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**Conference and Euromech Colloquium #480**

**on**

**High Rayleigh Number Convection**

4 - 8 Sept., 2006, ICTP, Trieste, Italy

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# Homogeneous Rayleigh-Benard convection

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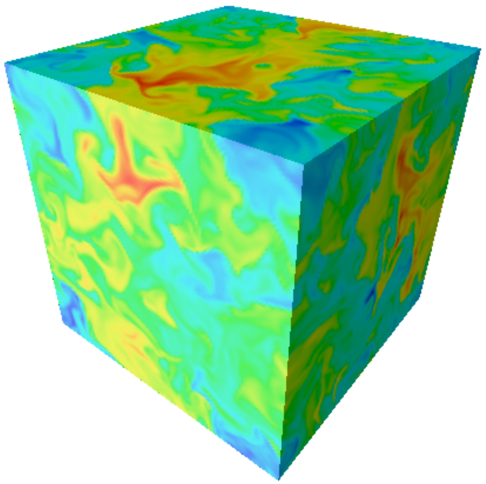
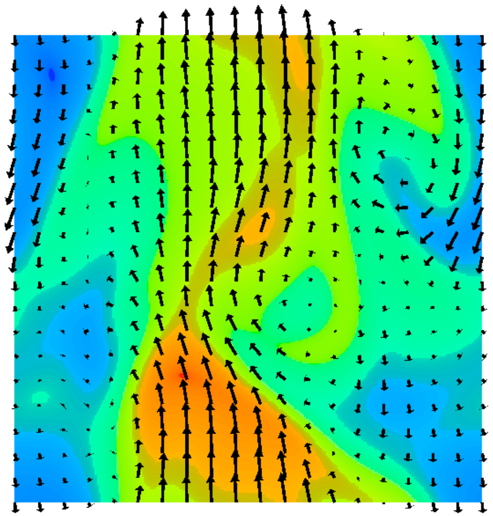
# Homogeneous Rayleigh-Bénard convection

## scaling, heat transport and structures





E. Calzavarini

and F. Toschi, D. Lohse, R. Tripiccione, C. R. Doering, J. D. Gibbon, A. Tanabe

**Euromech Colloquium #480 on High Rayleigh Number Convection**
September 4-8 2006 Trieste (IT)

# Motivation

- Small scale fluctuations ( Bolgiano-Obhukov or Kolmogorov ).
- Asymptotic (high Ra) scaling of Heat flux ( Kraichnan regime).
- Flow structures: role of the bulk compared to BL.

An unbounded (periodic) model of a convective cell may help:

Homogeneous Rayleigh-Bénard system

first studied by V.Borue and S.A.Orszag J.Sci.Comp. **12** 305 (1997).

# Outline

0. The Homogeneous RB model
1. Small scale fluctuations
2. Scaling of heat flux and Re vs. Ra and Pr
3. Dynamics of the flow

# Rayleigh-Bénard cell

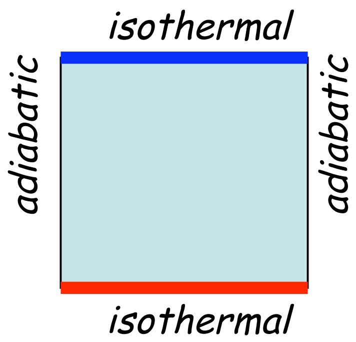
**Boussinesq set of equations:**

$$\partial_t \boldsymbol{v} + (\boldsymbol{v}\cdot\boldsymbol{\partial})\,\boldsymbol{v} = -\frac{\boldsymbol{\partial} p}{\rho} + \nu\,\boldsymbol{\partial}^2\boldsymbol{v} + \alpha g (T - T_0)\,\hat{x}_3$$

$$\boldsymbol{\partial}\cdot\boldsymbol{v} = 0$$

$$\partial_t T + (\boldsymbol{v}\cdot\boldsymbol{\partial})\,T = \kappa\,\boldsymbol{\partial}^2 T$$

**Standard Boundary conditions:**
Isothermal horizontal plates,
adiabatic vertical walls and
no-slip walls for velocity




$$T = T_h \quad \text{on the plane } (x_1, x_2, 0),$$

$$T = T_l \quad \text{on the plane } (x_1, x_2, H),$$

$$\boldsymbol{v} = 0 \quad \text{on the boundaries.}$$

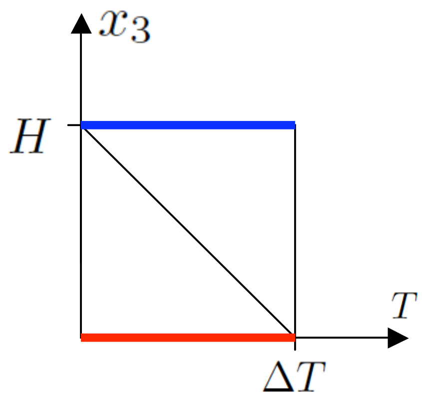
Temperature decomposition: static profile + fluctuation

$$T(\boldsymbol{x}) \equiv \theta(\boldsymbol{x}) + T_c(x_3) \equiv \theta(\boldsymbol{x}) - \frac{\Delta T}{H}x_3$$

Top-bottom thermal gap

Cell height

fluctuation $\vartheta$ vanishes on top/bottom plates

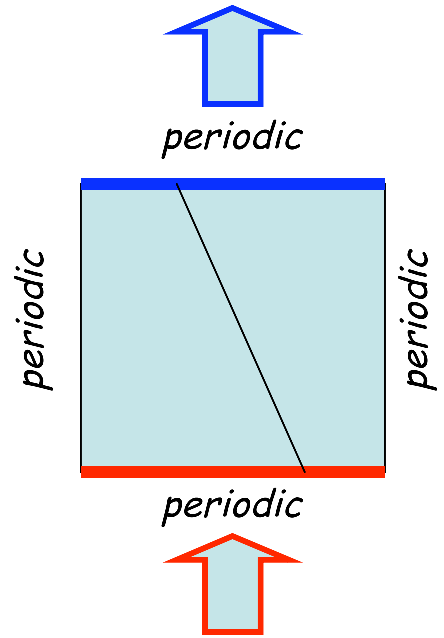


# No boundary: The homogeneous Rayleigh-Bénard model

1) 3-Dimensional cubic domain of size $H$.

2) $\boldsymbol{v}$ and $\theta$, periodic on the boundaries.

3) Keep the linear temperature background




Boussinesq system

$$\partial_t \boldsymbol{v} + (\boldsymbol{v}\cdot\boldsymbol{\partial})\,\boldsymbol{v} = -\frac{\boldsymbol{\partial} p}{\rho} + \nu\,\boldsymbol{\partial}^2\boldsymbol{v} + \alpha g\theta\,\hat{\boldsymbol{x}}_3$$

$$\partial_t \theta + (\boldsymbol{v}\cdot\boldsymbol{\partial})\,\theta = \kappa\,\boldsymbol{\partial}^2\theta + \frac{\Delta T}{H}\,\boldsymbol{v}\cdot\hat{\boldsymbol{x}}_3$$

*In turbulent conditions* ⇒ *model for the RB bulk*

# 1. Small scale fluctuations

Scaling of structure functions

$$S_p(r) \equiv \langle [(\boldsymbol{v}(\boldsymbol{r}+\boldsymbol{x}) - \boldsymbol{v}(\boldsymbol{x})) \cdot \boldsymbol{r}]^p \rangle$$

$$T_p(r) \equiv \langle [(\theta(\boldsymbol{r}+\boldsymbol{x}) - \theta(\boldsymbol{x}))]^p \rangle$$

