The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
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International Atomic
Energy Agency

SMR.1771 - 16

Conference and Euromech Colloquium #480

on

High Rayleigh Number Convection

4 - 8 Sept., 2006, ICTP, Trieste, Italy

Analogies between thermal convection and shear flow: RB Nusselt number, TC torque and pipe friction coefficient

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RB Nusselt number, TC torque, pipe friction

Analogies between

thermal convection and shear flows

by

Bruno Eckhardt, Detlef Lohse, SGn

Philipps-University Marburg and University of Twente

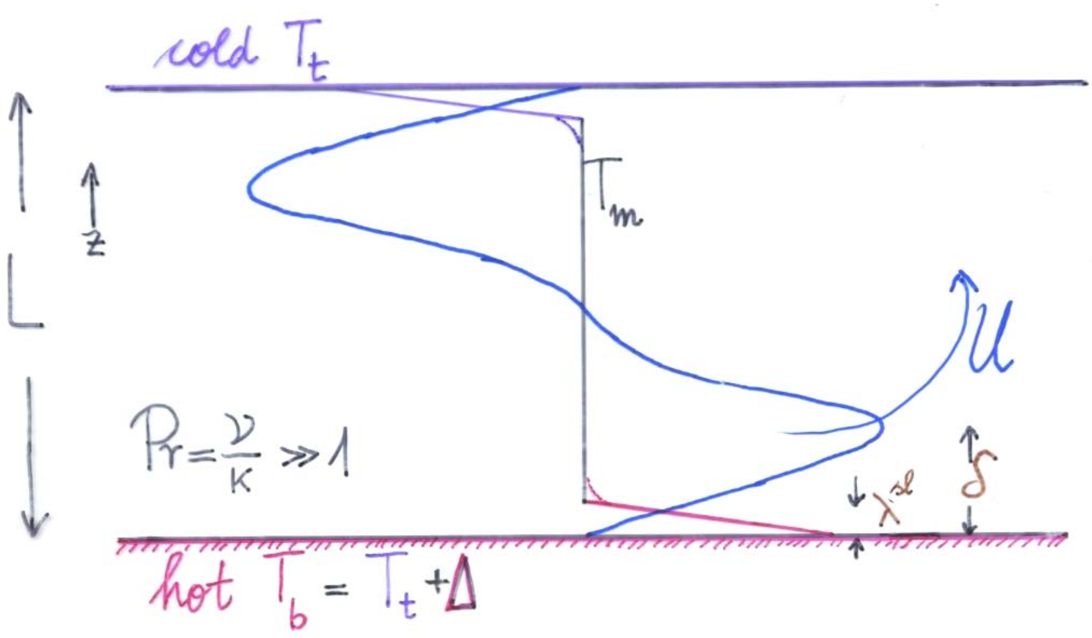
Nusselt number scaling in RB

$$Nu \propto Ra^{\beta}$$

result: $\beta = \beta\,(Ra, Pr; NOB)$

variable scaling exponent

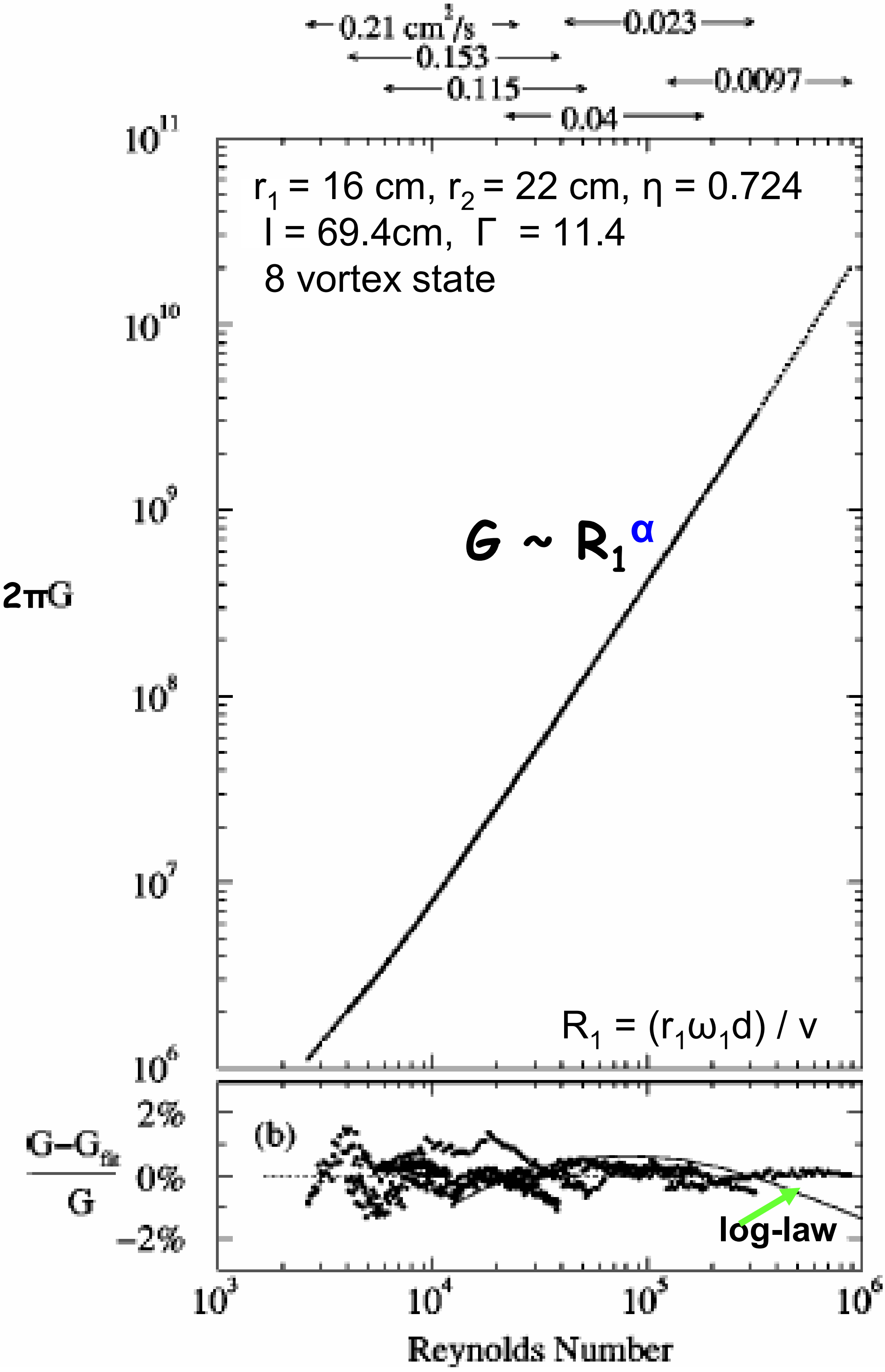
Physical origin of variable exponent
Varying weight of BL and bulk



