal points in the band structure of graphene



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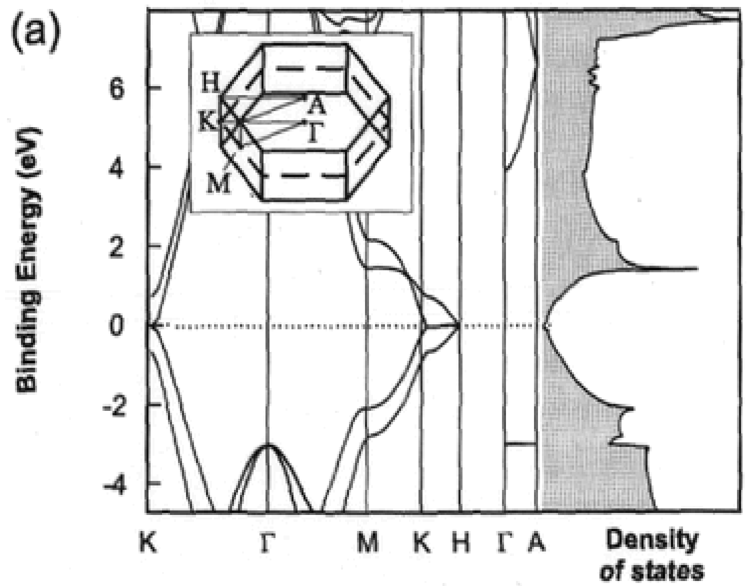

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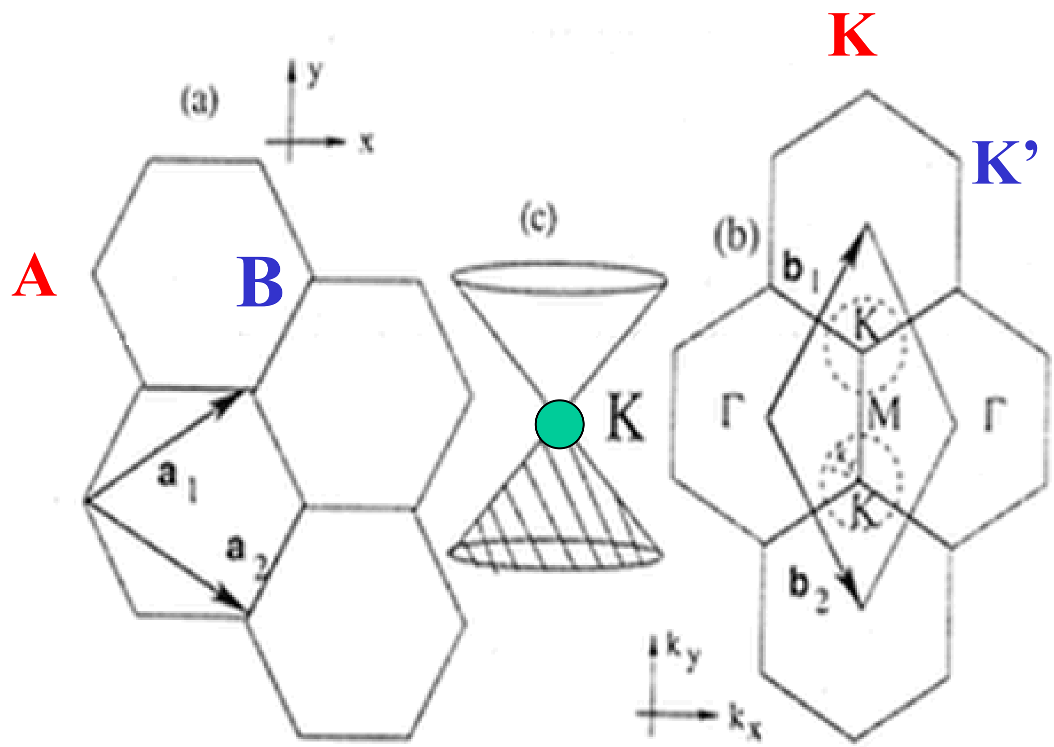
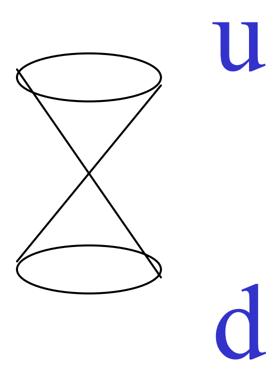
ν(E)

Massless Dirac fermions in graphene

- Continuous description of qp near the nodal points



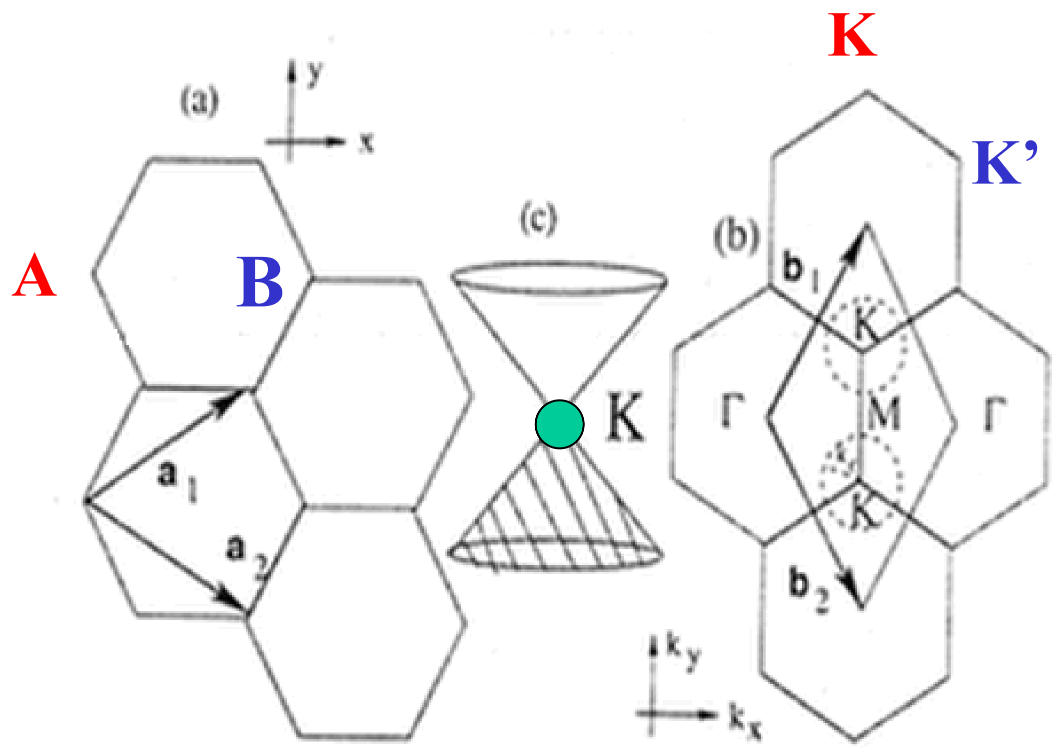


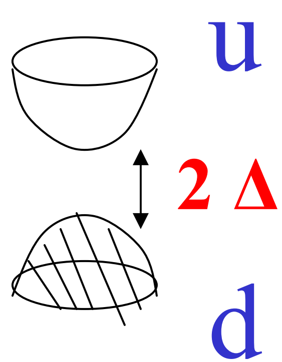
- Dirac (bi-) spinors: $\Psi = (u^{\sigma}_{A}, d^{\sigma}_{A}, u^{\sigma}_{B}, d^{\sigma}_{B})$

- The Dirac Hamiltonian: $H = \psi^{+} i\, v_F \gamma \partial\, \psi$

Massive Dirac fermions in graphene

•Continuous description of qp near the nodal points





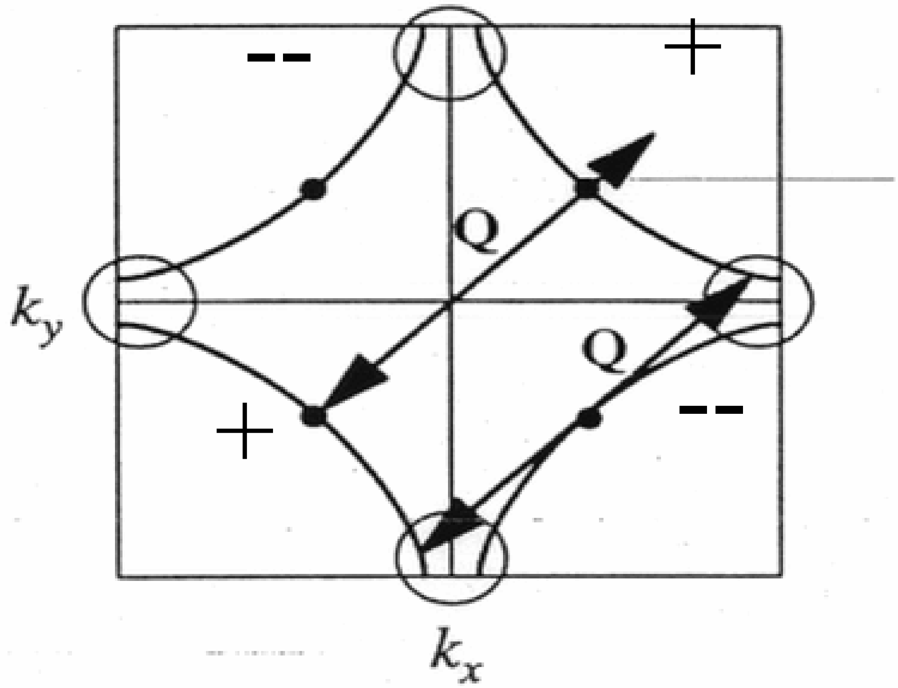
•Dirac (bi-) spinors: $\Psi = (u^{\sigma}_{A}, d^{\sigma}_{A}, u^{\sigma}_{B}, d^{\sigma}_{B})$

• The Dirac Hamiltonian: $H = \psi^{+} (i\, v_F \gamma \partial + \Delta)\, \psi$

Dirac fermions in other condensed matter systems

• **d-wave** superconductors

(high T_c cuprates, heavy fermion materials,..)



