The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1771 - 25

Conference and Euromech Colloquium #480

on

High Rayleigh Number Convection

4 - 8 Sept., 2006, ICTP, Trieste, Italy

Numerical study on non-Oberbeck-Boussinesq effects in Rayleigh-Bernard convection

K. Sugiyama
University of Twente
Enschede
The Netherlands

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Conference and Euromech Colloquium #480
On High Rayleigh Number Convection
Miramare, Trieste, Italy, September 4-8, 2006

Numerical Study on Non-Oberbeck-Boussinesq Effects in Rayleigh-Bénard Convection

Kazuyasu Sugiyama[1]
Enrico Calzavarini[1]
Francisco Fontenele Araujo[1]
Siegfried Grossmann[2]
Detlef Lohse[1]

[1]Department of Applied Physics, University of Twente, The Netherlands
[2]Fachbereich Physik der Philipps-Universität, Germany

Oberbeck-Boussinesq (OB) approximation

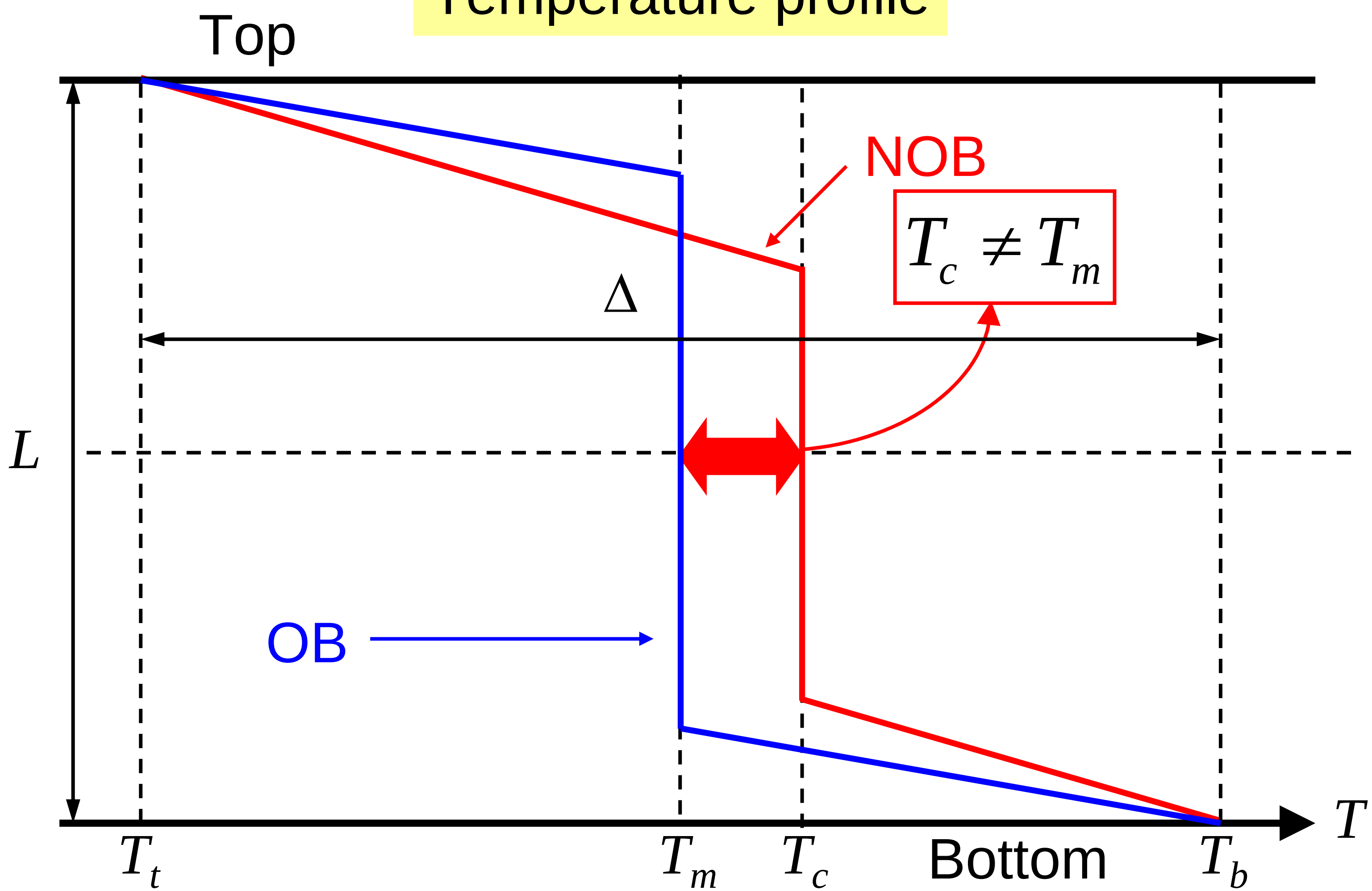
$\beta,\ \kappa,\ \nu$: constant $\quad Ra = \dfrac{\beta g \Delta L^3}{\nu\kappa}, \qquad Pr = \dfrac{\nu}{\kappa}$

Non-Oberbeck-Boussinesq (NOB) approximation

$\beta(T),\ \kappa(T),\ \nu(T)$

$$Ra = \frac{\beta_m g \Delta L^3}{\nu_m \kappa_m}, \qquad Pr = \frac{\nu_m}{\kappa_m}, \qquad \Delta = T_b - T_t$$

Temperature profile



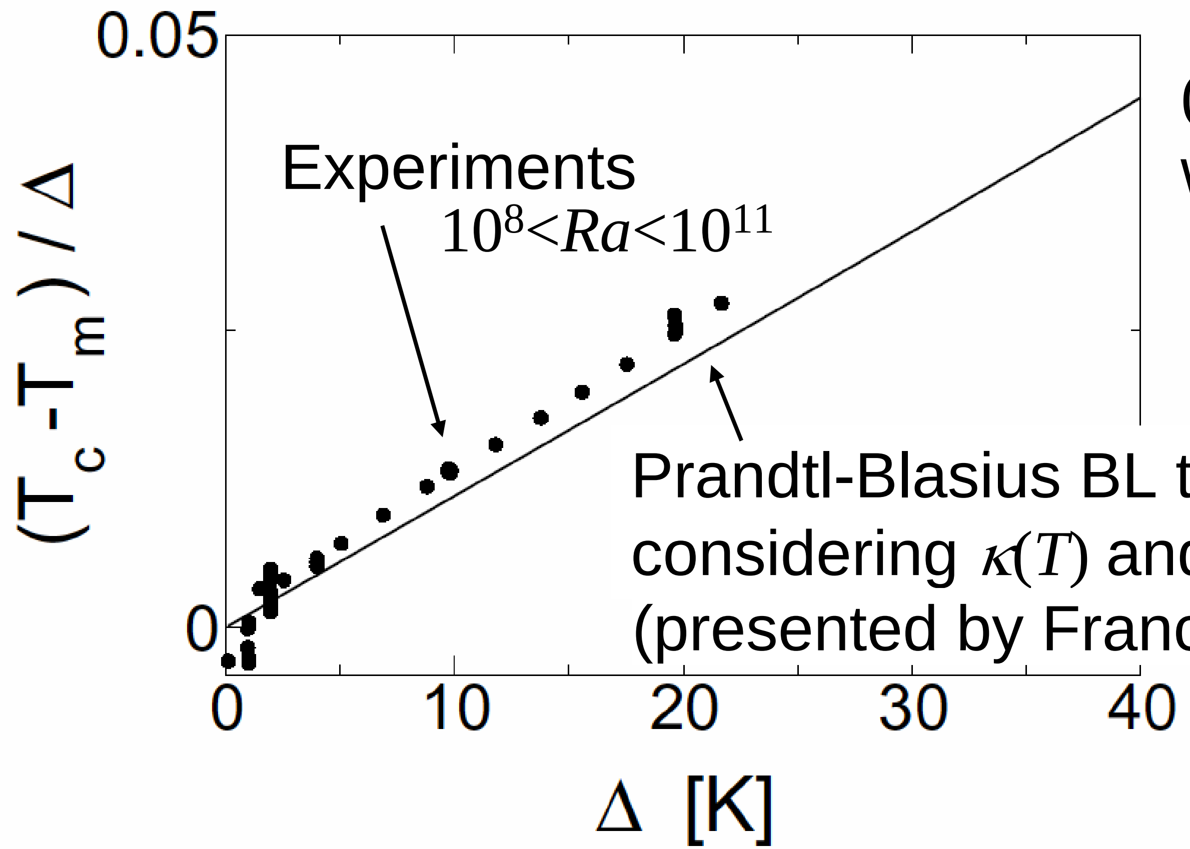
Experiments of temperature deviation $T_c - T_m$

Water

- Wu & Libchaber (1991, *Phys. Rev. A*, **43**)
- Ahler, Brown, Fontenele Araujo, Funfschilling, Grossmann & Lohse (2006, *J. Fluid Mech.* in press)

Glycerol

- Zhang, Childress & Libechaber (1997, *Phys. Fluids*, **9**)



$(T_c - T_m)/\Delta$ increases with increasing Δ.

Prandtl-Blasius BL theory considering $\kappa(T)$ and $\nu(T)$ (presented by Francisco Fontenele Araujo)

BL theory well describes NOB T_c in experiment

BL theory

assuming a laminar flow everywhere,
ignoring the temperature dependence of β,
determining unique T_c for given Δ, T_m, $\kappa(T)$ and $\nu(T)$.

Questions

How consistent T_c-agreement for various Ra?

