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High Rayleigh Number Convection

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Effect of the Earth's Corolis force on the large-scale circulation of turbulent Rayleigh-Benard convection

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Effect of the Earth's Coriolis force on the large-scale circulation of turbulent Rayleigh-Bénard convection

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Physics of Fluids., submitted.

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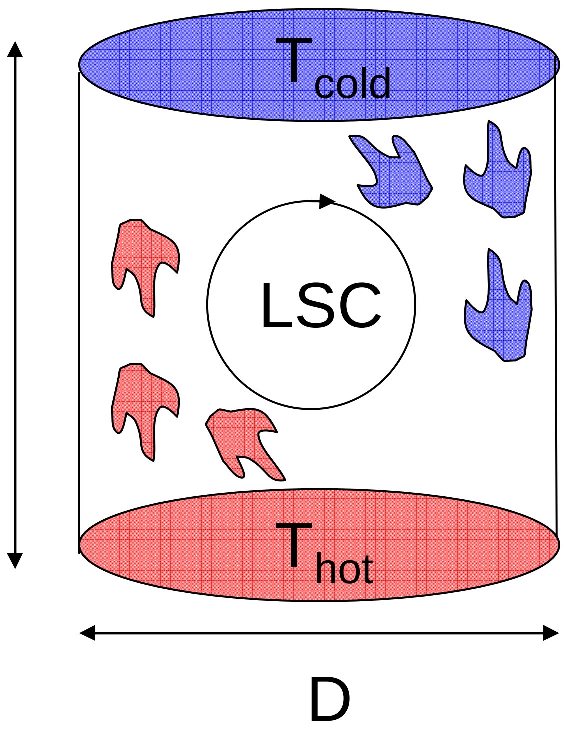
Turbulent Rayleigh-Bénard convection in cylindrical samples of aspect ratio 1

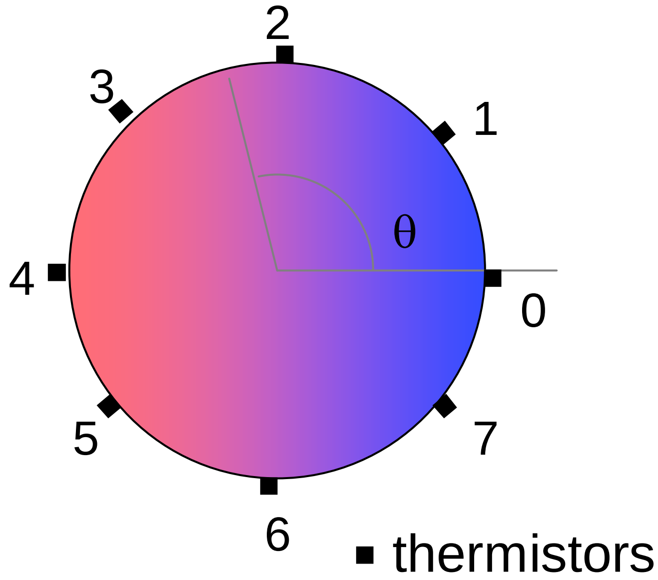
aspect ratio $\Gamma = D/L = 1$
Prandtl number Pr = ν/κ = 4.4 (water)
Rayleigh number R = $g\alpha\Delta T L^3/\kappa\nu = 3 \times 10^8$ to 10^{11}

2 samples:
L = 25 cm
&
L = 50 cm



$\Delta T = T_{hot} - T_{cold}$

Finding the orientation θ_0 of the large-scale circulation (LSC)