$\Delta \sim (\cos k_x - \cos k_y)$

$\varepsilon \sim (\cos k_x + \cos k_y)$

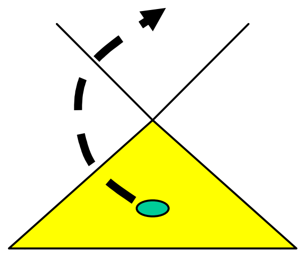
$E(k) = (\varepsilon^2 + \Delta^2)^{1/2} \sim$

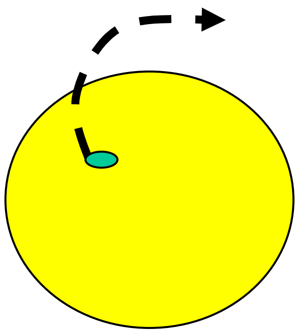
$\sim (v_x^2 k_x^2 + v_y^2 k_y^2)^{1/2}$

•**Dichalcogenides** (2D **f-CDW**);
•**He3** (3D **p-wave SF**) ,
•…

Nodal Dirac points vs extended Fermi surface

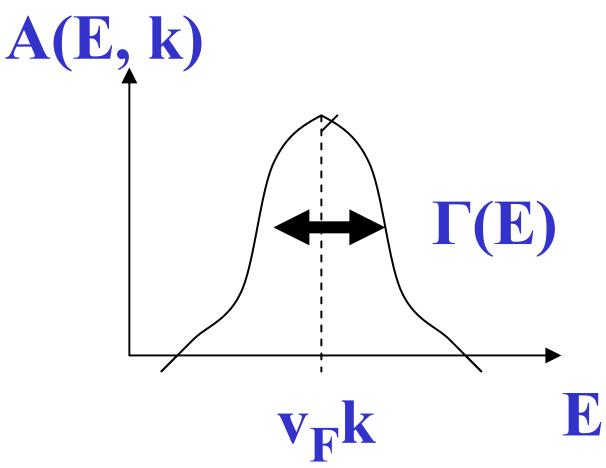
	Dirac points	Fermi surface
•Quasiparticle dispersion	$E(k) \sim k$	$E(k) \sim (k - k_F)$
•Specific heat	$C(T) \sim T^2$	$C(T) \sim T$
•Density of states	$\nu(E) \sim E$	$\nu(E) \sim Const$
•Electron interaction	$U(q) \sim 1/q$	$U(q) \sim Const$
•Conductivity	$\rho(T=0) = Const$	$\rho(T=0) \sim n_e/n_{imp}$
•Landau levels	$E_N \sim (H\, N)^{1/2}$	$E_N \sim H\,(N+1/2)$



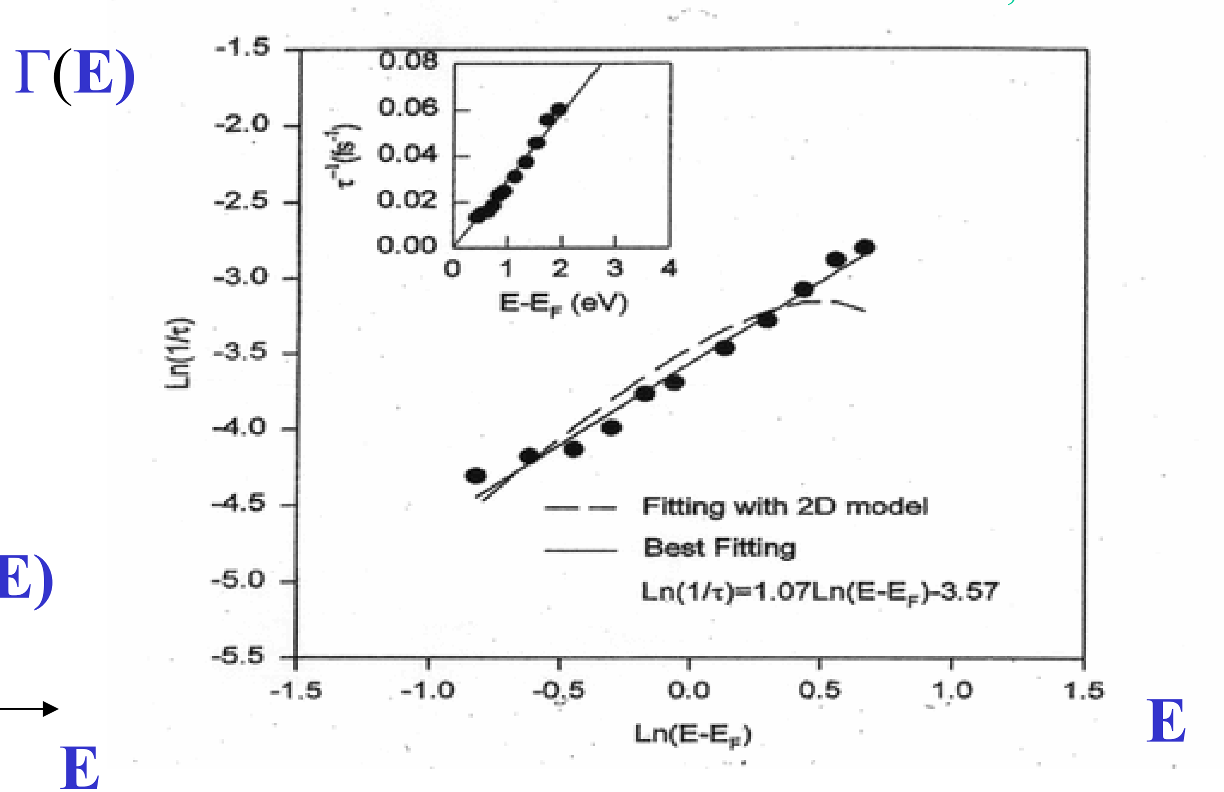


Quasiparticle damping in layered graphite

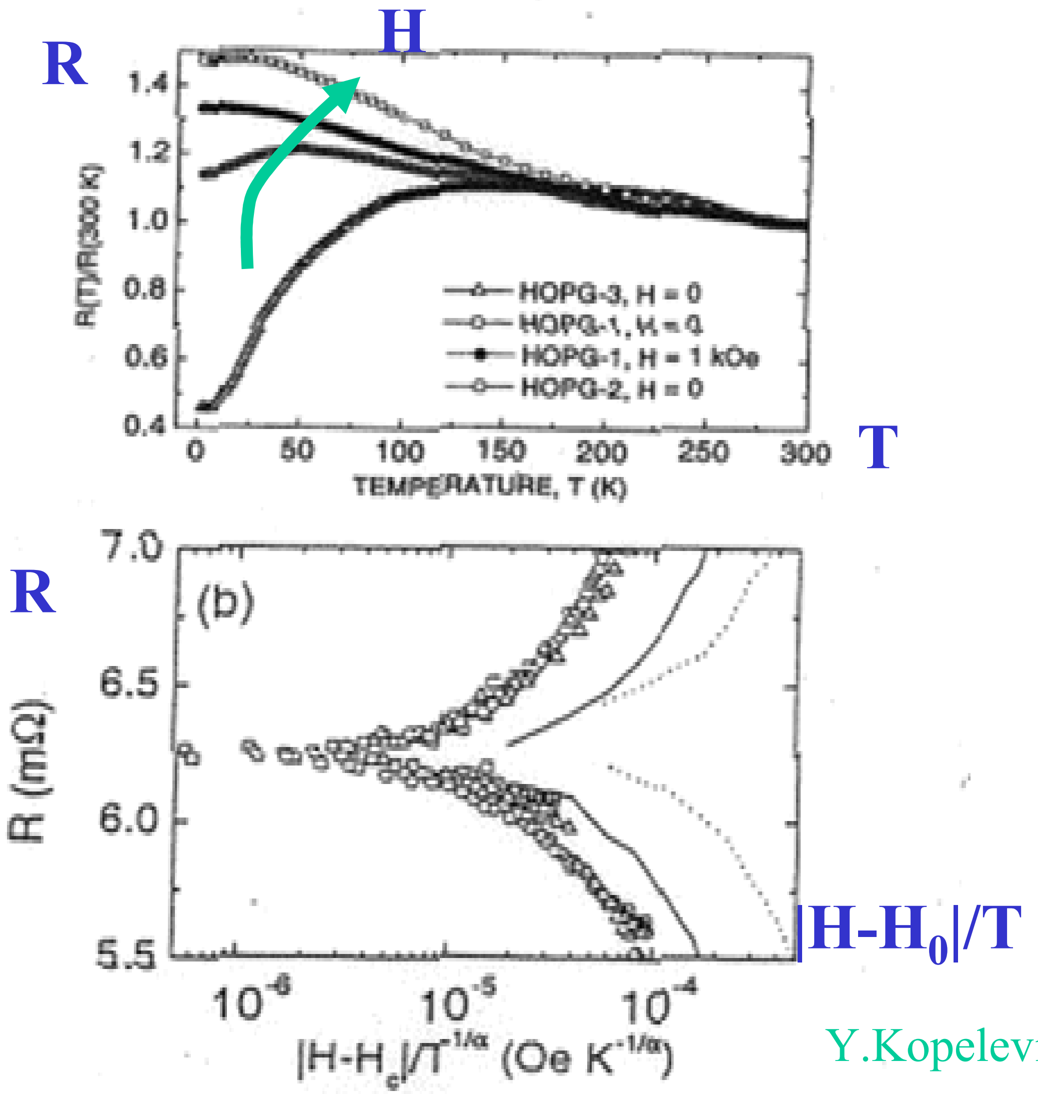
- Fermi liquid: $\Gamma(E) \sim E^2 \ (\times \ln E)$



