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on
High Rayleigh Number Convection

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Cascades, Decoherence & Plumes
in turbulent thermal convection

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Cascades, Decoherence and Plumes in Turbulent Thermal Convection

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Several issues in turbulent convection

- **Heat transport**

- **Statistics of turbulent fluctuations**

- **Flow structure and dynamics**

- **Coherent structures**
Specific issues addressed in this talk:

- **Cascades of velocity and temperature fluctuations**
  Measurements of real-space velocity and temperature structure functions

- **De-coherence of LSC:**
  Cessations and Flow mode transitions

- **Plumes:**
  Extraction and characterization of plumes as individual geometrical and thermal objects.
Part 1

Cascades of velocity and temperature fluctuations

A long-standing issue:

What is the mechanism that drives the velocity and temperature cascades in turbulent thermal convection, or, in general, buoyancy-driven thermal turbulence?

Different dynamics may be manifested as different scaling laws for the velocity and temperature structure functions.

What are the true scaling exponents of the structure functions?
Bolgiano-Obukhov Scaling (1959)

\[ \eta \ll \ell \ll L \]

\[ v_\ell = v(x + \ell) - v(x) \]

\[ S_p(\ell) = \langle v^p_\ell \rangle \]

\[ T_\ell = T(x + \ell) - T(x) \]

\[ R_p(\ell) = \langle T^p_\ell \rangle \]

Large scale

Buoyancy Effect is more important than energy dissipation

Small scale

\[ \ell < \ell_B \]

\[ S_p(\ell) \sim \ell^{p/3} \]

\[ R_p(\ell) \sim \ell^{p/3} \]

K41

\[ \ell > \ell_B \]

\[ S_p(\ell) \sim \ell^{3p/5} \]

\[ R_p(\ell) \sim \ell^{p/5} \]

BO59
Scaling behavior in convective thermal turbulence: previous works

Many studies have been made on this subject.

Procaccia & Zeitak, RPL(1989); L’vov, PRL (1991)
Wu, Kadanoff, Libchaber & Sano, PRL(1990);
Castaing, PRL (1990)
Shraiman & Siggia, PRA(1990);
Grossmann & Lohse, PRL(1991);
Tong & Shen PRL(1992);
Brandenburg PRL (1992); Yakhot, PRL (1992);
Kerr, JFM(1996)
Ashkenazi & Steinberg, PRL(1999);
Glazier et al. Nature (1999), Mashiko,Tsuji, Mizuno & Sano, PRE(2004);
Skrbek, Niemela, Sreenivasan & Donnelly,Nature (2000), PRE(2002);
Calzavarini, Toschi & Tripiccione, PRE (2002).
Zhou & Xia, PRL(2001) Shang & Xia, PRE(2001);
Calzavarini, Toschi & Tripiccione, PRE(2002);
Camussi & Verzicco, Euro. J. Mech. (2004);

Several theoretical models give BO scaling and many experimental and numerical results found apparent BO scaling exponent

But there are also many theoretical arguments and numerical evidences against it
In the studies that appear to have observed BO59, some are based on very limited scaling range or data precision, while many others based on data obtained in the time/frequency domain. All theoretical predictions are in space domain.

To determine the true scaling properties of the temperature and velocity fields direct real-space measurements of the structure functions are needed.
Experimental Setup:

Center: few plumes

Sidewall: plume dominant region

Two regions may have different scaling behavior!

Velocity measurement

Ra = 7x10^9, Pr = 4.3

PIV vector maps: 7500
(1 hour with data rate 2.2 Hz)

Spatial resolution: 0.66 mm

Temperature measurement

Ra = 1x10^{10}, Pr = 4.3

Step Motor

Wheatstone Bridge

Lock-in Amplifier

Lock-in Amplifier

Dynamic Signal Analyzer

l: from 0.5 mm to 90 mm
Each l: Data rate=32Hz, Data length: 1,152,000
PIV can acquire a large amount of data for statistical calculation over a reasonable time frame.

**Center:** 61x61x7500 data points

- # of velocity differences:
  - 24,700,000 for smallest r in inertial region.
  - 12,600,000 for largest r in inertial region.

