



The Abdu Salam  
International Centre for Theoretical Physics

United Nations  
Educational, Scientific  
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International Atomic  
Energy Agency

SMR.1771 - 14

**Conference and Euromech Colloquium #480**  
**on**  
**High Rayleigh Number Convection**

4 - 8 Sept., 2006, ICTP, Trieste, Italy

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**Recent experiments on turbulent RBC  
by the Santa Barbara group**

G. Ahlers  
University of California at Santa Barbara  
USA

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These are preliminary lecture notes, intended only for distribution to participants

# Non-Boussinesq Effects on Turbulent Rayleigh-Benard Convection in Gases

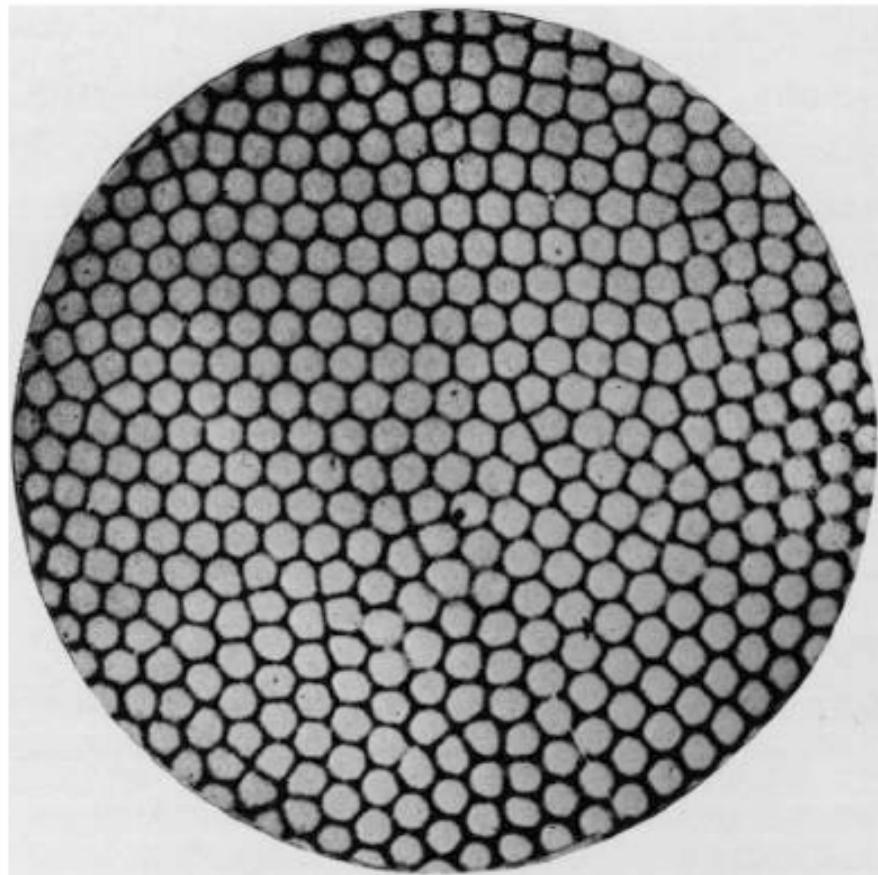
Guenter Ahlers  
Denis Funfschilling

Department of Physics  
and iQCD  
University of California  
Santa Barbara CA USA

# Pattern Formation in Convection: The Legacy of Henry Benard, Lord Rayleigh and Fritz Busse

# Les Tourbillons Cellulaires dans une Nappe Liquide

Henri Benard



Revue generale des Sciences XII, 1261 (1900)

Ph.D. March 15 1901 College de France



THE  
LONDON, EDINBURGH, AND DUBLIN  
PHILOSOPHICAL MAGAZINE  
AND  
JOURNAL OF SCIENCE.

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[SIXTH SERIES ]

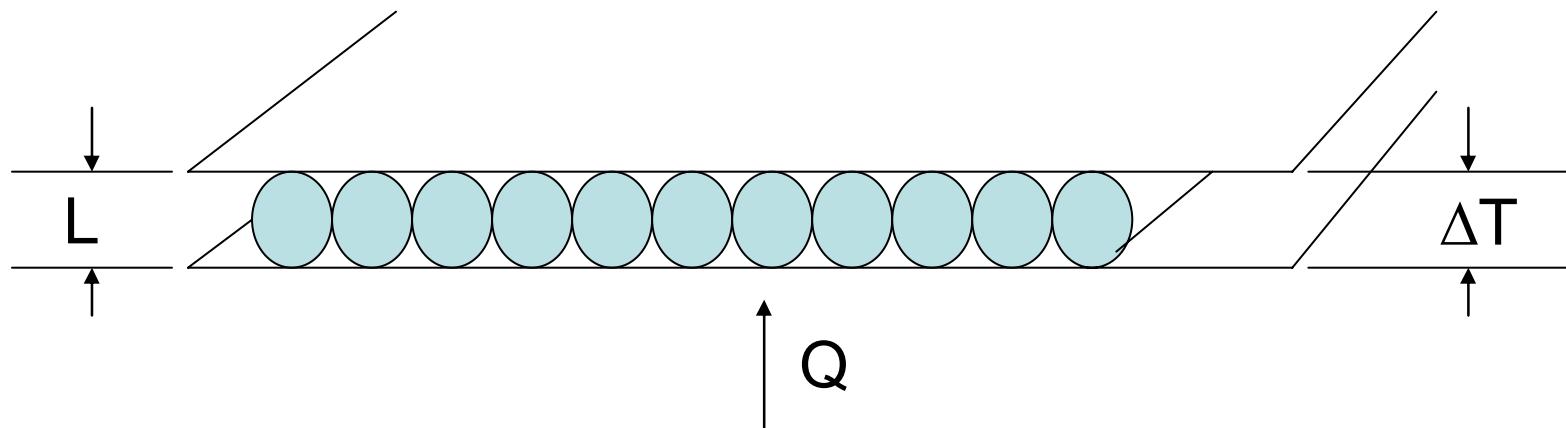
Lord Rayleigh

DECEMBER 1916.

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LIX. *On Convection Currents in a Horizontal Layer of Fluid, when the Higher Temperature is on the Under Side.*  
*By Lord RAYLEIGH, O.M., F.R.S.\**

THE present is an attempt to examine how far the interesting results obtained by Bénard † in his careful and skilful experiments can be explained theoretically.



$$\varepsilon = \Delta T / \Delta T_c - 1$$

Rayleigh carried out a stability analysis of the quiescent fluid layer. He used slip boundary conditions at top and bottom in order to permit an analytic solution.

He found that the bifurcation is stationary (i.e. that the lowest eigenvalue is real), and calculated the neutral curve  $R_c(k)$  and  $k_c$ . However, this does not give the nature of the bifurcation and the pattern above onset.

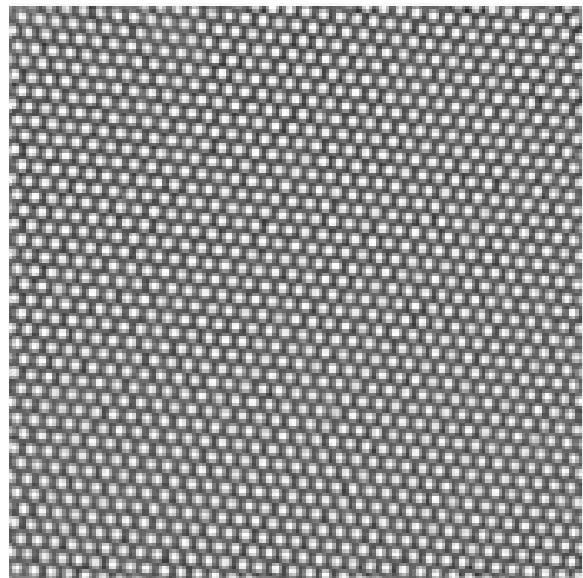
**Next milestones:**

nonlinear effects:

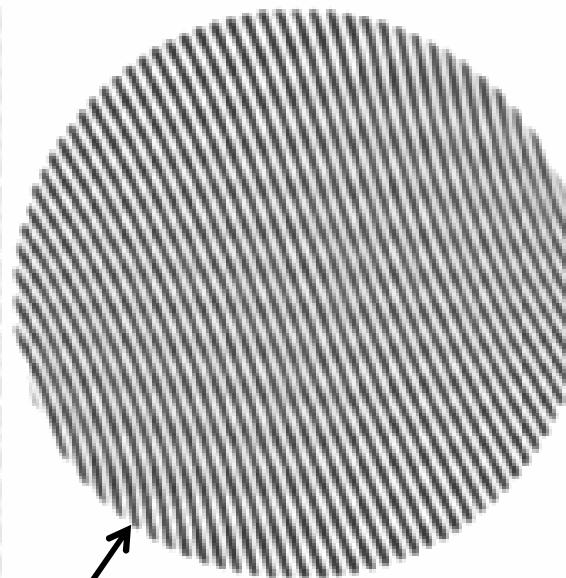
nature of the bifurcation  
pattern just above onset

Schluter, Lortz, and **Busse** (1965):  
ridgid BCs, Boussinesq samples:

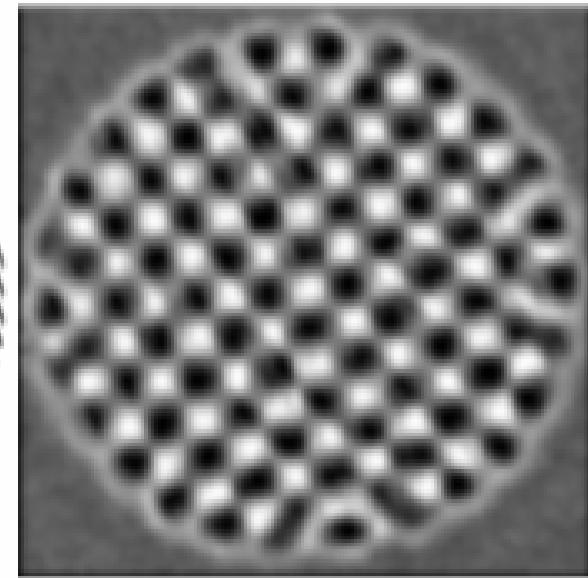
super-critical  
rolls (or stripes)



Non-Boussinesq



Boussinesq

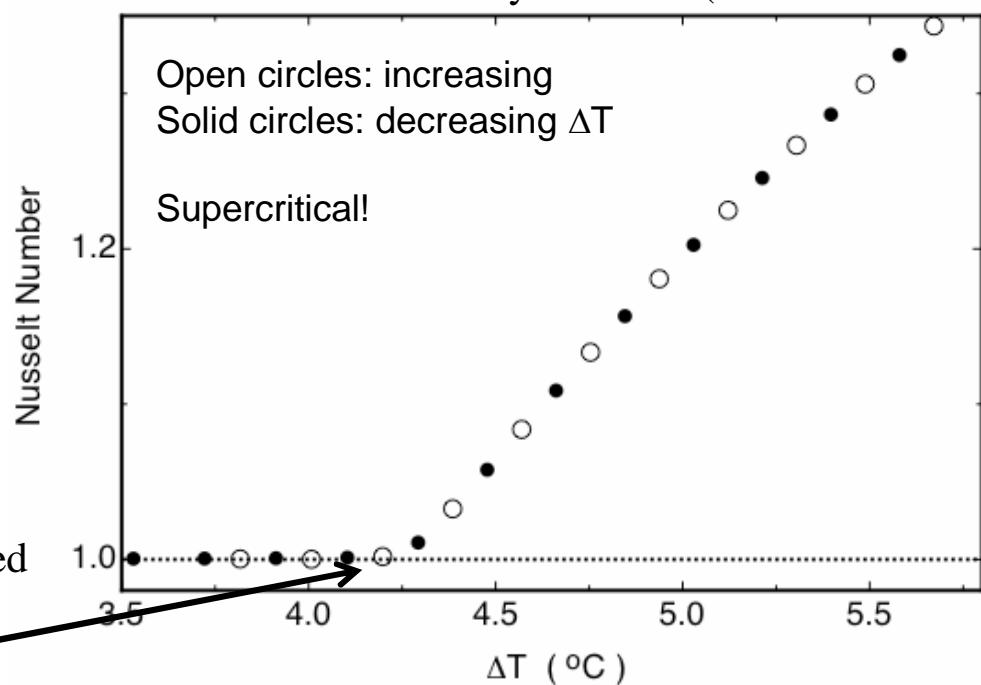


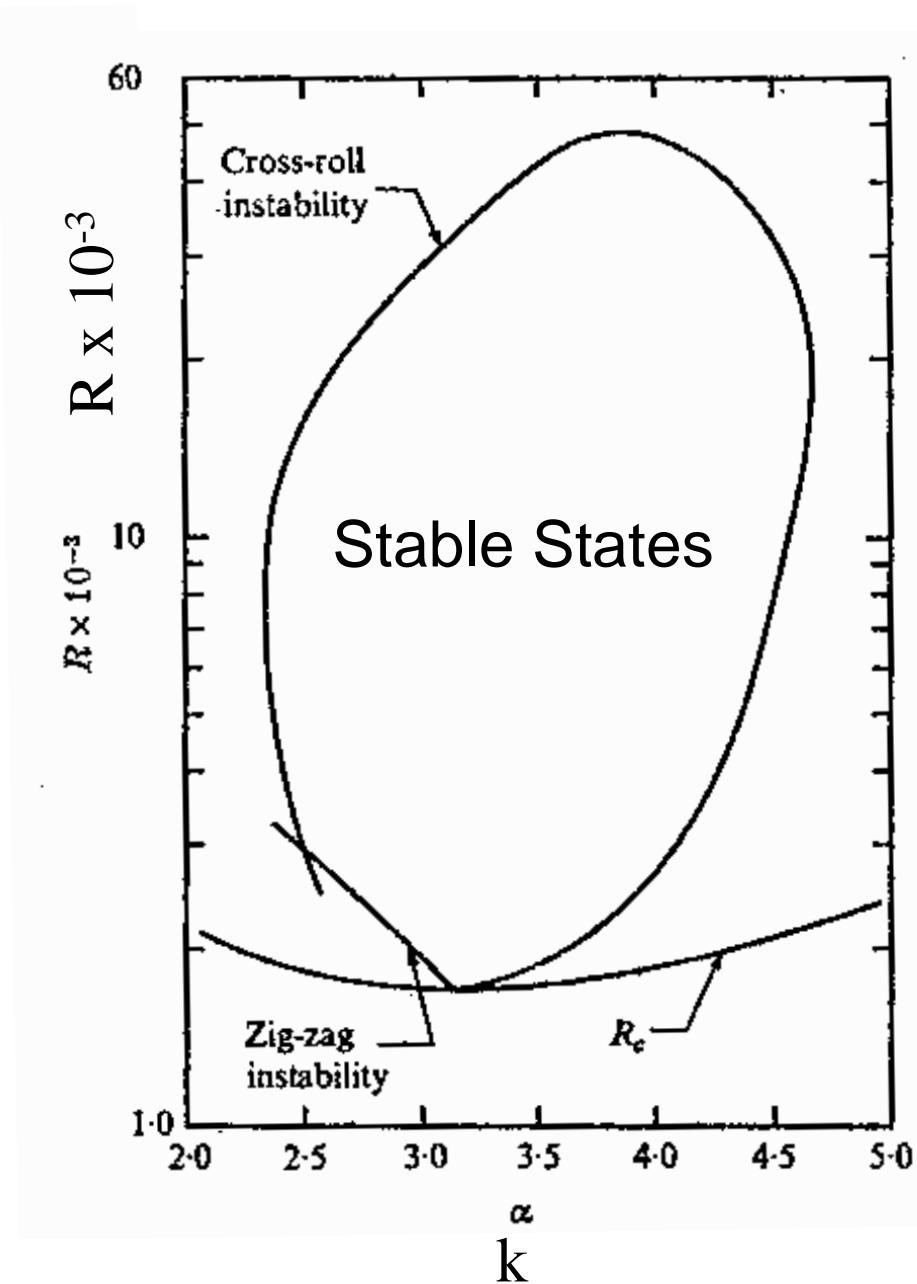
Binary mixtures (diffusion driven)