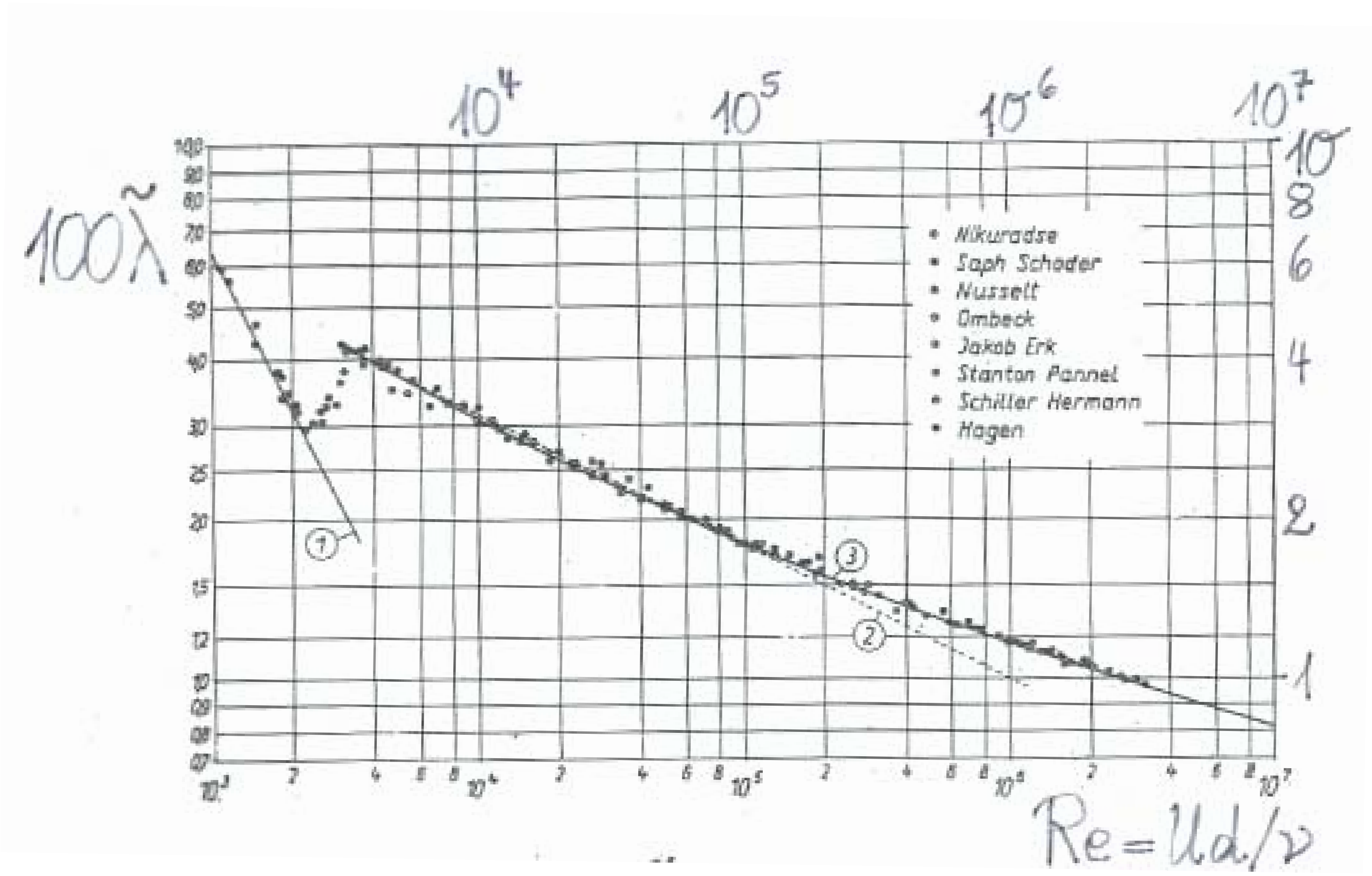
$$\lambda^{sl} = \frac{L}{2\ Nu} \qquad \delta = \frac{aL}{\sqrt{Re}}$$

Torque $2\pi G$ versus R_1 in TC

Lewis and Swinney, PRE 59, 5457 (1999)



Pipe flow
Skin friction coefficient $\tilde{\lambda} = 4\ c_f$



from Hermann Schlichting,
Grenzschichttheorie, Fig.20.1

Global relations in RB

$$\frac{J}{J_{lam}} = Nu = \frac{\langle u_z \theta \rangle_{A,t} - \kappa \partial_z \langle \theta \rangle_{A,t}}{\kappa \Delta L^{-1}}$$

$$= \tilde{\varepsilon}_\theta = \frac{\varepsilon}{\kappa \Delta^2 L^{-2}}$$

$$\frac{\varepsilon_w}{\nu^3 L^{-4}} = \tilde{\varepsilon}_w = Pr^{-2} Ra (Nu - 1)$$

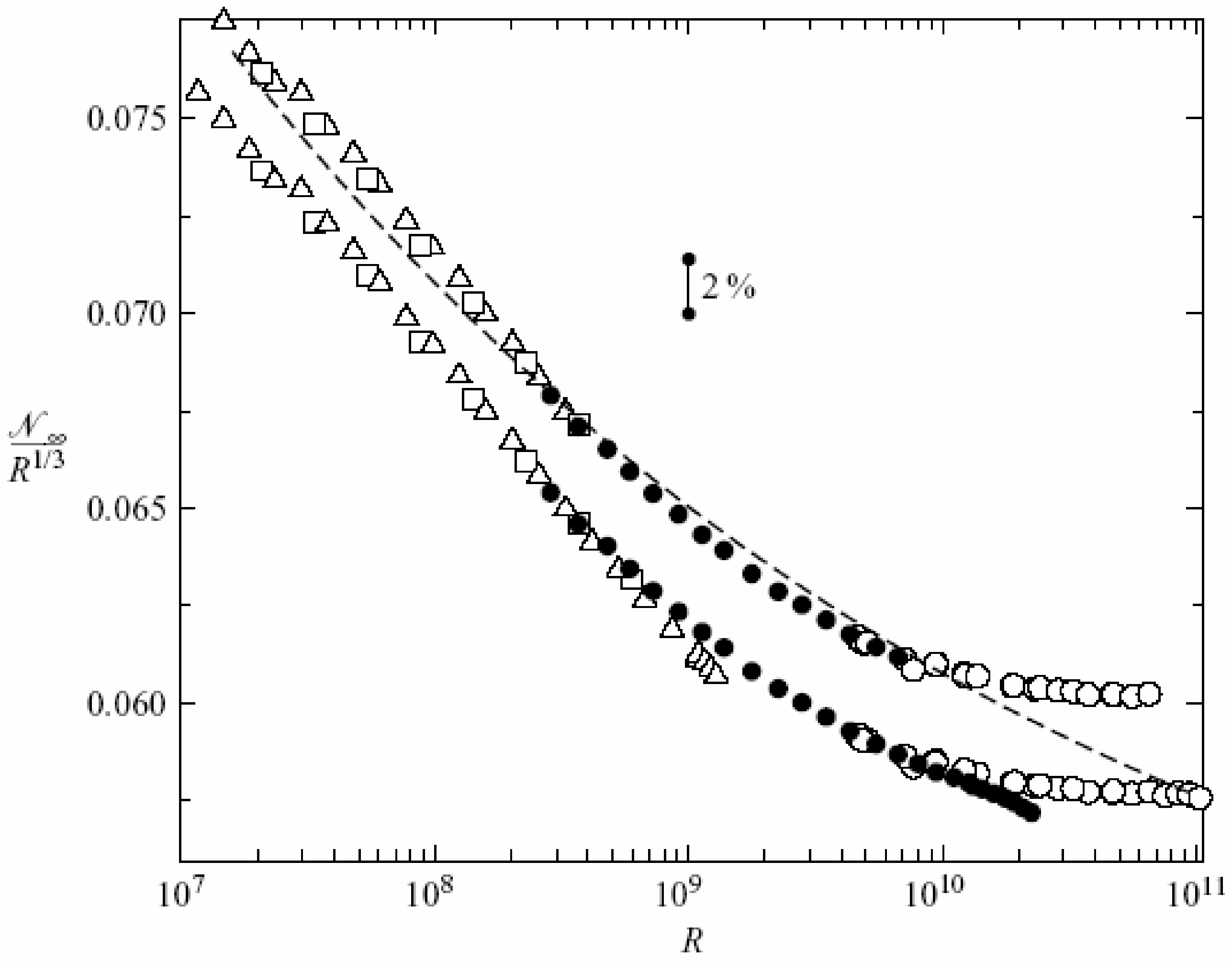
BL-bulk decomposition

$$
\begin{aligned}
Pr^{-2} Ra Nu &= c_1 Re^{5/2} + c_2 Re^3 \\
Nu &= c_3 \sqrt{Re Pr f(\frac{\lambda}{\delta})} + c_4 Pr Re f(\frac{\lambda}{\delta})
\end{aligned}
$$

Boundary layer thicknesses

$$
\delta / L = a/\sqrt{Re} \,, \quad \lambda^{sl}/L = 1/2Nu
$$

Global dissipation balance !