**Sidewall:** 61x30x7500 data points

- # of velocity differences:
  - 15,100,000 for smallest r in inertial range.
  - 3,720,000 for largest r in inertial region.
Definition of structure functions (SF)

Longitudinal SF of vertical velocity

\[ S_{p}^{L,w}(r) = \left\langle \left| w(x, z + r) - w(x, z) \right|^p \right\rangle \]

Transverse SF of vertical velocity

\[ S_{p}^{T,w}(r) = \left\langle \left| w(x + r, z) - w(x, z) \right|^p \right\rangle \]

Longitudinal SF of horizontal velocity

\[ S_{p}^{L,u}(r) = \left\langle \left| u(x + r, z) - u(x, z) \right|^p \right\rangle \]

Transverse SF of horizontal velocity

\[ S_{p}^{T,u}(r) = \left\langle \left| u(x, z + r) - u(x, z) \right|^p \right\rangle \]
Data convergence and SFs in cell center

velocity

\[
\begin{align*}
(\delta u_r)^p(\delta u_r) & \\
\frac{\delta u_r}{\sigma_{\delta u_r}} & \\
10\eta & \quad L
\end{align*}
\]

\[
\begin{align*}
S_{\tilde{T}_r}(r) & \\
R_{\tilde{T}_r}(r)
\end{align*}
\]

\[
\begin{align*}
\eta \approx 0.4 \text{ mm} \\
\ell_B \approx 5 \text{ mm} \\
L = 14.4 \text{ mm}
\end{align*}
\]
SFs in center

velocity

temperature
SFs near sidewall  
Inertial range is longer than in center

**velocity**

**temperature**
### SF exponents

<table>
<thead>
<tr>
<th></th>
<th>Center</th>
<th>Sidewall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\zeta_{p,w}$</td>
<td>$\zeta_{p}$</td>
</tr>
<tr>
<td>1</td>
<td>0.35±0.01</td>
<td>0.35±0.01</td>
</tr>
<tr>
<td>2</td>
<td>0.68±0.01</td>
<td>0.68±0.01</td>
</tr>
<tr>
<td>3</td>
<td>0.99±0.01</td>
<td>1.00±0.01</td>
</tr>
<tr>
<td>4</td>
<td>1.25±0.02</td>
<td>1.27±0.02</td>
</tr>
<tr>
<td>5</td>
<td>1.53±0.03</td>
<td>1.51±0.03</td>
</tr>
<tr>
<td>6</td>
<td>1.81±0.04</td>
<td>1.73±0.04</td>
</tr>
<tr>
<td>7</td>
<td>2.02±0.06</td>
<td>1.96±0.06</td>
</tr>
<tr>
<td>8</td>
<td>2.30±0.10</td>
<td>2.09±0.10</td>
</tr>
</tbody>
</table>

Velocity fluctuations in central region appear to be isotropic.
Statistical characteristics in cell center

Condition for local isotropy:

\[ S_2^T = S_2^L + \frac{1}{2} r \frac{\partial}{\partial r} S_2^L \]

\[ S_2^T = (1 + 0.68 / 2) S_2^L = 1.34 S_2^L \]

\[ w: \quad S_2^T / S_2^L = 1.31 \pm 0.01 \]

\[ u: \quad S_2^T / S_2^L = 1.34 \pm 0.02 \]

Cell center: nearly isotropic.
Scaling behavior in cell center

Velocity: homogeneous and isotropic Navier-Stokes turbulence with intermittency corrections.

Temperature: a passive scalar in homogeneous and isotropic NS turbulence.

\[ \zeta_p = p / 9 + 2 \left[ 1 - \left( \frac{2}{3} \right)^{p/3} \right] \]

\[ \zeta_p = 0.06p + 0.8(1 - 0.63^p) \]
Scaling behavior near sidewall: vertical direction

Neither BO59 nor K41
Dimension model

\[ \frac{\partial \tilde{u}}{\partial t} + \tilde{u} \cdot \nabla \tilde{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \tilde{u} - g \alpha \Delta T \]

Both dissipation and buoyancy influence the cascade.

Assume \( \varepsilon_v \) and \( \alpha g \Delta \) are equally important:

\[ v_\ell^p \sim \ell^\alpha (\alpha g \Delta \varepsilon_v)^\beta = \ell^{\frac{2p}{5}} (\alpha g \Delta \varepsilon_v)^\frac{p}{5} \sim \ell^{\frac{2p}{5}} \]

\[ T_\ell^p \sim \ell^\alpha (\alpha g \Delta \varepsilon_v)^b \varepsilon_\theta^c = \ell^{\frac{3p}{10}} (\alpha g \Delta \varepsilon_v)^\frac{p}{10} \varepsilon_\theta^2 \sim \ell^{\frac{3p}{10}} \]
Scaling behavior near sidewall: vertical direction

High order structure functions are influenced by intermittency effect, low order SFs can better test the dimensional model.
Excellent agreement between our model with experimental results for fractional SFs,
Summary for Part 1

Cell center: Turbulent flow is locally isotropic

Velocity behaves the same as in homogeneous and isotropic NS turbulence. SF exponents may be described by the hierarchy model for velocities (SL94).

