The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1771 -21

Conference and Euromech Colloquium #480

on

High Rayleigh Number Convection

4 - 8 Sept., 2006, ICTP, Trieste, Italy

Dissipation rates in Rayleigh-Benard convection

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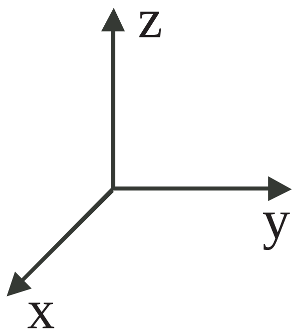
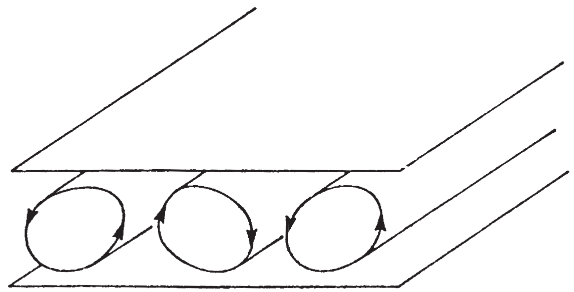
These are preliminary lecture notes, intended only for distribution to participants

Dissipation Rates in Rayleigh-Bénard Convection

A. Tilgner, University of Göttingen

with T. Hartlep, A. Ickler and S. Schmitz





Exact Relations

$$\epsilon_u = \frac{1}{V}\int \nu(\partial_i u_j)(\partial_j u_i)dV$$

$$\epsilon_\theta = \frac{1}{V}\int \kappa(\partial_i \theta)(\partial_i \theta)dV$$

$$\epsilon_u = \frac{\nu^3}{d^4}(Nu - 1)Ra/Pr^2$$

$$\epsilon_\theta = \kappa\frac{\Delta T^2}{d^2}Nu$$

Estimates for the dissipation

$$\epsilon_u = \epsilon_{u,bl} + \epsilon_{u,bulk} = \frac{1}{V}\int_{bl} \nu(\partial_i u_j)(\partial_j u_i)dV + \frac{1}{V}\int_{bulk} \nu(\partial_i u_j)(\partial_j u_i)dV$$

$$\epsilon_\theta = \epsilon_{\theta,bl} + \epsilon_{\theta,bulk} = \frac{1}{V}\int_{bl} \kappa(\partial_i \theta)(\partial_i \theta)dV + \frac{1}{V}\int_{bulk} \kappa(\partial_i \theta)(\partial_i \theta)dV$$

$$\epsilon_{u,bulk} \propto \frac{U^3}{d} = \frac{\nu^3}{d} Re^3$$

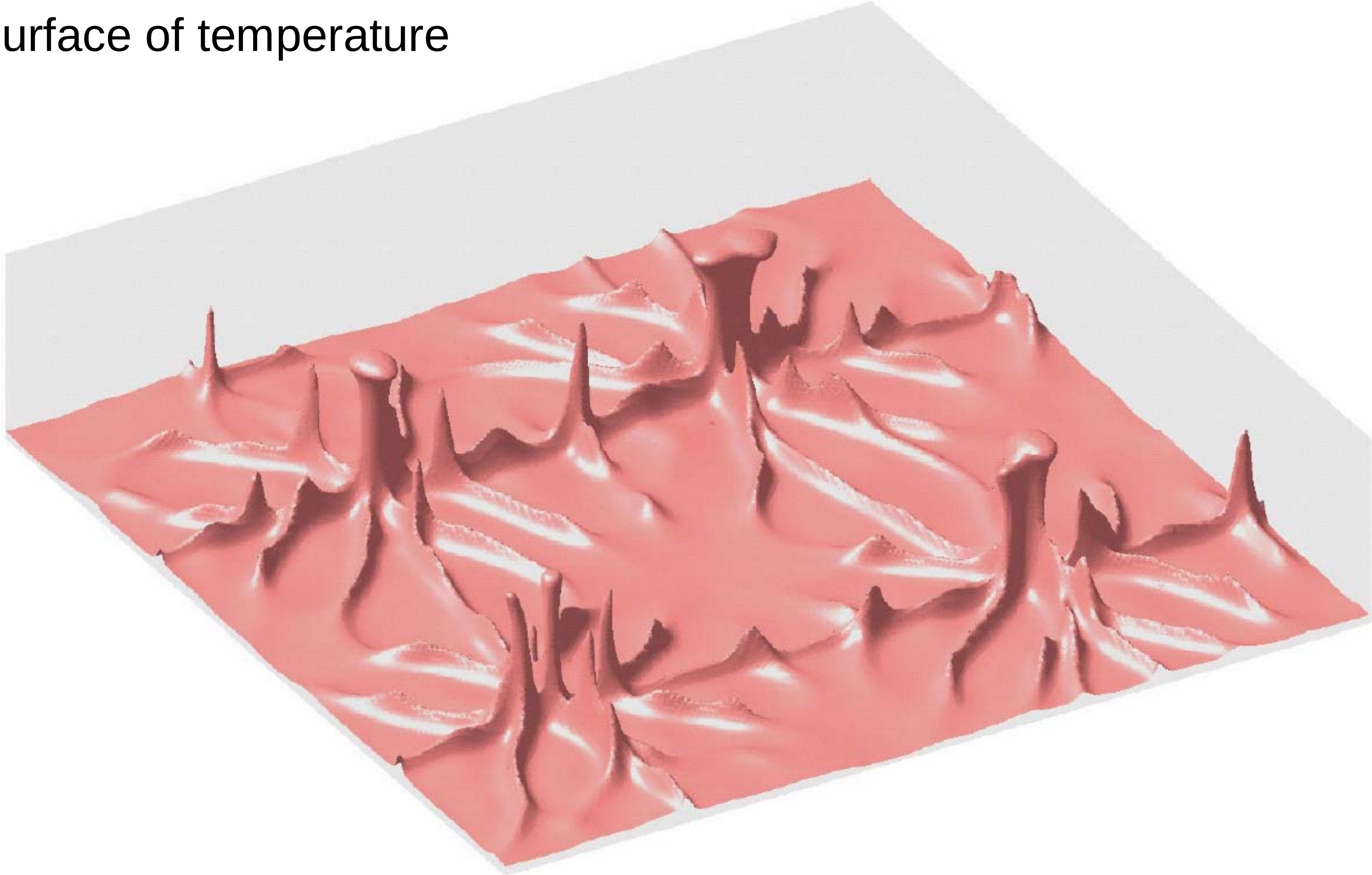
$$\epsilon_{u,bl} \propto \nu \frac{U^2}{\delta_v^2}\frac{\delta_v}{d} \propto \frac{\nu^3}{d^4} Re^{5/2}$$

