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on

High Rayleigh Number Convection

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Spatial distribution of the local thermal dissipation rate in turbulent Rayleigh-Bernard convection

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Measured local thermal dissipation rate in turbulent Rayleigh-Bènard convection

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OUTLINE:

1. Dissipations in Rayleigh-Bénard convection
2. Measurement of the local thermal dissipation rate
3. Experimental results: spatial distribution and fluctuations
4. Summary

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1. Dissipations in Rayleigh-Bénard Convection

For isotropic turbulence, energy cascades from the large system scale L_0 to the small viscous dissipation scale η, at which $Re_\eta = \eta u_\eta/\nu \approx 1$.

In the inertial range $\eta < r < L_0$, the kinetic energy cascades at a constant rate ε_u without dissipation and thus $u(r) \approx (\varepsilon_u r)^{1/3}$.

Van Gogh's painting
Starry Night (1889)

Important Kolmogorov scales:

$$u_\eta = (\varepsilon_u \eta)^{1/3}$$

$$\eta = (\nu^3 / \varepsilon_u)^{1/4}$$

$$\varepsilon_u = \nu (u_\eta / \eta)^2 = \nu^3 / \eta^4$$

For turbulent Rayleigh-Bénard convection, we have

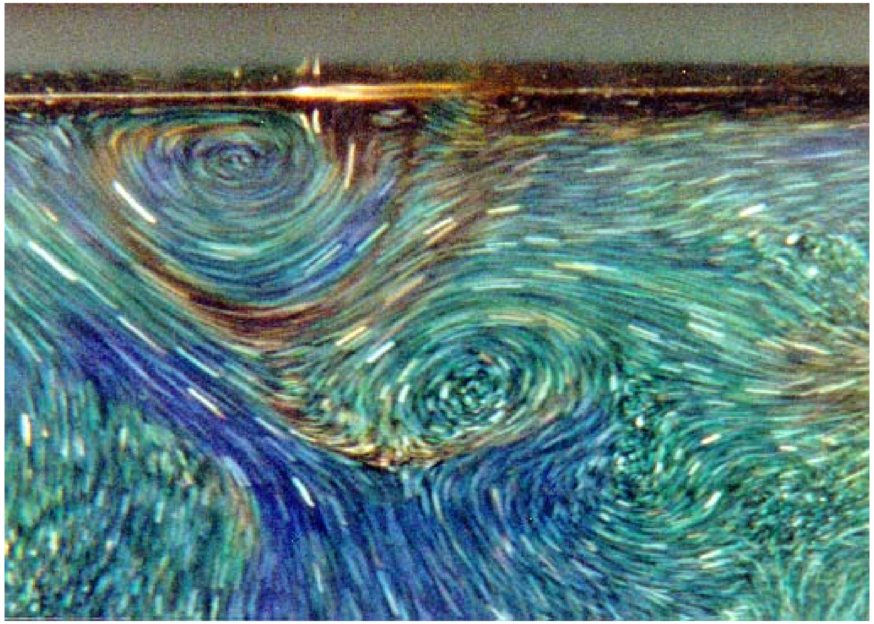
- two local variables, $\mathbf{v}(\mathbf{r},t)$ and $T(\mathbf{r},t)$, and two corresponding dissipation rates, ε_u and ε_T.
- two exact relations:

$$\varepsilon_N = \langle \varepsilon_N(\mathbf{r},t) \rangle_{V,t} = Nu(Ra, \Pr)$$

$$\varepsilon_N(\mathbf{r},t) = \frac{\varepsilon_T(\mathbf{r},t)}{\kappa(\Delta T / H)^2}$$

$$\varepsilon_0 = (Nu - 1) Ra \Pr^{-2}$$

$$\varepsilon_0 = \frac{\langle \varepsilon_u(\mathbf{r},t) \rangle_{V,t}}{\nu^3 / H^4}$$



Flow visualization of turbulent thermal convection in water at $Ra = 2.6\times10^9$ (6.5 cm × 4.0 cm)

Understanding heat transport, Nu(Ra,Pr), in turbulent convection through spatial decomposition of the dissipation rates ε_u and ε_T

Phenomenology I: boundary *versus* bulk (GL, JFM, 2000; PRL, 2001)

$$\varepsilon_u = \varepsilon_{u,BL} + \varepsilon_{u,bulk}$$

$$\varepsilon_T = \varepsilon_{T,BL} + \varepsilon_{T,bulk}$$

$$\varepsilon_{u,BL} \sim \nu (u/\lambda_u)^2 (\lambda_u / H)$$

$$\varepsilon_{T,BL} \sim \kappa (\Delta T / \lambda_T)^2 (\lambda_T / H)$$

$$\varepsilon_{u,bulk} \sim u^3 / H$$

$$\varepsilon_{T,bulk} \sim (u\varphi) \Delta T^2 / H$$

Phenomenology II: background *versus* plumes (GL, PoF, 2004)

$$\varepsilon_{u,BL} \sim \nu (u/\lambda_u)^2 (\lambda_u / H)$$

$$\varepsilon_{T,pl} \sim \kappa (\Delta T / \lambda_T)^2 (\lambda_T / H) N_{pl}^{sheet}$$

$$\varepsilon_{u,bulk} \sim u^3 / H$$

$$\varepsilon_{T,bg} \sim (u\varphi) \Delta T^2 / H$$

2. Measurement of the local thermal dissipation rate

Instantaneous local viscous dissipation rate:

$$\varepsilon_u(\mathbf{r},t) = \nu \sum_{i,j} \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}$$

Instantaneous local thermal dissipation rate:

$$\varepsilon_T(\mathbf{r},t) = \kappa \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 \right]$$

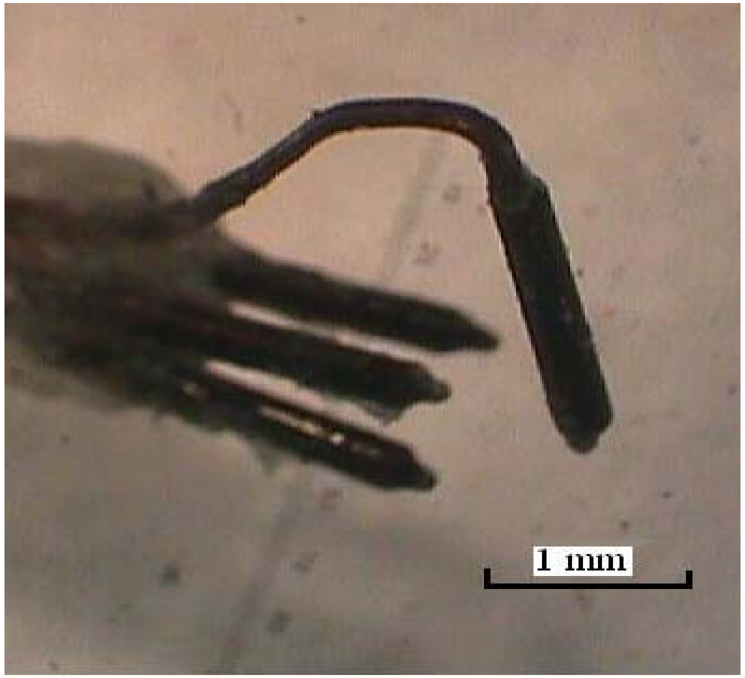
Time-averaged local thermal dissipation rate:

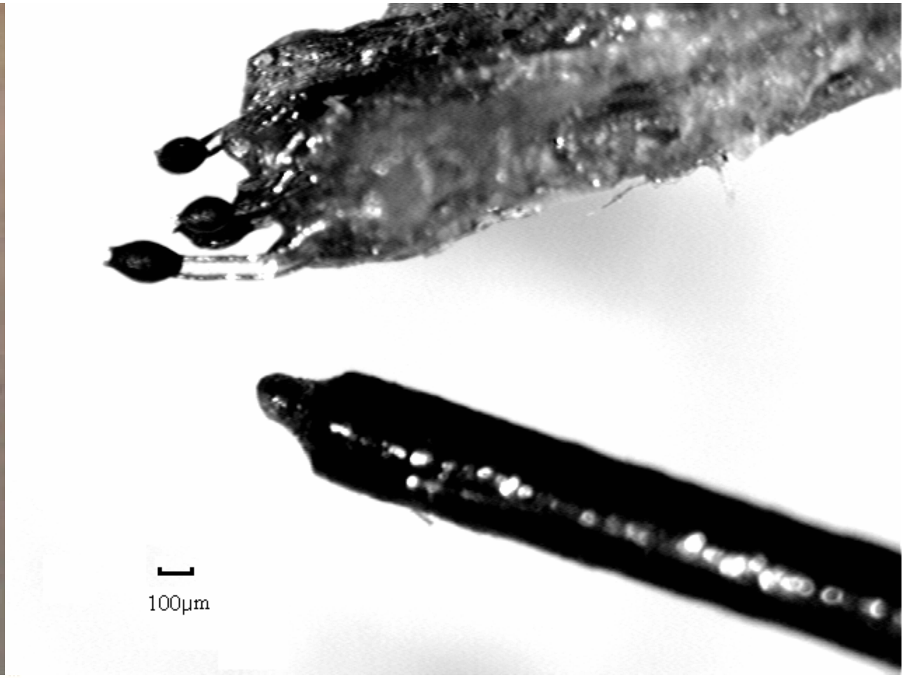
$$\langle \varepsilon_T(\mathbf{r},t) \rangle_t = (\varepsilon_T)_m(\mathbf{r}) + (\varepsilon_T)_f(\mathbf{r}) = \kappa \sum_i \left\langle \left(\frac{\partial \overline{T}}{\partial x_i}\right)^2 \right\rangle_t + \kappa \sum_i \left\langle \left(\frac{\partial \tilde{T}}{\partial x_i}\right)^2 \right\rangle_t$$