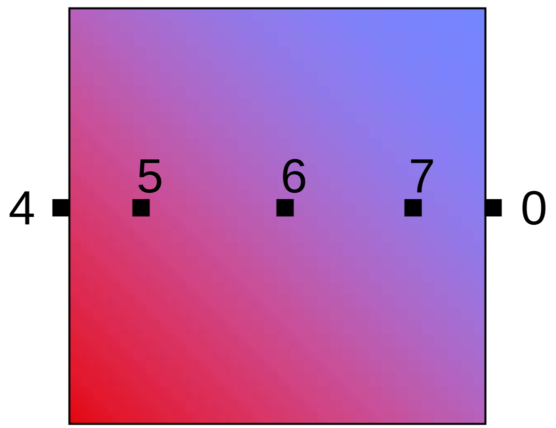


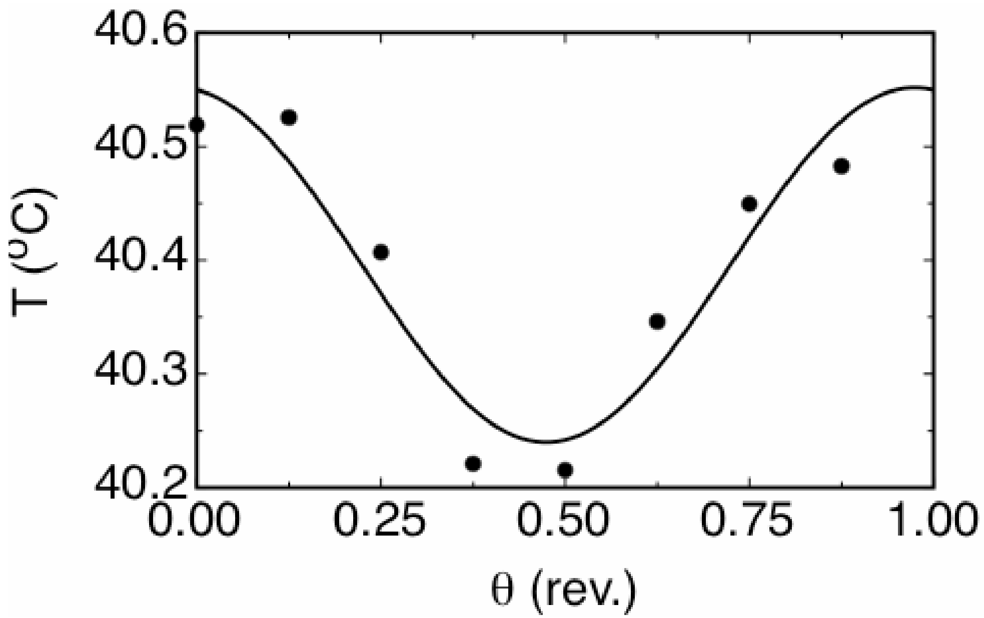
$$T(t) = T_0 + \delta \cos(\theta - \theta_0)$$