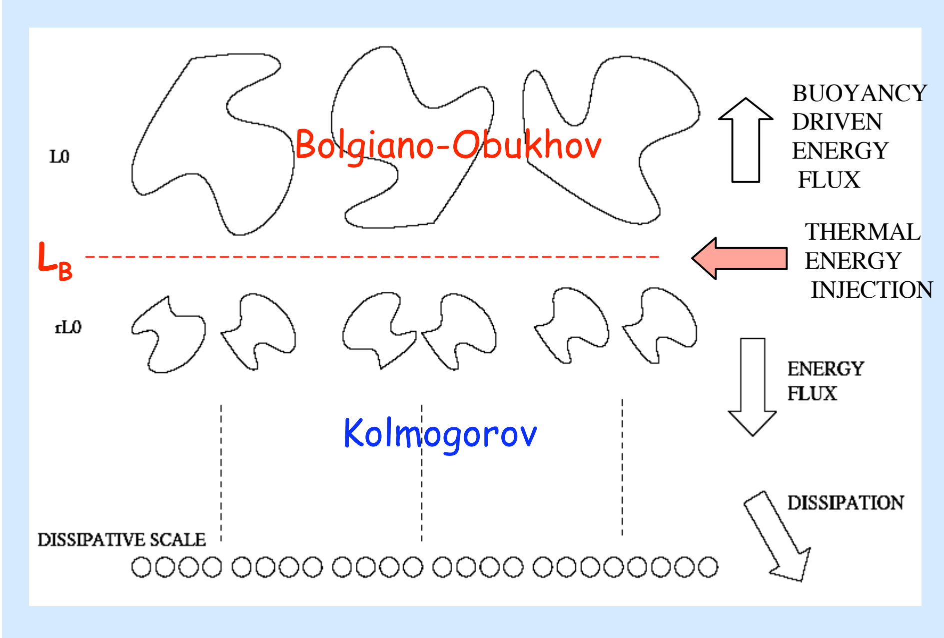
# Energy cascade in convective turbulence

Energy injection is mainly at an intermediate scale $L_B$

2 inertial sub-ranges

$$\eta << L_B << L_0$$

$$L_B \equiv \epsilon_v^{5/4} \epsilon_\theta^{-3/4} (\alpha g)^{-3/2}$$




Kolmogorov '41 scaling in:

$$\eta << r << L_B \qquad S_p(r) \sim r^{p/3}, \quad T_p(r) \sim r^{p/3}$$

Bolgiano-Obukhov scaling in:

$$L_B << r << L_0 \qquad S_p(r) \sim r^{3p/5}, \quad T_p(r) \sim r^{p/5}$$

# Energy cascade in the RB cell

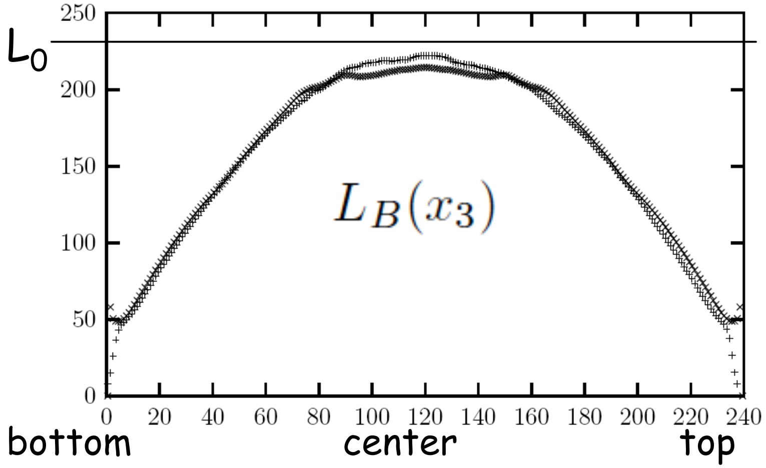
## Real RB systems are highly non-homogeneous:

- Bolgiano scale $L_B$ is position dependent
- $L_B$ reaches its maximum at the cell center

Evidences found in experiments:

E. Ching, K. Chui, X. Shang, X. Qui, P. Tong, and K. Xia, J. Turb **5**, 027 (2004)

and numerics :




Where $L_B$ is small (in the BL) Bolgiano-Obukhov is likely to be detected.

In the bulk Kolmogorov scaling

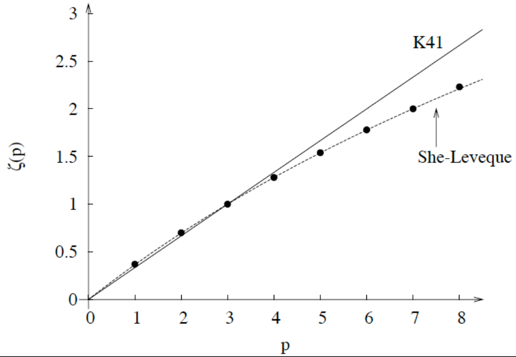
R. Benzi, F. Toschi, and R. Tripiccione, J. Stat. Phys. **93**, 901 (1998).

E.Calzavarini, F.Toschi, R.Tripiccione, Phys.Rev.E, **66**, 016304 (2002).

# Energy cascade in Homogeneous RB

1) The Bolgiano length is comparable to the integral scale: $L_B \sim L_0$ .

2) The Kolmogorov scaling regime + Intermittency shows up for velocity structure functions, T (or $\vartheta$ ) behaves as a passive tracer.




The exact relations:

$$\frac{L_B}{L_0} \simeq \frac{Nu^{1/2}}{Ra^{1/4} Pr^{1/4}}$$

$L_B$ remains constant if:

$$Nu \sim Ra^{1/2} Pr^{1/2} \ ,$$

HRB turbulence not distinguishable
from other anisotropic large scale forced flows.

SO(3) analysis: L.Biferale, E.Calzavarini, F.Toschi, R.Tripiccione, Europhys. Lett., **64**, 461 (2003).

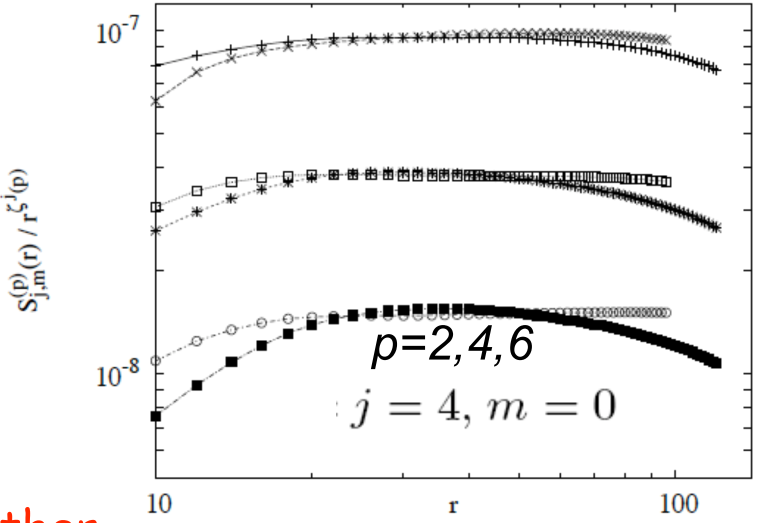
# What remains of the anisotropic forcing?

It is possible to disentangle isotropic from anisotropic mean fluctuations through the tool of SO(3) group decomposition of physical observables

$$S^{(p)}_{jm}(r) \equiv \int S^{(p)}(\boldsymbol{r})\, Y_{jm}(\hat{\boldsymbol{r}})\, d^3r \qquad S^{(p)}_{jm}(r) \simeq c_{jm} \cdot r^{\zeta^j(p)}$$

We measured the behavior of anisotropic components, in HRB turbulent convection.

