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Miniworkshop on
New States of Stable and Unstable Quantum Matter
(14 - 25 August 2006)

Fermi Surfaces that Wobble and Pop

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Fermi surfaces that wobble and pop

Pomeranchuk and Topological Fermi Surface Instabilities from Central Interactions

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Credits

This work: (mostly...) — cond-mat/0601103 (to appear in *PRB*) — **J. Quintanilla**, **A.J. Schofield**

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Discussions: J.M.F. Gunn (Birmingham), M.W. Long (Birmingham), J.F. Chalker (Oxford), N.I. Gidopoulos (ISIS), Gil Lonzarich (Cambridge), Andrew Green (St. Andrews), Klaus Capelle (Sao Carlos), Steve Simon (Lucent), Andrew Ho (Imperial).

S.A. Grigera (St. Andrews), S. Ramos (Birmingham), W.J.L. Buyers (NRC, Canada), Martyn Bull (ISIS).

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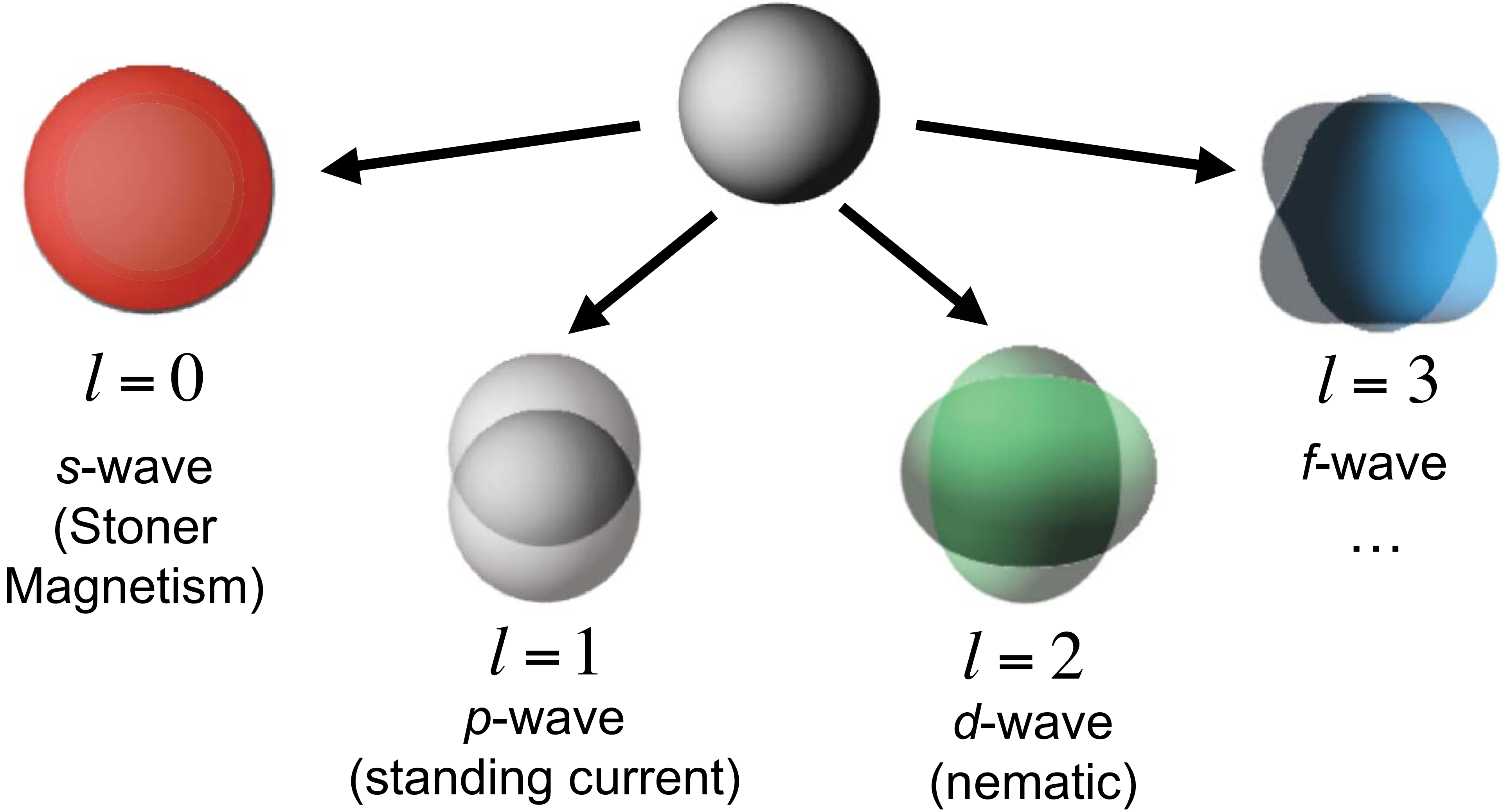
The Pomeranchuk Instability

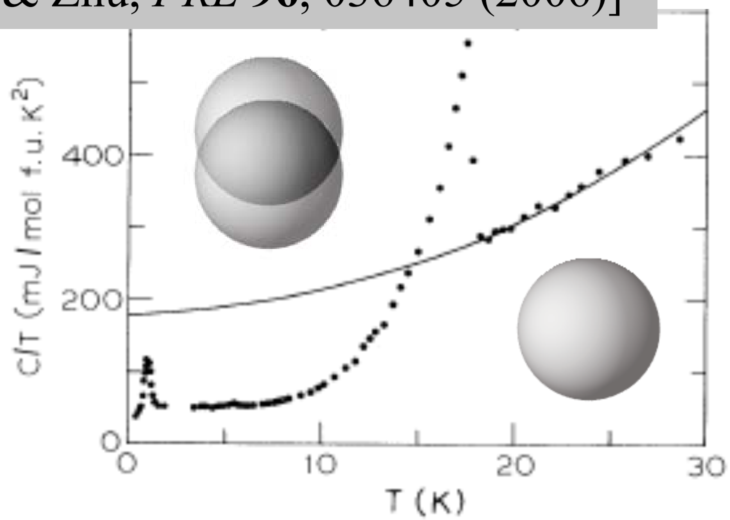


cond-mat/0601103

Unconventional Phase Transitions

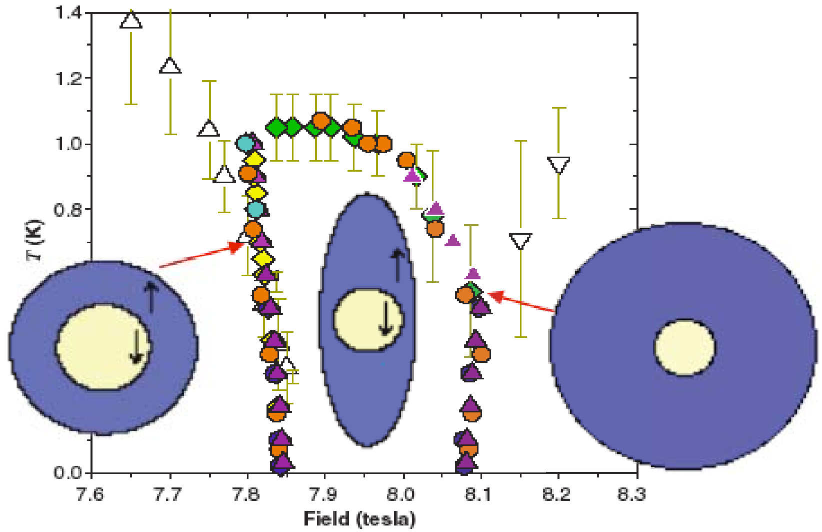
URu_2Si_2

[Varma & Zhu, *PRL* **96**, 036405 (2006)]



