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on

High Rayleigh Number Convection

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A comparison of constant heat flux and constant temperature thermal convection

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These are preliminary lecture notes, intended only for distribution to participants
A Comparison between Constant Temperature and Constant Heat Flux

Turbulent Thermal Convection

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Motivation

For $Ra \geq 10^9$ differences of $Nu$ between numerical simulations and "similar" experiments up to 20%
Motivation

The differences can not be ascribed to:

Prandtl number  (Ahlers & Xu 2001, Xia et al. 2002)

Shape of the cell  (cylindrical $\Gamma=1/2$ in all cases)

Non-Boussinesq effects  (irrelevant at $Ra\approx10^9$)

Sidewall conduction  (important only at low $Ra$)

Plates finite conductivity  (corrections too small)

Is there a fundamental difference?
In most experimental set-ups upper and lower plates are heated and cooled by different methods.

The lower plate has a constant heat flux surface and its heat capacity keeps the temperature constant.
Motivation

Constant temperature if:

\[(\rho C \lambda)_{\text{plate}} >> (\rho C_p \lambda)_{\text{fluid}}\]

(Schlichting 2000, p.507)

In thermal convection, however, \(\lambda_{\text{eff}} = Nu \lambda_{\text{fluid}}\) and eventually

\[(\rho C \lambda)_{\text{plate}} = (\rho C_p Nu \lambda)_{\text{fluid}}\]

**Small Biot number** \(B = (H e/ \lambda)_{\text{plate}} << 1\)

In thermal convection, however, \(H = Nu \lambda_{\text{fluid}}/h\) and

\[B = Nu \lambda_{\text{fluid}} e / (\lambda_{\text{plate}} h)\]

(the inverse of the plate correction parameter “X”)

In Niemela et al. (2000) and Chavanne et al. (2001) \(B << 1\) even for \(Nu = 10000\)

Conditions for steady convection!
Motivation

Convection is strongly unsteady

\[ \theta_{\text{wall}} = \text{const} \]
\[ \text{Pr}=0.7 \quad \text{Ra}=2 \times 10^8 \]
\[ \theta=0.8 \quad \theta_{\text{max}} \]

Mean flow “rotations” and “cessations”

Formation of line plumes

Aspect ratio $\Gamma=1/2$ enhances unsteadiness
Motivation

Verzicco (2004) and Brown et al. (2005) found similar plate corrections

Brown et al. (2005): Experiments in water and acetone ($Pr \approx 4$) in cylindrical cells with $0.4 < \Gamma < 3$

Verzicco (2004): Simulations at $Pr = 0.7$ in a cylindrical cells with $\Gamma = 0.5$

Correction $\text{Nu}_\infty = \text{Nu} \cdot f(X)$; $f(X) = 1 - \exp[-(aX)^b]$  

(Brown $a = 0.275$, $b = 0.39$) (Verzicco $a = 0.25$, $b = 0.33$)

$\text{Nu}_\infty$ is the limit for $e \to 0$ 
($X \to \infty$)

The two problems, however, do not converge to the same limit
Classical RB problem

Non-dimensional Navier-Stokes equations with the Boussinesq approximation

\[
\frac{Du}{Dt} = -\nabla p + \theta \hat{x} + \left( \frac{Pr}{Ra} \right)^{\frac{1}{2}} \nabla^2 u, \quad \nabla \cdot u = 0
\]

\[
\frac{D\theta}{Dt} = \frac{1}{(PrRa)^{\frac{1}{2}}} \nabla^2 \theta
\]

\[Ra = \frac{g \alpha \Delta h^3}{\nu k}\]

On input:

\[Pr = \frac{\nu}{k}\]

\[\Gamma = \frac{L}{h}\]

On output:

\[Nu = \frac{h}{\lambda \Delta}\]

Forcing parameter

Fluid properties

Geometric parameter
Present problem

Non-dimensional Navier-Stokes equations with the Boussinesq approximation

\[ \frac{Du}{Dt} = -\nabla p + \theta \hat{z} + \left( \frac{Pr}{Ra_q} \right)^{\frac{1}{2}} \nabla^2 u \]

\[ \nabla \cdot u = 0 \]

\[ \frac{D\theta}{Dt} = \frac{1}{(Pr Ra_q)^{\frac{1}{2}}} \nabla^2 \theta \]

\[ Ra = \frac{g \alpha q^4 h}{\nu \kappa} \]

'forcing' parameter

\[ Pr = \frac{\nu}{\kappa} \]

fluid properties

\[ \Gamma = \frac{d}{L} \]

geometric parameter

On input:

\[ \begin{align*}
Ra & = \frac{Ra_q}{Nu} \\
Pr & = \frac{\nu}{\kappa} \\
\Gamma & = \frac{d}{L}
\end{align*} \]

On output:

\[ \begin{align*}
Nu & = \frac{q h}{\Delta} \\
T_h & = \frac{Ra_q}{Nu}
\end{align*} \]
Numerical Code

Direct numerical simulation of the unsteady 3D Navier-Stokes equations with the Boussinesq approximation

Equations discretized in cylindrical coordinates

Central second-order accurate finite-difference in space and time

Third-order Runge-Kutta scheme for the time advancement of the solution

Elliptic equation solved by a direct method: trigonometric expansions in the azimuthal direction and FISHPACH in the other directions

Swartzrauber (1974)

Code in fortran with OMP directives for parallel computing on shared memory computers (efficient simulations up to 16 processors)

In order to avoid resolution issues the same grids (number of nodes and spatial distribution) as in Verzicco & Camussi (2003) have been used.
Simulations at $Pr=0.7$ in a cylindrical cell at $\Gamma=10$

Shishkina & Wagner (2006) & Present results

Ra = $10^5$ & Nu=4.1 & Nu=4.23±0.18
Ra = $10^6$ & Nu=8.2 & Nu=8.37±0.22
For $Ra \geq 10^9$ simulations closer to experiments.

