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on

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A comparison of constant heat flux and constant temperature thermal convection

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A Comparison between Constant Temperature and Constant Heat Flux Turbulent Thermal Convection

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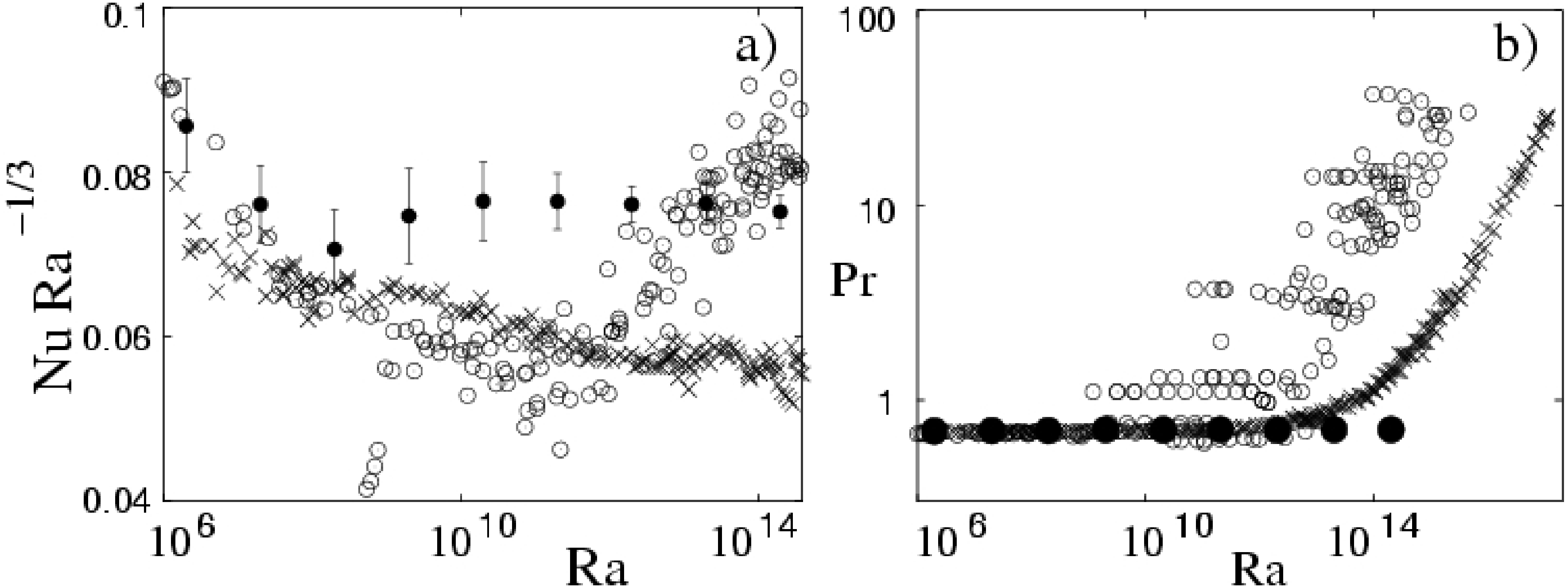
Motivation



• Amati et al. 2005 o Chavanne et al. 2001 x Niemela et al. 2000

For Ra≥10^9 differences of **Nu** between numerical simulations and *"similar"* experiments up to 20%

Motivation

The differences can not be ascribed to:

Prandtl number (Ahlers & Xu 2001, Xia et al. 2002)

Shape of the cell (cylindrical Γ=1/2 in all cases)

Non-Boussinesq effects (irrelevant at Ra$\approx 10^9$)

Sidewall conduction (important only at low Ra)

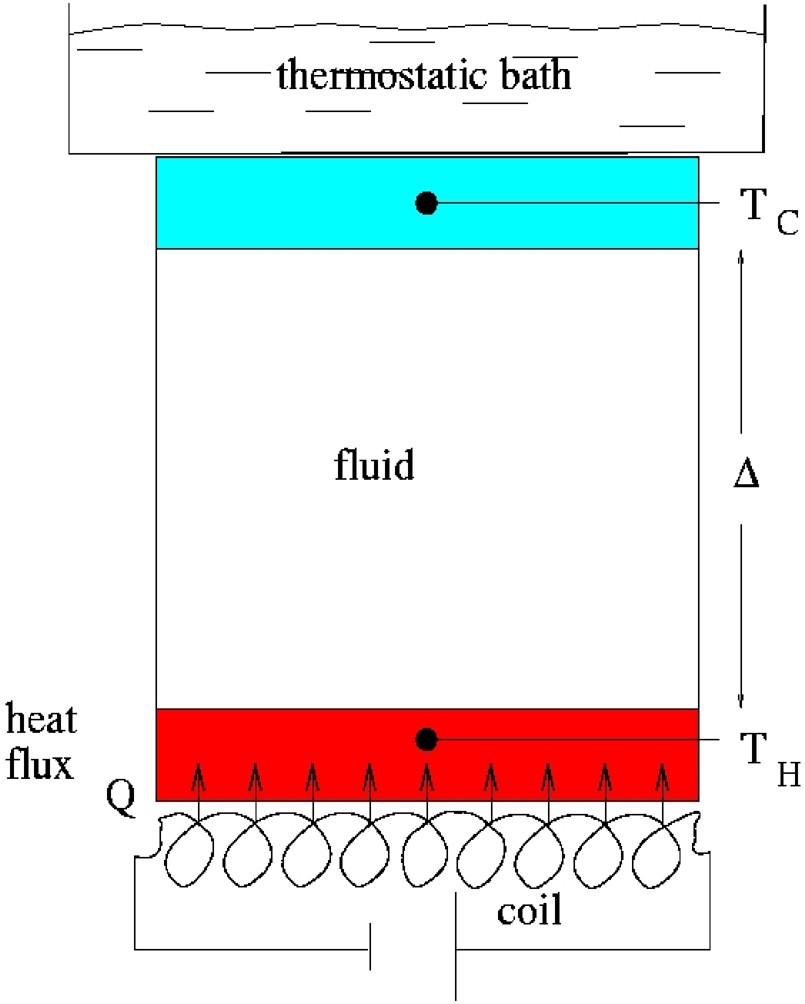
(Ahlers 2001, Roche et al. 2001,Verzicco 2002, Niemela & Sreenivasan 2003)

Plates finite conductivity (corrections too small)

(Chaumat et al. 2002,Verzicco 2004, Chillà et al. 2004, Brown et al. 2005)

Is there a fundamental difference?

Motivation



In most experimental set-ups upper and lower plates are heated and cooled by different methods

The lower plate has a constant heat flux surface and its heat capacity keeps the temperature constant

Motivation

Constant temperature if:

$(\rho\ C\ \lambda)_{\text{plate}} >> (\rho\ C_p\ \lambda)_{\text{fluid}}$

(Schlicting 2000, p.507)

In thermal convection, however, $\lambda_{\text{eff}} = Nu\ \lambda_{\text{fluid}}$ and eventually $(\rho\ C\ \lambda)_{\text{plate}} = (\rho\ C_p\ Nu\ \lambda)_{\text{fluid}}$

Small Biot number $B=(H\ e/\ \lambda)_{\text{plate}} <<1$

In thermal convection, however, $H = Nu\ \lambda_{\text{fluid}}/h$ and

$B = Nu\ \ \lambda_{\text{fluid}}\ e\ /(\lambda_{\text{plate}}\ h)$ (the inverse of the plate correction parameter "X")

In Niemela et al. (2000) and Chavanne et al. (2001) B <<1 even for Nu = 10000

Conditions for steady convection!

