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on

High Rayleigh Number Convection

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Flow reversals in Rayleigh-Benard convection

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Flow reversals in Rayleigh Benard Convection

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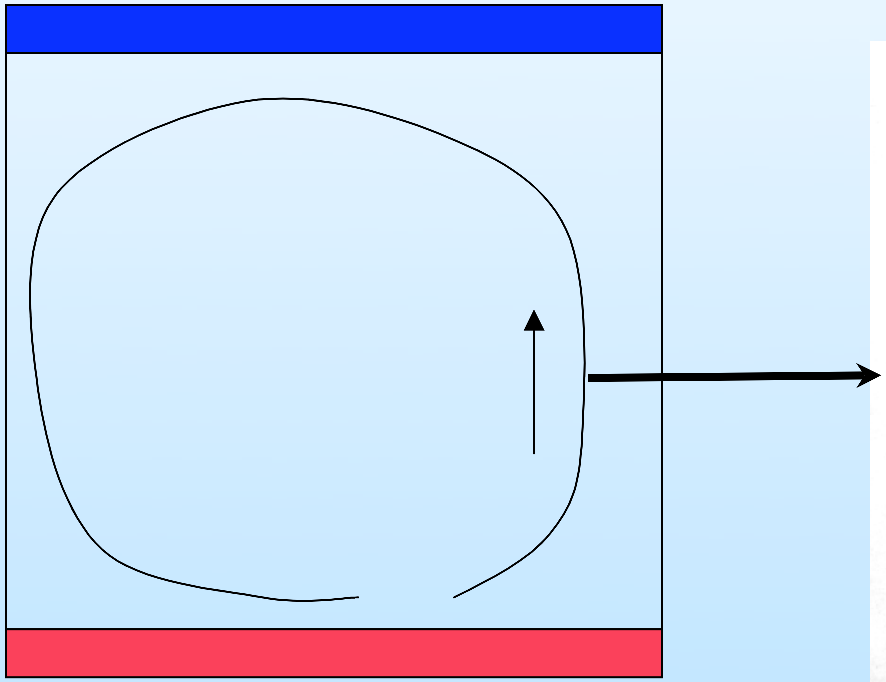
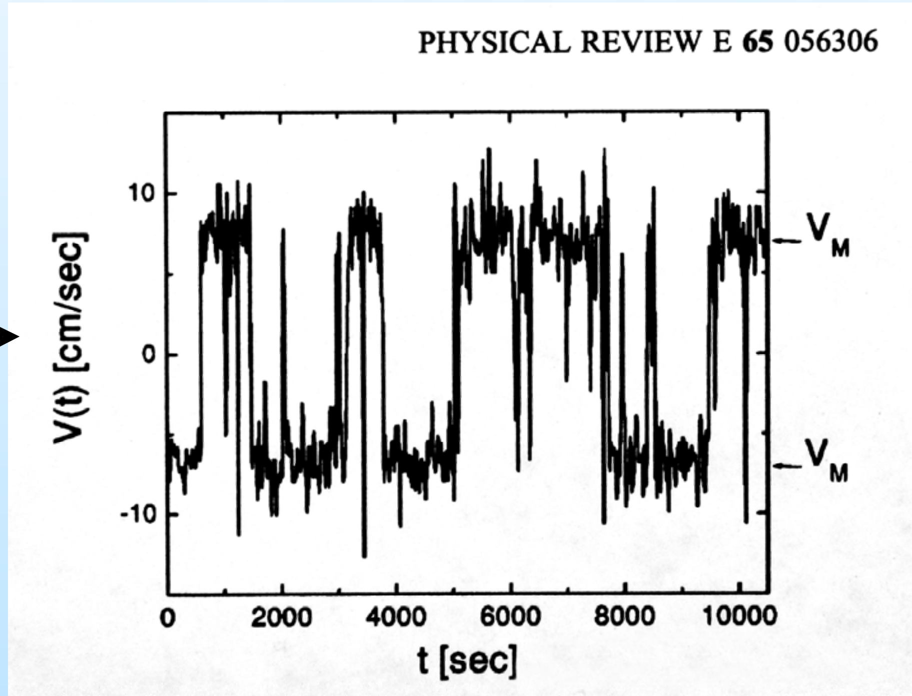
A “simple” complex system: thermal convection

For increasing ΔT, the system develops a large scale turbulent wind and one may observe flow reversals (Cioni, Ciliberto, Sommeria JFM 1997).

