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Miniworkshop on
New States of Stable and Unstable Quantum Matter
(14 - 25 August 2006)

Metal to Metal Transitions

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Metal to Metal Transitions

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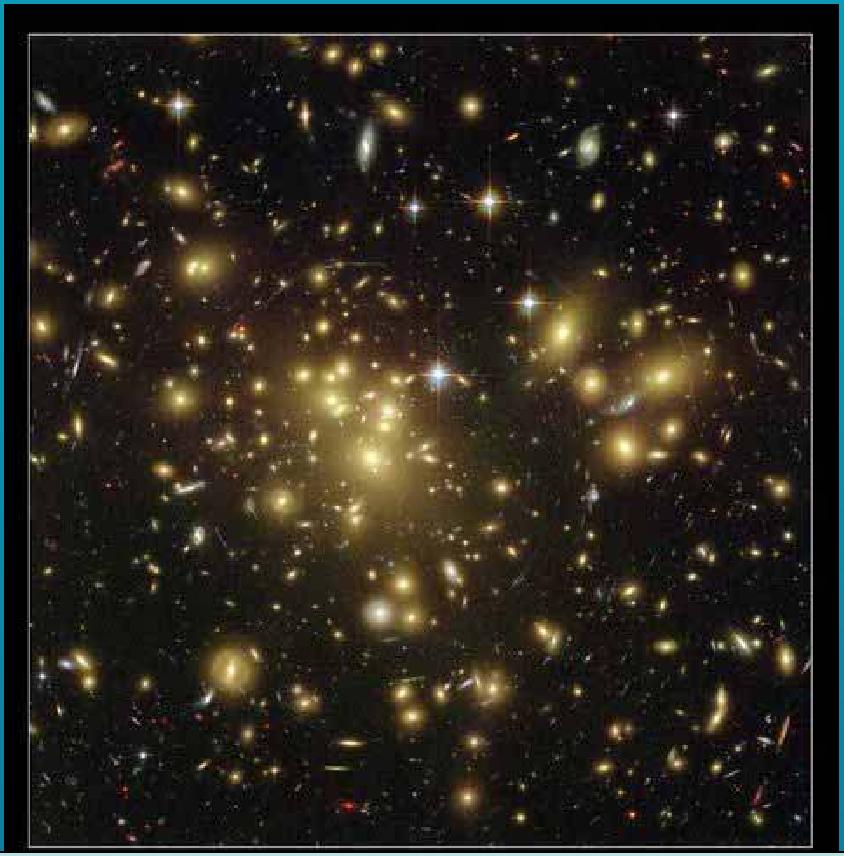
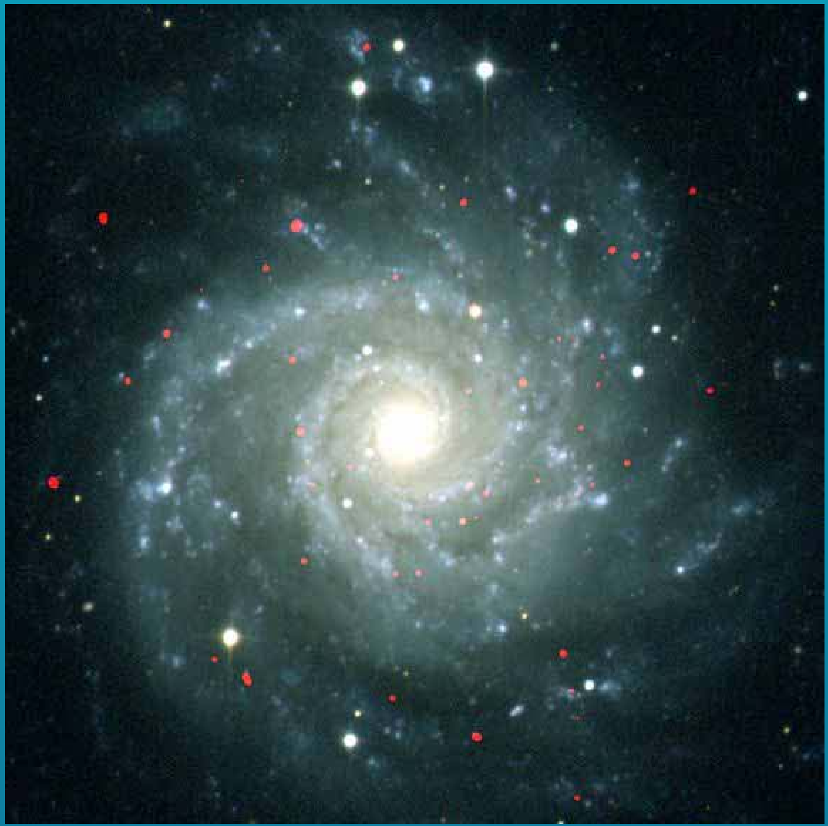
- Probing the transitions between metallic states
  - using transport
  - using mean field theory
  - using tunnelling

The dark matter problem I



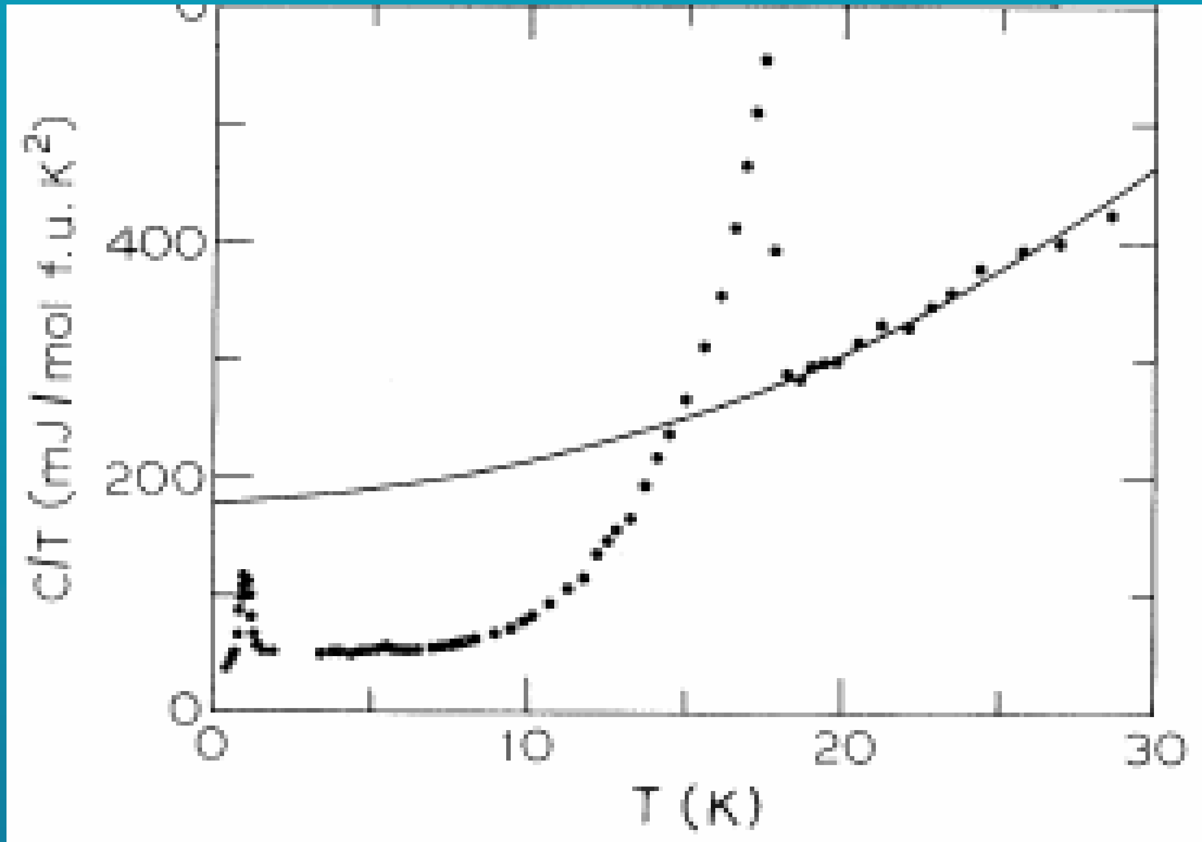


• Zwicky (1933) Viral theorem in clusters,

• Rubin & Ford (1965) galactic rotation curves, …

Dark Matter: has a gravitational effect but is transparent to the current observational probes… unless you know what to look for.

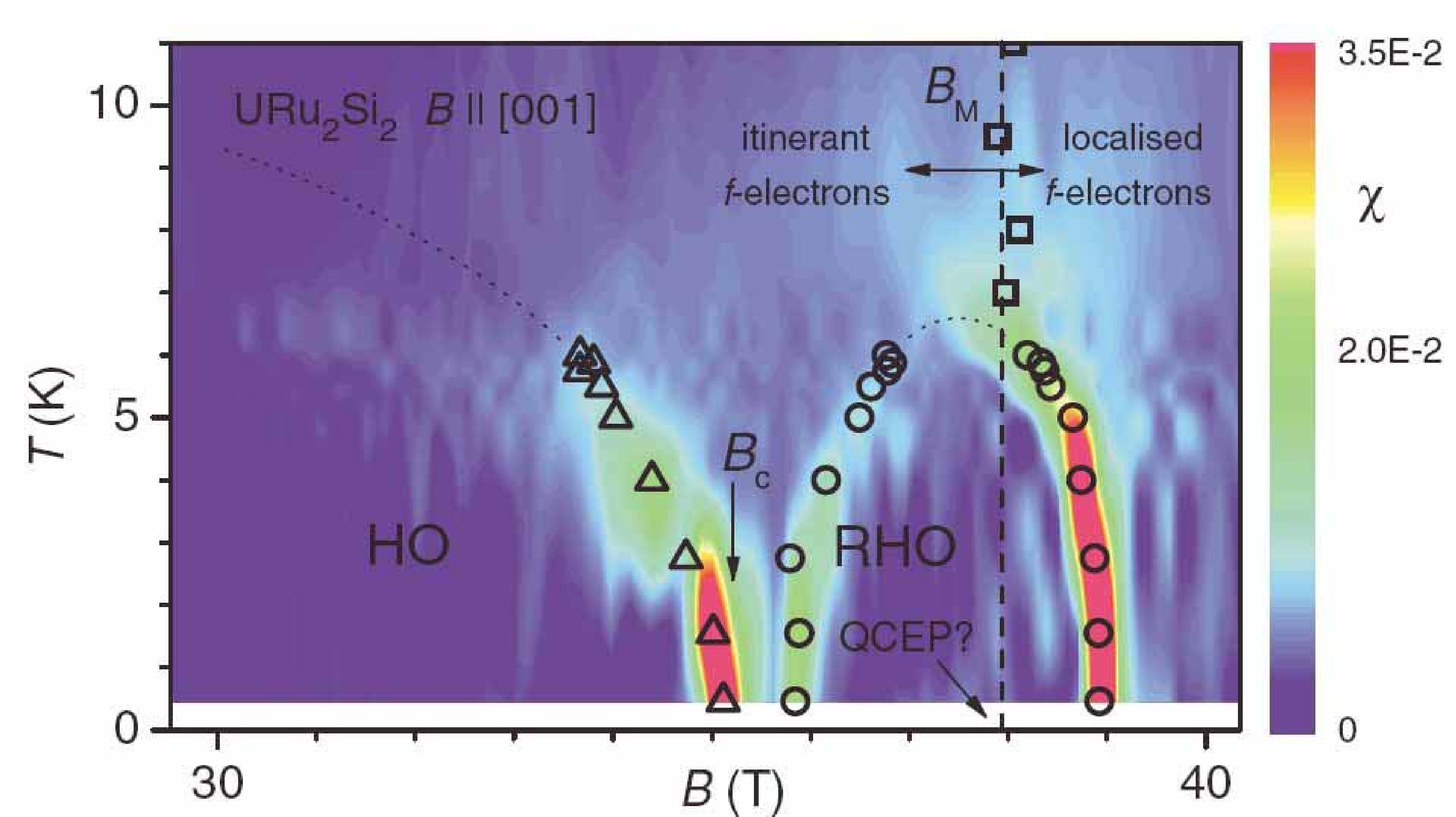
The dark matter problem II



URu_2Si_2: T. T. M. Palstra, A. A. Menovsky, J. van den Berg, A. J. Dirkmaat, P. H. Kes, G. J. Nieuwenhuys and J. A. Mydosh Physical Review Letters **55**, 2727 (1985)

- URu_2Si_2

The dark matter problem II



URu_2Si_2: N. Harrison, M. Jaime, and J. A. Mydosh, Phys. Rev. Lett. **90**, 096402 (2003).

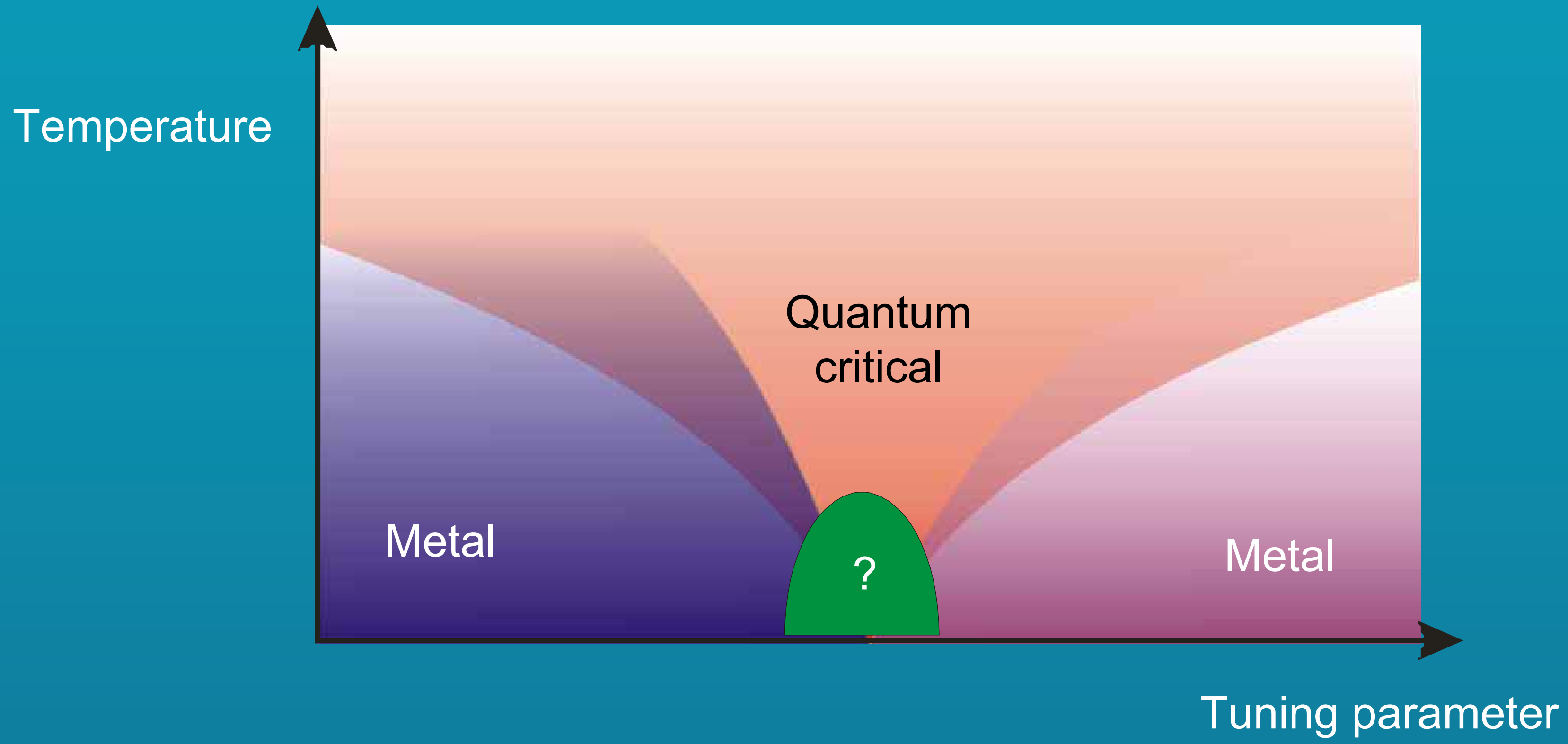
- URu_2Si_2 , Sr3Ru2O7 , cuprates(?) …

Condensed dark matter: matter that has a thermodynamic effect but whose order parameter is transparent to current probes.

An approach to the condensed dark matter problem

- Typically a metal–to–metal transition
  - Fermi surface changes
  - Well defined at T=0
- Fermi surface change at T=0: the heavy fermion quantum critical point?
  - Signatures in transport.
- What other changes could occur in interacting systems?
  - Mean field analysis
- How could one detect them directly?
  - A tentative proposal

Assumptions:

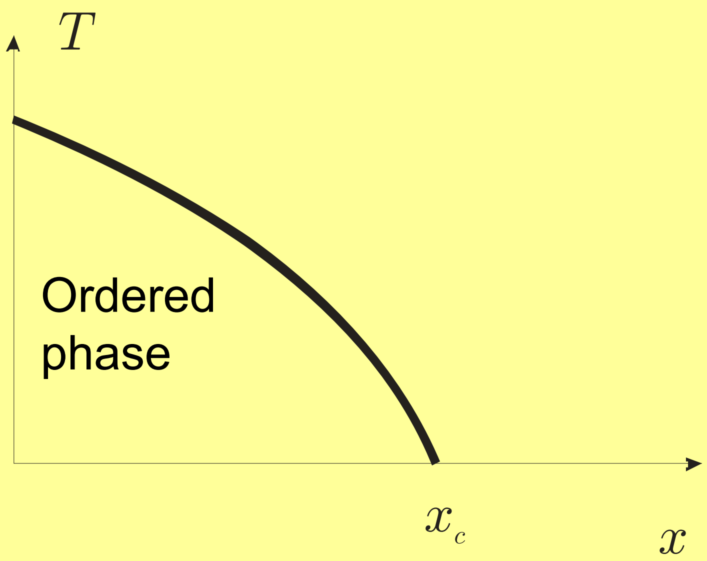


- Living at or near $T=0$
- Elastic (impurity) scattering dominates over inelastic
- Interested in characterizing metallic states, not the quantum critical region

Example 1: Fermi surface changes at an itinerant QCP

Conventional view: an itinerant picture of density wave order developing in a metal.
J. A. Hertz Phys. Rev. B **14**, 1165 (1976).