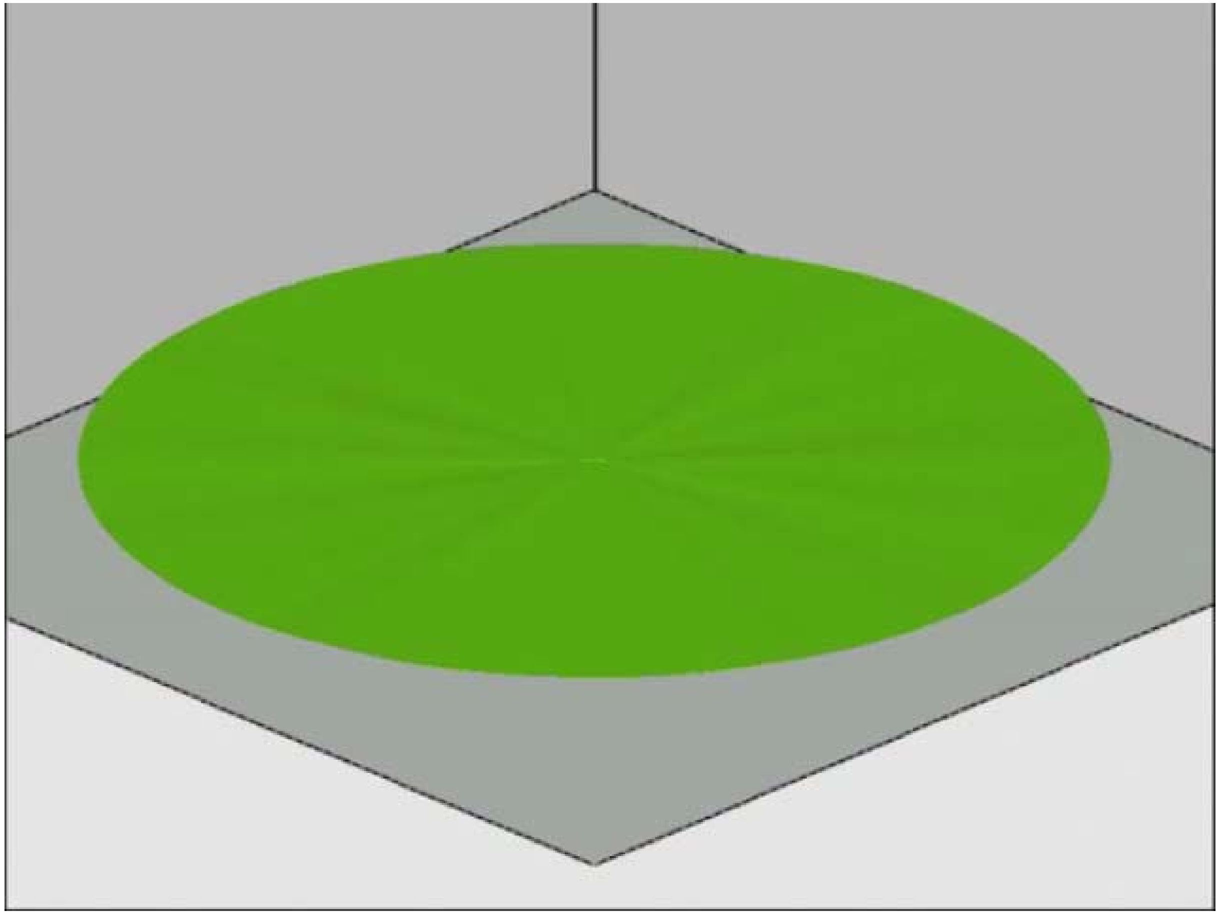
Motivation

θ_{wall}=const
Pr=0.7 Ra=2x10^8
θ=0.8 θ_{max}

Convection is strongly unsteady

Mean flow "rotations" and "cessations"

Formation of line plumes



Aspect ratio Γ=1/2 enhances unsteadiness

Motivation

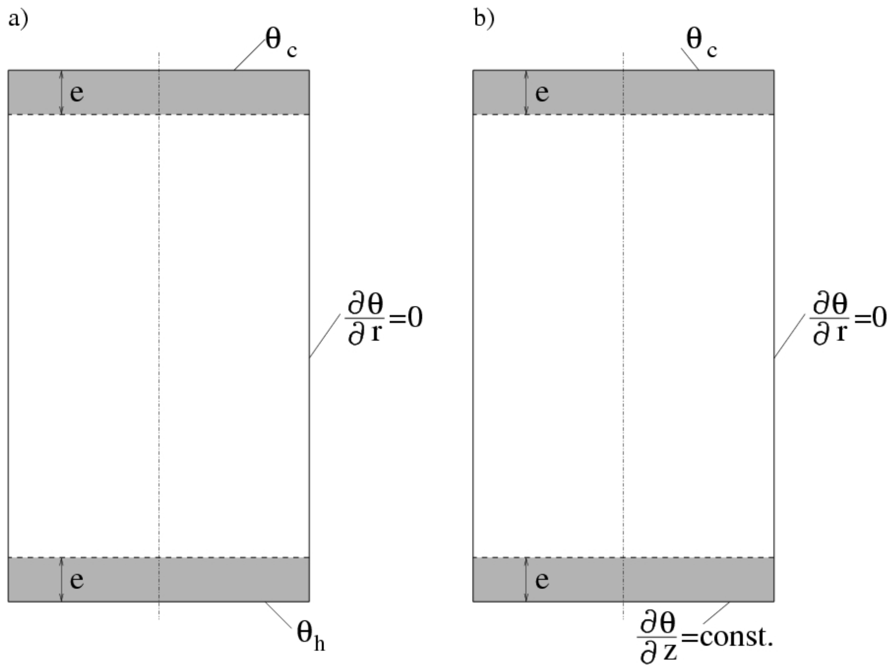
Verzicco (2004) and Brown et al. (2005) found similar plate corrections

Brown et al. (2005): Experiments in water and acetone (Pr≈4)
in cylindrical cells with $0.4<\Gamma<3$

Verzicco (2004): Simulations at Pr=0.7 in a cylindrical cells with $\Gamma=0.5$

Correction $Nu_\infty = Nu\, f(X)$; $f(X) = 1-\exp[-(aX)^b]$ $X = \frac{\lambda_w}{Nu \lambda_f} \frac{h}{e}$

(Brown a=0.275 b=0.39) (Verzicco a=0.25, b=0.33)



Nu_∞ is the limit for $e \to 0$
$(X \to \infty)$

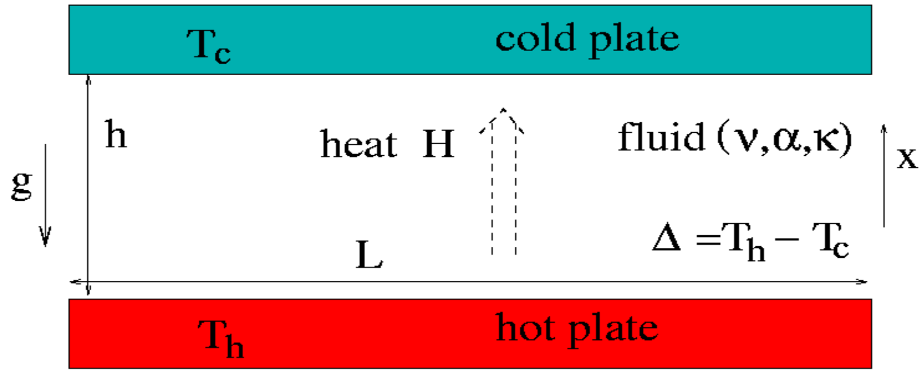
The two problems, however,
do not converge to the same limit