Hexagons, as predicted  
by **Busse** (1967) for  
non-Boussinesq RBC

Rolls, as predicted  
by **Busse** et al. (1965)  
for Boussinesq RBC

Supercritical, as predicted  
by **Busse** et al. (1965)  
for Boussinesq RBC





In the 1970's and thereafter Fritz, largely with his long-time friend Richard Clever, calculated the various stability boundaries for the straight-roll state. This gave us the **Busse Balloon** which soon became the “playground” of experimentalists and theorists with an interest in nonlinear physics and pattern formation.

No extremum principle

Any state inside the Balloon is attainable if the phase of the pattern is pinned, e.g. by sidewalls, in an experiment.

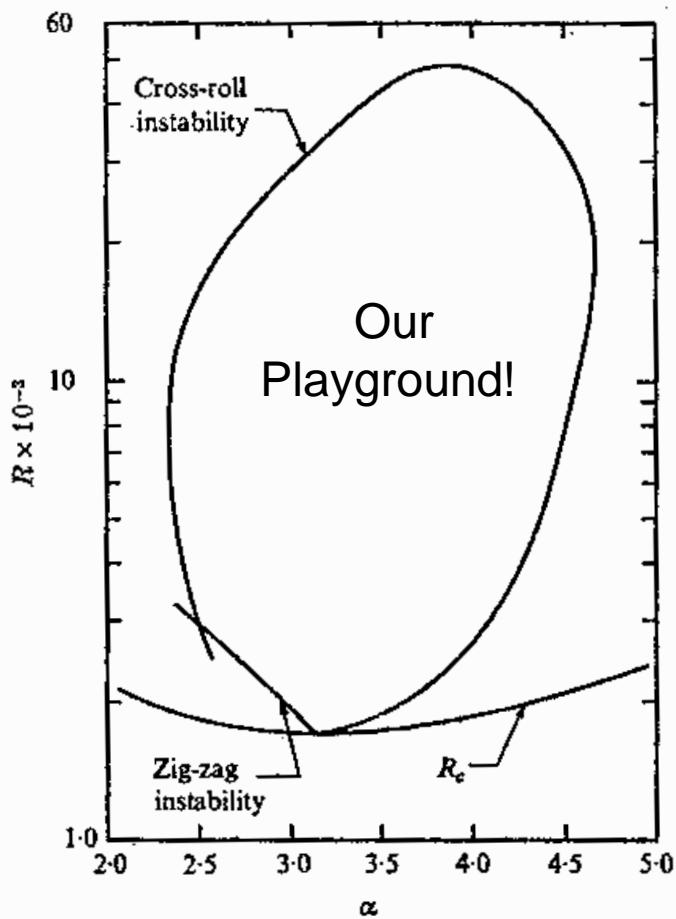
Wavenumber-selection processes occur when the phase can slip at some point in the pattern

Spatio-temporal chaos can be found

Many other things:

upper bounds for heat or momentum flux  
in turbulent flows  
geophysical fluid dynamics  
convection in rotating systems  
dynamo theory  
turbulent RB convection

# Our Heroes:



Henri Benard



Lord Rayleigh

AND

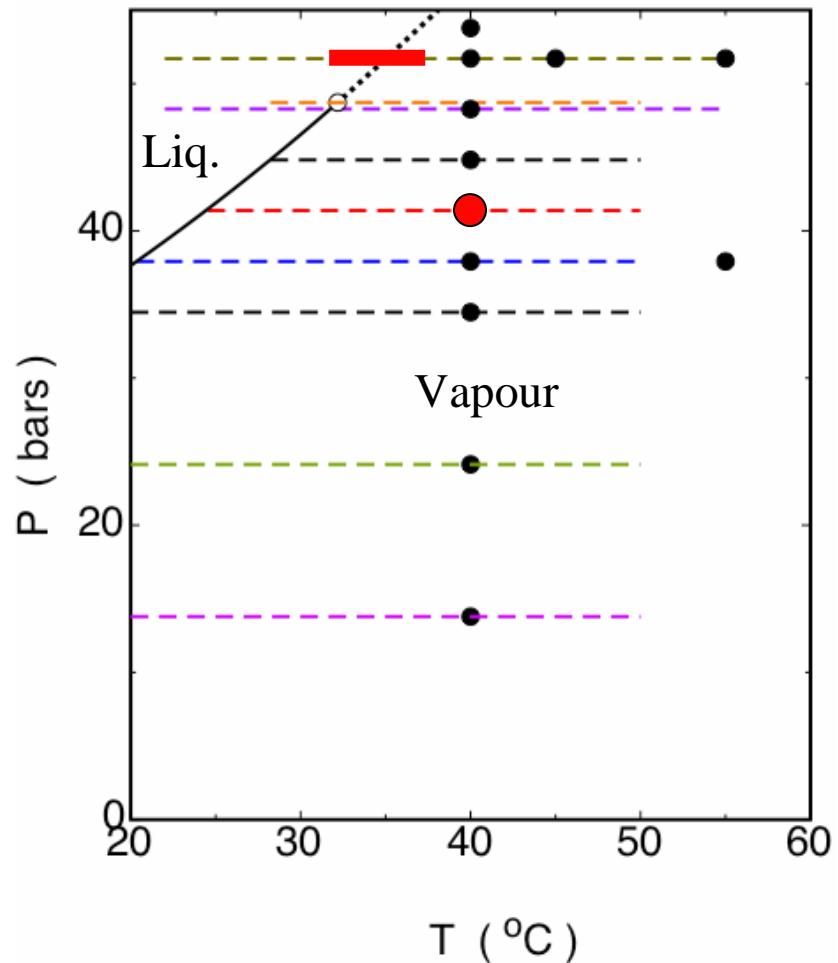
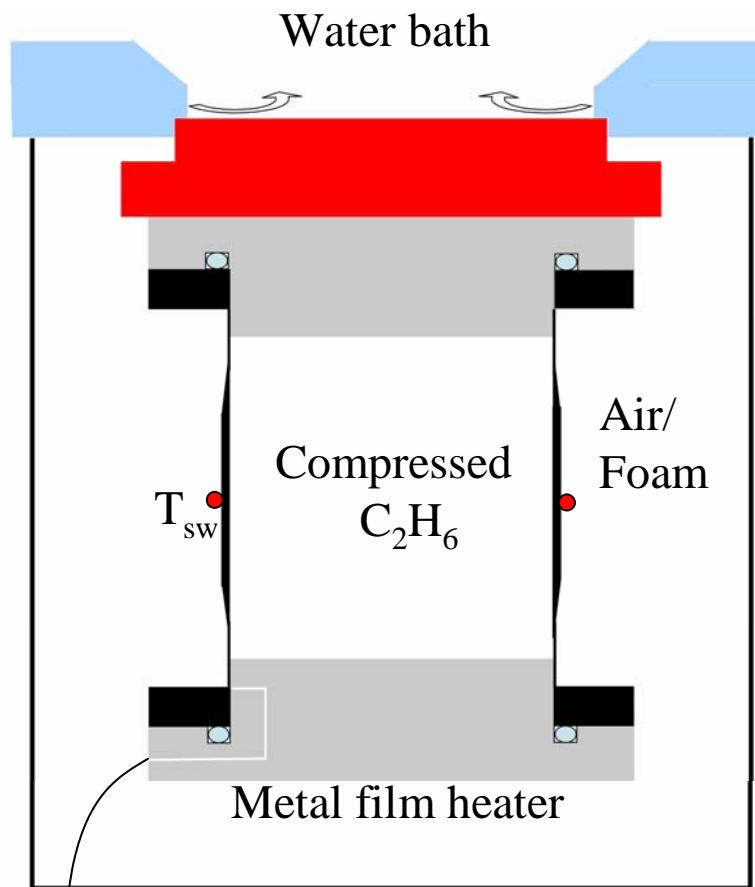


Fritz Busse

Thank you, Fritz, and  
best wishes for many more  
Birthdays and  
many more  
papers in *JFM* and elsewhere!

# Turbulent convection in Gases under pressure and Near their critical points

Denis Funfschilling and G.A.

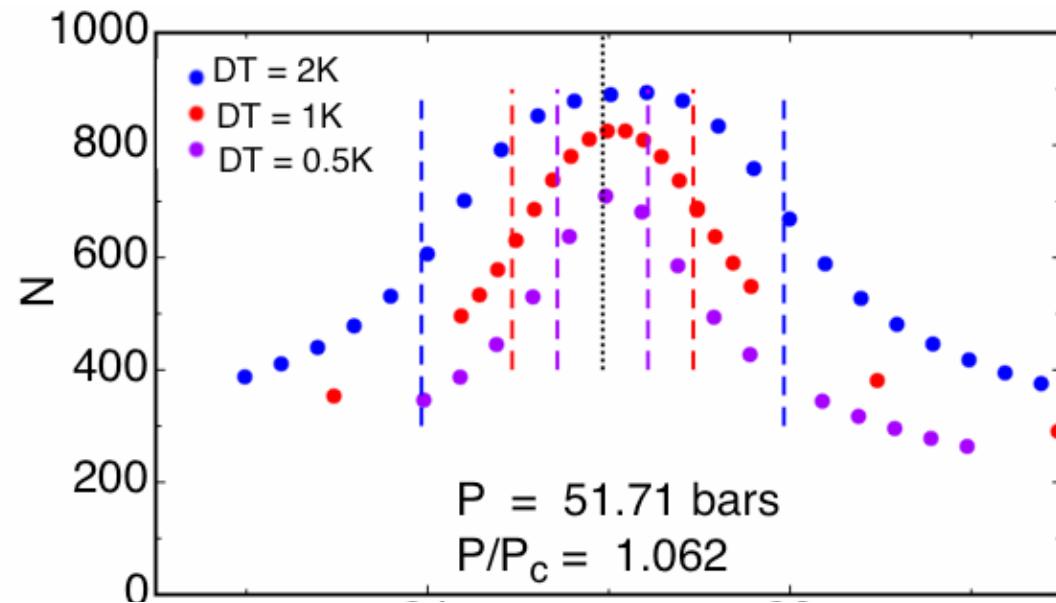
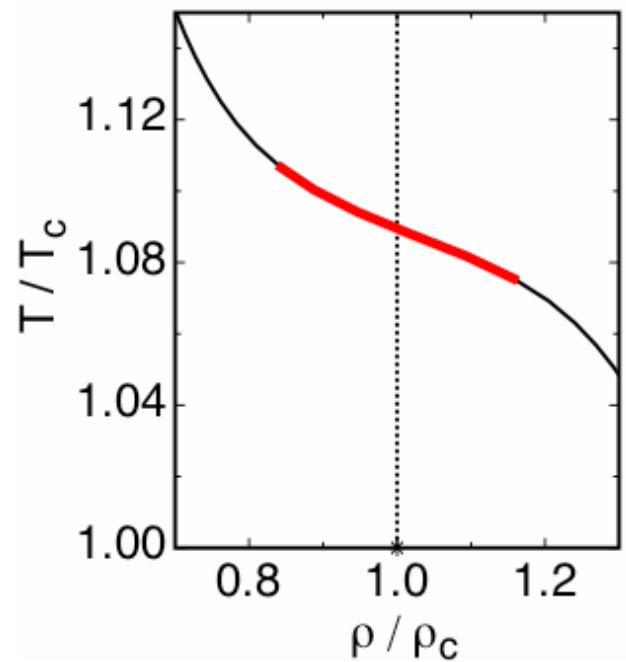