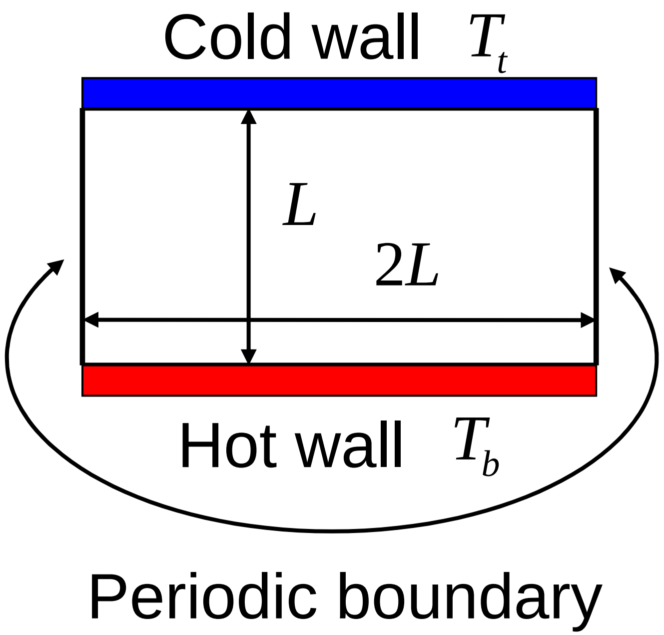
How to scale NOB Nu?

⇨

- Performing 2D direct simulations of NOB-RB convections in water and glycerol.
- Making comparisons with the BL theory and the available experimental data for $T_c - T_m$ and $Nu_{\mathrm{NOB}}/Nu_{\mathrm{OB}}$.

Simulation conditions

2D RB Flow



Mean temperature

$$T_m (= (T_b + T_t)/2) = 40^{\circ}\mathrm{C}$$

- Prandtl number $Pr = \dfrac{\nu_m}{\kappa_m}$,

$Pr_m = 4.398$ for water

$Pr_m = 2495$ for glycerol

Parameter

- "Non-Boussinesqness"

$$\Delta = T_b - T_t$$

$\Delta \le 60\mathrm{K}$ for water

$\Delta \le 50\mathrm{K}$ for glycerol

- Rayleigh number

$$Ra = \frac{\beta_m g \Delta L^3}{\nu_m \kappa_m}$$

$Ra_m \le 10^8$ for water

$Ra_m \le 10^7$ for glycerol

Variations of material properties

	β_b / β_t	ν_b / ν_t	κ_b / κ_t
Water (Δ=60K)	5	0.3	1.14
Glycerol(Δ=50K)	1.08	0.03	1.02

Governing equations

Continuity

$$\nabla \cdot \mathbf{u} = 0,$$

Navier-Stokes

$$\rho_m \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) = -\nabla p + \nabla \cdot \left\{ \rho_m \boxed{\nu(T)} \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right\} + \rho_m g \left(1 - \frac{\boxed{\rho(T)}}{\rho_m} \right) \mathbf{e}_y,$$

Heat transfer

$$\rho_m c_{pm} \left(\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T \right) = \nabla \cdot \left(\rho_m c_{pm} \boxed{\kappa(T)} \nabla T \right).$$

Simulation method

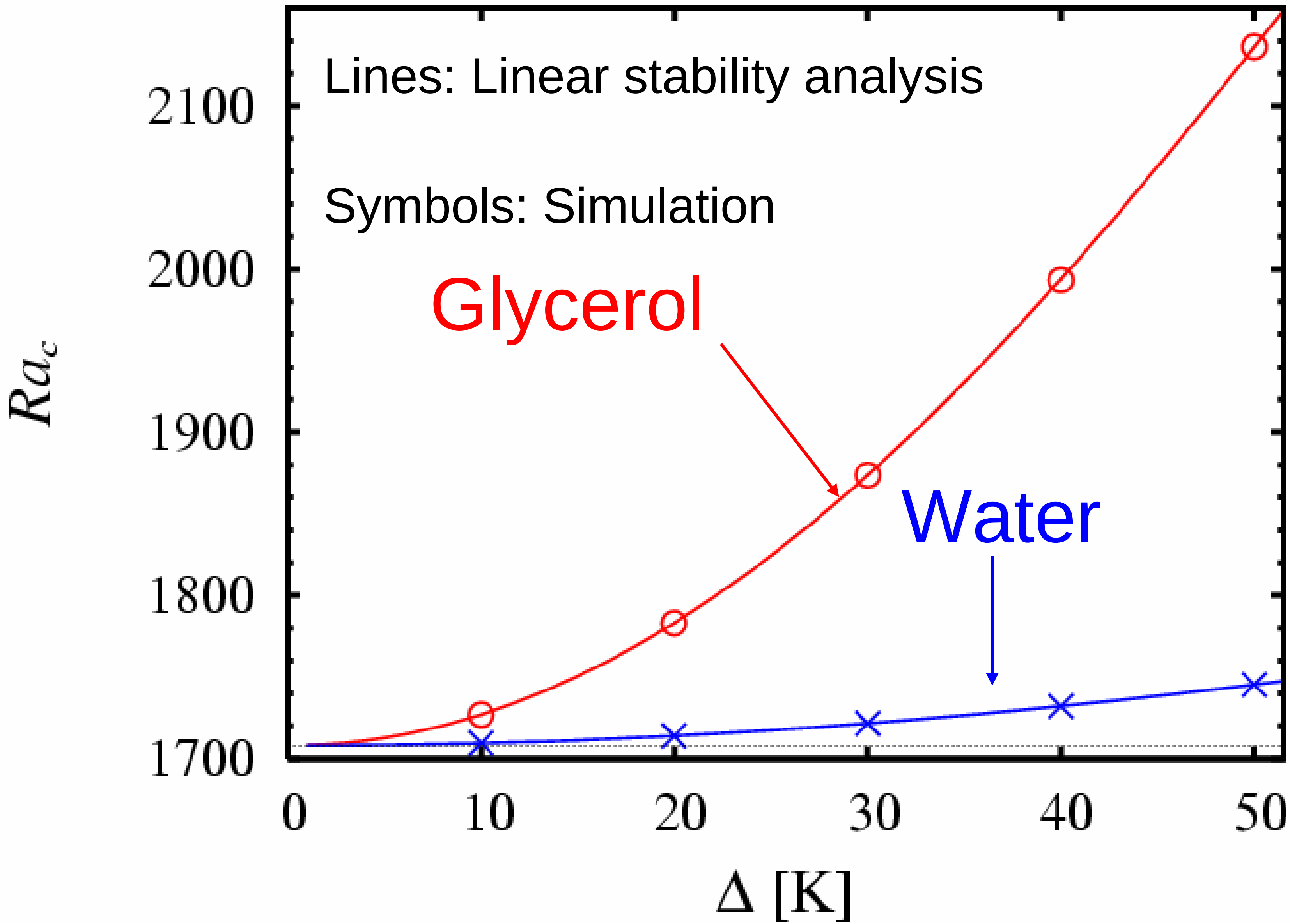
4th-order FDM (Highly energy conservative discretization by Kajishima *et al*., 2001, *JSME Int. J. B*, 44)

Statistical convergence (sampling time)

(uncertainty of *Nu*) ~ about 0.1%

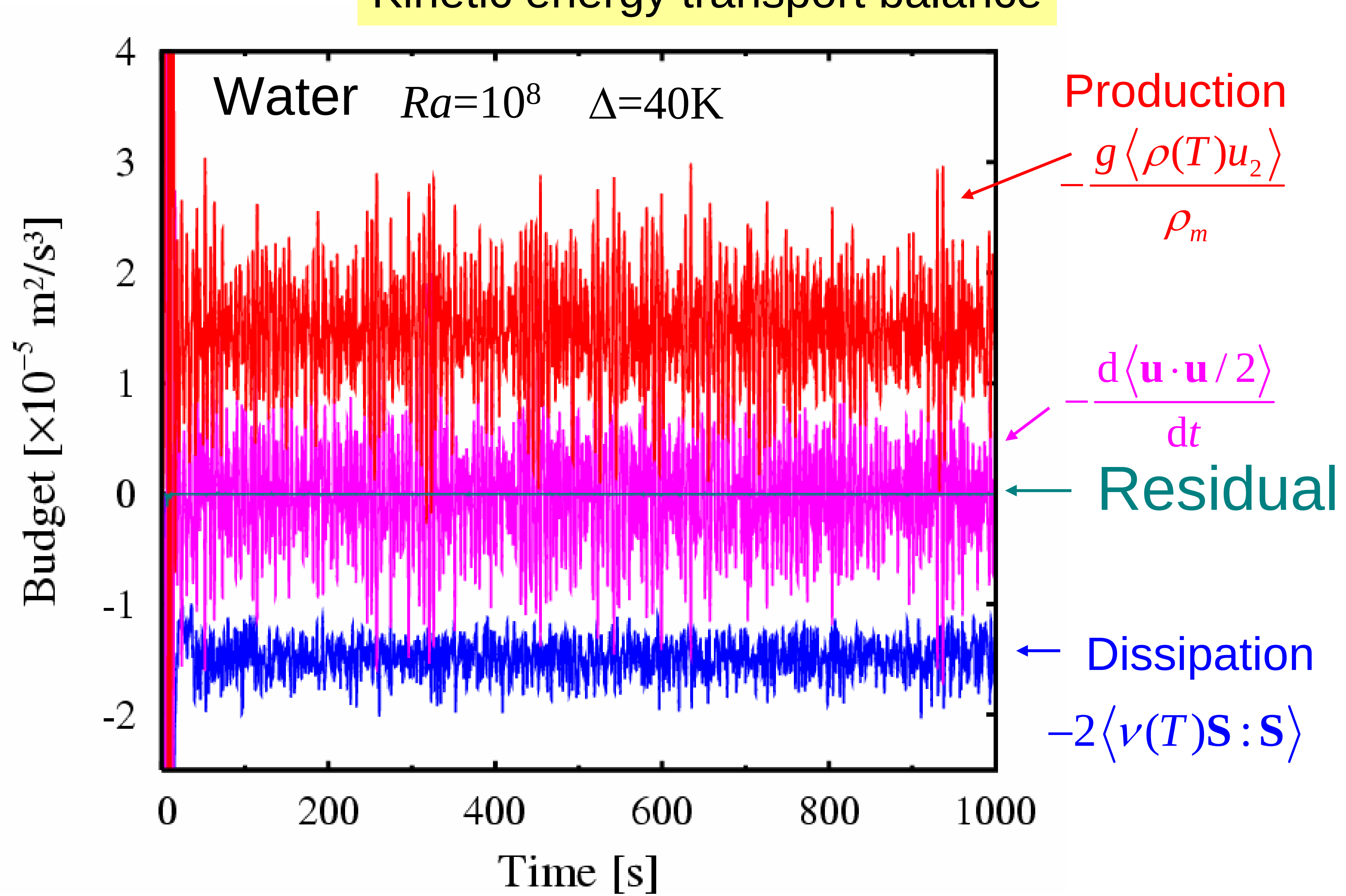
Code validation (Non-Boussinesqness)