Comparison between the HRB and The **Random Kolmogorov Flow**




HRB turbulence is not distinguishable from other anisotropic large scale forced flows.

# 2.
# Heat flux and Reynolds number vs. Ra and Pr

# The Ultimate regime of convection

In asymptotically ( Ra -> ∞ ) high turbulent conditions

Independency of the heat flux on viscosity
and thermal diffusivity is expected

$$Nu \sim Ra^{1/2} Pr^{1/2} \ ,$$

Never detected in standard RB experiments or DNS

First predicted by: R.Kraichnan (1962) + log corrections

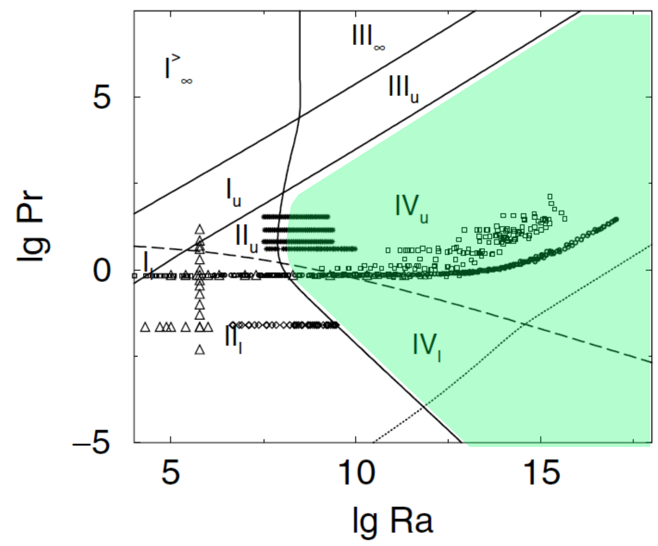
Also proposed by Spiegel (1972) and consistent with Grossman & Lohse (2000).

# Asymptotic scaling prediction

## Grossmann-Lohse model (2000)

A) Decomposition:

$$\epsilon_u = \epsilon_{u,bulk} + \epsilon_{u,BL}$$
$$\epsilon_\theta = \epsilon_{\theta,bulk} + \epsilon_{\theta,BL}$$




B) Dimensional estimate:

$$\epsilon_{v,bulk} \sim \frac{U^3}{H}, \qquad \epsilon_{v,BL} \sim \nu \frac{U^2}{\lambda_v^2}\frac{\lambda_v}{H},$$

$$\epsilon_{\theta,bulk} \sim \frac{U\Delta T^2}{H}, \qquad \epsilon_{\theta,BL,} \sim \kappa \frac{\Delta T^2}{H^2}(RePr)^{1/2}$$

+ exact relations from the spatial and time averaging of equations of motion

C) In asymptotic regime bulk contributions dominate:

$$Nu \sim Ra^{1/2}Pr^{1/2}\ , \qquad Re \sim Ra^{1/2}Pr^{-1/2}$$

# Our Direct Numerical Simulations

## Database for HRB

| Pr | Ra | $T_{eddy}$ |
|---|---|---|
| 1 | $1.4\ 10^7$ | 89 |
| 1 | $4.5\ 10^6$ | 104 |
| 1 | $2.2\ 10^6$ | 58 |
| 1 | $8.6\ 10^5$ | 74 |
| 1 | $2.2\ 10^5$ | 58 |
| 1 | $9.6\ 10^4$ | 166 |
| | | |
| 4 | $1.4\ 10^7$ | 156 |
| 3 | $1.4\ 10^7$ | 78 |
| 1 | $1.4\ 10^7$ | 89 |
| 1/3 | $1.4\ 10^7$ | 98 |
| 1/10 | $1.4\ 10^7$ | 65 |

Numerical resolution
$L \times L \times L = 240^3$

Performed on
APEmille
128 processors
~4 hours per $T_{eddy}$
Total: ~150 days

Strong fluctuations
requires a large number of $T_{eddy}$
for convergence of the averages

# Measure of Nu and Re

Nusselt definition:

$$Nu = \frac{1}{\kappa \Delta T \ L^{-1}} \left( \langle u_3 T \rangle_{A,t}(z) - \kappa \langle \partial_3 T \rangle_{A,t}(z) \right)$$

a) $$Nu = \frac{\langle u_3 \theta \rangle_{V,t}}{\kappa \Delta T \ L^{-1}} - 1$$

b) $$Nu = \epsilon_u \ L^4 \ Pr^2 / Ra$$

c) $$Nu = \epsilon_\theta \ L^2 / (\kappa \ \Delta T^2)$$

Thermal and kinetic dissipations:

$$\epsilon_\theta = \kappa \left\langle (\partial_i \theta)^2 \right\rangle_V$$

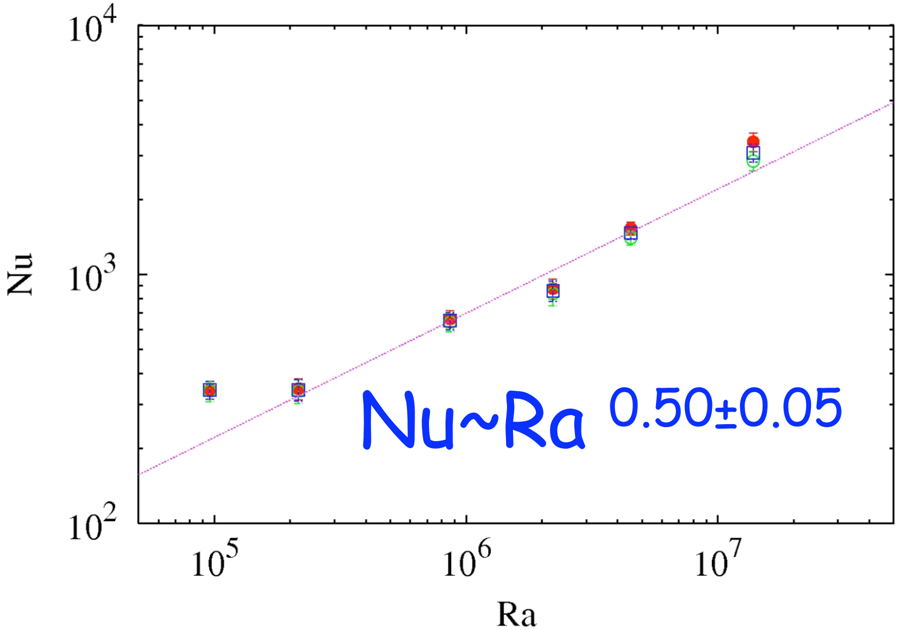
$$\epsilon_u = \nu \left\langle (\partial_i u_j)^2 \right\rangle_V$$