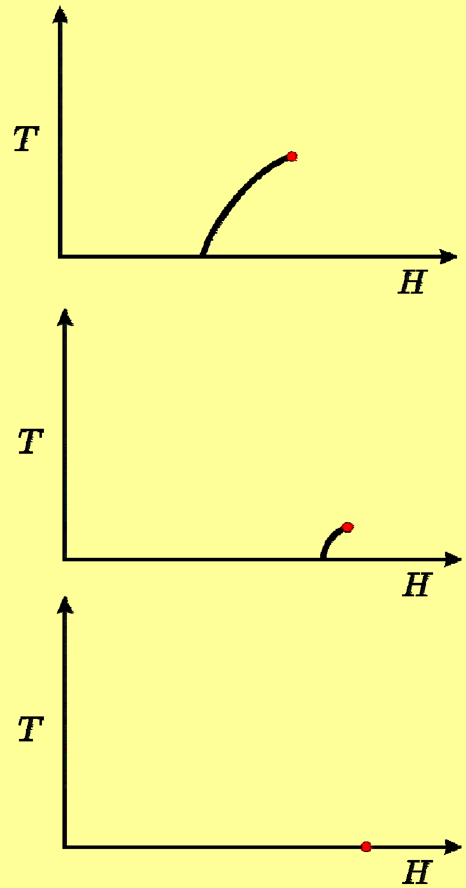
Continuous phase transition driven to T=0



Example: **$CePd_2Si_2$**
Antiferromagnetism tuned by pressure.

[S. R. Julian *et. al.* J. Phys. C. (1996)]

Critical end-point driven to T=0



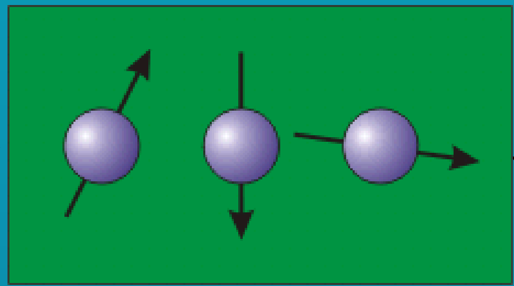
Example: **$Sr_3Ru_2O_7$** Metamagnetism tuned by magnetic field angle

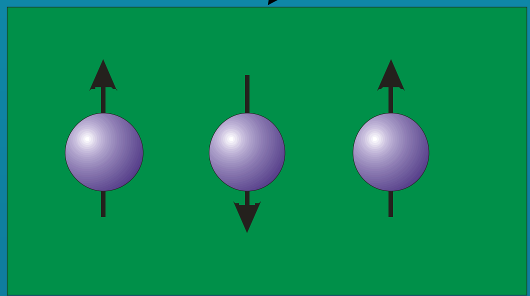
[S.A. Grigera *et al.*, Science (2001)]

Example 1: Fermi surface change at a heavy fermion QCP

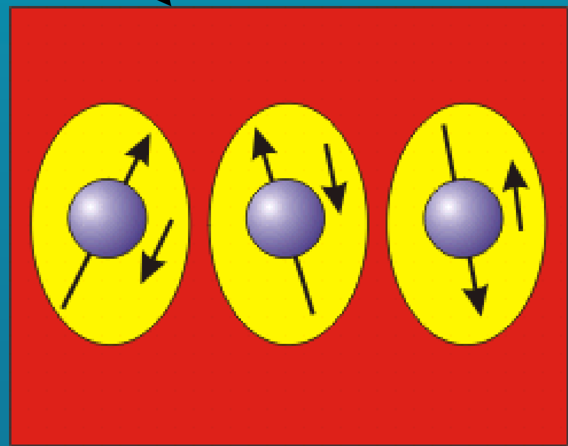
P. Coleman, C. Pépin, Q. Si & R. Ramazashvili; J. Phys. C. **13** R723 (2001)

High temperature
free magnetic moments
+ conduction electrons
$N_e=N_c$



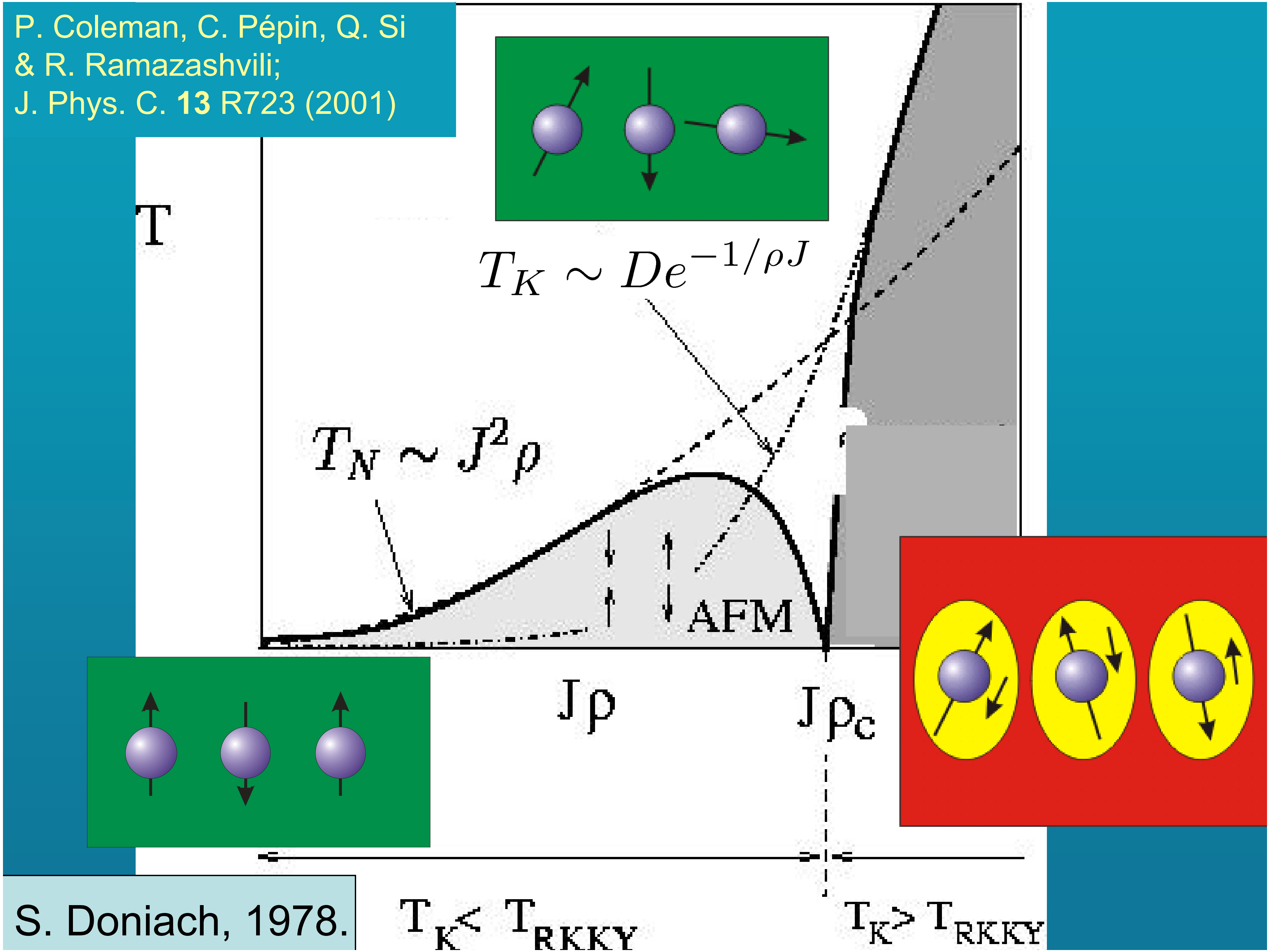


OR



Low temperatures
Free spins order, and
$N_c=N_c$ (small Fermi volume)

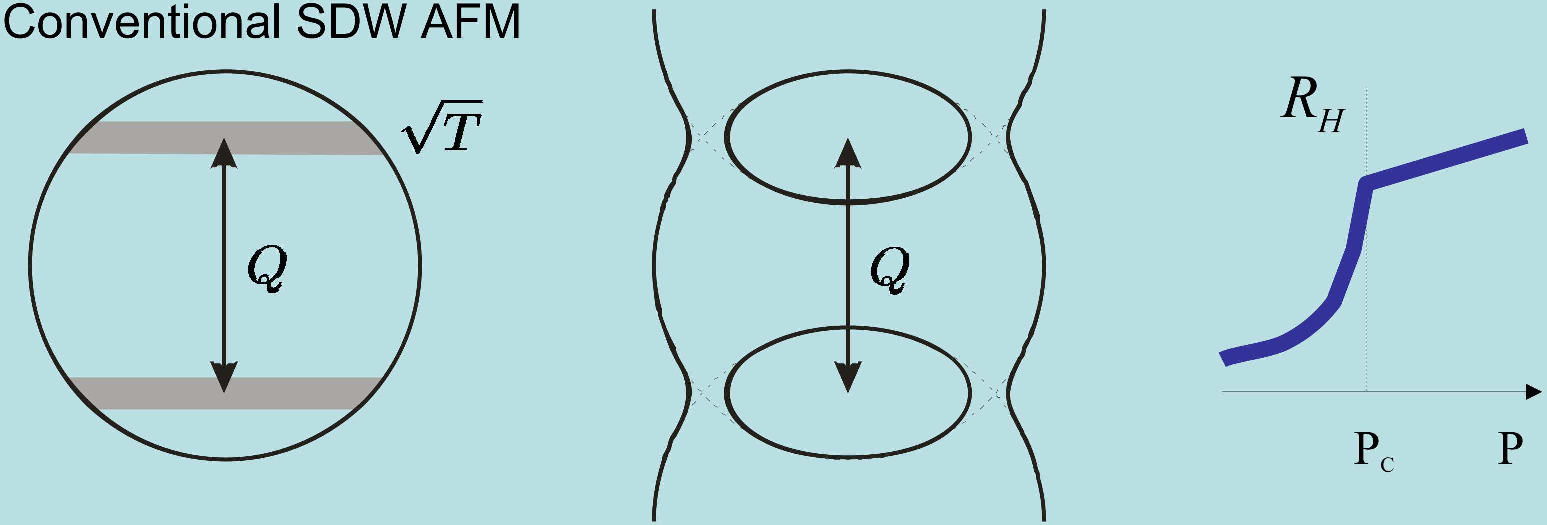
Low temperatures
No free spins but very heavy
electrons ($m \sim 10^3\ m_e$) and
$N_e=N_c+N_f$ (large Fermi volume)


P. Coleman, C. Pépin, Q. Si
& R. Ramazashvili;
J. Phys. C. 13 R723 (2001)
T
$T_K \sim De^{-1/\rho J}$
$T_N \sim J^2\rho$
AFM
Jρ
Jρc
S. Doniach, 1978.
$T_K < T_{RKKY}$
$T_K > T_{RKKY}$

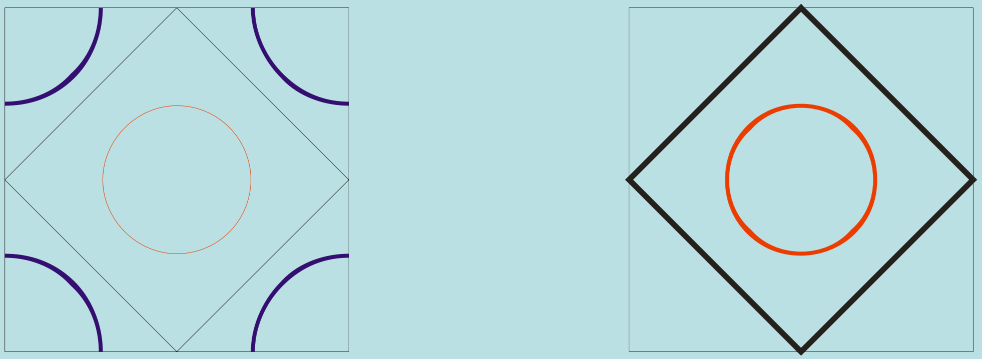
Hall effect as a discriminator:

P. Coleman, C. Pépin, Q. Si & R. Ramazashvili; J. Phys. C. **13** R723 (2001)

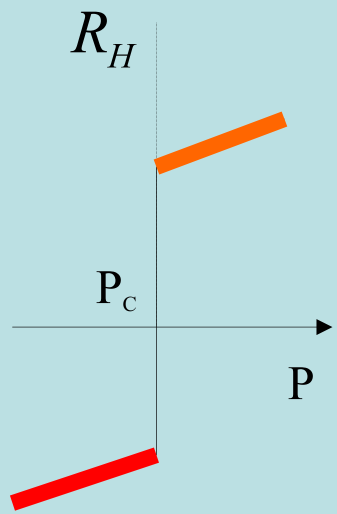
(1) Conventional SDW AFM



(2) Unconventional heavy fermion AFM



$$\frac{2v_F}{(2\pi)^d} = n_e + 1 \qquad\qquad \frac{2v_F}{(2\pi)^d} = n_e$$



Previous work appears to confirm assumption:

M. R. Norman, Qimiao Si, Ya. B. Bazaliy and R. Ramazashvili

Phys. Rev. Lett. **90**, 116601 (2003). Phys Rev B **69**, 144423 (2004).

Boltzmann transport theory in the relaxation time approximation (*T*=0)

$$e\vec{v}_k \cdot \vec{E}\partial_\epsilon n_F + e\vec{v}_k \times \vec{B} \cdot \vec{\nabla}_k g_k = -\frac{g_k}{\tau}, \quad n_k = n_F + g_k$$

Method of solution: “Jones-Zener” expansion:

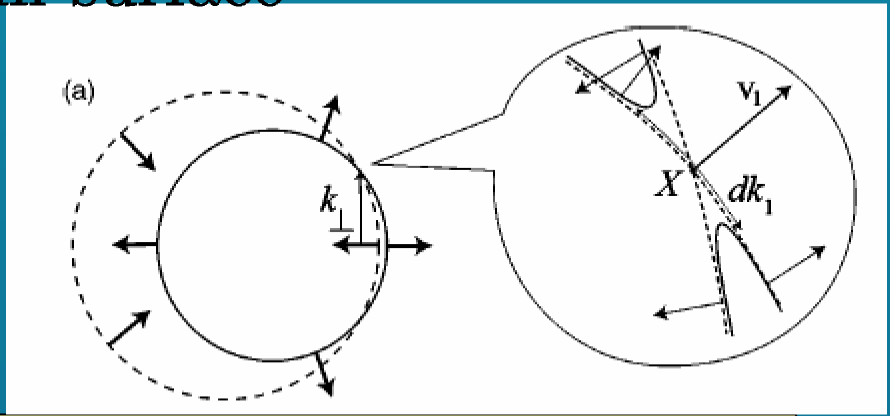
$$J_{ij} = \sigma_{ij} E_j \text{ where } \sigma_{ij} = \sigma_{ij}^{(0)} + \sigma_{ij}^{(1)}(B) + \sigma_{ij}^{(2)}(B^2) + \cdots$$

A weak-field expansion

$\sigma_{ij}^{(\alpha)}$ is an integral over the Fermi surface

$$\sigma_{ij}^{(1)} = \epsilon_{\alpha\beta\gamma} \frac{e^3\tau^2 B_\alpha}{4\pi^3\hbar} \int \frac{dS_F}{v_F} v_i v_\gamma \frac{\partial^2\epsilon}{\partial k_j \partial k_\beta}$$

c.f. Ong construction:
Ph. Ong, PRB 1991

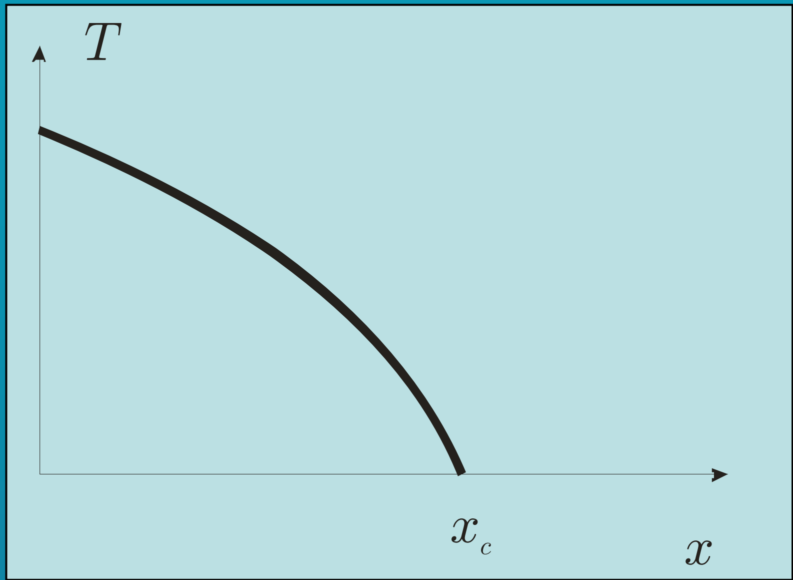


Conclusion: all changes are linear in the gap (at most) – so smooth.

Is magneto-transport really continuous at a density wave QCP?

J. Fenton and A. J. Schofield, Phys. Rev. Lett. **95** 247201 (2005)

At a quantum critical point…



… an energy scale is driven to zero.

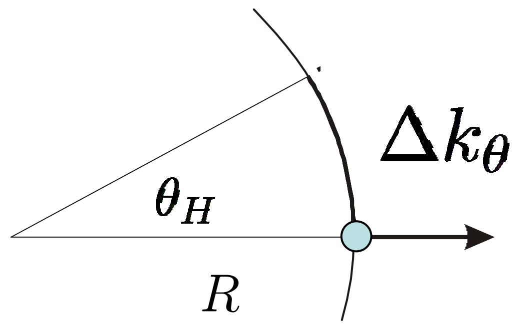
Then we "probe" the system.
Assumption: The probe is "weak" – but weak with respect to what?

What if the probe has to be weak with respect to the energy scale being driven to zero? Could we use this?

See also work by *A. G. Green and S. L. Sondhi,* Phys Rev. Lett. **95** 267001 (2005)

Specific example: is the weak-field expansion valid at a QCP?