S.Hu et al, '96



Apparent (semi)metal-insulator transition in layered graphite



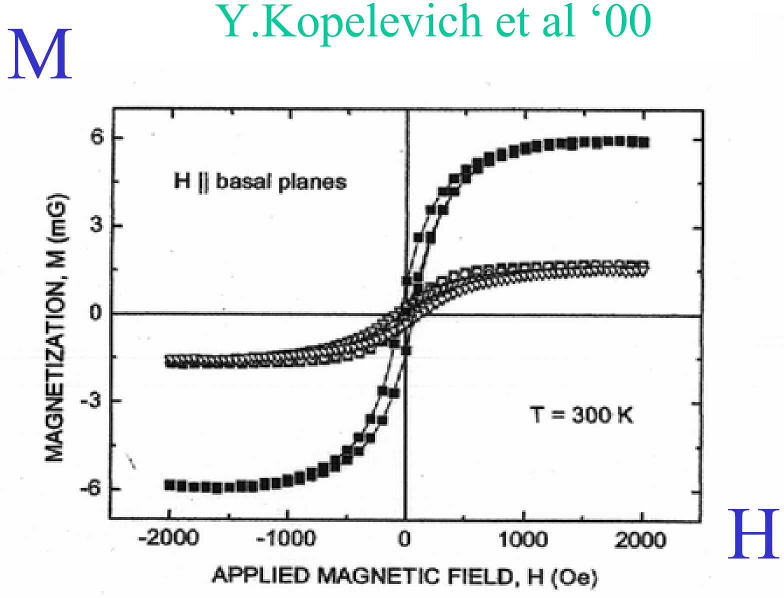
Y.Kopelevich et al, ‘00

Weak ferromagnetism in layered graphite

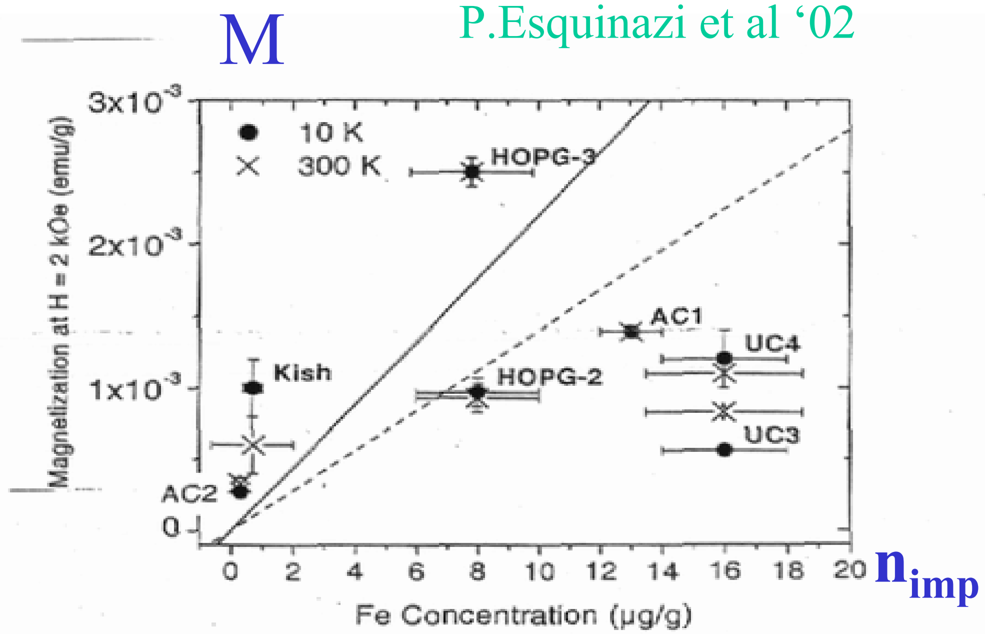
•Small (albeit robust) magnetic moment:

M ~ 0.03-0.05 μ_B/carrier, Tc~500K

Y.Kopelevich et al '00



P.Esquinazi et al '02



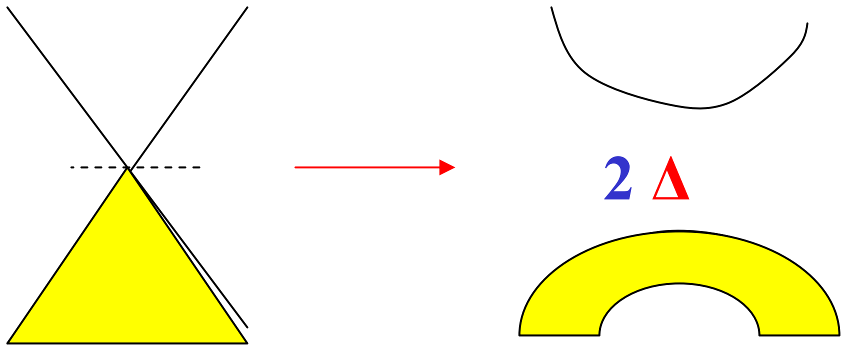
Mechanisms:

•Single-particle (magnetic impurities; structural defects, edges, H-bonds)

•Many-body (Coulomb interactions) ???

Conjecture: latent excitonic insulator transition in graphene

- The experimental evidence might indicate a possible **quantum-critical** behavior due to a nearby quantum phase transition
- In the **ordered** phase, the Dirac fermions develop a **gap**: $\Delta > 0$
- The gap can be induced by magnetic field: $\Delta(H)$



$E(k) = v_F\, k$ $\quad\quad$ $E(k) = v_F\, (k^2 + \Delta^2)^{1/2}$

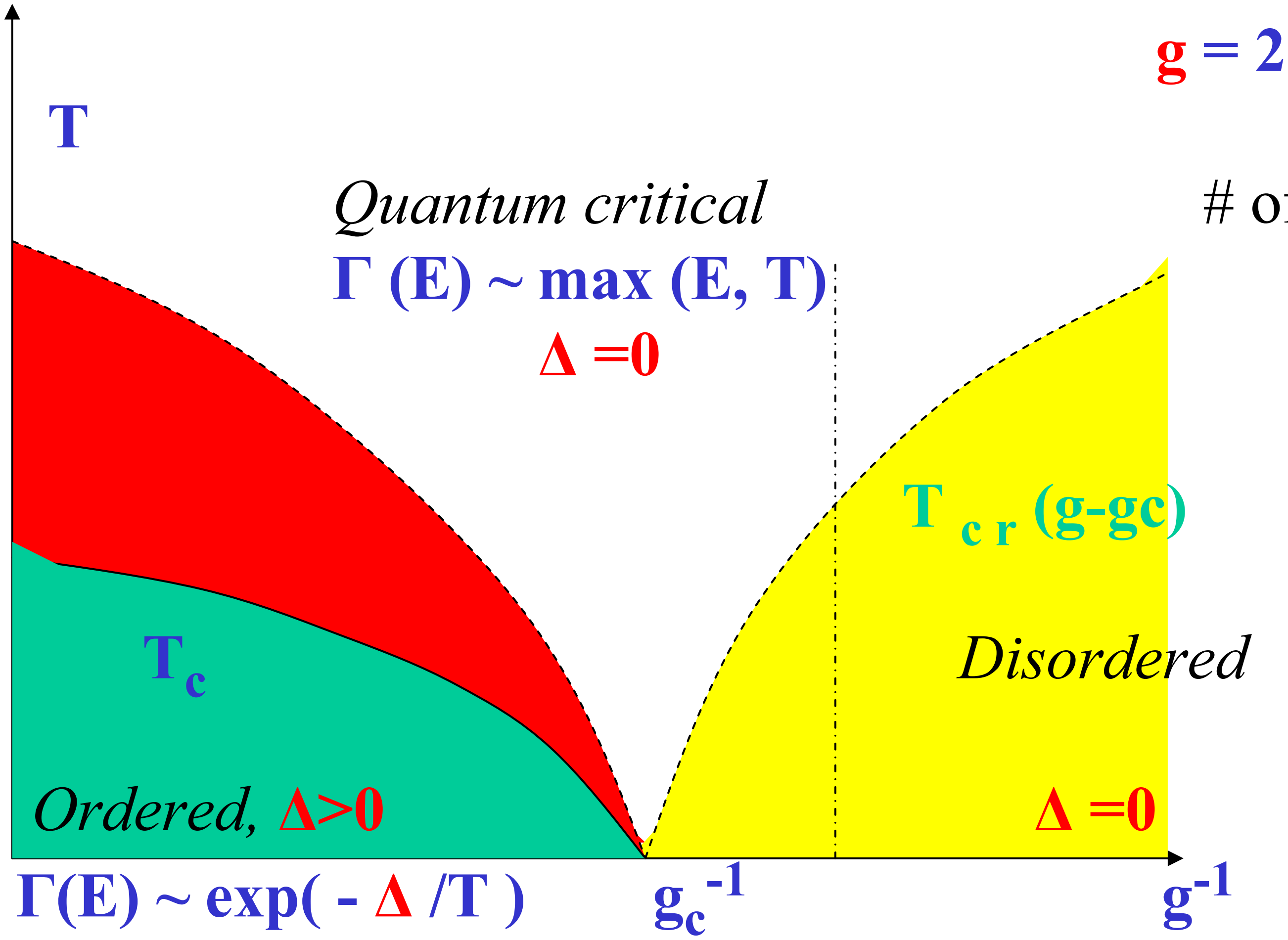
Possible quantum-critical behavior in graphene

Coulomb coupling

$g = 2\pi N e^2 / \varepsilon v_F$

# of fermion species

N=2



T=0 quantum critical point at $g=g_c$

Coulomb interacting Dirac fermions

- The Hamiltonian of graphene (no disorder):

$$H = iv_F \sum_{\alpha=1,2} \int_{\mathbf{r}} \Psi_\alpha^\dagger [\hat{\sigma}_x \nabla_x + (-1)^\alpha \hat{\sigma}_y \nabla_y] \Psi_\alpha$$

$$+ \frac{v_F}{4\pi} \sum_{\alpha,\beta=1,2} \int_{\mathbf{r}} \int_{\mathbf{r}'} \Psi^\dagger{}_\alpha(\mathbf{r}') \Psi_\alpha(\mathbf{r}') \frac{g}{|\mathbf{r}-\mathbf{r}'|} \Psi^\dagger{}_\beta(\mathbf{r}) \Psi_\beta(\mathbf{r})$$

- Dirac fermion propagator:

$$\hat{G}_\alpha^R(\omega, \mathbf{p})^{-1} = (\omega + \mu)\hat{1} - v_F(\hat{\sigma}_x p_x + (-1)^\alpha \hat{\sigma}_y p_y)$$

- Effective interaction: $$U^R(\omega, \mathbf{q}) = \frac{(g_0/q)}{1 + (g_0/q)\,\Pi^R(\omega, \mathbf{q})}.$$

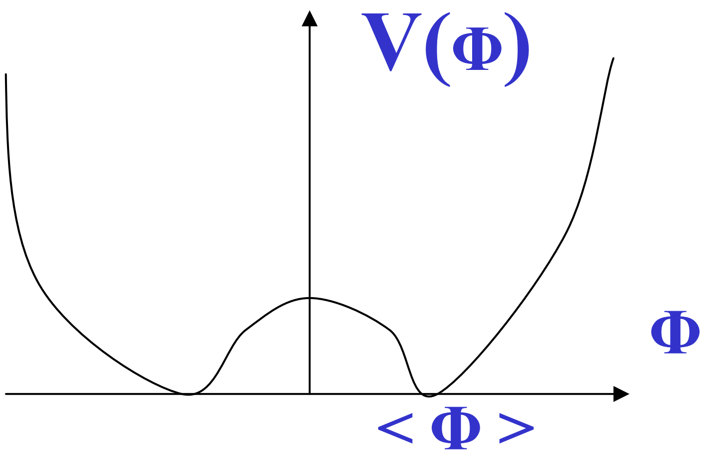
$$\Pi^R(\omega, \mathbf{q}) \approx \frac{1}{4v_F} \frac{\mathbf{q}^2}{\sqrt{v_F^2 \mathbf{q}^2 - (\omega + i0)^2}}, \quad max[\omega, v_F q] > T$$

$$\approx \frac{2T \ln 2}{\pi v_F}\left(1 - \frac{\omega}{\sqrt{(\omega + i0)^2 - v_F^2 \mathbf{q}^2}}\right), \quad max[\omega, v_F q] < T$$

Where do the Dirac fermions get their masses from?