Note: unlike the simulations, experiments have a plate between the heater ($q=\text{const.}$) and the fluid.

Classical “puzzle” still unsolved.
Results

Because the attainment of the thermal equilibrium is computationally expensive and the cost increases with $Ra$.

Unlike the "constant temperature case" the mean flow temperature is not known in advance and it must be found as part of the solution.

Even starting from a guess initial temperature the CPU requirements become soon too large.
Results

Near wall dynamics \((\theta =\text{const})\)

\[
\left( \frac{\partial \theta}{\partial z} \right)_\text{wall}
\]

\[
\theta = 0.8 <\theta_{\text{wall}}>
\]

\(u_z\)

The plate is swept on the sides of a plume
The wall temperature gradient increases above the average

The fluid below the plume is stagnant

The flow can provide any heat flux by making the thermal b.l. thinner
Results

Near wall dynamics \((q=\text{const})\)

The plate cools down during the formation of a plume

The wall temperature decreases below the average

The resulting plumes are colder and carry less heat

\[ \theta_{\text{wall}} \]

\[ \theta = 0.8 \langle \theta_{\text{wall}} \rangle \]

\[ u_z \]
Results

Wall temperature fluctuations increase for $q=\text{const}$

Bulk temperature fluctuations do not show the same increase

Bulk vertical velocity fluctuations are smaller for $q=\text{const}$

Thermal plumes are weaker for $q=\text{const}$
Results

Mean flow profiles similar to $\theta = const$

The same behaviour as in Verzicco & Camussi (2003)
Results

Upper thermal b.l. thickness from peak r.m.s.
- upper plate

Lower thermal b.l. thickness from linear extrapolation of mean temperature profile
- lower plate

Qualitative agreement but systematic difference

\[
\frac{\delta \theta}{\theta'} \propto \frac{1}{2 Nu}
\]
A simple model

(line) plumes have the same thickness as the thermal boundary layer and a horizontal extension comparable with the cell size

\[ \theta_{\text{wall}} = \text{const} \]
\[ \Pr = 0.7 \quad Ra = 2 \times 10^8 \]

(Puthenveettil & Arakeri, 2005)
A simple model

Heat flux needed by a plume

\[ Q_p \approx \rho C_p \vartheta_p u S \]

Average heat flux through a surface element \( S \)

\[ Q_w \approx \lambda < \partial \theta / \partial z >_w S = Nu \lambda \Delta / h \]

If: \( \theta_p \approx \Delta \) (a plume is a piece of detached b.l.)

\[ u \approx g \alpha \Delta \delta \theta h \Gamma / \nu \] (buoyancy and drag in equilibrium)

(Castaing et al. 1989)

\[ \frac{Q_p}{Q_w} \sim \frac{Ra}{Nu^2} \]

which increases with \( Ra \) if \( Nu \sim Ra^\beta \) with \( \beta < 1/2 \)

Note \( \theta_p \approx \Delta \) \( \forall Ra \) only if \( \theta_{wall} = \text{const} \)
A simple model

If: $< \partial \theta / \partial z >_w = const$

a plume can not drain more heat than that provided by the wall

$$\frac{Q_p}{Q_w} \approx 1$$

The plume temperature $\theta_p$ can be computed

$$\theta_p \sim \Delta \frac{Nu}{Ra^{1/2}}$$

which decreases with $Ra$ if $Nu \sim Ra^\beta$ with $\beta < 1/2$

Note similar conclusions if $u \approx \sqrt{g \alpha \Delta h}$ (free fall velocity)

or $u \approx \sqrt[3]{g \alpha < u_z \theta' > h}$ (Hunt et al. 2003)
A simple model

\[ \frac{\theta'_b}{\theta'_w} \sim -\frac{1}{8} \]

is the fraction of wall temperature fluctuations that reach the bulk:
PLUMES?

\[ \frac{\theta'_b}{\theta'_w} \]

\[ \theta_p \sim \Delta \frac{Nu}{Ra^{1/2}} \]

with \( Nu \sim Ra^{1/3} \) yields

\[ \theta_p \sim \Delta Ra^{-1/6} \]

\[ q=\text{const} \]

\[ \theta=\text{const} \]
Conclusions

Constant heat flux and constant temperature surfaces do not give the same results (in terms of Nu) when the flow becomes strongly unsteady and turbulent (\(Ra \geq 10^9\) in a \(\Gamma=1/2\) cylindrical cell)

Constant heat flux surfaces produce colder plumes at high Ra thus yielding a reduced Nu

Results of numerical simulations for constant heat flux are closer to experiments: have experiments a constant heat flux surface?

The experiment by J. Niemela with the “copper sponge” seem to contradict the present findings

Temperature plate uniformity would be assured by high \(\lambda_w / \lambda_f\) and high \(\sqrt{\frac{(\rho C)_{w}}{(\rho C_p)}_{f}}\) (Chillà et al. 2004). In this respect the “Ilmeneau barrel” with a good lower plate would be a good set-up.