What is the expansion made with respect to? (Answer – the local Hall angle)



R radius of curvature of F.S.

$$\frac{\partial}{\partial t}\vec{p} = e\vec{v} \times \vec{B} \;\rightarrow\; \hbar\frac{\partial k_\theta}{\partial t} = ev_F B$$

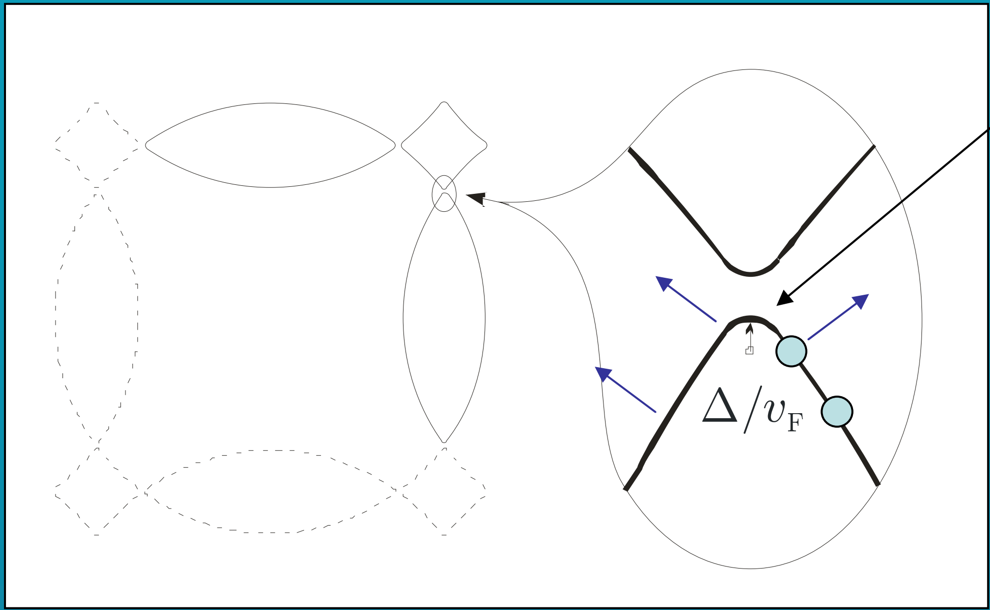
$$\Delta k_\theta = \frac{ev_F B\tau}{\hbar} \quad \Rightarrow \theta_H = \frac{ev_F B\tau}{\hbar R}$$

e.g. Usual magneto-conductance

$$\Delta J_x = J(B) - J(0) = J_0\ [\cos\theta_H - 1] \sim\ -J_0\ \theta_H^2$$

$\Rightarrow B^2$ magnetoconductance $\Rightarrow B^2$ magnetoresistance

But at an SDW/CDW critical point at T=0



Radius of curvature vanishes

$$R \rightarrow \frac{\Delta}{\hbar v_F} \rightarrow 0 \text{ as } \Delta \rightarrow 0$$

Condition for weak-field response:

$$\theta_H \ll 1$$

$$\Rightarrow \frac{e v_F^2 B \tau}{\Delta} \ll 1$$

$$\Rightarrow B \ll \frac{\Delta}{e v_F^2 \tau}$$

so weak-field region collapses as $\Delta \rightarrow 0$

What happens if B is greater than the weak-field scale?
All the quasi-particles which go round the "corner" between scattering events are deflected by a large Hall angle (determined by Fermi surface geometry and Q)

$$\text{fraction deflected: } \frac{\Delta k_\theta}{2\pi k_F} \propto |B\tau| \Rightarrow \frac{\Delta \rho}{\rho} \rightarrow |B\tau|$$

MR non-analytic in field

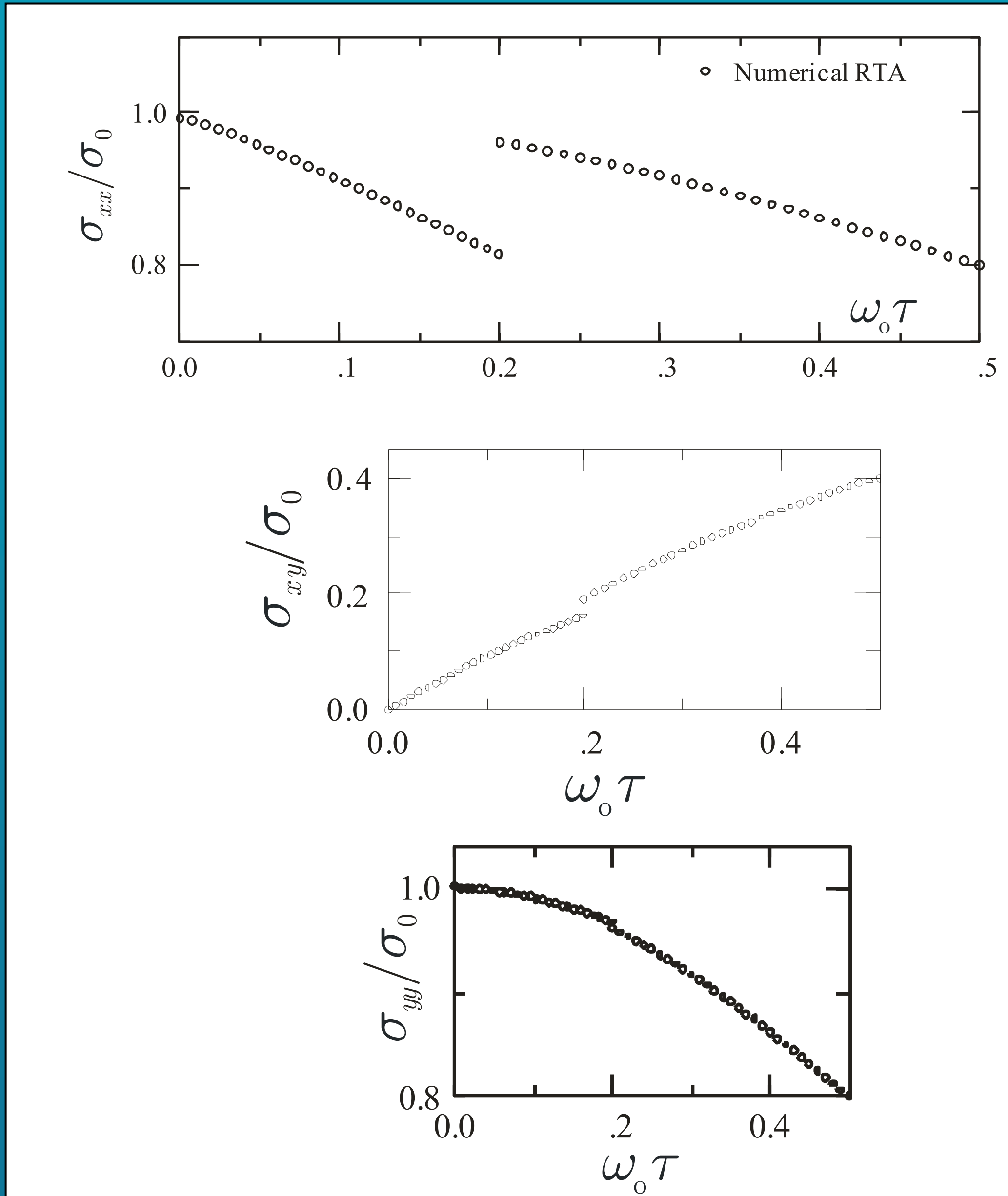
E.g. T=0 magneto-conductances at a field-driven DW QCP

Numerical results illustrating a density-wave quantum critical point at $\omega_c\,\tau = 0.2$

conductivity parallel to Q

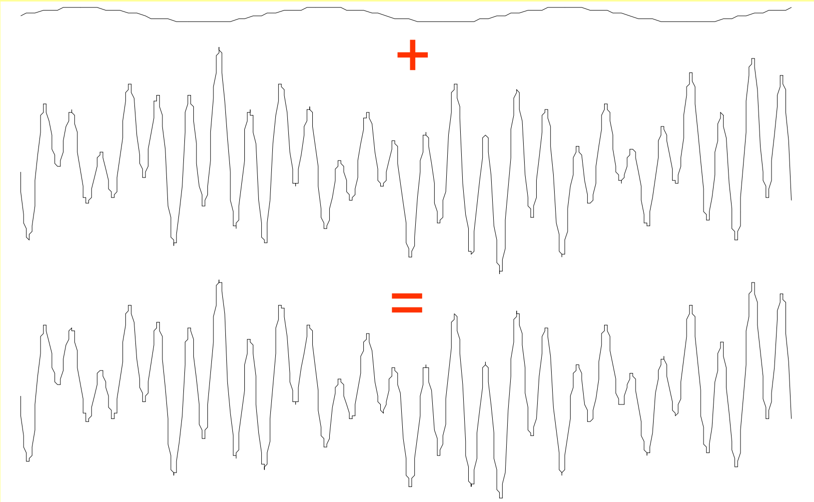
Hall conductivity

conductivity perpendicular to Q



Not the whole story: beyond the relaxation time approximation

• Role of disorder: if $\tau\Delta < 1$
then quasiparticles don't see the gap

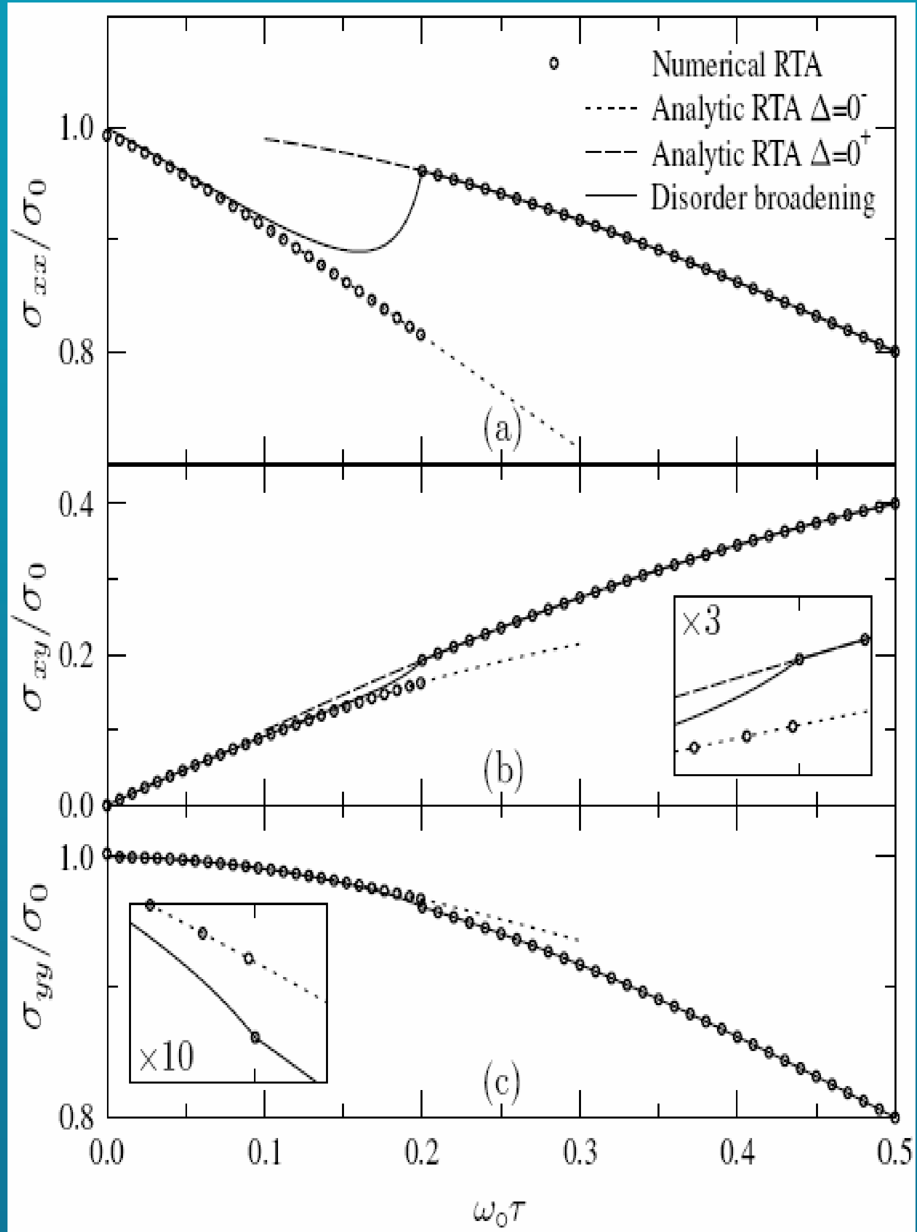


• Magnetic (Zener) breakdown

$$B > \frac{\Delta^2}{ev_F^2}$$

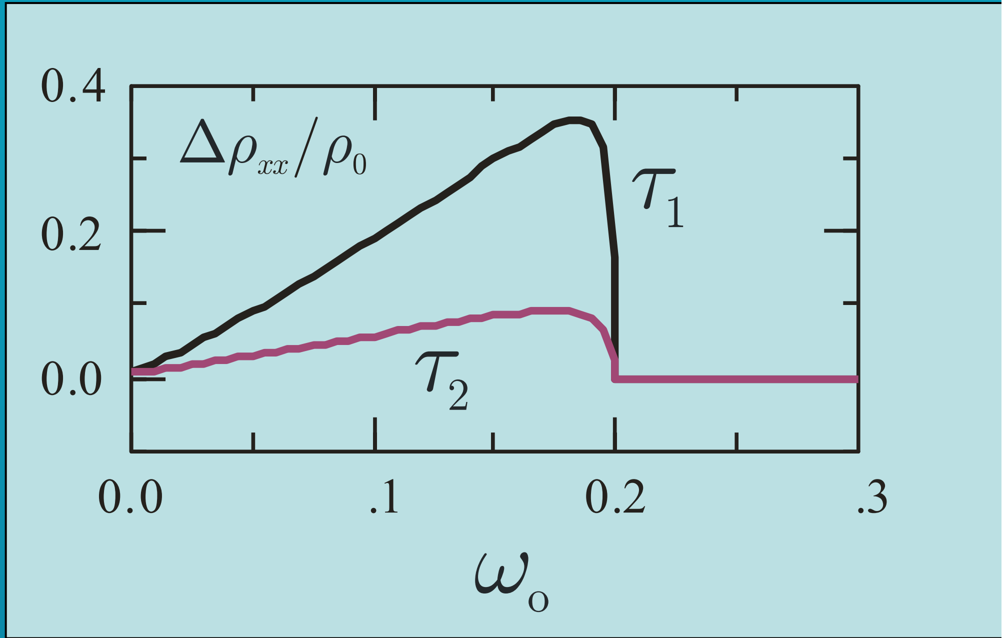
quasi-particles tunnel onto the original undistorted Fermi surface

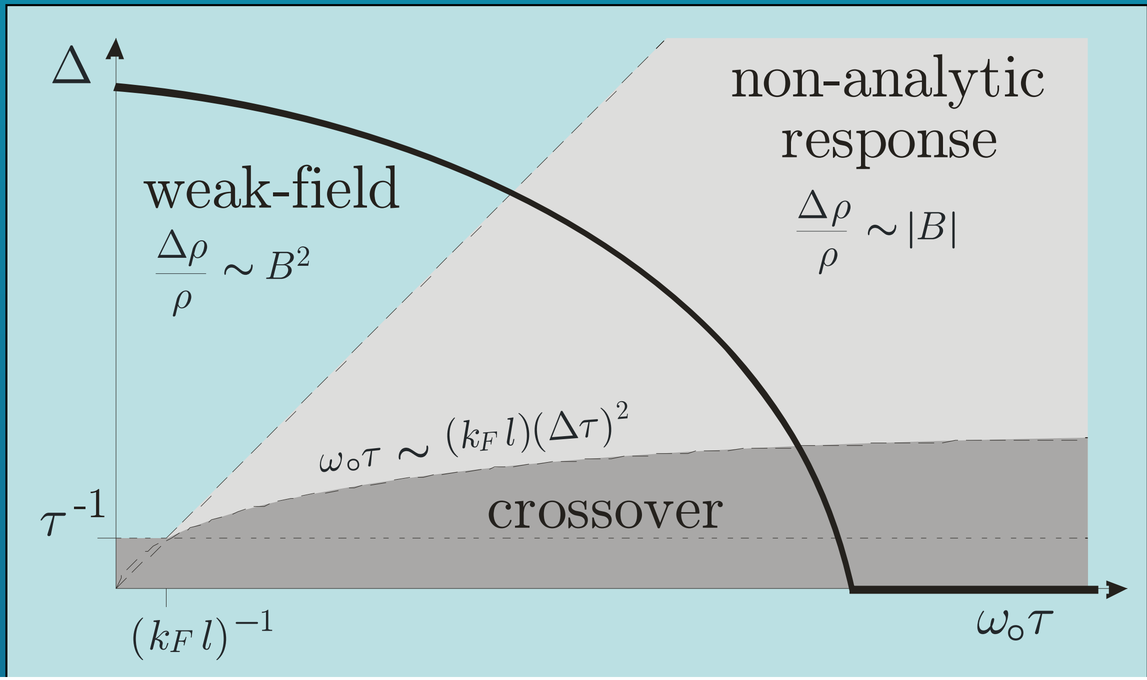
Both effects generate a cross-over region near the QCP.



Net result

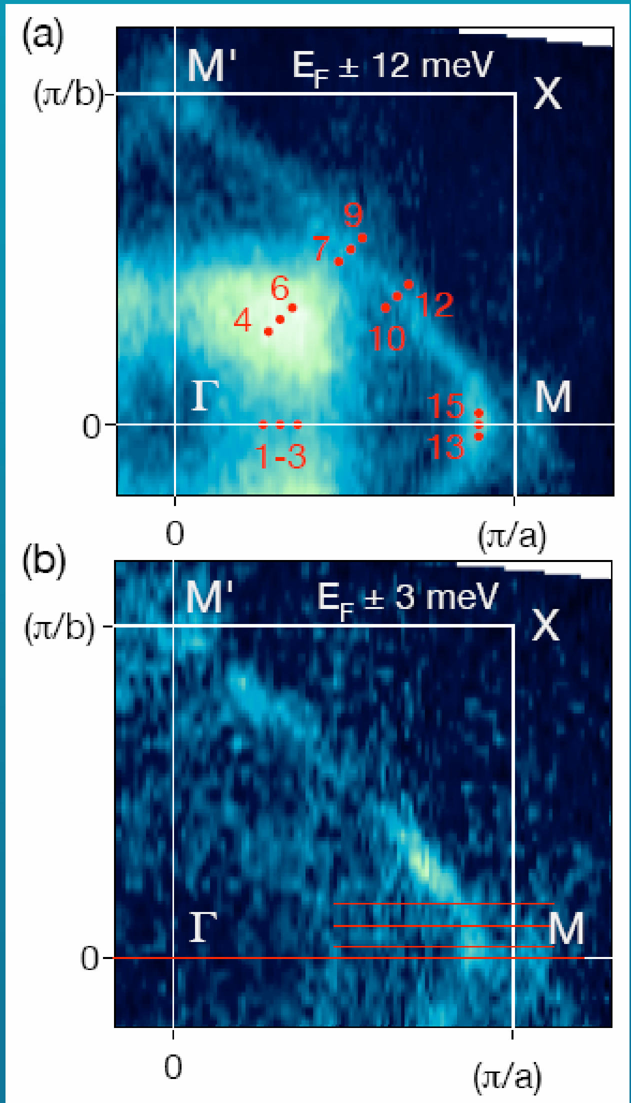
- Breakdown of weak-field assumption at a QCP
- Discontinuities in a transport in a field:
    - rounded by disorder
    - Most pronounced in MR