T_0 = average temperature

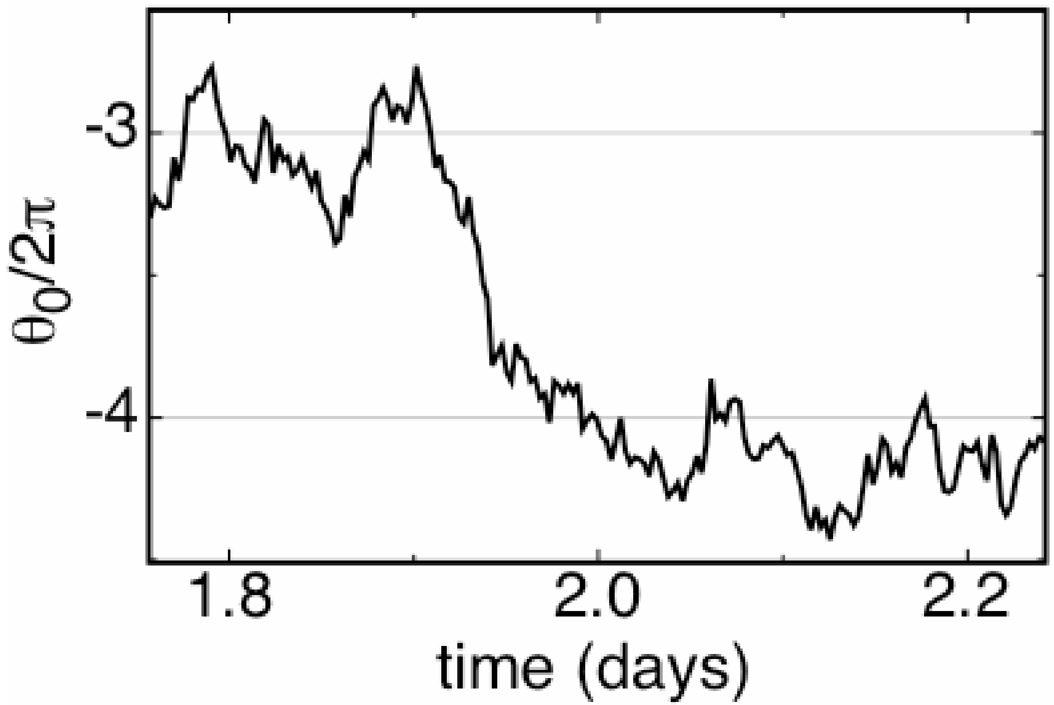
δ = amplitude of LSC

θ_0 = orientation of LSC



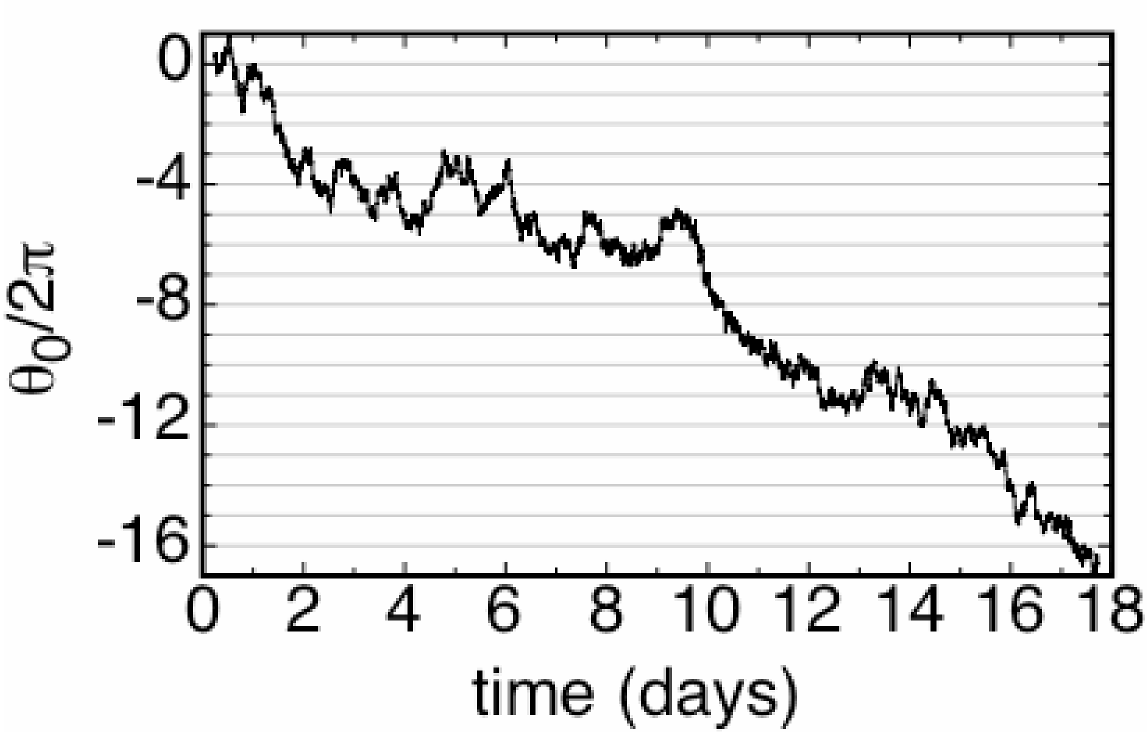


Revolutions