Classical RB problem

Non-dimensional Navier-Stokes equations with the Boussinesq approximation

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \theta \hat{x} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}} \nabla^2 \mathbf{u}, \qquad \nabla \cdot \mathbf{u} = 0$$

$$\frac{D\theta}{Dt} = \frac{1}{(Pr\,Ra)^{\frac{1}{2}}} \nabla^2 \theta$$



$$Ra = \frac{g \alpha \Delta h^3}{\nu k}$$ *`forcing' parameter*

On input: $$Pr = \frac{\nu}{k}$$ *fluid properties*

On output: $$Nu = \frac{H\ h}{\lambda\ \Delta}$$

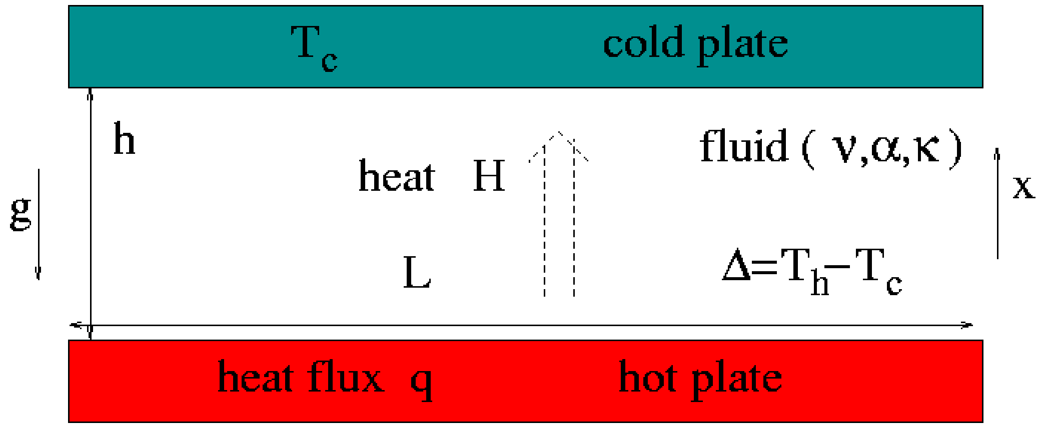
$$\Gamma = \frac{L}{h}$$ *geometric parameter*

Present problem

Non-dimensional Navier-Stokes equations with the Boussinesq approximation

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \theta \hat{z} + \left(\frac{Pr}{Ra_q}\right)^{\frac{1}{2}} \nabla^2 \mathbf{u} \qquad \nabla \cdot \mathbf{u} = 0$$

$$\frac{D\theta}{Dt} = \frac{1}{(Pr\,Ra_q)^{\frac{1}{2}}} \nabla^2 \theta$$



$$Ra = \frac{g\alpha q^4 h}{\upsilon\kappa}$$

`forcing' parameter

$$\left(Ra = \frac{Ra_q}{Nu} \right)$$

On input:

$$\Pr = \frac{\upsilon}{\kappa}$$

fluid properties

On output: T_h

$$\Gamma = \frac{d}{L}$$

geometric parameter

$$\left(Nu = \frac{qh}{\Delta} \right)$$

Numerical Code

Direct numerical simulation of the unsteady 3D Navier-Stokes equations with the Boussinesq approximation

Equations discretized in cylindrical coordinates

Central second-order accurate finite-difference in space and time

Third-order Runge-Kutta scheme for the time advancement of the solution

Elliptic equation solved by a direct method: trigonometric expansions in the azimuthal direction and FISHPACH in the other directions

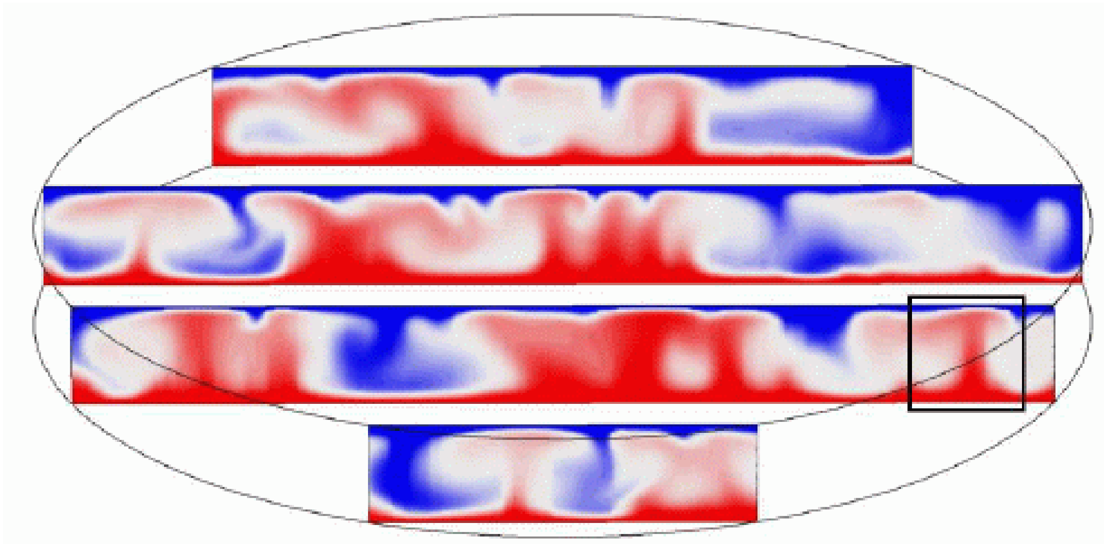
Swartzrauber (1974)

Code in fortran with OMP directives for parallel computing on shared memory computers (efficient simulations up to 16 processors)

In order to avoid resolution issues the same grids (number of nodes and spatial distribution) as in Verzicco & Camussi (2003) have been used.