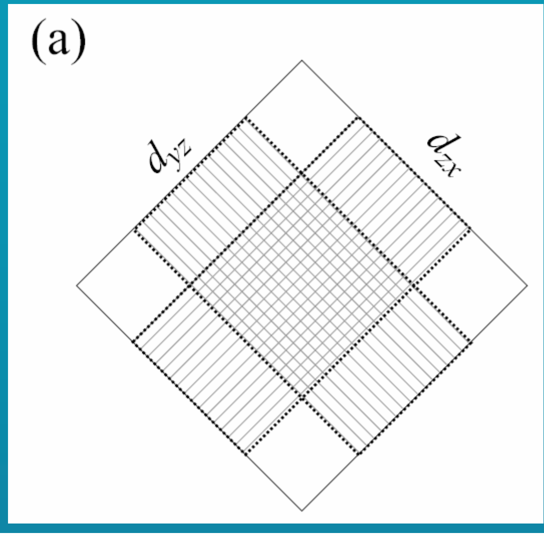


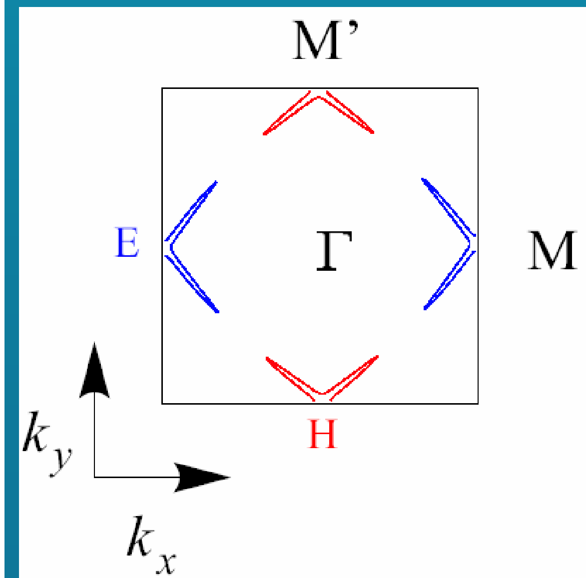


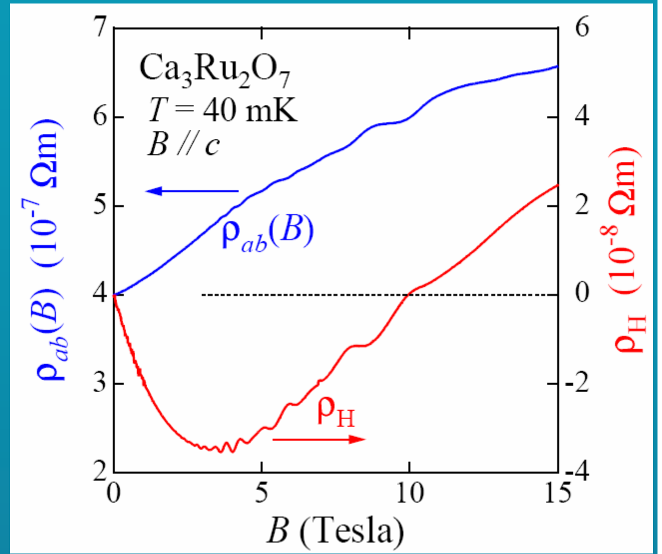
Experimental verification of non-analyticity: $Ca_3Ru_2O_7$

N. Kikugawa, A. Rost and A. P. Mackenzie (unpublished)
F. Baumberger *et al.* Phys. Rev. Lett. **96**, 107601 (2006)



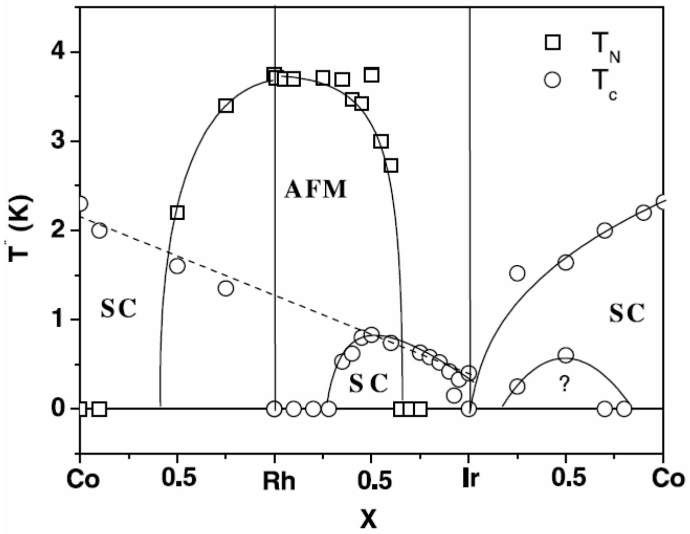




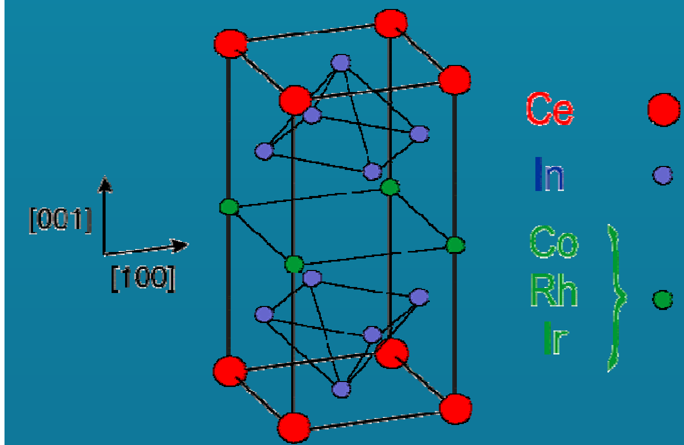


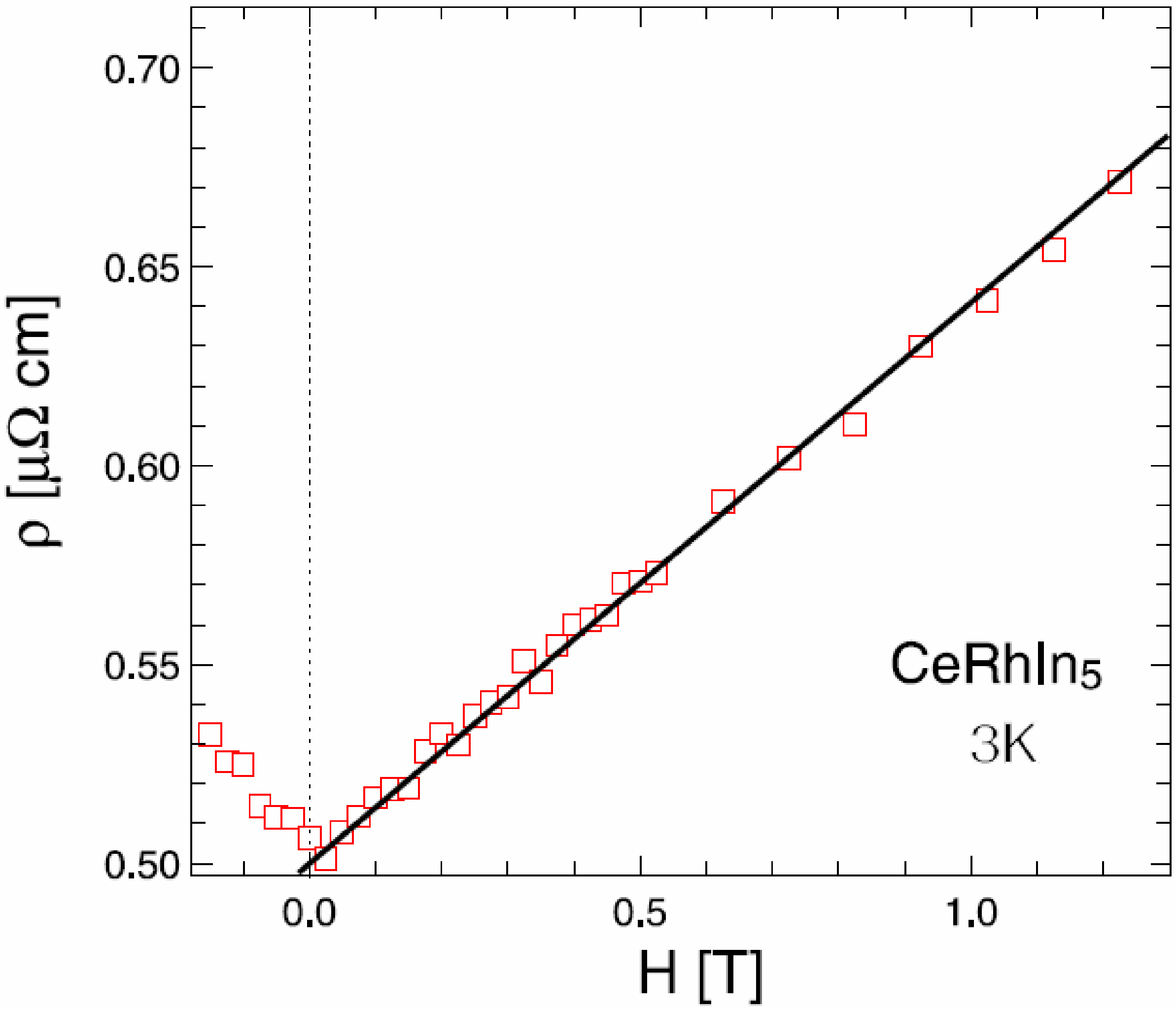
- Magnetoresistance is linear in B
- Quadratic for B<0.5T
- Quantum oscillations for B about 5T
- Minimum radius of curvature of FS less than 10% of Fermi surface perimeter

The smoking gun of a density wave in the heavy fermions?



P.G. Pagliuse et al.
Physica B 312, 129 (2002)





J. P. Paglione, PhD Thesis, University of Toronto (2005)

What can we say about transport at a heavy fermion QCP?

P. Coleman, J. B. Marston, and A. J. Schofield, Phys. Rev. B **72**, 245111 (2005)

- Very difficult to deal with Kondo physics and magnetism together.
- Treat the Kondo-Heisenberg lattice model in the large-*N* limit

$$H = \sum_{ij\sigma} -t_{ij}\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} - \frac{J}{N}\sum_{\alpha,\beta}\left(\hat{c}^\dagger_{j\alpha}\hat{f}_{j\alpha}\right)\left(\hat{f}^\dagger_{j\beta}\hat{c}_{j\beta}\right) - \frac{J_H}{N}\sum_{ij}\hat{f}^\dagger_{i\alpha}\hat{f}_{j\alpha}\hat{f}^\dagger_{j\beta}\hat{f}_{i\beta}$$

conduction electron dispersion — Kondo coupling — Heisenberg exchange

- Now imagine the lattice so frustrated that there is no *T*=0 magnetic order. [Senthil *et al.* PRB **69**, 035111 (2004)]
- At large-*N*: get an RVB spin liquid ground state. $\langle \hat{f}^\dagger_{i\alpha}\hat{f}_{j\alpha}\rangle = |\chi_{ij}|e^{i\theta_{ij}}$
  - spinons develop a dispersion
  - coupled to a gauge field: like EM θ_{ij}
- The Kondo term can develop a mean-field: $\langle \hat{c}^\dagger_{j\alpha}\hat{f}_{j\alpha}\rangle = V_j$

$$H = \sum_{ij\sigma} -t_{ij}\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} - \sum_{j\sigma} V_j \hat{f}^\dagger_{j\sigma}\hat{c}_{j\sigma} + \text{H.c.} - \sum_{ij\alpha}|\chi_{ij}|e^{i\theta_{ij}}\hat{f}^\dagger_{i\alpha}\hat{f}_{j\alpha} + \text{H.c.}$$

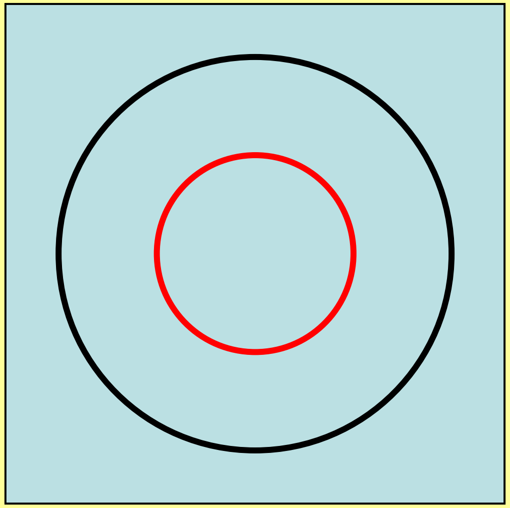
electron-spinon hybridization — spinon dispersion

Two possible phases:

Phases determined by mean-field theory:

T_K < J_H: V=0 (no hybridization)

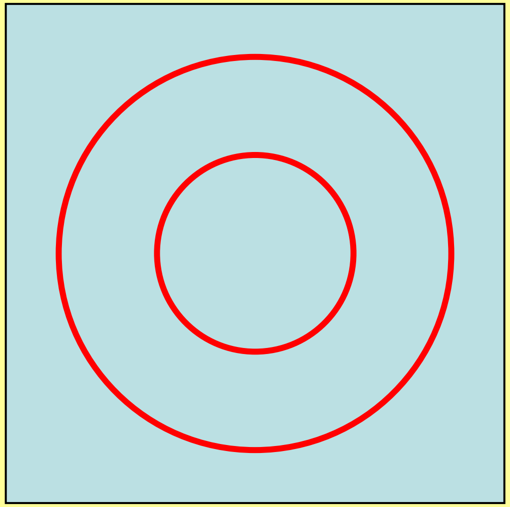
spin liquid state



Spinons are uncharged (don't see an applied EM field). Only conduction electrons respond to EM fields.

T_K > J_H: $V\neq$0 (hybridization)

heavy Fermi liquid



Spinons are charged (see an applied EM field).

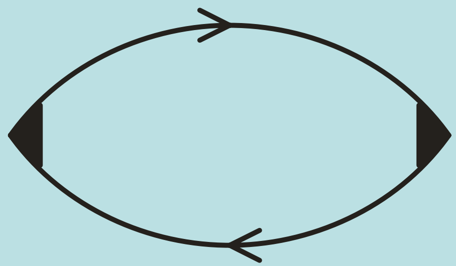
Separated by a second order phase transition

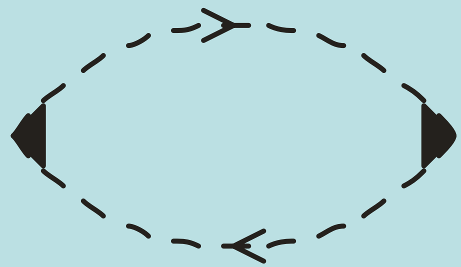
The Kondo effect can lead to a phase transition where the spinon Fermi surface suddenly acquires charge (e). What does that do to transport?

Case 1: If V=0 (the spin liquid state)

- Physical EM field (A) couples only to conduction electrons
- f-spinons see fictitious gauge field (θ)
- Integrate out particles to find conductivity

$$\mathcal{S}_0 = \frac{1}{2}\int \frac{d\omega}{2\pi}\left[-i\omega\sigma_1(\omega)e^2|A(\omega)|^2 - i\omega\sigma_2(\omega)|\theta(\omega)|^2\right]$$





$$\sigma_1(\omega) = \frac{\Omega_1^2}{\tau_1^{-1} - i\omega} \qquad \sigma_2(\omega) = \frac{\Omega_2^2}{\tau_2^{-1} - i\omega}$$

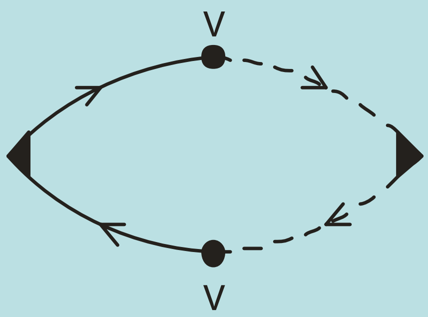
Physical conductivity is just σ_1

Wiedemann-Franz ratio is large: $\dfrac{\kappa_1 + \kappa_2}{e^2\sigma_1 T} = W\left[1 + \dfrac{\sigma_2}{\sigma_1}\right]$

Case 2: If $V \neq 0$ (the heavy Fermi liquid)

- f-spinons and c-electrons hybridize: negligible changes to the dispersion
- f-spinons now couple indirectly to the EM field (A)
- Integrate out particles to find new action:

$$\mathcal{S} = \mathcal{S}_0 + \frac{1}{2}\int \frac{d\omega}{2\pi} aV^2 |e\vec{A}(\omega) - \vec{\theta}(\omega)|^2$$



where $a =\sim \dfrac{n}{mtJ_H}$

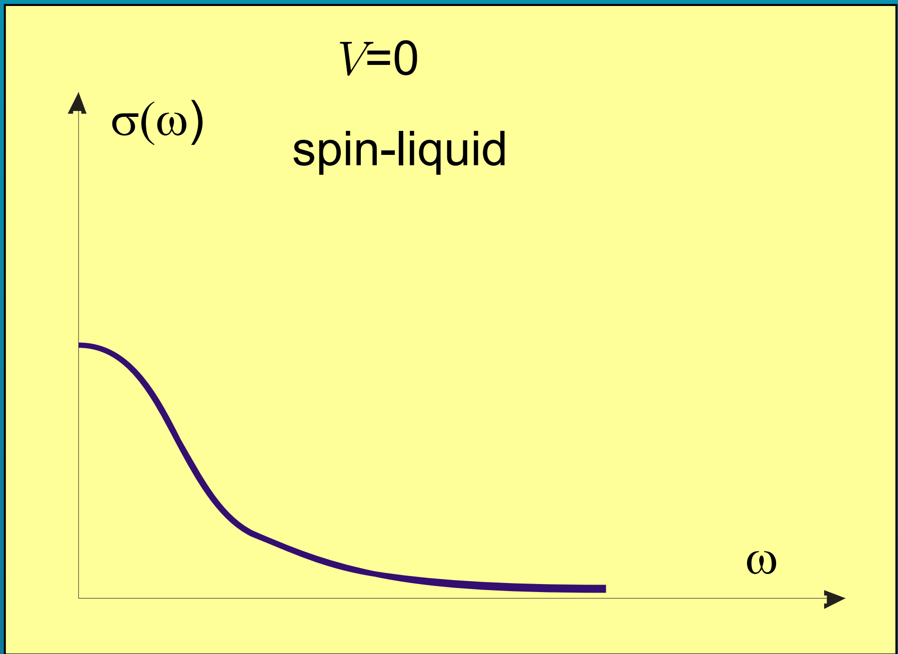
- Now integrate out the θ field to find the conductivity

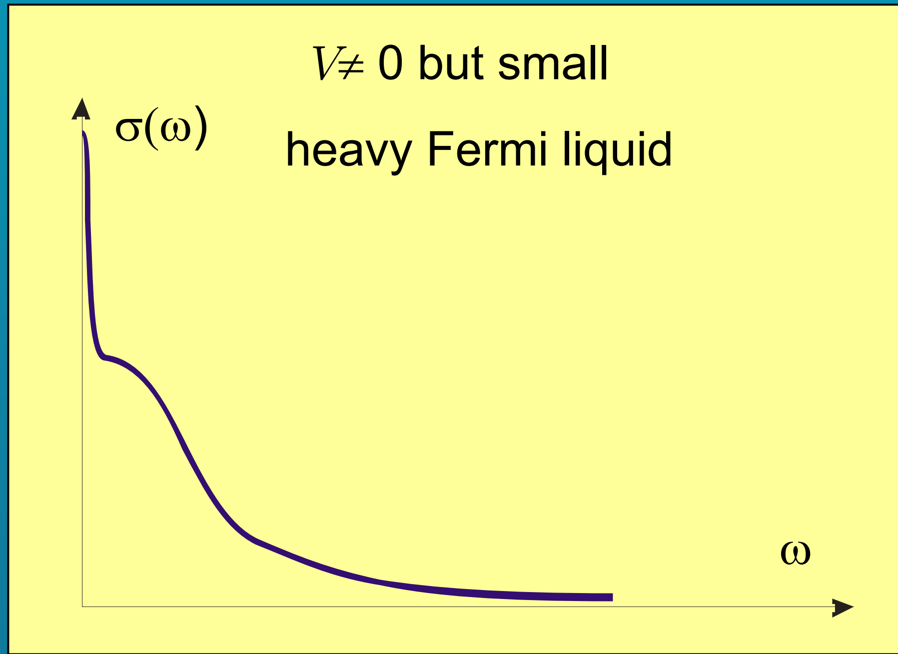
$$\sigma_+ = e^2 \left[\sigma_1(\omega) + \frac{aV^2}{\frac{aV^2}{\sigma_2(\omega)} - i\omega} \right]$$

So at a heavy fermion quantum critical point:

$$\sigma_+ = e^2 \left[\sigma_1(\omega) + \frac{aV^2}{\frac{aV^2}{\sigma_2(\omega)} - i\omega} \right]$$

- the d.c. conductivity **jumps**:
  $\sigma_+(0) = \sigma_1(0)+\sigma_2(0)$
  (charging a Fermi surface).
- Wiedemann-Franz recovered.
- but the Drude weight is very small (growing continuously).