P constant to +/- 1 mBar

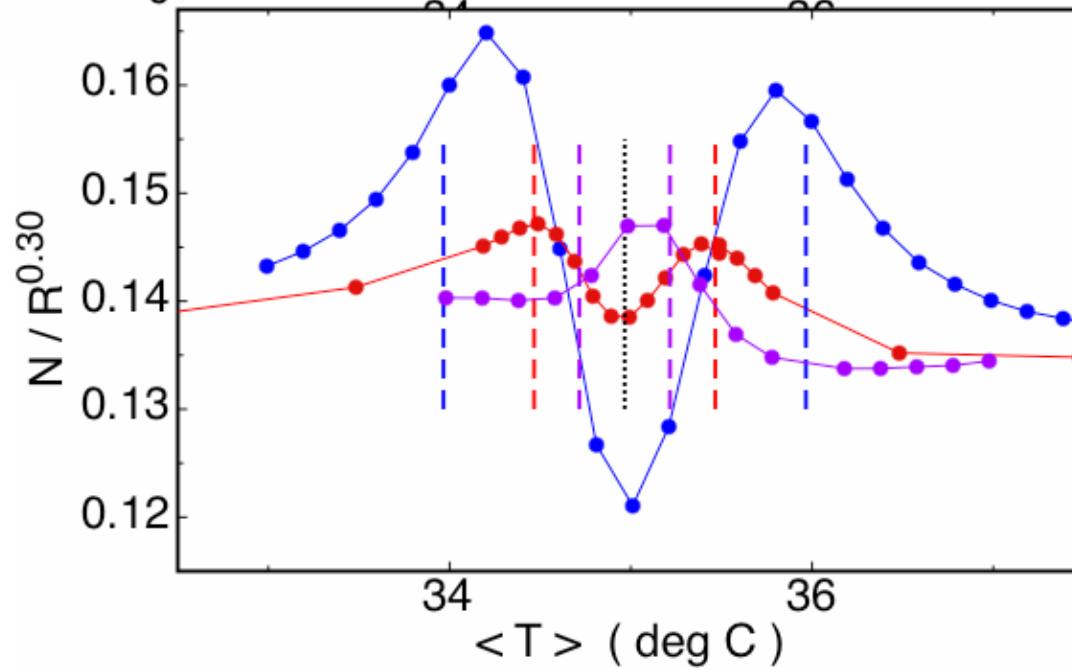
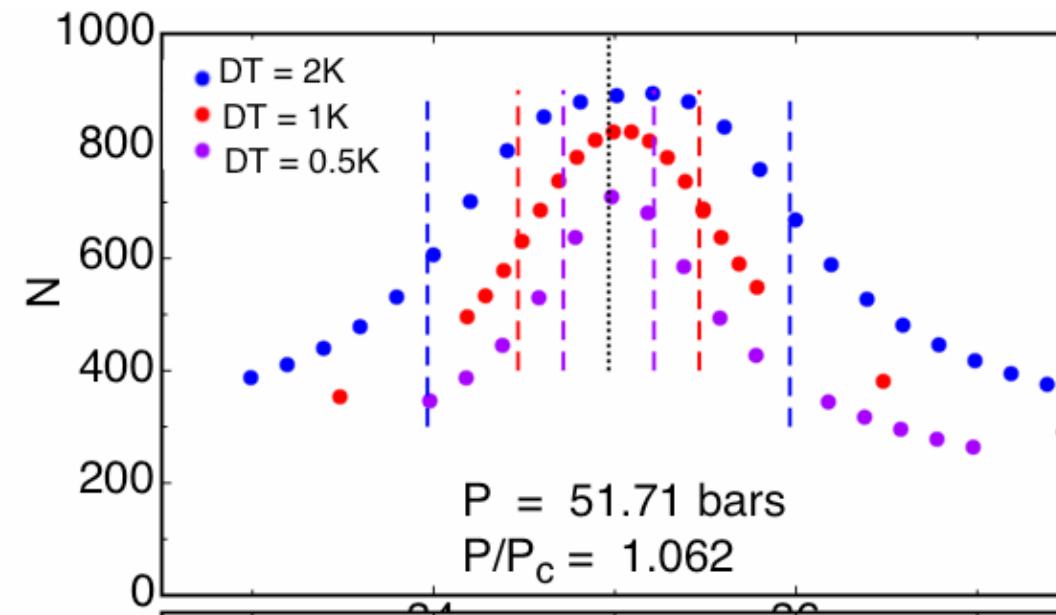
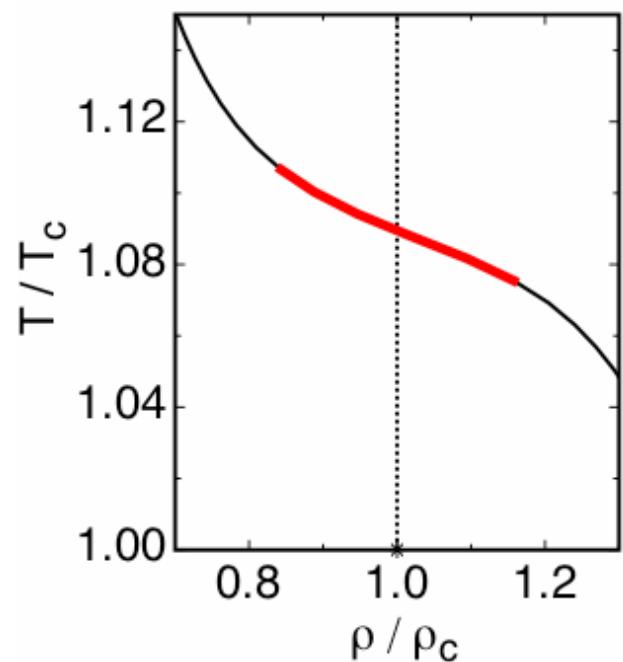
T constant to +/- 1 mK

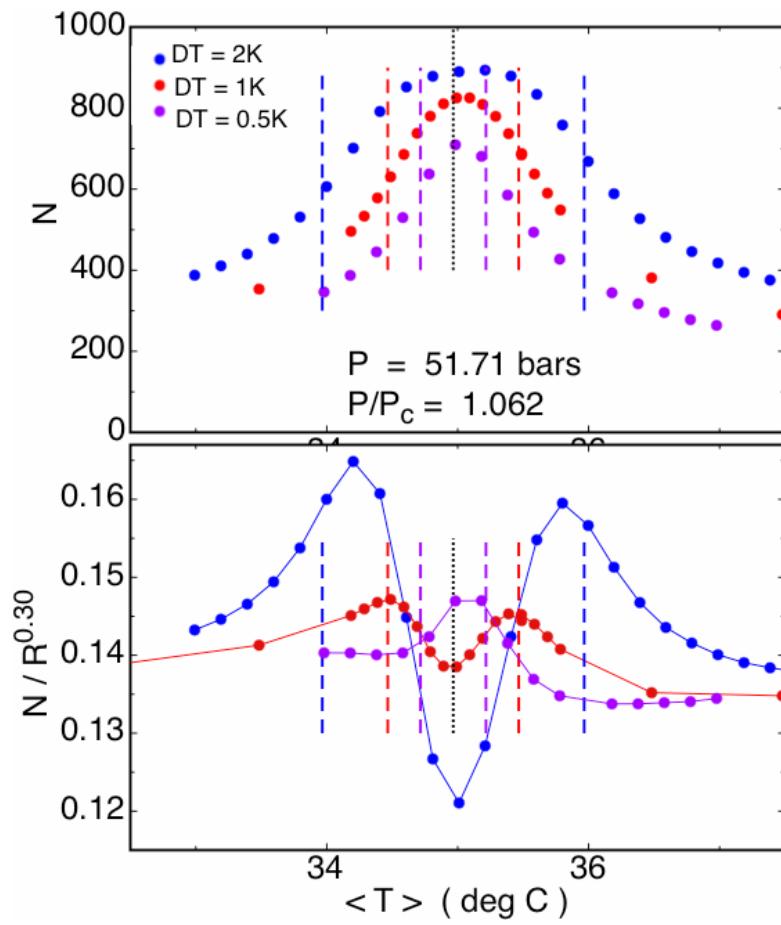
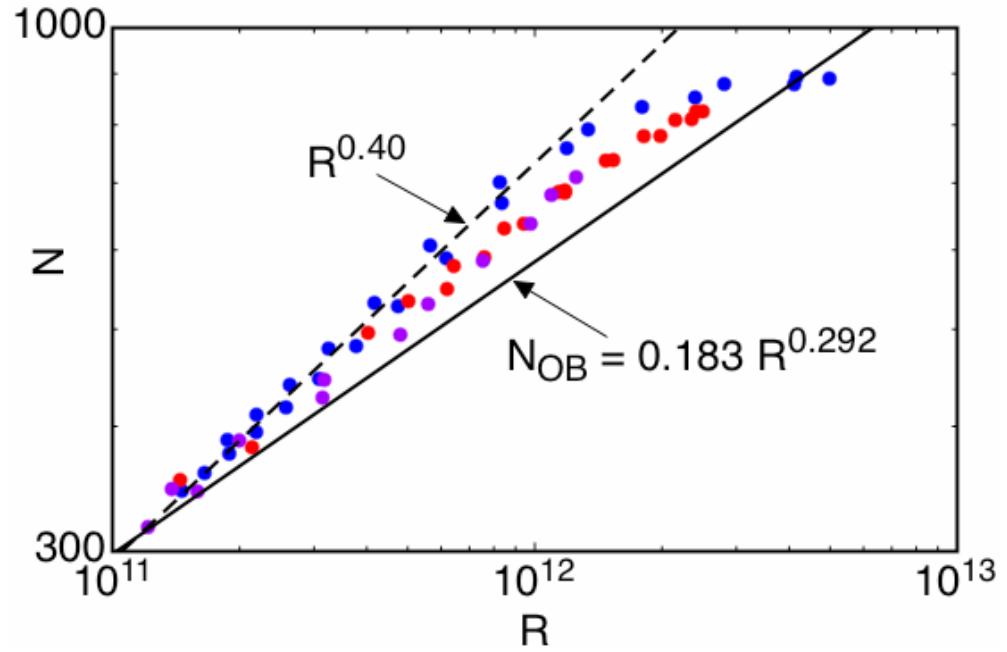
1.) Isobars at constant  $\Delta T$