R = $9x10^{10}$, large sample

Net Rotation



R = 9×10^{10}

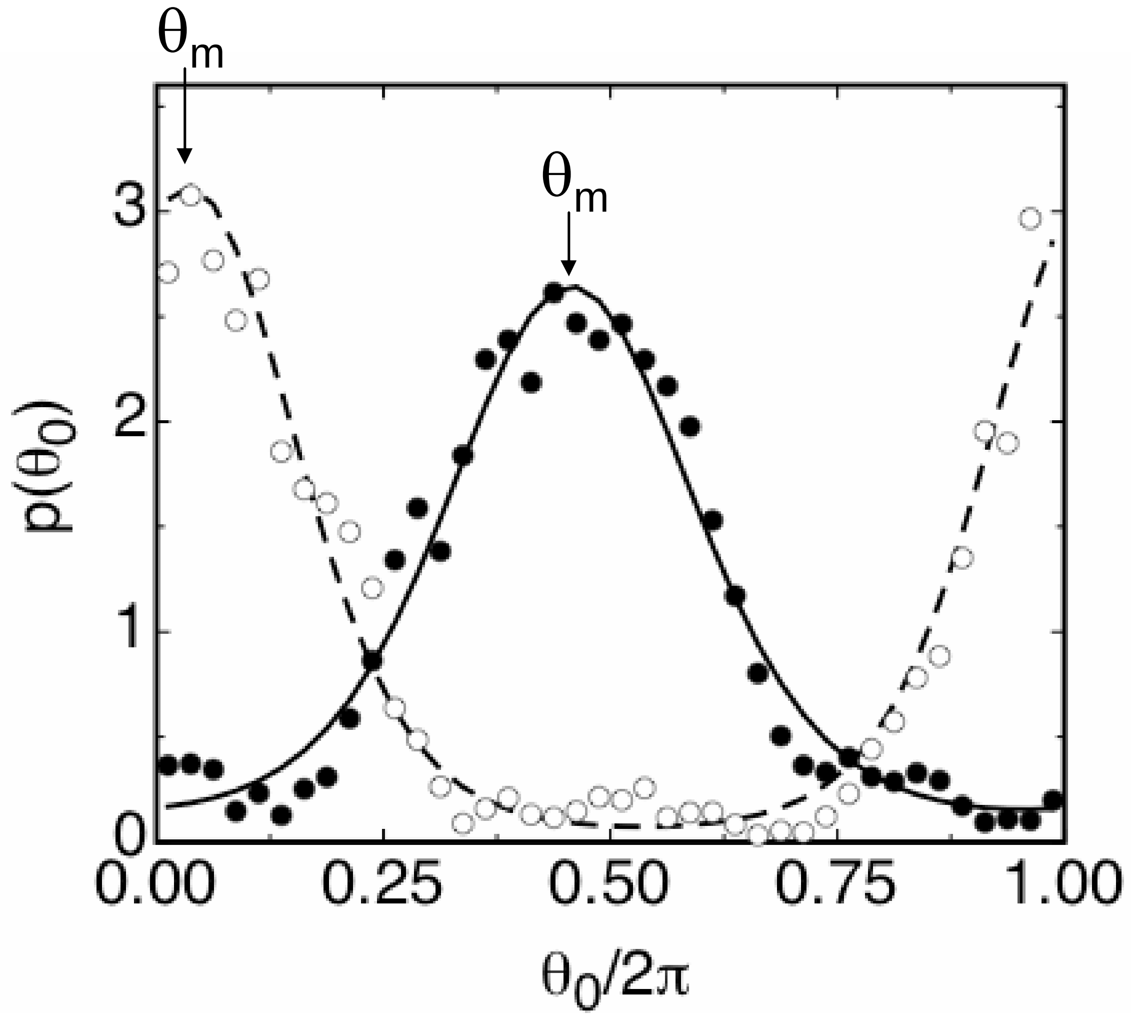
revolutions: 439

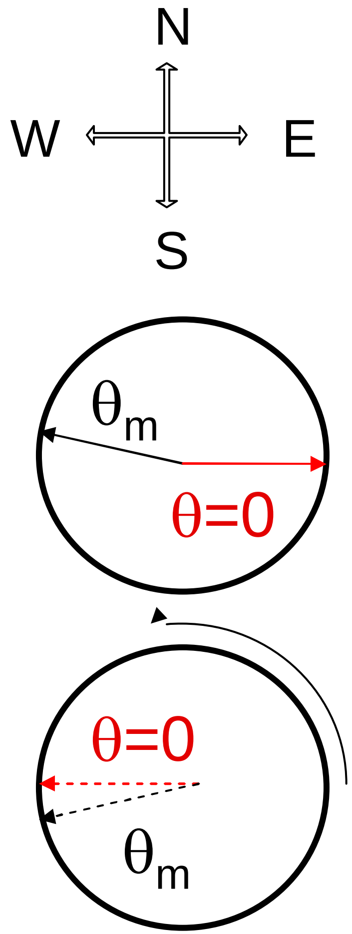
net rotation: -77 rev.