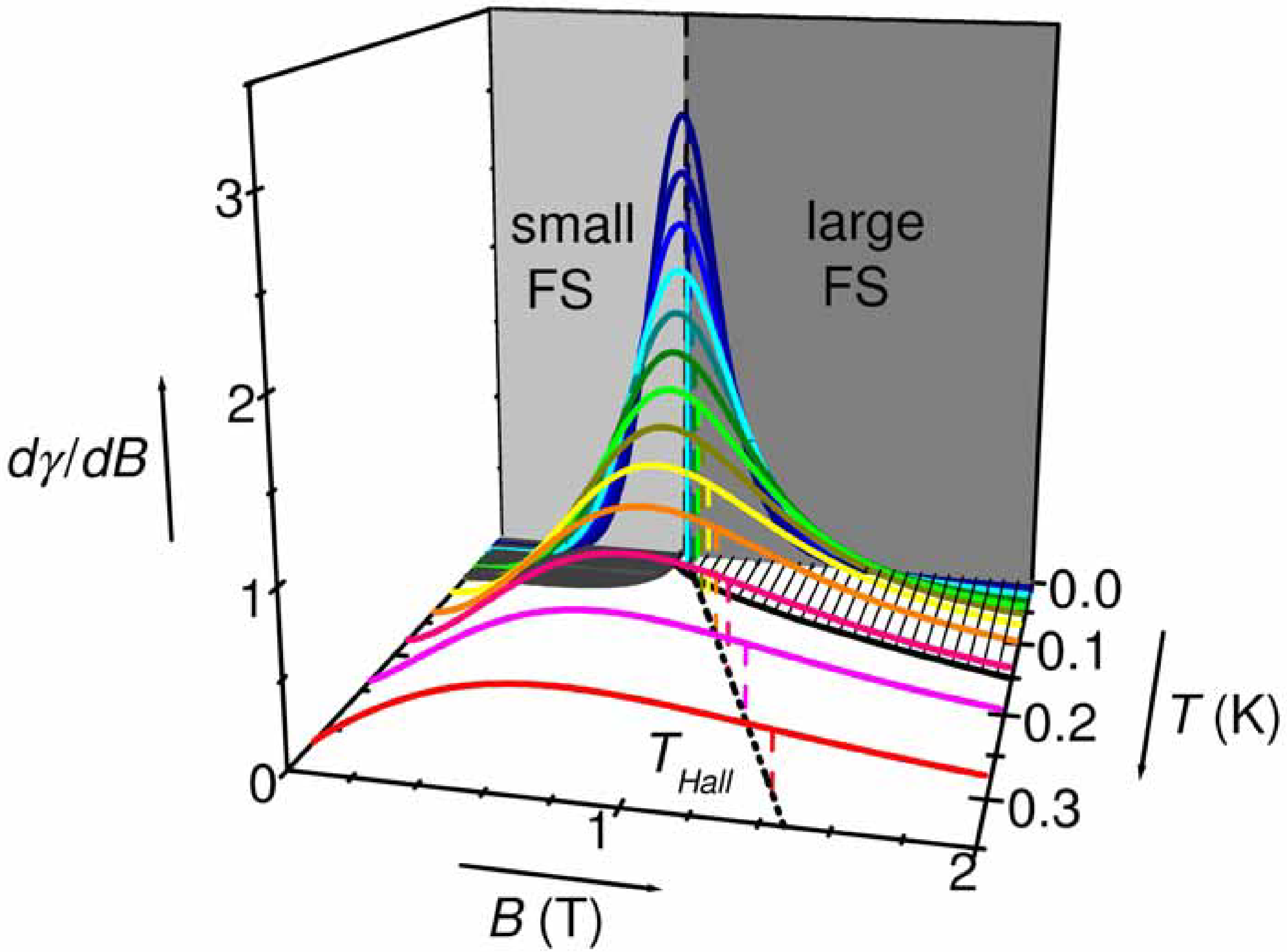
A "Gossamer" Fermi liquid

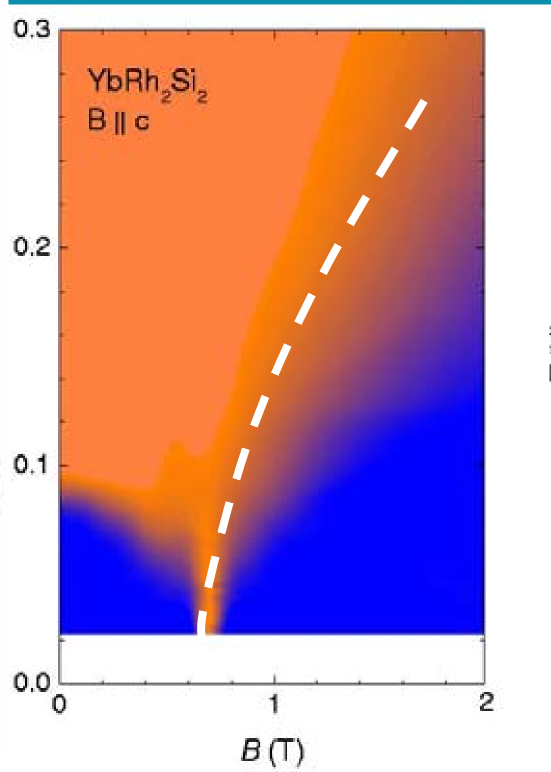
At high frequencies [$\omega > aV^2/\sigma_2(0)$] the "heavy" Fermi liquid decouples and appears light.

Experimental situation

Hall effect studies: in $YbRh_2Si_2$

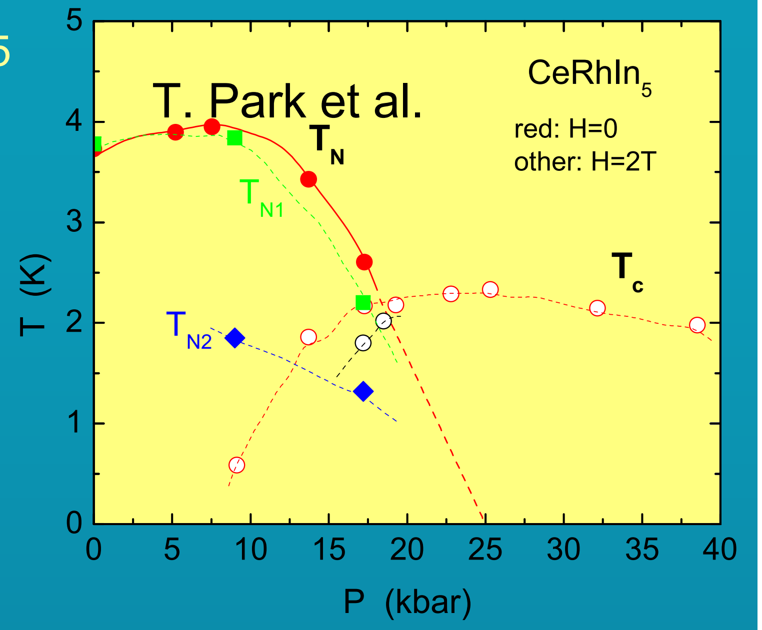
S. Paschen *et al*. Nature, **432**, 881 (2004).

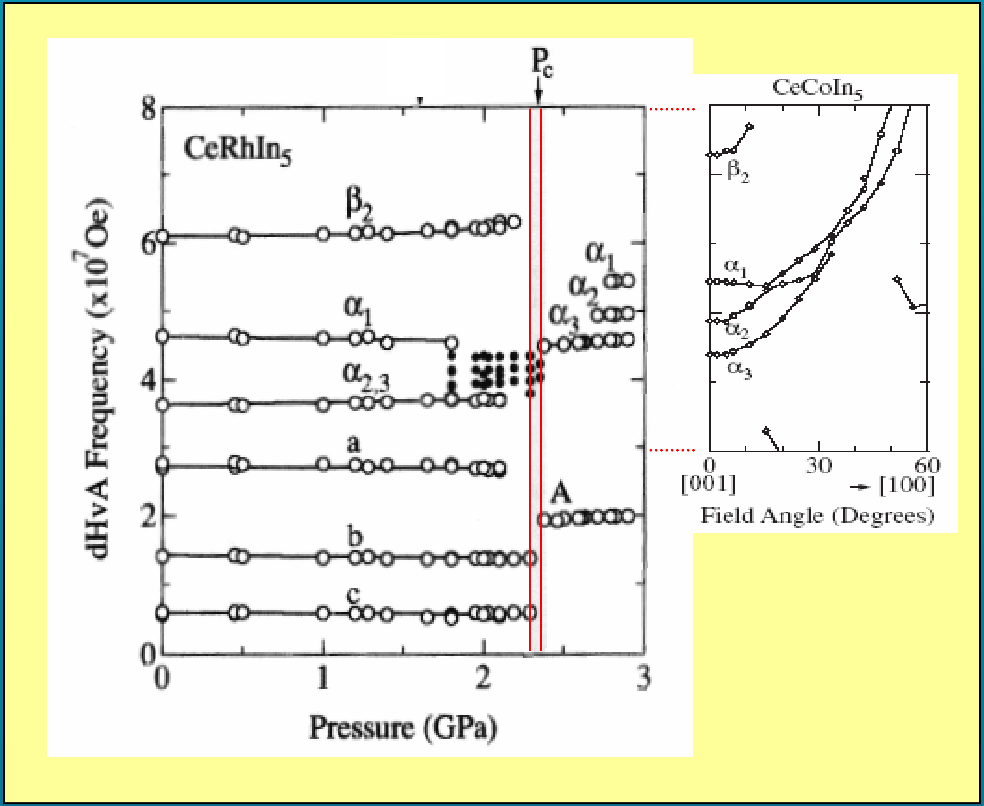


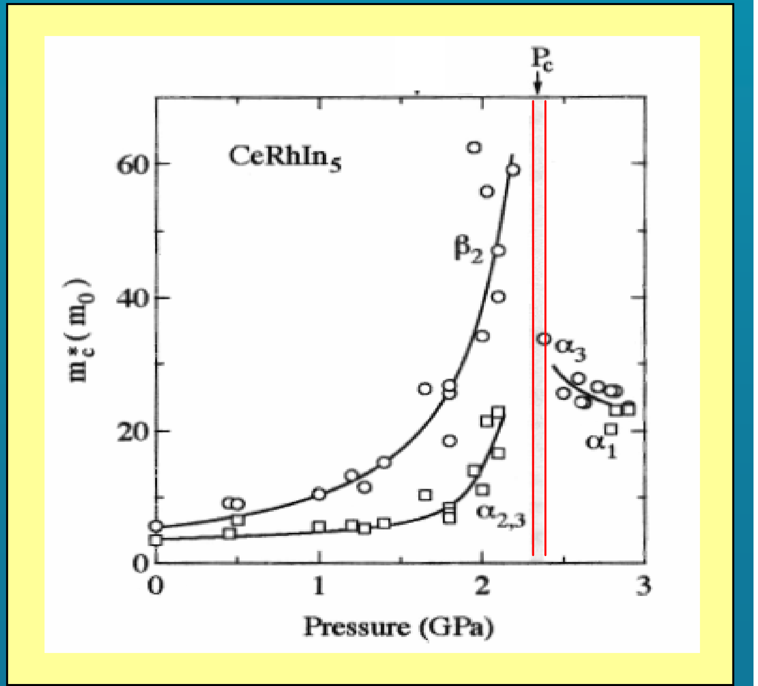


dHvA Fermi surface changes in $CeRhIn_5$

- Superconductivity masks a Neel AFM QCP at 23KBar – revealed in a B field.
- See mass divergences as the QCP is approached.
- dHvA frequences change abruptly to mirror those in $CeCoIn_5$.



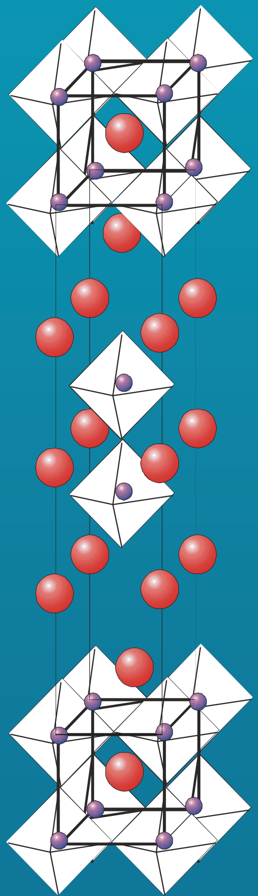


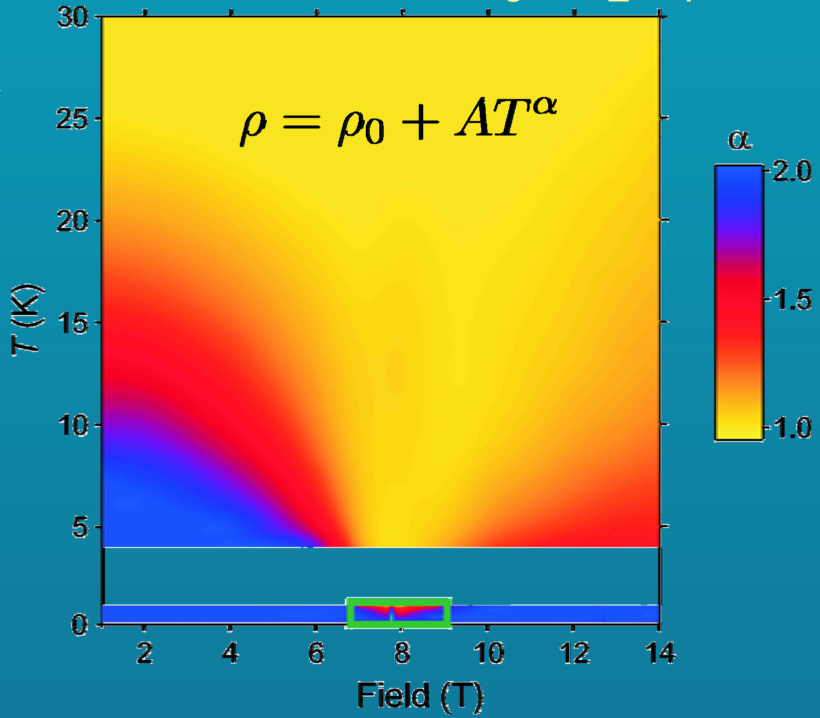


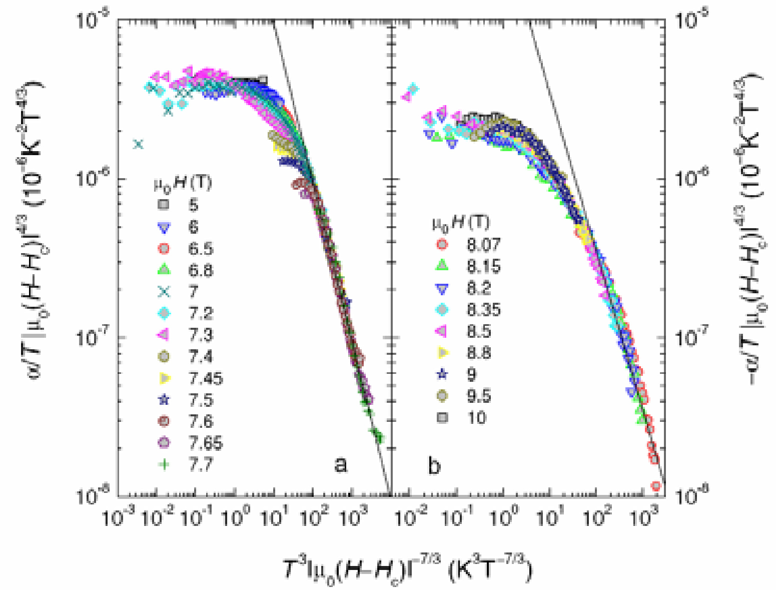
Y. Onuki *et al.* JPSJ **73**, 769 (2004); R. Settai *et al.* JPSJ **74**, 3016 (2005)

What other condensed “dark matter” candidates are there?
J. Quintanilla & A.J.Schofield, cond-mat/0601103

- Motivation: the vicinity of the metamagnetic quantum critical end point in $Sr_3Ru_2O_7$







S.A.Grigera, R.S.Perry, A.J.Schofield, M.Chiao, S.R.Julian, G.G.Lonzarich, S.I.Ikeda, Y.Maeno, A.J.Millis, A.P.Mackenzie,

Science, **294**, 329 (2001).