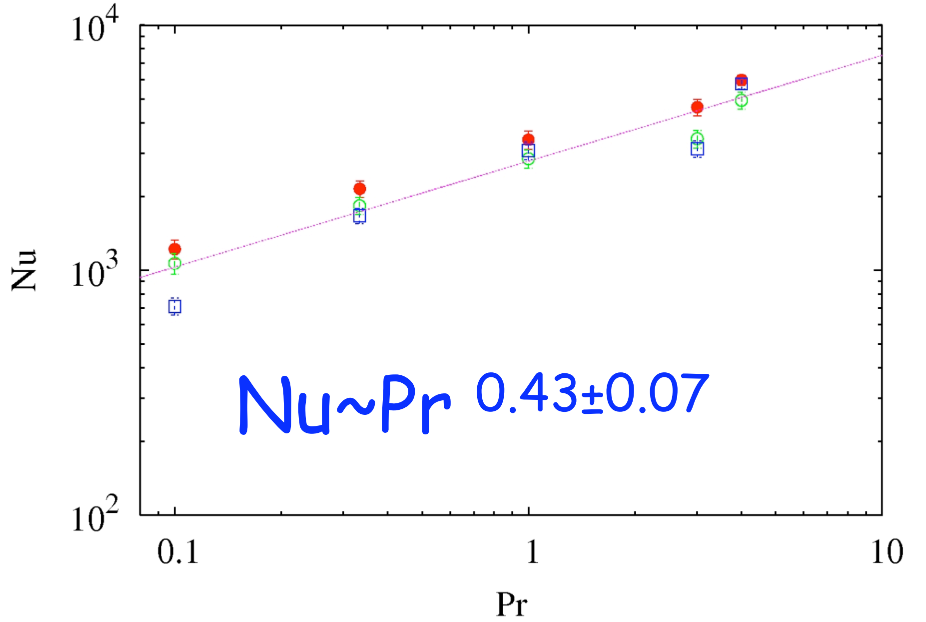
Reynolds is defined on fluctuation:

*note: no mean velocity profile in the system.*

$$Re = \frac{\left\langle \mathbf{u}^2 \right\rangle^{1/2} \ L}{\nu}$$

# Results: Nusselt vs. Ra & Pr




Nu evaluated in three independent ways:

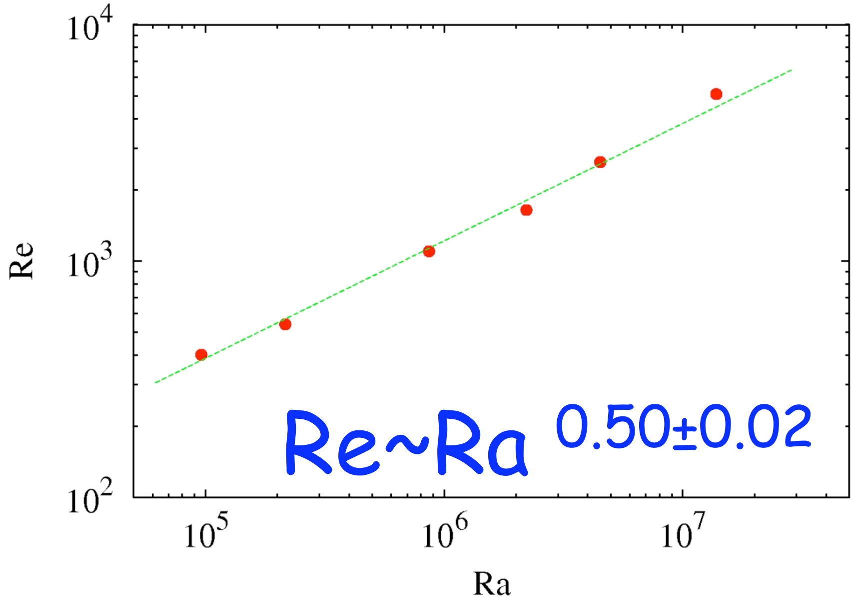
- ● (red) $Nu = \frac{\langle u_3 \theta \rangle_{V,t}}{\kappa \Delta T \ L^{-1}} - 1$
- ○ (green) $Nu = \epsilon_u \ L^4 \ Pr^2/Ra$
- □ (blue) $Nu = \epsilon_\theta \ L^2/(\kappa \ \Delta T^2)$

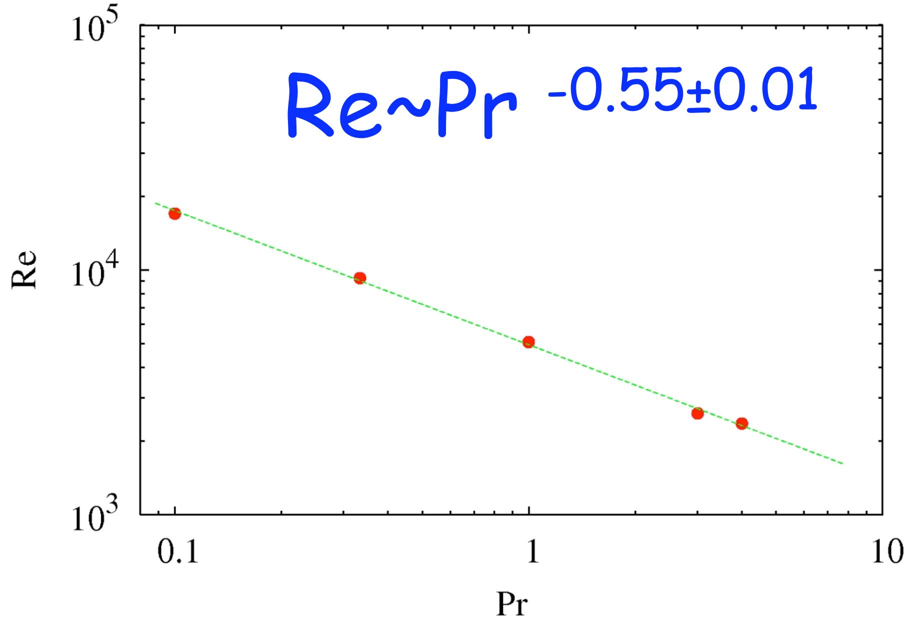
**Consistent with the "Ultimate regime"**

Although it happens at small Ra values: $Ra_{bulk} < Ra_{tb}$

D.Lohse, F.Toschi, Phys. Rev. Lett. **90**, 034502 (2003).
E.Calzavarini, D.Lohse, F.Toschi, R.Tripiccione , Phys. Fluids **17**, 5, 055107 (2005).