Total convective heat flux across the cell:

$$\mathrm{Nu} = \frac{1}{A} \iint \mathrm{J_z}(x,y)\, dxdy = \frac{1}{V} \iiint \langle \varepsilon_N(\mathbf{r},t) \rangle_t \, d\mathbf{r}$$

Temperature gradient probe:



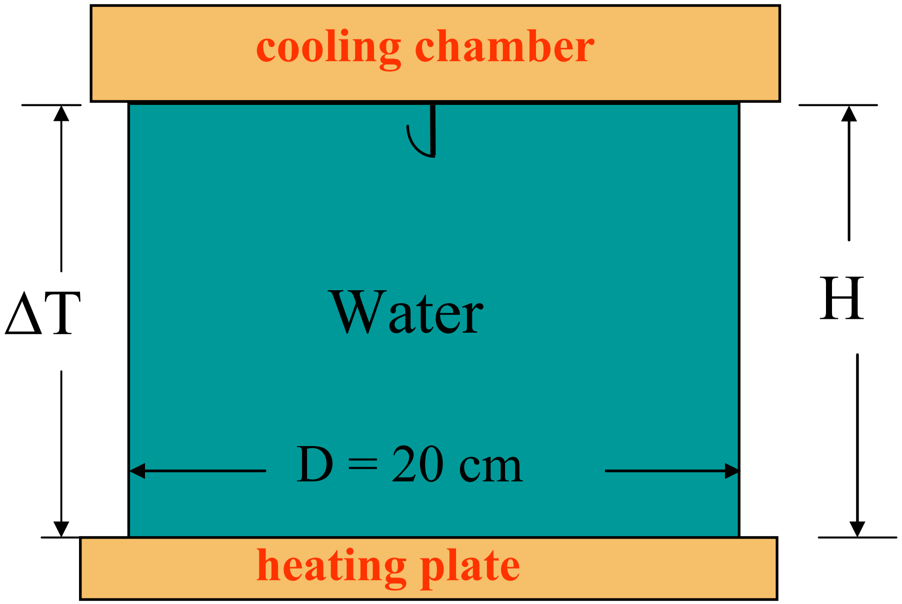


First probe: d = 0.17 mm
δx_i = 0.8 mm, $\delta T_{min} \approx$ 5 mK

Second probe: d = 0.11 mm
δx_i = 0.25 mm, $\delta T_{min} \approx$ 5 mK

$\lambda_T \approx$ 0.8 mm at Ra = 3.6×10^9

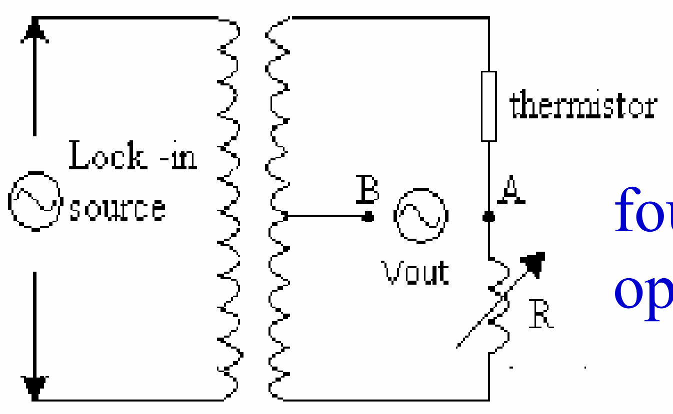
Rayleigh-Bénard convection cell



$$A = \frac{D}{H} = 1, 0.5$$

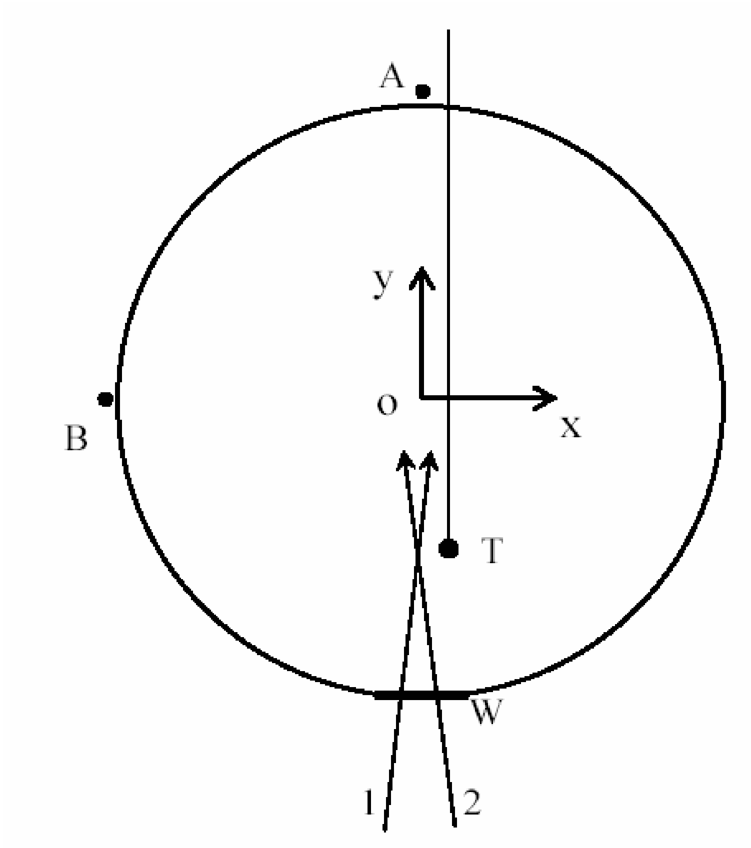
$$\Pr = \frac{\nu}{\kappa} \approx 5.4$$