Code Validation

Simulations at *Pr=0.7* in a cylindrical cell at *Γ=10*

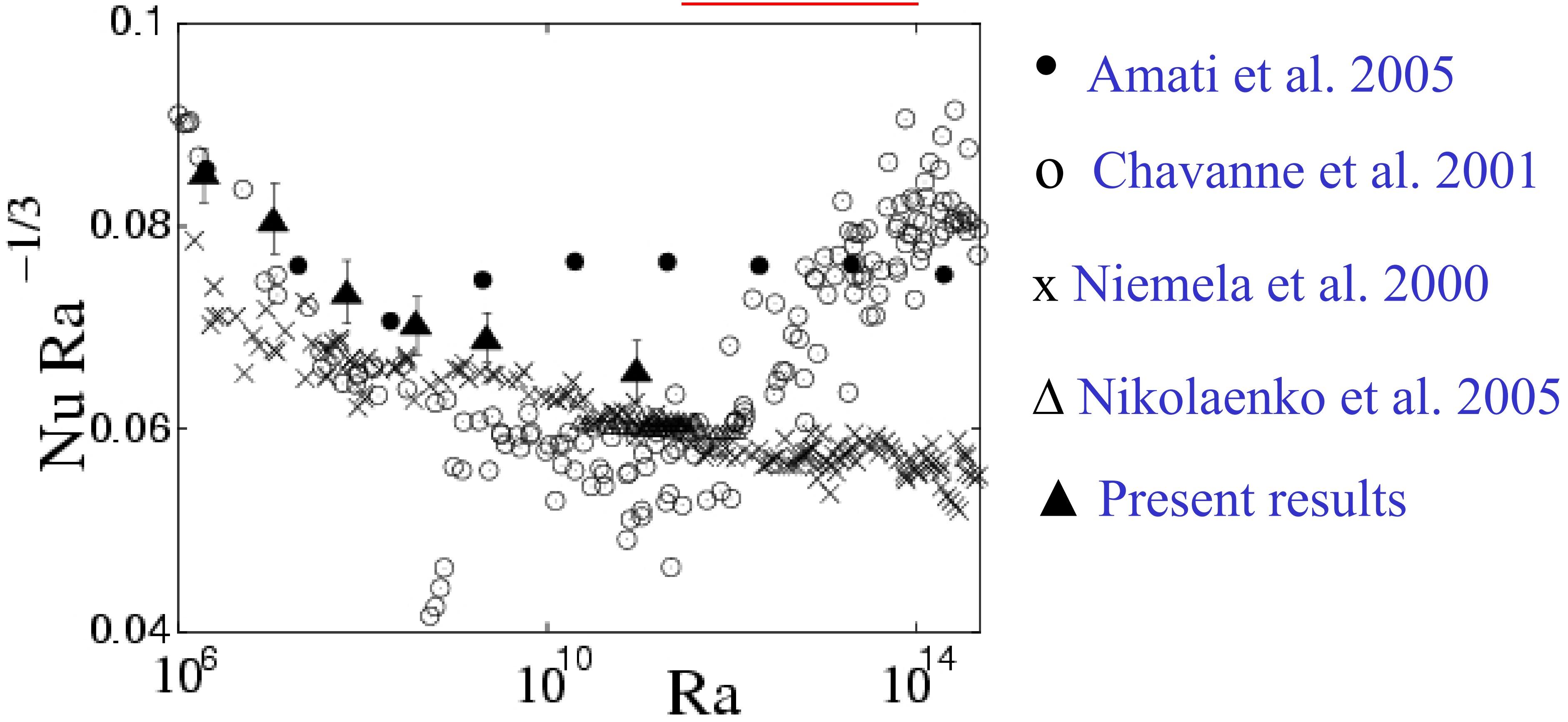


(111X513X193) grid

Shishkina & Wagner (2006)

	Shishkina & Wagner (2006)	Present results
$Ra = 10^5$	Nu=4.1	Nu=4.23±0.18
$Ra = 10^6$	Nu=8.2	Nu=8.37±0.22

Results

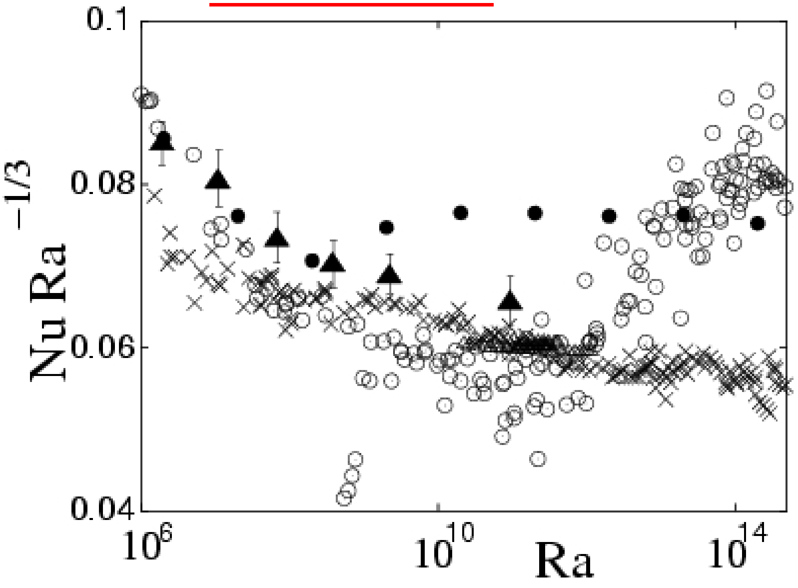


For Ra≥10^9 simulations closer to experiments.

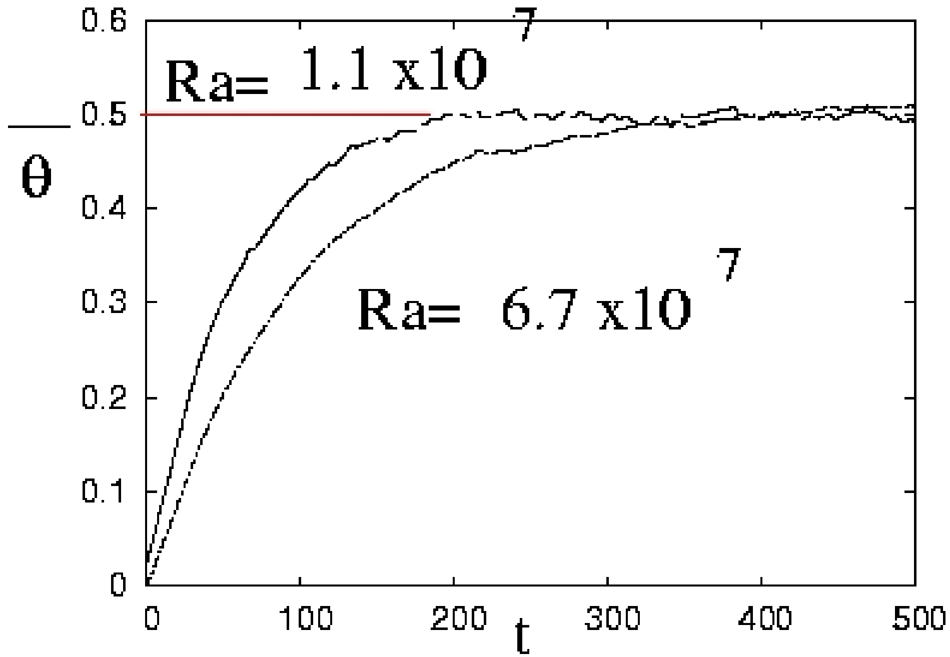
Note: unlike the simulations, experiments have a plate between the heater *(q=const.)* and the fluid.

Classical “puzzle” still unsolved.

Results



Why not higher *Ra*?



Because the attainment of the thermal equilibrium is computationally expensive and the cost increases with *Ra*.

Unlike the “constant temperature case” the mean flow temperature is not known in advance and it must be found as part of the solution.

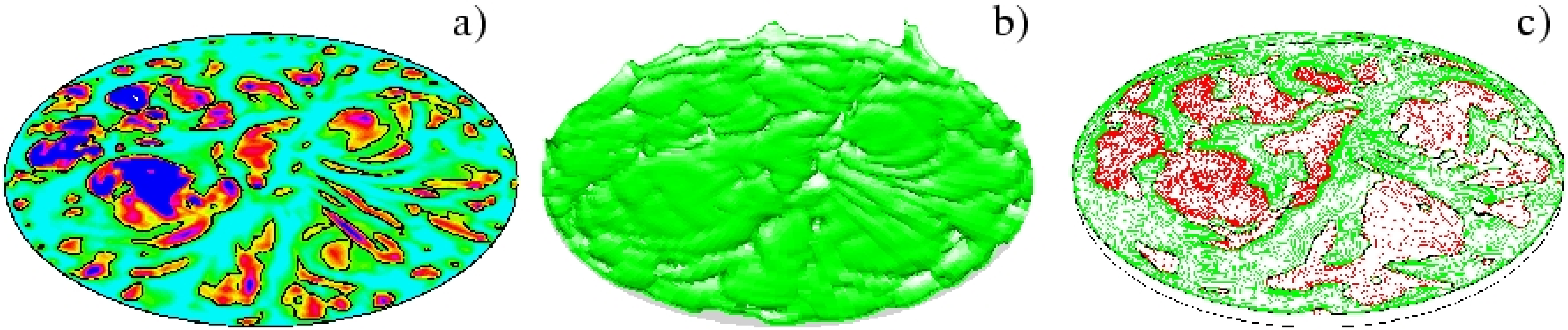
Even starting from a guess initial temperature the CPU requirements become soon too large.