$$\frac{\delta_v}{d} \propto d/Re^{1/2}$$

Suspicion: There might be a Pr-dependence in the estimate for the boundary layer

$$Pr = 15 \qquad Ra = 10^5$$

Isosurface of temperature



Numerical Methods

- The equations of motion:

$$\frac{\partial}{\partial t}\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla p + Pr\ \nabla^2\boldsymbol{v} + Ra\ Pr\ T\hat{\boldsymbol{z}}$$

$$\nabla\cdot\boldsymbol{v} = 0$$

$$\frac{\partial}{\partial t}T + (\boldsymbol{v}\cdot\nabla)T = \nabla^2 T$$

- Two control parameters:

$$Ra = \frac{g\alpha\Delta T d^3}{\kappa\nu} \quad , \quad Pr = \frac{\nu}{\kappa}$$

- The boundary conditions:

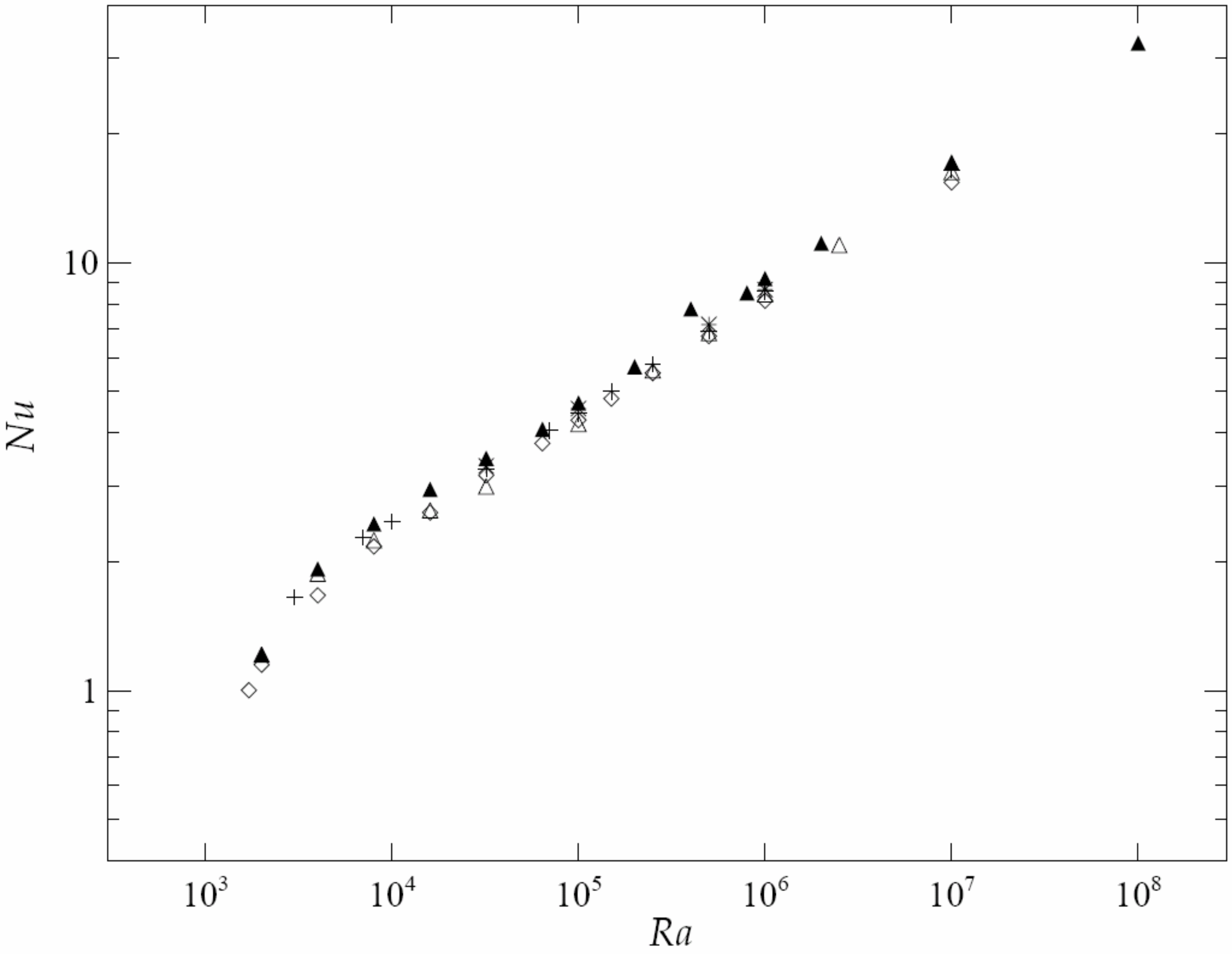
$$T(z=0)=1 \quad , \quad T(z=1)=0 \quad , \quad \boldsymbol{v}(z=0)=\boldsymbol{v}(z=1)=0$$