$$Ra = \frac{\alpha g H^3 \Delta T}{\nu \kappa}$$

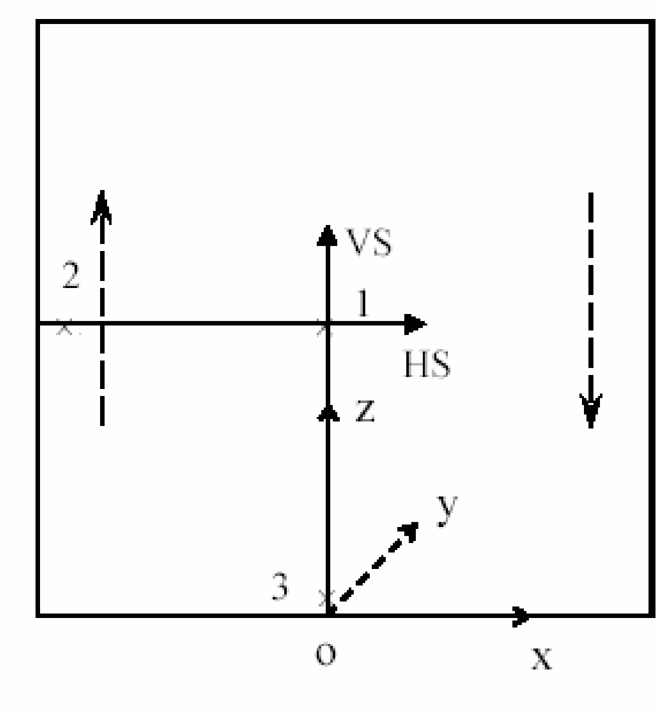


four ac bridges with lock-in amplifiers operated at f = 1kHz and Δf = 100 Hz

Simultaneous dissipation and velocity measurements

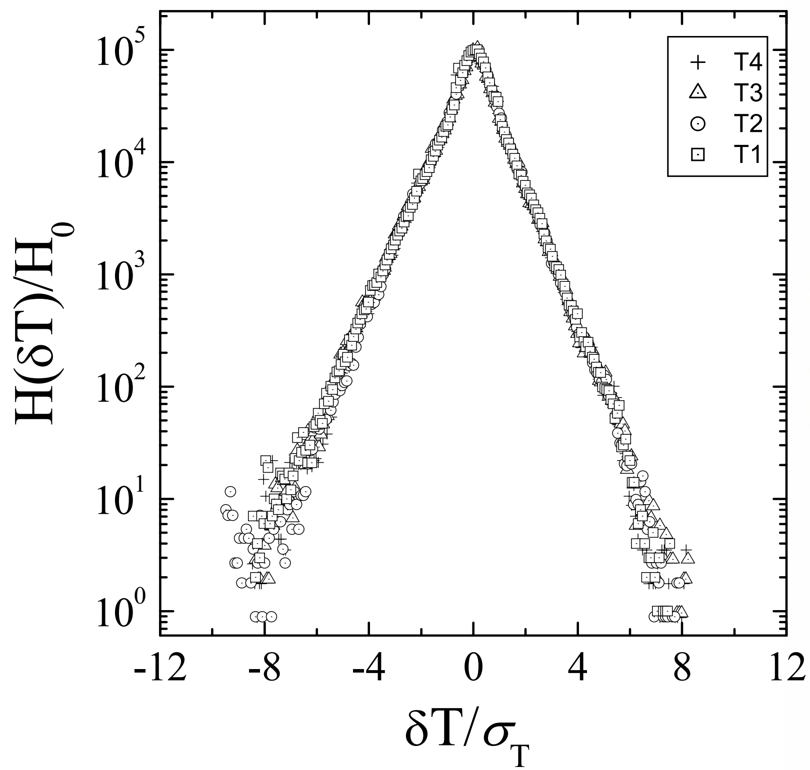


top view

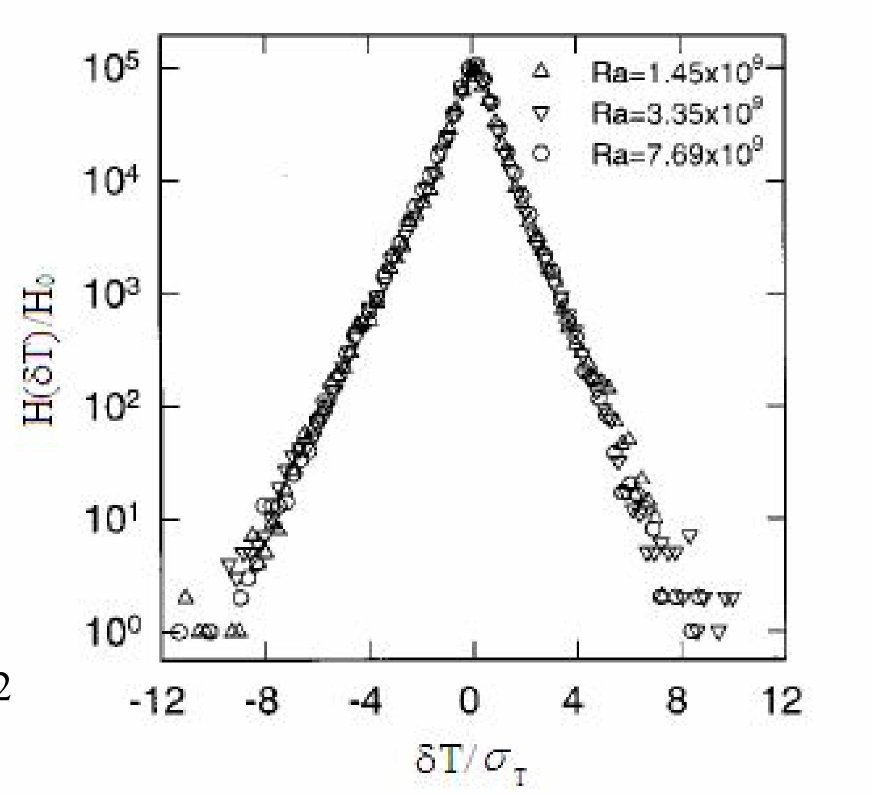


side view

Does the invasive probe perturb the local temperature field?

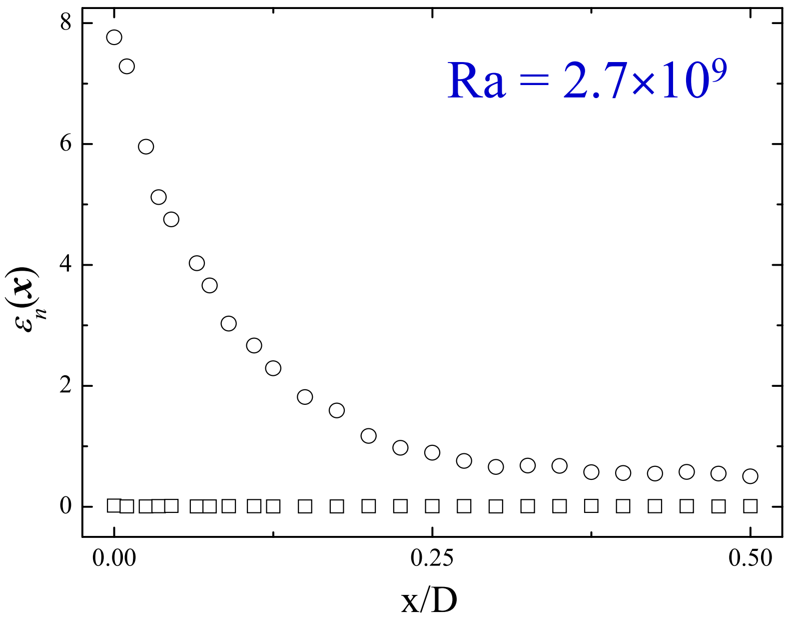


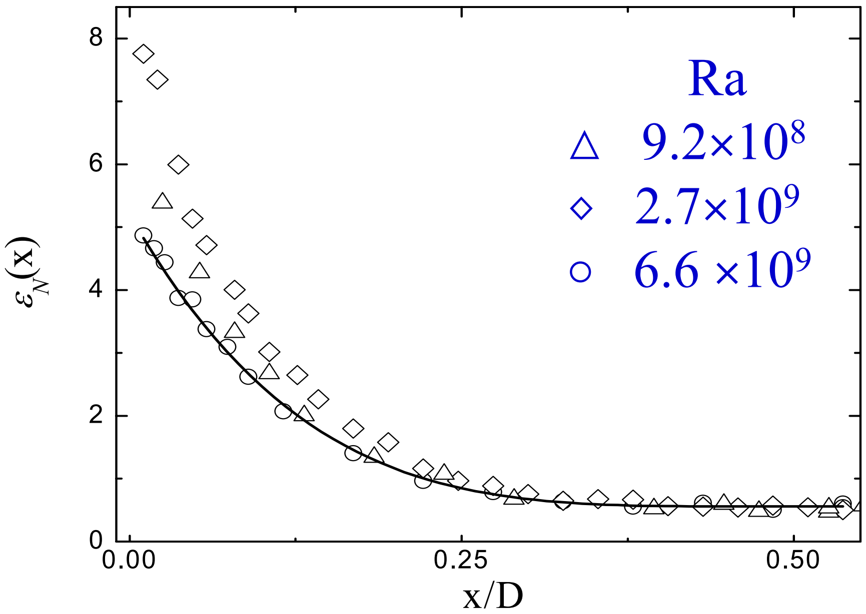
Multiple sensor measurements at the center of the A = 1 cell with Ra= 4.0×10^9