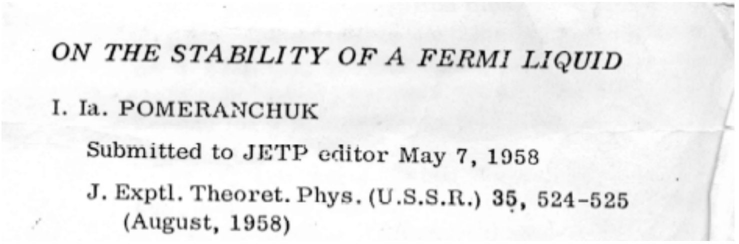
[Palstra et al.,
PRL **55**, 2727 (1985)]

$Sr_3Ru_2O_7$



[S.A.Grigera et al.,
Science **306**, 1154 (2004)]

Conditions for a Pomeranchuk instability



ON THE STABILITY OF A FERMI LIQUID

I. Ia. POMERANCHUK

Submitted to JETP editor May 7, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) 35, 524-525 (August, 1958)

$\delta E \; < \; 0 \; ?$

Pomeranchuk **instability condition**:

$$1 + \frac{1}{2l+1} F_l < 0$$

$l = 0, 1, 2, 3 \ldots$ symmetry of deformation

$F_0, F_1, F_2, F_3 \ldots$ Landau parameters

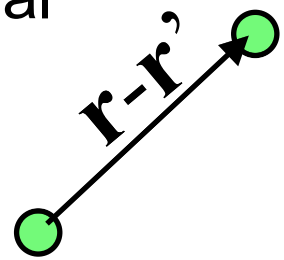
Conditions for a Pomeranchuk instability

Microscopic model:

$$H = \underbrace{\int d^3\mathbf{r} \sum_{\sigma} \hat{c}^{+}_{\mathbf{r},\sigma} \left[\frac{-\hbar^2 \nabla^2}{2m} - \mu \right] \hat{c}_{\mathbf{r},\sigma}}_{\text{free fermions}} + \underbrace{\frac{1}{2} \sum_{\sigma,\sigma'} \int d^3\mathbf{r} \int d^3\mathbf{r}' \hat{c}^{+}_{\mathbf{r},\sigma} \hat{c}^{+}_{\mathbf{r}',\sigma'} V\left(\left|\mathbf{r}-\mathbf{r}'\right|\right) \hat{c}_{\mathbf{r}',\sigma'} \hat{c}_{\mathbf{r},\sigma}}_{\textit{central}\text{ potential}}$$

free fermions

central potential



Trial ground state:

$$\left|\Psi\right\rangle = \prod_{\varepsilon_\sigma(\mathbf{k})<0} \hat{c}^{+}_{\mathbf{k},\sigma} \left|0\right\rangle$$

The dispersion relation $\varepsilon_\sigma(\mathbf{k})$ is determined **variationally**.

Conditions for a Pomeranchuk instability

$\varepsilon_\sigma(\mathbf{k})$ becomes anisotropic under the following instability condition:

$$\int d^3\mathbf{r} V(|\mathbf{r}|)\left[\underbrace{j_l(k_F|\mathbf{r}|)^2}_{} - j_1(k_F|\mathbf{r}|)^2\right] > \frac{2\pi^2\hbar^2}{mk_F}$$

$= 0$ for $l=1$

for $k_F r << \pi$: $\begin{cases} \approx 1 \text{ for } l = 0; \\ \approx 0 \text{ for } l = 1,2,3,\ldots \end{cases}$

$l = 0, \not{1}, 2, 3\ldots$

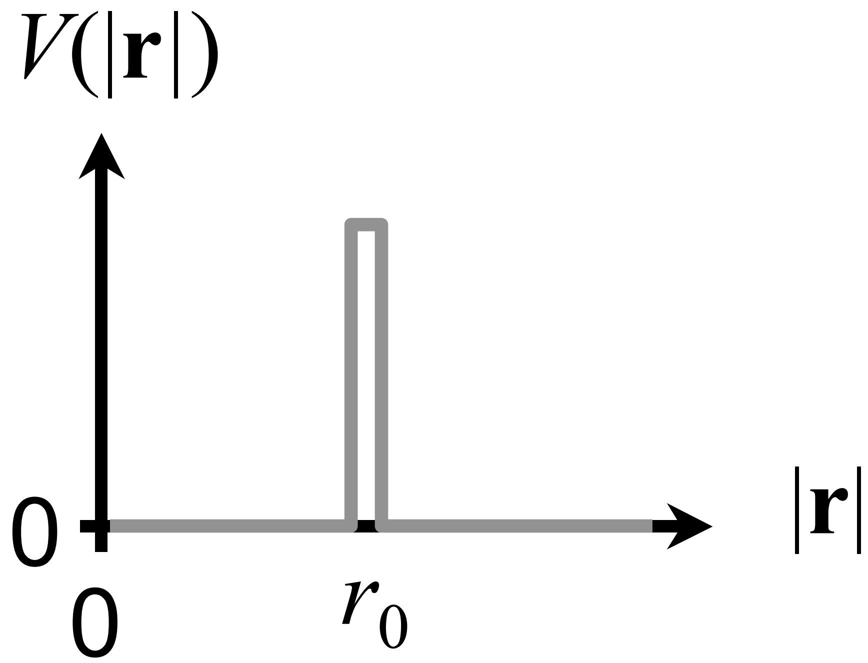
Pomeranchuk instability with l=1 **never takes place**.

l=2,3… requires **range of interaction $r_0 \sim \pi/k_F$** or larger.

A simple example

We can solve the theory analytically for **repulsion at a finite distance** r_0:

$$V(|\mathbf{r}|) = g\delta(|\mathbf{r}| - r_0)$$

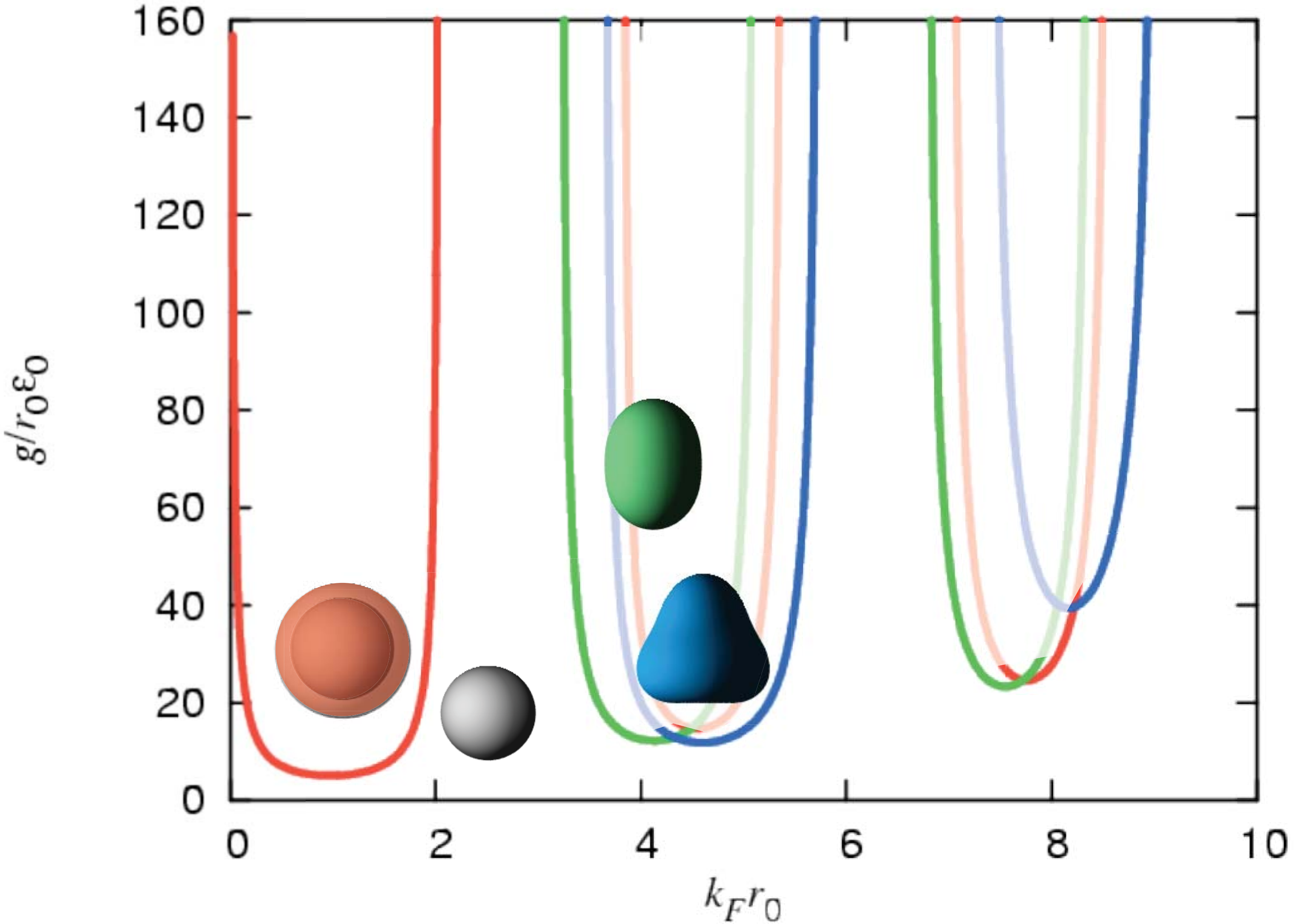


Characteristic length: r_0

Characteristic energies: g/r_0

$\hbar^2/2mr_0^2 \equiv \varepsilon_0$

A simple example



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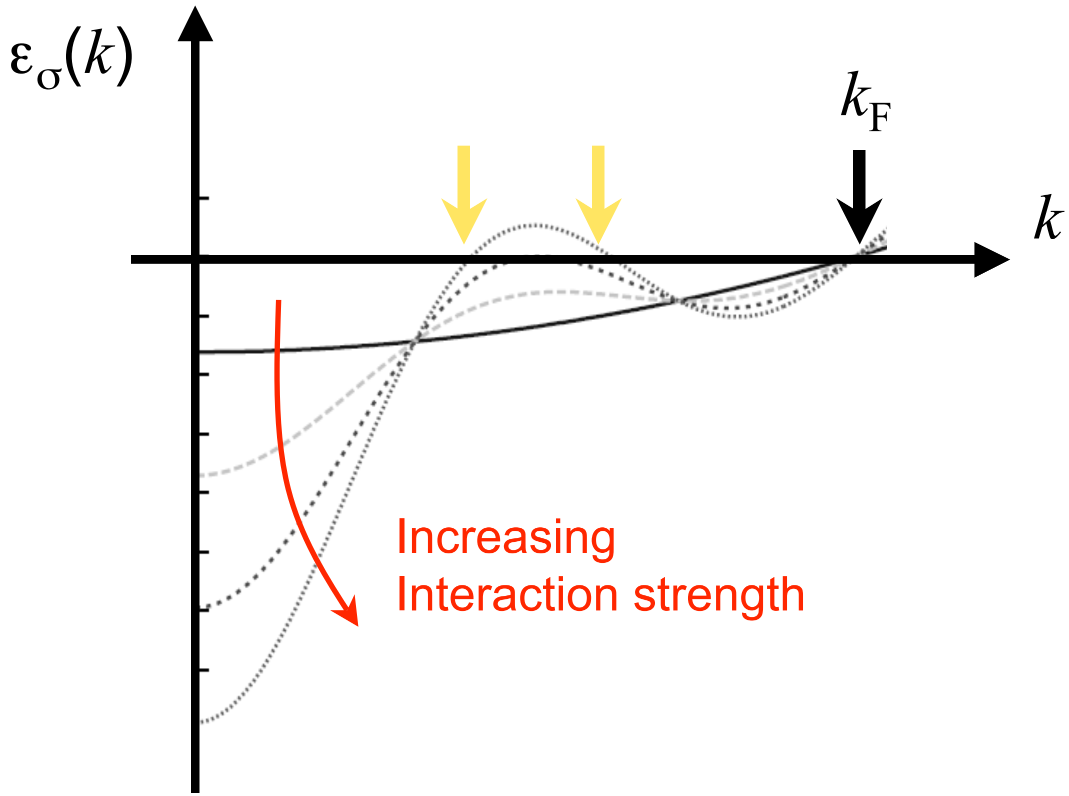
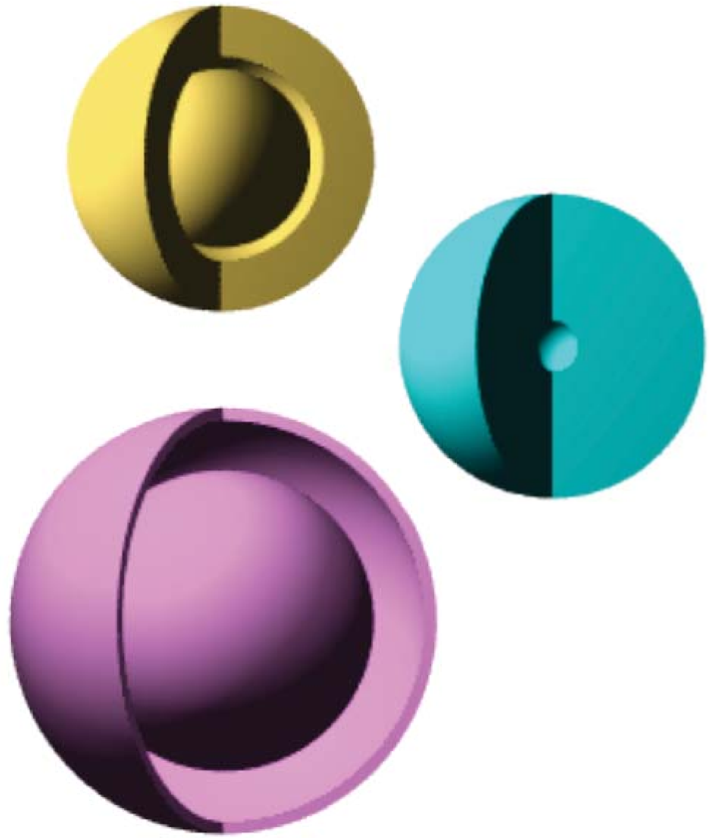
Topological phase transitions