Temperature behaves as a passive scalar in NS turbulence. SF exponents may be described by the hierarchy model for passive Scalars (RCBC96).

Sidewall: Turbulent flow is locally anisotropic.

Vertical velocity and temperature follow neither BO59 nor K41. SF exponents may be described by a dimension model treating buoyancy and viscous dissipation on an equal footing.

High order SF deviate from dimensional model due to intermittency.

Test of crossover effect. Constraints on future models

PRL (accepted)
A large-scale-circulation (LSC) exists in RB system

Cioni et al observed azimuthal rotation of the LSC in mercury convection (*JFM* 1997).

Niemela et al found reversals of LSC in helium (*JFM* 2001).

Flow reversal occurs in many turbulent flow systems.

Reversal of the LSC in turbulent convection may be related to natural phenomena such as the reversal of the geomagnetic polarity.
Reversal phenomenon

Two scenarios

- In-plane reversal
- 180° azimuthal rotation

Statistical analysis and physical models:

More recent experiments:
- Sun, Xi & Xia, *PRL* 2005, Xi, Zhou, & Xia (PRE 2006)

Ahlers group found both the in-plane reversal and 180° azimuthal rotation can occur and identified the ‘cessation’ phenomenon.

Xia group found the azimuthal motion of LSC plane consists of net rotations and erratic fluctuations. The residents phenomenon is observed.
Motivation

- The physical pictures in theoretical models appear to apply to unity aspect ratio geometry.

- Can cessations occur in small aspect ratio geometry?

- What happens to the LSC during a cessation?
The PIV measurement

Xi, Zhou and Xia, PRE (2006)
The Multi-thermistor measurement

\[ T_i = T_0 + A \cos\left(\frac{i\pi}{4} - \phi\right) \]

\( A(t) \) measures the flow strength (amplitude of the LSC)
\( \phi(t) \) is the azimuthal position where the hot side of the LSC goes up.

Brown, Nikolaenko, and Ahlers (PRL 2005)
Cessations in $\Gamma = \frac{1}{2}$ convection cell

Setup and parameter range:

*Sapphire cell*

- PIV, Multi-thermistor simultaneously
  \[ Ra = 5.7 \times 10^9, 1.5 \times 10^{10}, 3.3 \times 10^{10}, \text{Pr} = 5.3 \]
- Multi-thermistor
  \[ Ra = 1.4 \times 10^{10}, 1.5 \times 10^{10}, 1.6 \times 10^{10}, 3.0 \times 10^{10}, 3.3 \times 10^{10}, 4.3 \times 10^{10}, 7.2 \times 10^{10} \]
  \[ \text{Pr} = 4.9-5.3 \]

*Copper cell*

- Multi-thermistor, 34-day measurement. \[ Ra = 5.6 \times 10^{10} \]

The same statistical results are obtained from all these measurements
The examples of cessation in $\Gamma = \frac{1}{2}$ cell

PIV result

Muti-thermistor result
The definition of a cessation

Cessation is defined by $V_{LSC} < (V_{LSC})_l$

Duration $\tau_d$ of cessation is defined by $V_{LSC} < (V_{LSC})_h$

$$(V_{LSC})_h \quad (V_{LSC})_l = 0.1 < V_{LSC} >$$

$$(V_{LSC})_l \quad (V_{LSC})_h = 0.3 < V_{LSC} >$$

Cessation is defined by $A < (A)_l$

Duration of cessation $\tau_d$ is defined by $A < (A)_h$

$A_h$ $\quad A_l$

$A_l = 0.1 < A >$

$A_h = 0.3 < A >$
**Thresholds explored**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(V_{LSC})_l$</td>
<td>$(0.05 \sim 0.2)&lt;V_{LSC}&gt;$</td>
</tr>
<tr>
<td>$(V_{LSC})_h$</td>
<td>$(0.2 \sim 0.8)&lt;V_{LSC}&gt;$</td>
</tr>
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Similar statistics are obtained

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<td>$A_l$</td>
<td>$(0.05 \sim 0.2)&lt;A&gt;$</td>
</tr>
<tr>
<td>$A_h$</td>
<td>$(0.2 \sim 0.8)&lt;A&gt;$</td>
</tr>
</tbody>
</table>