Critical Rayleigh number vs. Δ

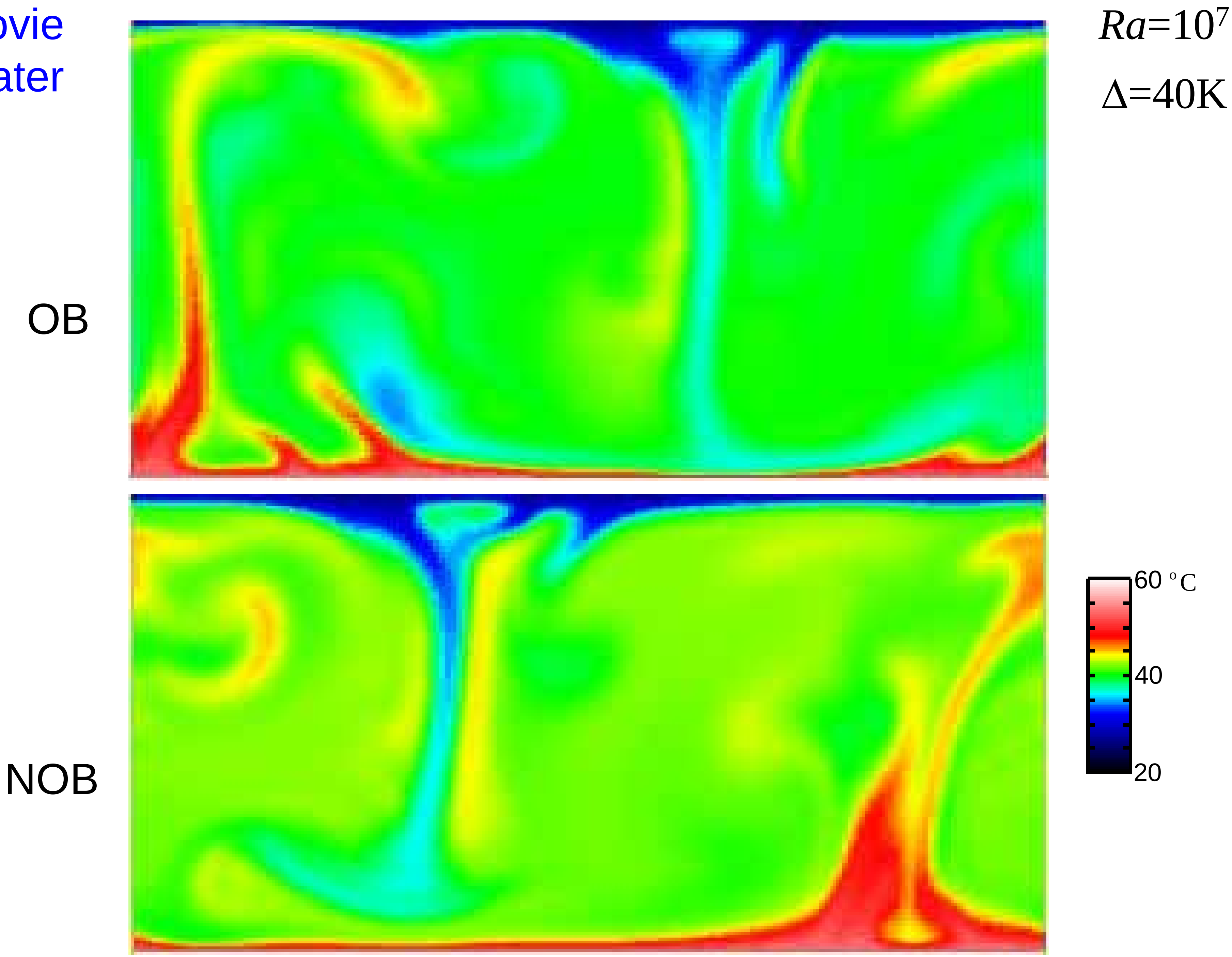


Code validation (numerical convergence)