- The Higgs mechanism:

$$L = i\,\bar{\psi}\,(\gamma\partial + \Phi)\psi + |\partial\Phi|^2 \;-\; V(\Phi)$$



$$\Delta \sim \langle \Phi \rangle \neq 0$$

fermion mass (gap)

$$L_{eff} = i\,\bar{\psi}\,(\gamma\partial \;+ \Delta\,)\,\psi$$

Alternative scenario: chiral symmetry breaking

- CSB in QED$_3$ T.Appelquist et al, '88

$$L = i \sum_{f=1}^{N} \bar{\psi}_f (\gamma\partial + A)\psi_f + F^2/2g \qquad F = \partial \times A$$

- Chiral rotation symmetry for **massless** fermions

$$\psi_f^{L,R} = (1 \pm \gamma_5)/2\, \psi_f^{L,R} \rightarrow \exp(i\gamma_5)\, \psi_f^{L,R}$$

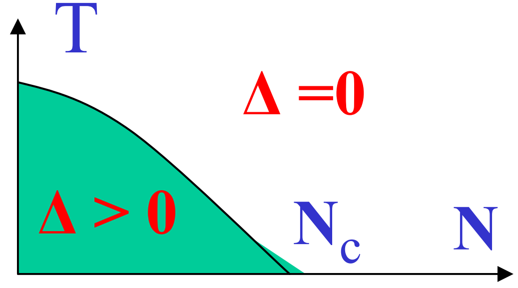
- CSB order parameter, U(2N)→U(N)×U(N)

$$\Delta \sim \sum_{f=1}^{N} \langle \bar{\psi}_f \psi_f \rangle$$

$$\Delta \neq 0, \quad N < N_c$$

$$\Delta = 0, \quad N > N_c$$

CSB phase transition



Excitonic pairing between nodal fermions

•Gap equation: $$\Delta_{\rm p} = \sum_{\rm q,\mp} U(0,{\rm p}-{\rm q}) \frac{(\mp)\Delta_{\rm q}}{2E_q}\theta(\mu \mp E_{\rm q})$$

•Vanishing DOS: **ν (E) ~ E**$^{\beta}$

•Singular (unscreened Coulomb) interaction: **V(q) = g/|q|**$^{\alpha}$

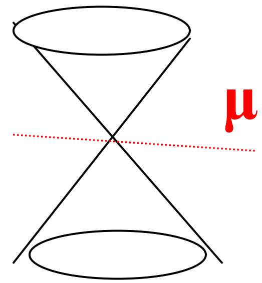
•Solutions: $$\frac{d^2\Delta(\epsilon)}{d\epsilon^2} + \frac{\alpha+1}{\epsilon}\frac{d\Delta(\epsilon)}{d\epsilon} + g\frac{\alpha\Delta(\epsilon)}{\epsilon^{2+\alpha-\beta}} = 0$$

•Pairing occurs **only** for couplings **g > gc**

• $\alpha = \beta > 0$: $$\Delta(\epsilon) = \Delta\left(\frac{\Delta}{\epsilon}\right)^{\alpha/2} \frac{\sin\left[\sqrt{\alpha g - \alpha^2/4}\,\ln(\epsilon/\Delta) + \delta\right]}{\sin\delta}$$

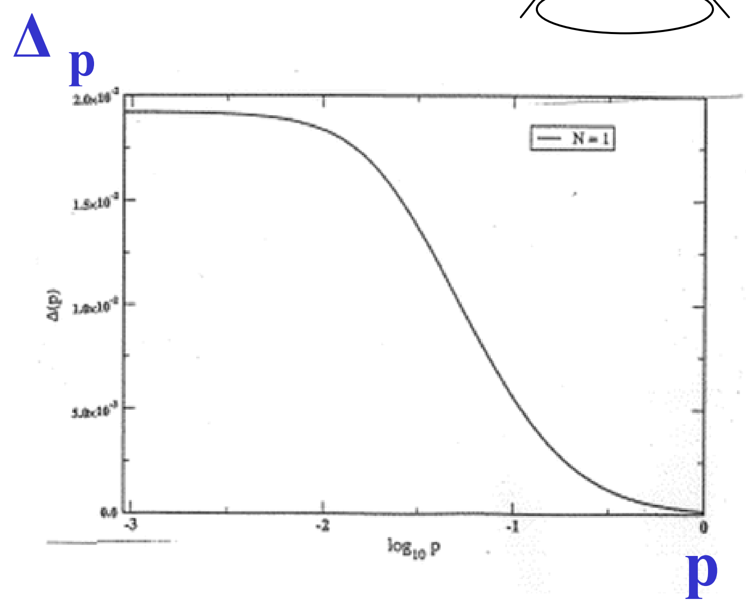
•Formal analogy: pairing between incoherent fermions in cuprates (A. Chubukov et al, '02)

Excitonic pairing in graphene: zero qp density



- Gap equation $\alpha = \beta = 1$

$V(q) = g/|q|$, $\nu(E) \sim E$



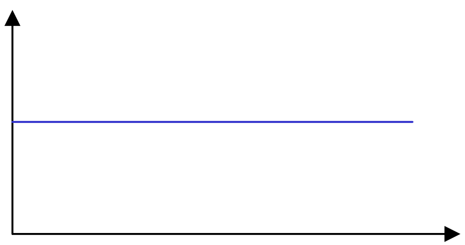
- A finite **critical** coupling: $g > g_c$

$\Delta \sim \exp(-\mathrm{Const} / (g - g_c)^{1/2})$

- Strong momentum dependence

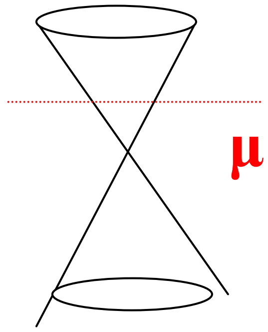
Cf. BCS: **$V(q) = g$, $\nu(E) \sim \mathrm{Const}$**

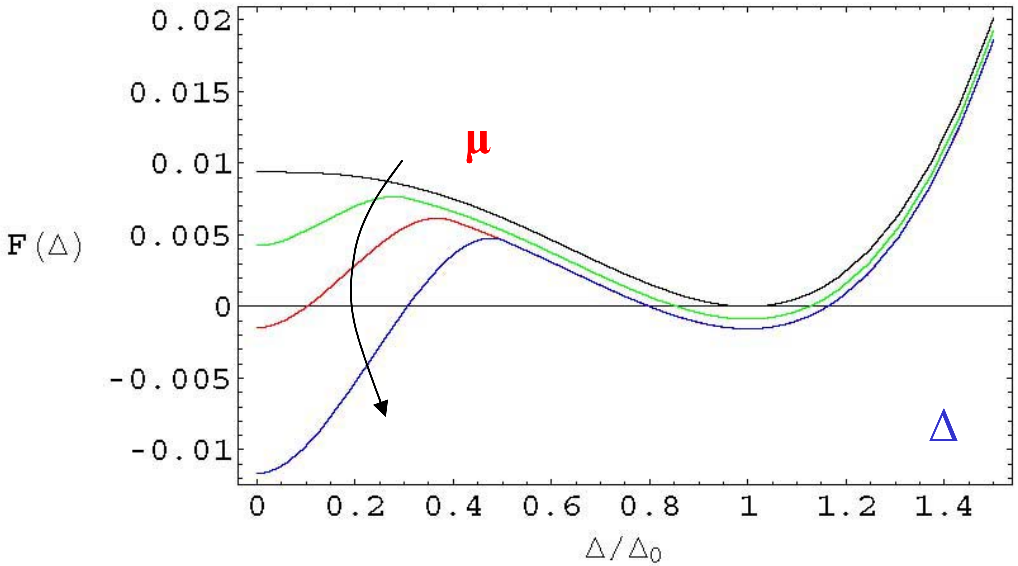
$\Delta p = \Delta \sim \exp(-\mathrm{Const} / g)$



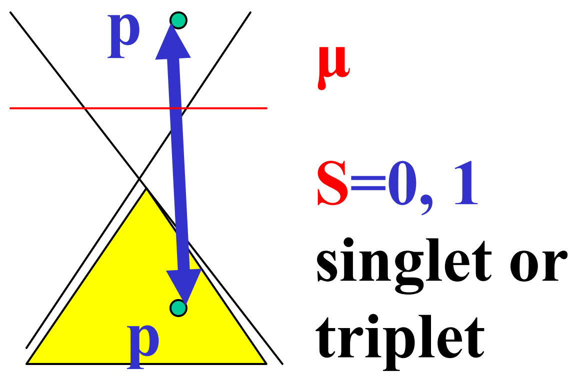
Excitonic pairing in graphene: finite qp density