Similar statistics are obtained

- PIV, multi-Ra: **466** cessations
- Multi-thermistor, multi-Ra: **1778** cessations
- Single-Ra, 34-day measurement: **1071** cessations

All give similar statistics

The number of cessations identified by the 3 rows of thermistors at different heights differ by less than 3%, and the statistics are all the same.
Examples of cessations, multi-thermistor (middle height)
Examples of cessations, multi-thermistor (bottom row)
$f_c(\Gamma = 1/2, 1/3) / f_c(\Gamma = 1) = 14$

$f_c(\text{PIV}) / f_c(\text{therm}) = 3.7 \sim 6$

Cessations in $\Gamma < 1$ cells occur an order of magnitude more frequent than in $\Gamma = 1$ cells.

$\Gamma = 1$ PIV data is reanalyzed using new definition (96 now vs. 28 in Xi et al, PRE 2006)

Brown and Ahlers, JFM 2006
Orientational change $\Delta \phi$ of LSC before and after a cessation

Uniform distribution in $\Gamma = 1$ cell (Brown and Ahlers, PRL05, JFM 2006; Xi, et al PRE 2006)

Cessations with $|\Delta \phi|$ close to 180 is 6 times more likely to occur than cessations with small $|\Delta \phi|$
Statistics of cessation

Cessation statistics in $\Gamma=1/2$ are similar to the $\Gamma=1$ case, except that of $\Delta\phi$

\[ p(\tau_1/<\tau_1>) = \exp(-\tau_1/<\tau_1>) \]

$\tau_1$ ---- time between successive cessations.

The straight line is:

\[ <\tau_1> = 46 \text{ min}, \quad <\tau_1>_{\text{max}} = 342 \text{ min}, \quad <\tau_1>_{\text{min}} = 0.7 \text{ min} \]

$\tau_d$ ---- duration of a cessation

\[ <\tau_d> = 46 \text{ s}, \quad \text{max} = 150 \text{ s}, \quad \text{min} = 18 \text{ s} \]

The statistics for $t_n$ are consistent with those of “reversals” found by Sreenivasan, Bershadskii and Niemela (PRE2002), and “crossings” by Xi, Zhou and Xia (PRE2006). But not those for $\tau_1$ (no power law for small $\tau_1$).
What happens during the cessation?

\[ \Gamma = 1 \]

Incoherent mean wind

Coherent mean wind
What happens during the cessation?

\[ \Gamma = \frac{1}{2} \]

Incoherent mean wind

coherent mean wind
Three incoherent modes causing cessations

Incoherent flow, radial mode

Incoherent flow, chaotic mode

Incoherent flow, competing mode
The measured time trace of orientation of LSC plane

Positive is counterclockwise

\[ \phi (2\pi) \]

\[ Ra = 5.6 \times 10^{10} \]
Possible scenario for the divergence of $\phi$ obtained at different heights

Flow mode transition between dominant and minor modes

Verzicco and Camussi, (JFM 2003)
More examples of flow-mode transitions

Duration of flow mode transition is less than 100 second.
The orientation trace in cells of different $\Gamma$

- $\Gamma = 1$ (PIV result)
- $\Gamma = 1/2$
- $\Gamma = 1/3$

Graphs showing the orientation $\phi$ over time $(\text{hr})$ for different Ra values:

(a) $\text{Ra} = 8.0 \times 10^8$  
(b) $\text{Ra} = 4.7 \times 10^9$  
(c) $\text{Ra} = 5.3 \times 10^{10}$  
(d) $\text{Ra} = 5.0 \times 10^{10}$
The plots of the traces in polar coordinate

\[ \Gamma = 2.3 \]

(a) \( V_{LSC} \) (cm/s)

\[ \phi \text{ (degree)} \]

(b) \( \Gamma = 1 \)

\[ \Gamma = 1/2 \]

(c) \( \Gamma = 1/2 \)

(d) \( \Gamma = 1/3 \)
The PDF of the orientation of LSC for different aspect ratio

PIV sapphire cell

Multi-thermistor result
Cessations occur an order of magnitude more frequently in small $\Gamma$ cells than in cell with $\Gamma = 1$. The frequency of occurrence has no obvious dependence on $Ra$.

The LSC likes to reverse its direction after a cessation in cells with $\Gamma = 1/2$.

Common features of the statistics properties based on pure-cessation events, rotations/reorientations, reversals, and crossings suggest that all these phenomena may share the same origin.

Constraints for theoretical models (SOC, dynamical systems, stochastic).