2.) Constant  $\langle T \rangle$ , varying  $\Delta T$

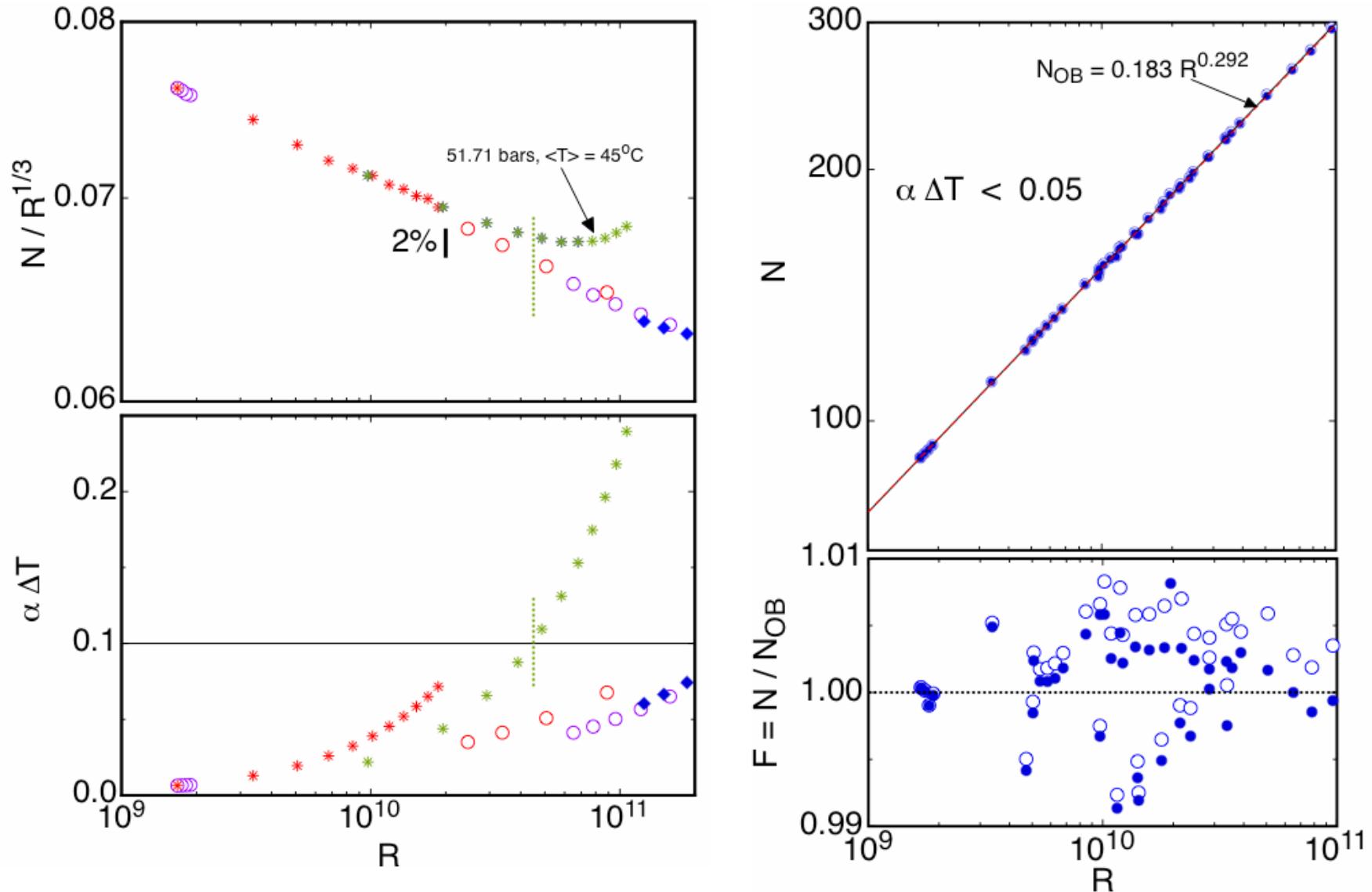


Isobars, constant Delta T,  
crossing the critical isochore



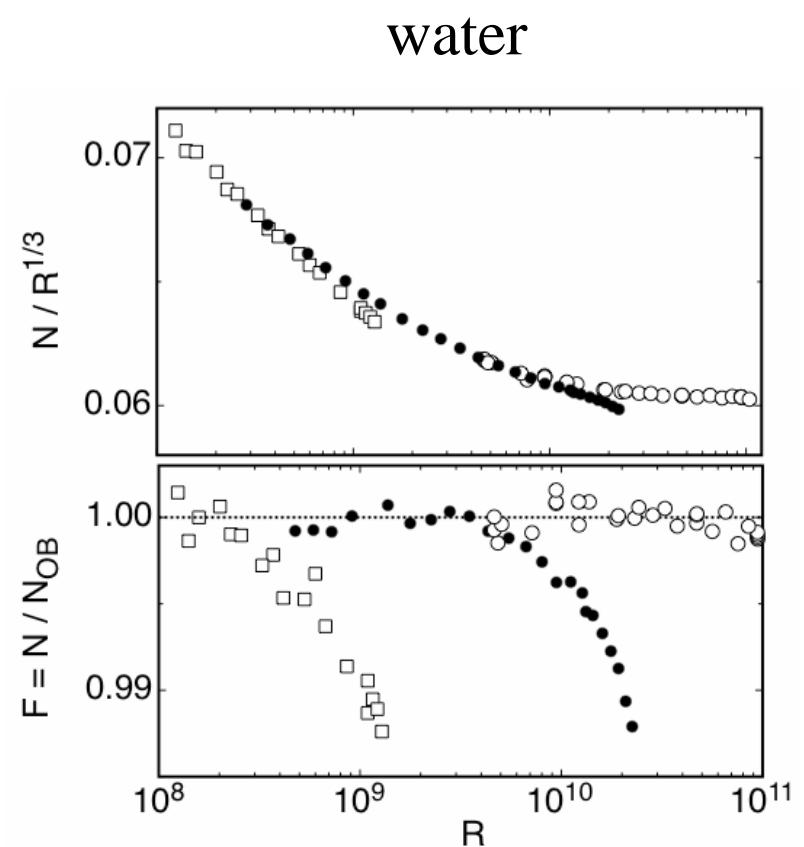
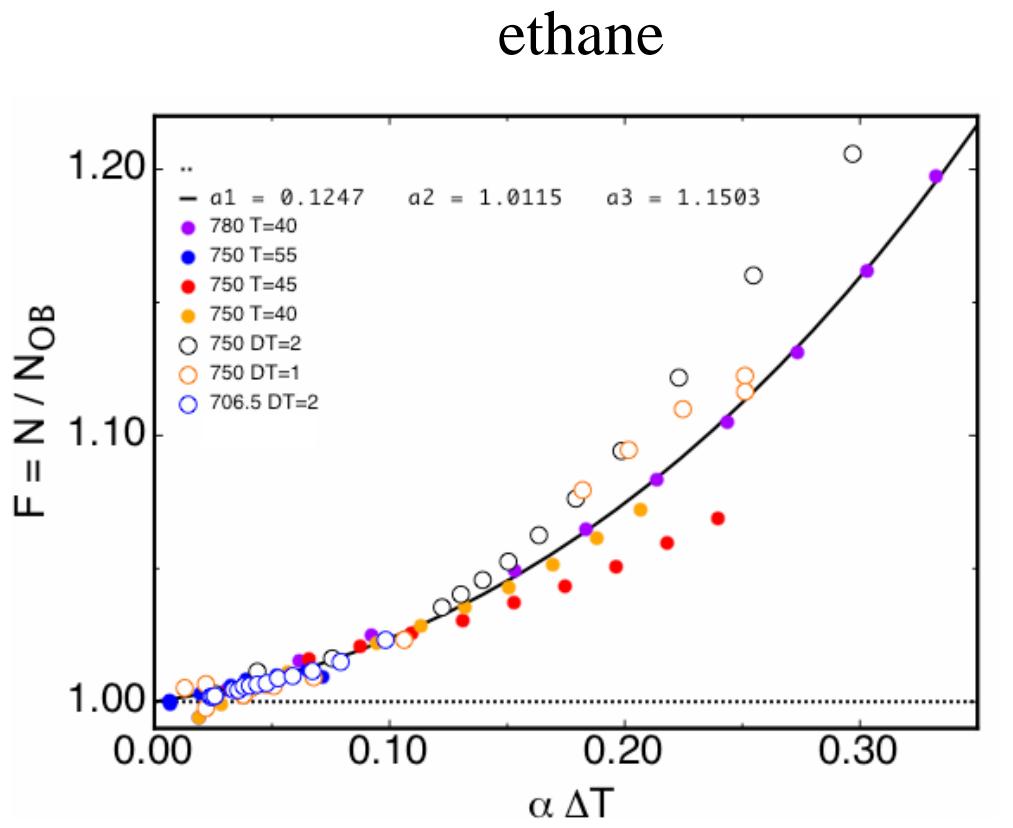


# Non-Oberbeck-Boussinesq (NOB) effects in the vapour phase



For NOB effects in liquids, especially water, see

G Ahlers, E. Brown, F. Fontenele Araujo, D. Funfschilling, S. Grossmann, and D. Lohse, J. Fluid Mech., in press.

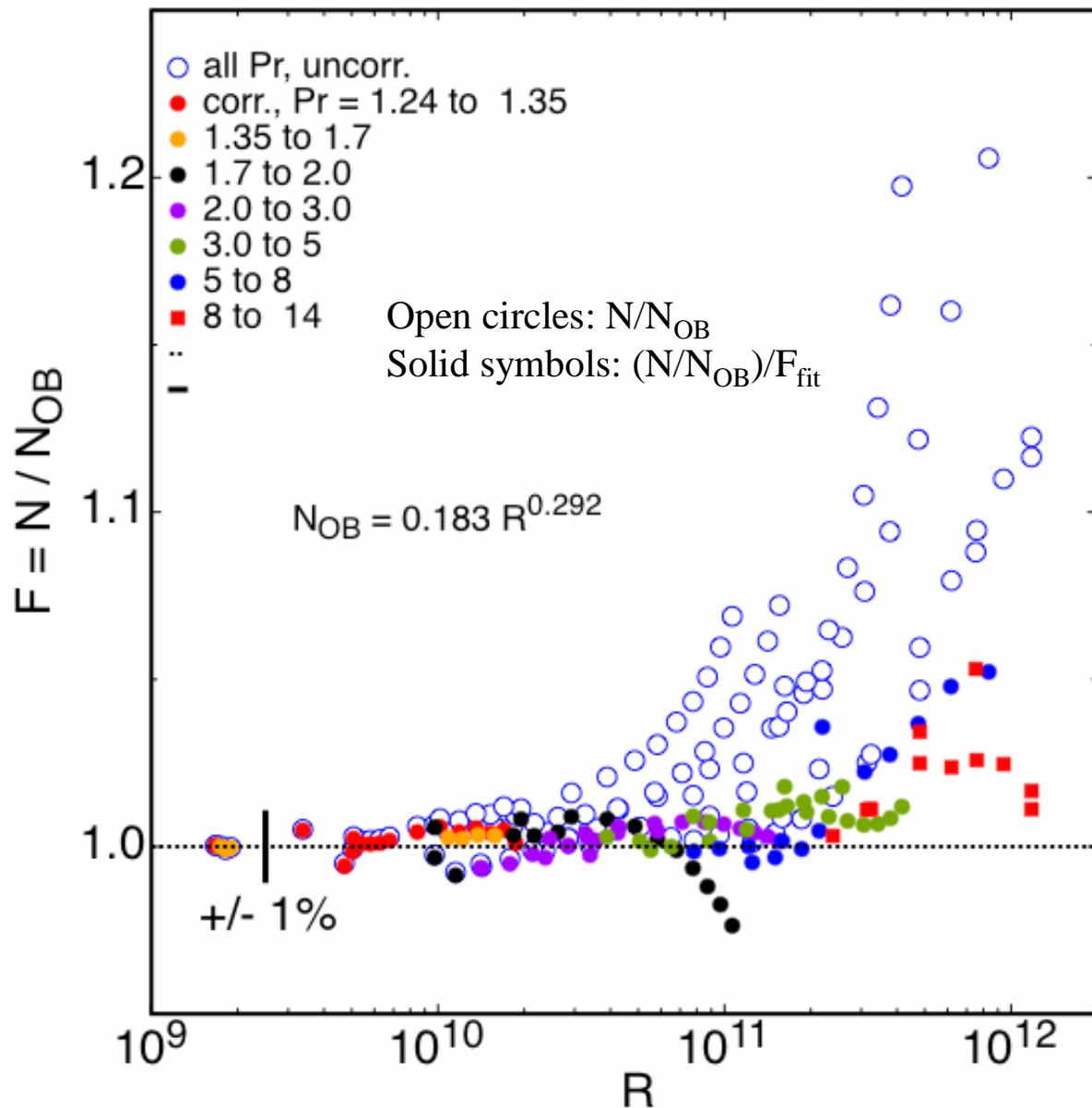


Solid line:

$$F_{fit} = 1 + a_1(\alpha \Delta T) + a_2(\alpha \Delta T)^2 + a_3(\alpha \Delta T)^3$$

$$a_1 = 0.125, a_2 = 1.01, a_3 = 1.15$$

From G Ahlers, E. Brown, F. Fontenele Araujo, D. Funfschilling, S. Grossmann, and D. Lohse, J. Fluid Mech., in press. .



Note:

No significant Prandtl-number dependence.

Agrees with

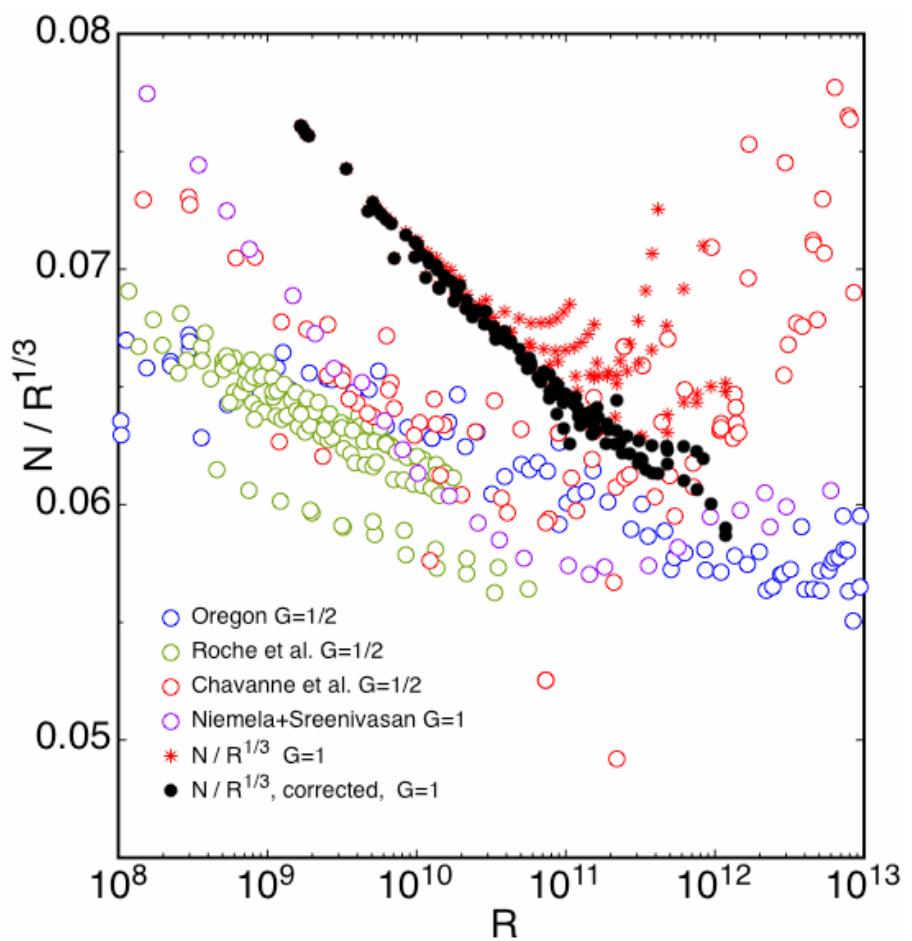
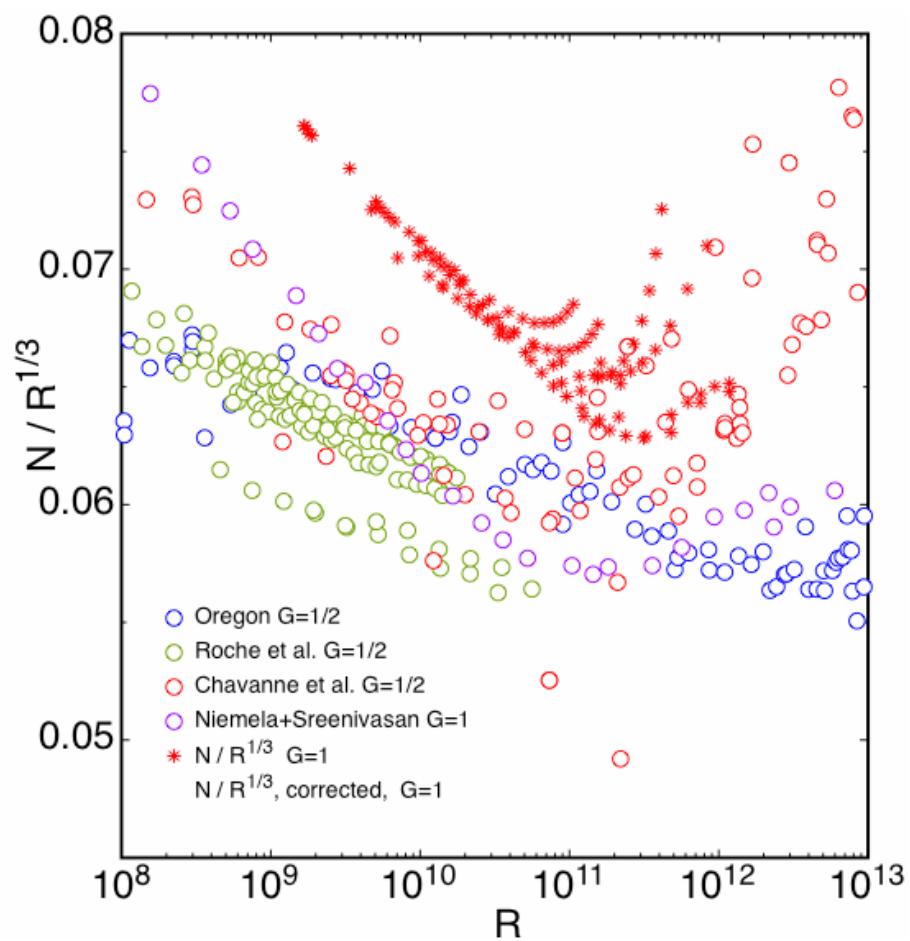
A.+ Xu, PRL **86**, 3320 (2001);