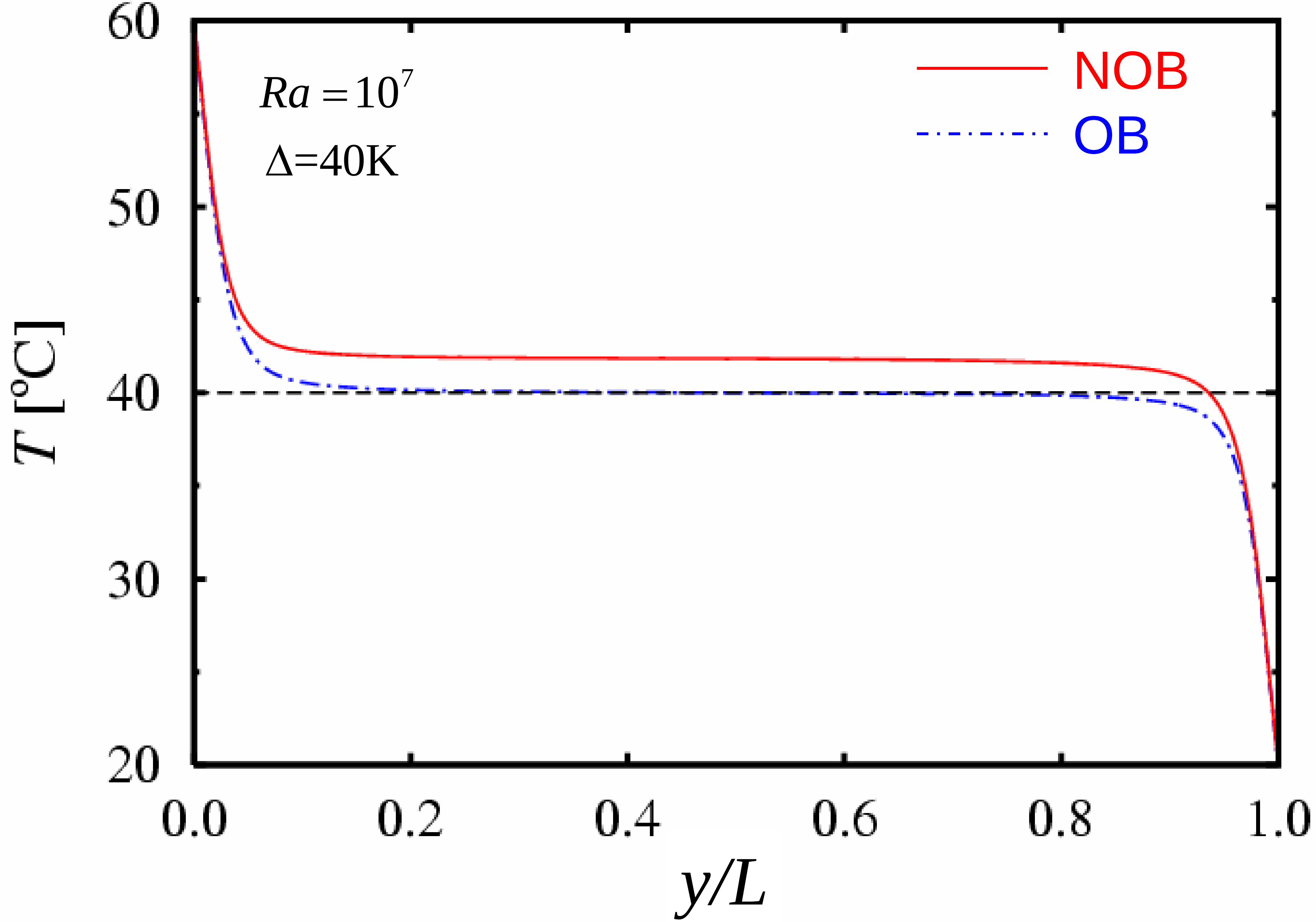
Kinetic energy transport balance



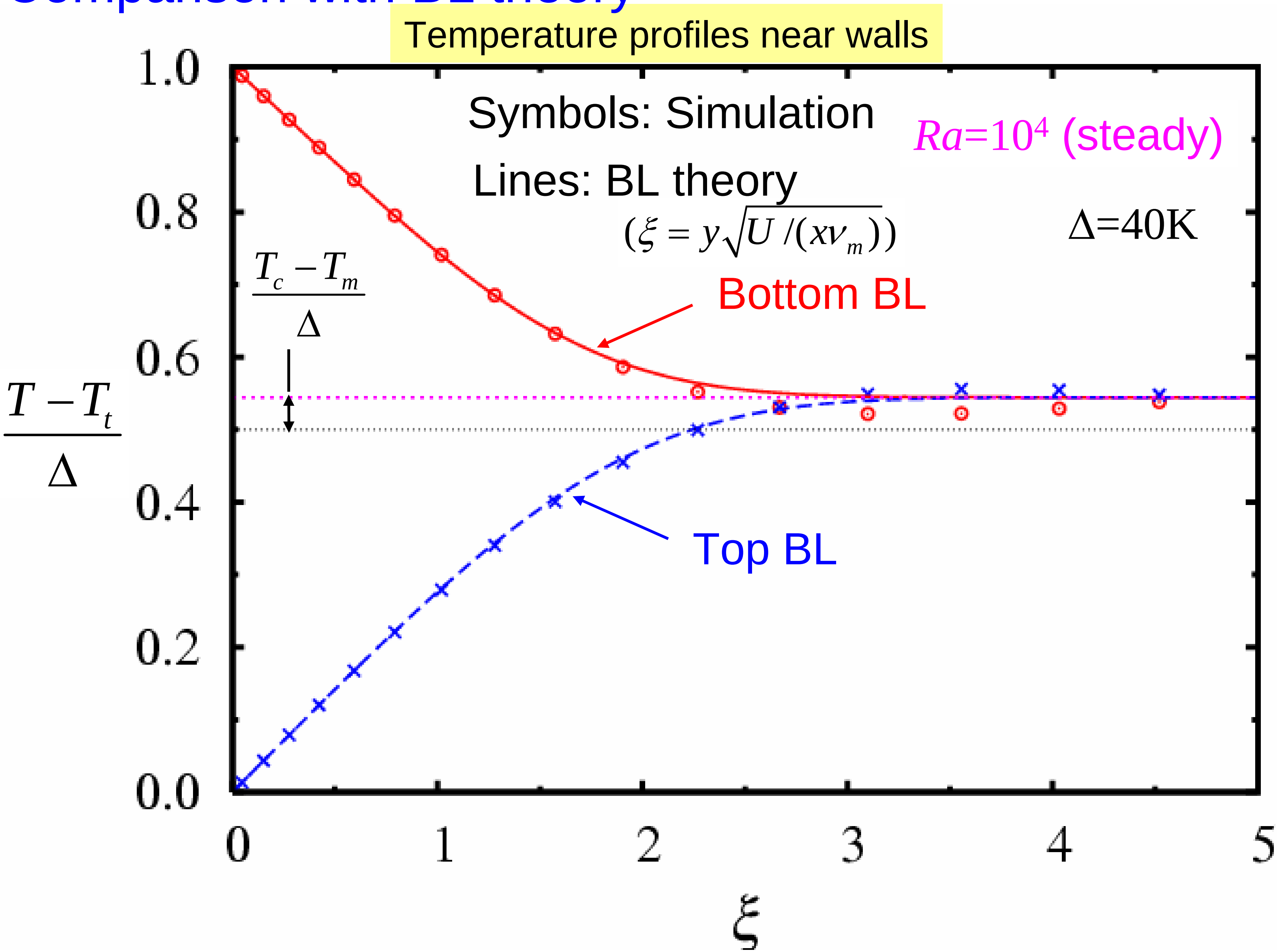
Movie
Water



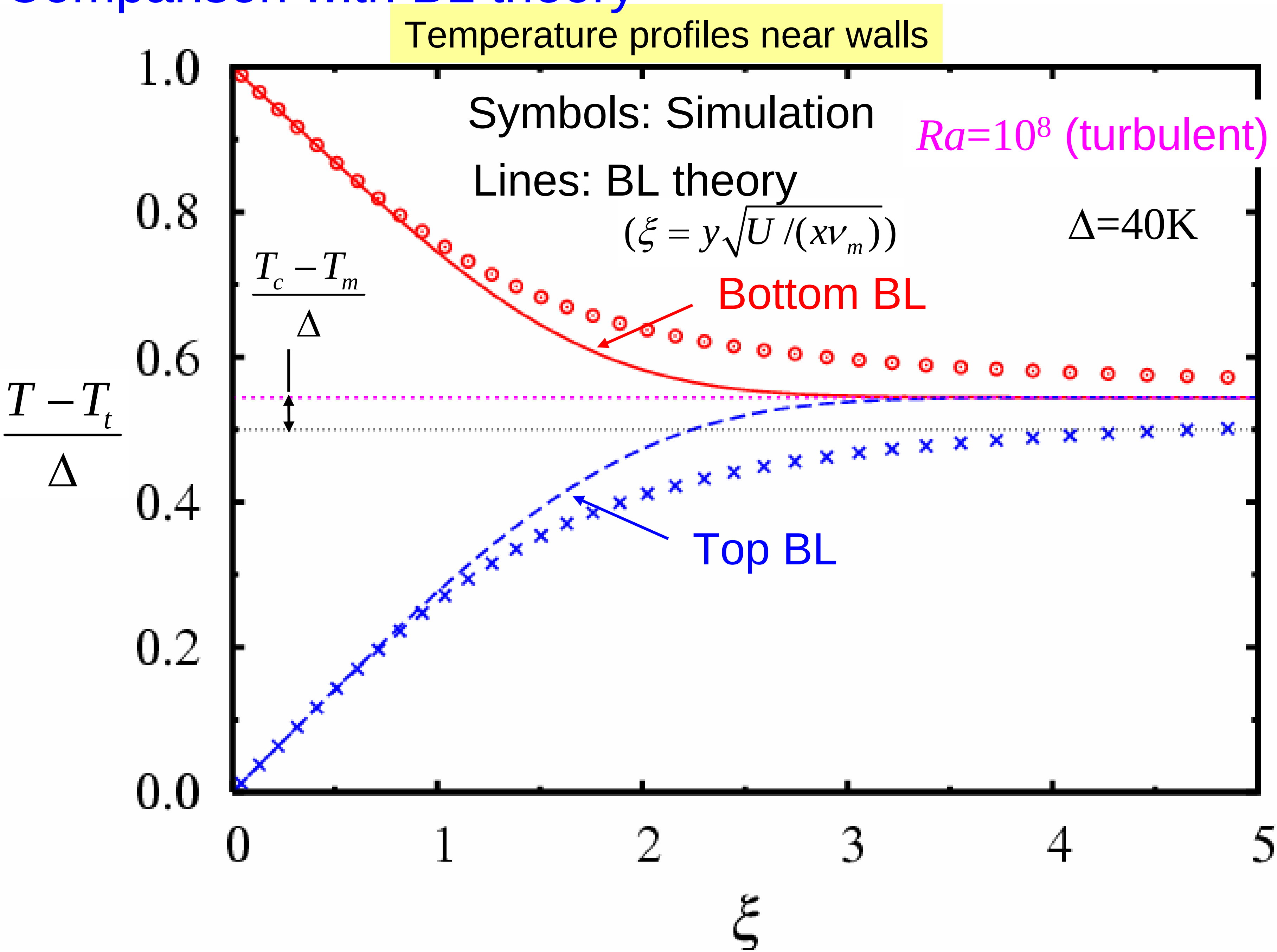
Temperature profile



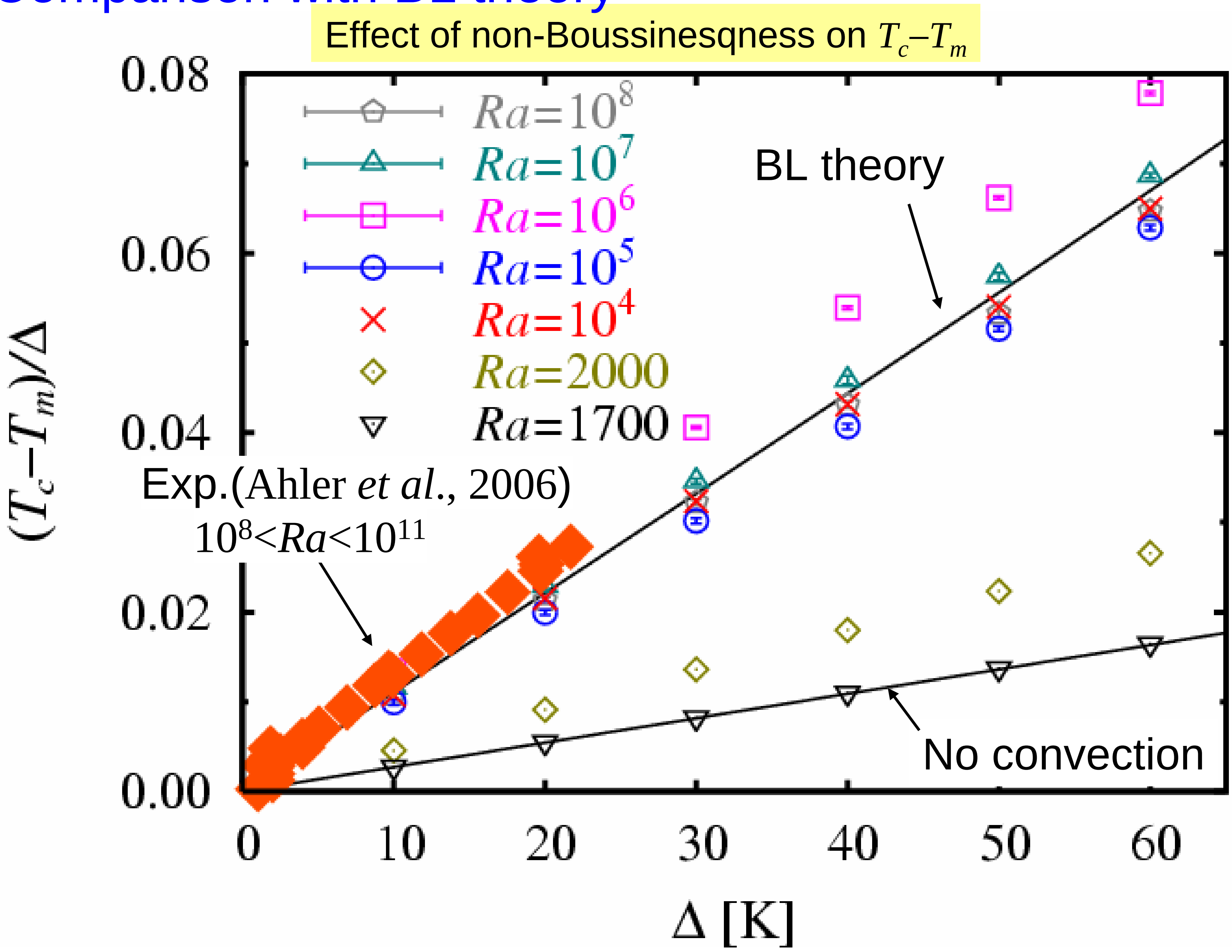
Comparison with BL theory



Comparison with BL theory

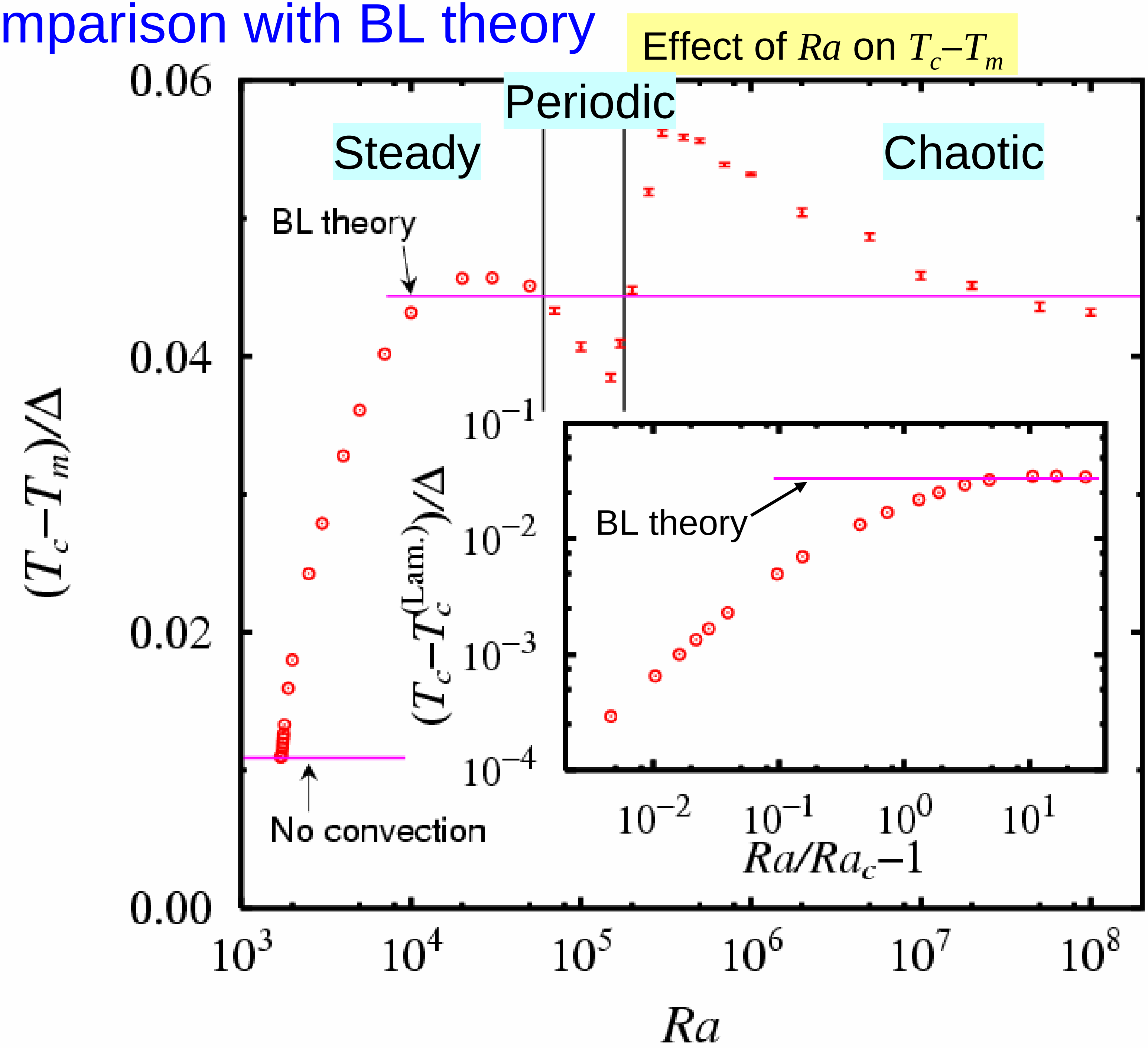


Comparison with BL theory



Comparison with BL theory

Δ=40K



BL theory well describes NOB T_c for $Ra >> Ra_c$

How to scale NOB Nu?

Nusselt number ratio (Ahlers *et al.*,2006)

$$\frac{Nu_{\mathrm{NOB}}}{Nu_{\mathrm{OB}}} = F_1 \cdot F_2$$

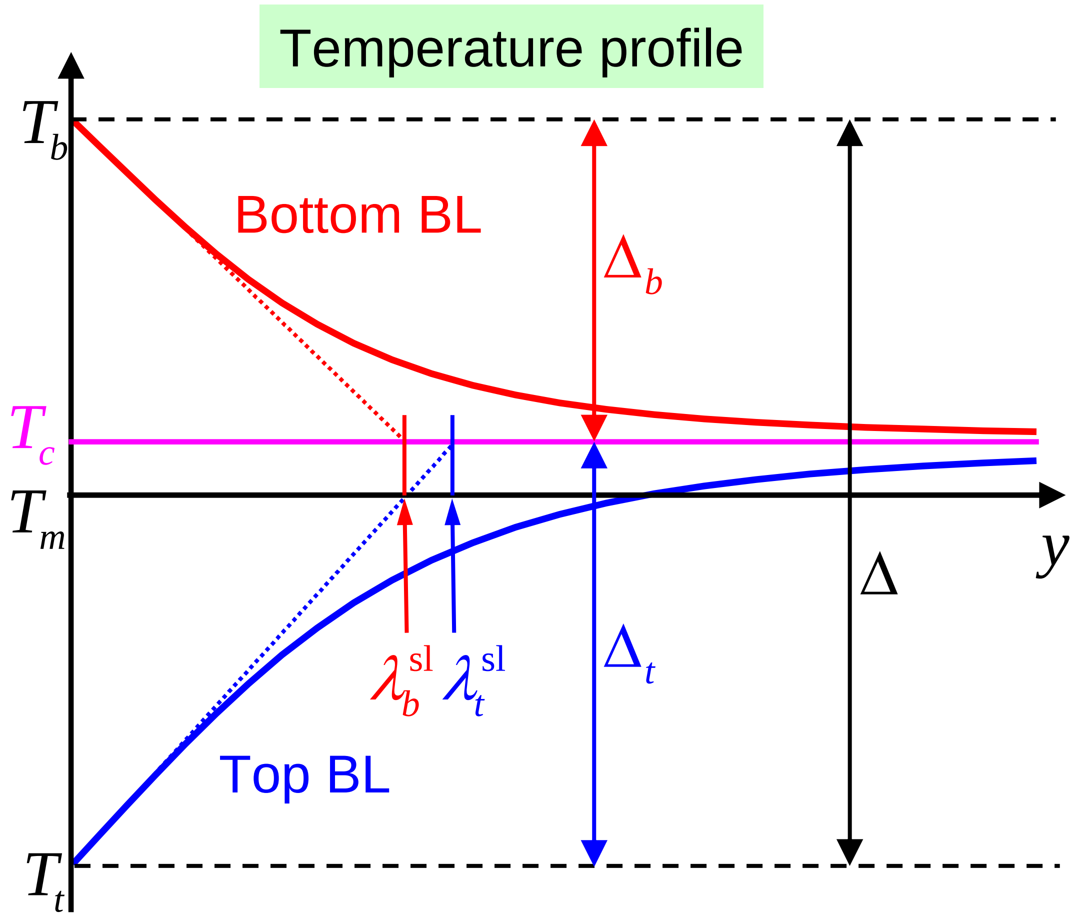
Change of thermal BL thickness

$$F_1 = \frac{2\lambda_{\mathrm{OB}}^{\mathrm{sl}}}{\lambda_t^{\mathrm{sl}} + \lambda_b^{\mathrm{sl}}}$$

Change of center temperature

$$F_2 = \frac{\kappa_t \Delta_t + \kappa_b \Delta_b}{\kappa_m \Delta}$$

$$= \frac{\underbrace{(\kappa_t - \kappa_b)}_{<0} T_c - \kappa_t T_t + \kappa_b T_b}{\kappa_m \Delta}$$



Nusselt number ratio (Ahlers *et al.*,2006)

$$\frac{Nu_{\mathrm{NOB}}}{Nu_{\mathrm{OB}}} = F_1 \cdot F_2$$

Change of thermal BL thickness

$$F_1 = \frac{2\lambda_{\mathrm{OB}}^{\mathrm{sl}}}{\lambda_t^{\mathrm{sl}} + \lambda_b^{\mathrm{sl}}} \approx 1 \, ?$$