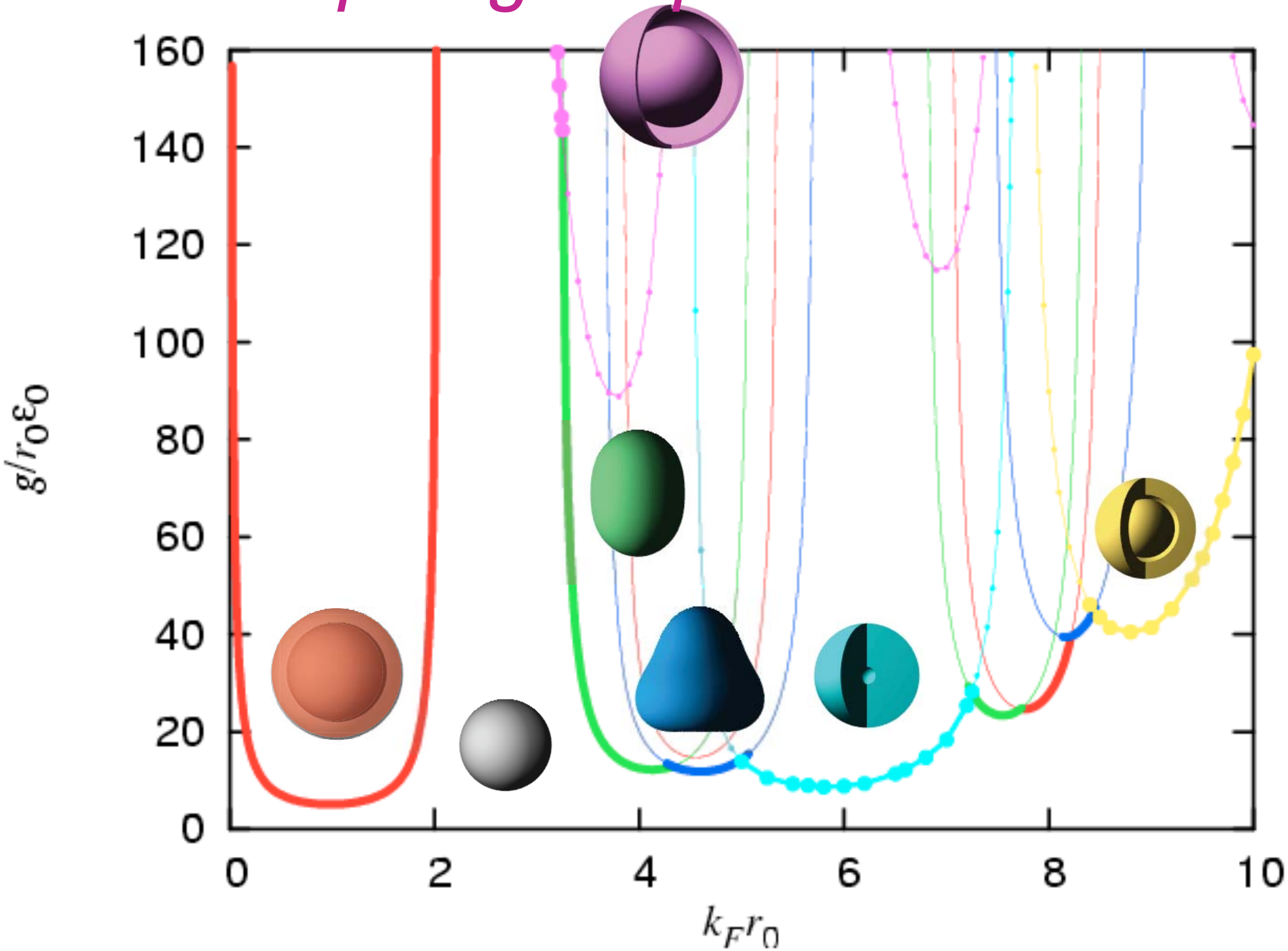
Even without symmetry breaking, interactions may change $\varepsilon_\sigma(\mathbf{k})$ qualitatively…

…even leading to changes of Fermi surface **topology**





Topological phase transitions



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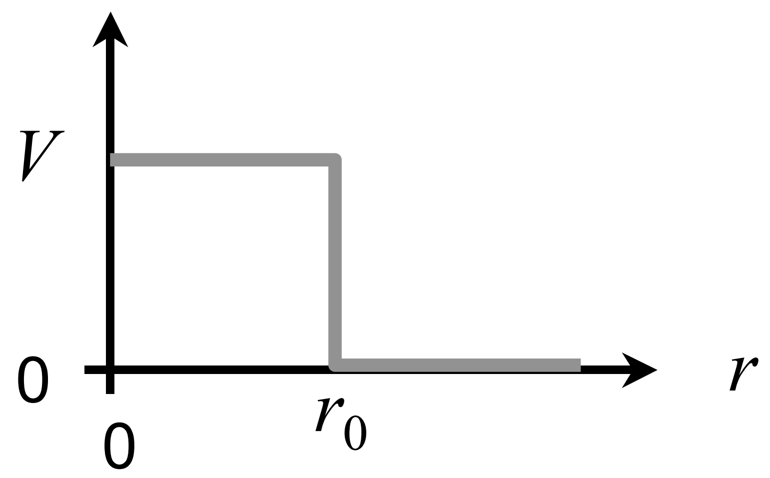
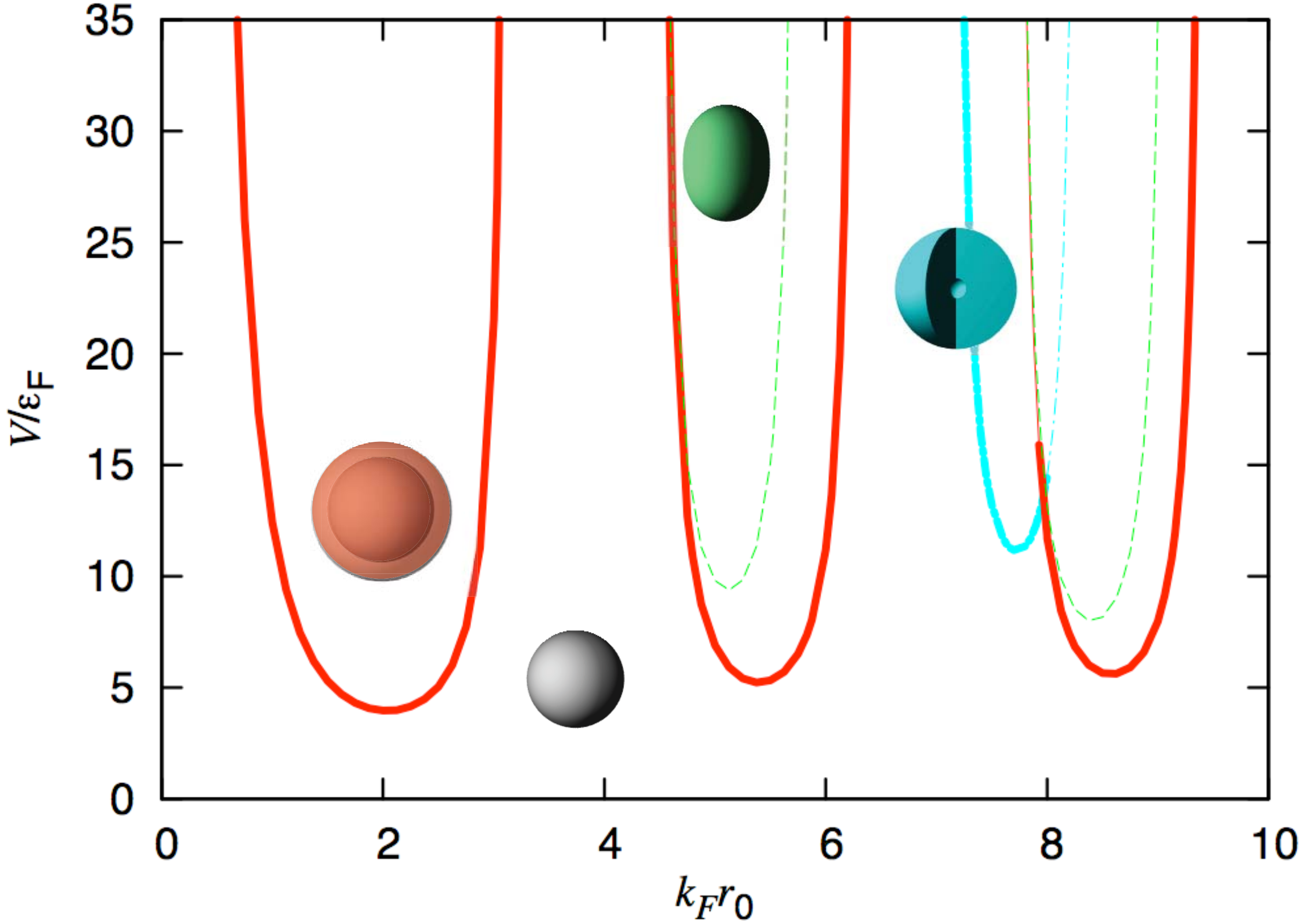
Another example: "hard" core potential