running time: 258 days

net rate: 0.30 +/- 0.08 rev./day

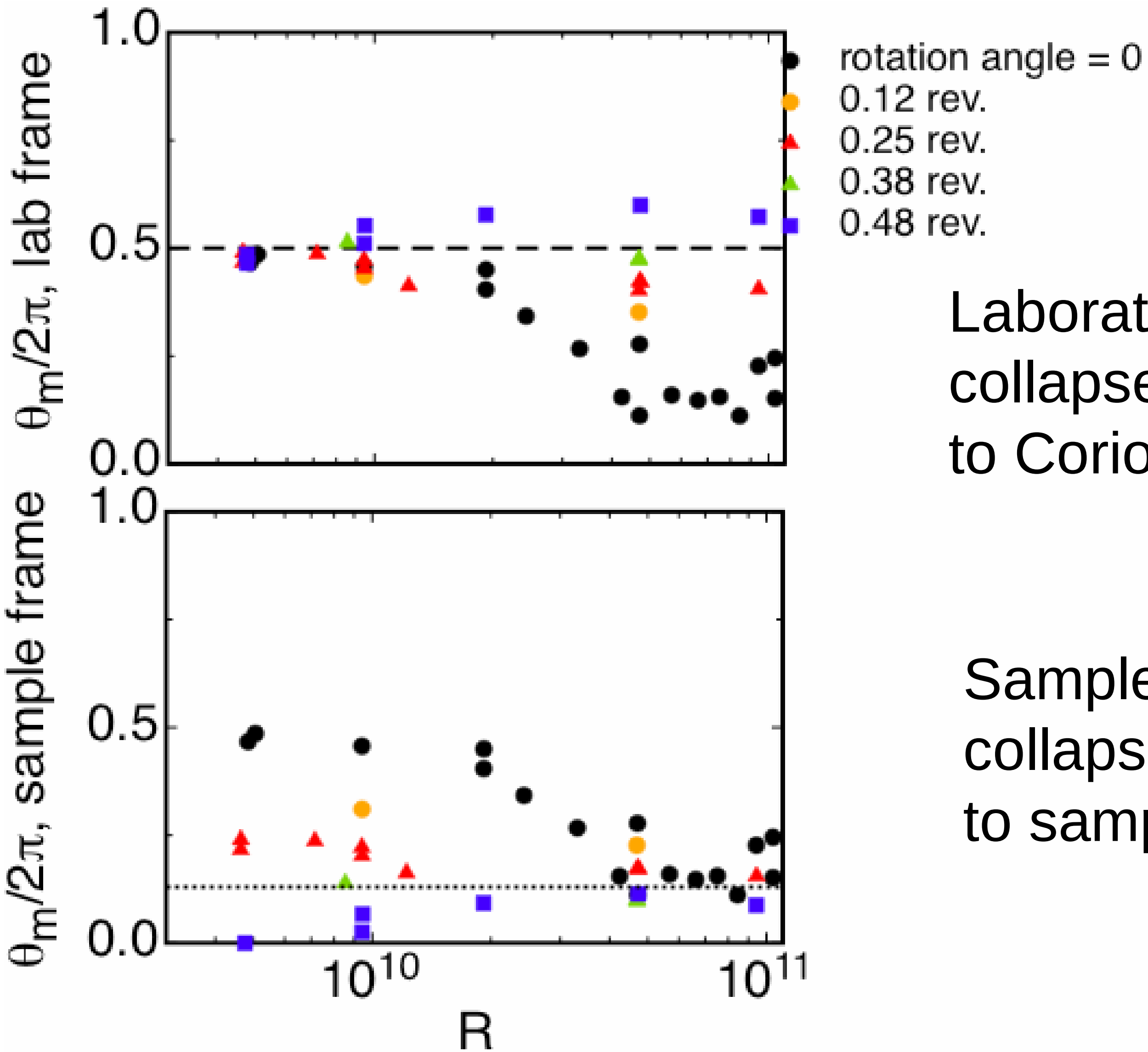
Preferred Orientation





R = 9×10^9, large sample

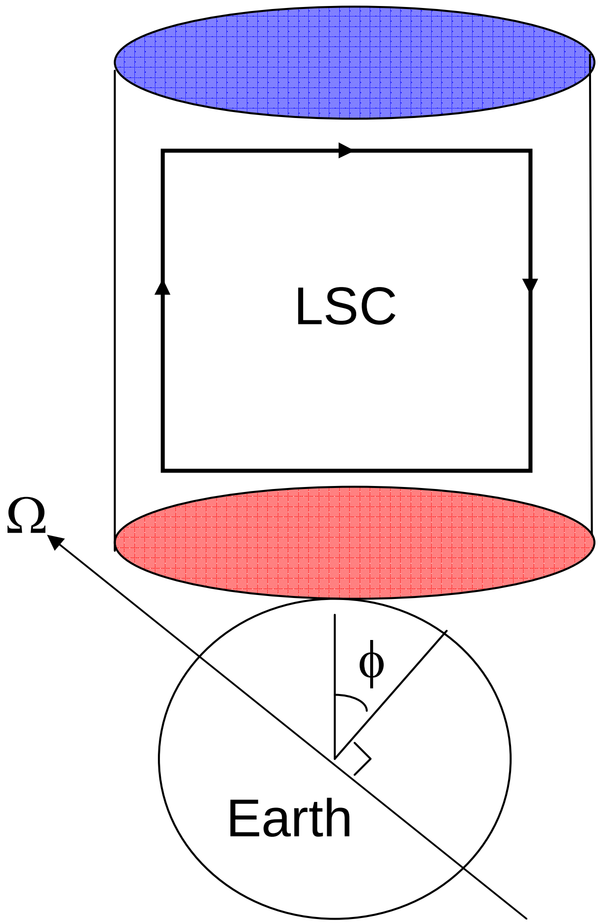
Preferred Orientation vs. R



Laboratory frame: data collapse at small R due to Coriolis force.

Sample frame: data collapse at large R due to sample asymetry.

Coriolis force model: Langevin equation



$\phi = 34^o$ in Santa Barbara

Coriolis force:

$$\frac{d\vec{u}}{dt} = -2\left(\vec{\Omega} + \frac{d\vec{\theta}_0}{dt}\right) \times \vec{u} - \frac{d^2\vec{\theta}_0}{dt^2} \times \vec{r}\,.$$