Ra, material property

Change of center temperature

$$F_2 = \frac{\kappa_t \Delta_t + \kappa_b \Delta_b}{\kappa_m \Delta}$$

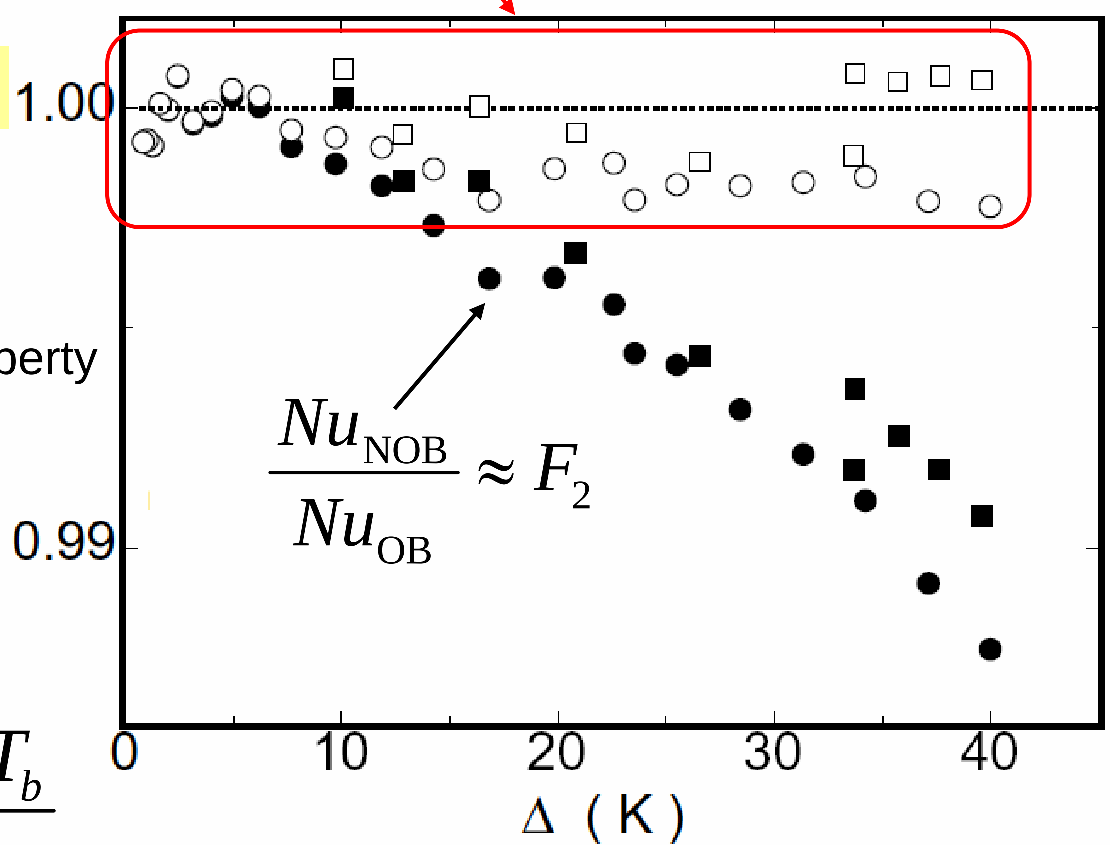
$$= \frac{(\kappa_t - \kappa_b) T_c - \kappa_t T_t + \kappa_b T_b}{\kappa_m \Delta}$$

$(\kappa_t - \kappa_b)$ <0

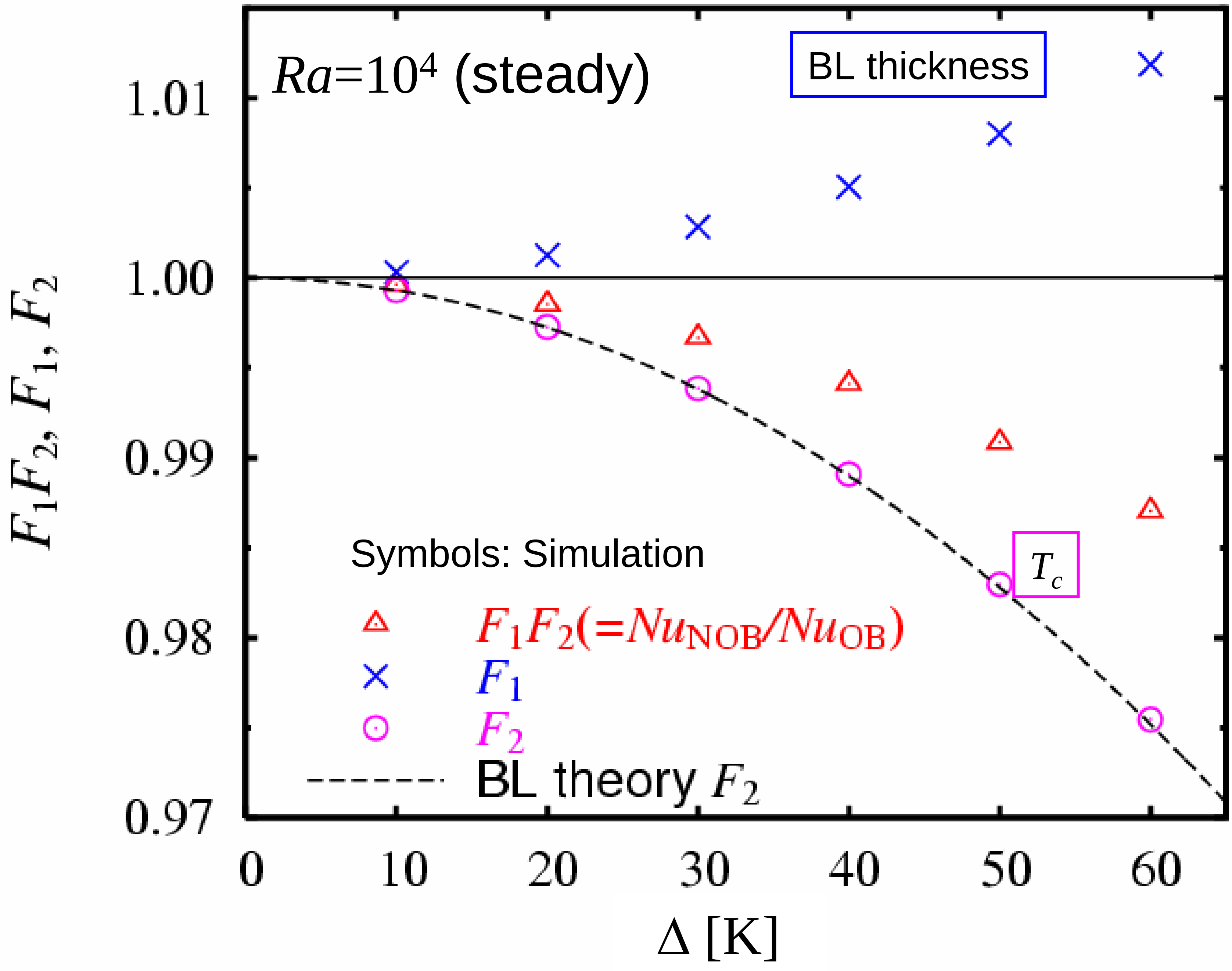
Nu ratio and F_1 vs. Δ (Ahlers *et al.*,2006)