Funfschilling, et al., JFM 536, 145 (2005)

water, Pr = 4.38, $10^7 < Ra < 10^{11}$

D ≈ 50, 25, 9 cm, all with Γ ≈ 1

Large Ra scaling

◇ $Pr^{-2} Ra(Nu-1) = \tilde{\varepsilon}_u \propto Re^3$, bulk

satisfied by $Nu \propto Ra^{1/3}$ and $Re \propto Ra^{4/9}$

if $Re \propto Ra^{1/2}$, Kolmogorov $\tilde{\varepsilon}_u \propto Ra^{9/3}$ fails

instead $\tilde{\varepsilon}_u \propto Re^{8/3}$

- then also Prandtl exponent balance changes !

  instead of $Re \propto Ra^{4/9} Pr^{-2/3}$, in IV_u

  Pr-exponent prediction is $Re \propto Ra^{1/2} Pr^{-3/4}$

  12% effects

$$Pr^{-2}\ Ra\ Nu = \sim Re^{8/9}$$

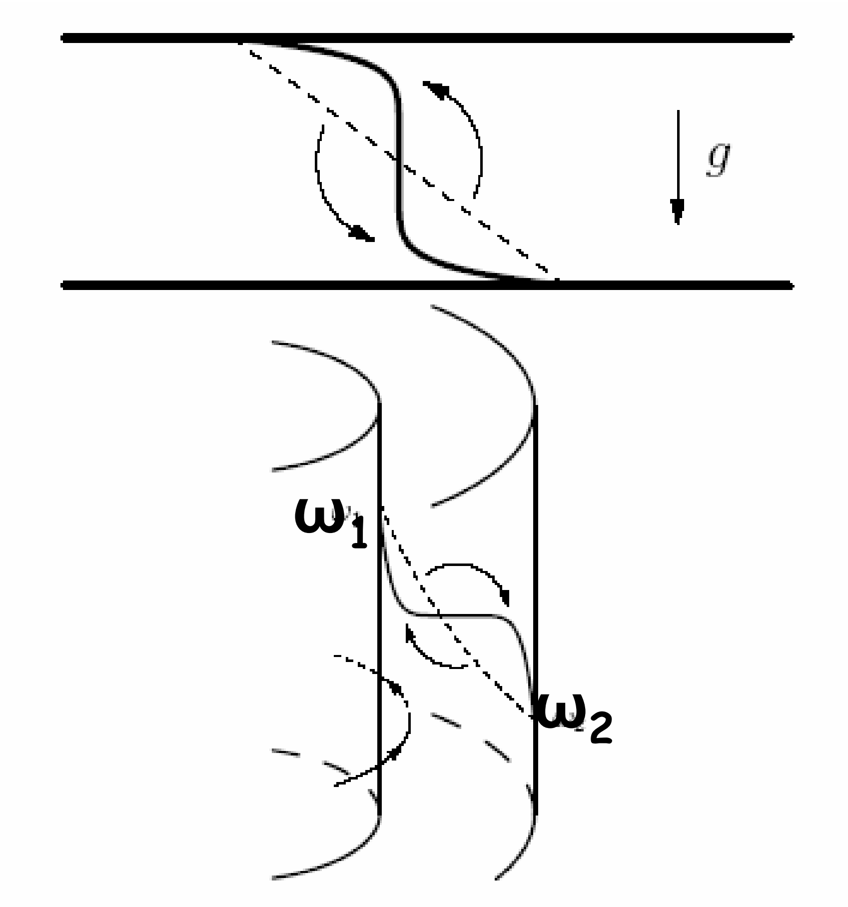
$$Nu = \sim Re^{1/2}\ Pr^{1/3} + \sim Re^{3/4}\ Pr^{1/2}$$

$$= \sim Re^{2/3}\ Pr^{4/9}$$

$$\rightarrow \quad Re \sim Ra^{1/2}\ Pr^{-7/9}$$

$$\rightarrow \quad Nu \sim Ra^{1/3}\ Pr^{-2/27}$$

Analogy RB and Pipe



T-profile ↔ u_φ or ω or L profile

wind U ↔ u_r and u_z

Relevant current density

$$J^\omega = J^\omega_{lam}\ N^\omega = \frac{r_1 F_1}{2\pi \ell \rho_{fluid}}$$
$$= \sigma_{r\varphi}(r_1)\cdot r_1^2$$
$$= \nu^2 G$$
$$= \nu \frac{r_1^2 r_2^2}{r_a}\frac{\omega_1}{d}\cdot\left(\frac{r_1\omega_1 d}{\nu}\right)^\beta$$

flux of longitudinal momentum through $A^{||}$:

$$J = r^3\left[\langle u_r\omega\rangle_{A^{||},t} - \nu\partial_r\langle\omega\rangle_{A^{||},t}\right]$$
$$= r\langle u_r L\rangle_{A^{||},t} - \nu r^3\partial_r\langle\omega\rangle_{A^{||},t}$$