Coriolis force on vertical legs causes θ_0 to align towards west

Coriolis force on horizontal legs causes net rotation

$$\dot{\theta}_0 \approx 2\Omega\cos\phi\sin\theta_0 - \Omega\sin\phi + f(t)$$

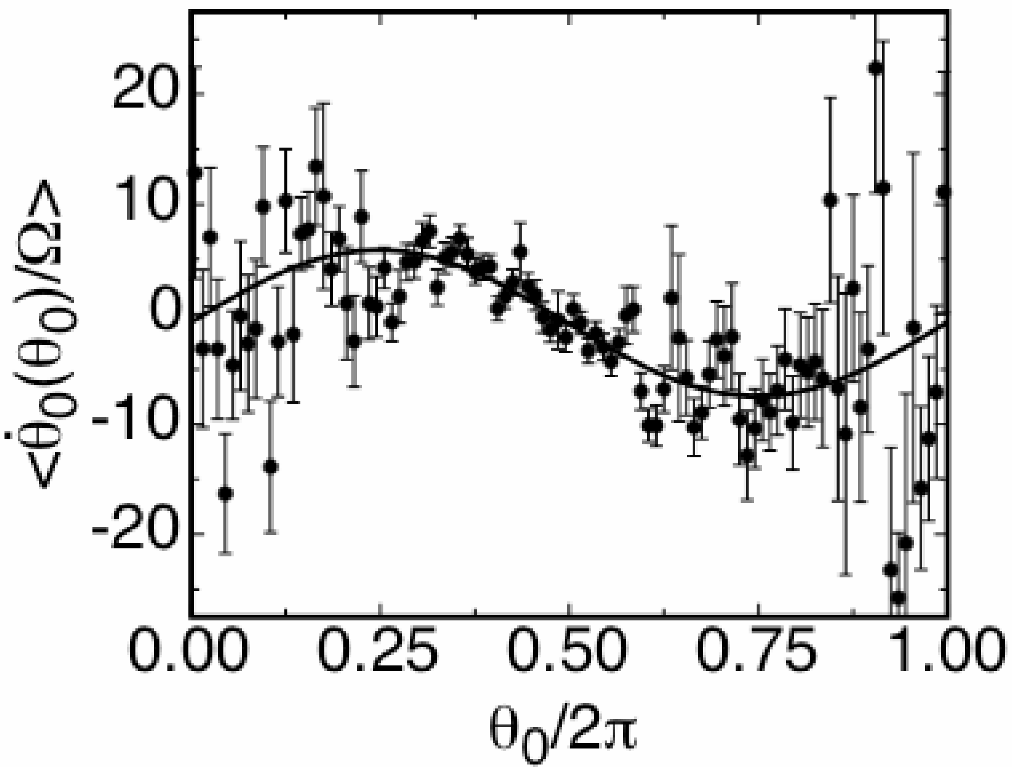
diffusive noise term

assumed steady state ($d^2\theta_0/dt^2 = 0$)

Fitting the preferred orientation term

$$\langle \dot{\theta}_0(\theta_0) \rangle_t = 2a\Omega \cos\phi \sin\theta_0 - \Omega \sin\phi$$