Single sensor measurements at the center of the A = 1 cell

3. Experimental results

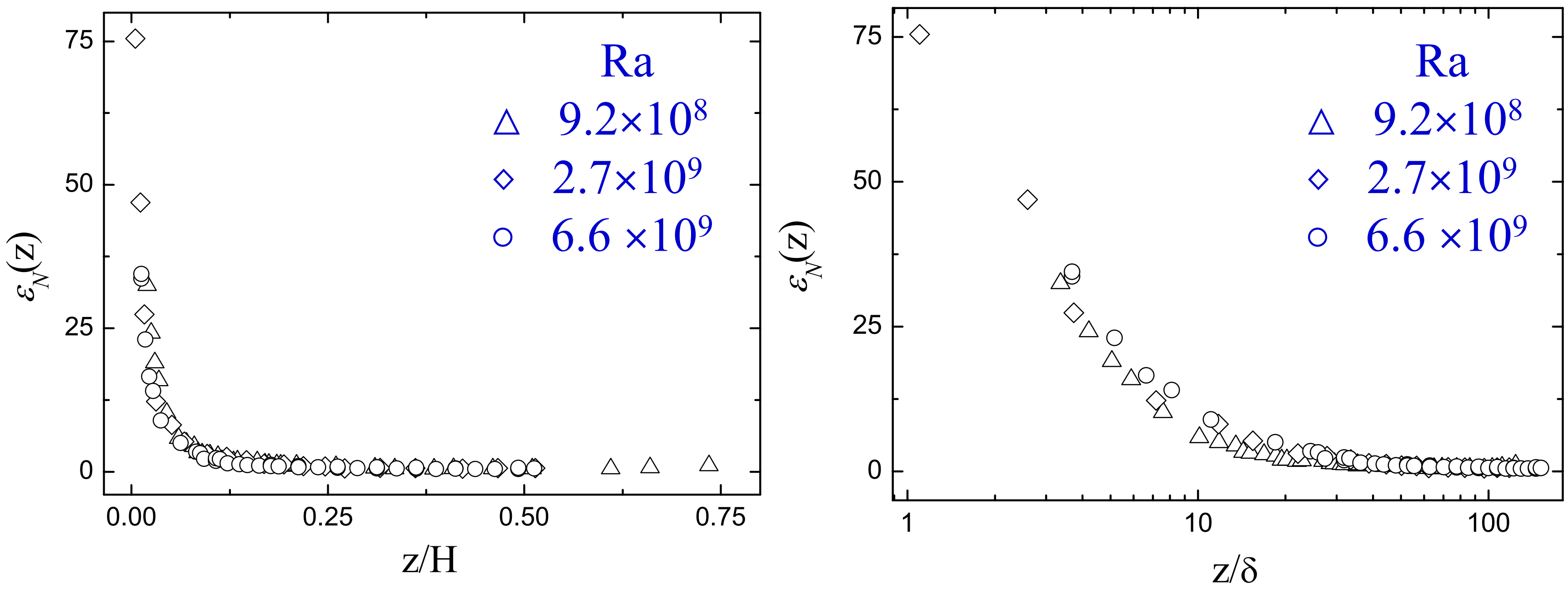




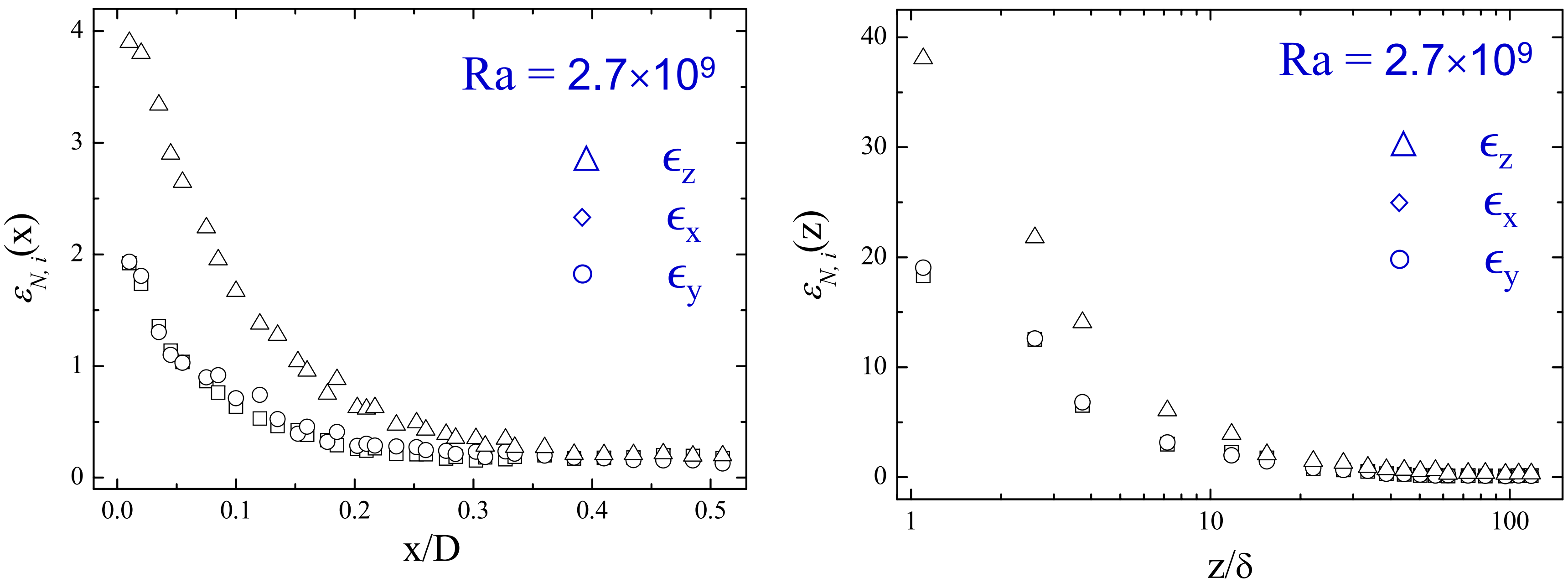
$$\varepsilon_N(\mathbf{r}) = \varepsilon_m(\mathbf{r}) + \varepsilon_f(\mathbf{r})$$

(i) In the bulk region, ϵ_f is dominant and ϵ_m is negligibly small.

(ii) Near the sidewall, ϵ_f is ~10 times larger than that at the cell center.

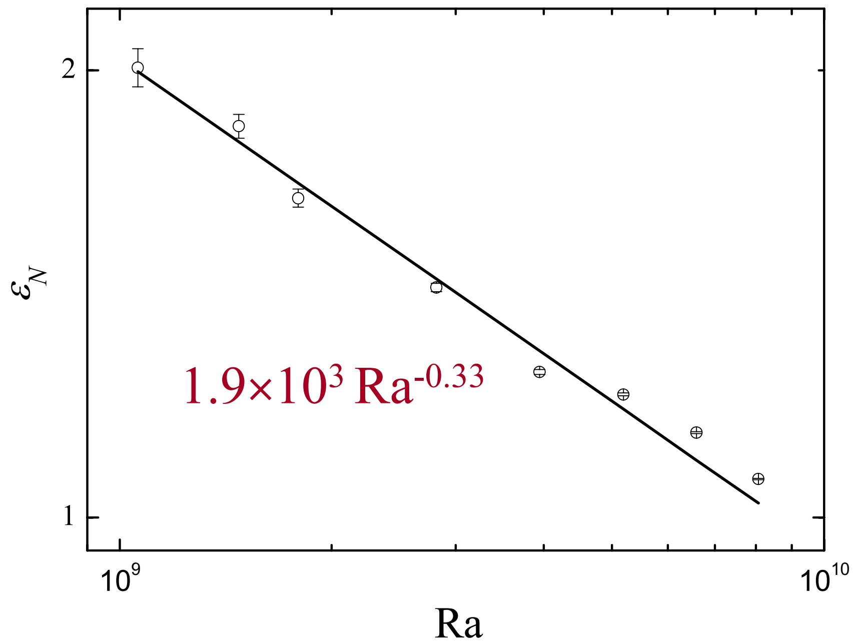


(iii) ϵ_f increases rapidly in the $1 \leq z/\lambda_T \leq 10$ region and is ~140 times larger than that at the cell center.

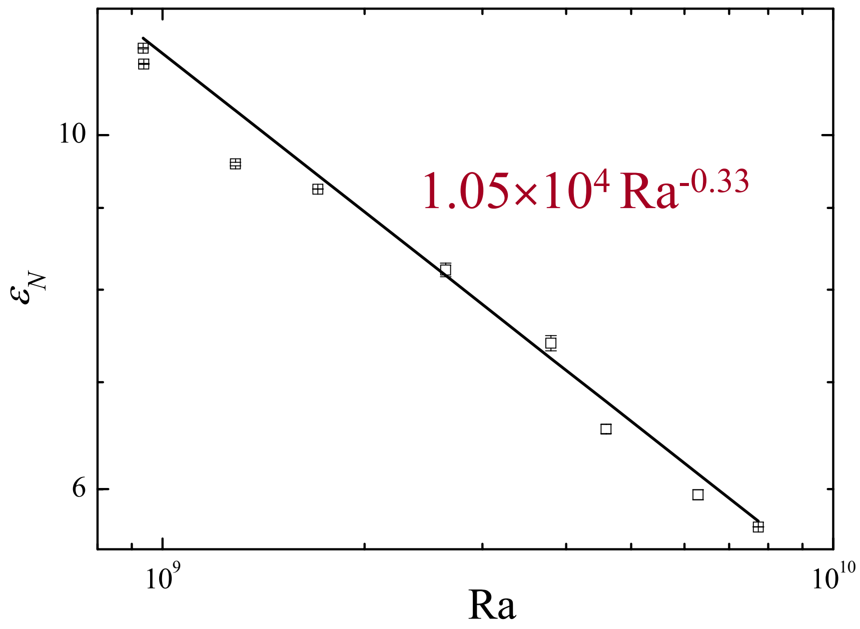


(iv) ϵ_f has three terms, $\epsilon_f = \epsilon_x + \epsilon_y + \epsilon_z$, and the dominant term is ϵ_z, which is twice larger than ϵ_x and ϵ_y.