derivation: average u_φ-equation over t and $A^{||}$

Excess dissipation rate

$$\varepsilon_w = \varepsilon - \varepsilon_{lam}$$

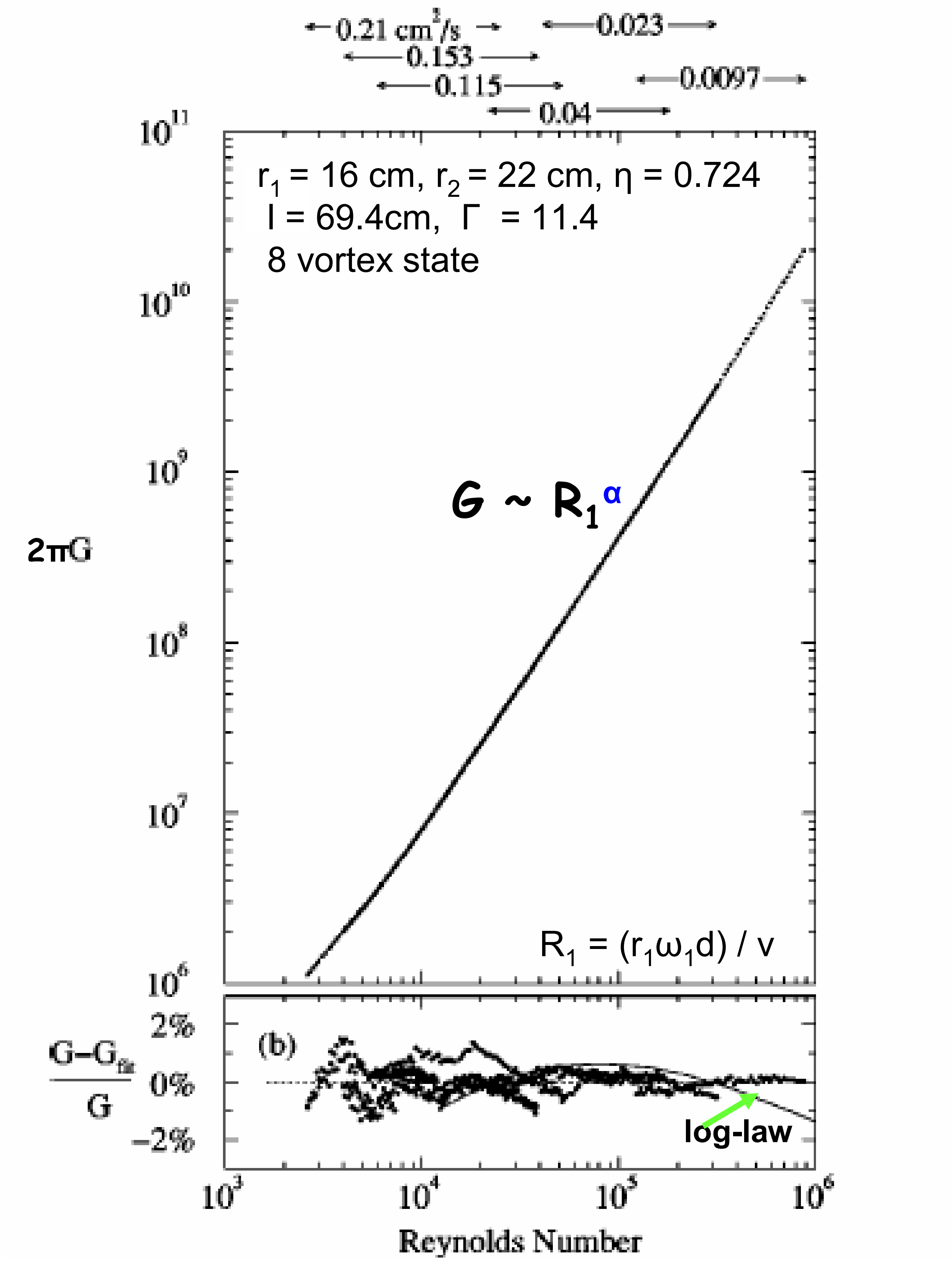
$$\frac{\varepsilon_w}{\nu^3 d^{-4}} = \tilde{\varepsilon}_w = \sigma^{-2}(\eta)\, Ta\, (N^\omega - 1)$$

$$\text{with} \quad \sigma(\eta) = \left(\frac{r_a}{r_g}\right)^4 = \left(\frac{(1+\eta)/2}{\sqrt{\eta}}\right)^4$$

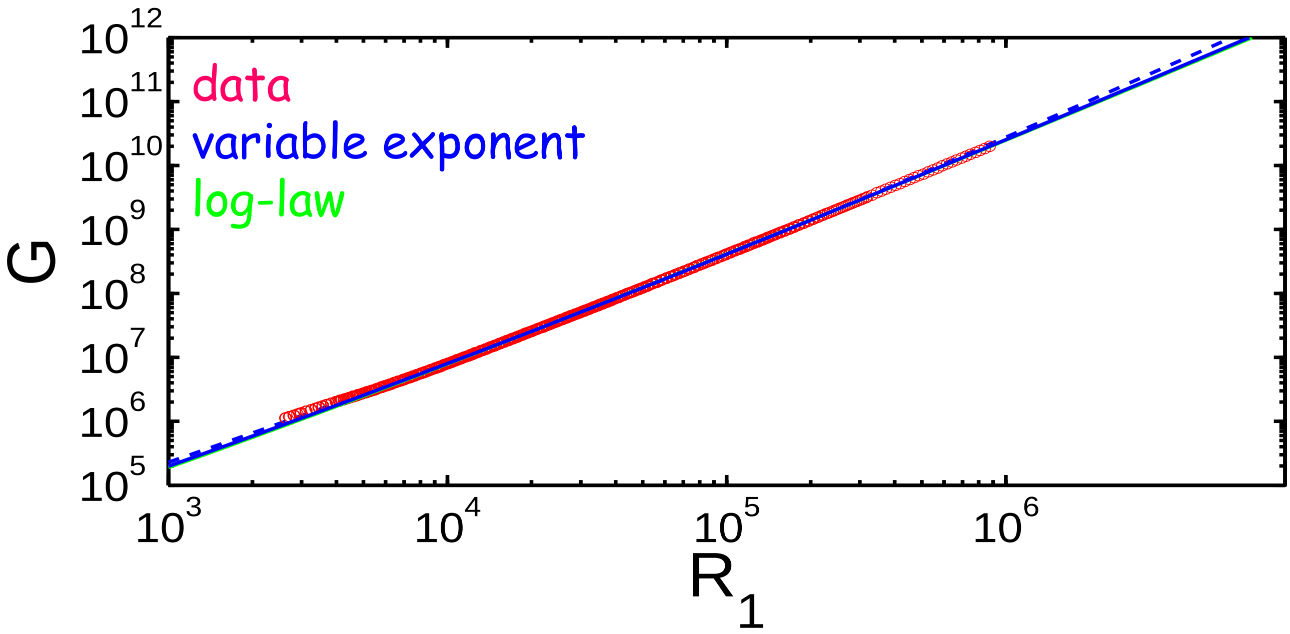
$$\text{and} \quad Ta = \sigma \left(\frac{r_a}{r_1}\right)^2 \cdot \left(\frac{r_1 \omega_1 d}{\nu}\right)^2 = \frac{r_a^6}{r_g^4 r_1^2} \cdot R_1^2$$

Torque 2πG versus R_1 in TC

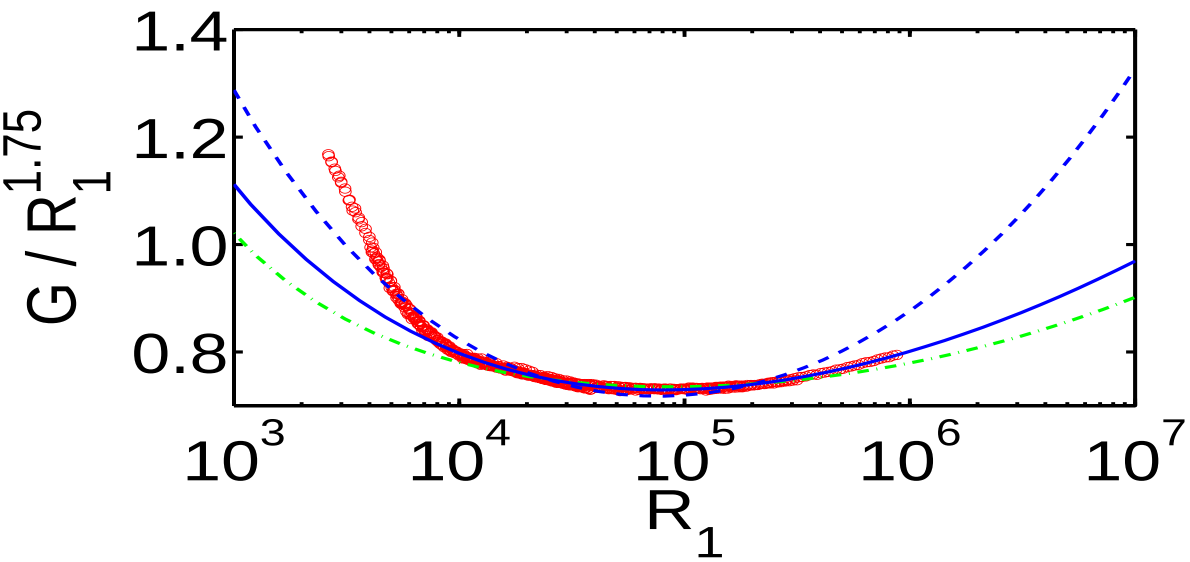
Lewis and Swinney, PRE 59, 5457 (1999)