# Reynolds vs. Ra & Pr

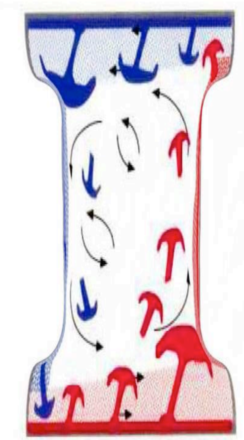


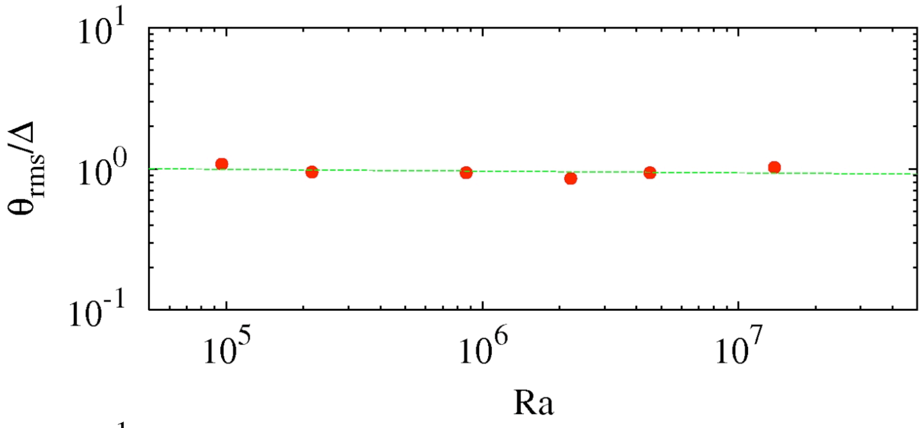

$$Re = \frac{\langle \mathbf{u}^2 \rangle^{1/2} \; L}{\nu}$$

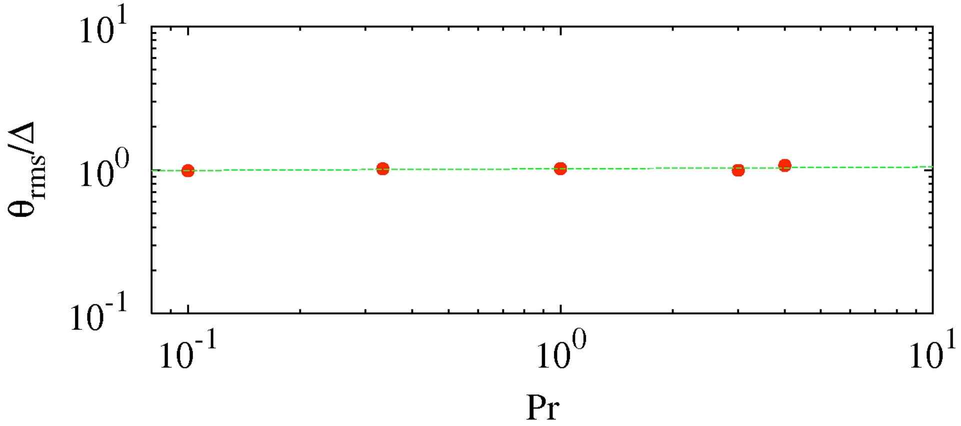
Turbulent convection in open vertical channels has same scaling!

( M. Gibert *et al.* "*High-Rayleigh-Number convection in a vertical channel*" *PRL 2006* )



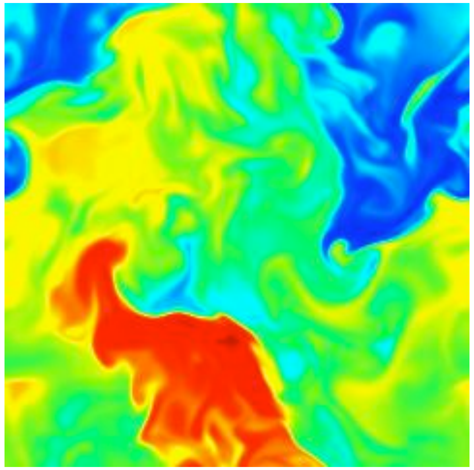
# Thermal fluctuations




In HRB thermal fluctuations dominated by the large-scale.

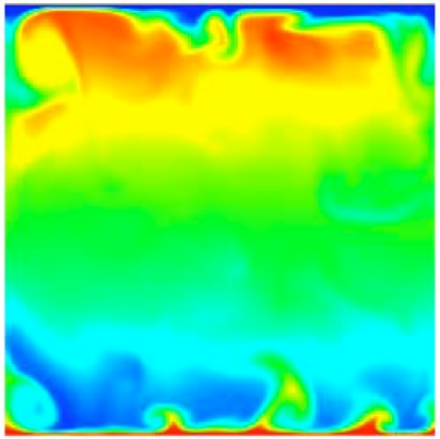
$$\frac{\langle \theta^2 \rangle^{1/2}}{\Delta T} = 1$$



$T = T_b(z) + \vartheta$

*Ra = 1.4 ·10⁷*
*Pr = 1*

No small coherent thermal structures (plumes), but **Large jets**



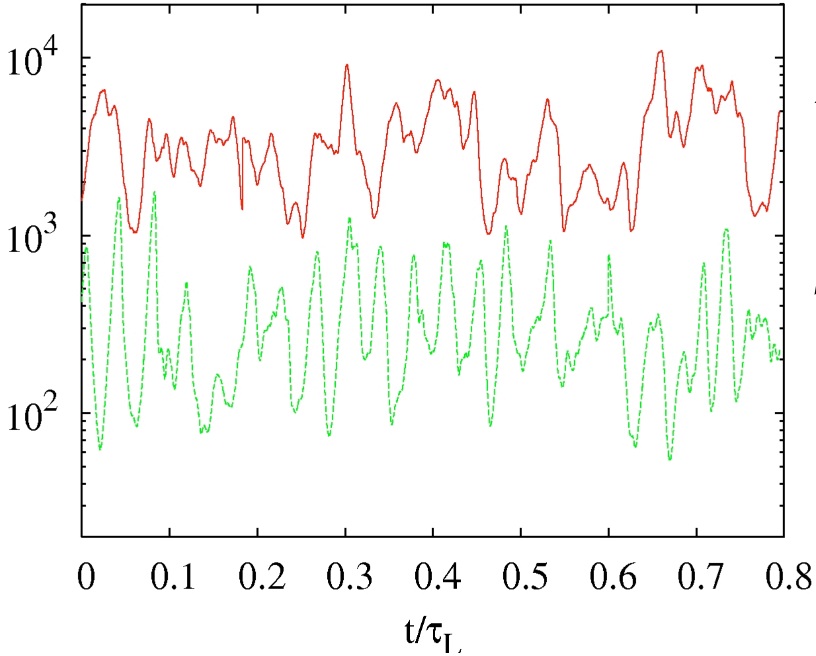
RB no-slip b.c.
*Ra = 3.5 ·10⁷*

# 3. Dynamics

# Nusselt time behavior

(top)
Ra = 1.4 · $10^7$
Pr = 1

(bottom)
Ra = 9.6 · $10^4$
Pr = 1



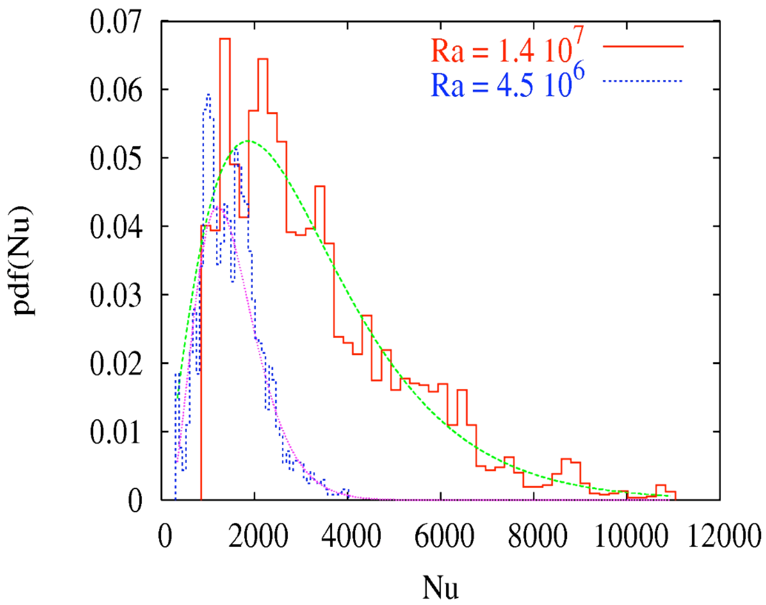

Nu(t)

Nusselt time behavior:
very large and skewed fluctuations

**Probability density function**

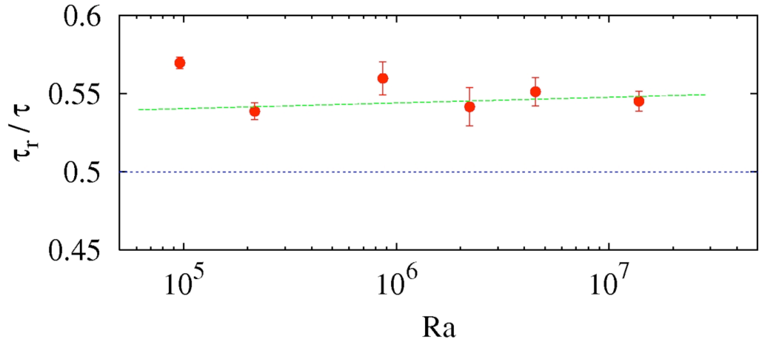
Fit with Gamma distribution
Pdf(x) = $x^a$ exp(-bx)