$10^8 < Ra < 10^{11}$

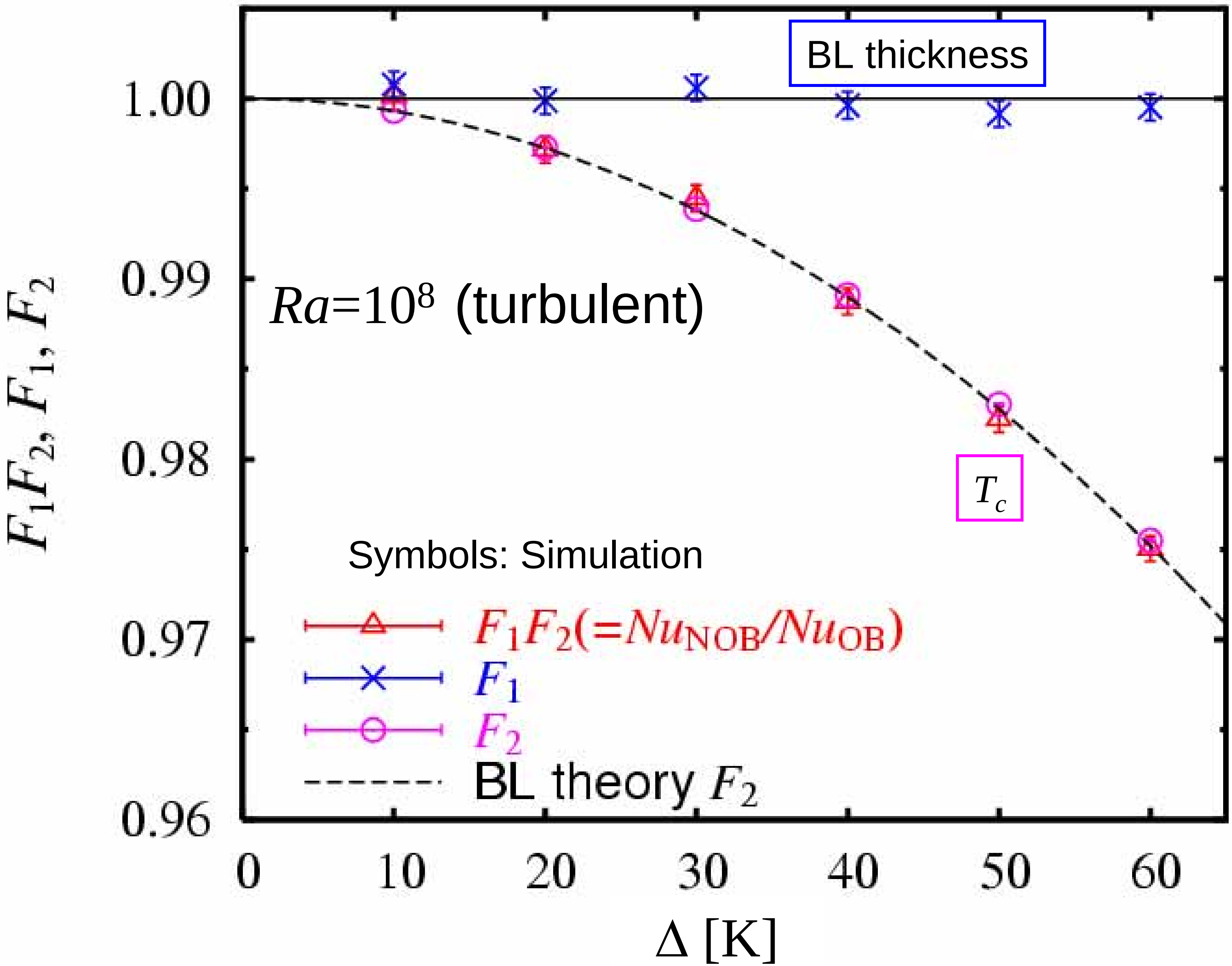
○□ $F_1 \approx 1$



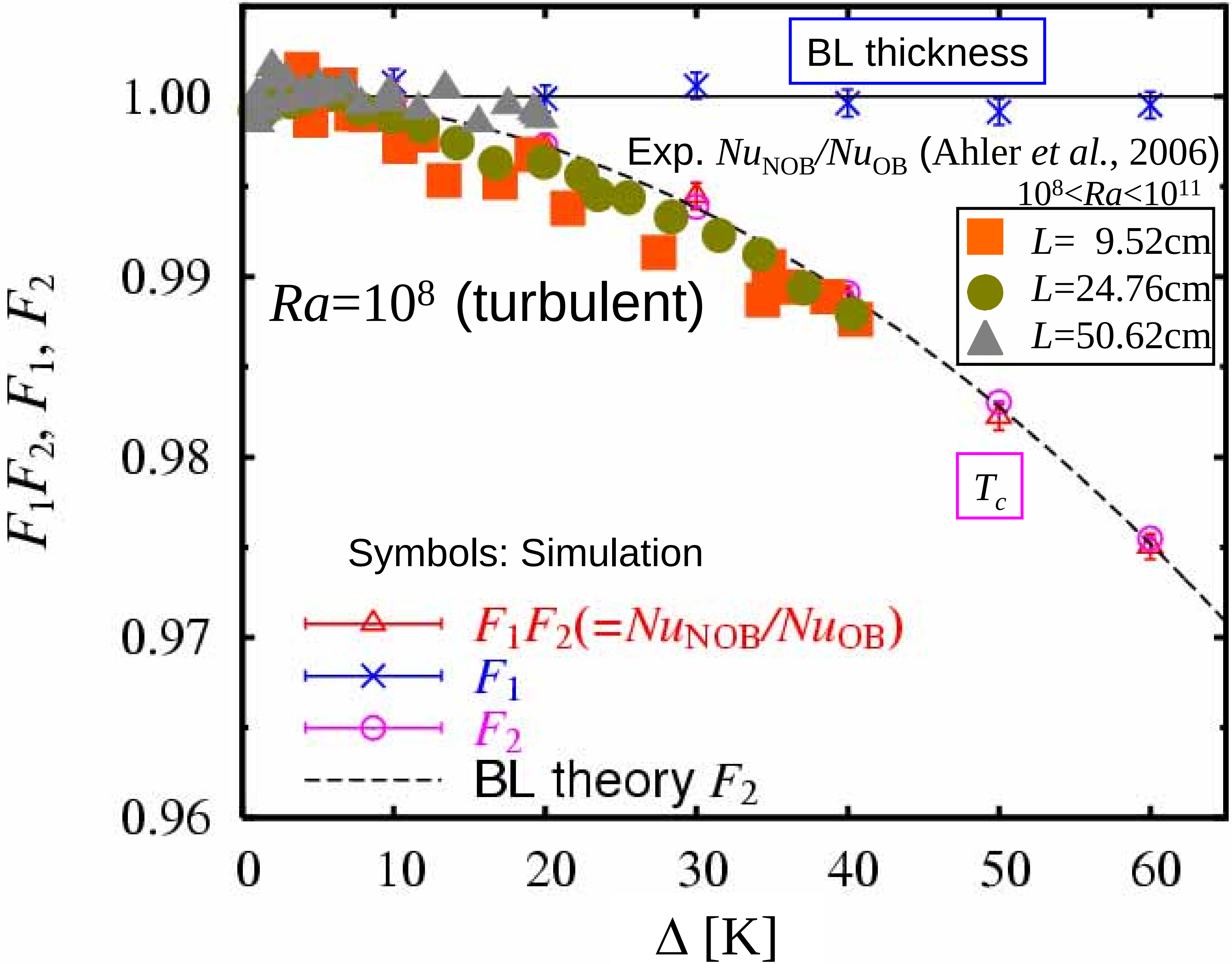
Effect of non-Boussinesqness on $Nu_{\mathrm{NOB}}/Nu_{\mathrm{OB}}$



Effect of non-Boussinesqness on $Nu_{\mathrm{NOB}}/Nu_{\mathrm{OB}}$



Effect of non-Boussinesqness on Nu_{NOB}/Nu_{OB}



Nu_{NOB}/Nu_{OB} for water at T_m=40^{o}C

consistent result among

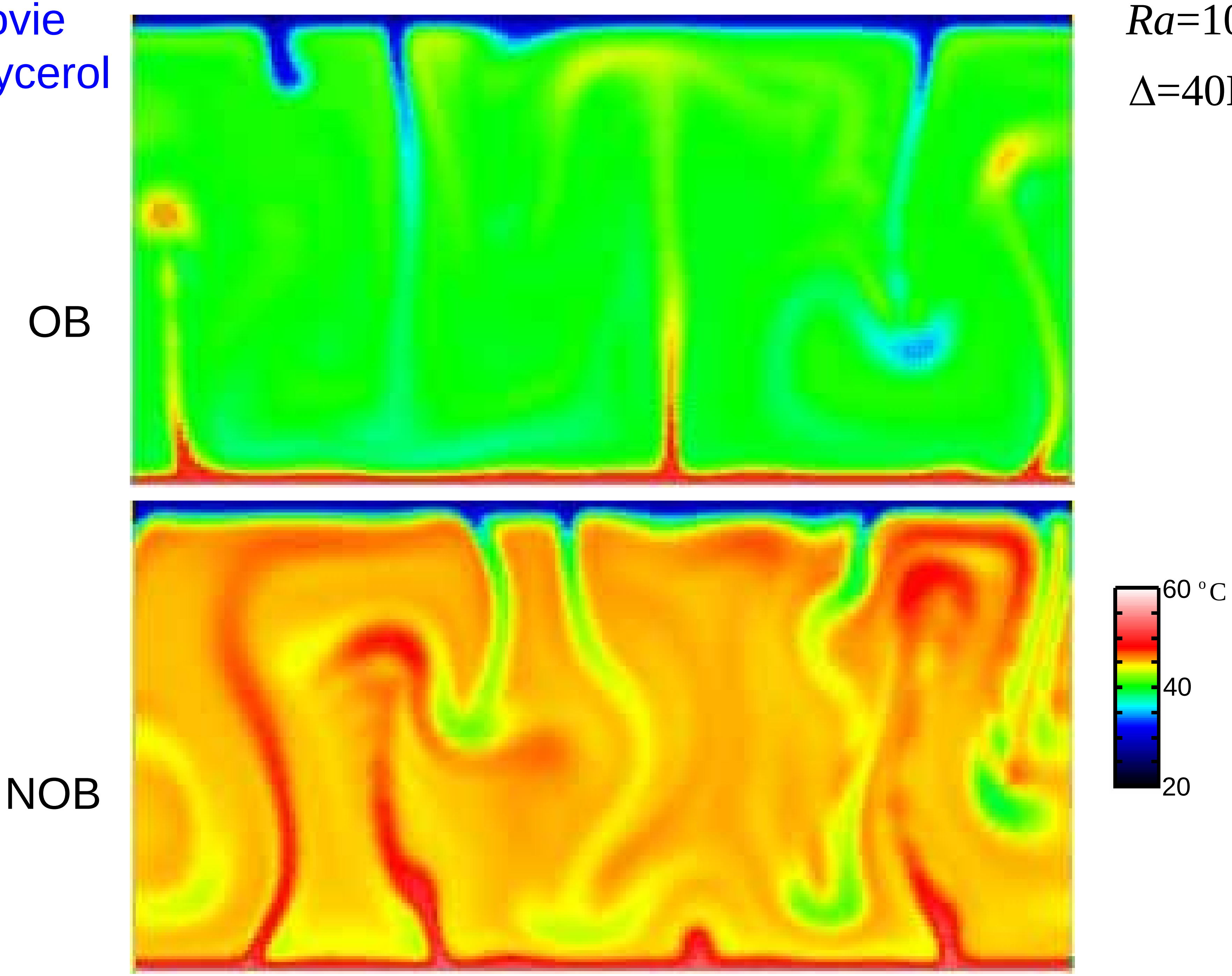
- Experiment ($10^8 < Ra < 10^{11}$)
- Simulation ($Ra = 10^8$)
- F_2 predicted by BL theory ($F_1 = 1$)

Universal for any fluid?