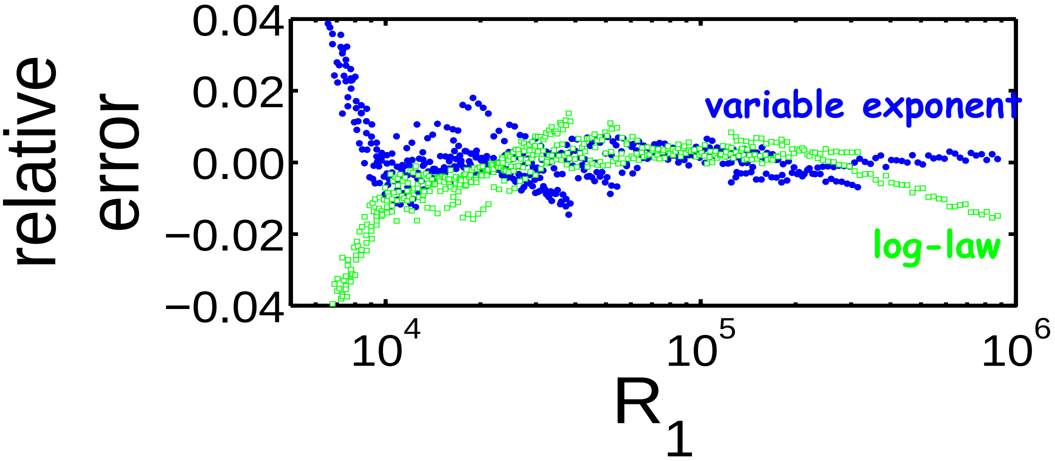
Torque $2\pi G$ versus R_1

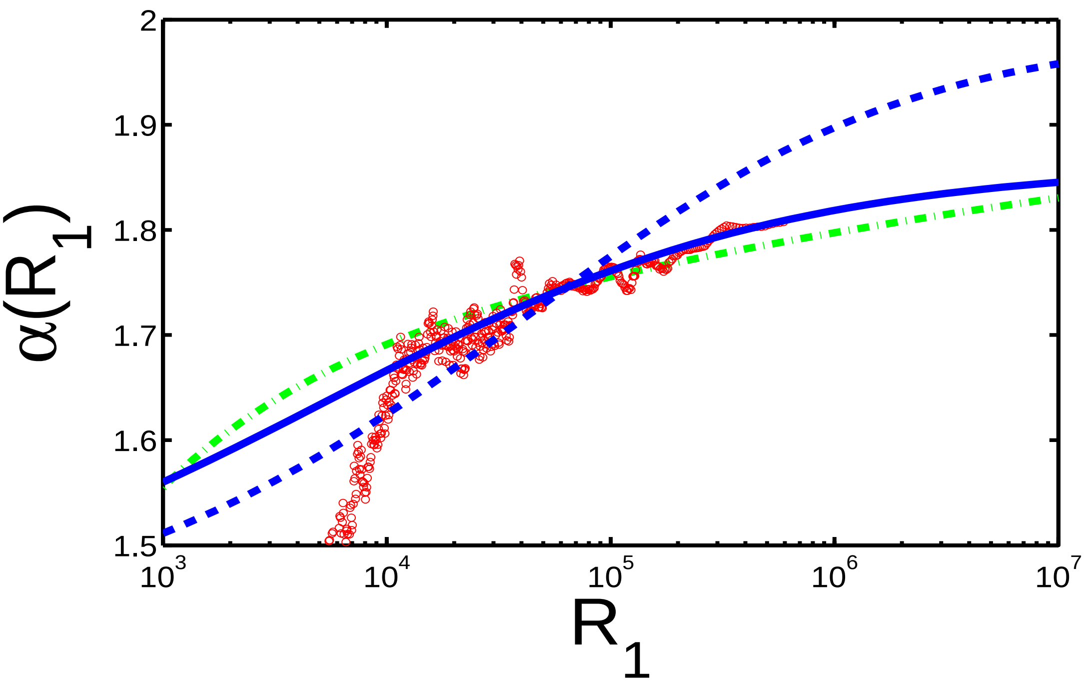


Reduced torque

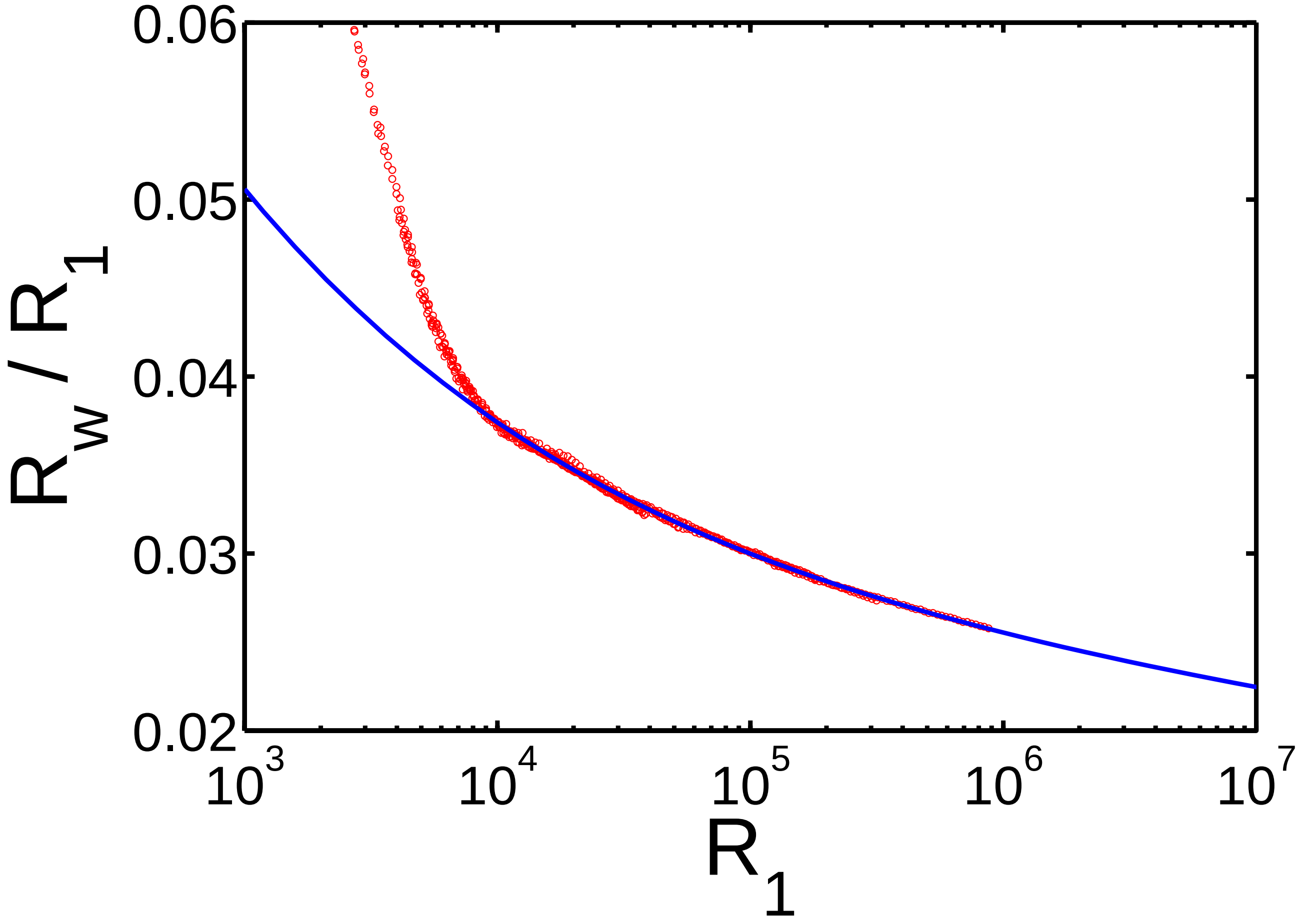


B.Eckhardt, SGn, D.Lohse, 2006


relative error
0.04
0.02
0.00
−0.02
−0.04
variable exponent
log-law
10^4
10^5
10^6
R_1


α(R_1)
2
1.9
1.8
1.7
1.6
1.5
10^3
10^4
10^5
10^6
10^7
R_1

Scaling of the wind Reynoldsnumber R_w