Periodic lateral boundary conditions $\rightarrow$ aspect ratio A

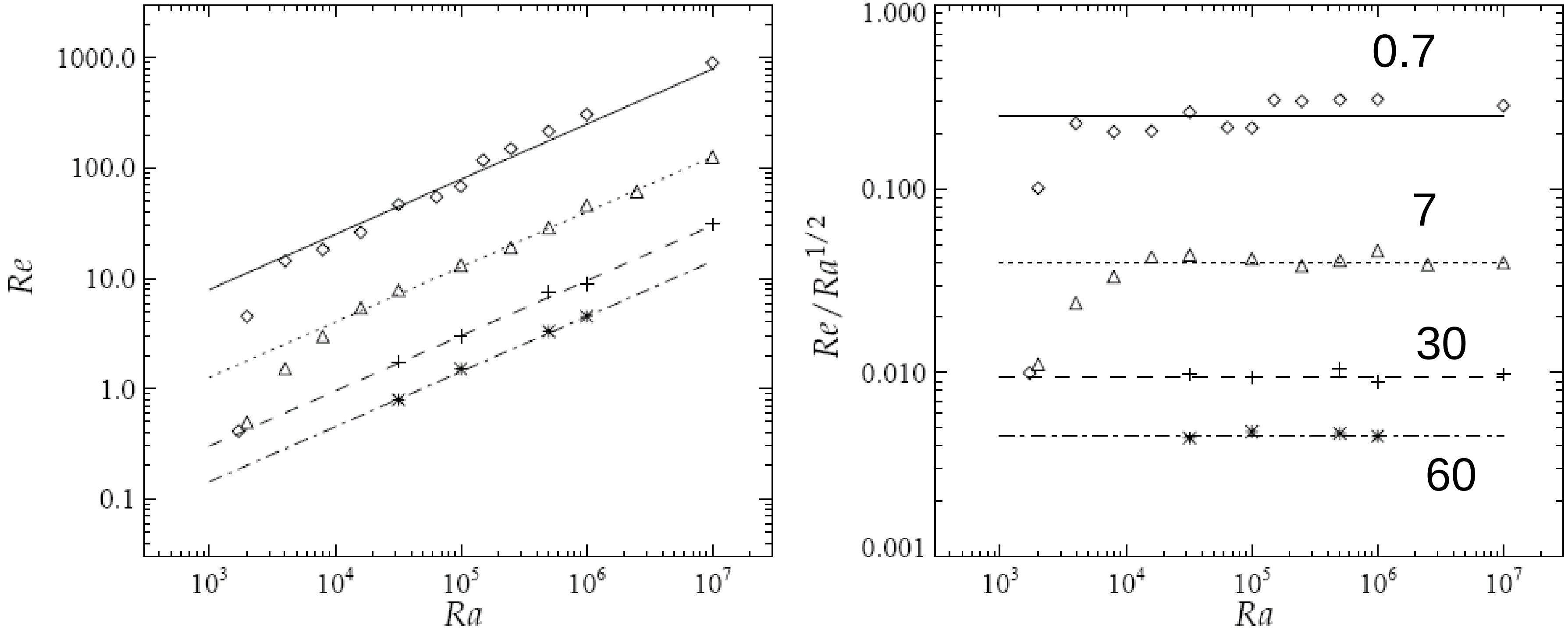
- Decomposition of $\boldsymbol{v}$:

$$\boldsymbol{v}=\nabla\times\nabla\times\Phi(\boldsymbol{r},t)\hat{\boldsymbol{z}}+\nabla\times\Psi(\boldsymbol{r},t)\hat{\boldsymbol{z}}+\boldsymbol{U}(z,t)$$