cold

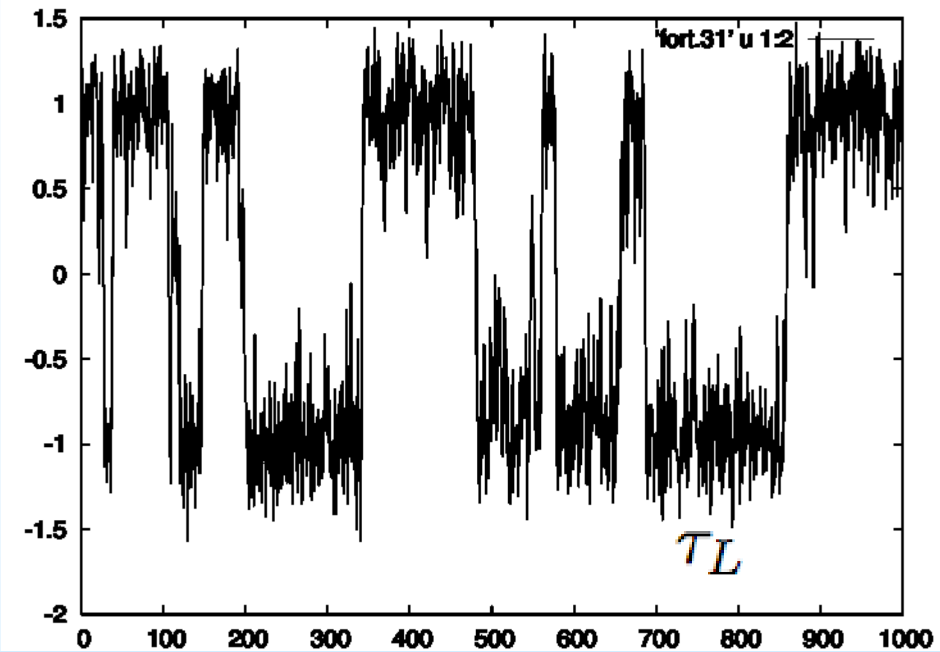


hot

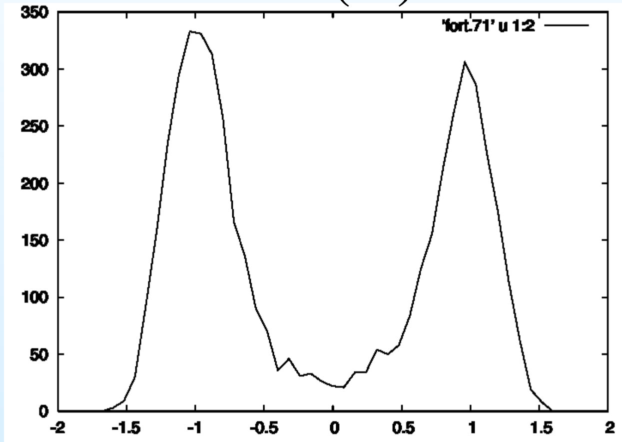


Sreenivasan, Bereshadskii, Niemela 2002

One could expect the following picture



P(U)



Random time (exit time) with probability distribution

$$P(\tau_L) = \lambda exp(-\lambda \tau_L)$$

Sreenivasan, Bereshadskii, Niemela 2002 reported

$$P(\tau_L) = \frac{A}{\tau_L} exp(-\lambda \tau_L)$$

Questions:

1) Why there are flow reversals in the large scale flow ?

2) Why the pdf of τ_L is a power law (for small enough τ_L)?

Aim of the talk is to answer the above questions by using DNS of RB turbulence and a lit bit of theory.

Same questions addressed by Fontenele Araujo et. al. (Phys. Rev. Lett. 2005) Brown et. al. (Phys. Rev. Lett. 2005), Sun et. al. (Phys. Rev. Lett. 2005, see Xia' talk).

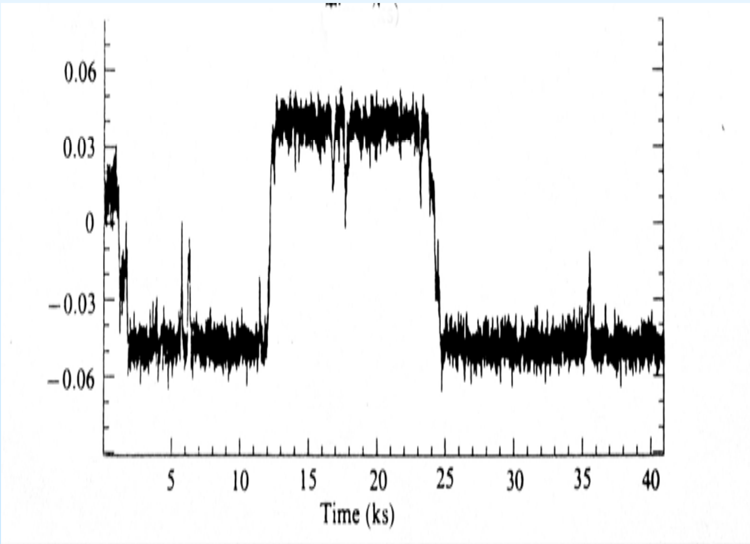
Why there are flow reversals in the large scale turbulent flow?