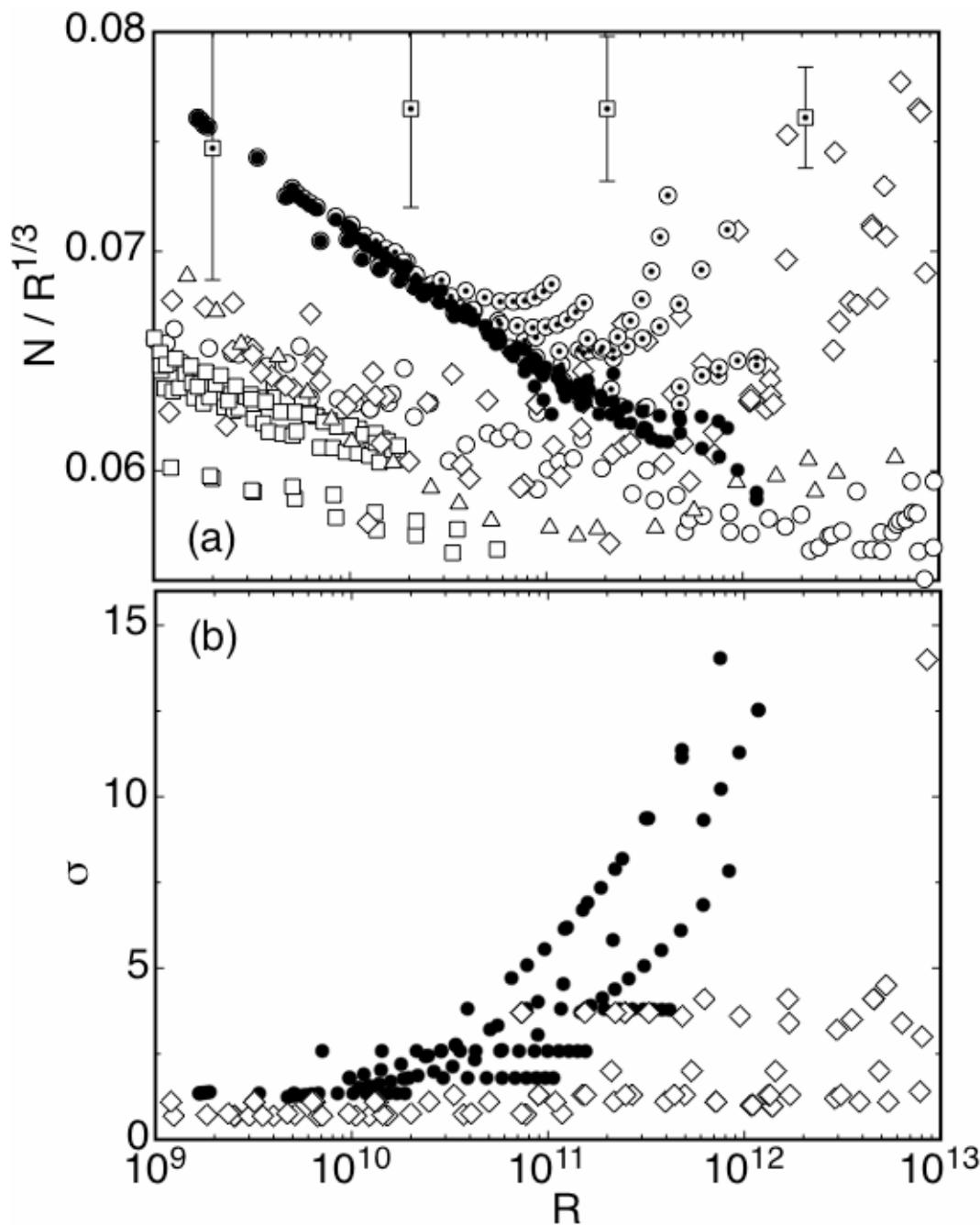
Roche et al., Europhys. Lett. **58**, 693 (2002);

Xia et al., PRL **88**, 064501 (2002).

Disagrees with

Ashkenazi and Steinberg,  
PRL **83**, 3641 (1999).



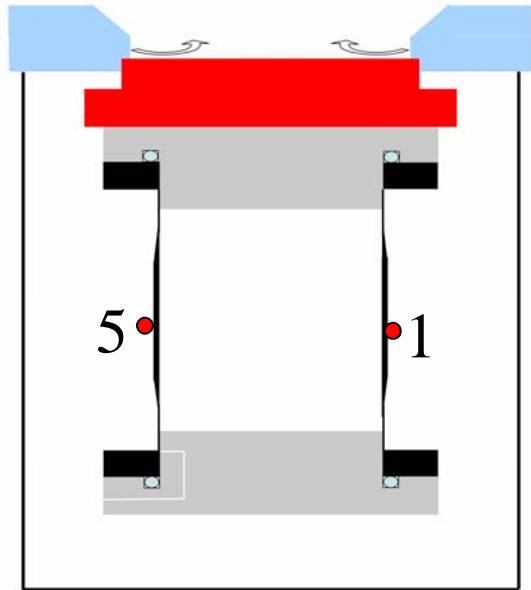


## Summary:

NOB effects cause an INCREASE of Nusselt;  
this differs from liquids like water where  $N$   
decreases, albeit by a much smaller amount.

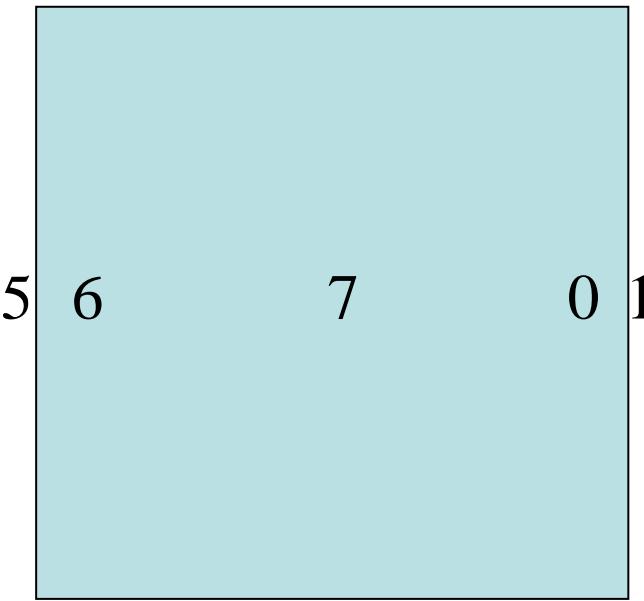
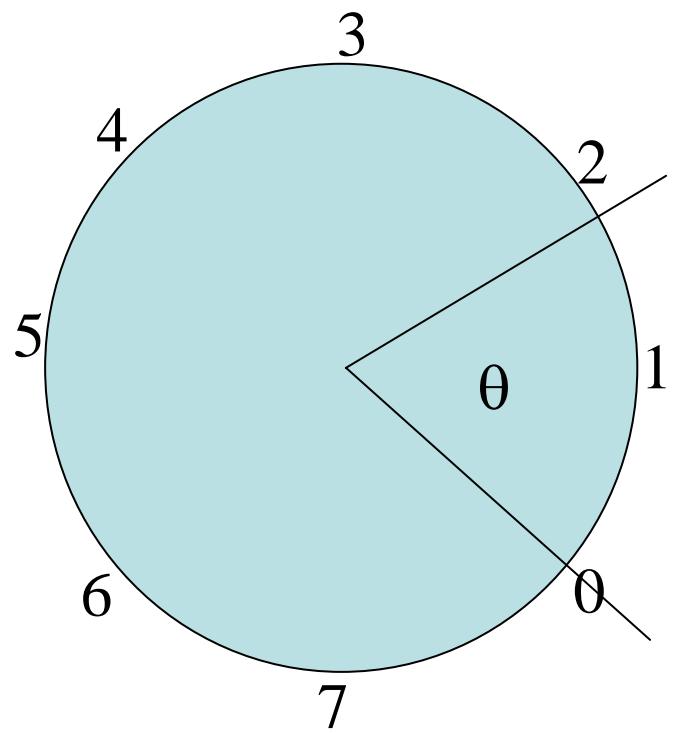
$\alpha\Delta T$  by itself does not completely determine  
the size of the NOB effect, but for  $\alpha\Delta T < 0.15$   
it does so reasonably well.

After NOB corrections, our data reach  
 $R \sim= 10^{12}$ , but are not yet sufficient to enter  
the range where the Oregon and the Grenoble  
data differ so dramatically.

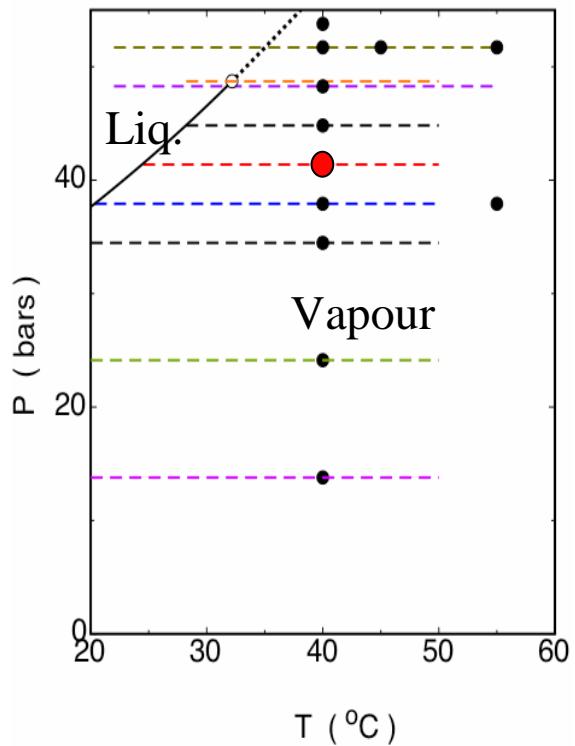


## Sidewall Thermometers

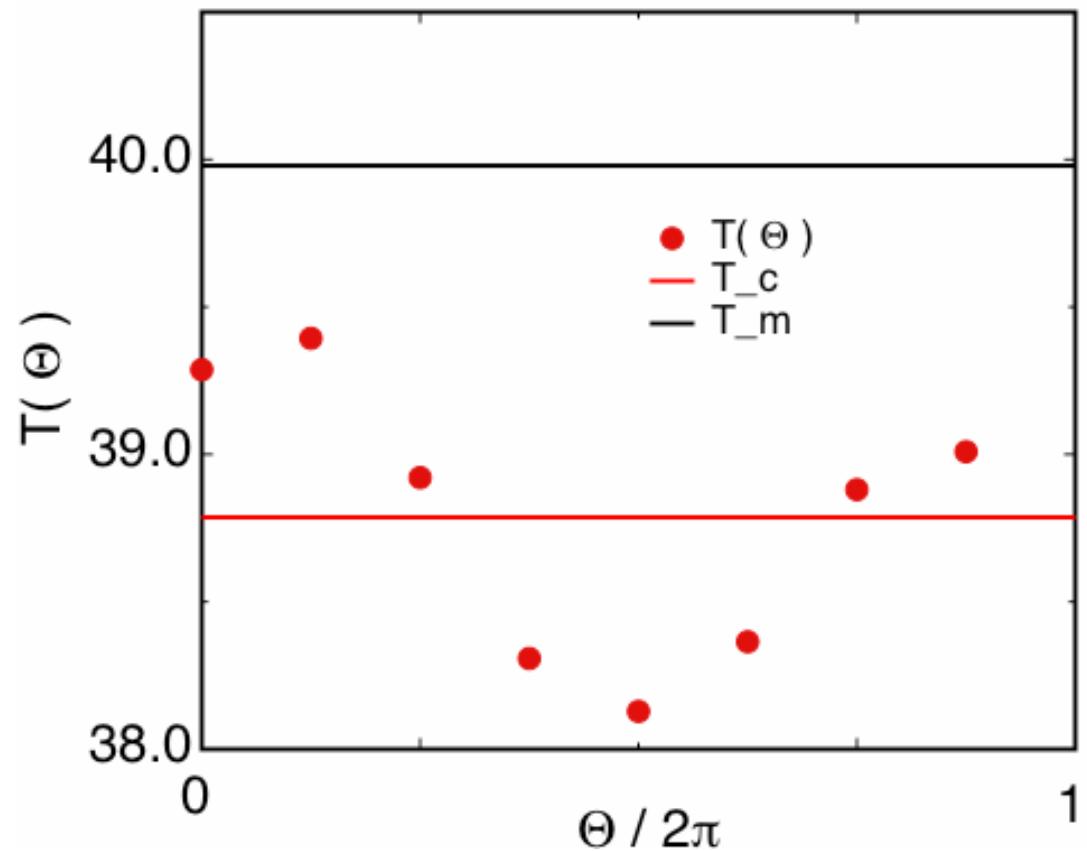
Side view



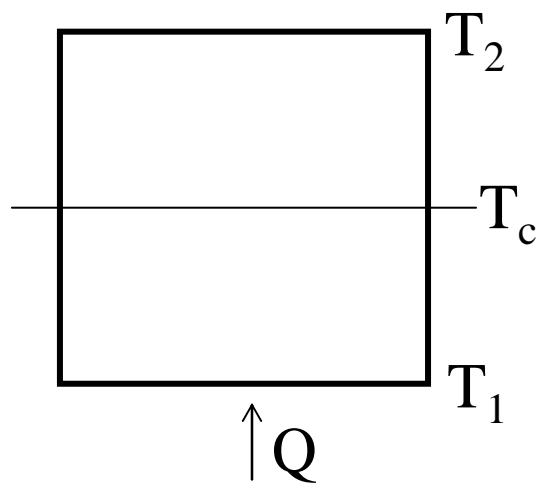
$L/2$  (middle)