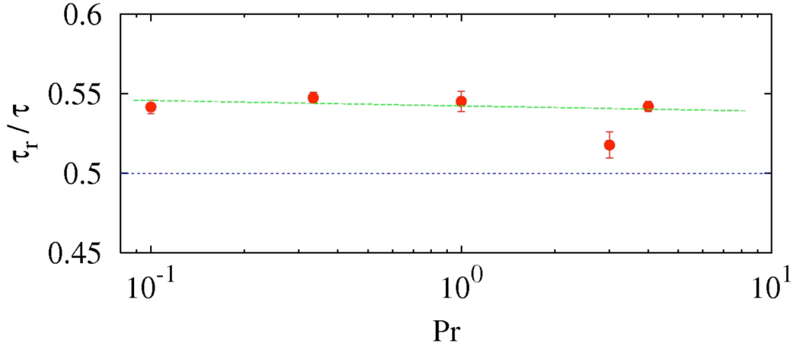
( S.Aumaitre & S.Fauve, Europhys. Lett. (2004) )



# More on Nusselt dynamics

Normalized rising time $\tau_r / \tau$




Independent from Ra and Pr and

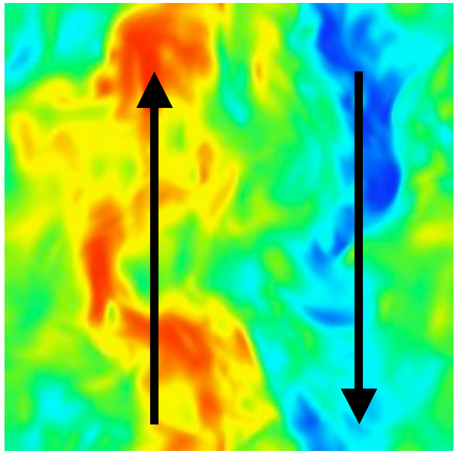
$\tau_r / \tau$ >0.5 mean: 54 %

Loading time longer than discharge time

**What is happening inside the flow?**

A typical snapshot:
Vertical section of vertical velocity

*Ra = 1.4 ·10⁷*
*Pr = 1*



Large and accelerating columnar structures are often present in the ascending phase of heat flux

*Already noticed by Borue and Orszag (1997)*

# A family of unstable solutions: **elevator modes**

Assumption:

*The solution does depend on x,y,t and not on the vertical coordinate z*

Boussinesq system decouples:

$$
\begin{aligned}
\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\partial} \boldsymbol{u} &= -\nabla p + \nu \Delta \boldsymbol{u} \\
\partial_t v_3 + \boldsymbol{u} \cdot \boldsymbol{\partial} v_3 &= \nu \Delta v_3 + \alpha g \theta \\
\partial_t \theta + \boldsymbol{u} \cdot \boldsymbol{\partial} \theta &= \kappa \Delta \theta + \frac{\Delta T}{H} v_3
\end{aligned}
$$

- Unforced 2-D N-S equation for the horizontal components.
- Vertical component $v_3$ and temperature linearly coupled and passively advected by the horizontal velocity components *which decrease in time.*

E.C., C. R. Doering, J. D. Gibbon, D. Lohse, A Tanabe and F. Toschi Phys. Rev. E **73** 035501 R (2006).

# Elevator modes

The periodic system admits a class of exact, separable and unstable solutions

$$\boldsymbol{u} = 0$$

$$v_3(x_1, x_2; t) = v_0 \, e^{\kappa\lambda t} \sin(\boldsymbol{k} \cdot \boldsymbol{x})$$

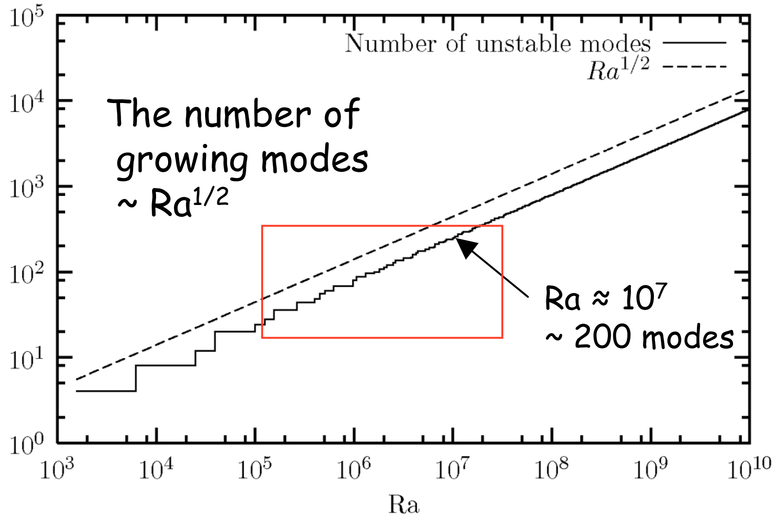
$$\theta(x_1, x_2; t) = \theta_0 \, e^{\kappa\lambda t} \sin(\boldsymbol{k} \cdot \boldsymbol{x})$$

$\Rightarrow$ $Nu \propto e^{2\kappa\lambda t}$

If Pr=1 $\quad \lambda = \frac{\sqrt{Ra}}{L^2} - \boldsymbol{k}^2 \qquad \mathbf{k} = \frac{2\pi}{L}(n_x, n_y)$

If $Ra > (2\pi)^4 \approx 1558 \Rightarrow \lambda > 0$

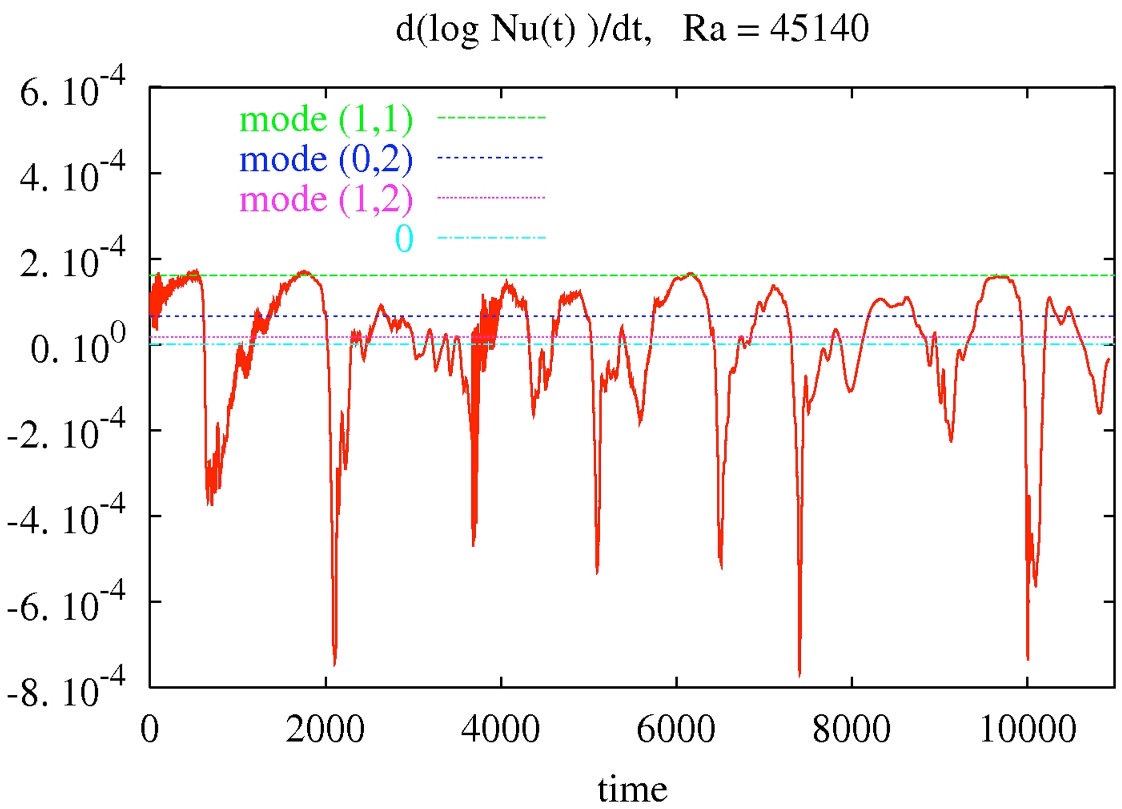
At high Ra the number of unstable modes is large (and the different modes corresponds to very near exponents)




Are they relevant for the dynamics in the Ra-range explored?