Results

Near wall dynamics *(θ =const)*

Pr=0.7 Ra=2x10^9

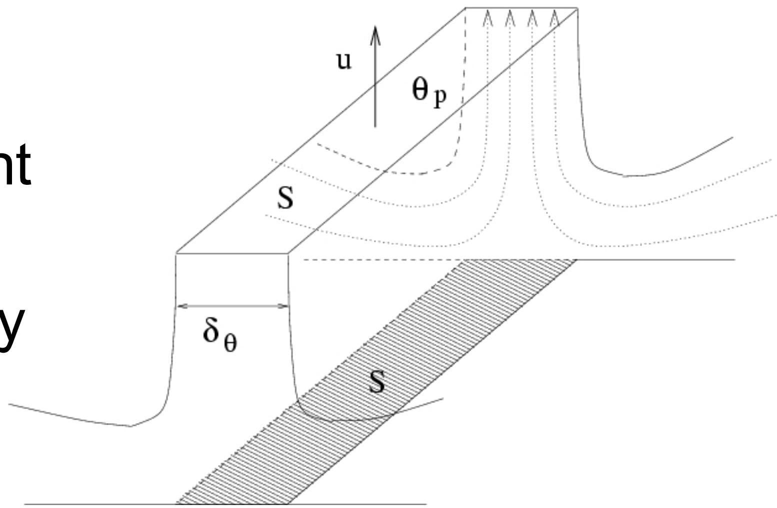
$(\partial\theta/\partial z)_{wall}$ $\theta = 0.8 <\theta_{wall}>$ u_z



The plate is swept on the sides of a plume
The wall temperature gradient increases above the average

The fluid below the plume is stagnant

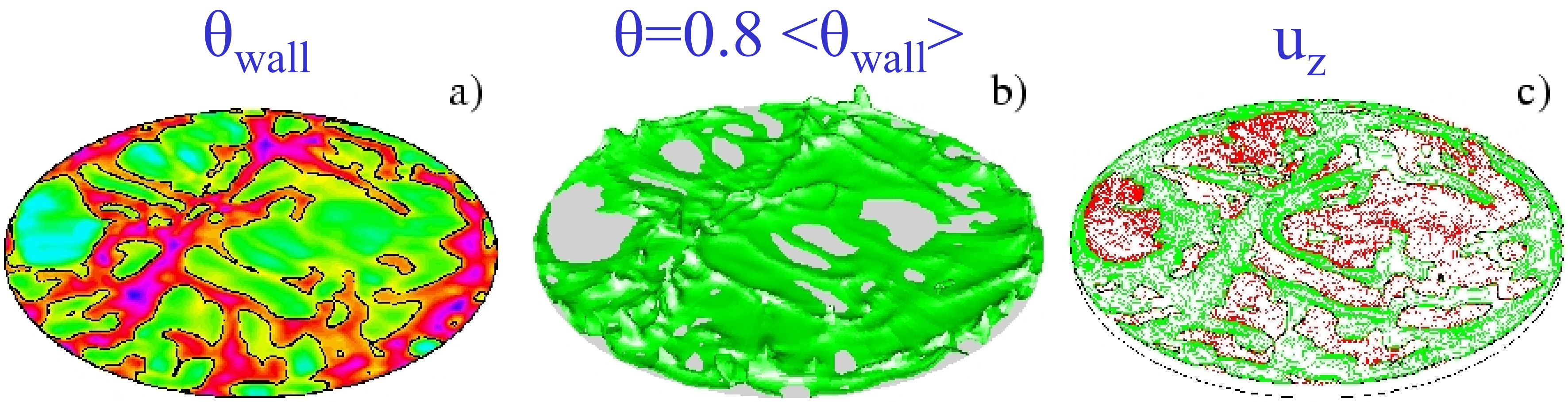
The flow can provide any heat flux by making the thermal b.l. thinner



Results

Near wall dynamics *(q=const)*

$Pr=0.7 \quad Ra=2.2x10^9$

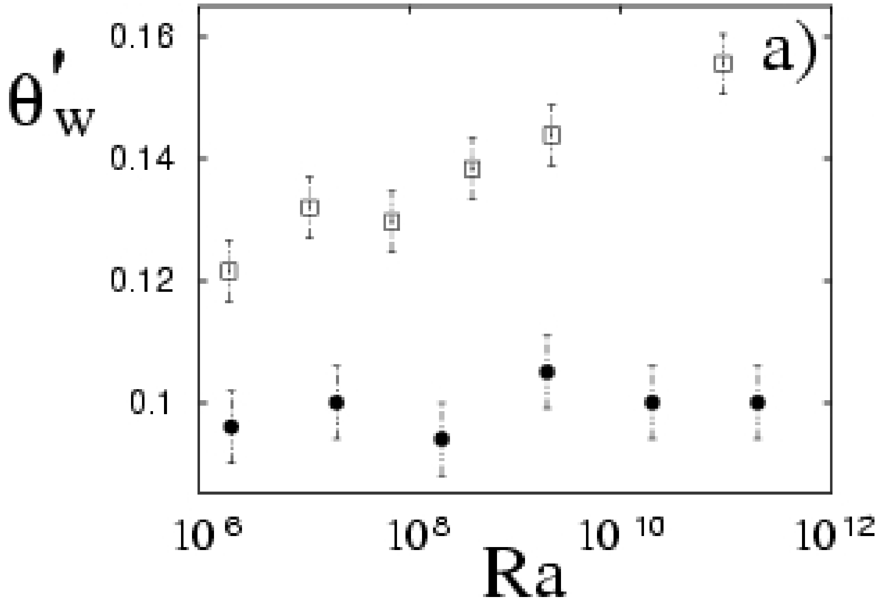


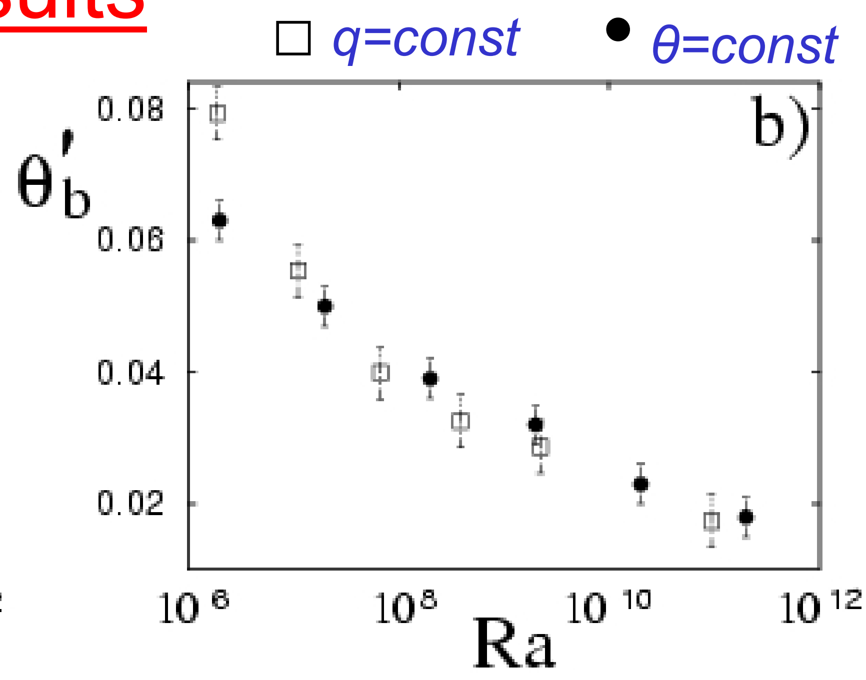
The plate cools down during the formation of a plume