P. Gegenwart, F. Weickert, M. Garst, R.S. Perry and Y. Maeno,

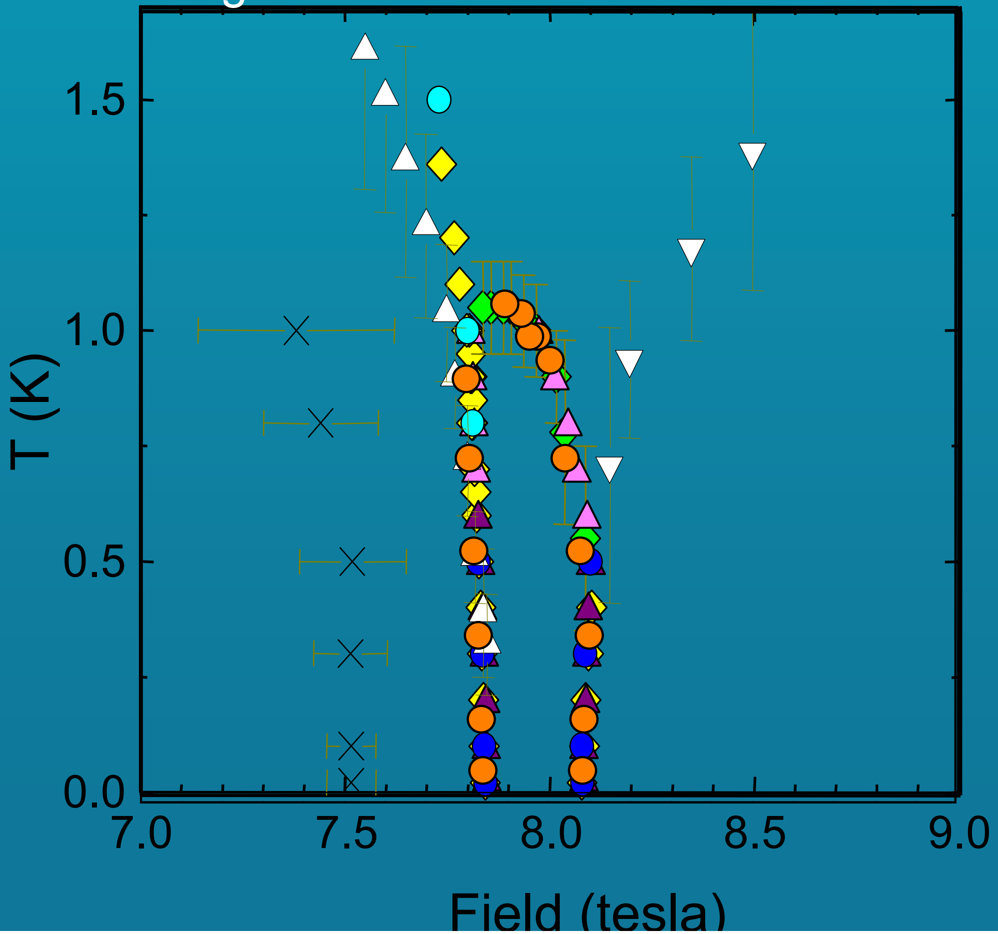
Phys. Rev. Lett. **96**, 136402 (2006).

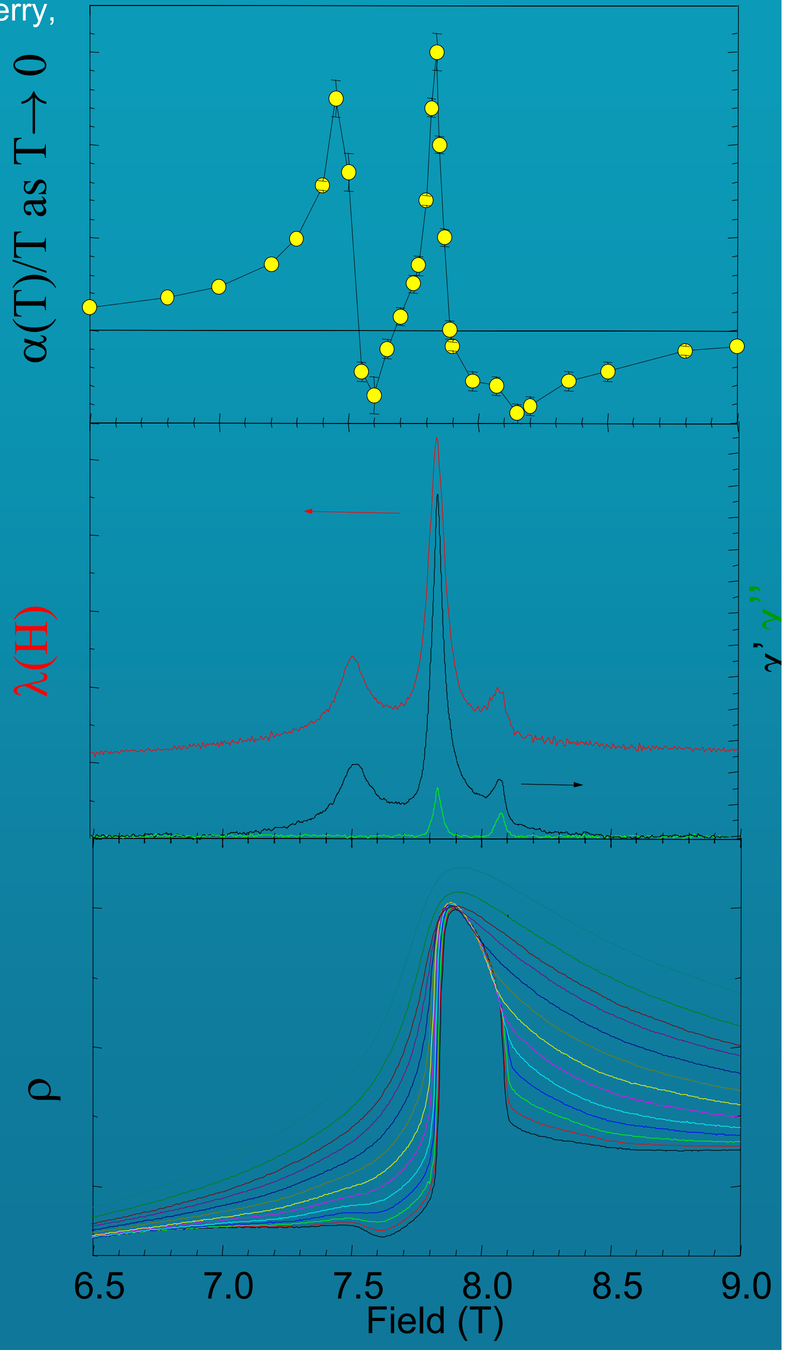
Near the "QCP"

S. A. Grigera, P. Gegenwart, R. A. Borzi, F. Weickert, A. J. Schofield, R. S. Perry, T. Tayama, T. Sakakibara, Y. Maeno, A. G. Green and A. P. Mackenzie

Science **306**,1154 (2004)

- Resistivity: $\partial\rho/\partial T$ and $\partial^2\rho/\partial T^2$
- Susceptibility: χ' and χ''
- Magnetostriction: $\lambda(H)$
- Thermal expansion: $\alpha(T)$
- Magnetisation





What about the anomalous region in $Sr_3Ru_2O_7$?

1. A theory based on metal physics: dHvA oscillations are observed both above and below the metamagnetic transition.

R.A. Borzi, S.A. Grigera, R.S. Perry, N. Kikugawa, K. Kitagawa, Y. Maeno and A.P. Mackenzie, Phys. Rev. Lett ***92****, 216403 (2004).*

2. Field-dependent transitions which only increase the moment.

3. A temperature dependent transition that does not give a sudden change of moment.

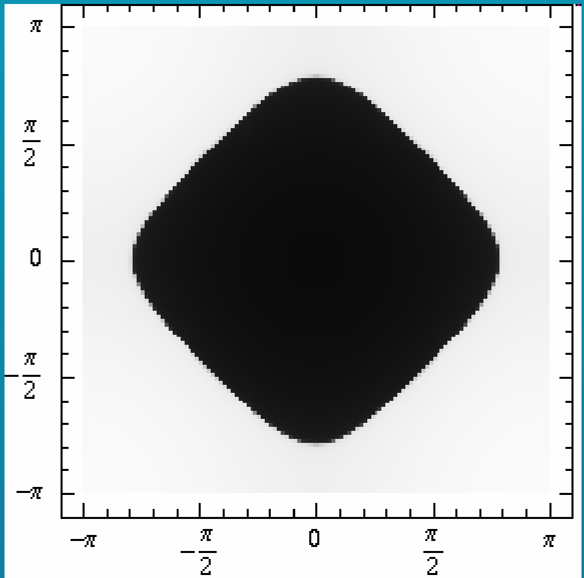
4. Something leading to the formation of domains, our only plausible explanation for the behaviour of the resistivity in the anomalous phase.

Fermi surface distortions: proximity to van Hove

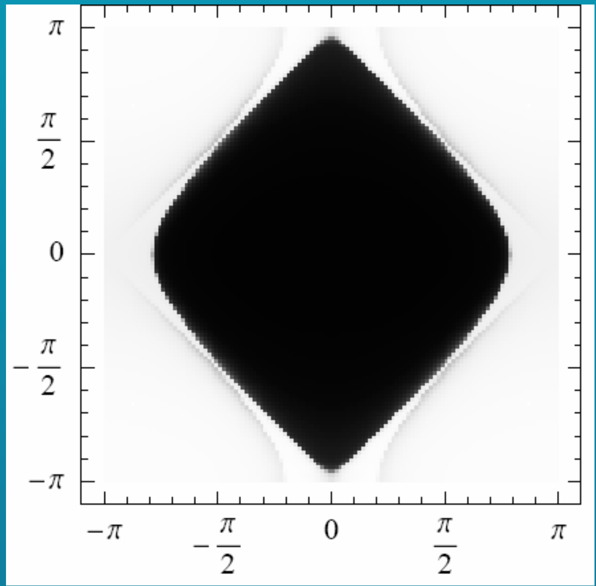
e.g. Cluster dynamical mean field theory of the 2D Hubbard model

E.C. Carter and A. J. Schofield, Phys. Rev. B **70**, 045101 (2004).

$n(k)$ for U=5t, N_e=0.8N

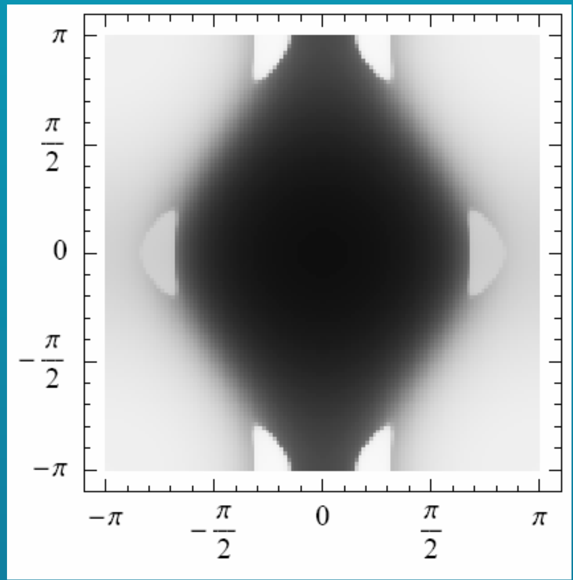


$n(k)$ for U=2t, N_e=0.9N



Fermi surface distorts

$n(k)$ for U=5t, N_e=0.9N



"Arced" Fermi surface and a partial gap

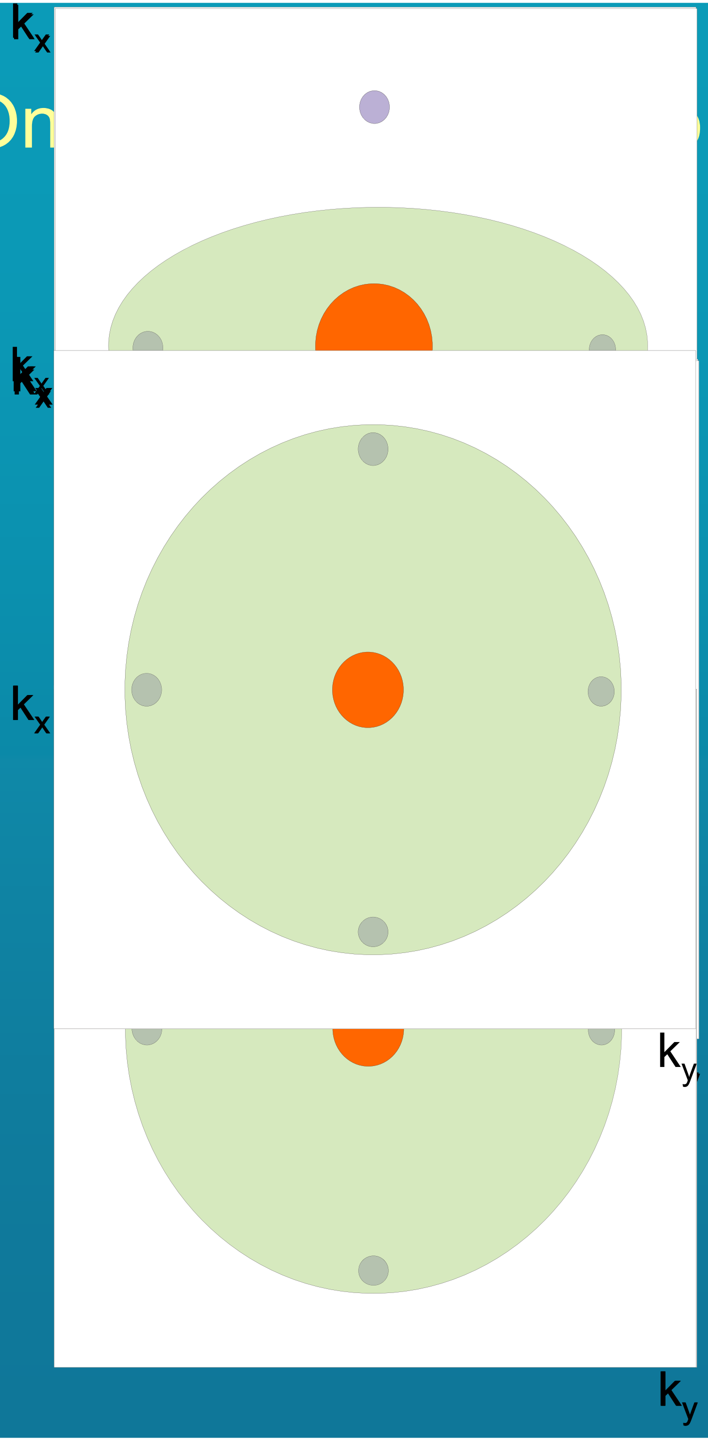
Order with internal angular momentum (k)

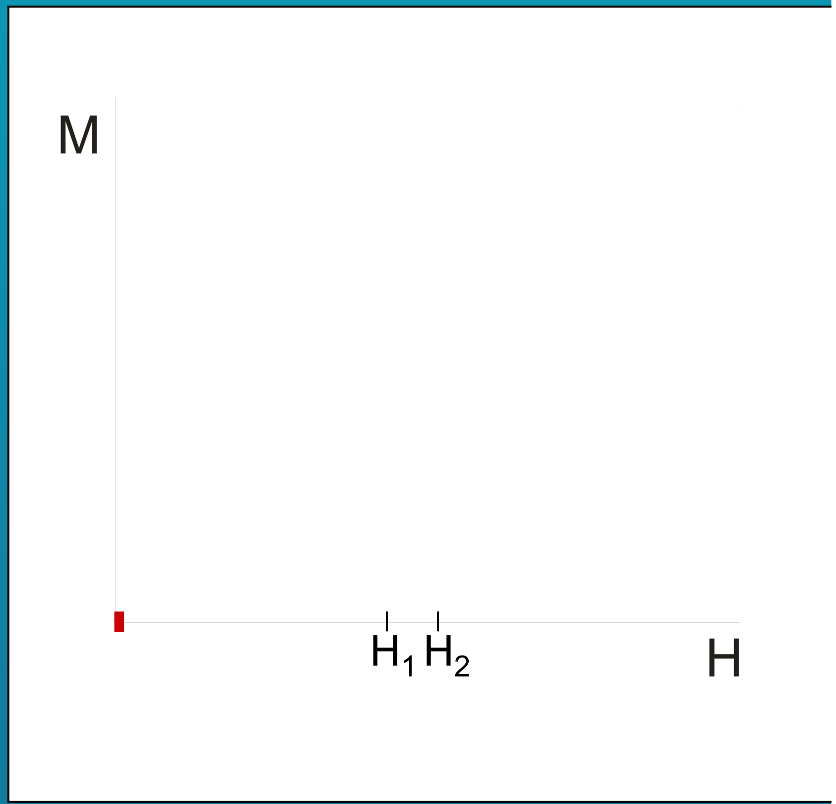
$$\Delta_{k,q} \sim \langle \hat{c}^\dagger_{k+\frac{q}{2},\sigma} \hat{c}^\dagger_{-k+\frac{q}{2},\bar{\sigma}} \rangle \qquad M^{\alpha\beta}_{k,q} \sim \langle \hat{c}^\dagger_{k+\frac{q}{2},\alpha} \hat{c}_{k-\frac{q}{2},\beta} \rangle$$