# Exponential growth and fast collapse

Ra = 4.5·10^4




$$\mathbf{k} = \frac{2\pi}{L}\,(n_x, n_y)$$

$$\frac{2\pi}{L}(0,1)$$

$$\frac{2\pi}{L}(1,1)$$

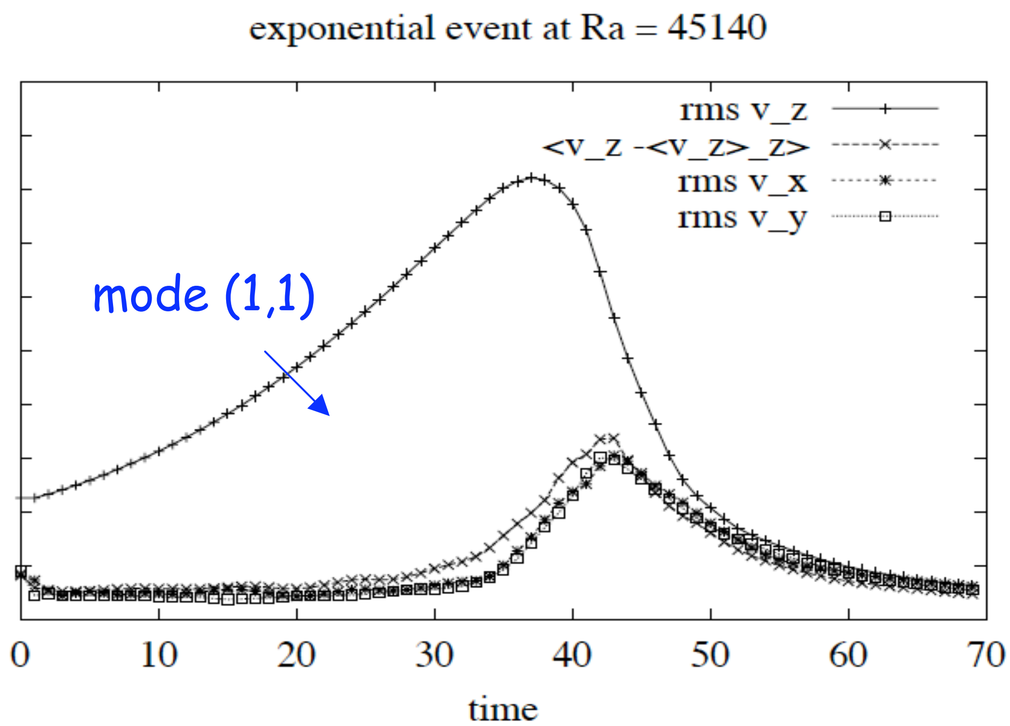
$$\frac{2\pi}{L}(0,2)$$

$$\frac{2\pi}{L}(1,2)$$

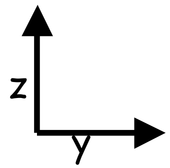
1. Elevator modes are present in the HRB system
2. Secondary instability mechanism leads to a chaotic statistically stationary dynamics.



# Exponential growth and fast collapse (2)




Temperature movie



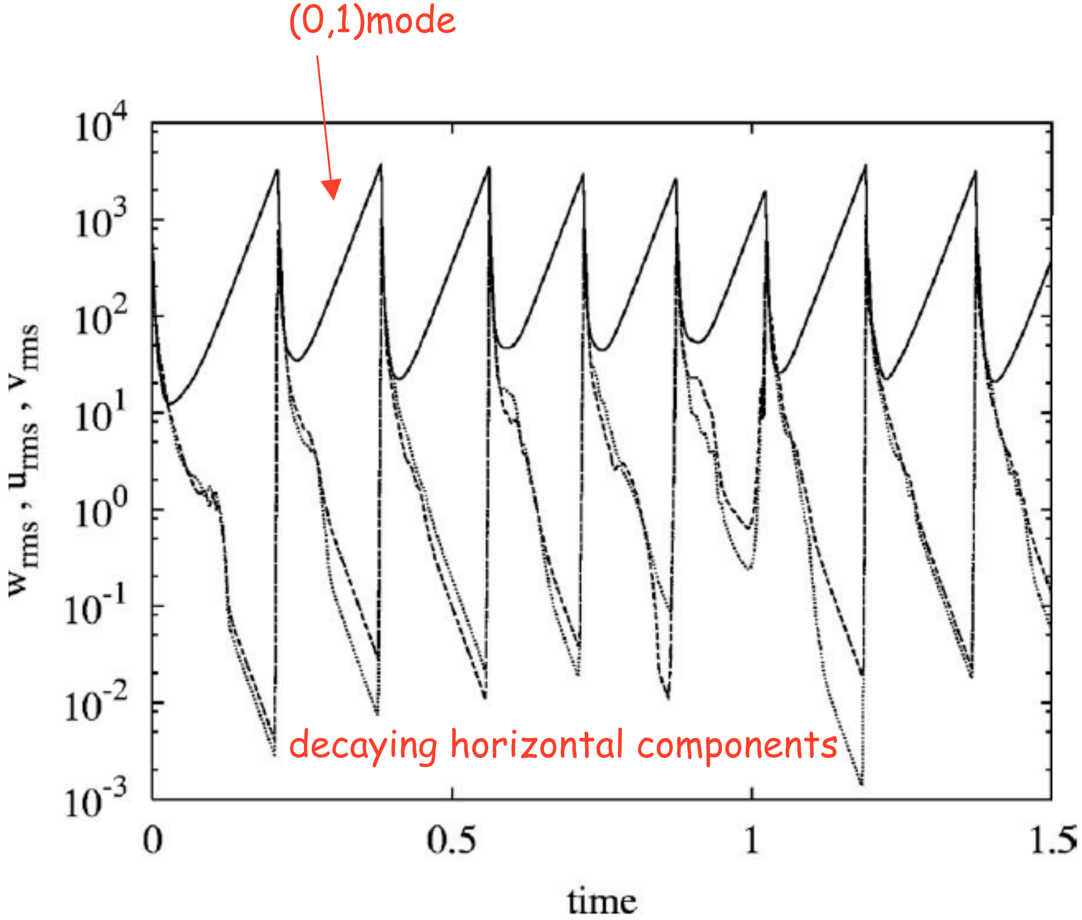

Similar to:

- Zero-Prandtl RB system at the onset of convection (O.Thual JFM 1992) and bursting mechanism (*Critical bursting* ) proposed by K. Kumar, P. Pal, S. Fauve Europhys. Lett. **74** 1020 (2006).