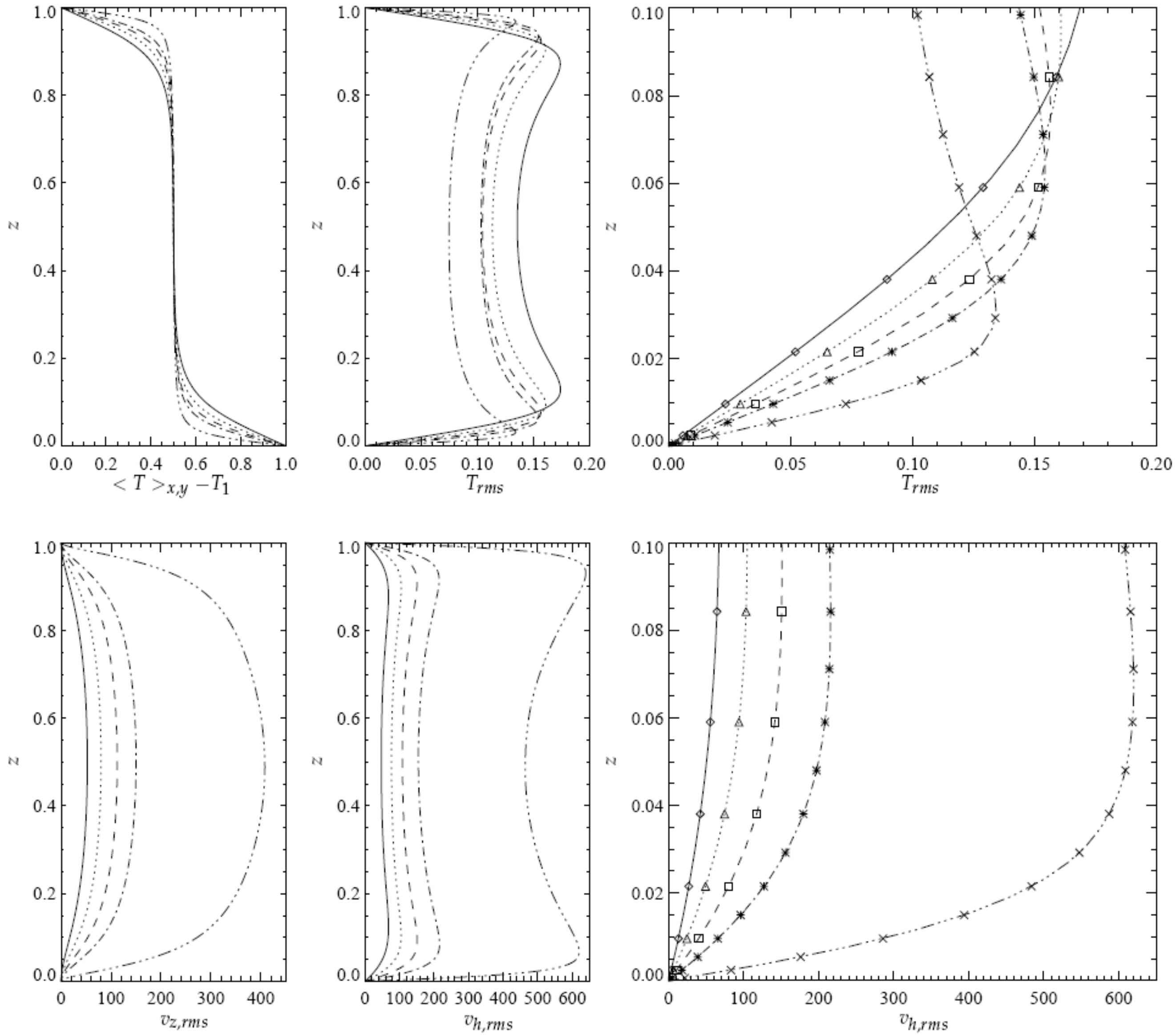
- Spectral method, Chebychev / Fourier



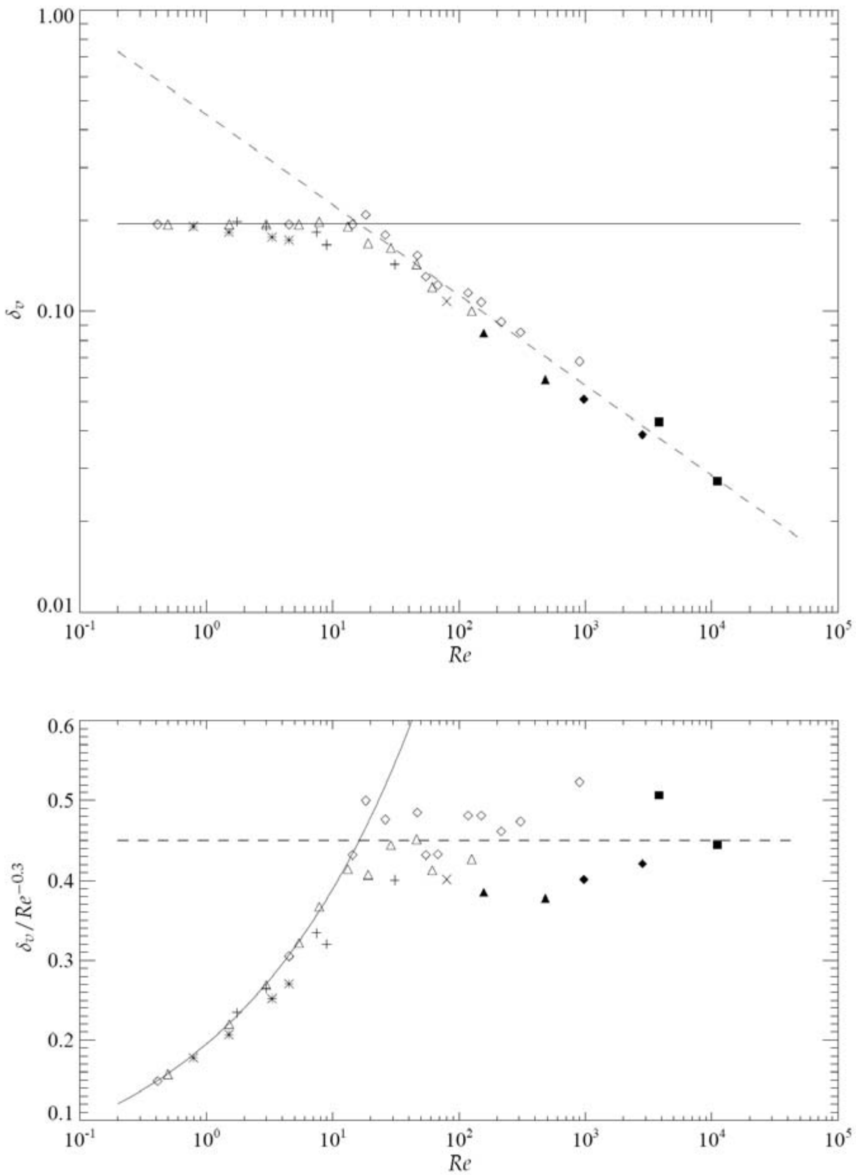
Pr ranges from 0.7 to 60
Aspect ratio 10



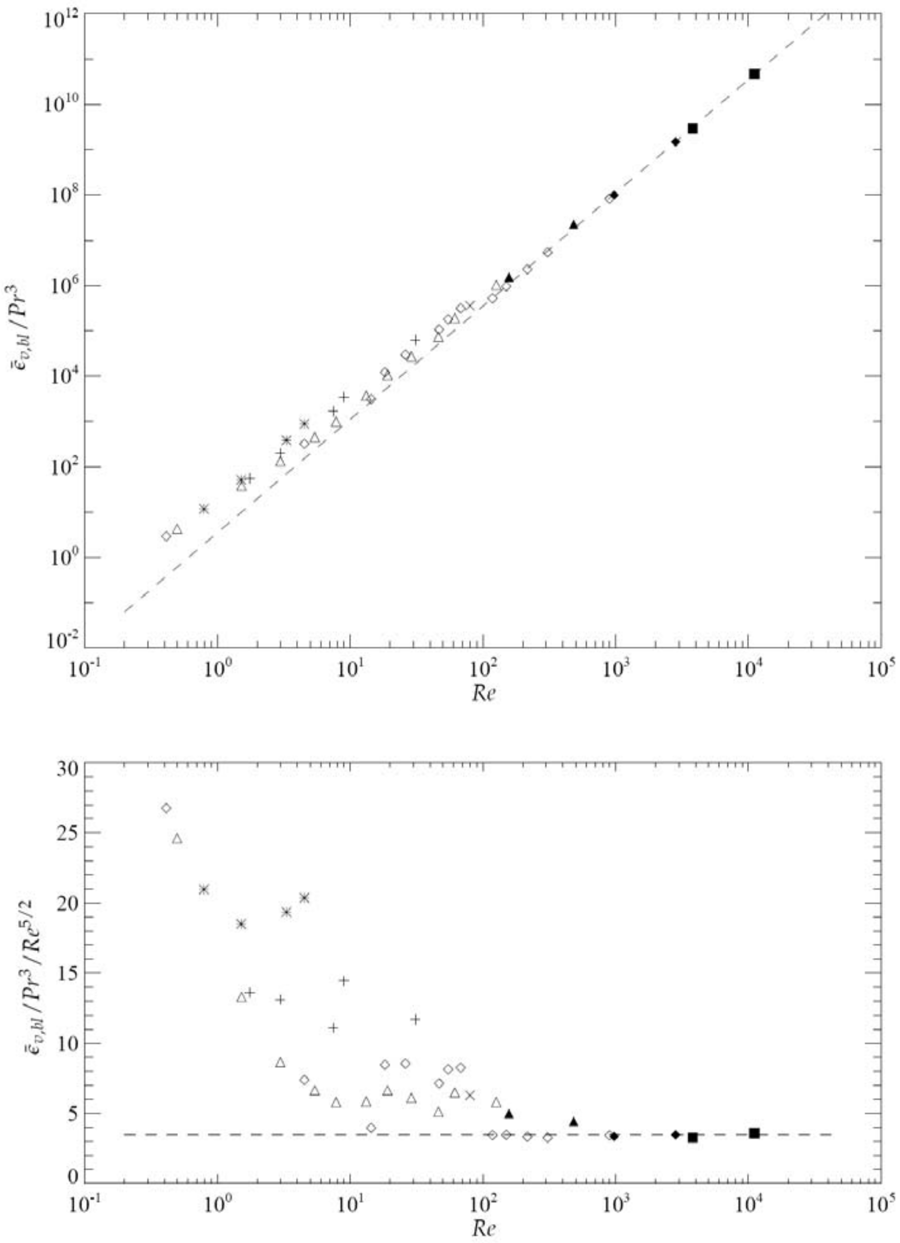
Re
1000.0
100.0
10.0
1.0
0.1
10^3
10^4
10^5
10^6
10^7
Ra
$Re/Ra^{1/2}$
1.000
0.100
0.010
0.001
0.7
7
30
60