At the cell center

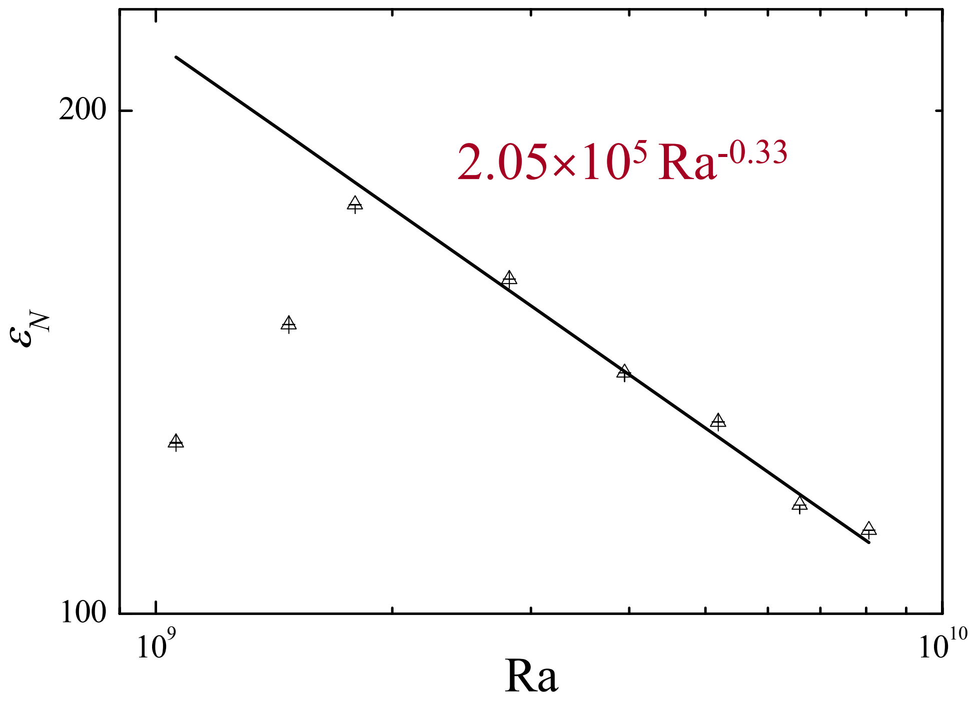


Near the sidewall



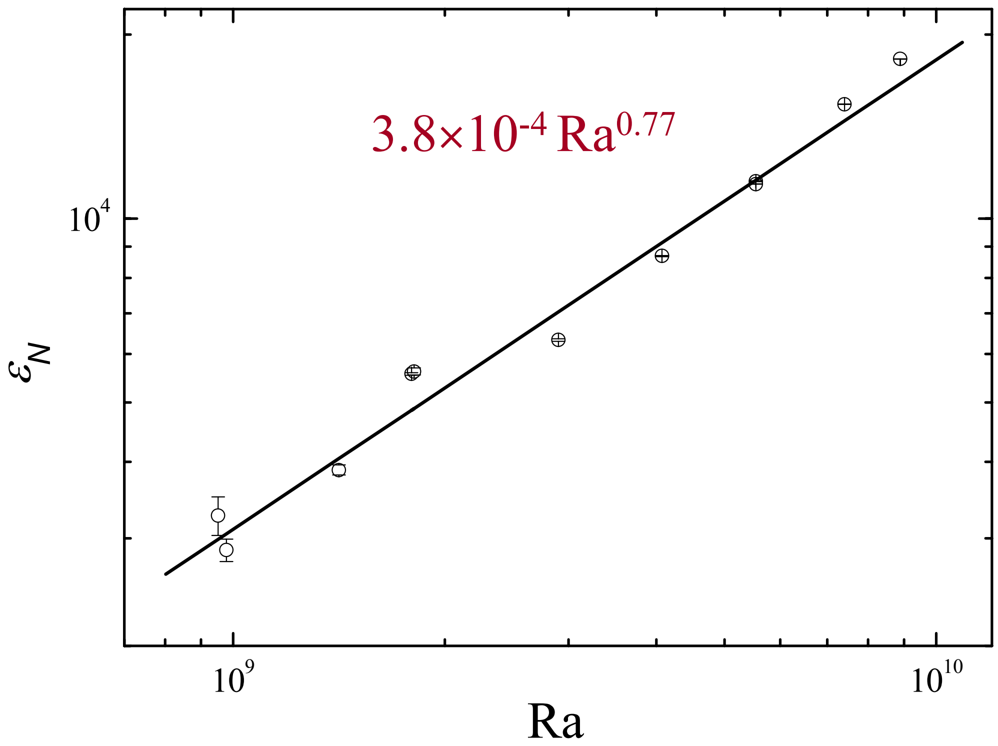
(v) $\epsilon_f \sim Ra^{-\alpha}$ with the exponent $\alpha \approx 0.33 \pm 0.03$ both at the cell center and near the sidewall.

Measured ϵ_f when the probe is placed ~1 mm above the lower conducting plate

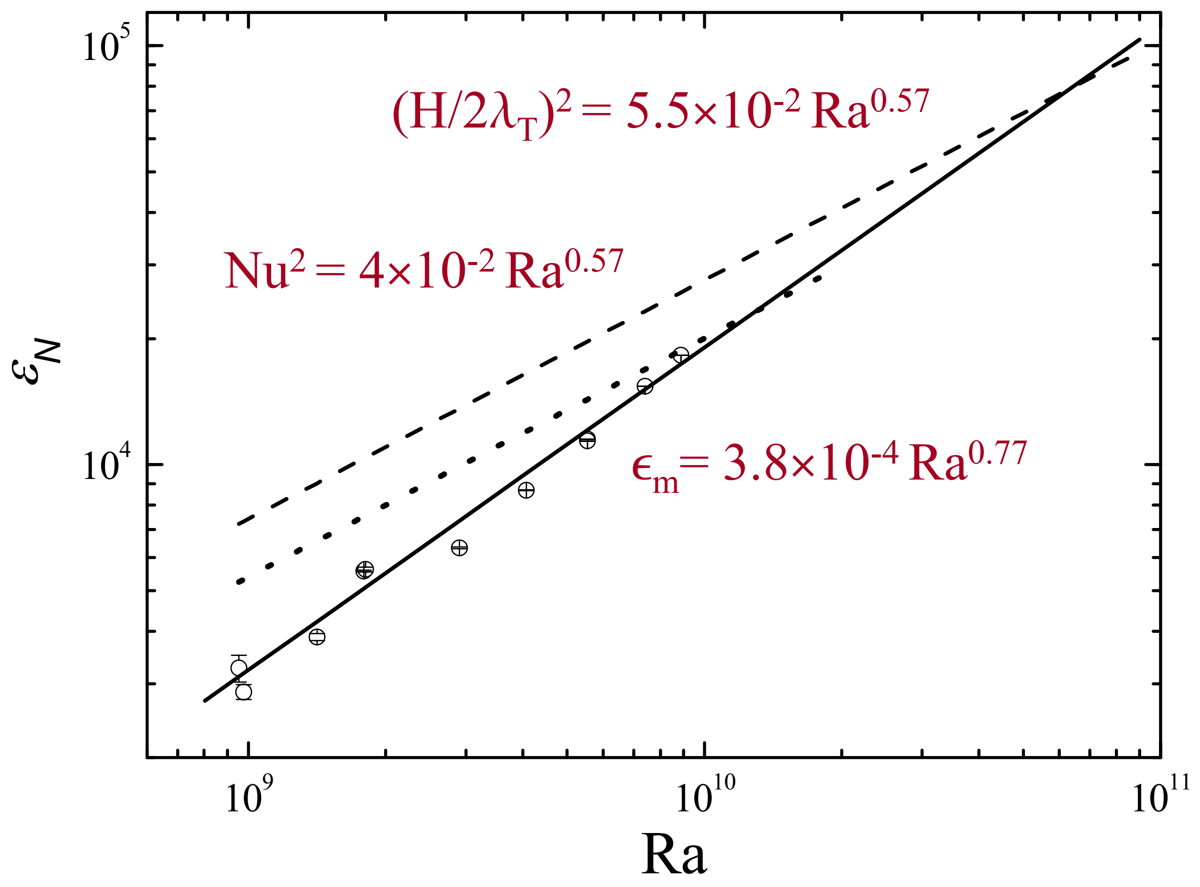


(vi) $\epsilon_f \sim Ra^{-\alpha}$ with the exponent $\alpha \approx 0.33 \pm 0.03$ near the lower conducting plate.