•Electron density dependence: first order transition



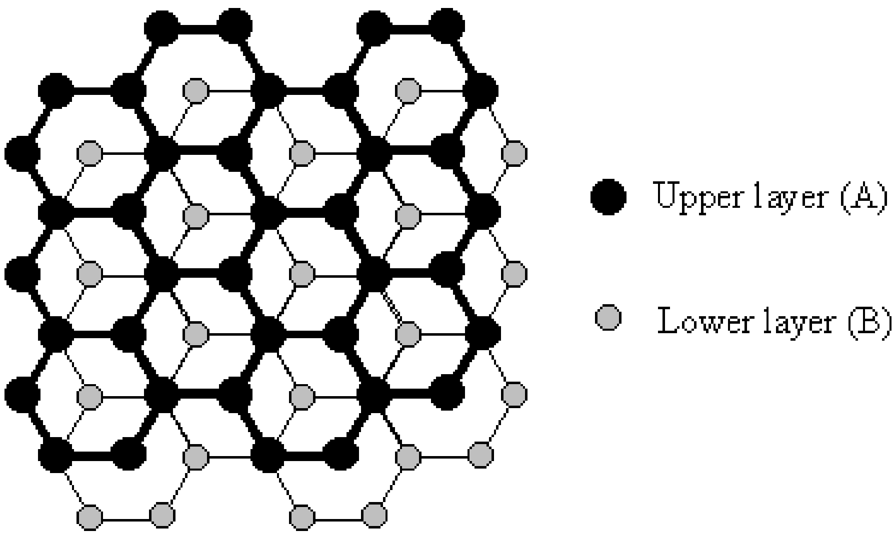


•Degeneracy between
singlet and triplet pairing is lifted



Observable manifestations of the CSB transition in HOPG

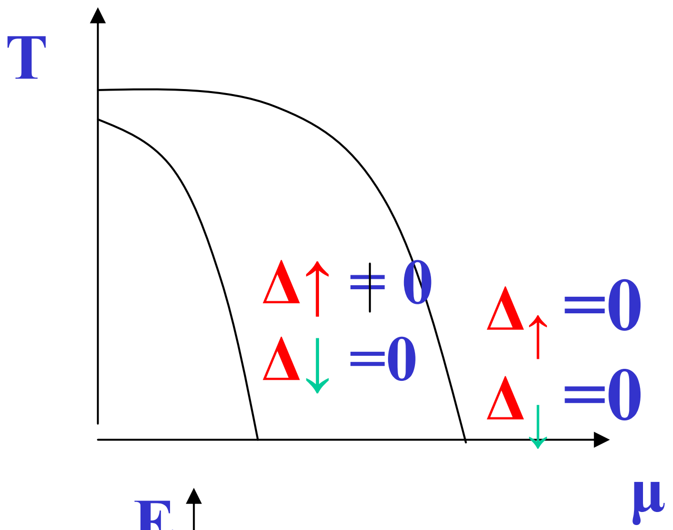
- Lifting of the A/B degeneracy
- Charge (and/or spin) density wave, **$Q=(\sqrt{3}/2,1/2,1)\pi/a$**
- CDW is further stabilized by inter-layer Coulomb repulsion (staggered stacking configuration)

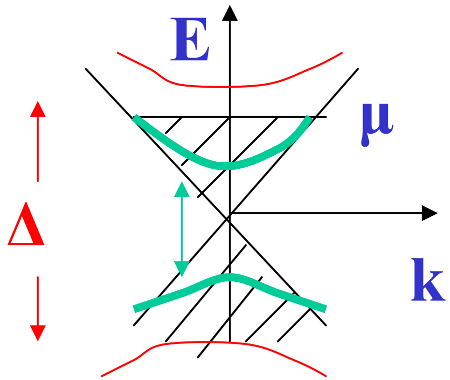


Excitonic mechanism of (weak)ferromagnetism

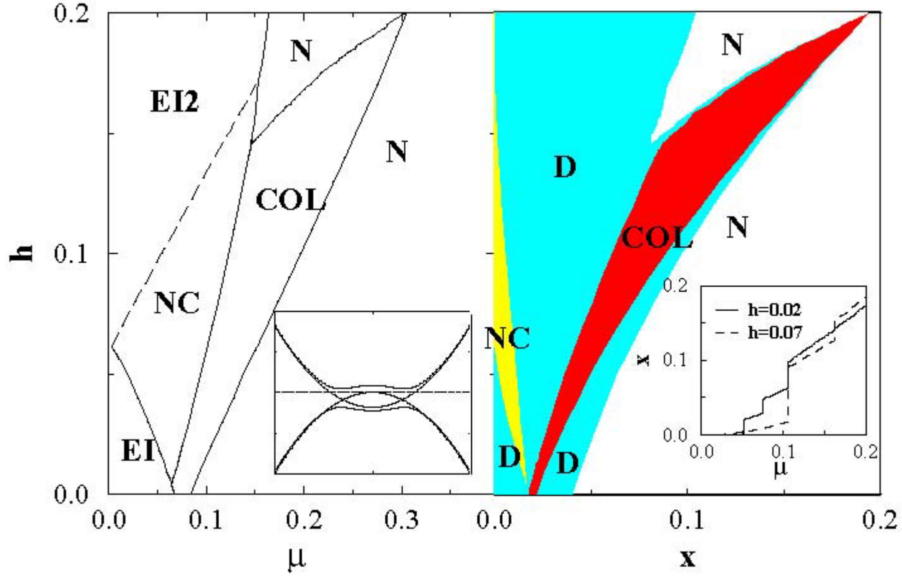
$$M\,(T) \sim f\,(\Delta\uparrow /T\,) - f\,(\Delta\downarrow /T\,)$$

$$f\,(x\,) = 1/(e^{x} + 1)$$





α=β=0 (BCS)

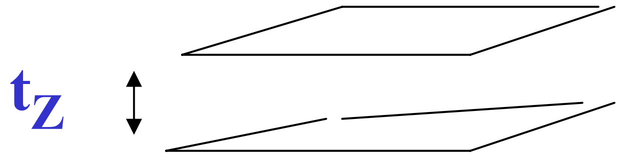


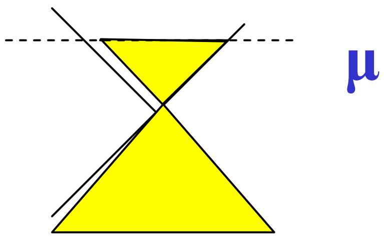
A. McDonald et al, '00

N = paramagnetic (semi)metal;
EI = excitonic insulator;
COL/NC = (non-)collinear ferromagnet

Excitonic insulator transition in realistic graphite?

• Finite Fermi surface
(layered structure, finite natural doping, disorder)





• Accuracy of the gap equation

QED_3 : Gap equation: $N_c = 3.2$
Monte Carlo simulations: $N_c < 2.0$ (>1.0 ?)

Graphene: Gap equation: $g_c \sim 5$
Actual values: $g \sim 3$ (HOPG)
~ 1.5 (graphene on SiO)

Effects of the Coulomb interactions: photoemission

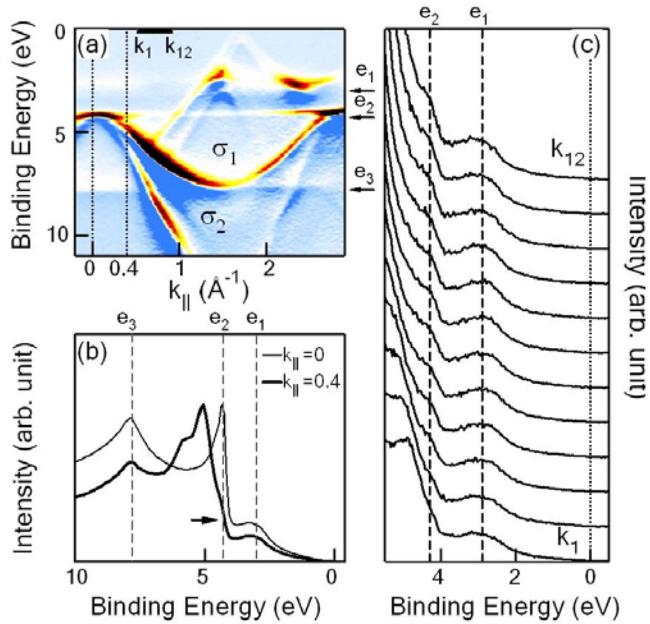
- Electron spectral function (ARPES)

g<gc

$$\Gamma(\epsilon, \mathbf{p}) \propto \theta(p_\mu^2)\frac{p_\mu^2}{max[\epsilon, v_F p]}\ln g, \quad max[\epsilon, v_F p] > T$$

$$\propto \theta(p_\mu^2)(\frac{p_\mu^2 T}{max[\epsilon, v_F p]})^{1/2}, \quad max[\epsilon, v_F p] < T$$

$P^2 = E^2 - \mathbf{p}^2$



A. Lanzara et al, '05

•Related problem:
quasiparticle width in d-wave cuprates

J. Paaske and DVK, '00
A.Chubukov and A.Tsvelik, '05

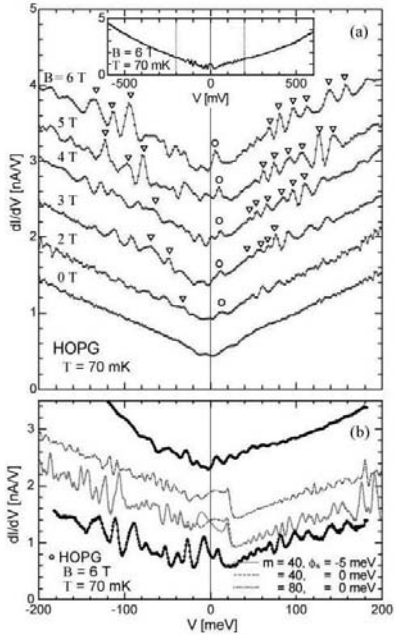
(**NOT** just **Γ (E) ~ max (E^3, T^3)**)

Effects of the Coulomb interactions: tunneling

• Tunneling DOS:

$$\nu(\epsilon) \approx -\frac{1}{\pi} Im Tr \int_{-\infty}^{\infty} \hat{G}_0^R(\mathbf{0},t) e^{-S(t)+i\epsilon t} dt$$

$$S(t) = \int_0^{\Lambda} \frac{d\omega}{4\pi} \sum_{\mathbf{q}} Im U(\omega,\mathbf{q}) \coth\frac{\omega}{2T} \int_0^t dt_1 \int_0^t dt_2 e^{-i\omega(t_1-t_2)} \left\langle e^{i\mathbf{q}(\mathbf{r}(t_1)-\mathbf{r}(t_2))} \right\rangle$$