„wind“ = u_r-amplitude increases less than control parameter $R_1 \sim u_\varphi$

$$R_w \sim R_1^{\,1-\xi}$$

$$\xi \approx 0.10\text{-}0.05$$

Cf. Eckhardt, SGn, Lohse, EPJ B18, 541 (2000): $\xi = 0.06$

u_φ-profiles measured by Fritz Wendt 1933

r_2 = 14.70cm, l = 40cm, Γ = 8.5 / 18 / 42

r_1 = 10.00cm / 12.50cm / 13.75cm

η = 0.68 / 0.85 / 0.94

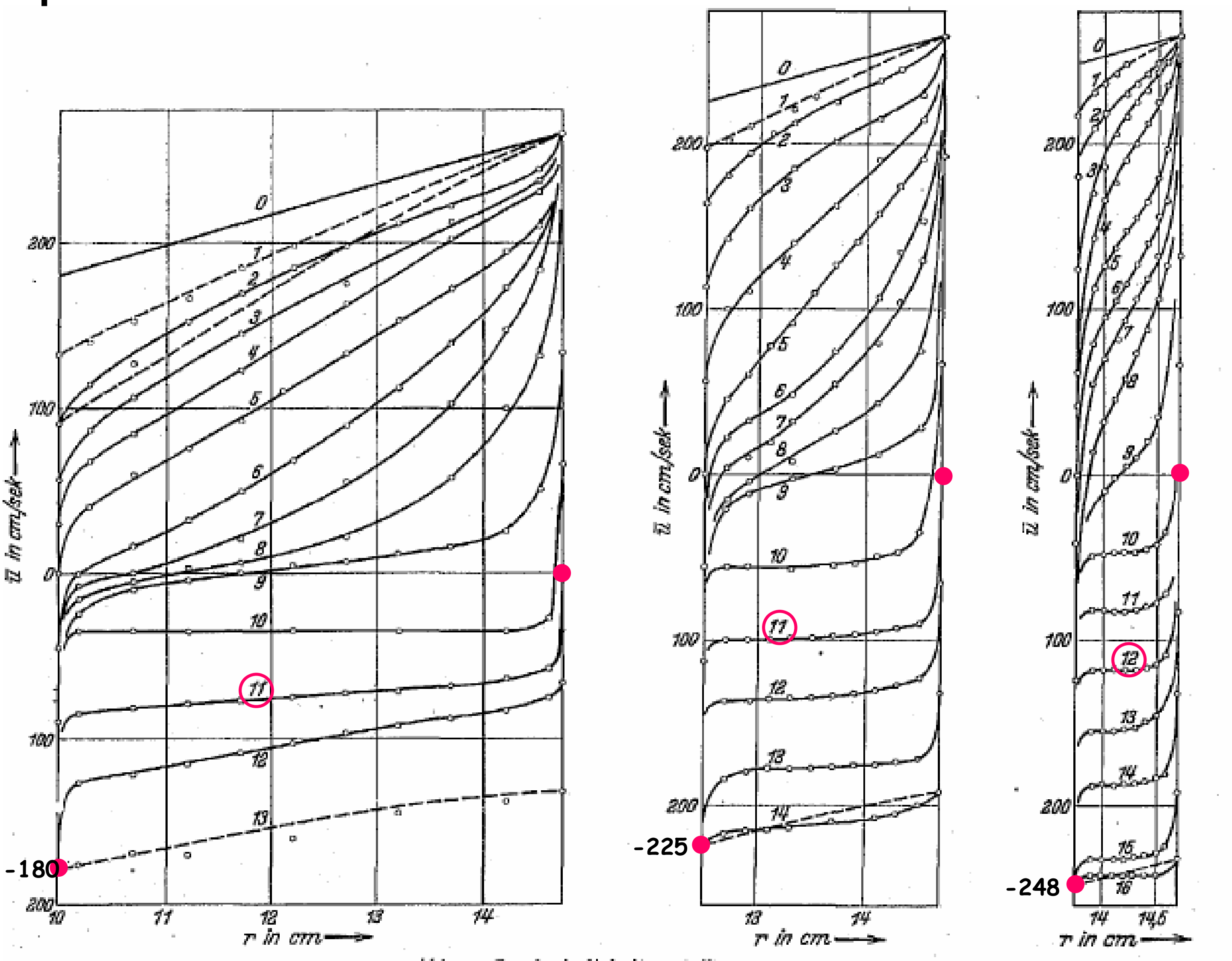
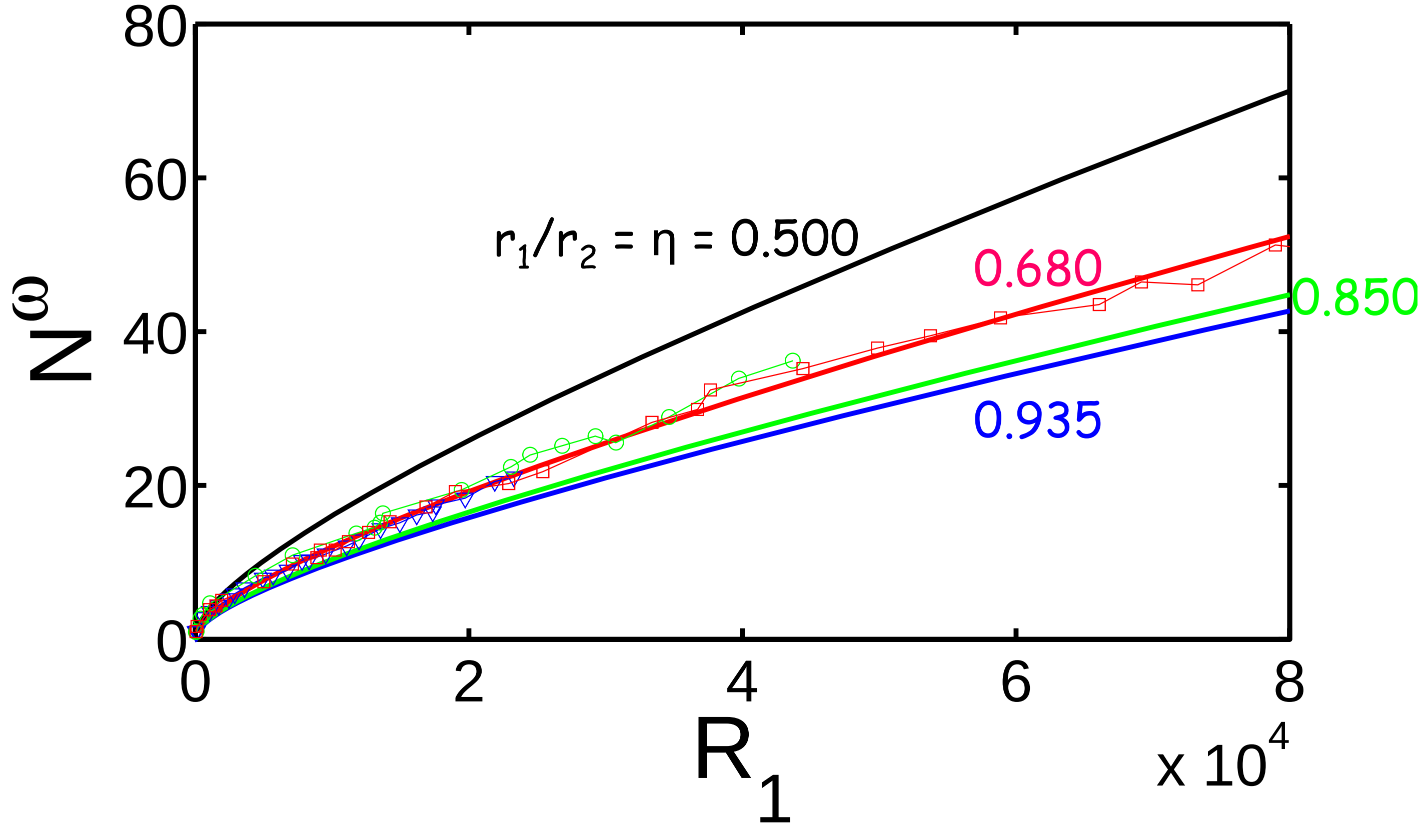