Inside the thermal boundary layer (almost touching the lower surface)



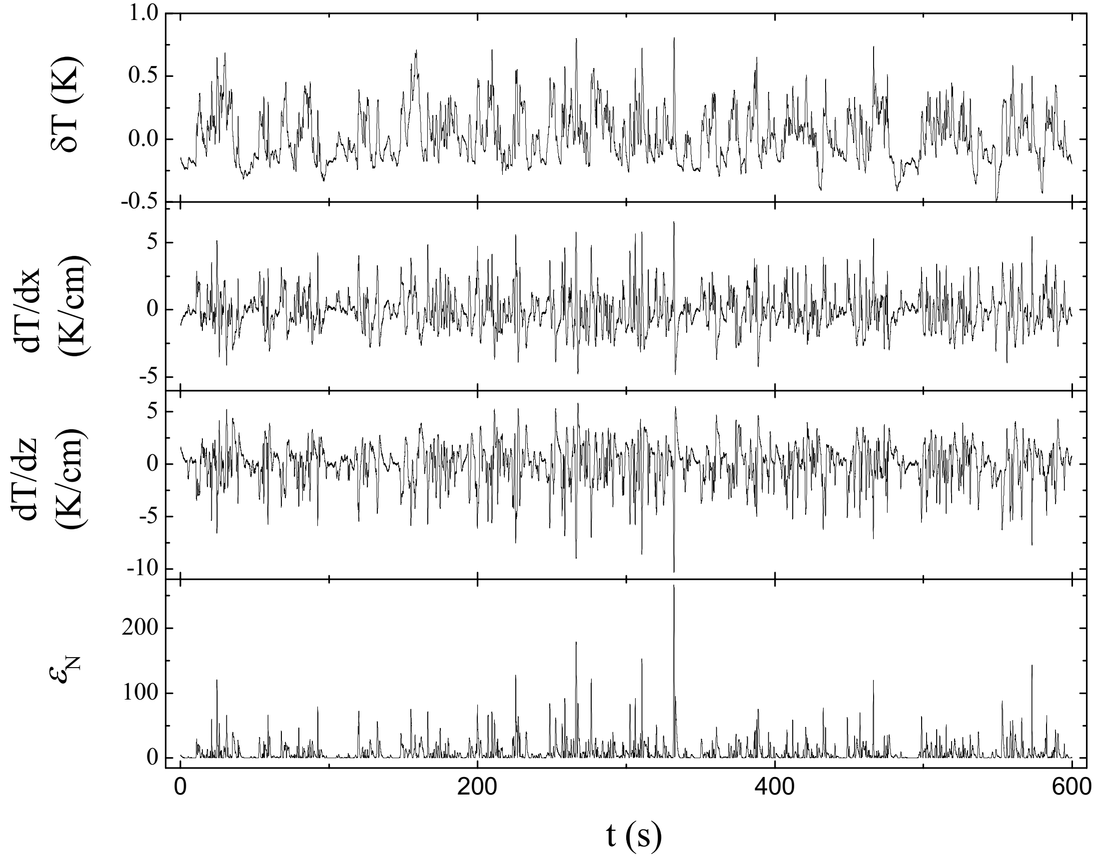
(vii) $\epsilon_m \sim Ra^{-\beta}$ with the exponent $\beta \approx 0.77 \pm 0.1$ inside the thermal boundary layer.



(viii) The line of $(H/2\lambda_T)^2$ intersects the line of $\epsilon_m = 3.8\times10^{-4}Ra^{-0.77}$ at $Ra = 6.4\times10^{10}$.

Fluctuations of the local thermal dissipation rate $\epsilon_f(\mathbf{r},t)$

Near the sidewall at Ra = 3.6×10^9



asymmetric

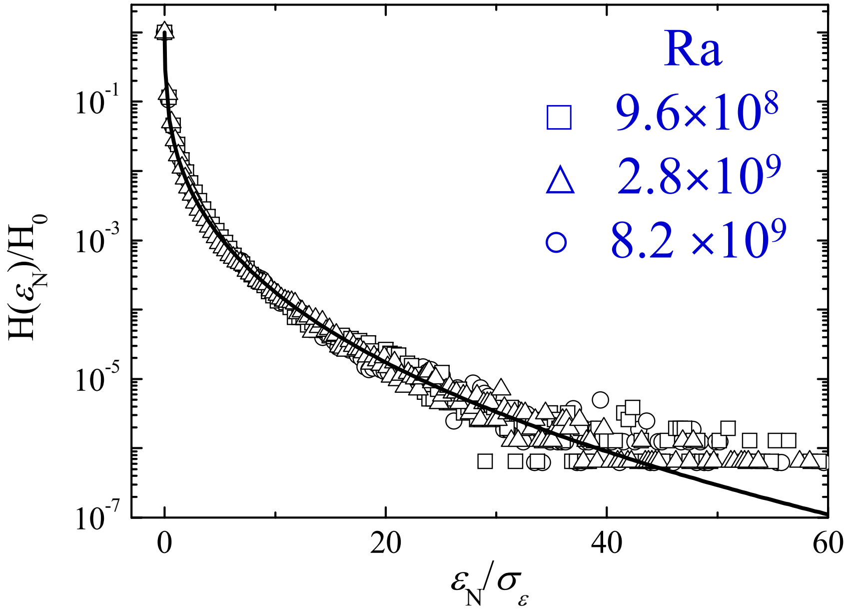
symmetric

slightly asymmetric

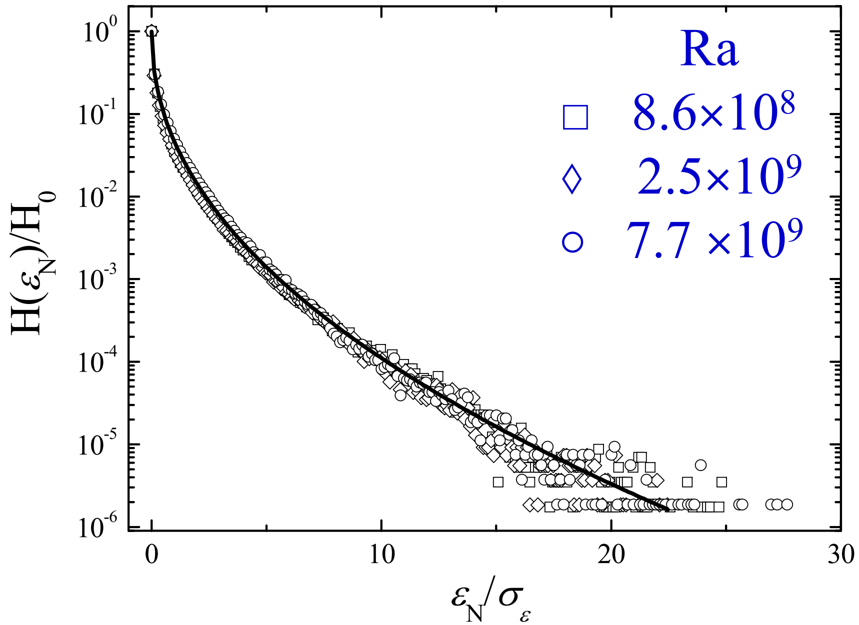
asymmetric

PDF of the local thermal dissipation rate $\epsilon_f(\mathbf{r},t)$

At the cell center



Near the sidewall



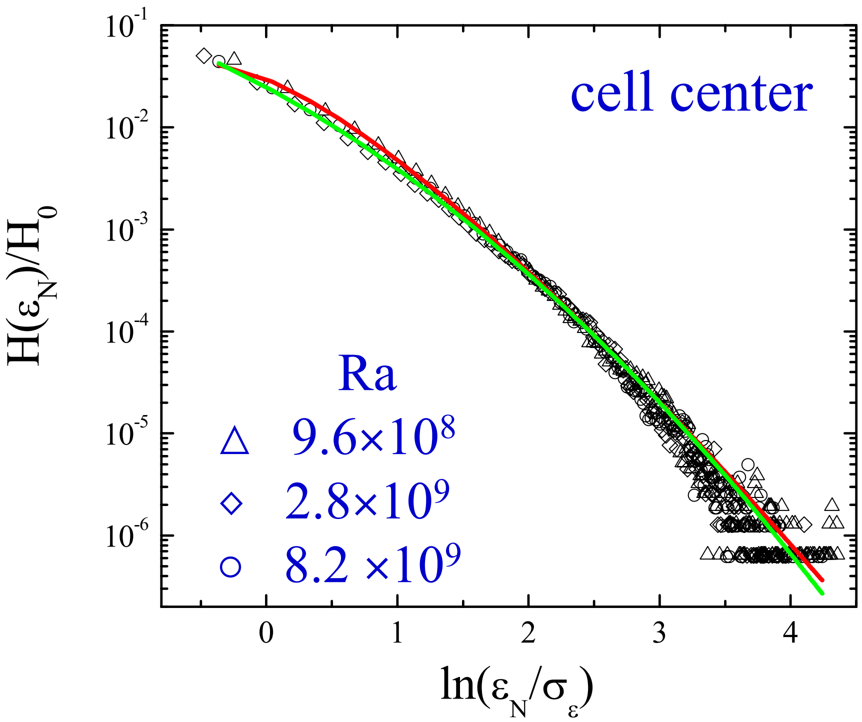
$$H(\varepsilon_f) = H_0 e^{-\alpha(\varepsilon_f/\sigma_\varepsilon)^\beta}$$