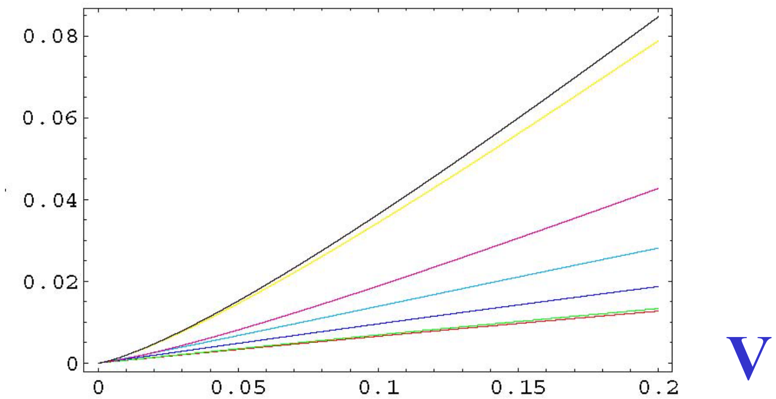


T.Matsui et al, '05

• Tunneling conductance:

$$G(V) \propto \frac{d}{dV} \int_0^{\infty} \mathcal{G}^R(\mathbf{0},t)\, \mathcal{G}_0^R(\mathbf{0},t)\, e^{iVt} dt$$

G(V,T) ~ max (V, T) $^{1+\eta(g)}$

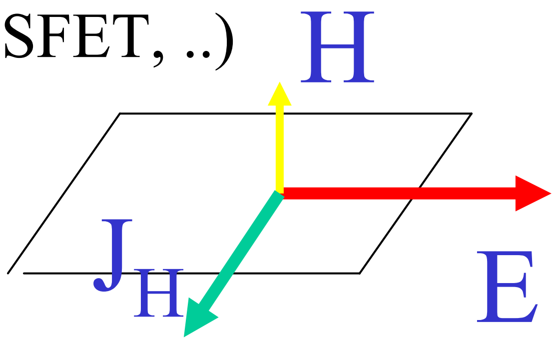


Coulomb interacting fermions in magnetic field

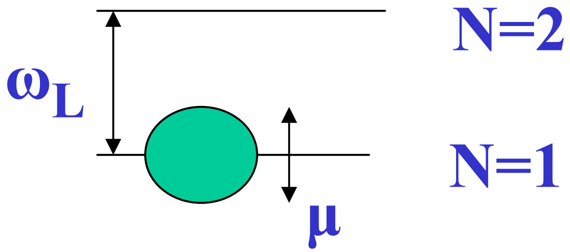
FQHE in ordinary 2DEG (**GaAs**, **Si**-MOSFET, ..)

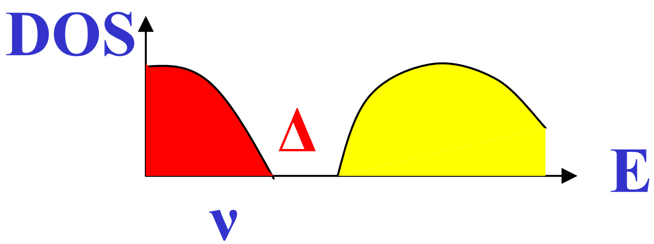
$$\sigma_H(T) = (e^2/h)\,\nu$$

$$\nu = p/(2q+1)$$



magnetic field must be **strong**, $\omega_L = eH/mc > \mu \sim n/m$



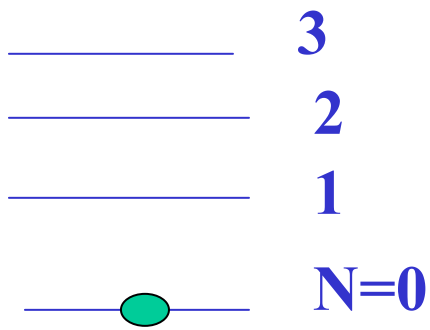


Spectral gap: $\Delta \sim e^2 n^{1/2}$

Field-induced Coulomb gap in a nodal fermion system

• Relativistic analog of FQHE: **magnetic catalysis in QED**

$$\Delta(p) = i\int \frac{d\omega d\mathbf{k}}{(2\pi)^3}\frac{\Delta(k+p)}{(\epsilon+\omega+i\delta)^2-\Delta^2(k+p)}$$
$$\frac{g e^{-((\mathbf{k}+\mathbf{p})^2+\mathbf{p}^2)/B}}{|\mathbf{k}| + \sqrt{B} g N \mathbf{k}^2 e^{-\mathbf{k}^2/2B}(B-\omega^2/2)^{-1}}$$

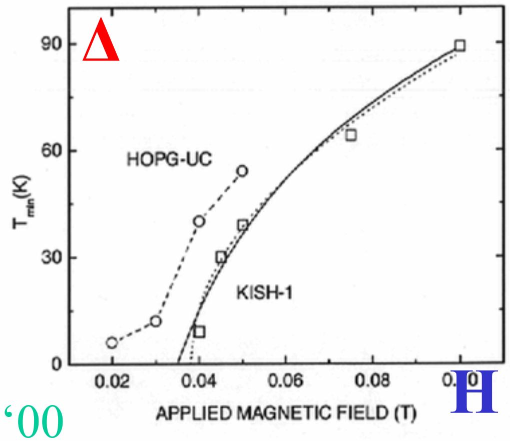


•Gap opens at **arbitrarily weak field:** $\Delta \sim H^{1/2}$

•Layered (quasi-2D) graphite:

$\Delta \sim (H - H_{cr})^{1/2}$

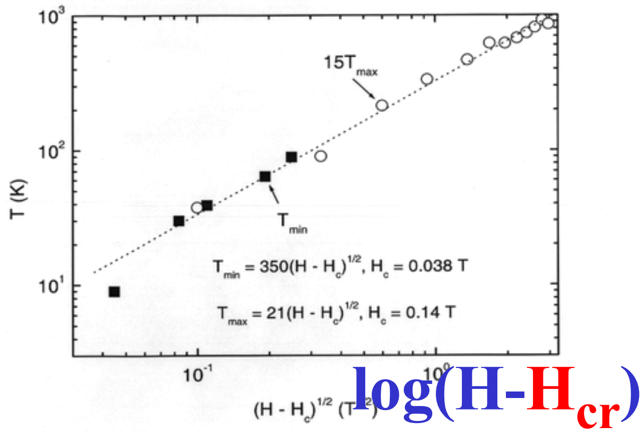
$H_{cr} \sim \max(\mu, t_Z)$



H.Kempa et al '00

log Δ

M.Sercheli et al '02



Weak magnetic fields: Dirac kinematics and Berry phase

•SdH oscillations:

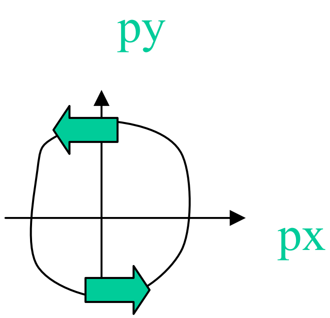
I. Lukyanchuk and Y. Kopelevich, ‘04

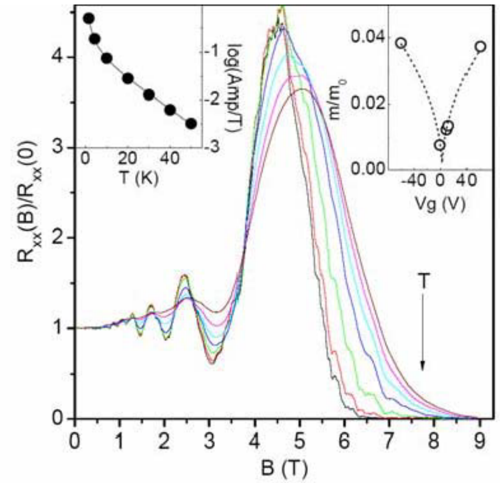
$$\Delta R_{xx} = R(B,T)\cos\left[2\pi\left(\frac{B_F}{B} + \frac{1}{2} + \beta\right)\right]$$

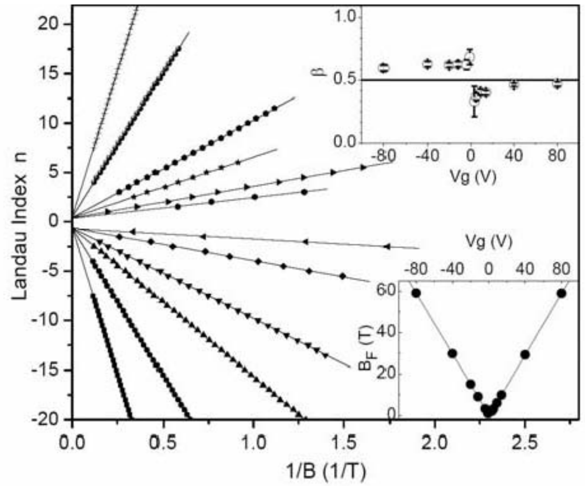
•Berry phase: $2\pi\beta = \pi$

P. Kim et al, ‘06

ψp → -- ψp

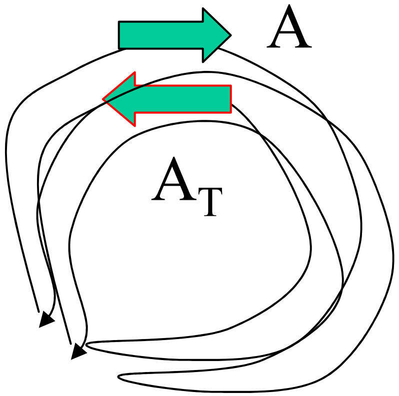






Weak magnetic fields: suppression of (anti)localization

Negative interference → WAL → Positive MR



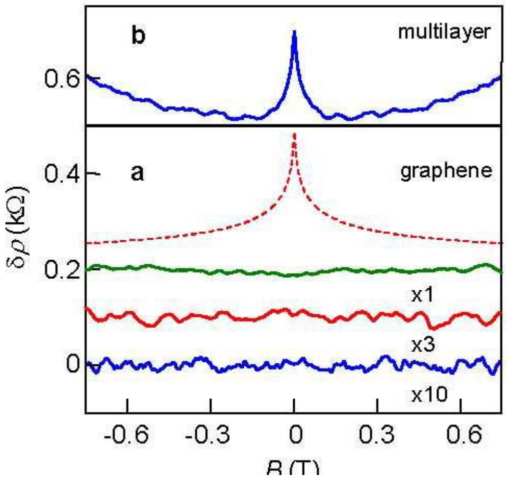
Berry phase π

$A_T = -A$

T. Ando and H. Suzuura, '02

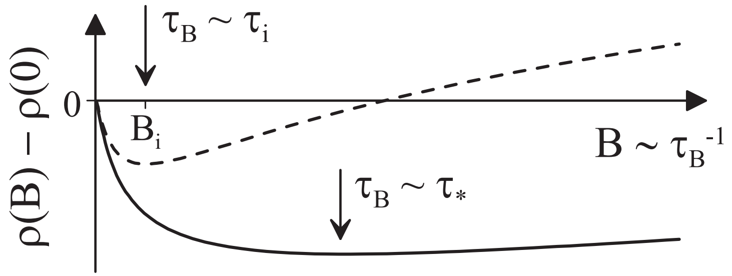
$\Delta\sigma_{WL}(H) < 0$

A. Geim et al '06



Crossover between WL and WAL?