Abb. 7. Geschwindigkeitsverteilungen.

$R_1=8.47 \cdot 10^4$	$4.95 \cdot 10^4$	$2.35 \cdot 10^4$
$N_\omega=M_1/M_{lam} \approx 52$	≈ 37	≈ 21
R_w ≈ 2 670	≈ 1 580	≈ 820
δ/d ≈ 1 / 100	≈ 1 / 80	≈ 1 / 58

η dependence of N^{ω}



Data **Δ 0.935**

○ 0.850

□ 0.680

(η_{LS} = 0.724)

Fritz Wendt, Ingenieurs-Archiv **4**, 577-595 (1933)

Momentum transport in pipe flow

$$
\begin{aligned}
J^{u_z} = J^{u_z}_{lam} N^{u_z} &= \frac{F_{drag}}{\pi a^2 \ell \rho_{fluid}} \\
&= \Delta p / \ell \\
&= (2/a)\ \sigma_{zr}(a) \\
&= 8 \nu a^{-2} U \cdot N^{u_z}
\end{aligned}
$$

flux of axial momentum through $A^{(=)}$:

$$
J^{u_z} = \frac{2}{r} \left[\langle u_r u_z \rangle_{A^{(=)},t} - \nu \partial_r \langle u_z \rangle_{A^{(=)},t} \right]
$$

derivation: average u_z-equation over t and $A^{(=)}$

Excess dissipation rate ε_w

$$\varepsilon = \frac{1}{2}\nu\,\langle(\partial_j u_i + \partial_i u_j)^2\rangle_V$$

$$= U J^{u_z} \; + \; \text{corrections} \;\; O(\ell^{-1})$$

$$\varepsilon_w \; = \; \varepsilon - \varepsilon_{lam} \; = \; U\, J^{u_z}_{lam}\,(N^{u_z} - 1)$$

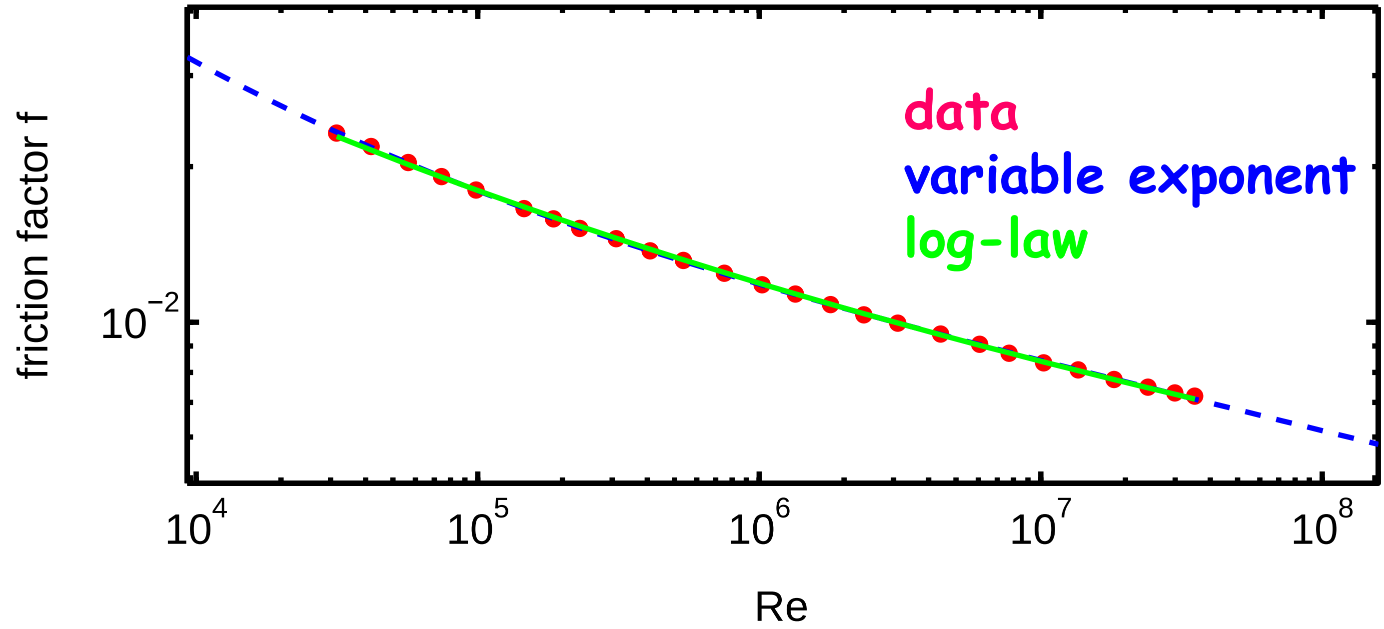
$$\tilde{\varepsilon}_w = 2\; Re^2\;(N^{u_z} - 1)$$