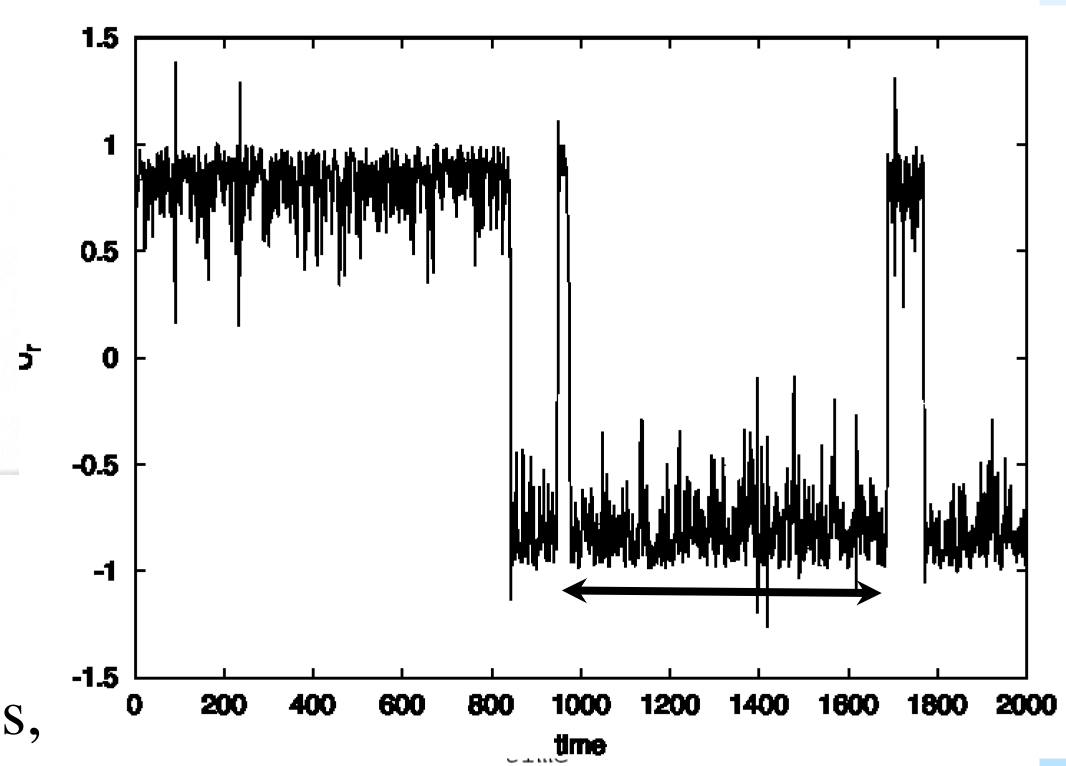
J. Sommeria JFM 1986
Experimental data on
2D turbulence

<u>It should be expected !</u>

R. B. PRL 2005

$$\partial_t U_r = U_r(1 - U_r^2) + \text{"}N.S.equations\text{"}$$





Turbulence acts as a <u>“noise”</u>
on the large scale flow and
induces jumps between states,

τ_L

Why the pdf of τ_L is a power law (for small enough τ_L)?

A power law behaviour of $P(\tau_L)$ means stronger fluctuations than expected.

<u>Observation</u>: let us consider the following 2D Landau-Ginzburg equation:

$$\partial_t \psi = m\psi - g\psi^3 + \nu \Delta \psi + \sqrt{\epsilon}\eta \qquad \psi = \psi(x,y,t)$$

$$\langle \eta(x,y,t)\eta(x_1,y_1,t_1)\rangle = \delta(x-x_1)\delta(x-x_2)\delta(t-t_1)$$

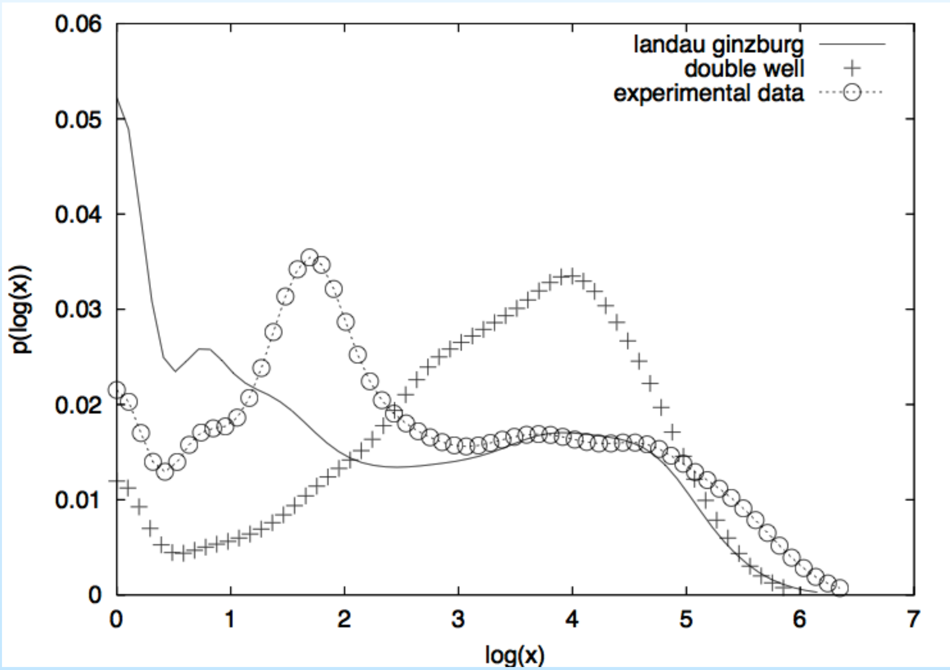
$$\psi_0 \equiv \langle \psi \rangle_s \equiv \frac{1}{L^2}\int dxdy\psi \qquad \psi = \psi_0(t) + \phi(x,y,t)$$

$$\partial \psi_0 = (m - 3g\langle \phi^2 \rangle_s)\psi_0 - g\psi_0^3$$

Dynamics of the space average field is controlled by the fluctuations. " *Flow reversals*" can be faster than expected (critical behaviour).

$$\partial \psi_0 = (m - 3g\langle \phi^2 \rangle_s)\psi_0 - g\psi_0^3$$

Probability distribution of $P(\log(\tau_L))$ for L-G, experimental data and D=0 double well potential. See also, Sreenivasan et. al. 2005.