$$\Delta T = 30^{\circ}$$

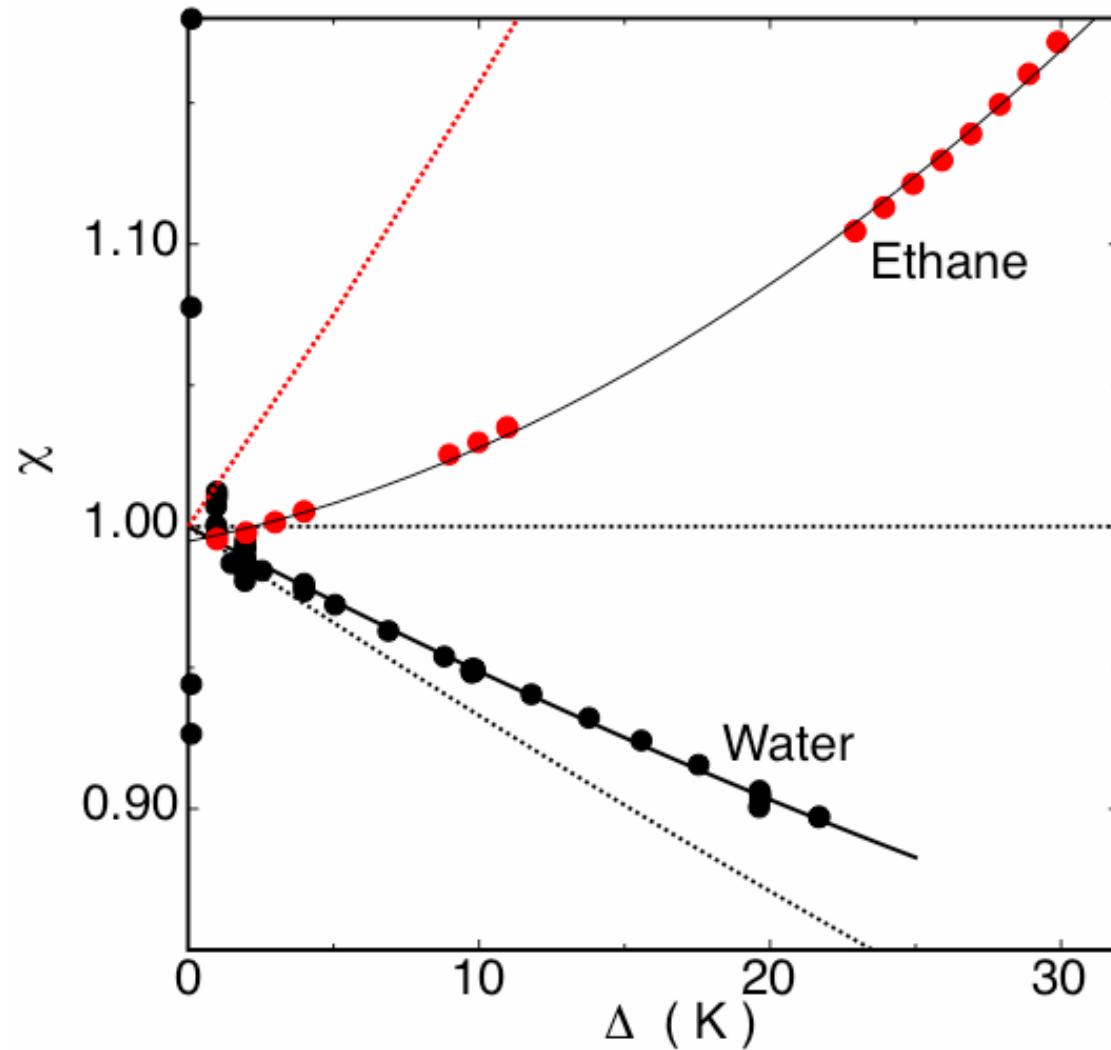


$$P = 41.37 \text{ bars}, P/P_c = 0.85, \langle T \rangle = 40^{\circ}\text{C}$$



$$\Delta = T_1 - T_2$$

$$\chi = \frac{T_1 - T_c}{T_c - T_2}$$



See Francisco Fontenele Araujo's talk yesterday.

## Summary:

The temperature  $T_c$  at the horizontal midplane is decreased by NOB effects. This differs from liquids, where  $T_c$  is increased. A quantitative comparison with Fontenele Araujo et al. has yet to be made.

# Reorientations of the large-scale circulation of turbulent RBC in aspect-ratio one samples

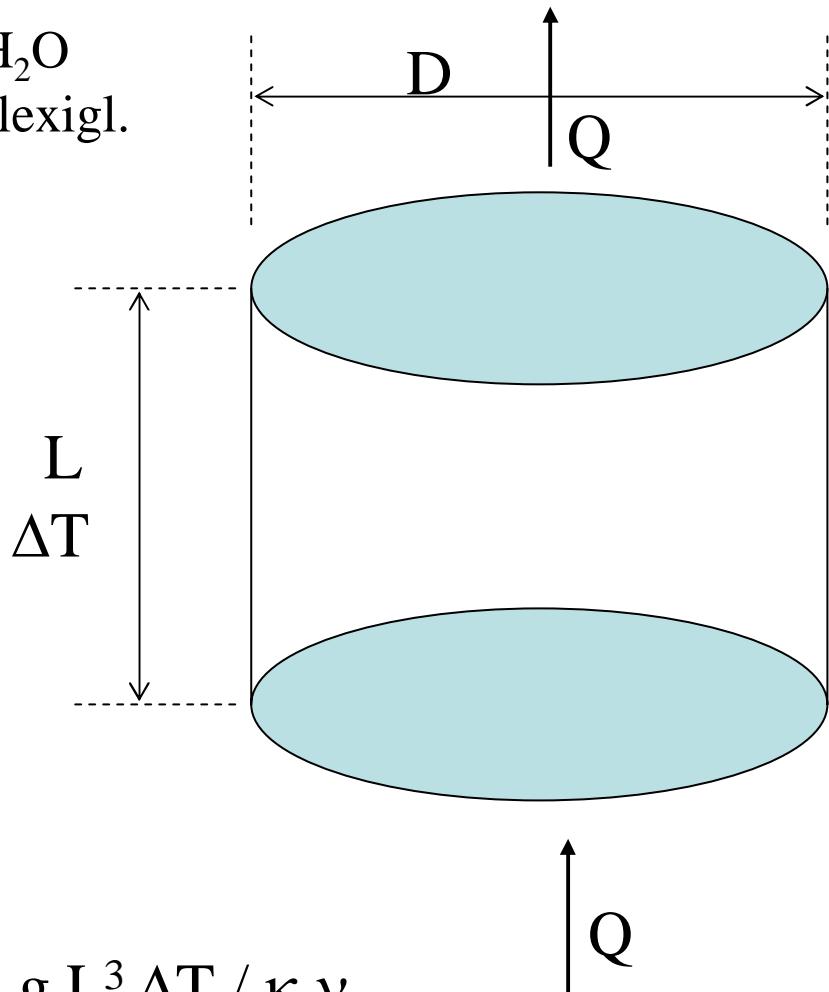
E. Brown, A. Nikolaenko, and G.Ahlers,  
Phys. Rev. Lett. 95, 084503 (2005).

E. Brown and G. Ahlers,  
J.Fluid Mech., in press.

For preprints or reprints, go to  
<http://www.nls.physics.ucsb.edu/>

# Rayleigh-Benard Convection Cell

Fluid: H<sub>2</sub>O  
Wall: Plexigl.



$$R = \alpha g L^3 \Delta T / \kappa v$$

$$\Gamma = D/L = 1.0$$

Three samples:

Small:  $D = L = 9$  cm

Medium:  $D = L = 25$  cm

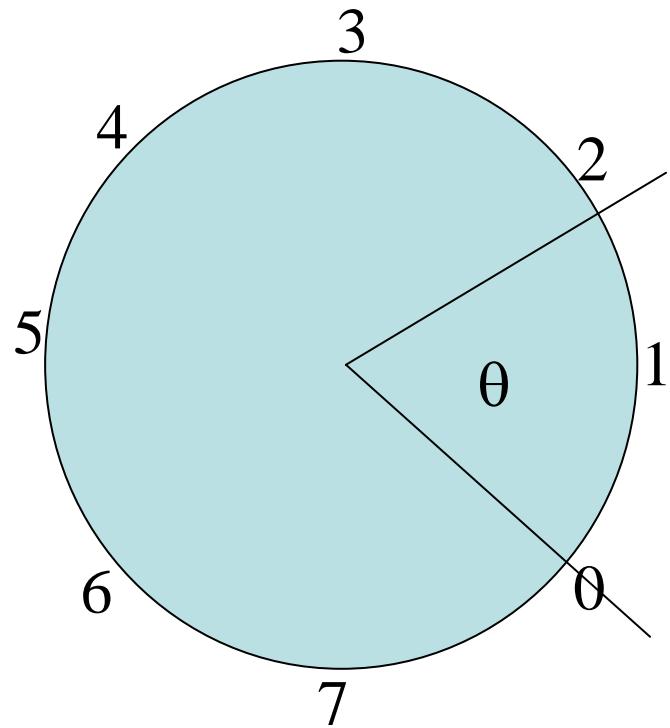
Large:  $D = L = 50$  cm

$$\Delta T = 20^\circ\text{C} : R = 10^{11}$$

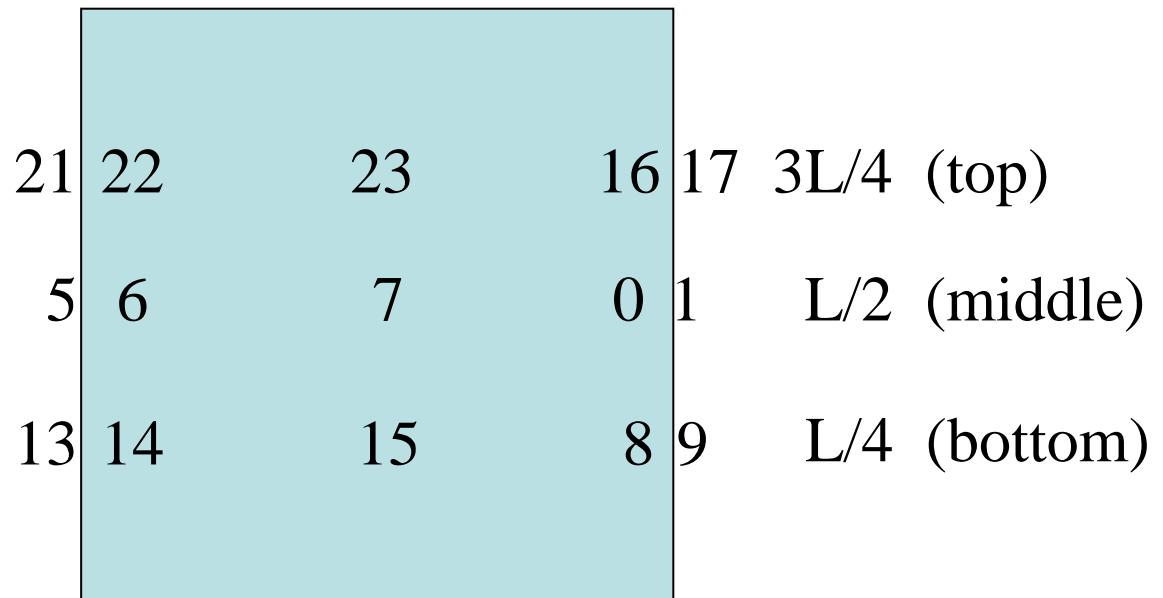
$$Q = 1500 \text{ W}$$

Prandtl No.  $\sigma = v/\kappa = 4.4$   
( H<sub>2</sub>O, 40 °C )

Top view



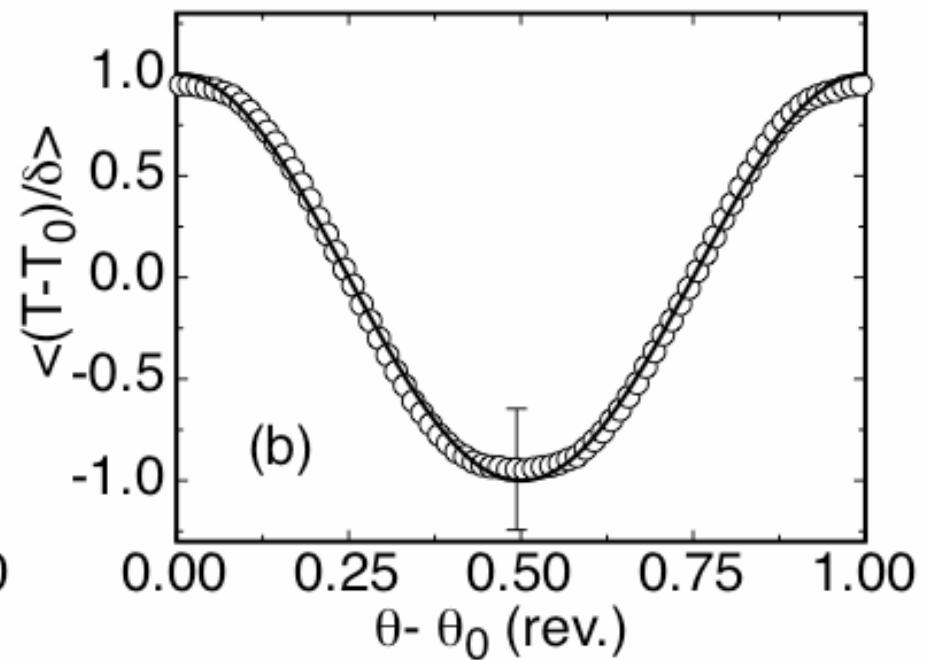
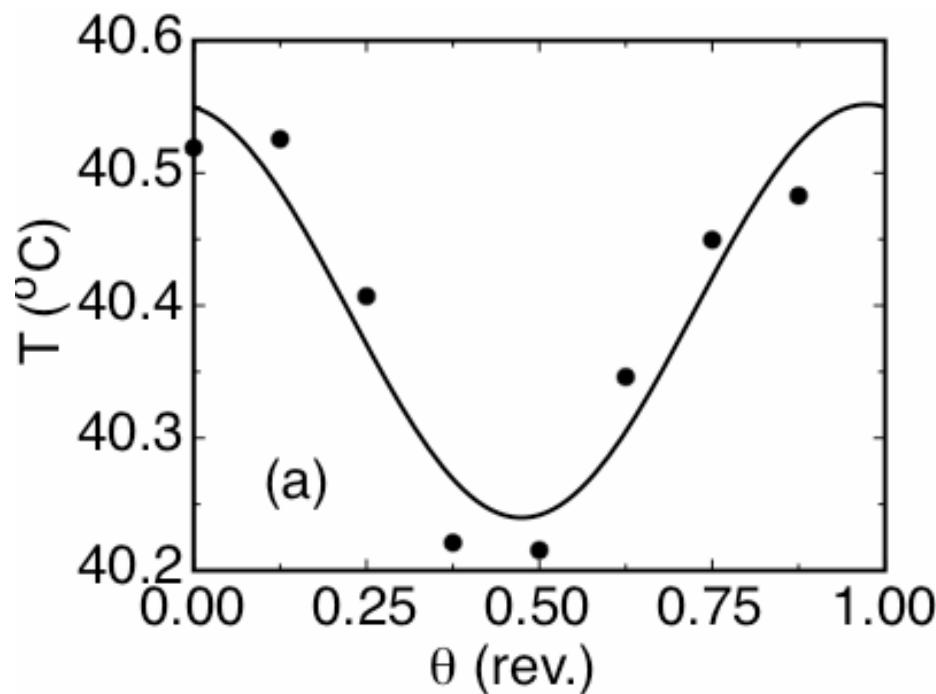
Side view