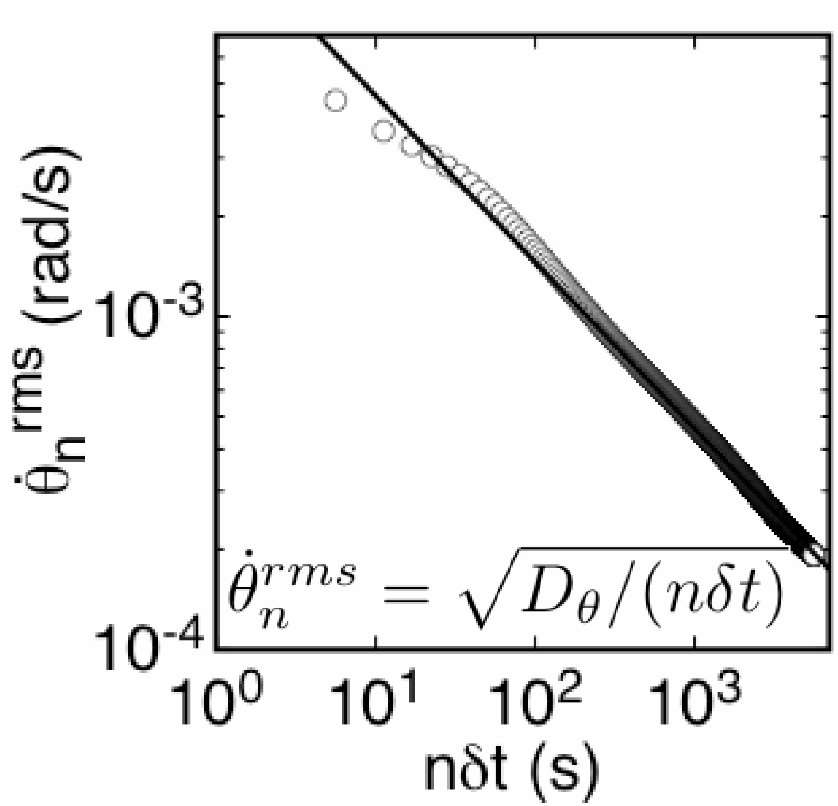
fit parameter

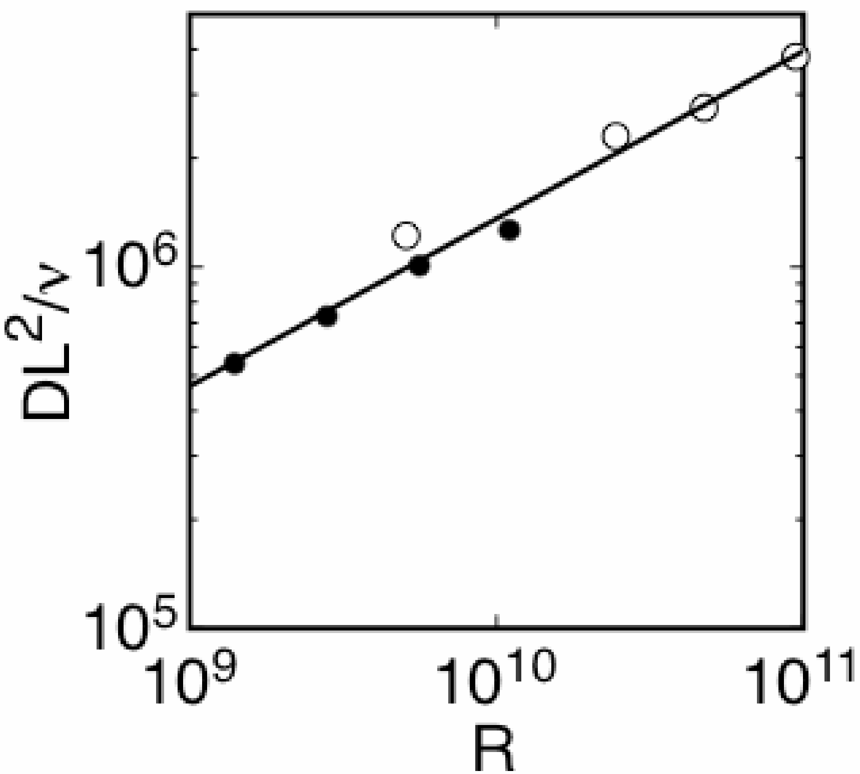


R = 5×10^9 , large sample

Diffusive noise

$$\dot{\theta}_n \equiv \frac{\delta\theta_0}{n\delta t}$$

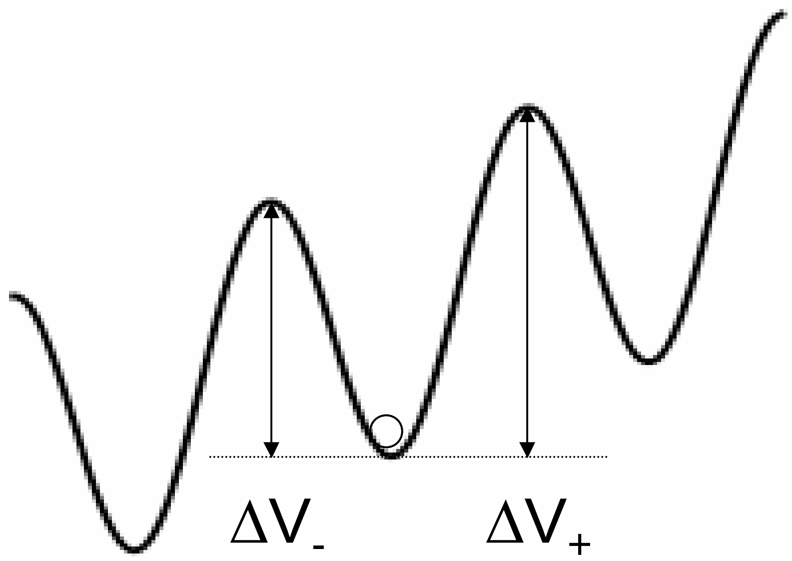




R = 5x10^9 , large sample

Diffusion in a potential well

$$V = -\int \dot{\theta}_0 d\theta_0 = 2\Omega \cos\phi \cos\theta_0 + \Omega\theta_0 \sin\phi$$



Arrhenius-Kramers problem

Using the Coriolis-force potential and the measured diffusivities, we can write the

Fokker-Planck equation

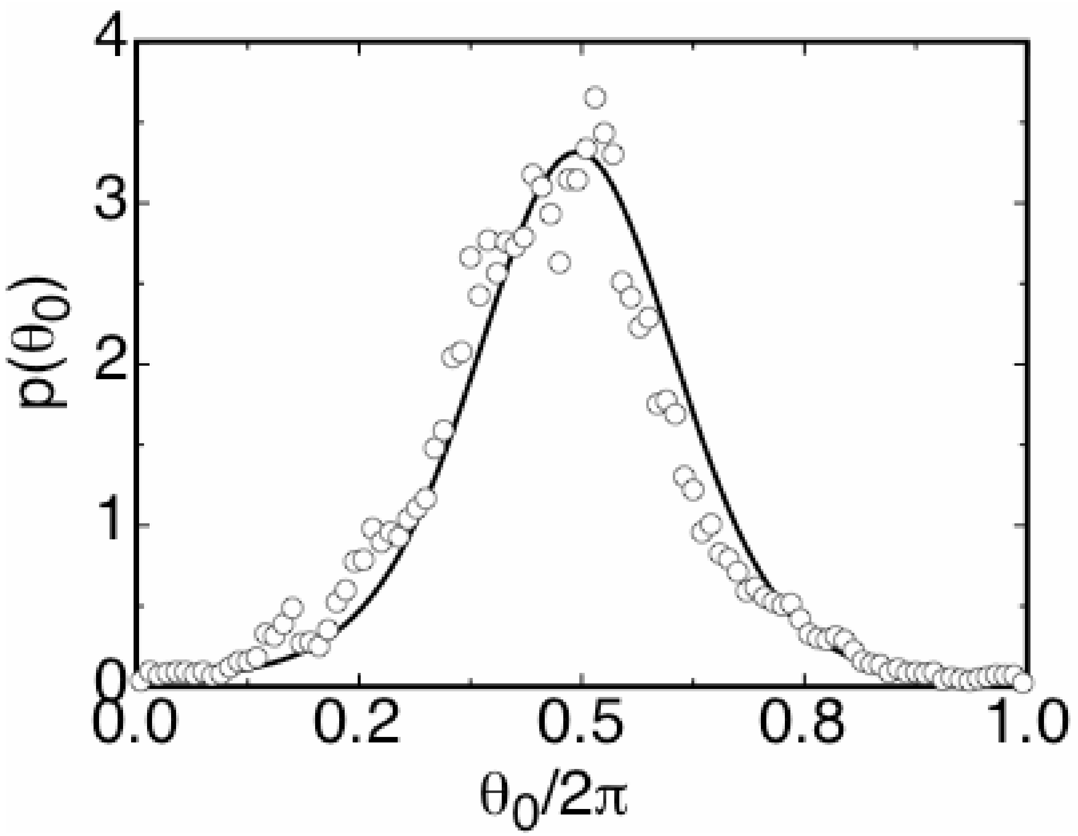
$$\frac{dp(\theta_0)}{dt} = \nabla[-p(\theta_0)\dot{\theta}_0(\theta_0) + D_\theta \nabla p(\theta_0)]$$

Assuming a steady-state and integrating yields

$$-p(\theta_0)\dot{\theta}_0(\theta_0) + D_\theta \nabla p(\theta_0) = -\frac{\omega_\mathcal{R}}{2\pi}$$

which can be numerically integrated to obtain $p(\theta_0)$ and ω_R with the requirement that $p(\theta_0)$ is normalized .