"Top hat" potential of radius r_0:

$$V(|\mathbf{r}|) = V \; \Theta(r_0 - |\mathbf{r}|)$$



Another example: “hard” core potential



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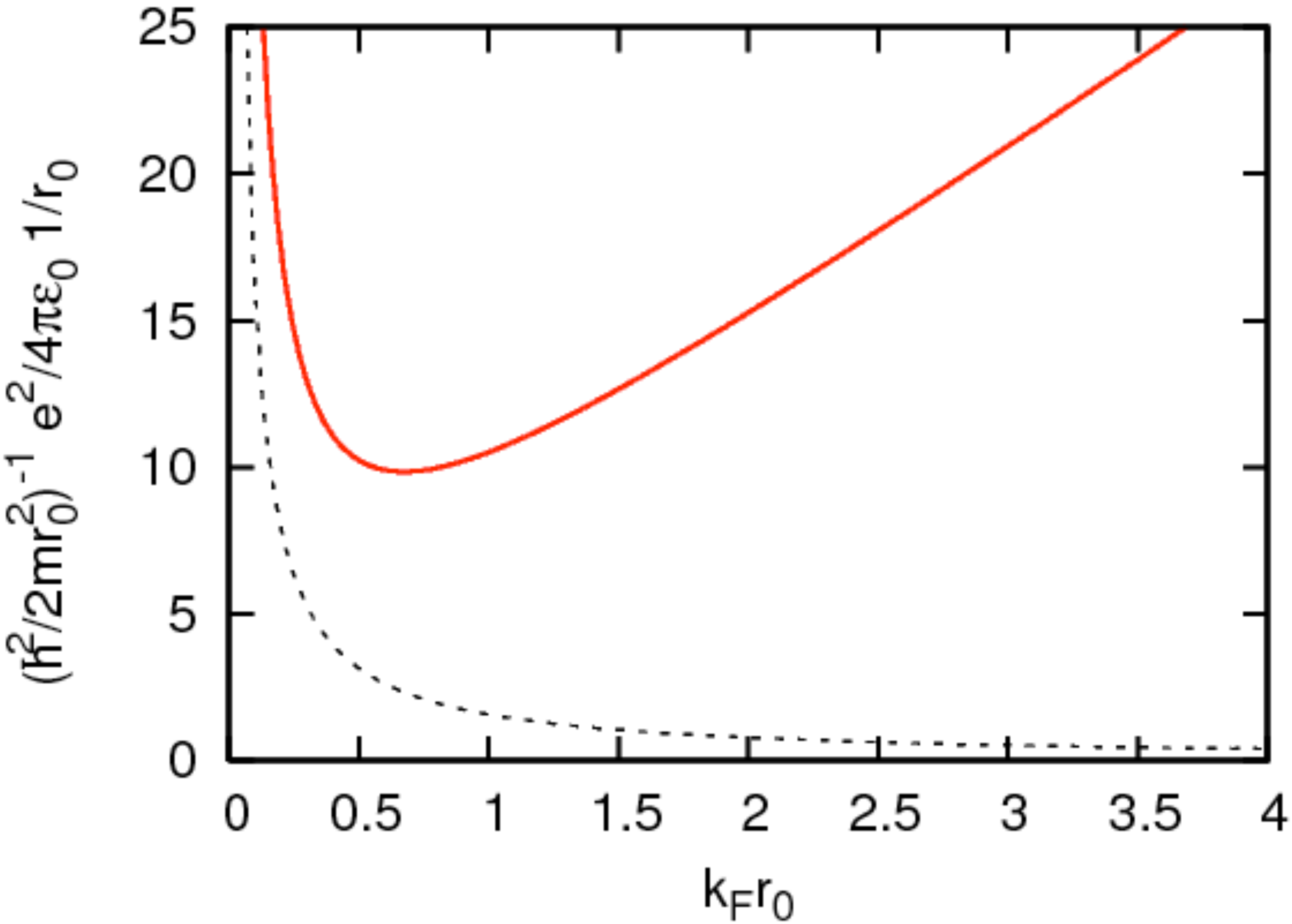
How "sharp" does the feature have to be?

Screened Coulomb Interaction:

$$V\left(|\mathbf{r}|\right)=\frac{e^2}{4\pi\epsilon_0^2}\frac{e^{-|\mathbf{r}|/r_0}}{|\mathbf{r}|}$$

How "sharp" does the feature have to be?



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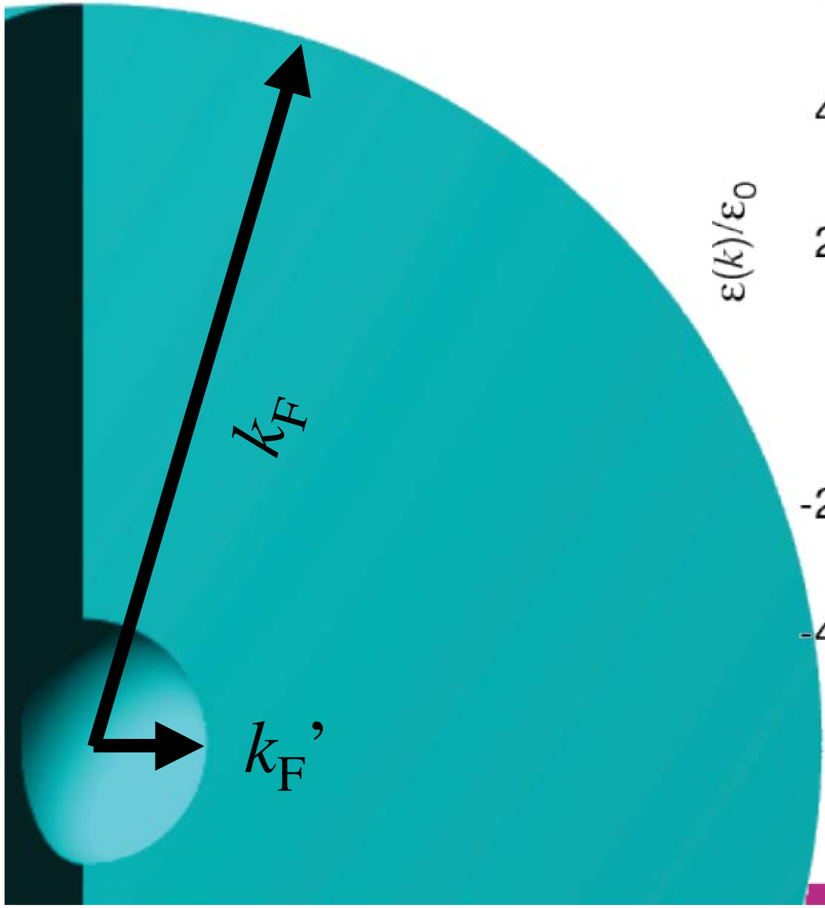