The wall temperature decreases below the average

The resulting plumes are colder and carry less heat

Results



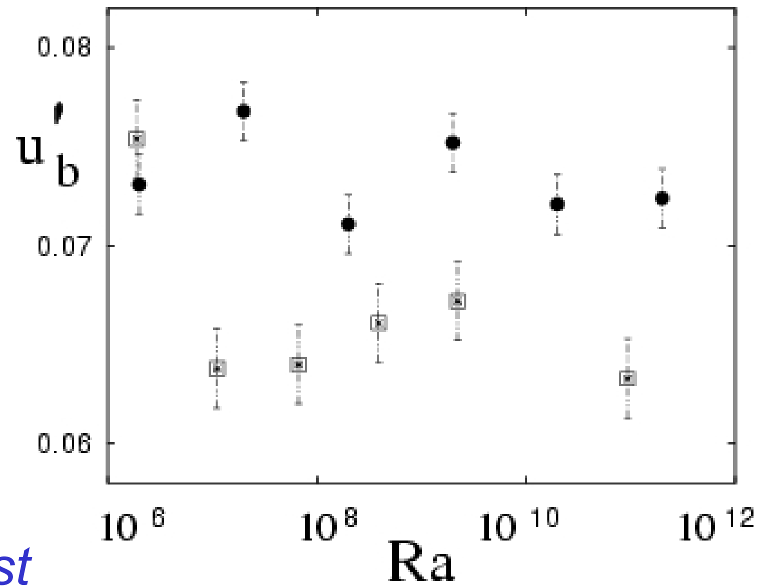


Wall temperature fluctuations increase for *q=const*

Bulk temperature fluctuations do not Show the same increase

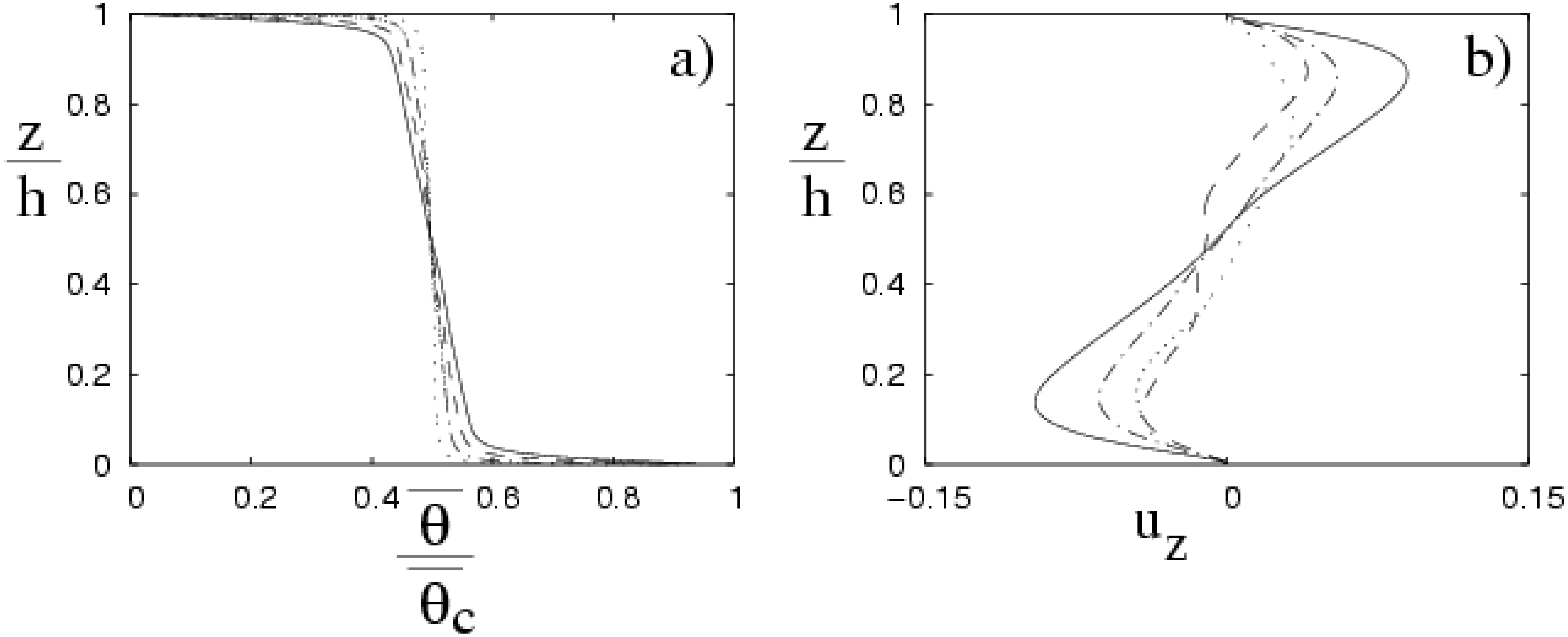
Bulk vertical velocity fluctuations are smaller for *q=const*

Thermal plumes are weaker for *q=const*



Results

Mean flow profiles similar to $\theta = const$



The same behaviour as in Verzicco & Camussi (2003)