If $P(\tau_L) \approx 1/\tau_L$ then $P(\log(\tau_L)) \approx$ const.

DNS of large scale flow in RB turbulence

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \theta \hat{z} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}} \nabla^2 \mathbf{u}, \qquad \nabla \cdot \mathbf{u} = 0,$$

$$\frac{D\theta}{Dt} = \frac{1}{(Pr Ra)^{\frac{1}{2}}} \nabla^2 \theta \qquad D/Dt \equiv \partial_t + \mathbf{u} \bullet \nabla$$

$$\theta = (T - T_c)/\Delta T \qquad \Delta T \equiv T_h - T_c$$

Aspect ratio=1, Pr = 0.7, cylindrical geometry.

Ra = 2 10^5 / 6 10^5

Ra too low to achieve good statistics (just one reversal in few days of numerical simulation)!

Main idea: increasing thermal fluctuations by white noise

$$\frac{D\theta}{Dt} = \frac{1}{(PrRa)^{\frac{1}{2}}} \nabla^2 \theta$$

$$\frac{D\theta}{Dt} = \frac{1}{(PrRa)^{\frac{1}{2}}} \nabla^2 \theta + \epsilon \sqrt{\Delta t}\, \eta(\vec{x}, t)$$

$$\epsilon \sim 0.1$$

Noise on temperature = increasing thermal diffusivity = decreasing Prandtl = increasing Reynolds number !