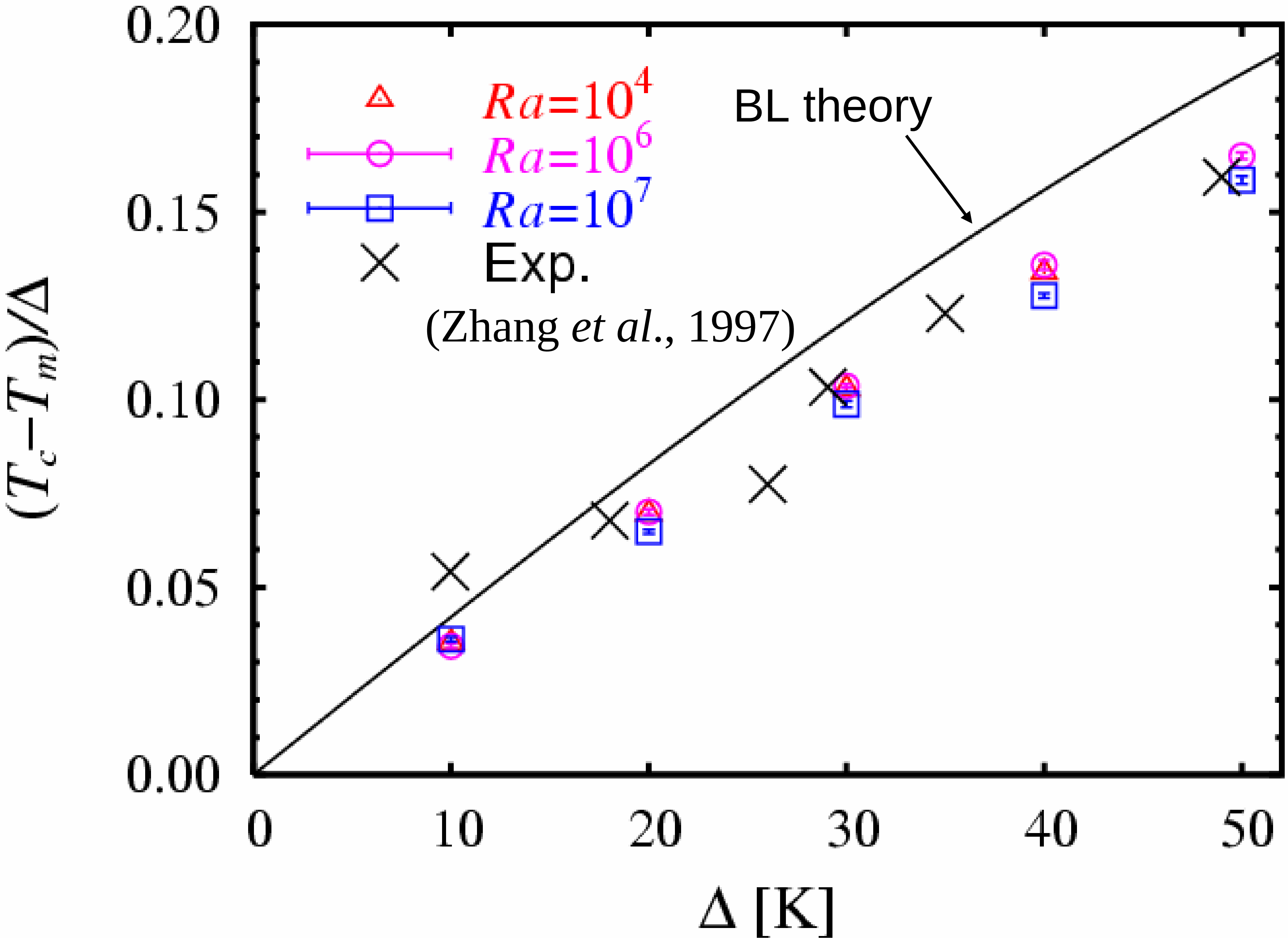
Movie
Glycerol

$Ra = 10^7$

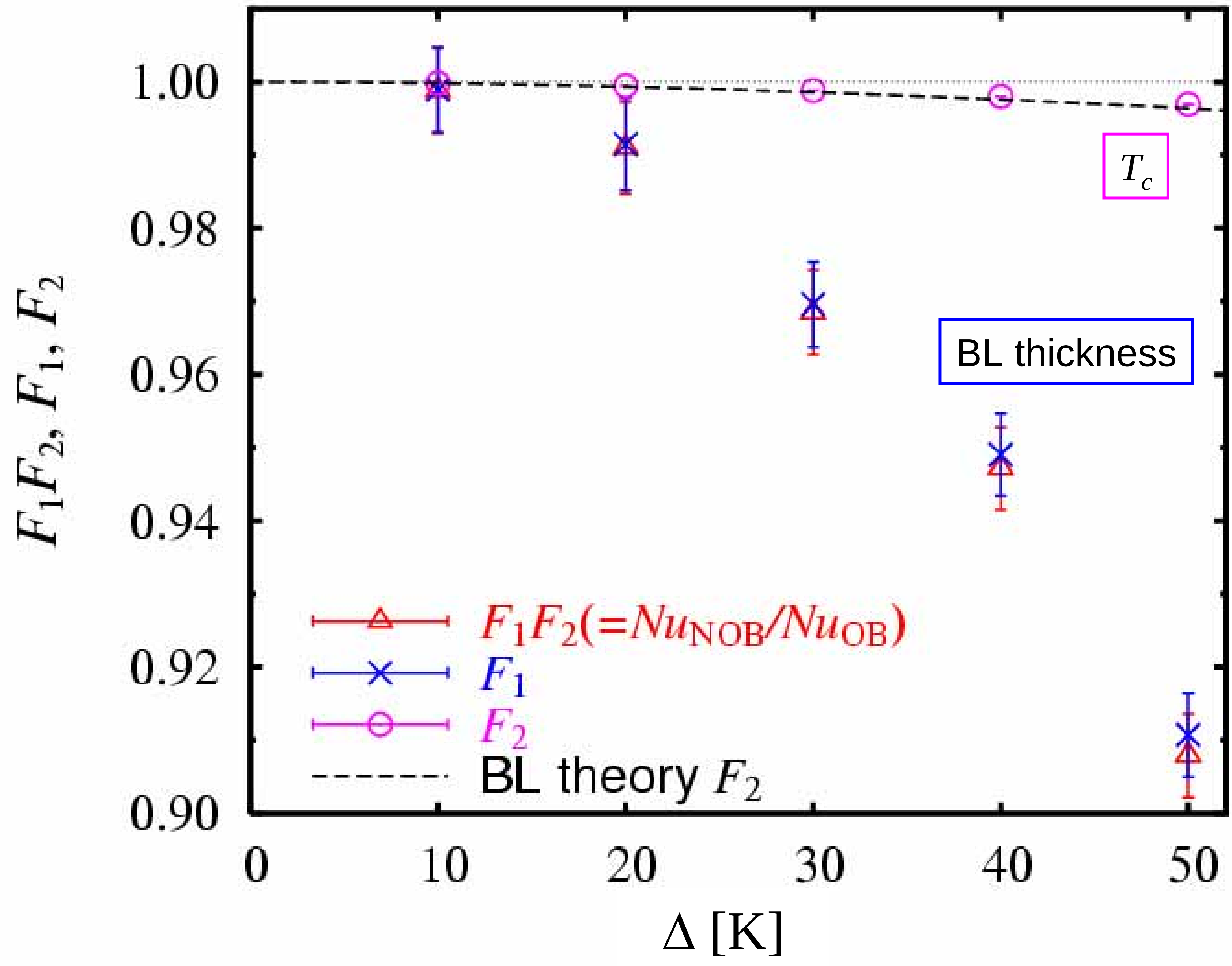
$\Delta = 40K$



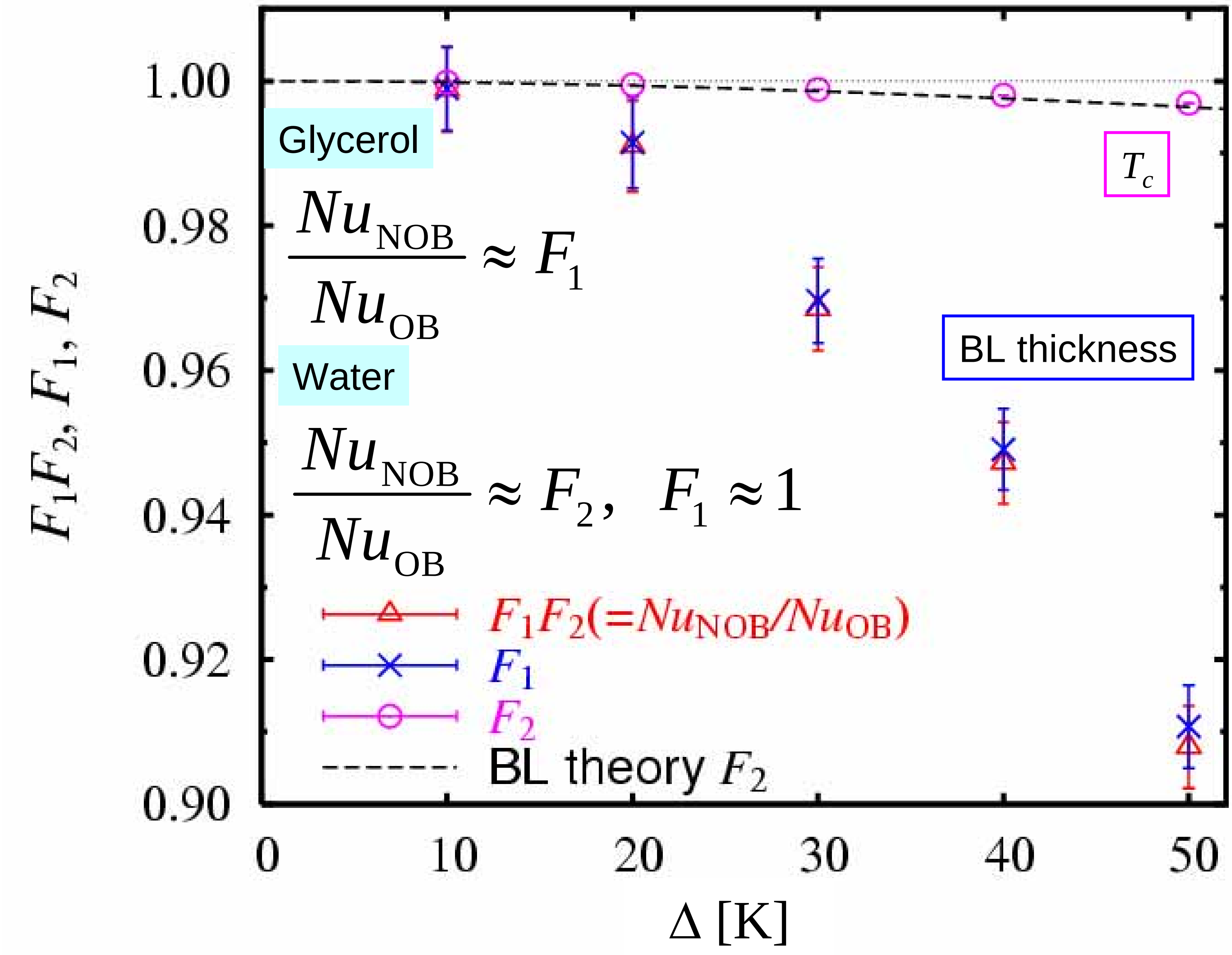
Effect of non-Boussinesqness on $T_c - T_m$ Glycerol



Effect of non-Boussinesqness on Nu_{NOB}/Nu_{OB} $Ra=10^7$



Effect of non-Boussinesqness on Nu_{NOB}/Nu_{OB} $Ra=10^7$



Conclusions (2D simulation of NOB-RB convection)

Center temperature T_c

- consistent with available experiments and BL theory. ("non-Boussinesqness" Δ)

Nusselt number ratio $Nu_{\mathrm{NOB}}/Nu_{\mathrm{OB}}=F_1 F_2$

- For water: dominated by F_2 (change of T_c)
- For glycerol: dominated by F_1 (change of thermal BL thickness)

F_1: dependent on material property.
How to explain?