$Pr = 0.7$ and $Ra = 10^5$, 2.5×10^5, 5×10^5 and 10^6



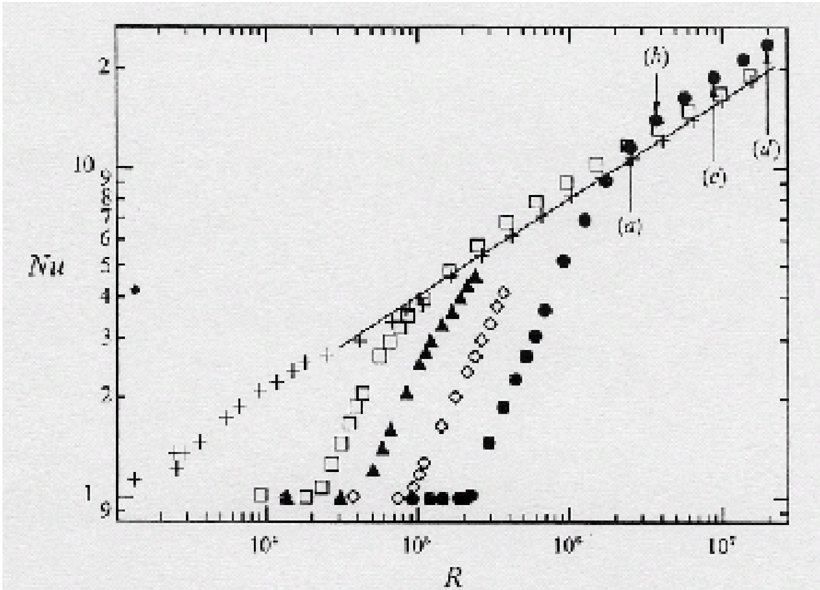
Viscous boundary layer thickness for Pr between 0.7 and 60