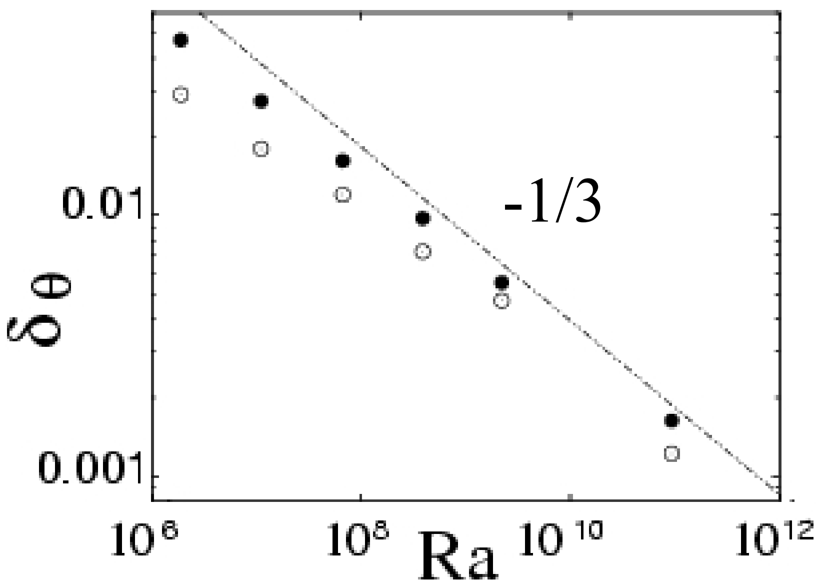
Results

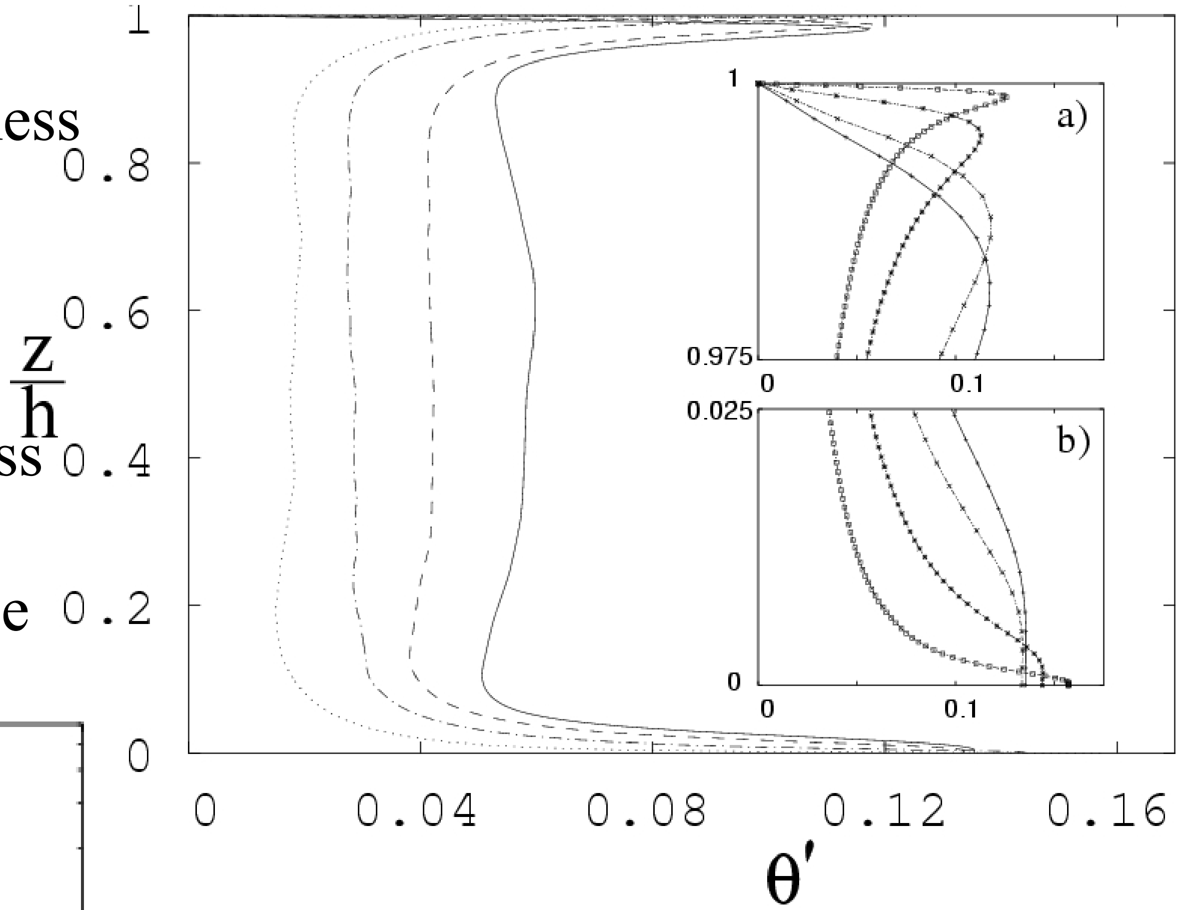
Upper thermal b.l. thickness
from peak r.m.s.

o *upper plate*

Lower thermal b.l. thickness
from linear extrapolation
of mean temperature profile

• *lower plate*



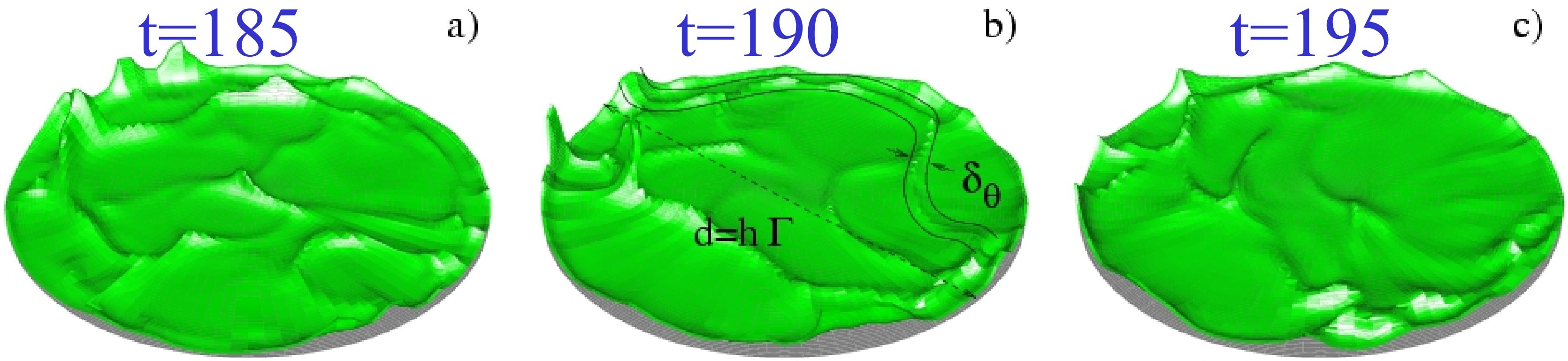


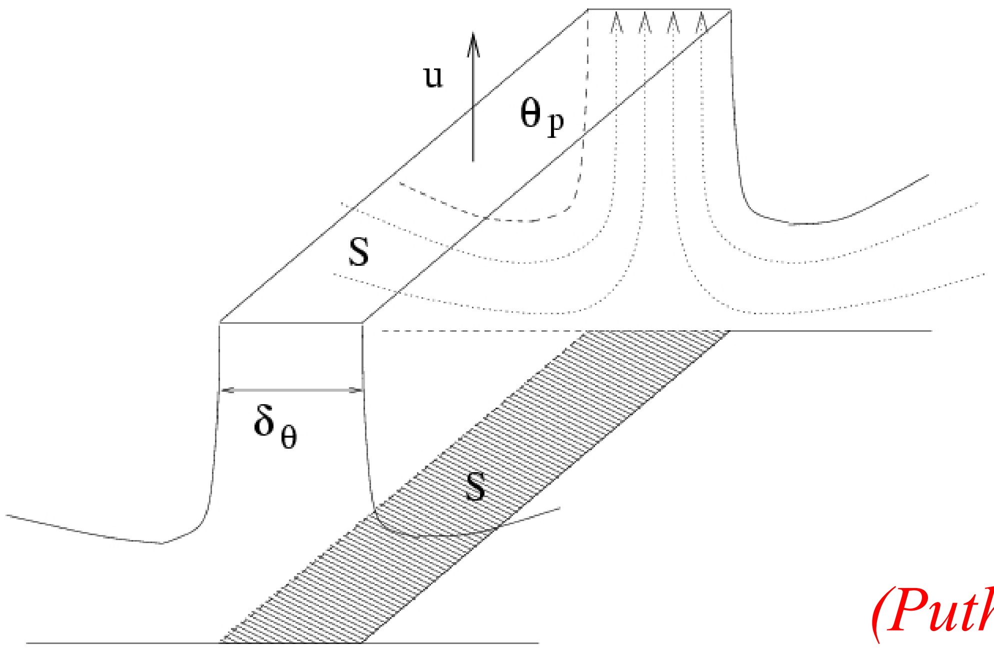
Qualitative agreement but systematic difference