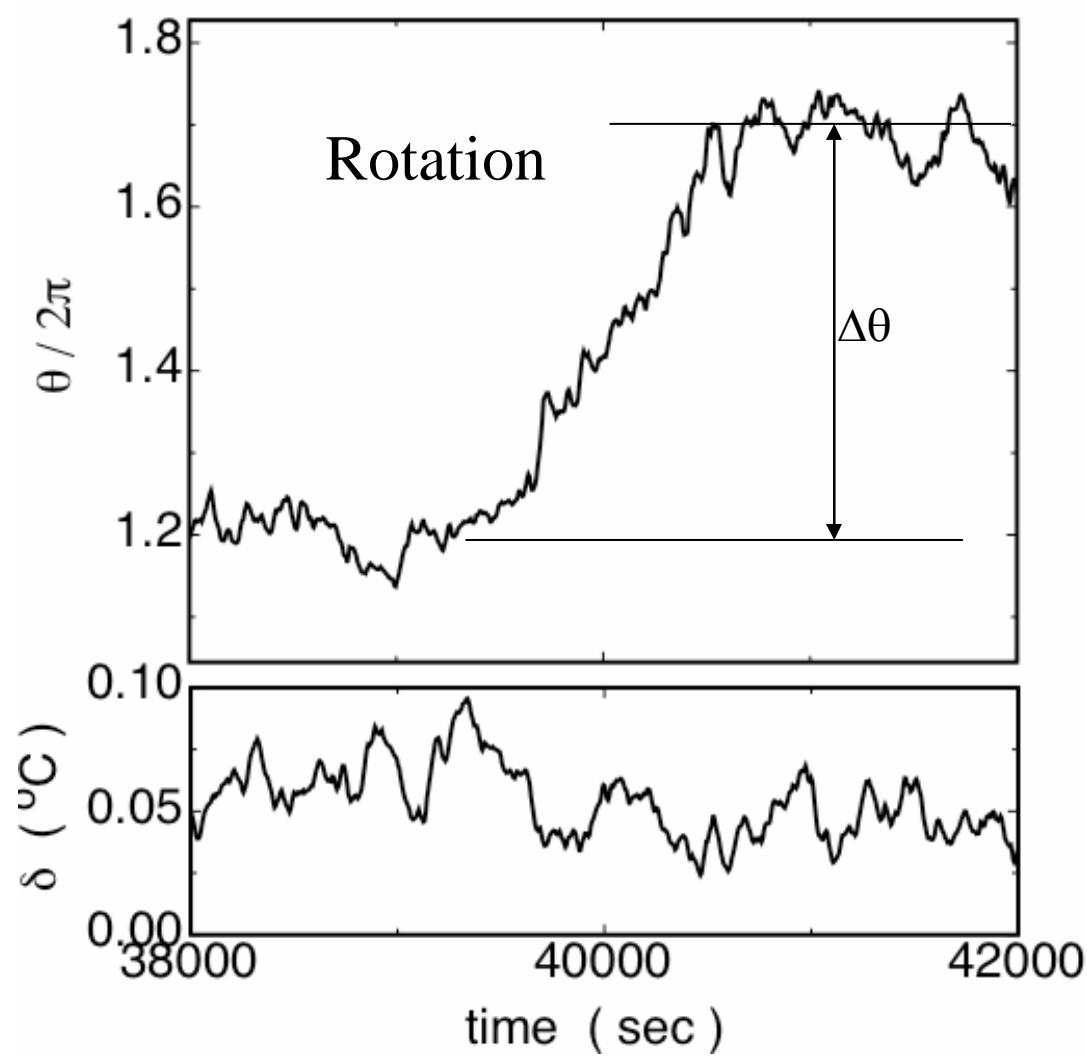


$$T_i = T_0 + \delta \cos( i\pi/4 + \theta_0 ), \quad i = 0, \dots, 7$$

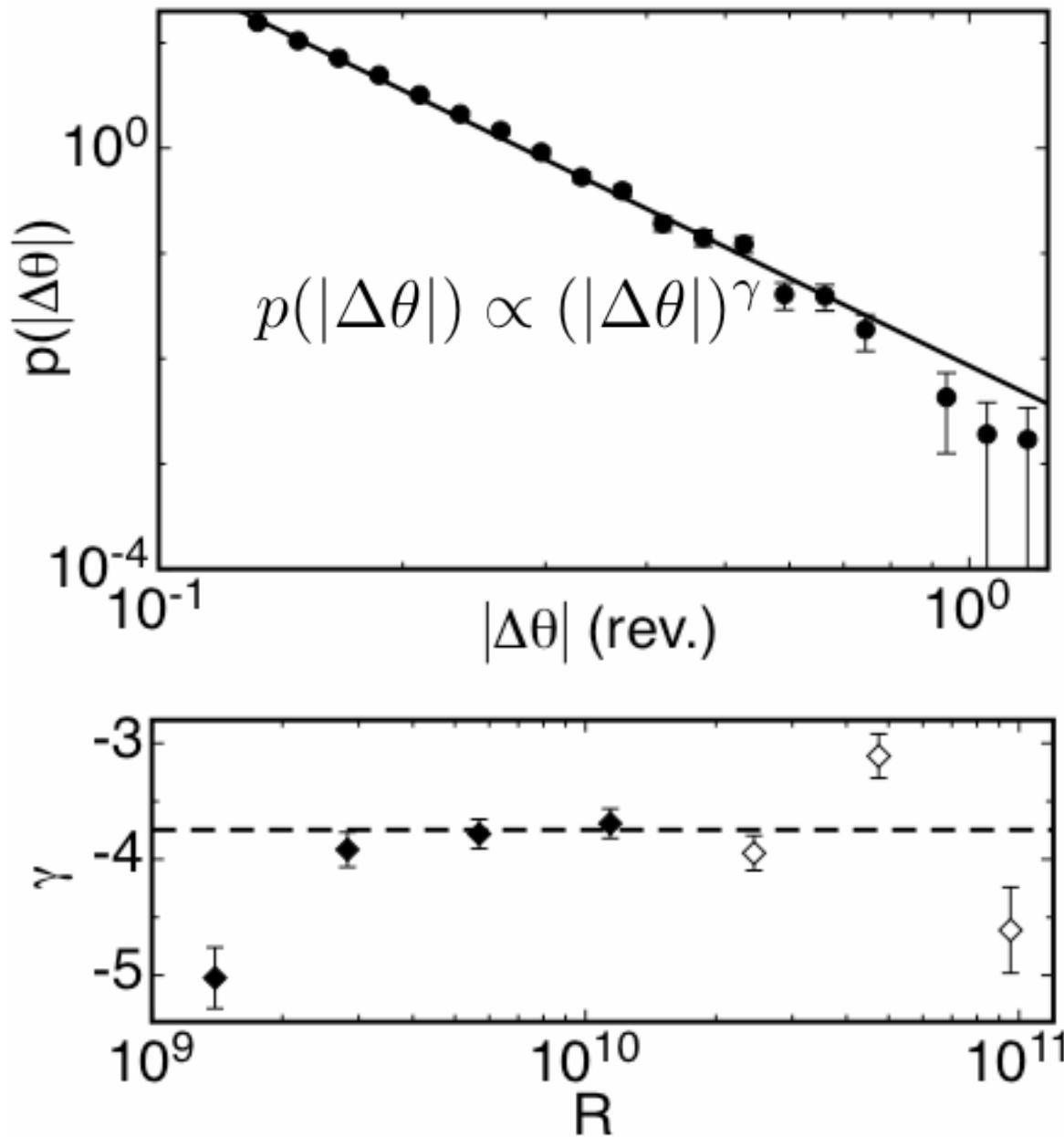
$$i = 8, \dots, 15$$

$$i = 16, \dots, 23$$

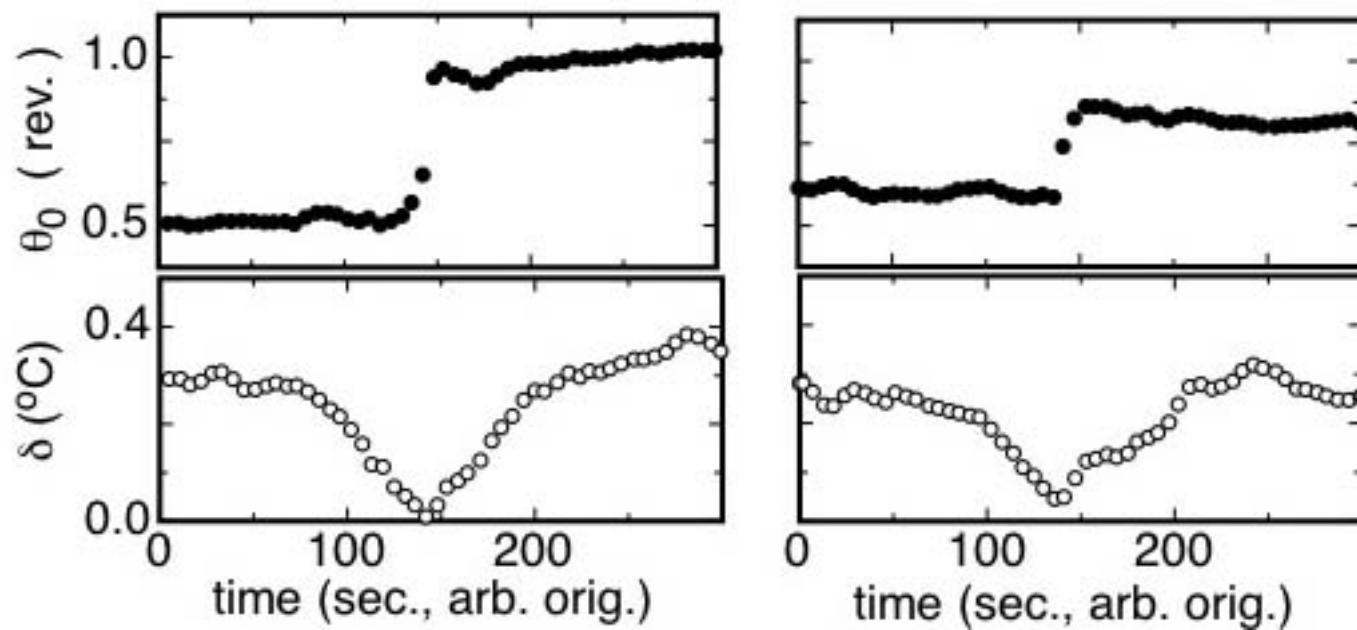




## Rotations

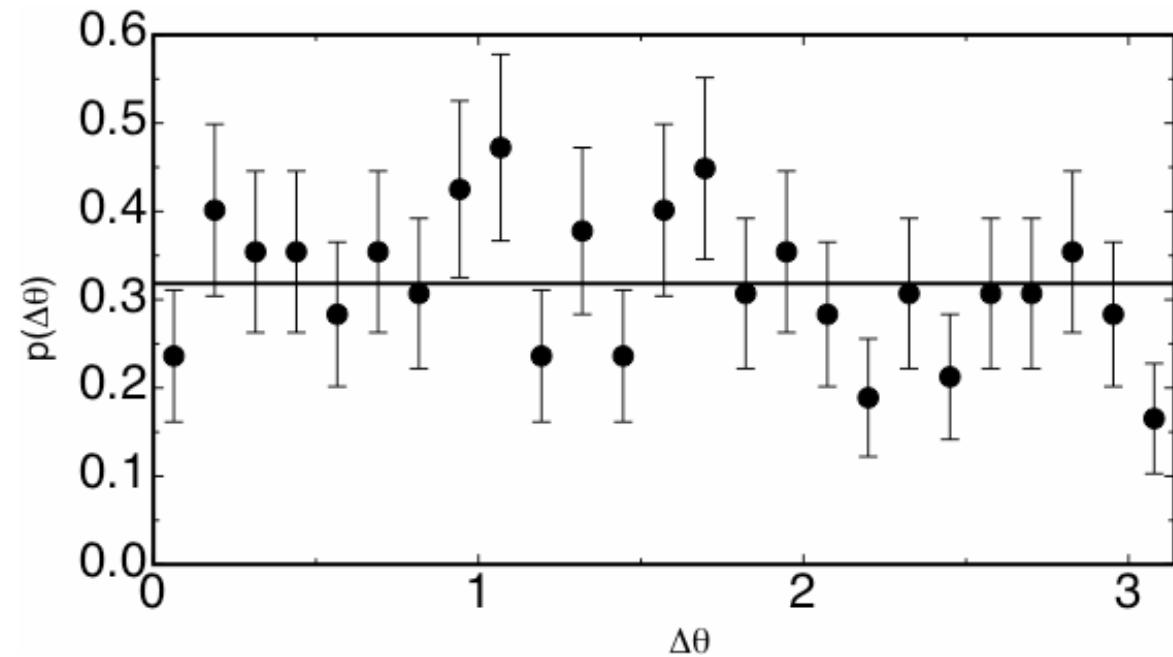
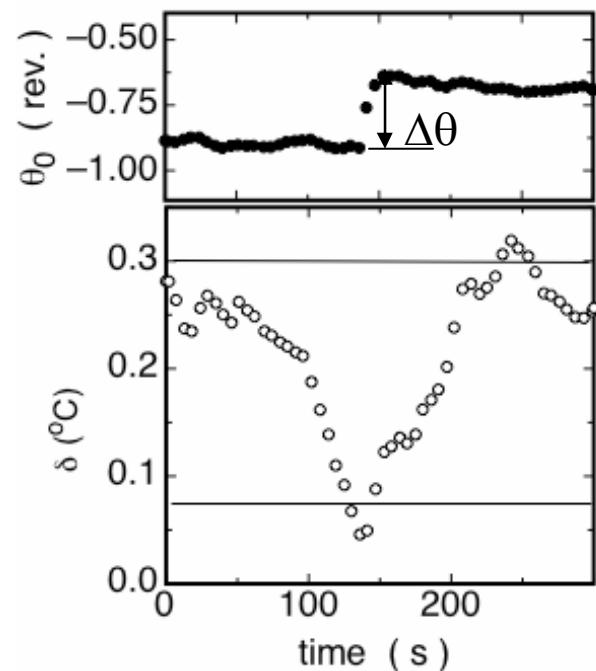