A cessation event is a process of de-coherence, or disorganization and reorganization of the LSC.

Flow-mode transitions may have been observed indirectly by temperature measurement.
Part 3

Morphological evolution and geometrical properties of turbulent thermal plumes
Mushroom-like Plumes

- Werne (1994)
- Zhang, Childress, Libchaber (1997)
- Du & Tong (1998)
- Stringano, Pascazio & Verzicco (2006)
- Parodi et al. (2004)
- Toschi (2006)
Sheet-like Plumes

Zocchi, Moses, Libchaber (1990)

Gluchman, Willaime, Gollub (1993)


String-like coherent structure
Haramina and Tilgner (2004)

Sheet-like plumes

Puthenveettil & Arakeri (2005)

Mush-like plumes
How do sheet-like plumes transform into mush-like ones?

This work represents the attempt to quantitatively characterize these coherent structures, which remain largely unexplored.
Experimental setup

**TLC**: Thermochromic liquid crystal microspheres (R29C4W), 50 μm mean diameter.

**Halogen lamp**

Red: 29-29.5°C; Green: 29.5-29.7°C; Blue: 29.7-33°C

Zocchi, Moses & Libchaber (1990); Gluchman, Willaime & Gollub (1993); Du & Tong (1998)
Sheet like plumes were observed near boundary layer.
Extraction of sheet-like plumes
Extraction of sheet-like plumes
Geometric characteristics: Area

4001 plumes extracted from 200 images.

Areas of plumes follow log-normal distribution: both sheet-like and mush-like plumes (Zhou and Xia, PRL 2002).
The lengths of sheet-like plumes also follow log-normal distribution.
Temperature of a plume

Red-Green-Blue (RGB)

Hue-Saturation-Intensity (HSI)

Hue: a measure of the peak wavelength
Saturation: a measure of the color contrast
Intensity: a measure of the total light intensity
Plumes statistics

‘heat’ contained in a plume

\[ Q_p = \sum c_p(T_i) \rho(T_i) S_{pix}(T_i - T_0) \]

Area of a pixel:
\[ S_{pix} = 0.16 \times 0.16 mm^2 \]

‘heat’ contained in a plume also has log-normal distribution.

The distribution of \( Q_p \) is determined by the geometric properties of the plume
Mush-like plumes

2.5 cm from top plate

9.3 cm from top plate

Mush like plumes were observed in bulk region.
**Vortical plumes**

Long-exposure shot gives both temperature and trajectories of the TLC particles.

- 2.5 cm from top plate
- 9.3 cm from top plate

Mush like plumes have intense vortical structures!

Where does vorticity come from?
Formation of mushroom-like plumes: interaction between sheet like plumes

Sheet-like plumes collide and convolute to form swirls that spiral down to become mushroom-lime plumes.
Plume clustering

Plumes merge and cluster as they move upward/downward.

2.5 cm from top plate

9.3 cm from top plate
Morphological evolution of sheet-like to mushroom-like plumes
Simultaneous vorticity & temperature measurement

The fluctuations of vertical vorticity and temperature are strongly correlated.
Vorticity share the same characteristics with temperature rather than with velocity. 

Ra-dependence of vorticity and temperature fluctuations share similar behavior.

PDF of temperature, velocity and vorticity

Temperature: Exponential
Velocity: Gaussian

\[
p(\omega/\omega_{\text{rms}}) = \begin{cases} 10^0 & \omega/\omega_{\text{rms}} = 0 \\ 10^{-1} & \omega/\omega_{\text{rms}} = 8 \\ 10^{-2} & \omega/\omega_{\text{rms}} = 4 \\ 10^{-3} & \omega/\omega_{\text{rms}} = 0 \\ 10^{-4} & \omega/\omega_{\text{rms}} = -8 \\ \end{cases}
\]
The maximum $\omega_{\text{rms}}$ suggests that strong vorticity fluctuations are associated with merging and clustering of plumes.

The mixing zone is roughly the same as that found from the skewness of plus and minus temperature increments.
Summary for Part 3

- Individual sheetlike plumes are extracted as 2D individual geometrical and thermal objects and their area, circumference, and `heat content' are found to all exhibit log-normal distributions.

- As the sheetlike plumes move across the plate they collide and convolute into spiraling swirls. These swirls then spiral downward to become mushroom-like plumes accompanied by strong vertical vorticity.

- The measured vorticity profile reveals a mixing region within which most of plume merging and clustering take place.

- The fluctuating vorticity is found to have the same exponential distribution and scaling behavior as the fluctuating temperature.