Geometry and ideal probes for temperature and velocity

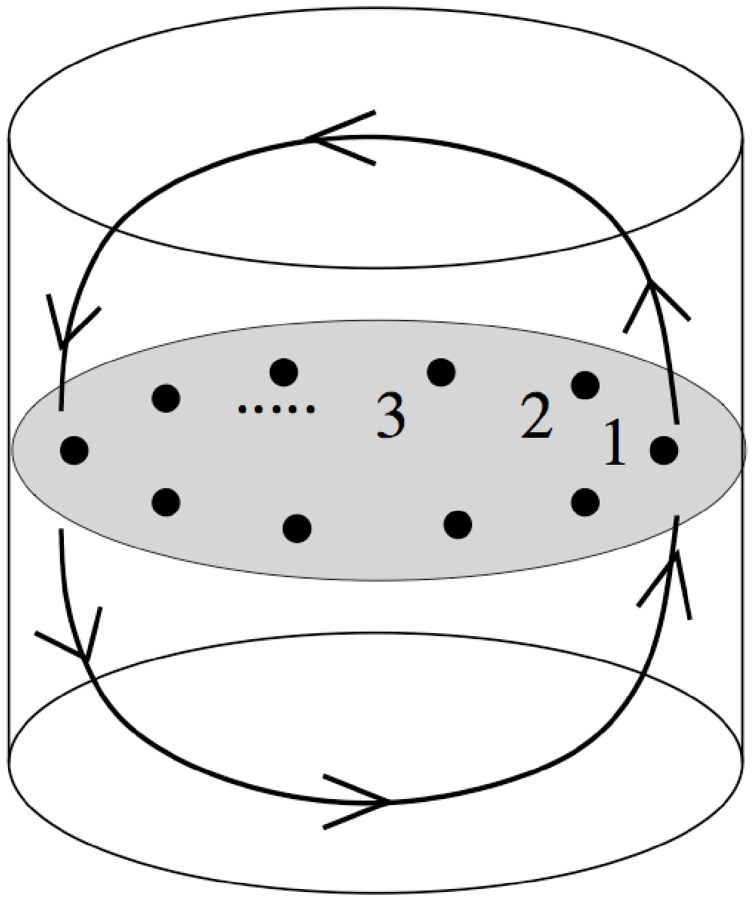
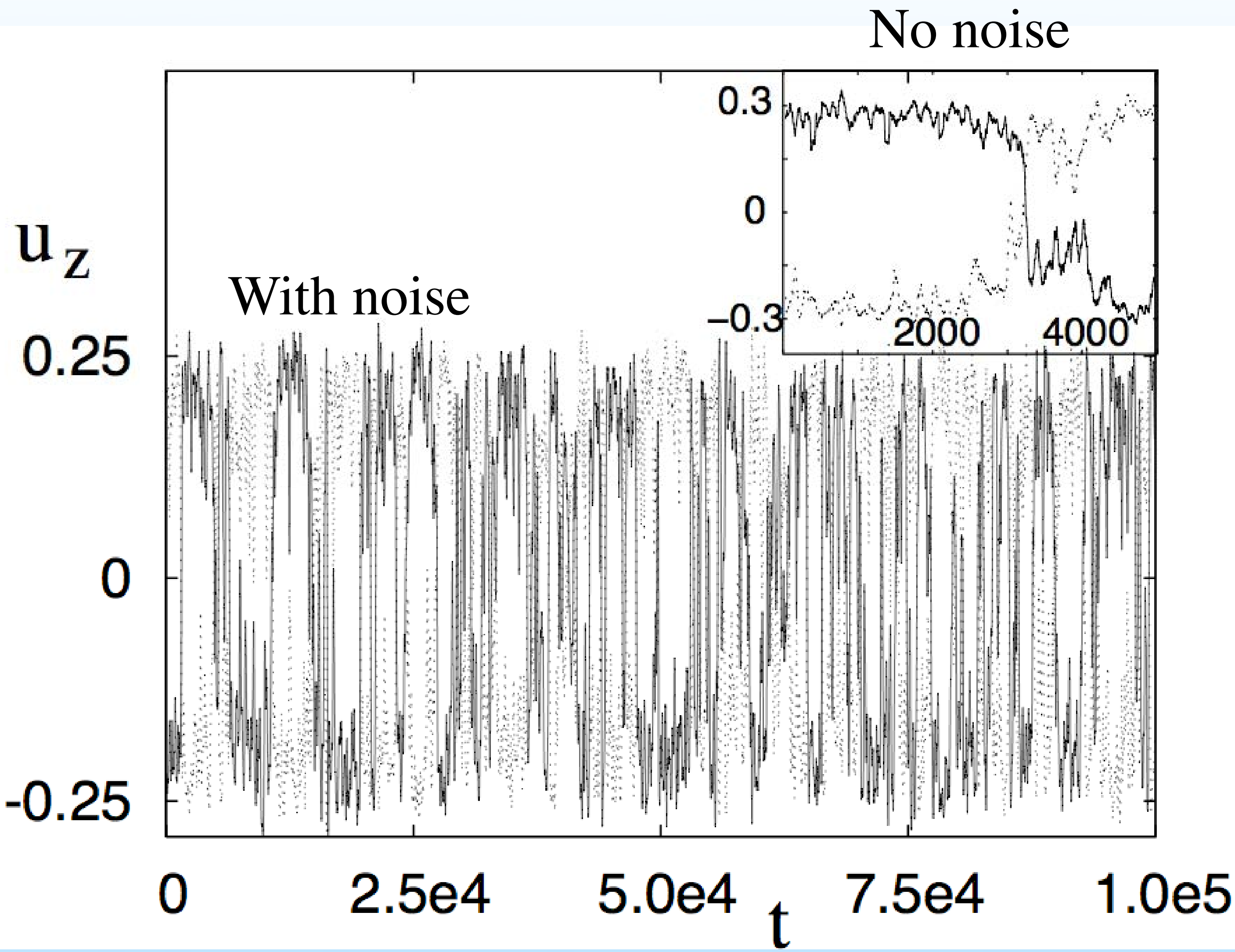
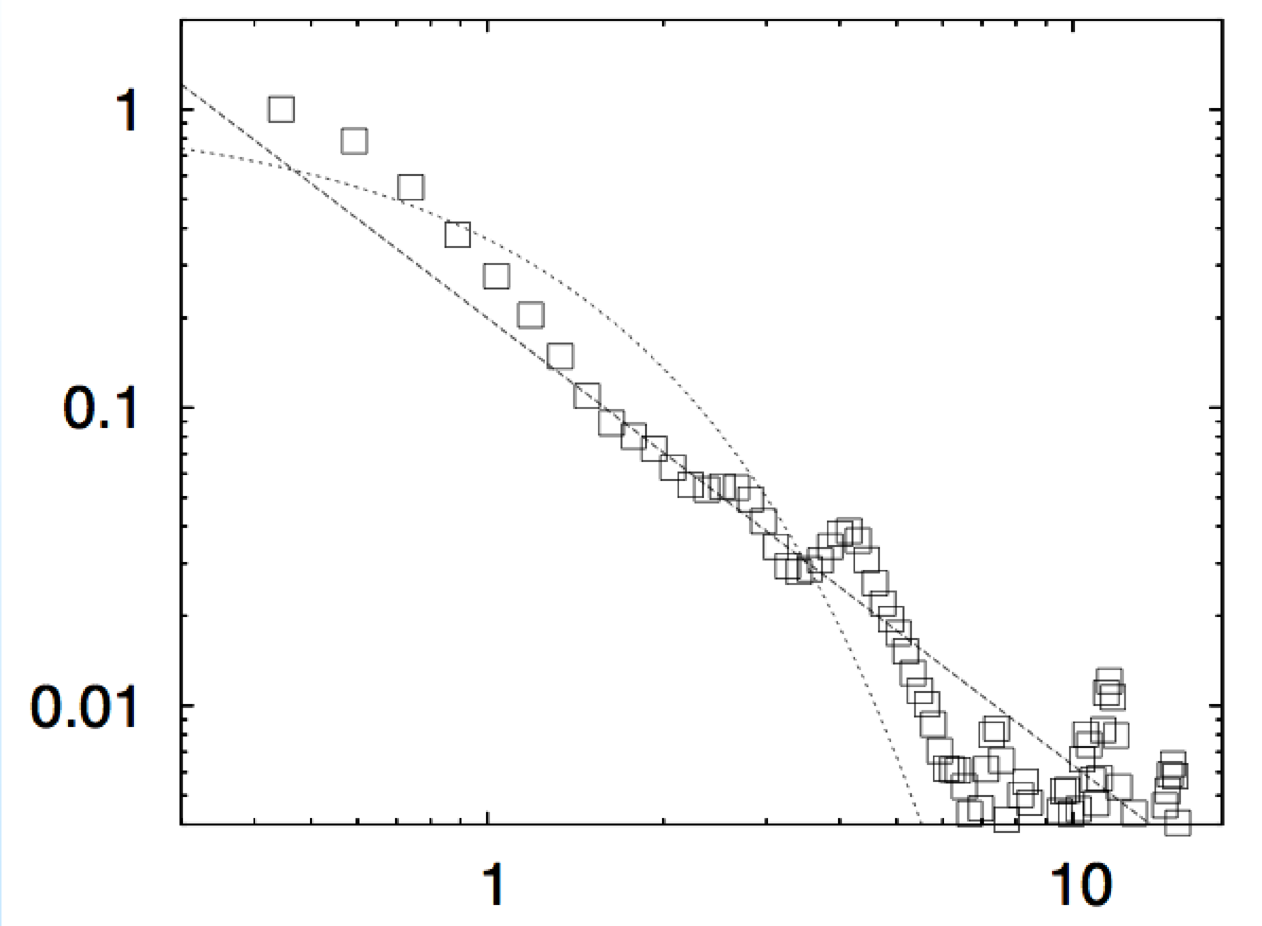