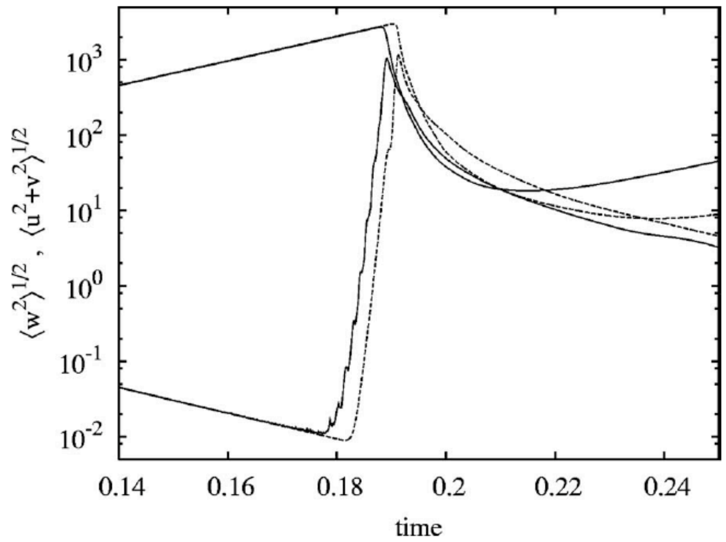
# Behavior at low Ra

At $Ra \gtrsim Ra_c$ only one unstable mode active -> (0,1)-mode

Secondary instability of growing mode still present
and very sensitive to numerical resolution and round-off errors.




double vs. single precision



# Summary/final remarks

1) Turbulent fluctuations in HRB are large scale forced and with statistical properties of homogeneous isotropic turbulence ( K41 + Intermittency)

2) Nu(Ra, Pr) and Re(Ra,Pr) compatible with asymptotic scaling $Nu \approx Ra^{1/2} Pr^{1/2}$ and $Re \approx Ra^{1/2} Pr^{-1/2}$.

3) Elevator modes are responsible of strong fluctuations (or bursting).

4) At small-Ra bursting dominate the dynamics and HRB is no longer a valid model for bulk RB behavior

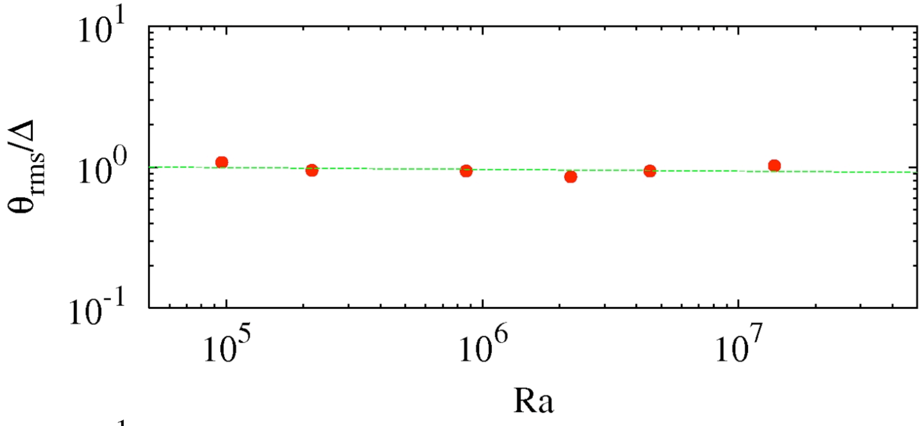
# References
## on the *homogeneous convective cell*

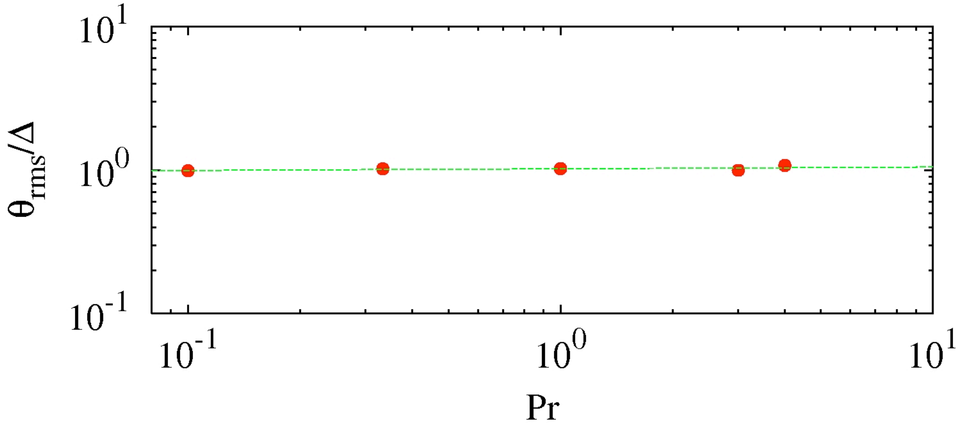
- V.Borue, S.A.Orszag, 1997 J.Sci. Comput. **12**, 305.
- A.Celani *et al.*, 2002 Phys. Rev.Lett. 88, 054503. For the **2-Dim case**
- D.Lohse, F.Toschi, 2003 Phys. Rev. Lett. **90,** 034502.
- L.Biferale, E.Calzavarini, F.Toschi, R.Tripiccione, 2003 Europhys. Lett., **64**, 461.
- E.Calzavarini, D.Lohse, F.Toschi, R.Tripiccione , 2005 Phys. Fluids **17,** 5, 055107.
- E.Calzavarini, C. R. Doering, J. D. Gibbon, D. Lohse, A Tanabe and F. Toschi 2006 Phys. Rev. E **73** 035501 R.

On random bursting:

- K. Kumar, P. Pal and S. Fauve, 2006 Europhys. Lett. **74**, 6, 1020.

# Thermal fluctuations



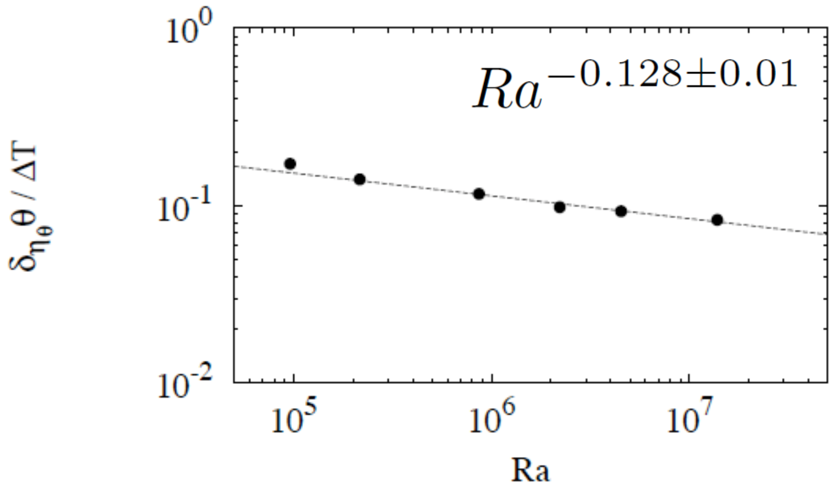

$$\frac{\langle\theta^2\rangle^{1/2}}{\Delta T} = 1$$

While G-L predicts:

$$\frac{\langle\theta^2\rangle^{1/2}}{\Delta T} \sim (Pr\ Ra)^{-1/8}$$

with the assumption:

$$\delta_{\eta_\theta}\theta \simeq \langle\theta^2\rangle^{1/2}$$




In HRB thermal fluctuations are dominated by the large-scale.

$$\epsilon_\theta \simeq \kappa\left(\delta_{\eta_\theta}\theta/\eta_\theta\right)^2 \text{ and } \eta_\theta \simeq \left(\kappa^3/\epsilon_v\right)^{1/4}$$