$$\text{width of } u_z\text{-profile } \lambda^{sl} \propto a/N^{u_z}$$

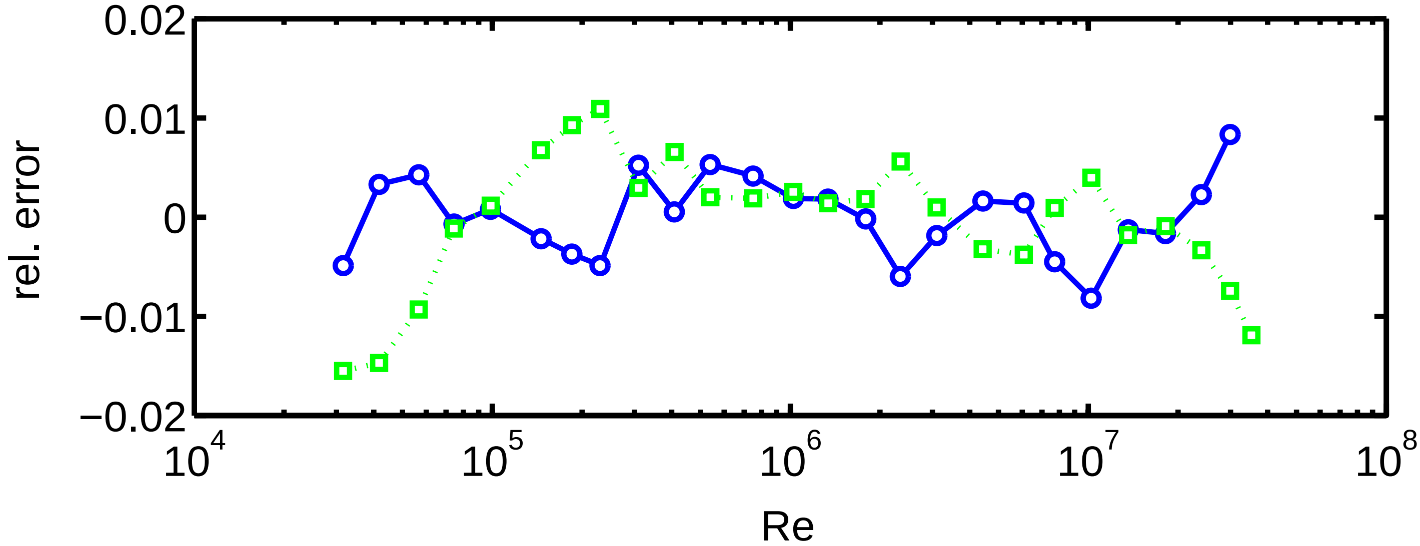
$$\text{width of } u_r\text{-profile } \delta \;\propto a/\sqrt{Re_w}$$

$$\frac{\text{shear stress}}{\text{dynamic stress}} = c_f = \frac{16}{Re}\cdot N^{u_z} \;\propto\; Re^{-1+\beta}$$

Friction factor c_f pipe flow

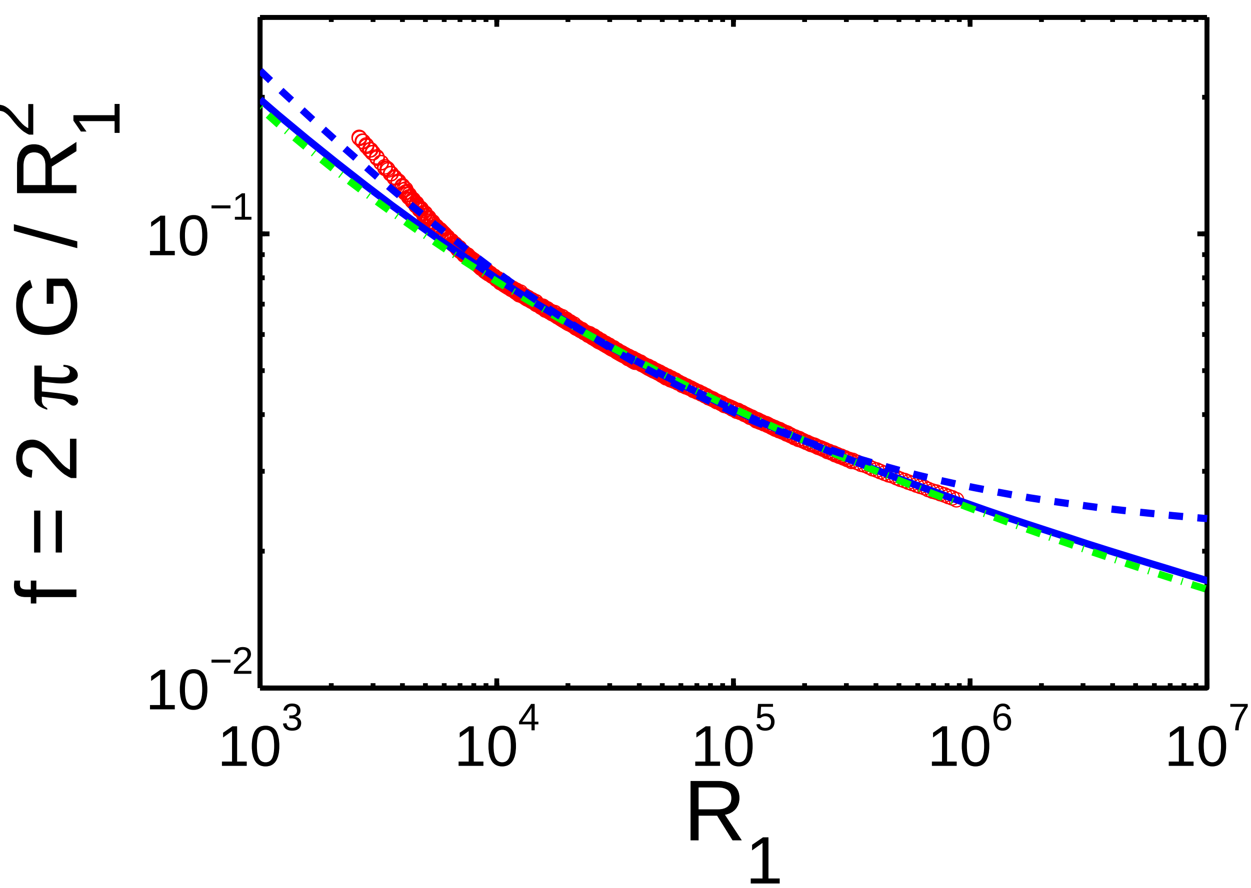


data: Zagarola, Smits, PRL 78, 239 (1997)



B.Eckhardt,SGn,D.Lohse, 2006

(Formal) Friction factor for TC



Summary

1. Exact analogies between RB, TC, Pipe
   quantities $Nu = Q/Q_{lam}$, $M_1/M_{1,lam}$, $\Delta p/\Delta p_{lam}$ or c_f
   Navier-Stokes based

2. Variable exponent power laws $M_1 \sim R_1^{\alpha}$ with $\alpha(R_1)$ etc
   due to varying weights of BLs relative to bulk

3. Wind BL of Prandtl type, width $\sim 1/\sqrt{Re_w}$,
   time dependent but not turbulent, explicit scale L
   transport BL width $\sim 1/Nu$

4. N, N^{ω}, N^{u} as well as ε_w decomposed into BL and bulk
   contributions and modeled in terms of Re_w

5. Exact relations between transport currents N
   and dissipation rates $\varepsilon_w = \varepsilon - \varepsilon_{lam}$:
   $\varepsilon_w = Pr^{-2} Ra (Nu-1), = \sigma^{-2} Ta (N^{\omega}-1), = 2Re^2(N^{u}-1)$

6. Perspectives:

Verify/improve by higher experimental precision.
Explain large Ra or (in TC) R_1 behaviour, very small η.
Measure/calculate profiles $\omega(r)$, u_r, u_z, etc and N's, ε_w's.
Determine normalized correlations/fit parameters c_i.
Include non-Oberbeck-Boussinesque or compressibility.