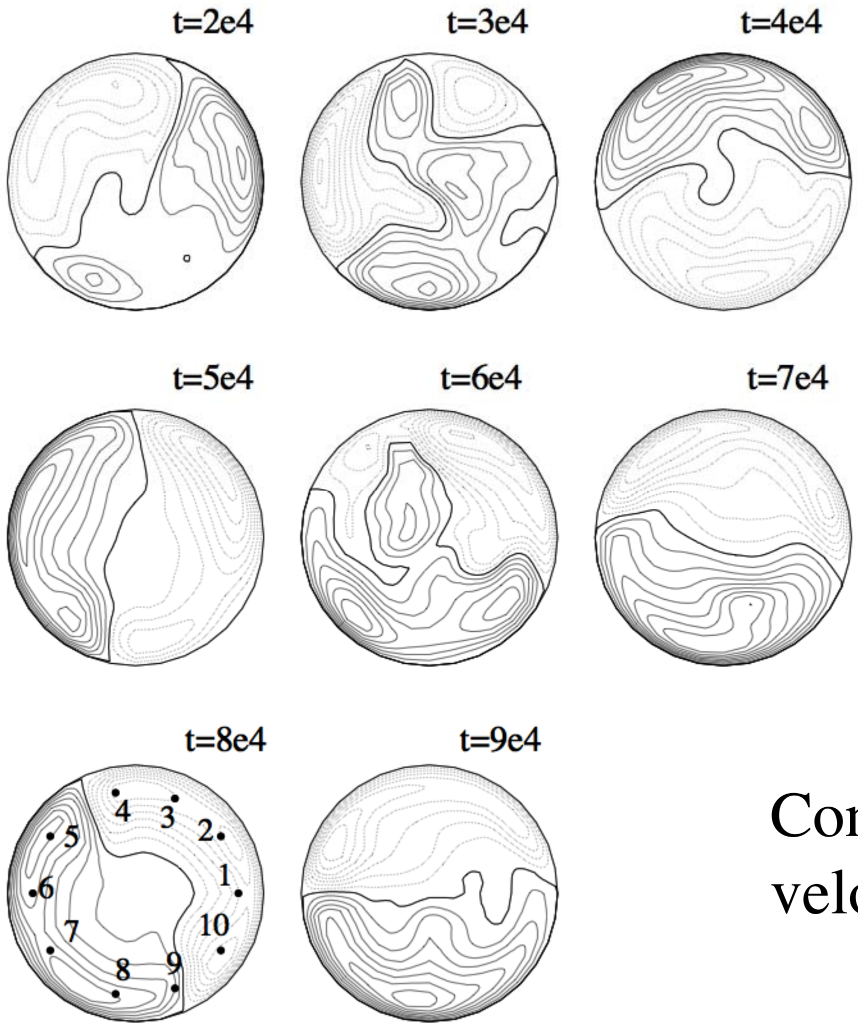
Figure 1: Sketch of the cell.


No noise
0.3
0
−0.3
2000
4000
u_z
With noise
0.25
0
-0.25
0
2.5e4
5.0e4
t
7.5e4
1.0e5