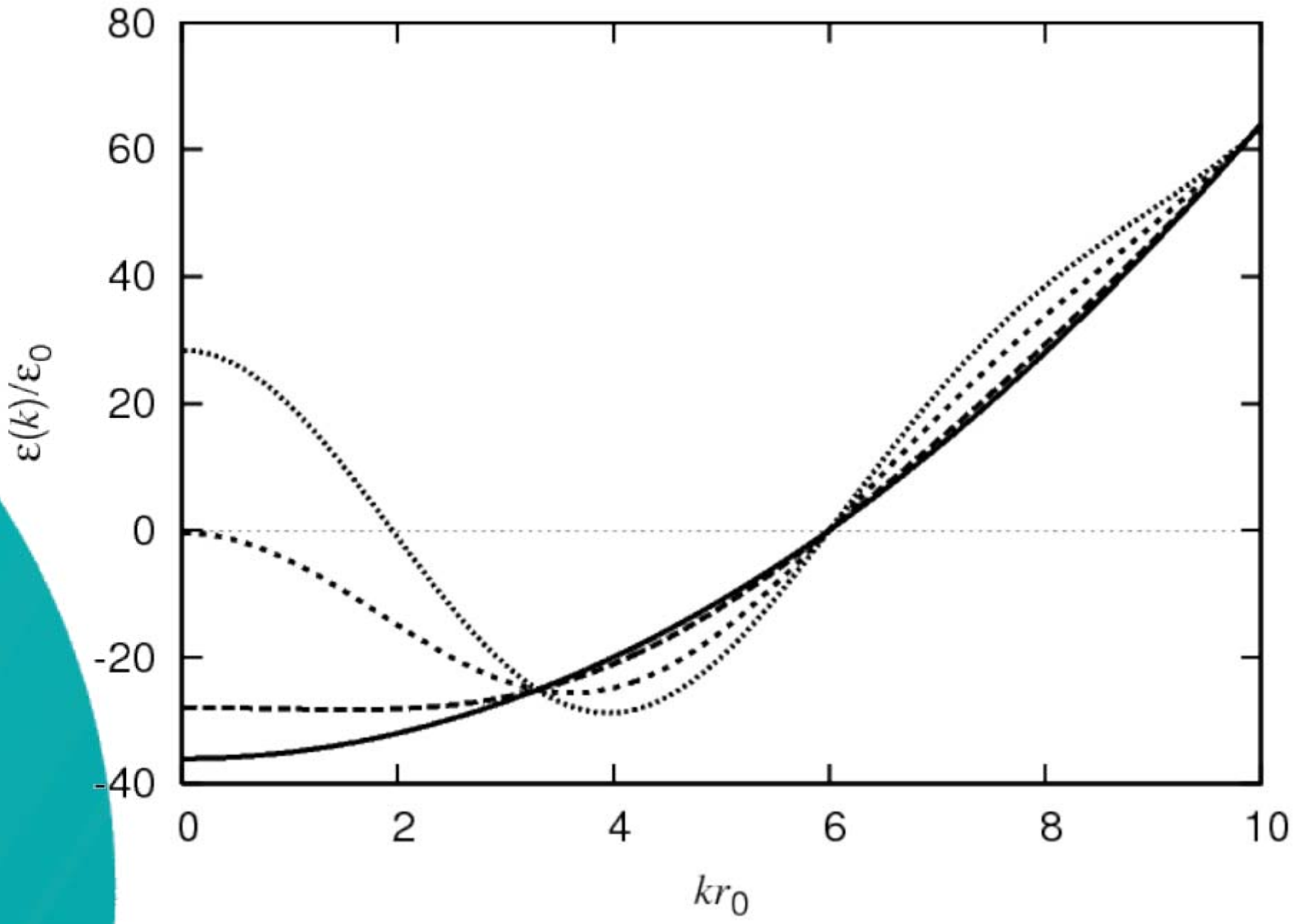

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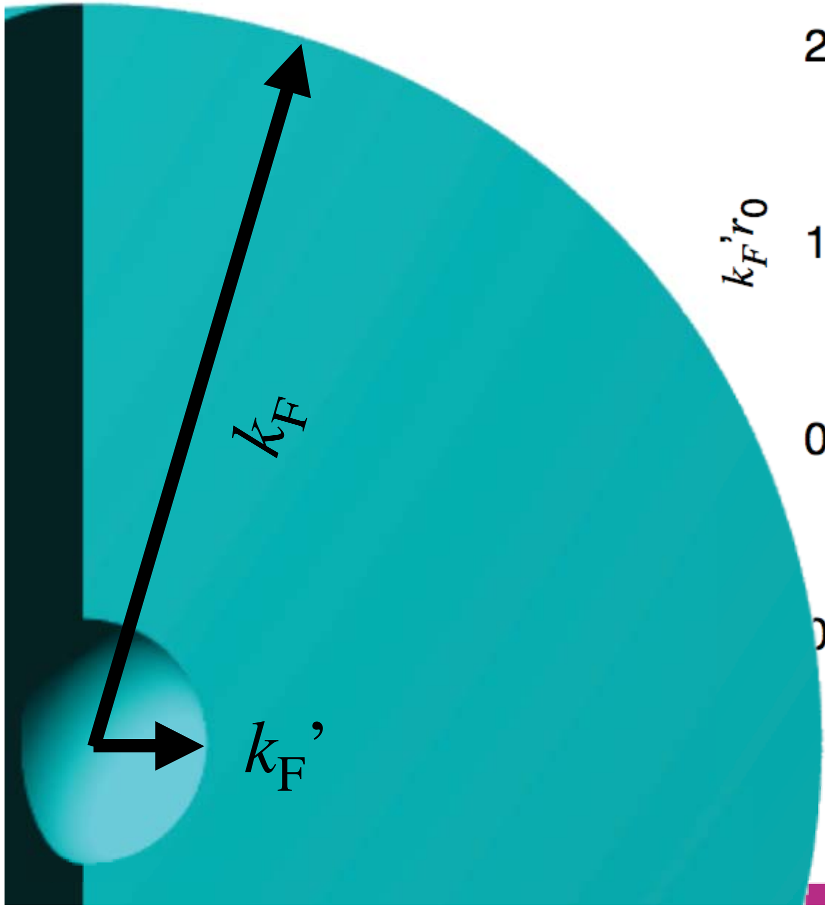