• agrees with $$\frac{\delta_\vartheta}{h} = \frac{1}{2Nu}$$

A simple model

θ_{wall}=const
Pr=0.7 Ra=$2x10^8$

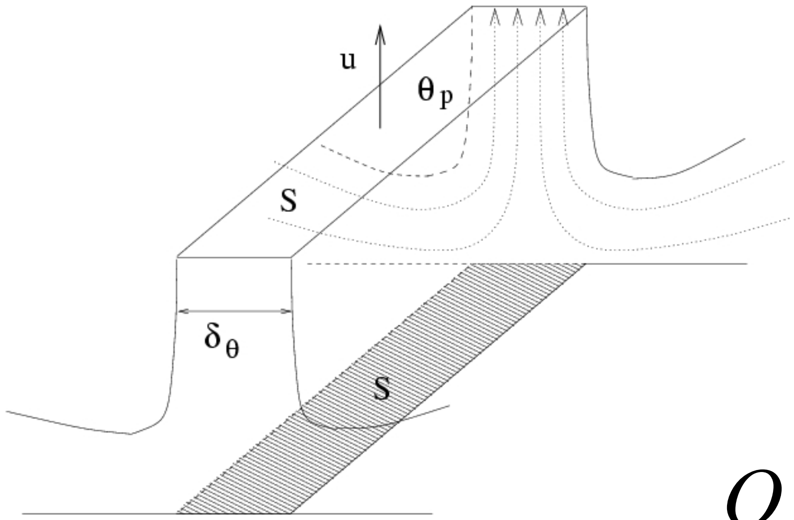




(line) plumes have the same thickness as the thermal boundary layer and a horizontal extension comparable with the cell size

(Puthenveettil & Arakeri, 2005)

A simple model



Heat flux needed by a plume

$$Q_p \approx \rho C_p \vartheta_p u S$$

Average heat flux through a surface element S

$$Q_w \approx \lambda < \partial\theta / \partial z >_w S = Nu \lambda \Delta / h$$

If: $\theta_p \approx \Delta$ (a plume is a piece of detached b.l.)

$u \approx g\alpha\Delta\delta_\theta h\Gamma / \upsilon$ (buoyancy and drag in equilibrium)

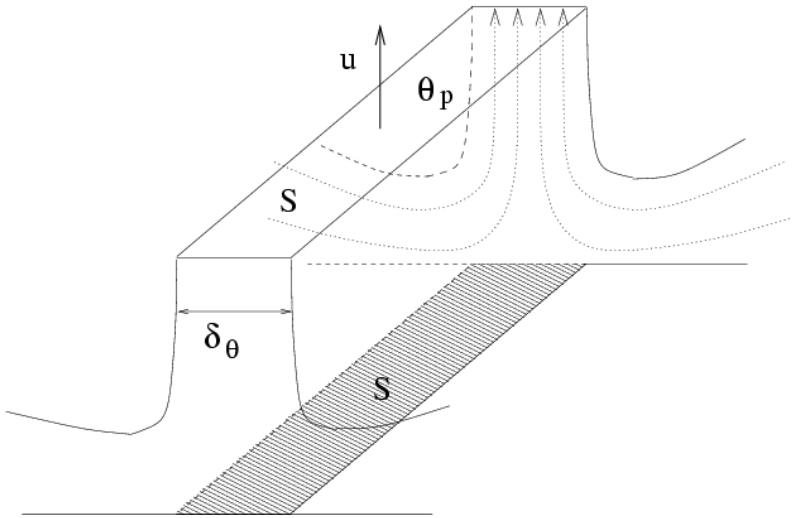
(Castaing et al. 1989)

$$\frac{Q_p}{Q_w} \sim \frac{Ra}{Nu^2}$$

which increases with Ra if $Nu \sim Ra^\beta$ with $\beta < 1/2$

Note $\theta_p \approx \Delta \quad \forall Ra$ only if $\theta_{wall} = const$

A simple model



If: $< \partial\theta / \partial z >_w = const$

a plume can not drain more heat than that provided by the wall

$$\frac{Q_p}{Q_w} \approx 1$$

The plume temperature θ_p can be computed

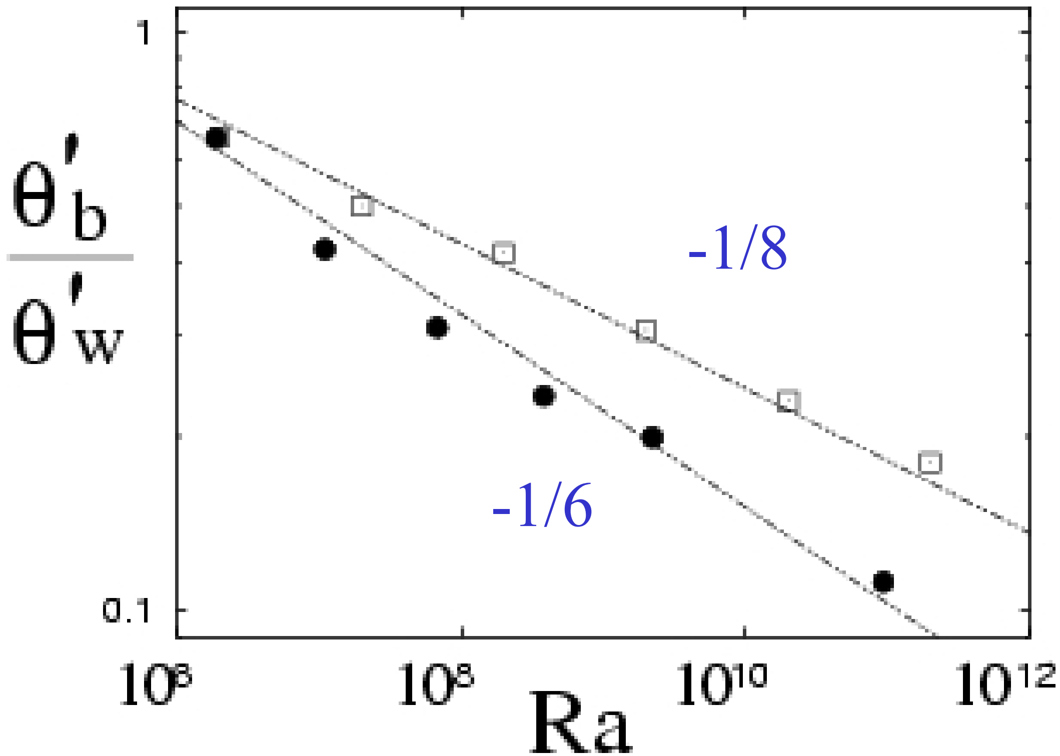
$$\theta_p \sim \Delta \frac{Nu}{Ra^{1/2}}$$

which decreases with Ra if $Nu \sim Ra^{\beta}$ with $\beta < 1/2$

Note similar conclusions if $u \approx \sqrt{g\alpha\Delta h}$ (free fall velocity)

or $u \approx \sqrt[3]{g\alpha < u_z^{'}\theta^{'} > h}$ *(Hunt et al. 2003)*

A simple model



$\frac{\theta'_b}{\theta'_w}$ is the fraction of wall temperature fluctuations that reach the bulk: PLUMES?

● *q=const*

□ *θ=const*

$$\theta_p \sim \Delta \frac{Nu}{Ra^{1/2}}$$ with $Nu \sim Ra^{1/3}$ yields $$\theta_p \sim \Delta Ra^{-1/6}$$

Conclusions

Constant heat flux and constant temperature surfaces do not give the same results (in terms of Nu) when the flow becomes strongly unsteady and turbulent ($Ra \geq 10^9$ in a $\Gamma=1/2$ cylindrical cell)

Constant heat flux surfaces produce colder plumes at high Ra thus yielding a reduced Nu

Results of numerical simulations for constant heat flux are closer to experiments: have experiments a constant heat flux surface?

The experiment by J. Niemela with the *"copper sponge"* seem to contradict the present findings

Temperature plate uniformity would be assured by high λ_w / λ_f and high $\sqrt{(\rho C)_w / (\rho C_p)_f}$ *(Chillà et al. 2004)*. In this respect the "Ilmeneau barrel" with a good lower plate would be a good set-up.