DVK, '06; E. McCann et al, '06



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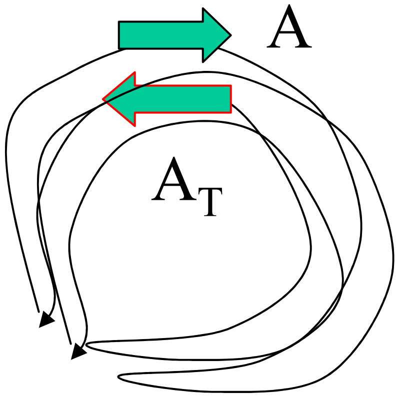
T. Ando and H. Suzuura, '02



Berry phase π

AT= - A

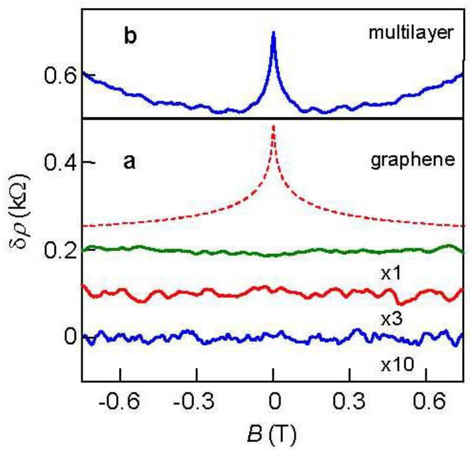
$\Delta\sigma_{WL}(H) < 0$

Long-range scattering
(Coulomb and/or topological defects)?

A. McDonald and K. Nomura '06;

F. Guinea and A. Morpurgo, '06

A. Geim et al '06

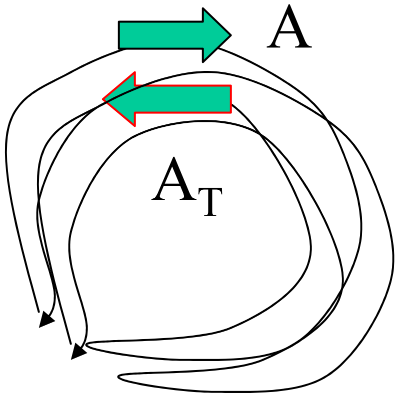


$<VV>\sim 1/q^2$

Weak magnetic fields: suppression of (anti)localization

Negative interference → WAL→ Positive MR

T. Ando and H. Suzuura, '02



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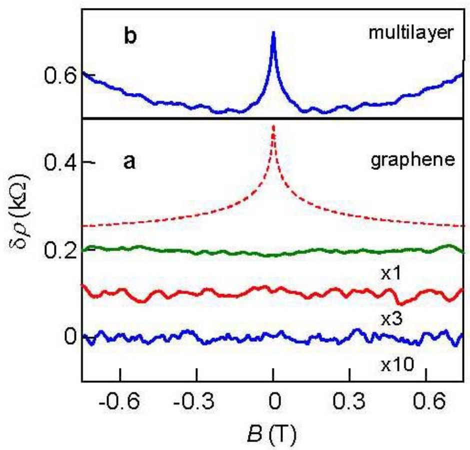
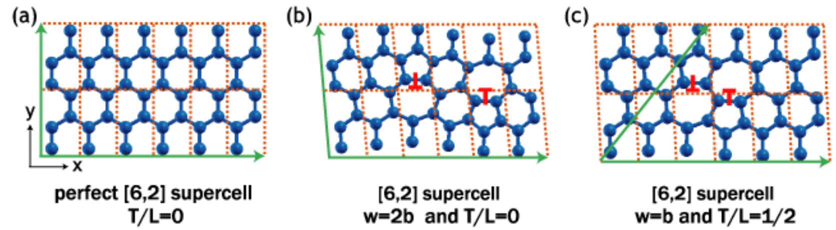


$<AA>\sim 1/q^2$



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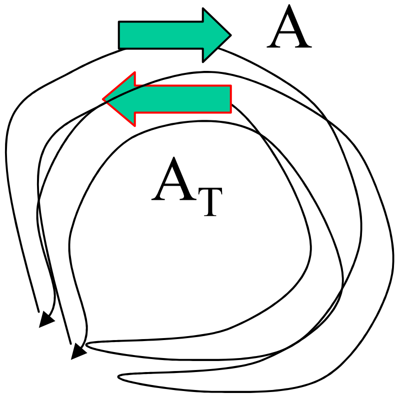

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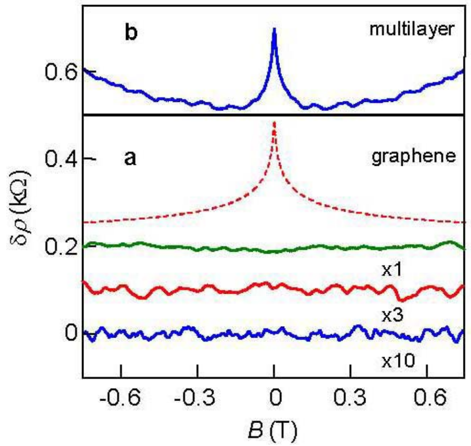
A. Geim et al ‘06



Umklapp scattering
(substrate potential, incommesurate CDW,..)

$\Delta\sigma_{WL}(H) = 0$

DVK, ’06

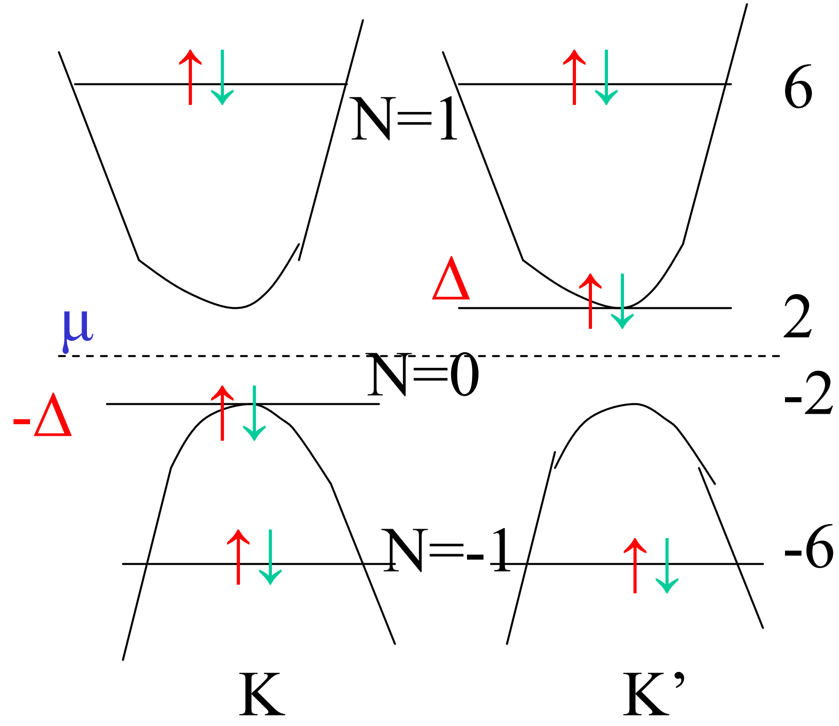
Intermediate fields: Integer quantum Hall Effect

- Dirac Landau levels:

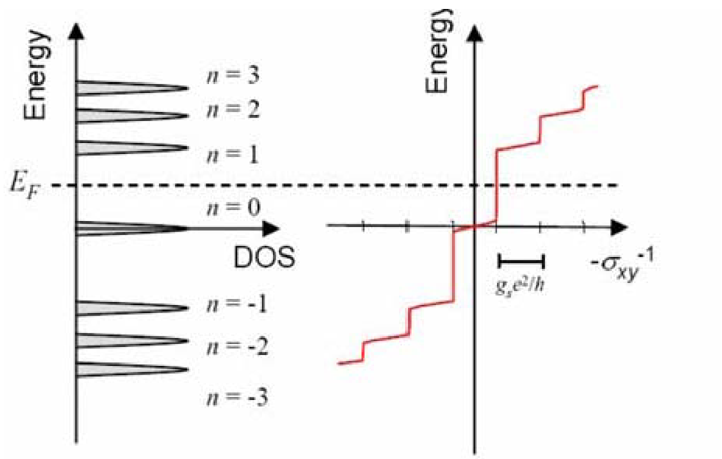
$E_N = v_F\,(2NH + \Delta^2)^{1/2}$

- "Anomalous" IQHE:

$\sigma_H(T) = 4(e^2/h)(N + 1/2)$



μ



σH

H < 10T Experiment suggests:

$\Delta(H<10T) = 0$

A.Geim et al '05
P. Kim, et al '05

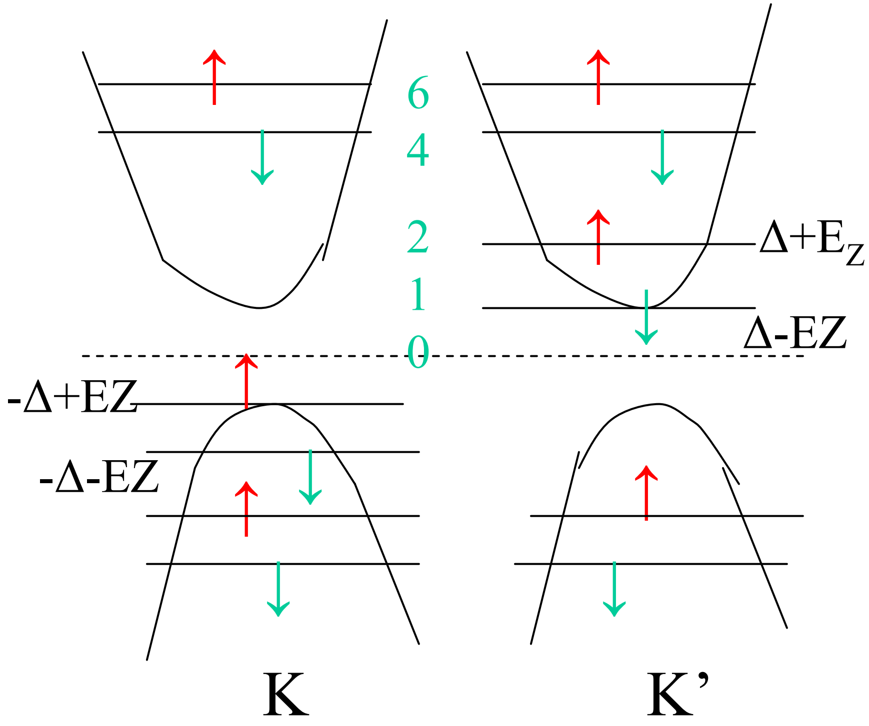
Strong fields: evidence for a magnetic field-induced gap?

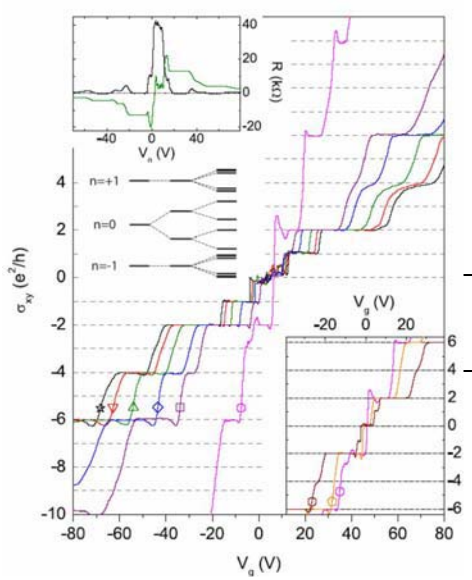
•Spin and valley splitting→ new plateaus at LLL (N=0)

$\sigma_H(T)=(e^2/h)(0, 1, 4)$

•NO plateaus at (3, 5)

•Lifting of the A/B degeneracy





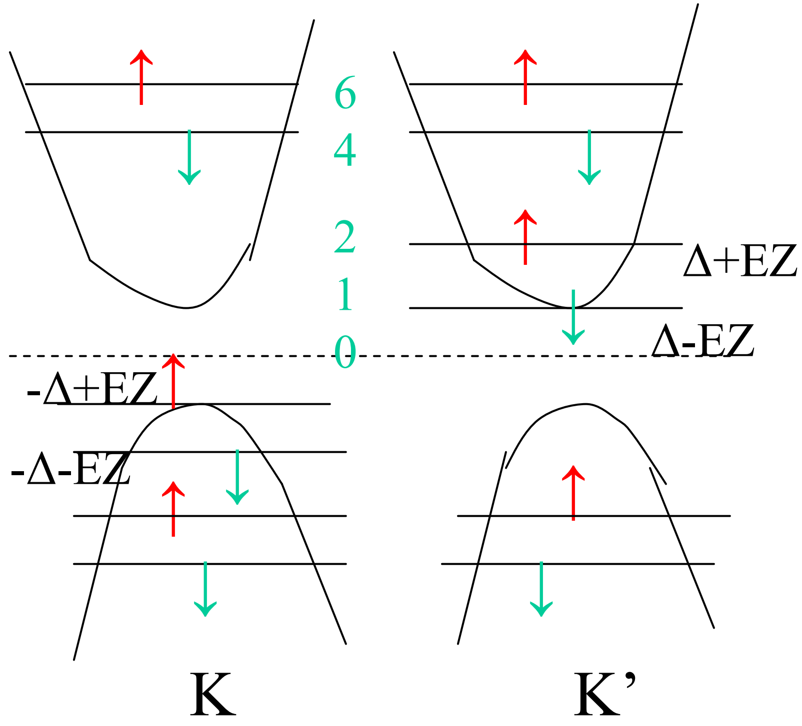
H=45T

P. Kim et al, '06

Strong fields: evidence for a magnetic field-induced gap?

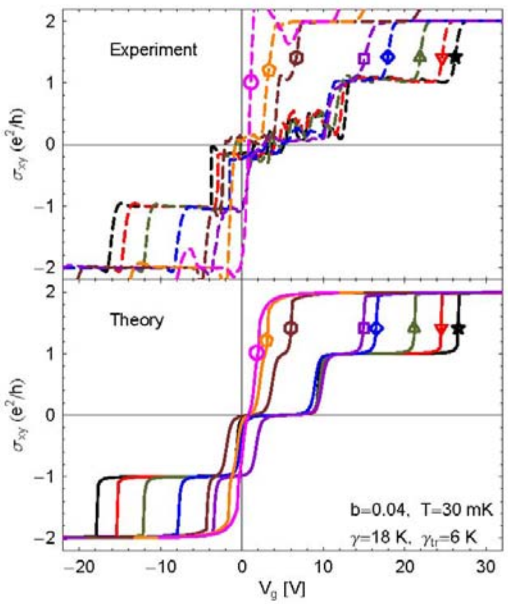
•Spin and valley splitting
 new plateaus at LLL (N=0)

$\sigma_H(T)=(e^2/h)(0, 1, 4)$



•Lifting of the A/B degeneracy

•Good fit:
 Δ>0



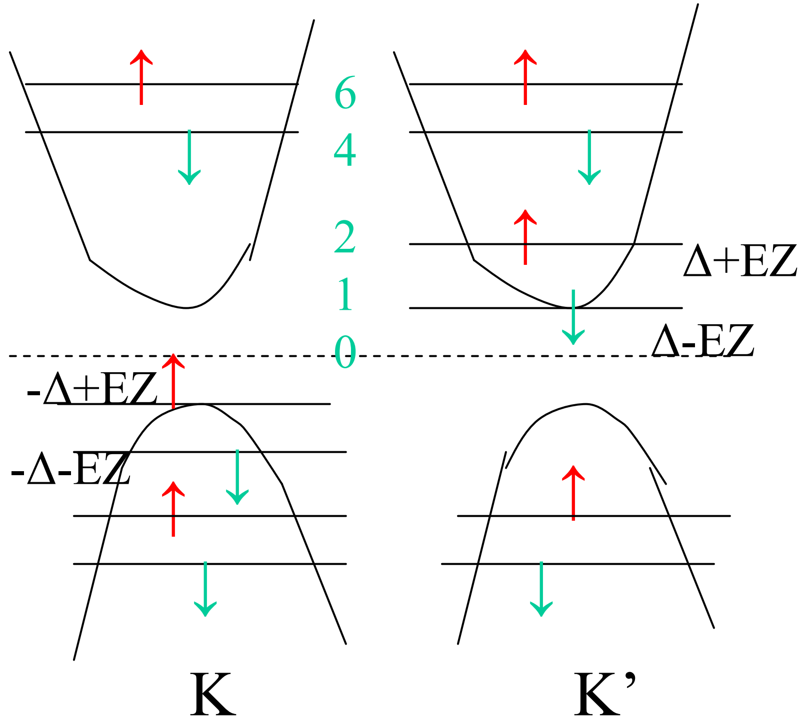
V.P.Gusynin et al, '06

Strong fields: evidence for a magnetic field-induced gap?

- Spin and valley splitting
- New plateaus at LLL (N=0)

$\sigma_H(T)=(e^2/h)(0, 1)$,
but NOT at (3, 5)

- Lifting of the A/B degeneracy
($\Delta>0$??)



- Alternative mechanisms:

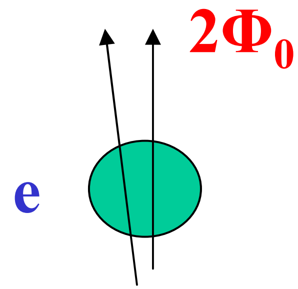
QH Ferromagnetism
A.McDonald et al, '06; M.P.E. Fisher et al, '06,…
Janh-Teller splitting
J. Fuchs and P. Lederer, '06

Composite Dirac fermions and FQHE in graphene

- Statistical flux attachment: composite Dirac fermions
- Effective field: **$H' = H - 4\pi n = H/(2N+1)$**
- Effective filling factor: **$\nu' = N$ [$\nu = N/(2N+1)$]**
- Spectral gap: **$\Delta \sim E_{N+1} - E_N \sim H/N$**
- Metal-like (gapless) CDF states: $\nu = -3/2, -1/2, 1/2, 3/2$
- Paired (gapful) CDF states: $\nu = -1, 0, 1$
- FQHE as IQHE of CDF: not only **$\nu = 2N/(2N+1)$,**

But also **$\nu = N/(2N+1) + N-1/(2N-1)$,**

and **$\nu = 2N/(2N+1) + 2(N-1)/(2N-1) - 2$**



DVK, ‘06

Conclusions

•Due to the linear energy spectrum and unscreened Coulomb interactions, many properties of graphene are **markedly different** from those of the conventional 2DEG;

•2D Dirac fermions in graphene have a propensity towards excitonic pairing (a non-relativistic counterpart of **chiral symmetry breaking);**

•Further interest in fundamental properties of graphene is needed to ascertain the **status** of the existing theoretical predictions;

•Possible technological **applications** (switching,…)?