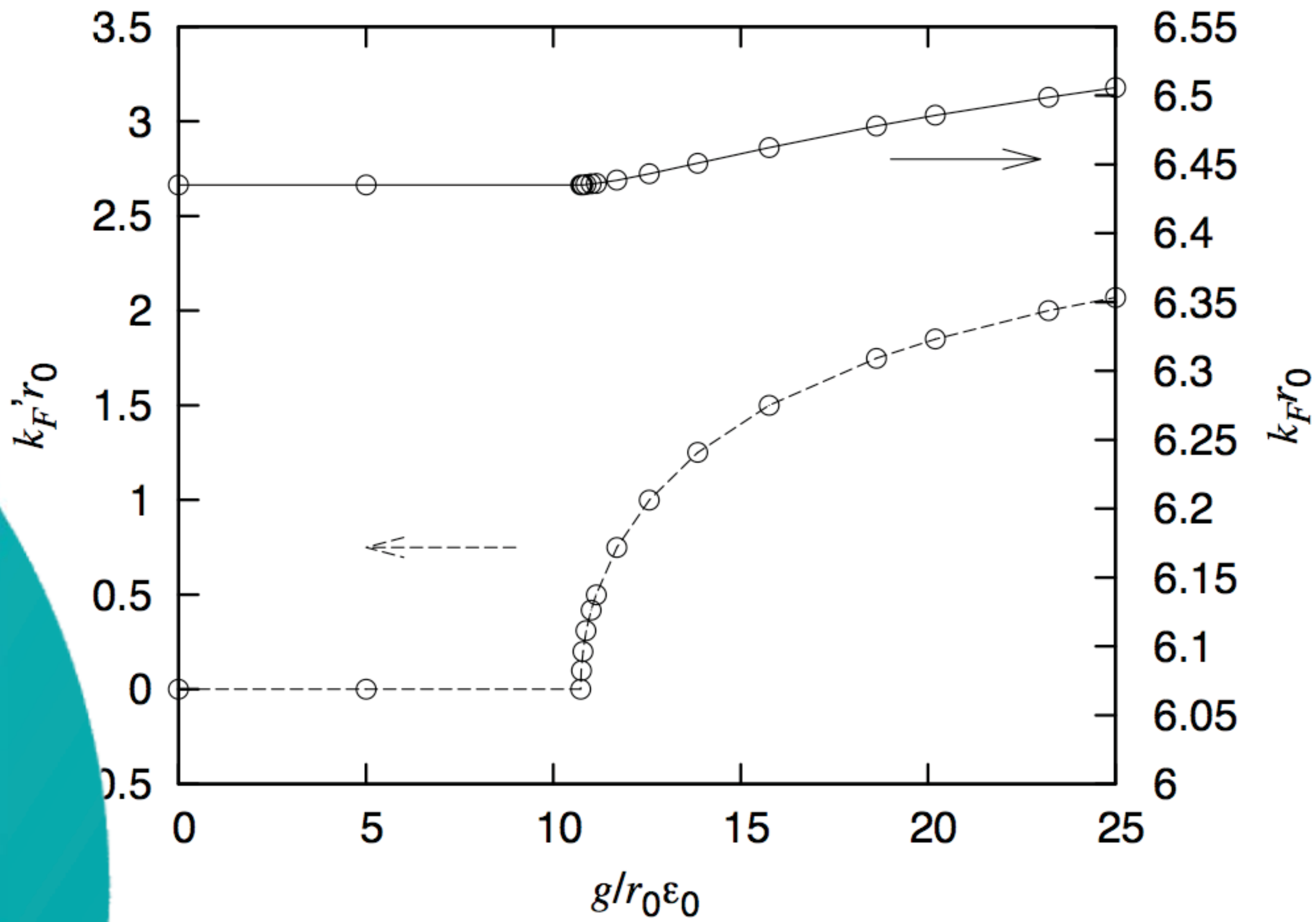

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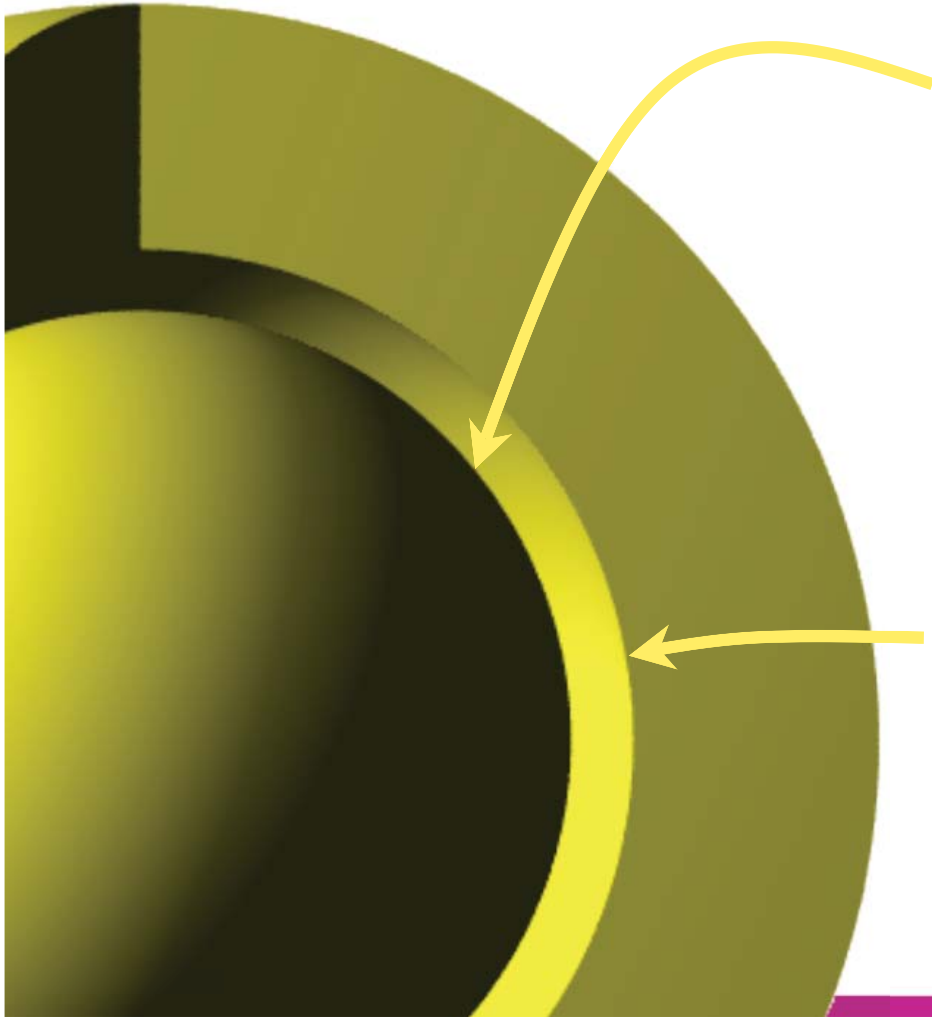
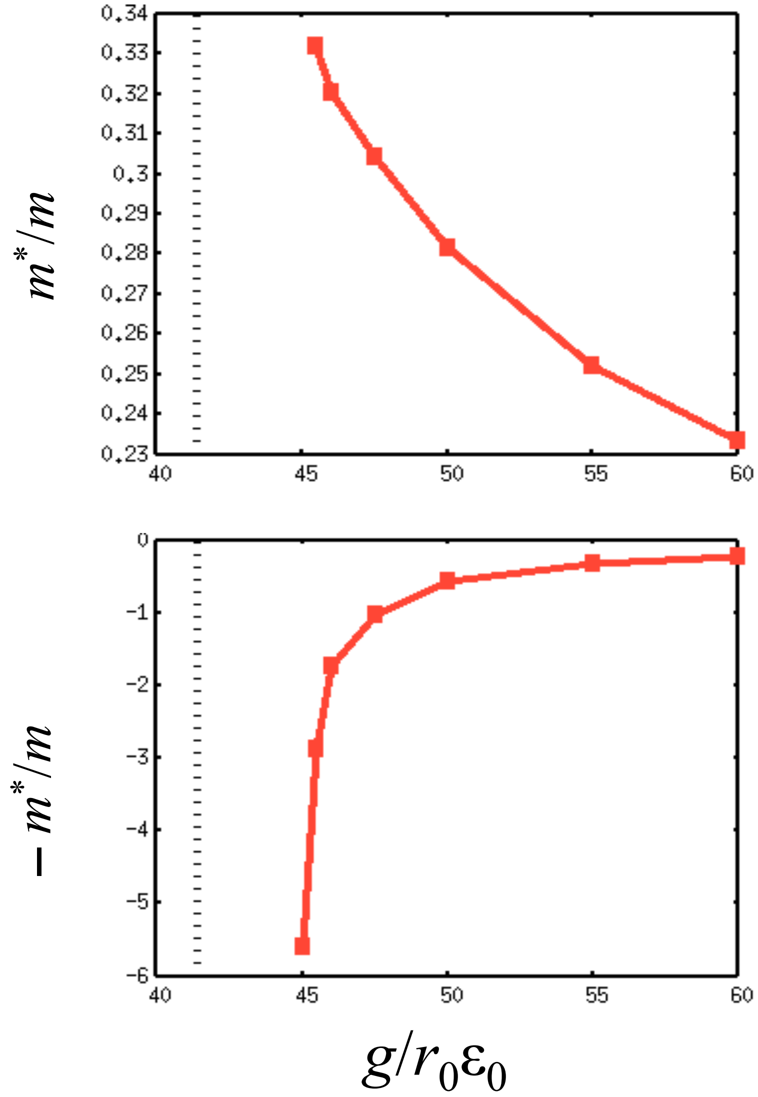
Topological phase transitions