On

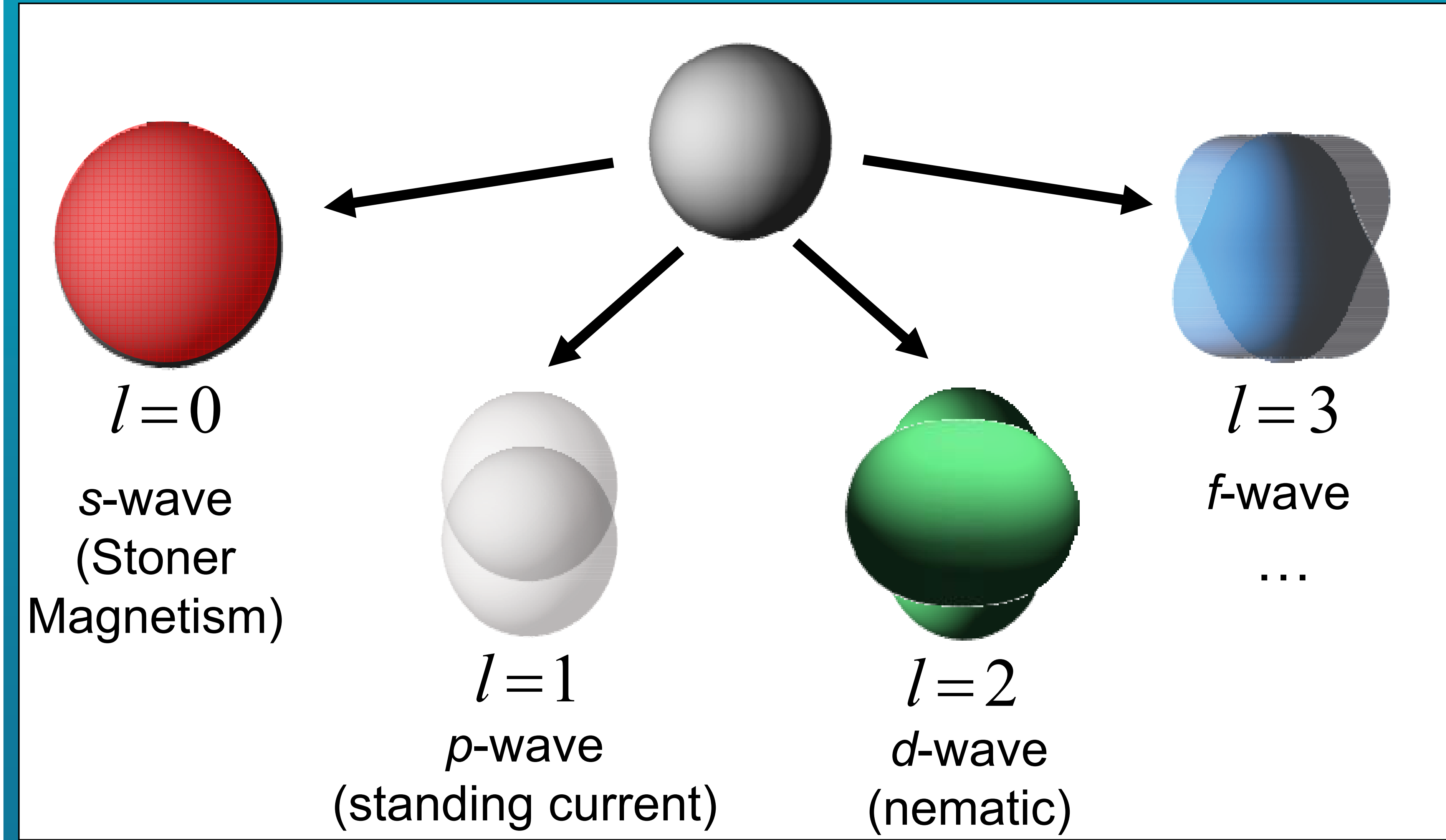
Spin up FS   Spin down FS

Van Hove points



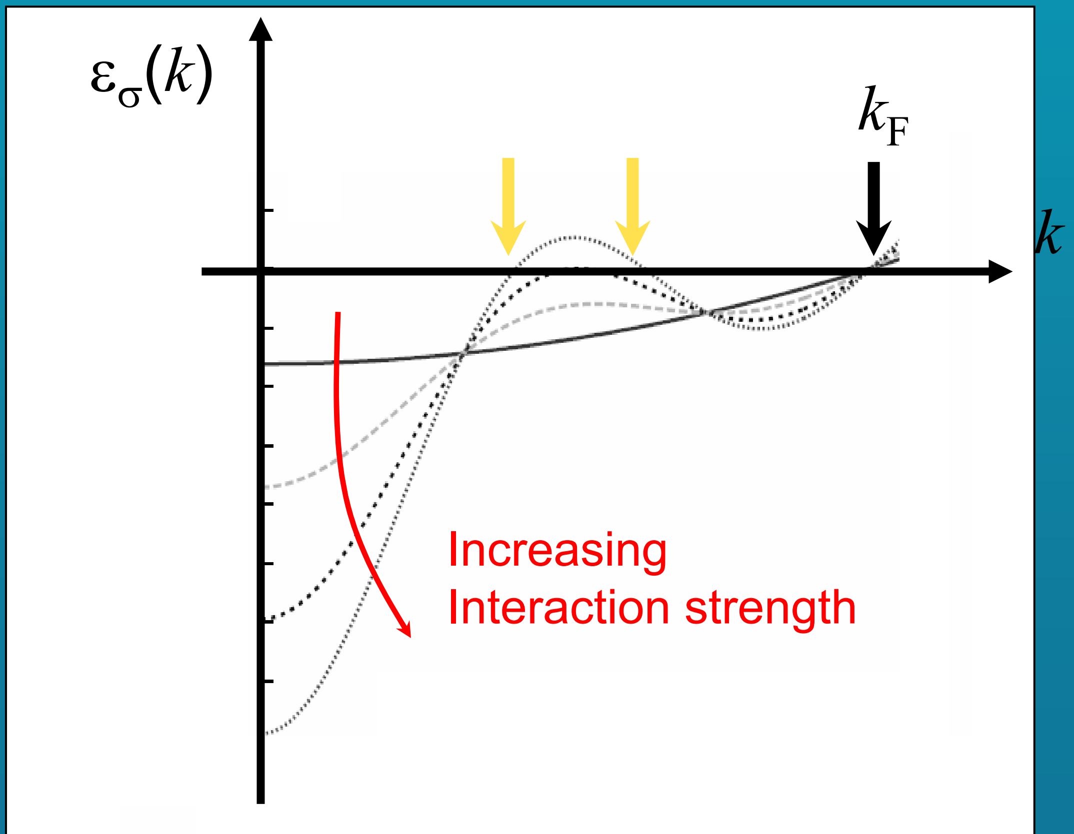


Other candidates for metal-to-metal transitions: Pomeranchuk (1958) instabilities

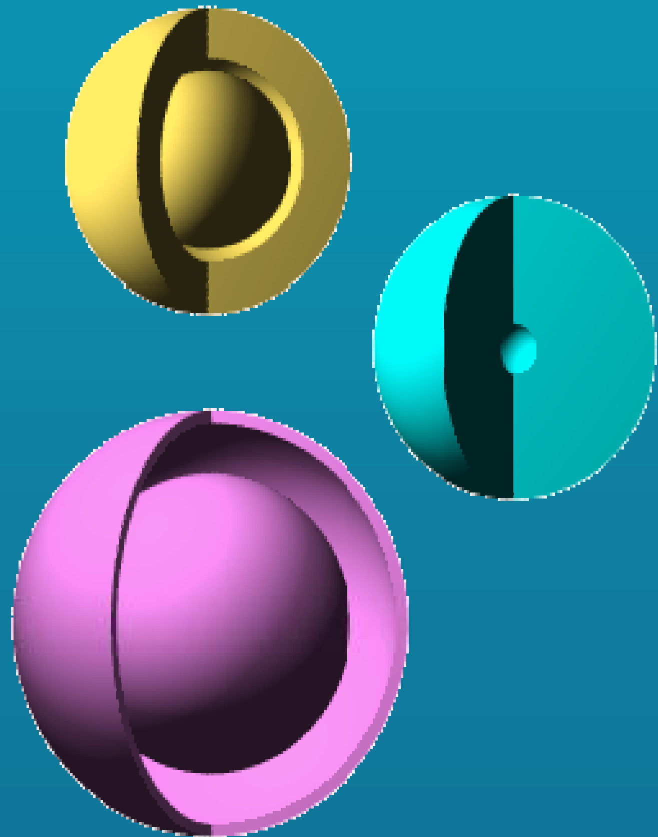


Topological Fermi surface transitions
J. Quintanilla and A. J. Schofield, cond-mat/0601103

Even without symmetry breaking, interactions may change $\varepsilon_\sigma(\mathbf{k})$ qualitatively…



…even leading to changes of Fermi surface **topology**

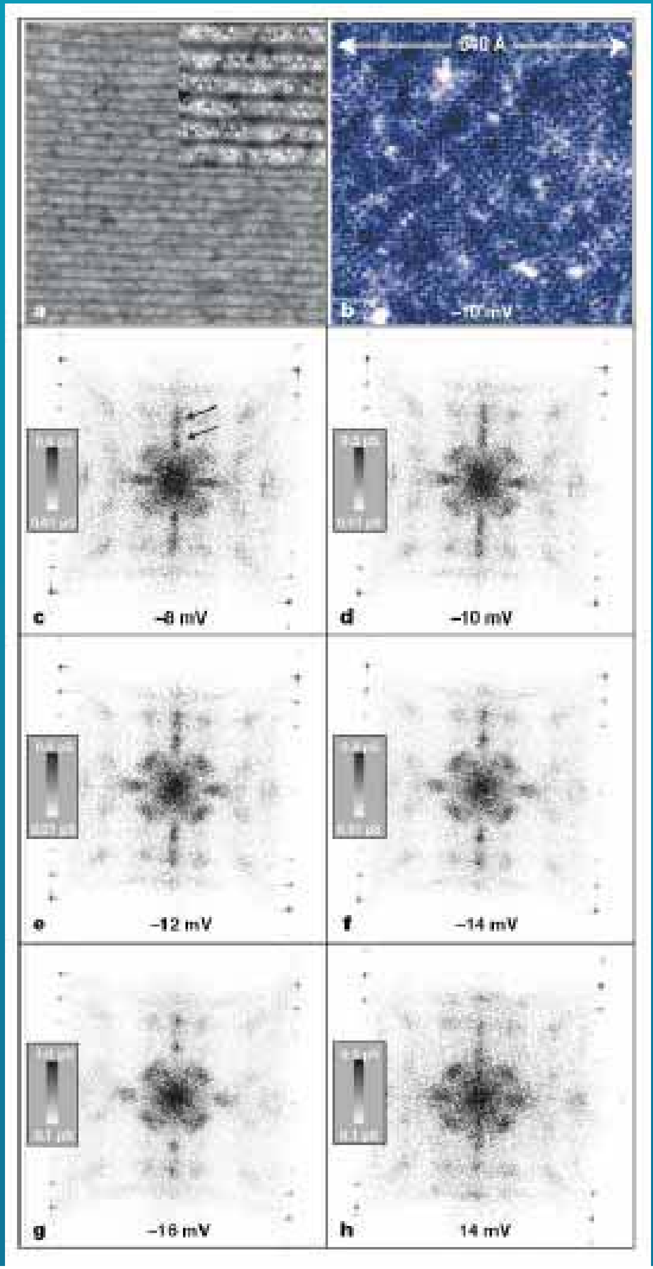


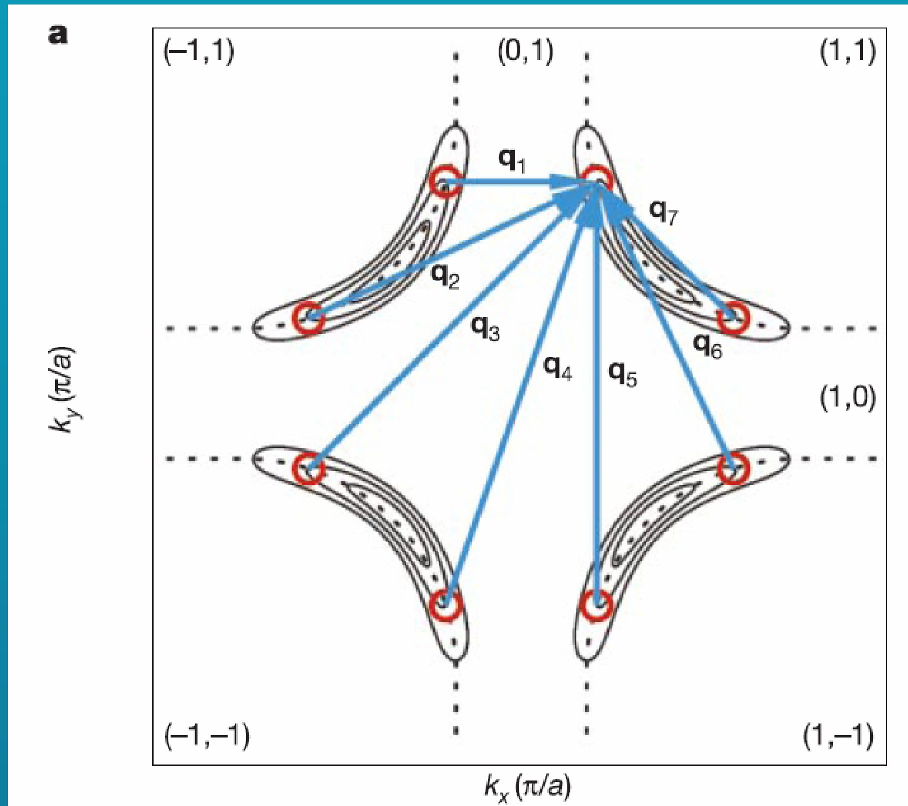
Similar to that described by Kun Yang and Subir Sachdev, PRL **96,** 187001 (2006)

Finally: How to detect small Fermi surface changes
A. J. Schofield, C. A. Hooley & J. Quintanilla

- Typically volume preserving:
  - can't use dHvA in 2D.
- Likely to form domains:
  - need a local probe
- Would like to probe the Fermi surface itself
  - μSR and NMR too indirect.

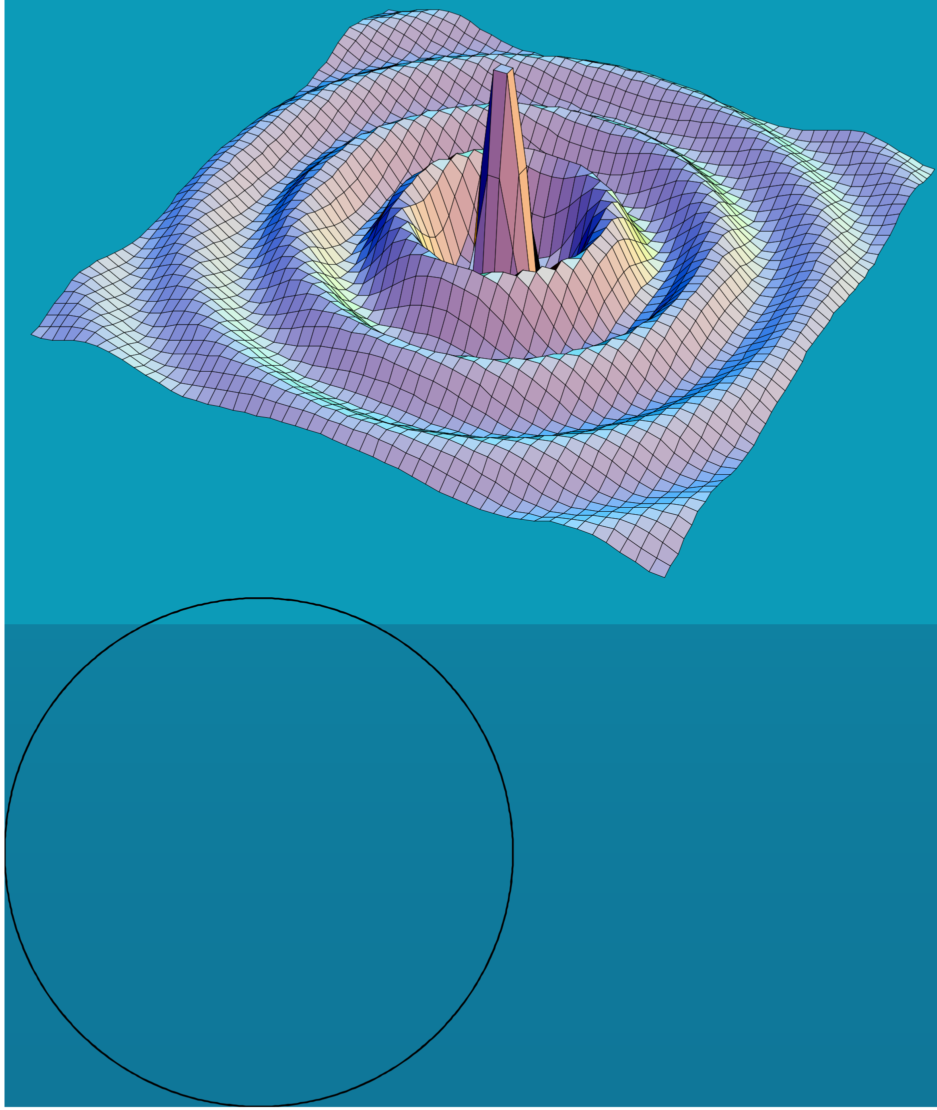
The inspiration: local interference phenomena

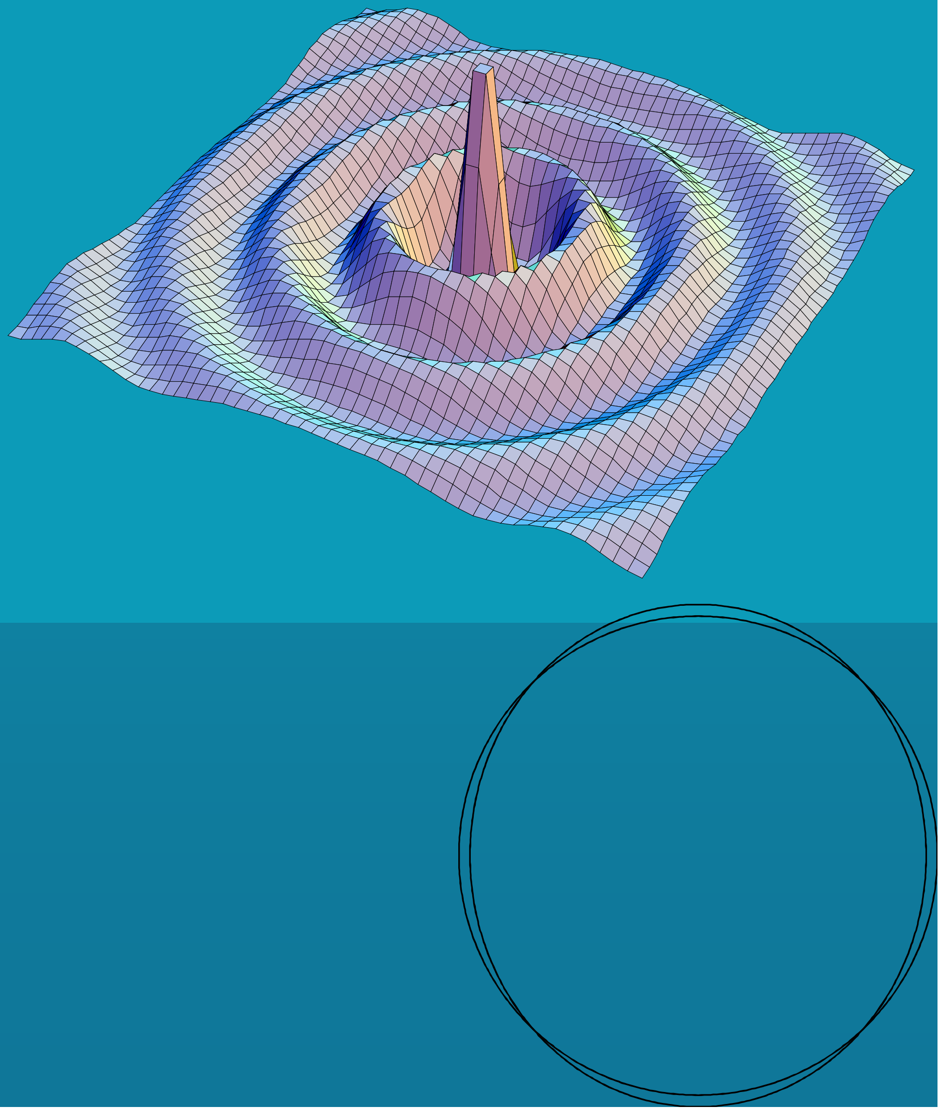




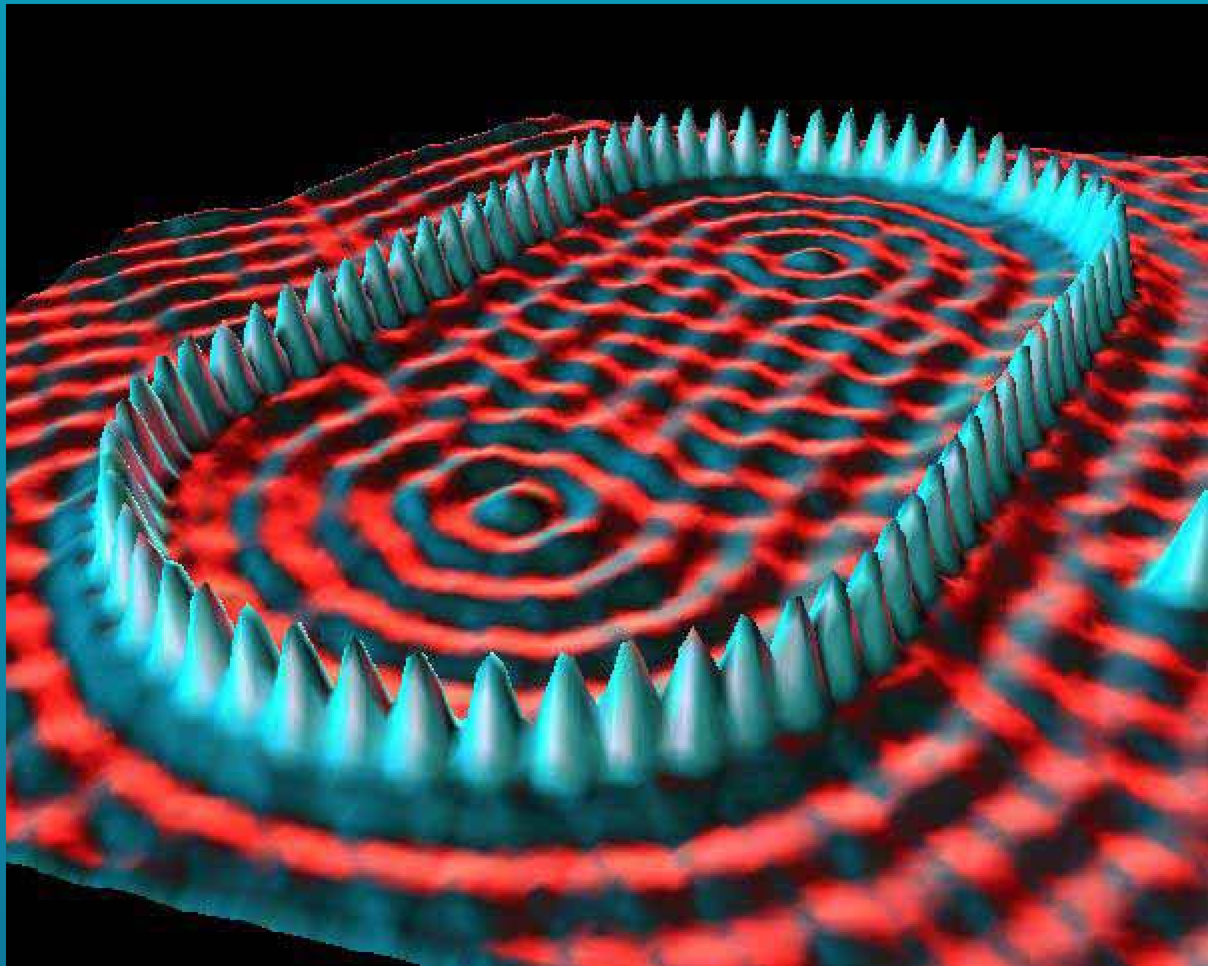
K. McElroy, R. W. Simmonds, J. E. Hoffman, D.-H. Lee, J. Orenstein, H. Eisaki, S. Uchidak and J. C. Davis
Nature **422**, 592-596, (2003)