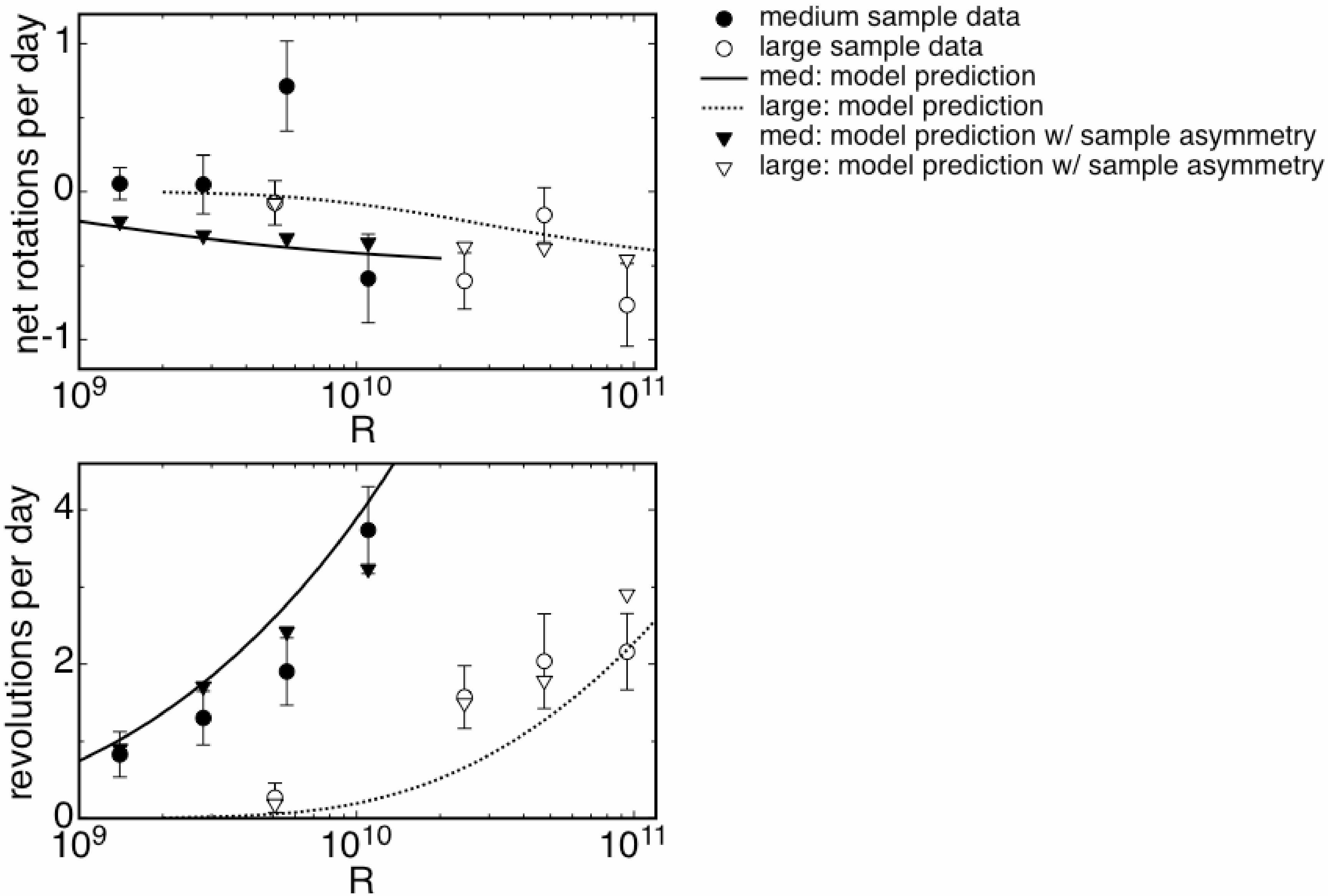
Probability distribution of θ_0



No adjustable parameters !

R = 5×10^9 , large sample

Net rotation and revolutions



Summary

- We found the meandering of the LSC to be diffusive
- We observed a slow net rotation of the LSC.
- We observed a preferred orientation of the LSC fixed in the laboratory frame in the west.
- We modeled the effect of the Coriolis force on the LSC and were able to accurately calculate the size of the above effects, as well as $p(\theta_0)$ and the rate of revolutions.

Asymmetry of the top plate cooling system