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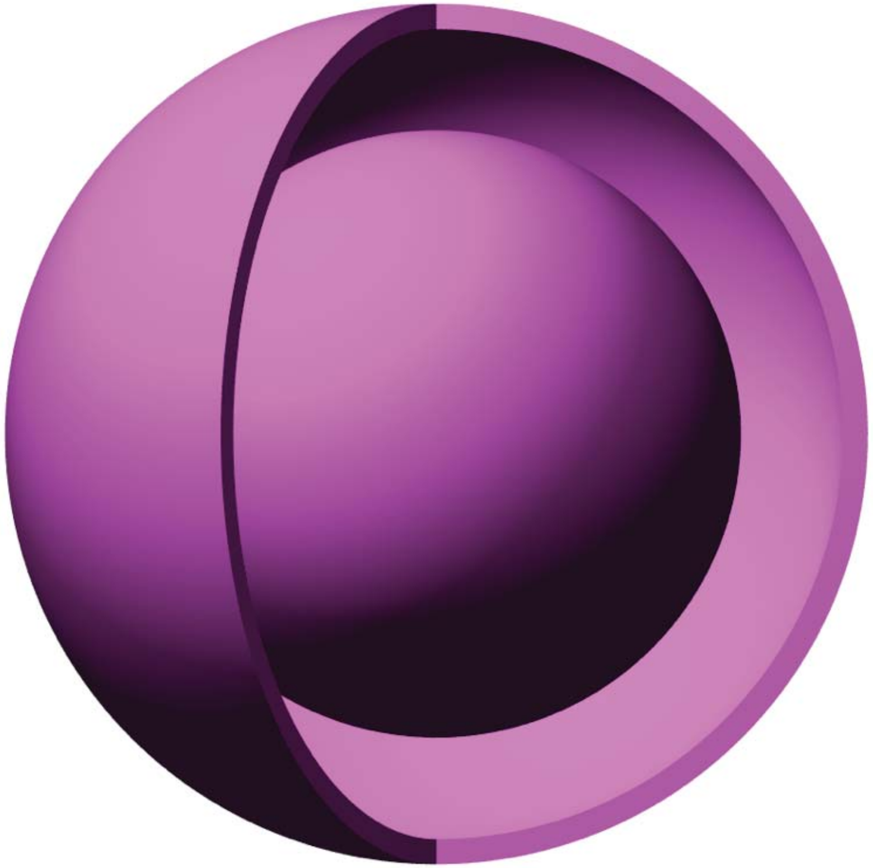
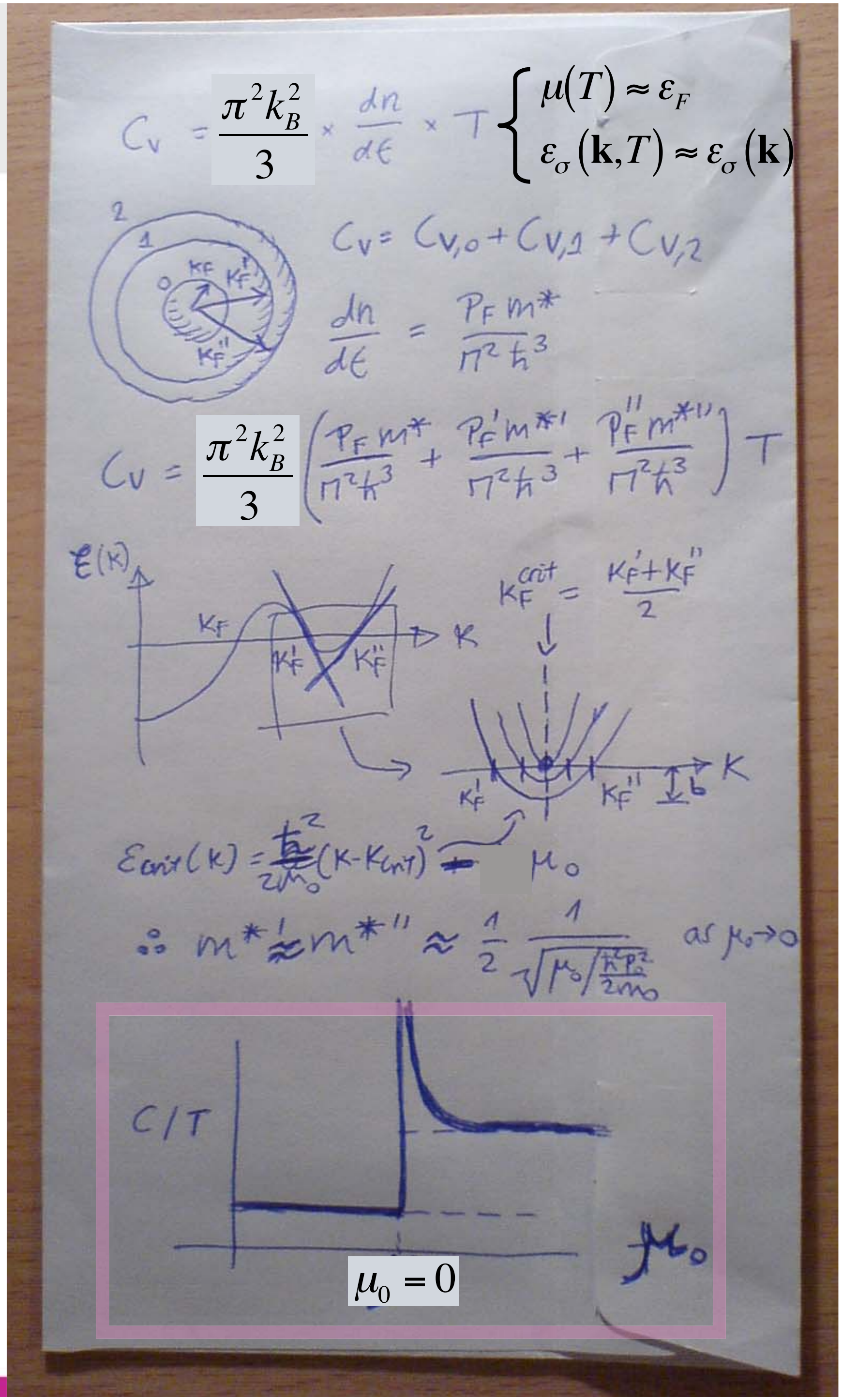
Specific heat signature of topological Fermi surface shape instability





Jorge Quintanilla

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Conclusion
and work in progress…

What could be happening ?

Pomeranchuk instability: a subtle symmetry-breaking phase transition, and therefore a candidate for hidden order.

Even subtler: **topological Fermi surface instabilities**.

Why would it happen ?

Key ingredient: **characteristic length** scale $r_0 \gtrsim k_F^{-1}$.

A guide to realisations? (optical lattices, MOSFETS…)

How would we find out ?

Topological may have **unusual thermod. signatures**.

Nesting, STM, coupling to the lattice…

CCLRC ISIS

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Thanks!