A single impurity may not be sensitive enough



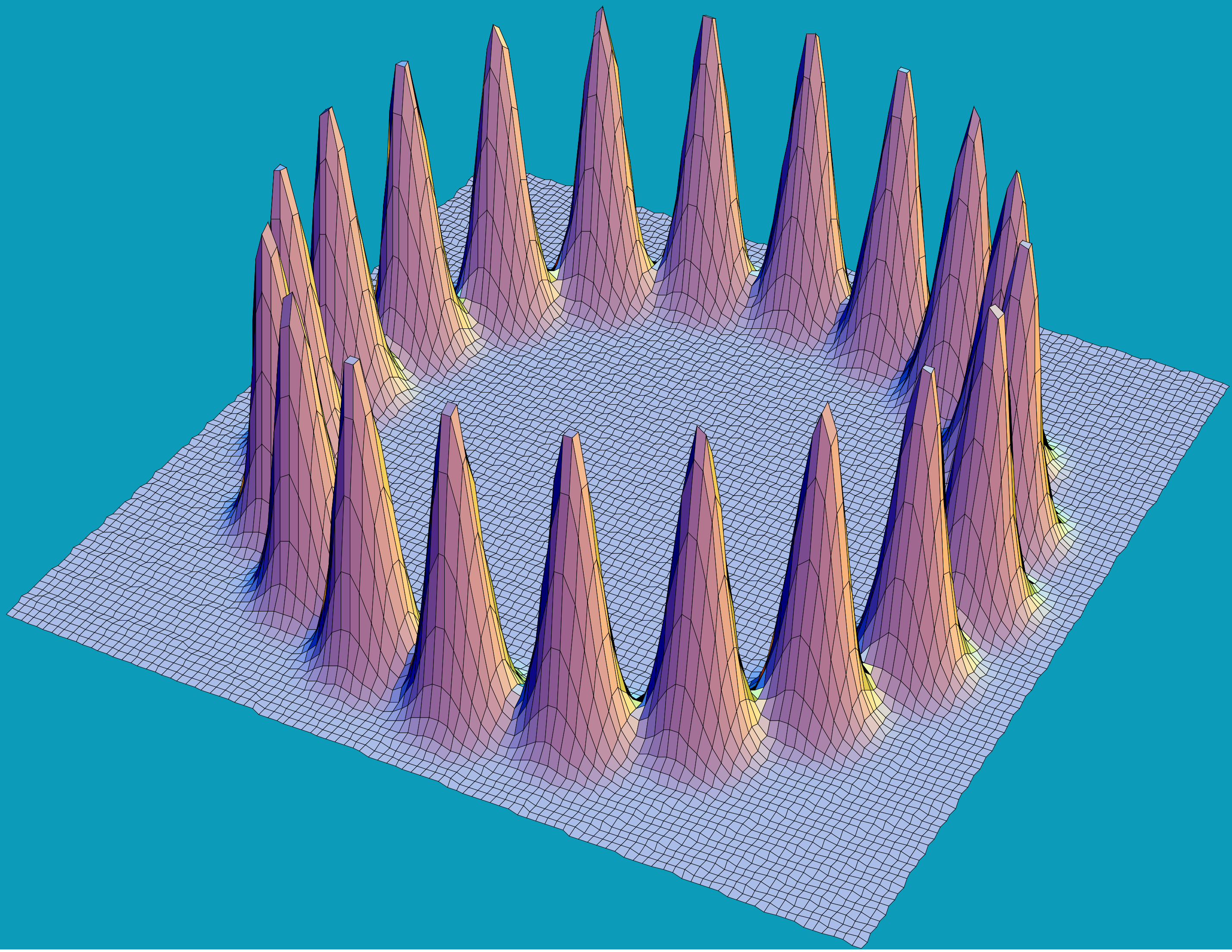


The inspiration: local interference phenomena

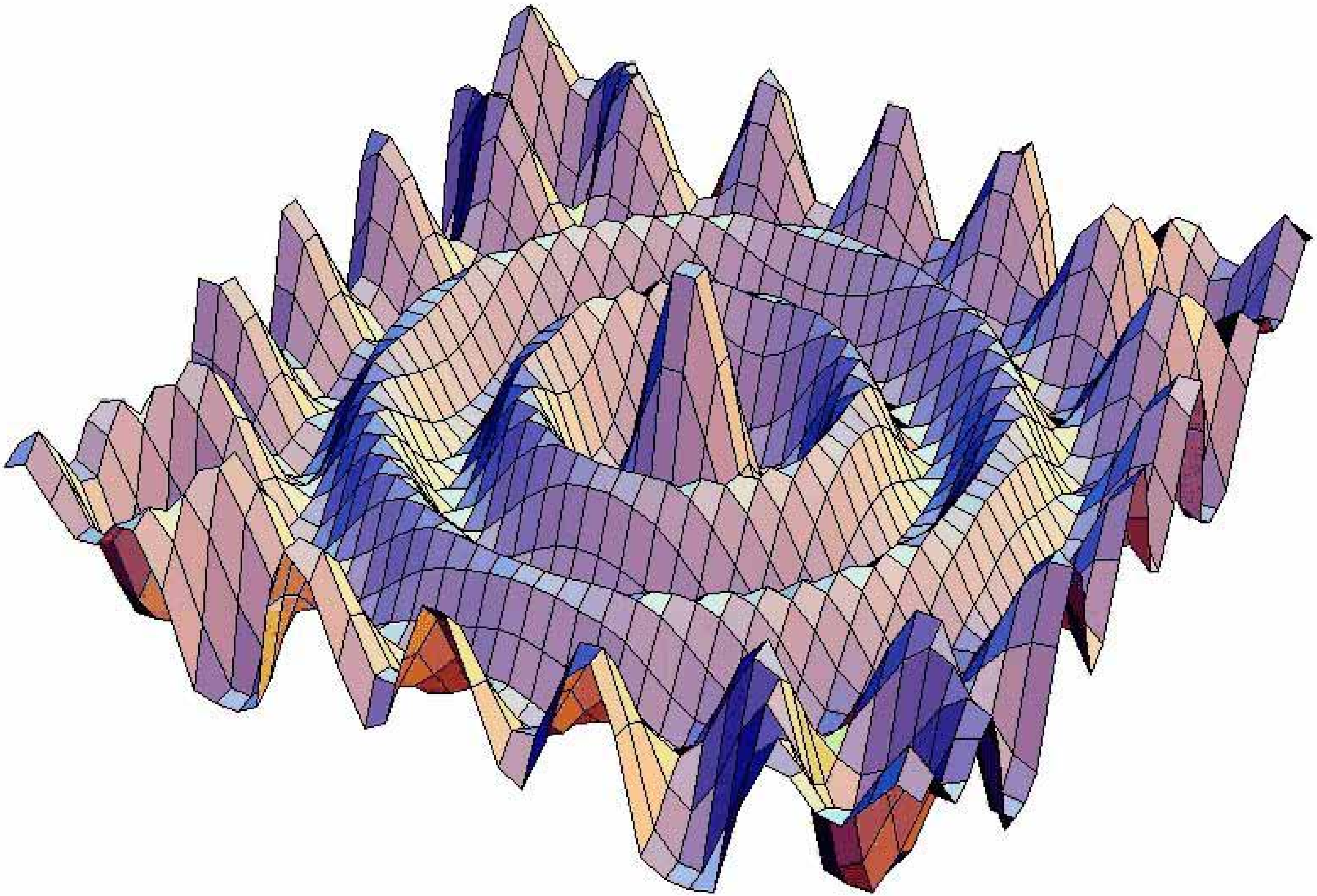


M.F. Crommie, C.P. Lutz, D.M. Eigler, E.J. Heller.
Surface Review and Letters **2** (1), 127-137 (1995).

Create a “Friedel Resonance” structure

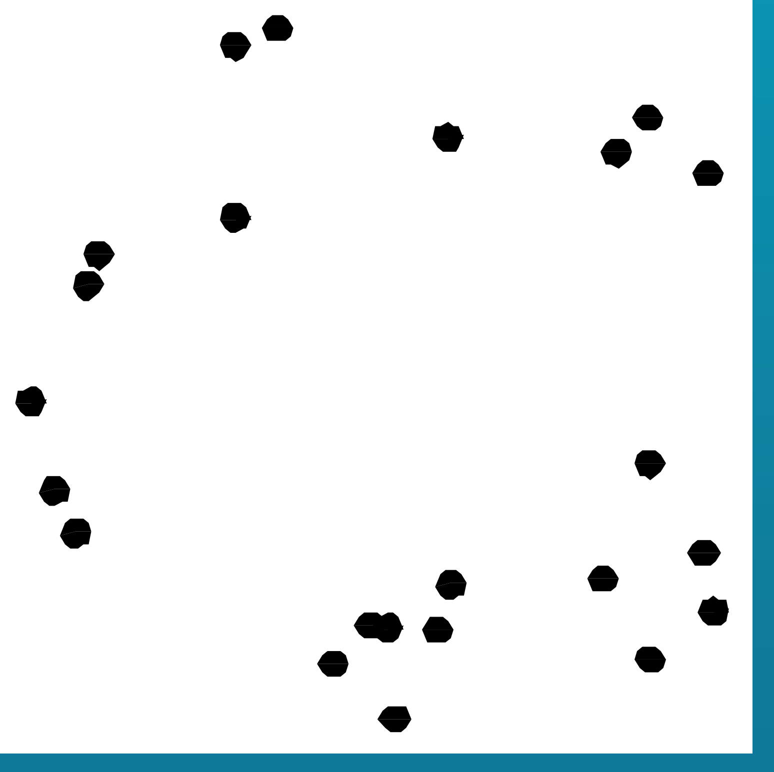


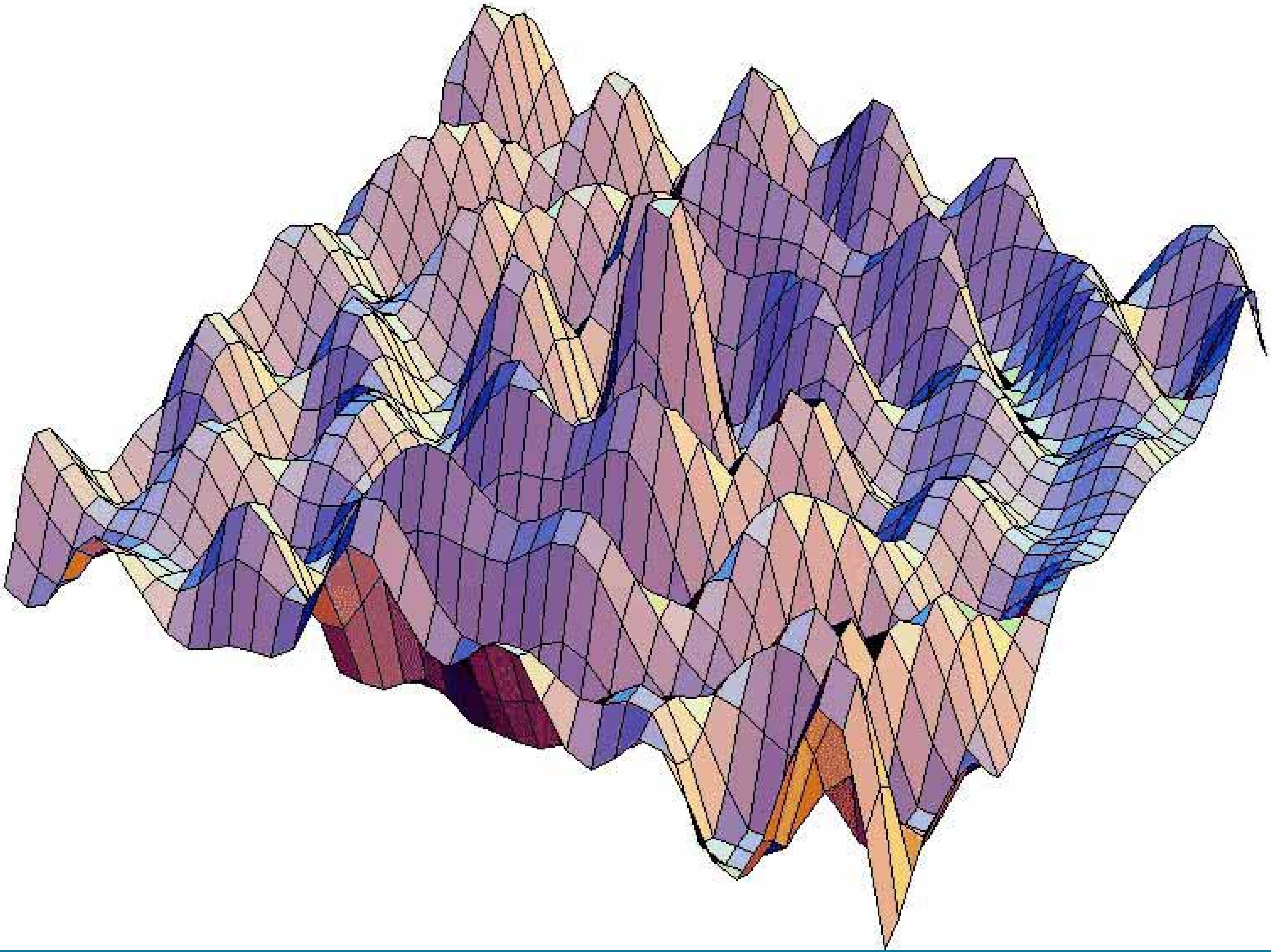
Evolving through an instability



Some issues

- Influence of the structure on the transition:
  - "random" array of positive and negative scatters





Conclusions

- The challenge of understanding metal-to-metal transitions
- The richness of possible transitions – if the conditions are right.
- Delicate changes in the Fermi surface can be probed:
  - Use a collapsing scale to amplify the probe: *e.g.* transport at finite magnetic field at a density wave transition.
  - dHvA is great for extremal-area-changing, field-insensitive transitions…but that is limiting
- Could nanotechnology be useful here?

“Those who cannot remember the past are condemned to repeat it.”

George Santayana, *The Life of Reason, Volume 1, 1905*

ICTP Trieste 18 June – 27 July 1990

1D Models

Application of the Landau-Luttinger liquid formulation to the study of the magnetic properties of the 1D Hubbard model: *Carmelo, Horsch, Bares, Ovchinnikov*

Correlation functions in the 1D large-U Hubbard model: *Shiba and Ogata*

The Hubbard U=∞Model in 1D – slave boson method: *Schmeltzner*

Correlated fermions in 1D: *Schultz*

Frustrated Quantum AFM

Questions, Controversies and Frustration in quantum AFM: *Chandra, Coleman, Ritchie*

A new approach to the dynamics of holes and spins: *Ichinose and Matsui*

A single hole in a quantum AFM: Self-consistent greens function approach: *Martinez and Horsch*

Large N-expansion for frustrated and doped quantum AFM: *Sachdev and Read*

Phenomenology etc

Frustrated spin (J-J’) systems do not model the properties of HiTc ssuperconductors: *Bacci, Gagliano and Nori*

Strong correlation transport and coherence: *Kotliar*

Superconductivity in high and very high magnetic fields: *Tesanovic*

Raman scattering in Mott-Hubbard systems: *Shastry and Shraiman*

2D Hubbard and tJ Models

Numerical studies of strongly correlated electron models: *Daggotto*

Study of phase separation in the Hubbard model: *Moreo*

Hard-core slave boson description of generalized flux phases: *Nori, Zimanyi and Abrahams*

tJ Model for triplet holes: *Oles, Zaanen and Drchal*

Hole-hole correlations in the 2D Hubbard model: *Sorella, Parola and Tosatti*

SO(4) Symmetries of the Hubbard model and experimental consequences: *S.-C.Zhang*

Other related models

Parquet eqns for self-consistent field theory: *Bickers*

Models of correlated fermions in the slave boson approximationfor the copper oxide systems: *Entel, Behera, Zielinski and Kaufmann*

Variational theory of the correlated Fermi-liquid state in the Kondo lattice model: *Fazekas and Shiba*

Phase separation and superconductivity in the U=∞ limit of the extended multiband Hubbard model: *Grilli, Raimondi, Castellani, Di Castro and Kotliar*

Quantum Hall Effect and anyon superconductivity

Composite Chern-Simons gauge boson in anyon gas: *Van Hieu and Hung Son*

Charge-vortex binding, fractional quantum Hall effect and anyon superconductivity: *D.-H Lee*

Collective field theory applied to the fractional quantum Hall effect: *Sakita, Sheng and Z.-B. Su*