Viscous dissipation in the viscous boundary layer for Pr between 0.7 and 60

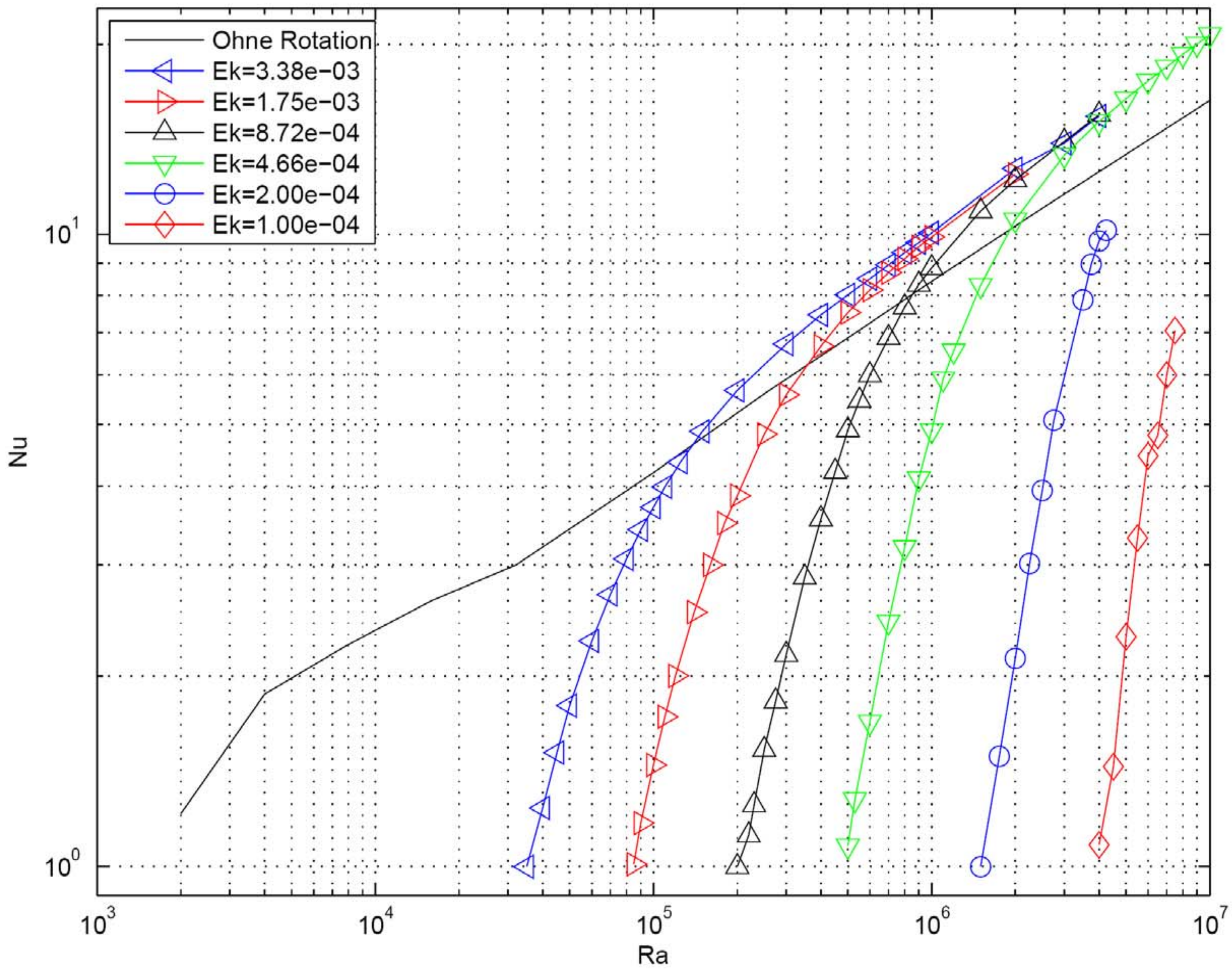
Rotating Convection

Zhong, Ecke, Steinberg, J. Fluid Mech. (1993)

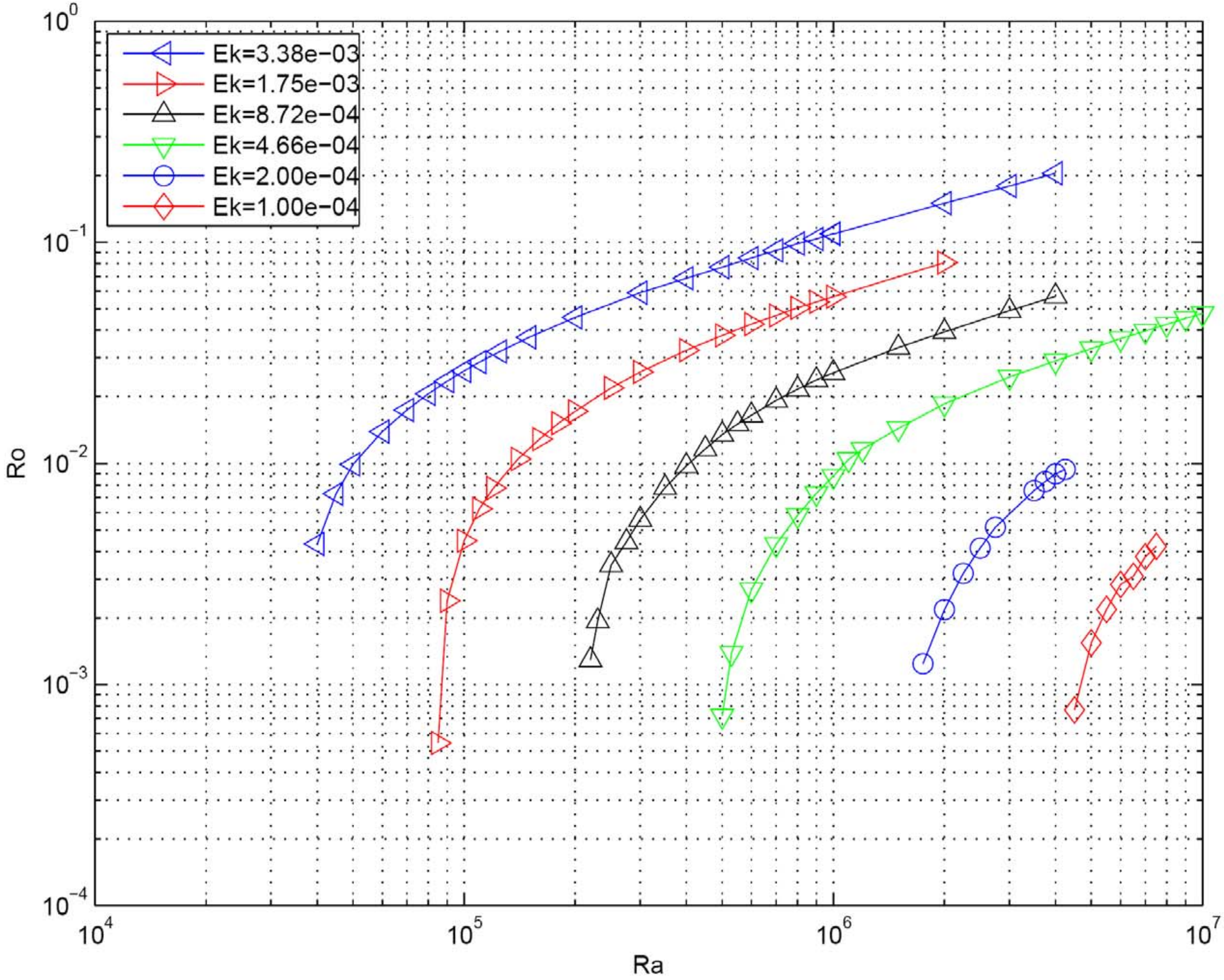


$$Nu \propto Ra \cdot Ek \quad ?$$

$$Ek = \nu/(\Omega d^2)$$



Ohne Rotation
Ek=3.38e-03
Ek=1.75e-03
Ek=8.72e-04
Ek=4.66e-04
Ek=2.00e-04
Ek=1.00e-04
Nu
Ra
10^1
10^0
10^3
10^4
10^5
10^6
10^7



Ek=3.38e-03
Ek=1.75e-03
Ek=8.72e-04
Ek=4.66e-04
Ek=2.00e-04
Ek=1.00e-04
Ro
Ra
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^4
10^5
10^6
10^7

ϵ_u in rotating convection:

- Boundary layer: $\epsilon_{u,bl} \propto \frac{\nu^3}{d^4} Re^2 Ek^{-1/2}$
- Bulk:

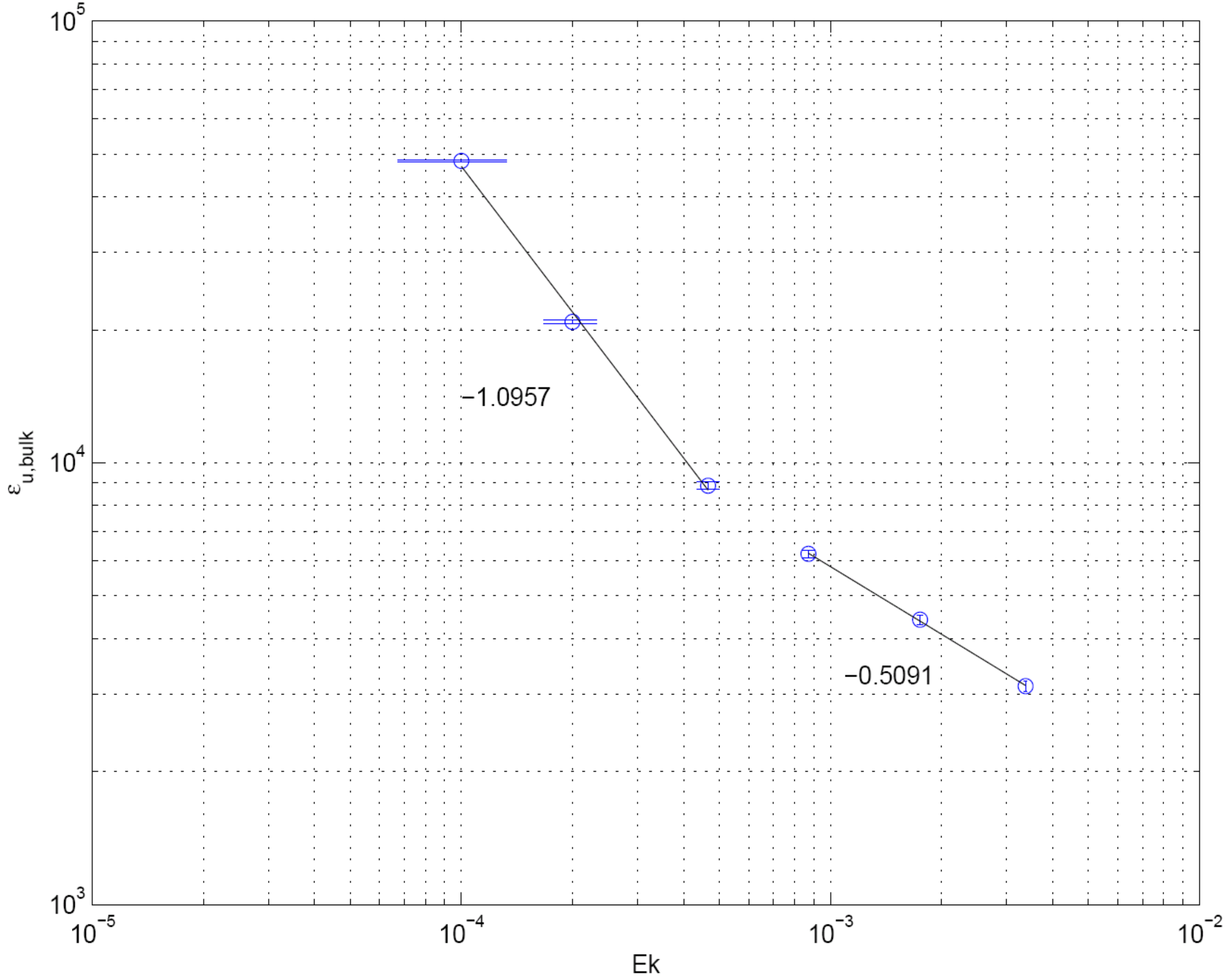
If viscous and nonlinear terms are small:

Redistribution of energy on the time scale $1/\Omega$

$\epsilon_{u,bulk} \propto \Omega U^2$

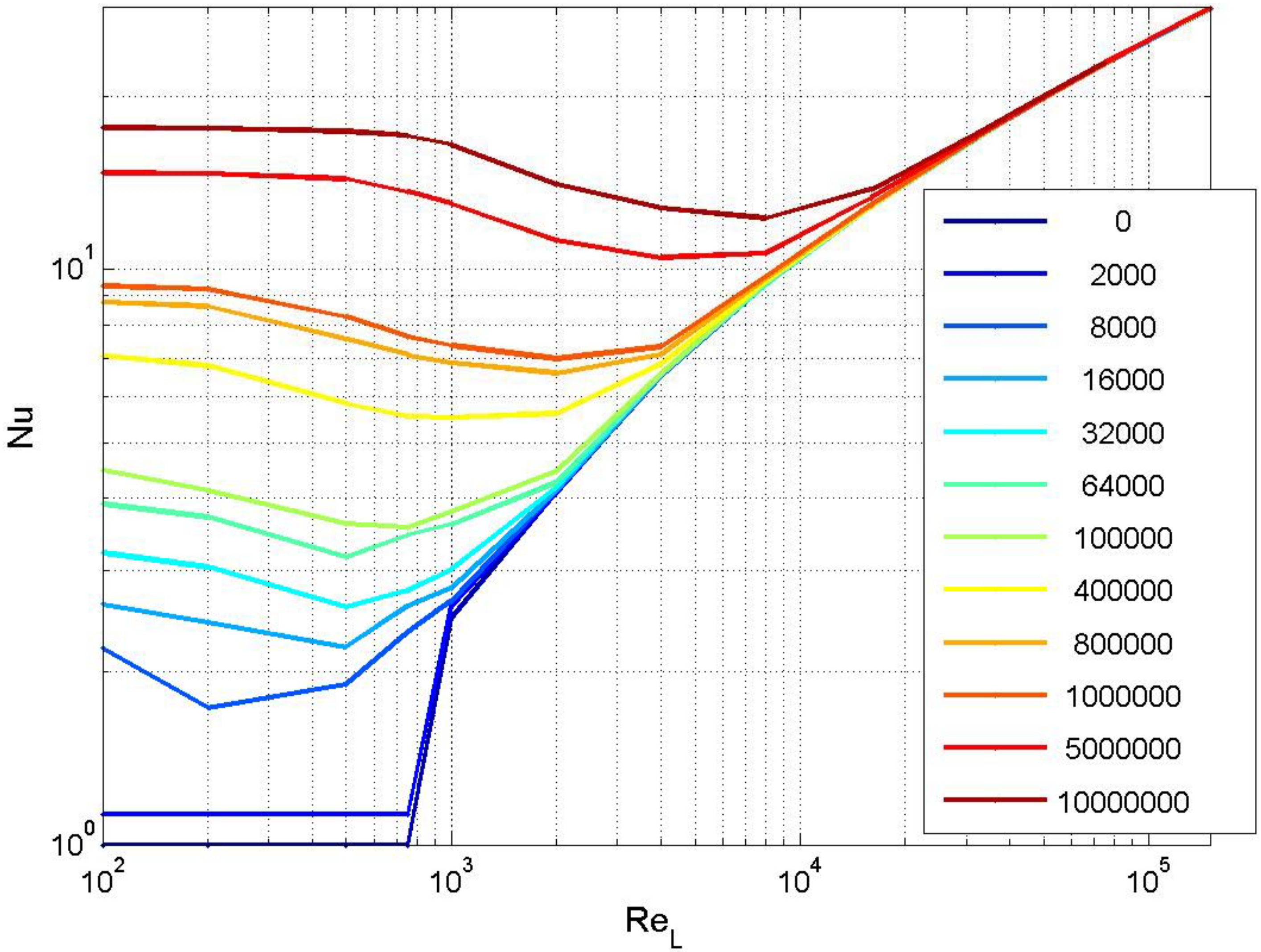
$U = U_{convection} + U_{Ekman} = A \cdot Re + B \cdot Re \cdot Ek^{1/2}$