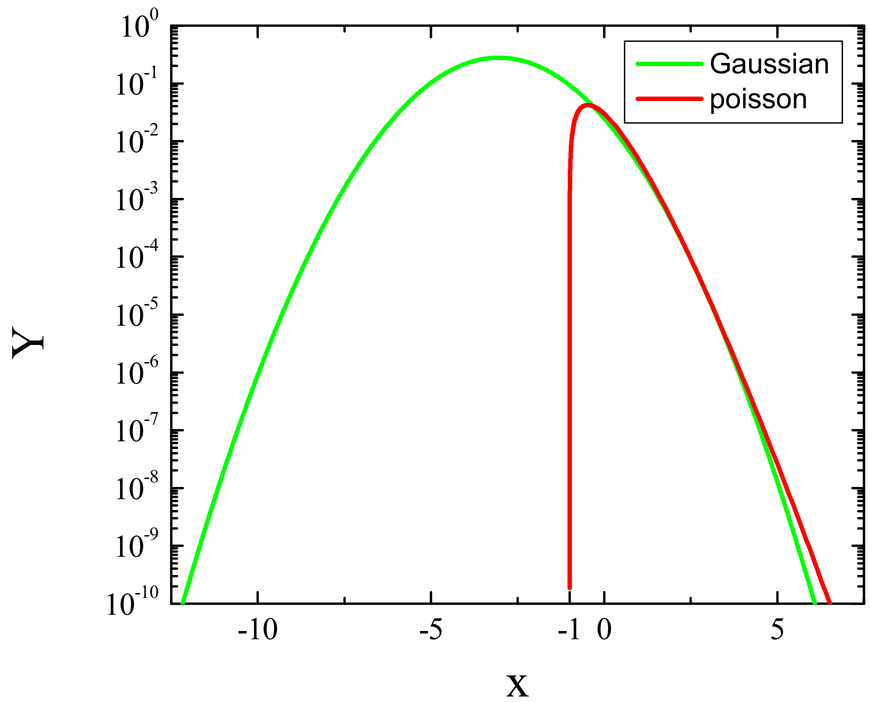
with $\alpha = 3.9$ and $\beta = 0.35$

$$H(\varepsilon_f) = H_0 e^{-\alpha(\varepsilon_f/\sigma_\varepsilon)^\beta}$$

with $\alpha = 3.1$ and $\beta = 0.47$

Functional form of the measured $H(\epsilon_f)$



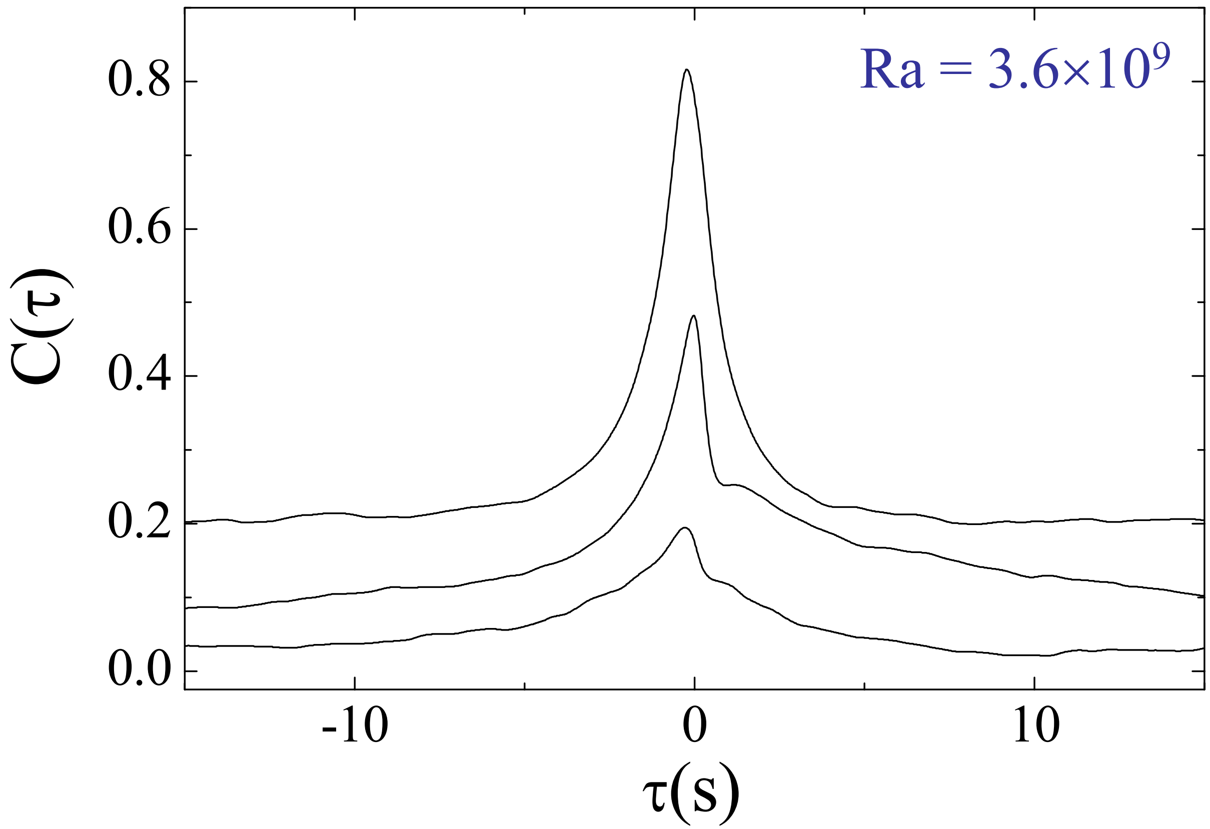


$$H(\varepsilon_f) = H_0 e^{-[\ln(\varepsilon_f/\sigma_\varepsilon)-m]^2/2\gamma^2}$$

$$H(\varepsilon_f) = H_0 \frac{e^{-\lambda}\lambda^{\log(\varepsilon_f/\sigma_\varepsilon)}}{[\log(\varepsilon_f/\sigma_\varepsilon)]!}$$

$m = -3.04$, $\gamma = 1.38$ and **$\lambda = 0.16$**

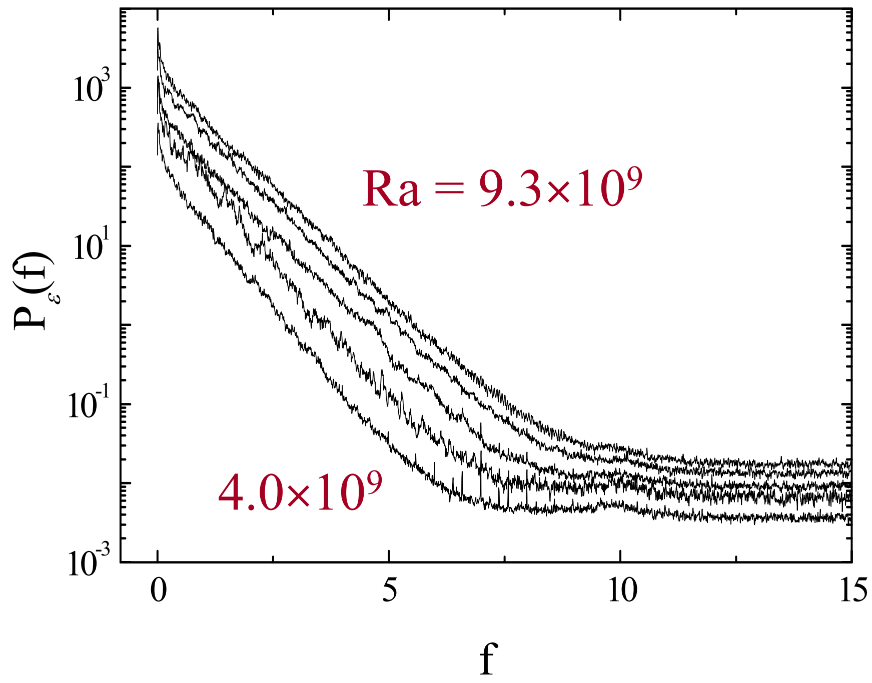
Cross-correlation function $C(\tau)=\langle \varepsilon_N(t)\delta T(t+\tau)\rangle / \sigma_T\sigma_\varepsilon$