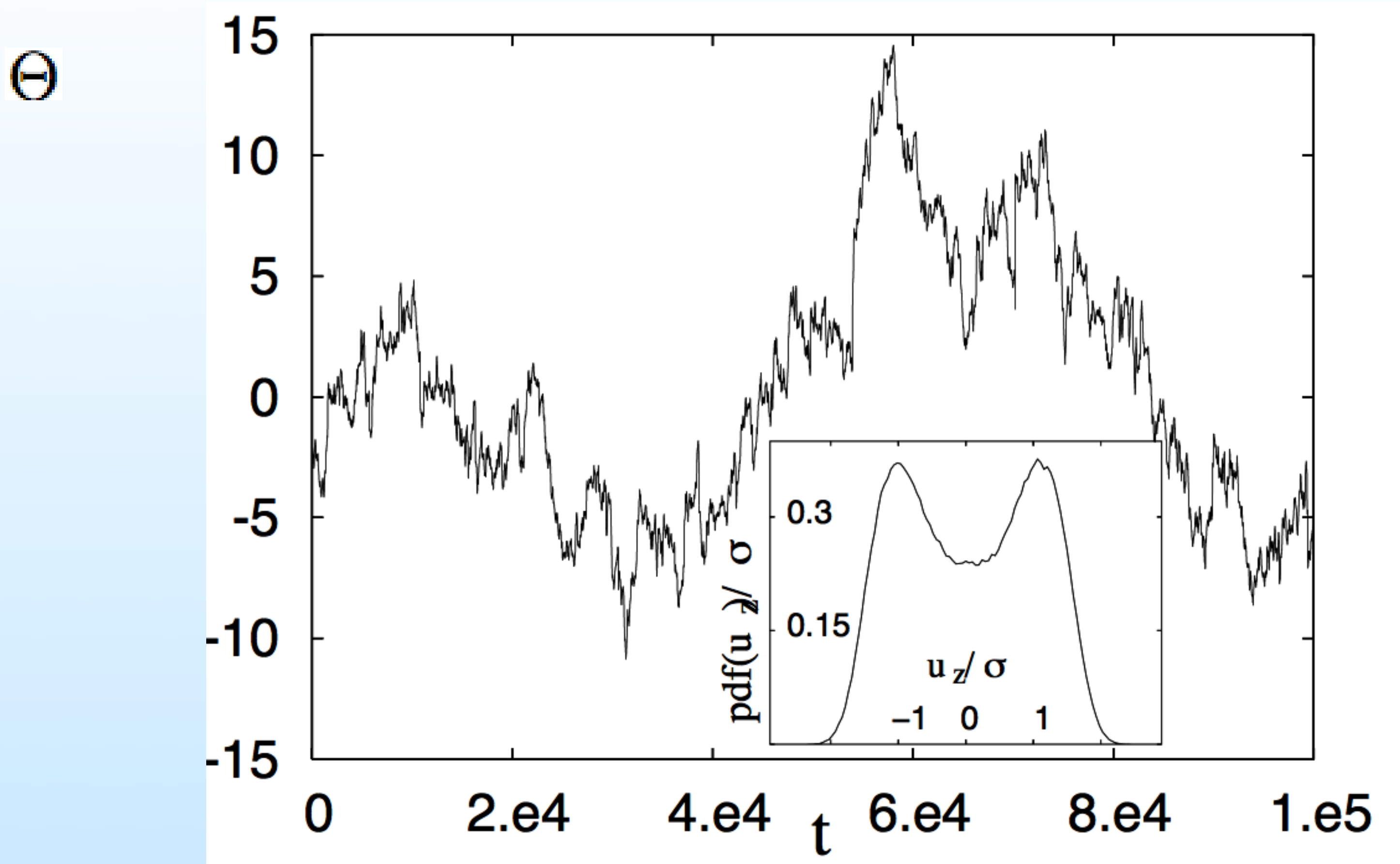


- Probability distribution of τ_L for the flow reversal, obtained by considering the signal in one probe. Good agreement with τ_L^{-1} (dashed line)

Reversal and reorientations



Contour plot of the vertical velocity for different times

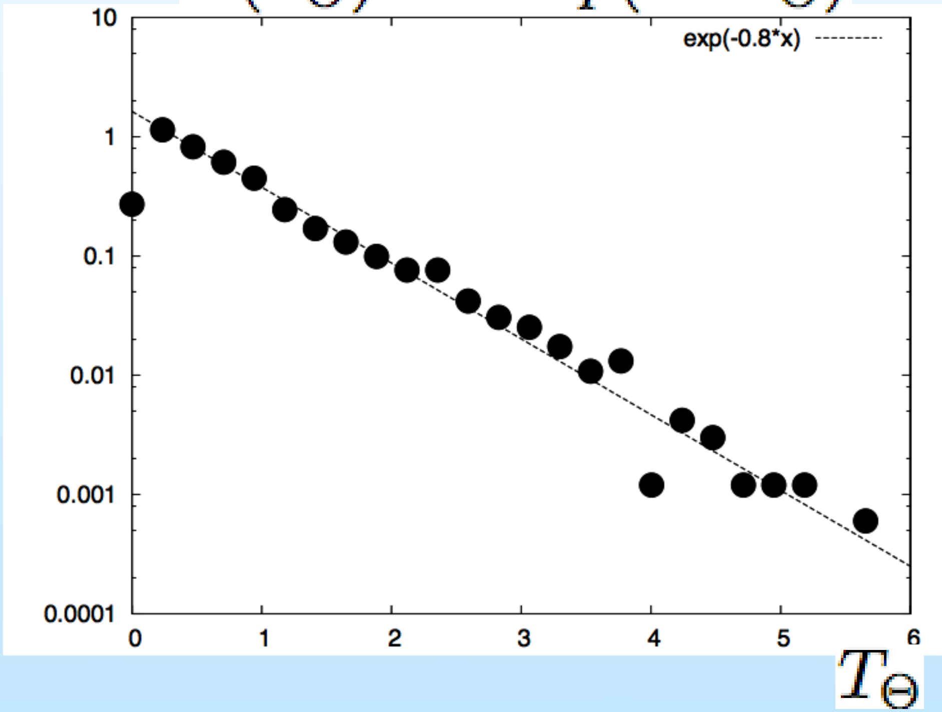


Estimated direction Θ of the interface between positive and negative vertical velocity u_z as a function of time (*same as in Cioni et. al. 1997, Brown et. al. 2005, Xia'talk*).