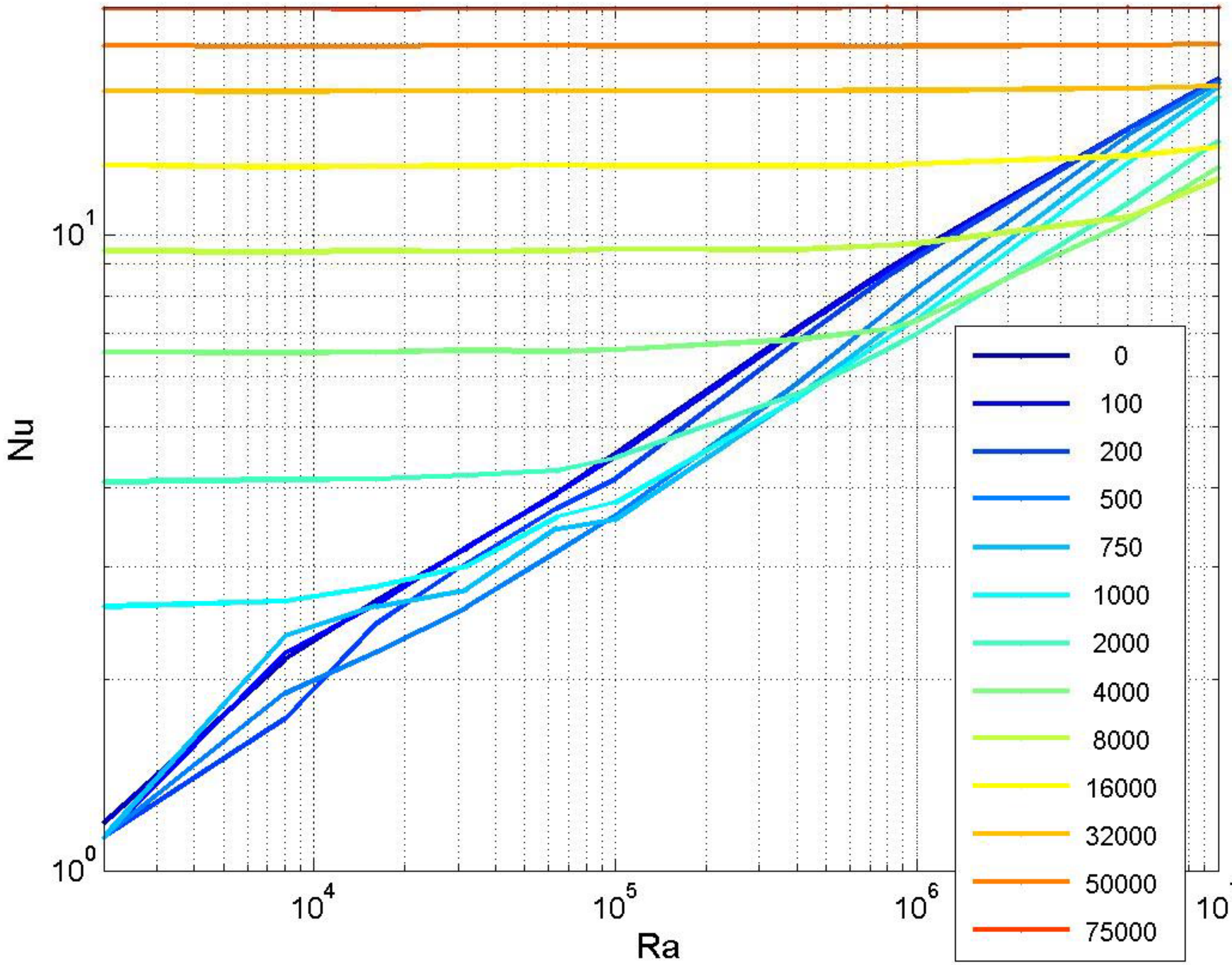
$\epsilon_{u,bulk} \propto A^2 \cdot Re^2 \cdot Ek^{-1} + 2A \cdot B \cdot Re^2 \cdot Ek^{-1/2} + B^2 Re^2$



10^5
10^4
10^3
ε_u,bulk
−1.0957
−0.5091
10^-5
10^-4
10^-3
10^-2
Ek

Sheared Convection





Nu
Ra
10^0
10^1
10^4
10^5
10^6
10^7
0
100
200
500
750
1000
2000
4000
8000
16000
32000
50000
75000

Summary

- Numerical simulation allow us to determine dissipation rates
- So far, there is no need to improve the usual estimates for the dissipation rates in standard Rayleigh-Bénard convection
- Simple estimates for the dissipation rates for rotating convection at low Ekman numbers seem to be adequate
- Sheared convection is more challenging