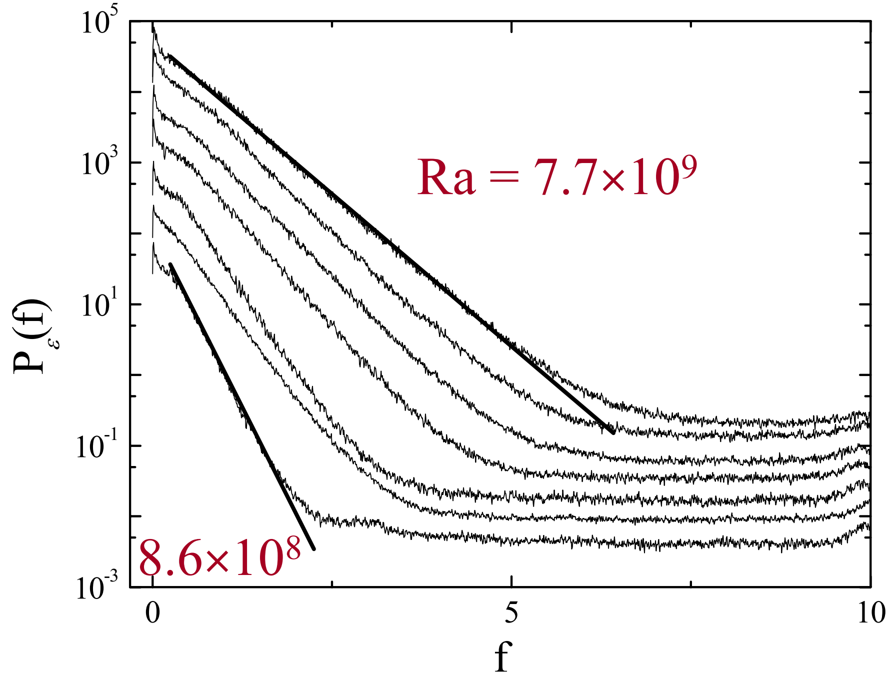
Correlation amplitude increases from C(0) = 0.2 (cell center) to C(0) = 0.4 (near the sidewall) and to C(0) = 0.6 (near the lower conducting plate).

Power spectrum of the local thermal dissipation rate $\epsilon_f(\mathbf{r},t)$

At the cell center



Near the sidewall



$$P_\varepsilon(f) = P_0 e^{-(f/f_C)}$$

4. Summary

- Measured thermal dissipation rate can be decomposed into two terms: $\epsilon_N(\mathbf{r}) = \epsilon_m(\mathbf{r}) + \epsilon_f(\mathbf{r})$, with the mean dissipation $\epsilon_m(\mathbf{r})$ concentrating in the thermal boundary layers and the fluctuations $\epsilon_f(\mathbf{r})$, which are produced by the detached thermal plumes, occupying mainly in the plume-dominated bulk region.

- Measured $\epsilon_f(\mathbf{r}) \sim Ra^{-\alpha}$ with $\alpha \approx 0.33$ at the cell center and near the sidewall and lower conducting plate.

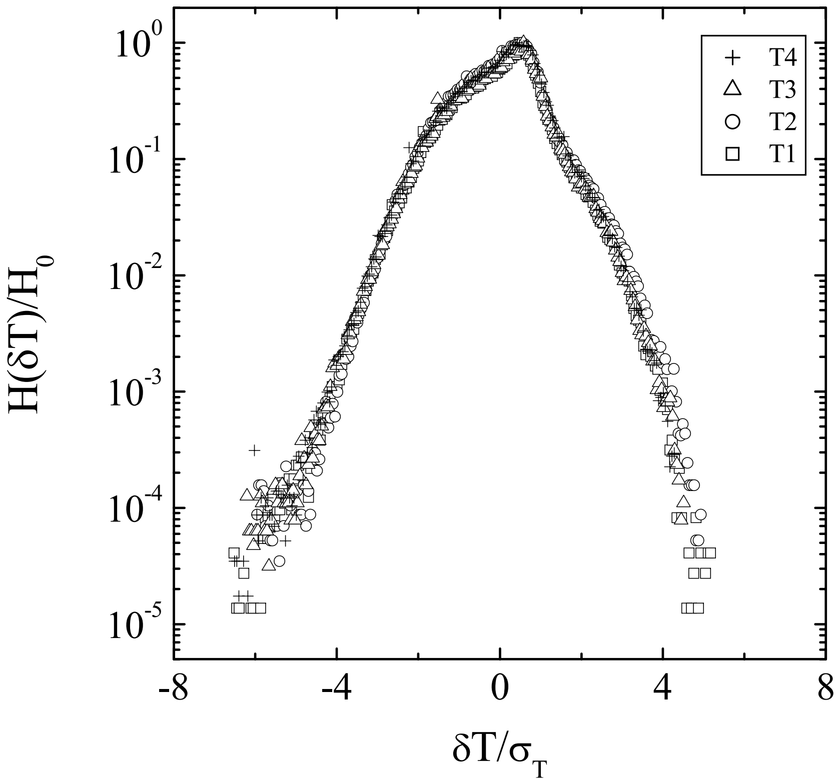
- Measured $\epsilon_m(\mathbf{r}) \sim Ra^{-\beta}$ with $\beta \approx 0.77$ inside the thermal boundary layer.

- At $Ra = 2.7\times10^9$, $\epsilon_f \approx 0.1Nu$ and $\epsilon_m = \epsilon_N - \epsilon_f \approx 0.9Nu$.

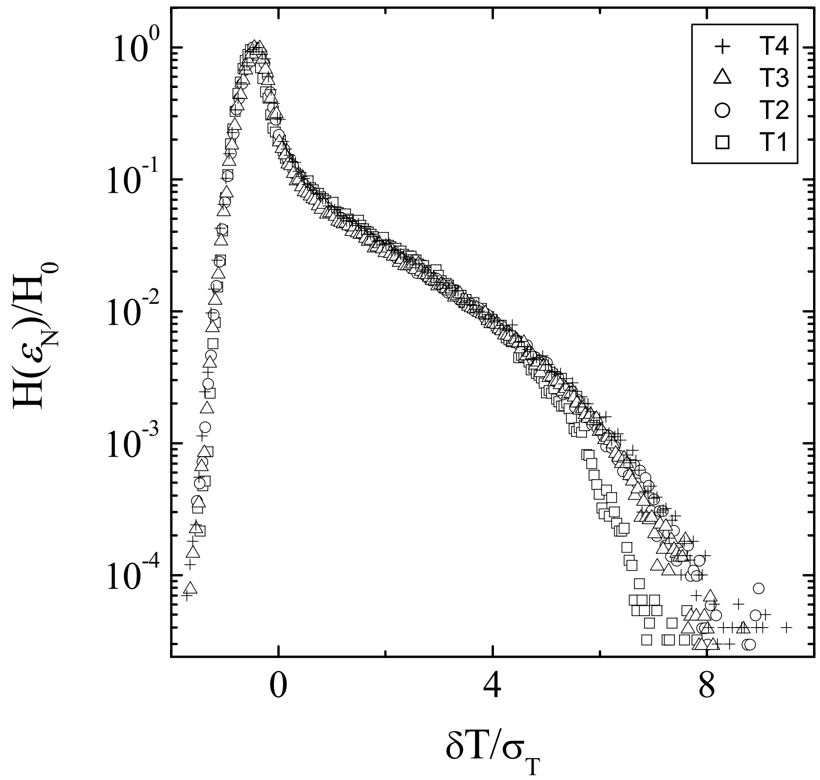
4. **Summary** - continued

- Measured histogram of $\epsilon_f(\mathbf{r},t)$ has a log-normal or log-Poisson form.
- Cross-correlation amplitude C(0) increases from 0.2 (cell center) to 0.4 (near the sidewall) and to 0.6 (near the lower conducting plate).
- Power spectrum of $\epsilon_f(\mathbf{r},t)$ has a simple exponential form $P_\epsilon(f) = P_0\exp(-f/f_c)$.
- The experiment reveals an interesting interplay between the mean dissipation and the fluctuations. A minimum modeling of the spatial distribution of $\epsilon_N(\mathbf{r})$ requires a decomposition of $\epsilon_N(\mathbf{r})$ into two terms with $\epsilon_m(\mathbf{r})$ in the thermal boundary layer ($z/\lambda_T \leq 1$) and $\epsilon_f(\mathbf{r})$ in the bulk region ($1 < z/\lambda_T \leq 10$).

Does the invasive probe perturb the local temperature field?



Multiple sensor measurements near the sidewall at Ra= 3.6×10^9



Multiple sensor measurements near the lower conducting plate at Ra= 3.6×10^9