Following Brown et. al. 2005, we define T_Θ as the time between two events for which the following both conditions are satisfied:

1) $|\Theta(t+\Delta t)- \Theta(t)| > \Theta_s$
2) $|[\Theta(t+dt)- \Theta(t)]/dt| > \Theta_{dt}$

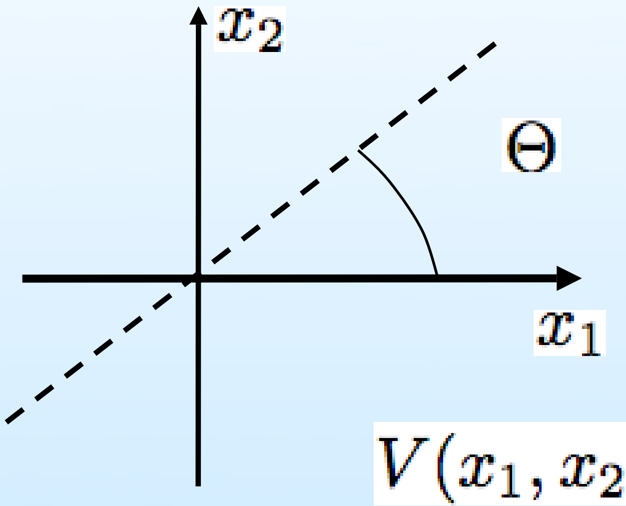
$$P(T_\Theta) = \lambda exp(-\lambda T_\Theta)$$



Same as in Brown et. al. 2005

Preliminary conclusion: adding white noise to RB turbulence in the temperature field enhances fluctuations (*expected)* with the right dynamical behaviour.

A simple model for the observed statistics



$$\frac{dx_i}{dt} = -\frac{\partial V}{\partial x_i} + noise$$

$$V(x_1, x_2) = -\alpha(x_1^2 + x_2^2)[R^2 - (x_1^2 + x_2^2)]$$

$$u_z = \frac{x_2}{\sqrt{x_2^2 + A}} \sim \frac{sin(\Theta)}{\sqrt{sin^2(\Theta) + A/R^2}}$$

A is a measure of the interface thickness

Equation for u_z

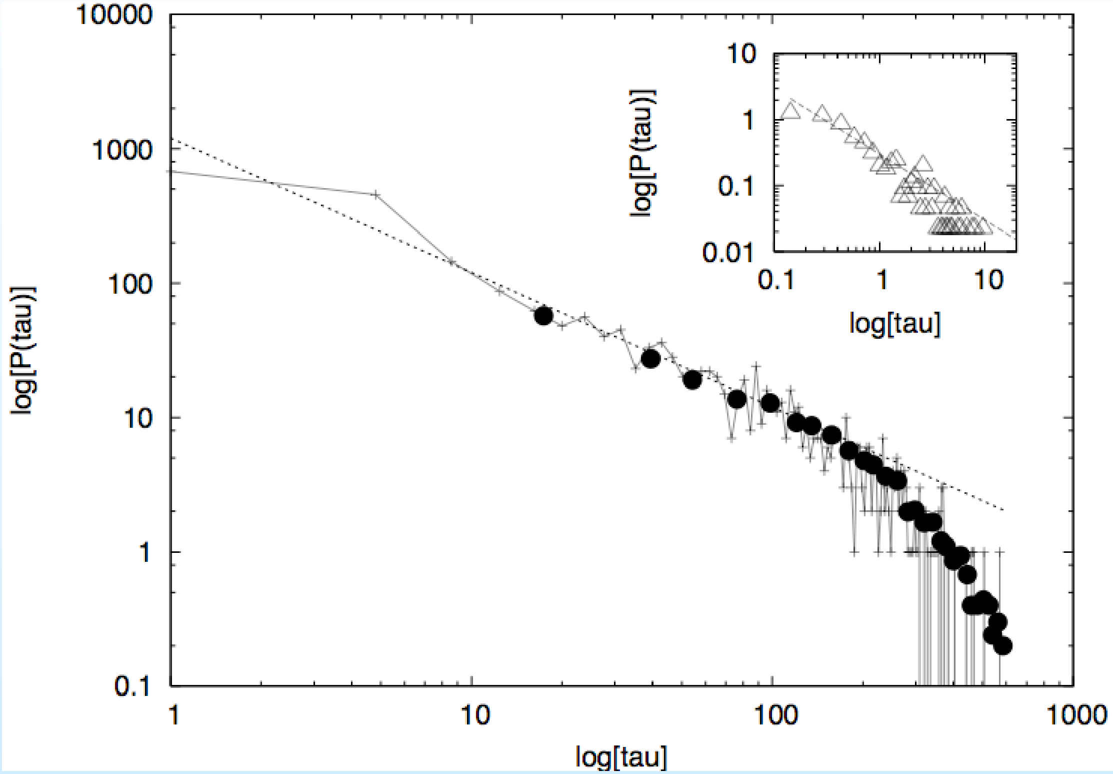
$$\frac{du_z}{dt} = au_z - (a + 4\alpha A)u_z^3 + noise$$

$$a = \alpha(2R^2 - 4x_1^2)$$

The parameter ***a*** can change sign.

The statistics of flow reversals is no longer defined by the large deviation theory (no need to be exponential!).

Similarity with the L-G equation.



Insert, data from DNS and

$$u_z = \frac{sin(\Theta(t))}{\sqrt{sin^2(\Theta) + 0.1}}$$

Probability distribution of τ_L for flow reversals. Numerical simulation of the simple model (black circles) against experimental data of Sreenivasan et. al. (2002).

Conclusions

- We have been able to reproduce existing experimental data by using DNS plus noise.
- Flow reversasl and reorientations can be explained in terms of the interface dynamics $\Theta(t)$.
- We propose a simple model to explain the observed statistcs; the model shows some links to the statistical properties of critical systems.