## Cessation

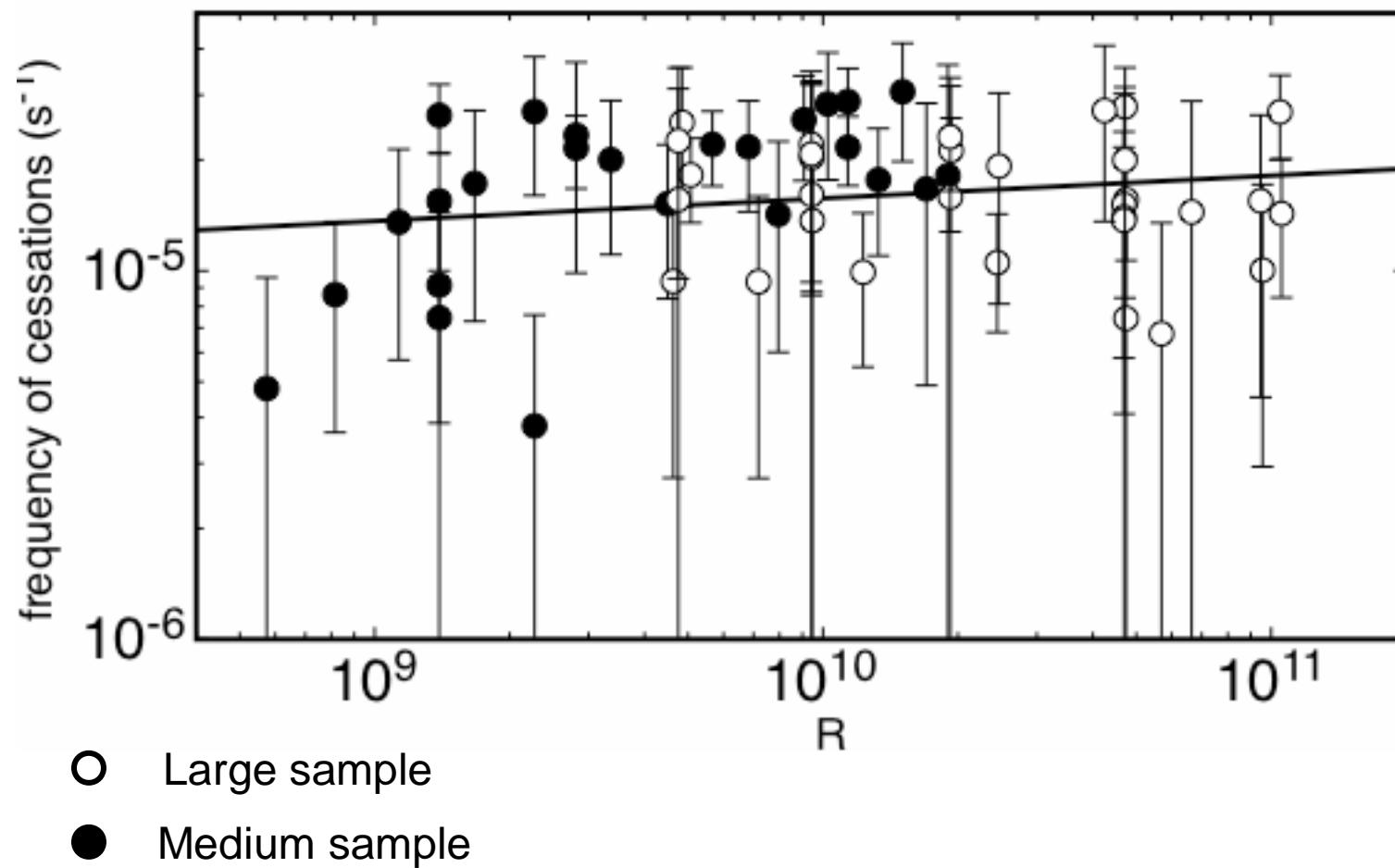


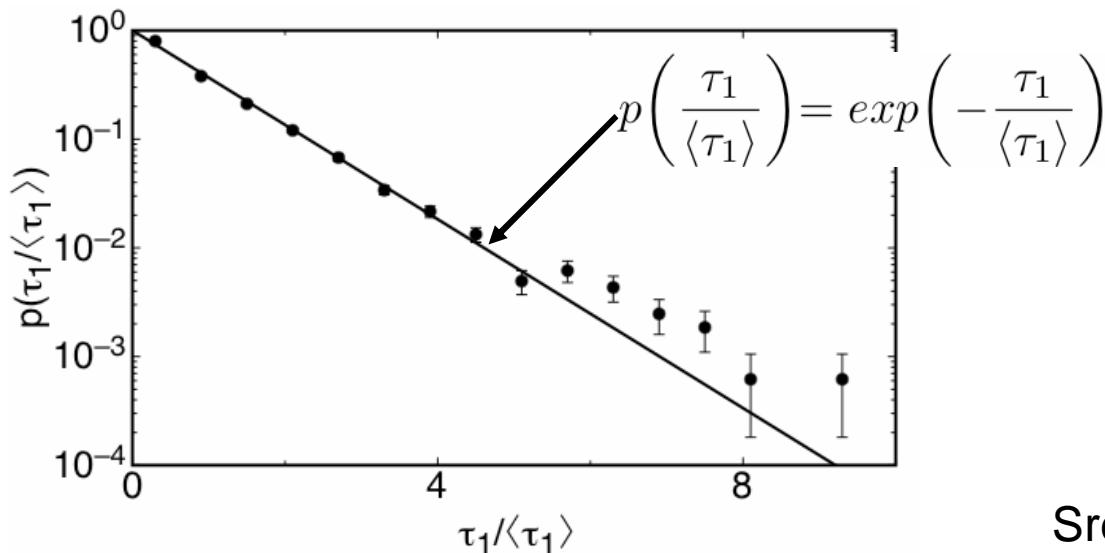
# Cessations

probability distribution of  $|\Delta\theta|$  for reorientations with  $\delta/\langle\delta\rangle < 0.25$

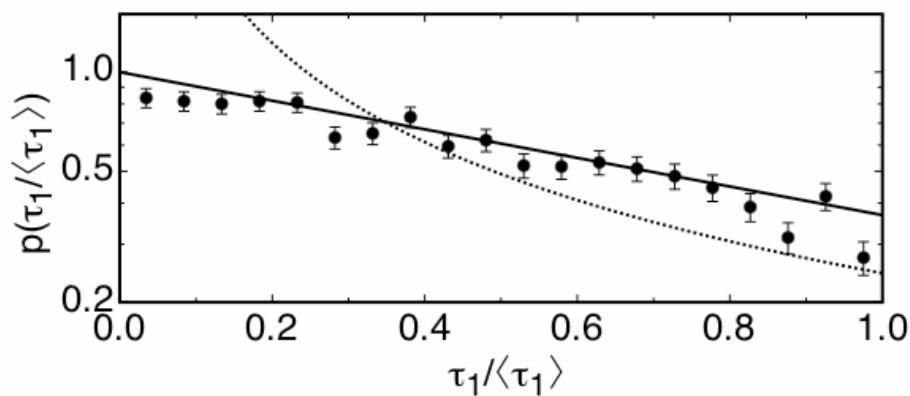


## Properties in the time domain





Poisson distributed,  
in agreeent with  
Sreenivasan et al.  
for large  $\tau_1$

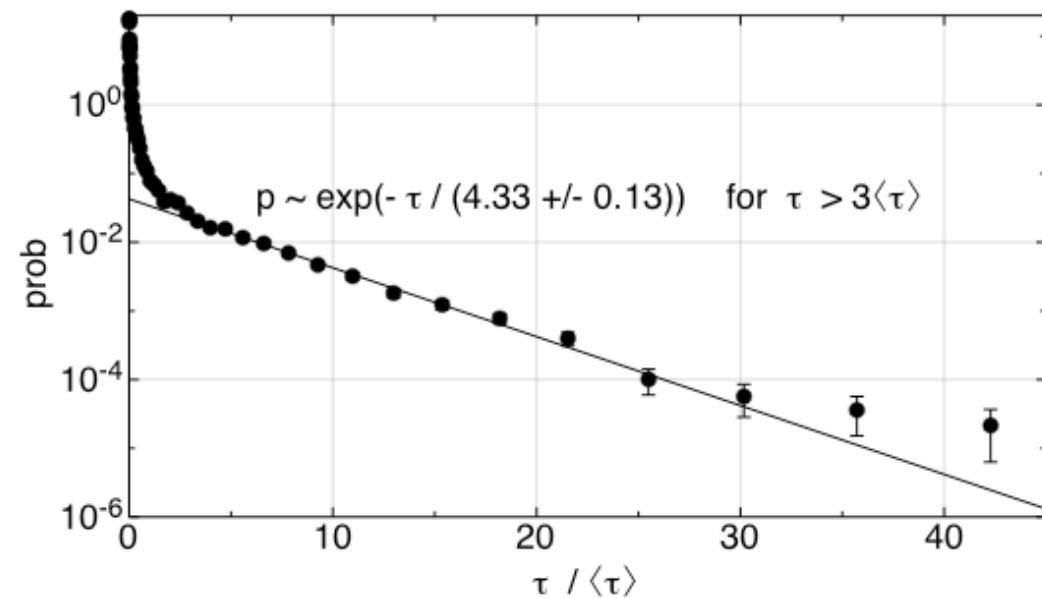
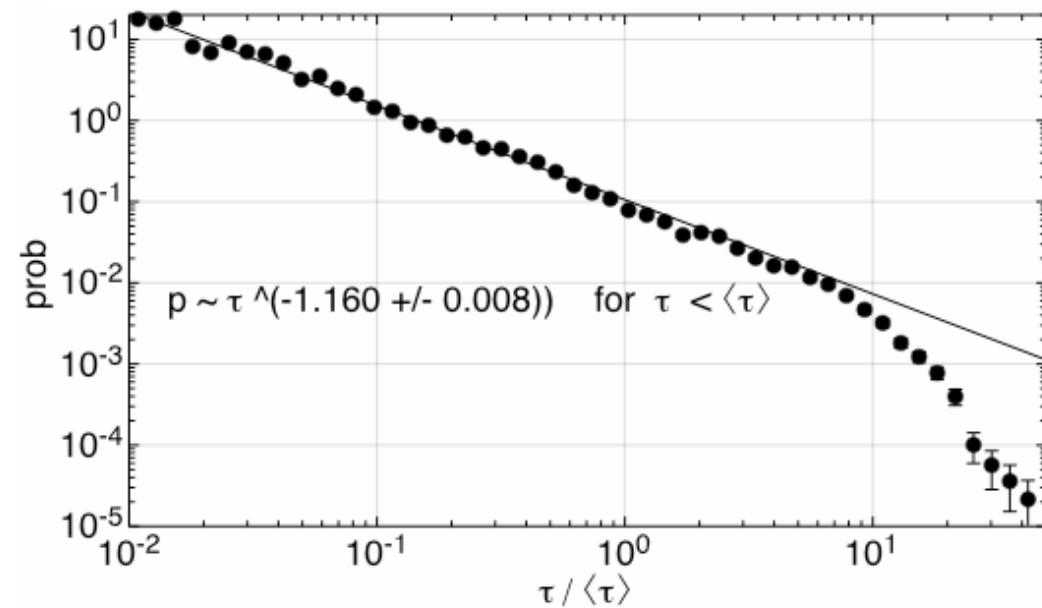
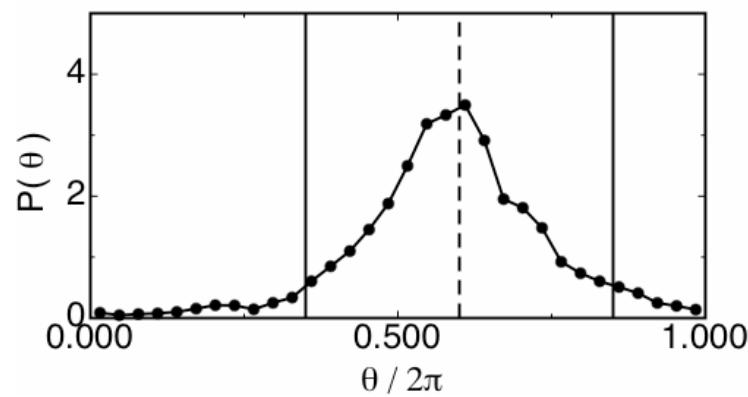


Sreenivasan et al.  
found power law  
 $P(\tau_1/\langle \tau_1 \rangle) \sim \tau_1^{-1}$   
for small  $\tau_1$   
(dotted line)

Following  
K, Sreenivasan, A. Bershadskii, and J. Niemela, Phys. Rev. E 65, 056306 (2002):

- $\tau_1$  = time between  $i^{\text{th}}$  and  $(i+1)^{\text{th}}$  re-orientation

probability distribution of the time interval  $\tau$  between reversals,  
using a criterion more similar to SBN's.



- A.) The azimuthal dynamics of the LSC is diffusive, with interspersed reorientations of relatively short duration.**
- B.) LSC reorientations can occur via**
  - 1.) rotation of the vorticity vector (“rotation”)**
  - 2.) shrinking of the vorticity vector, followed by re-development with a new orientation (“cessation”)**
- C.) Cessation is followed by re-development of the LSC in a circulation plane with an arbitrary new orientation, i.e.  $P(\Delta\theta) = 1/\pi$ .**
- D.) Rotation through an angle  $\Delta\theta$  has a powerlaw probability distribution  $P(\Delta\theta) \sim \Delta\theta^{-\gamma}$  with  $\gamma \sim 4$  (i.e. rotations through smaller angles are more likely). “Heavy tail” distrib. as in forest fires and stock markets.**
- E.) Rotations and Cessations are each Poisson distributed in time.**
- F.) Small-amplitude high-frequency diffusive “jitter” of  $\theta$  can give a power-law distribution in time for small time intervals  $\tau$  as observed by Sreenivasan et al.**