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on

High Rayleigh Number Convection

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Thermal plumes non-Boussinesq convection Rayleigh Taylor instability

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Thermal plumes, non-Boussinesq convection, Rayleigh-Taylor instability

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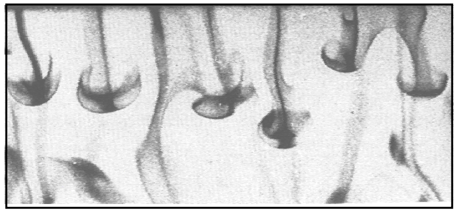
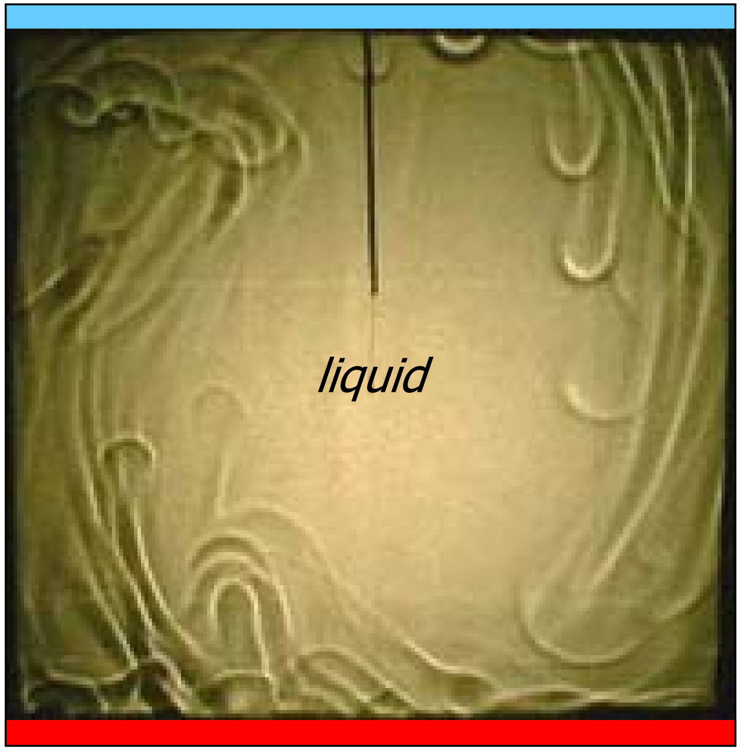
Non-Boussinesq convection

Compressible fluid in gravity field
heated from the bottom and cooled from the top

instabilities



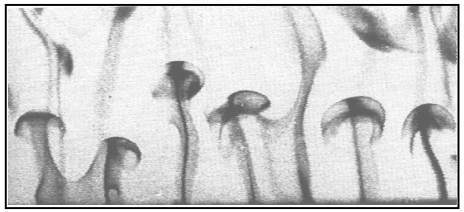
cooling



heating

thermal wind

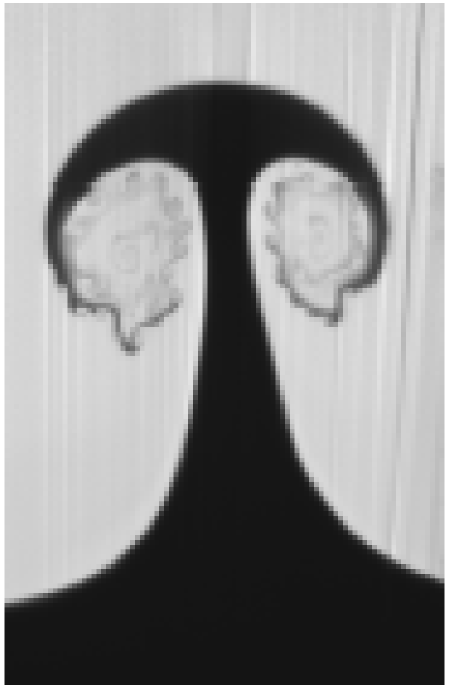
thermal plumes



instabilities

Rayleigh-Taylor instability

Two fluids of different densities are accelerated against the density gradient



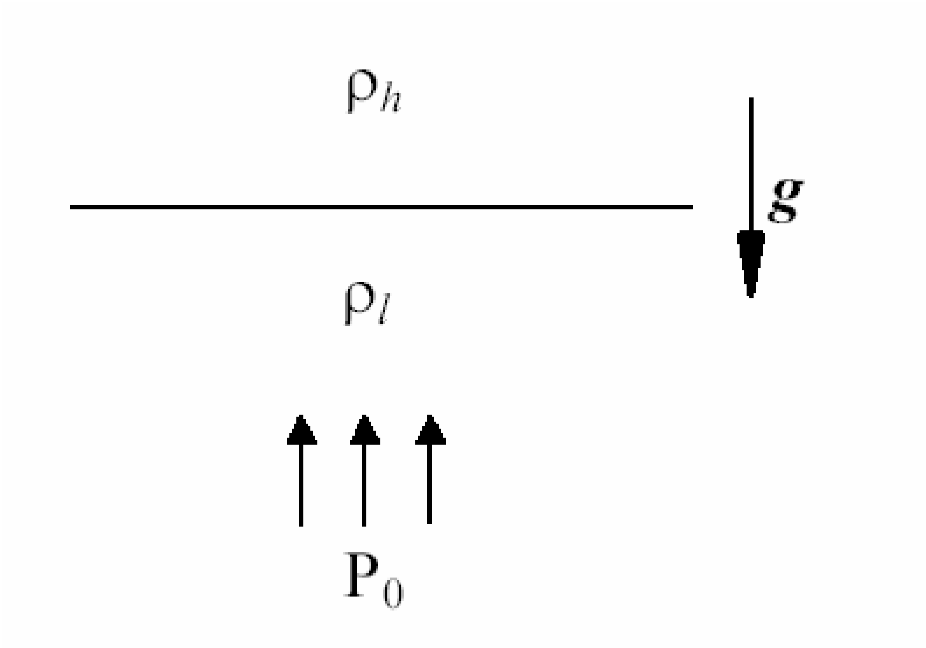
$\rho_h/\rho_l = 3, \ \lambda \sim 6cm, \ t \sim 10ms$

Jacobs et al[2004]

$(\rho_h - \rho_l)/\rho_h \sim 10^{-3}, \ \lambda \sim 1cm, \ t \sim 1s$

Ramaprabhu & Andrews[2004]

Rayleigh-Taylor instability

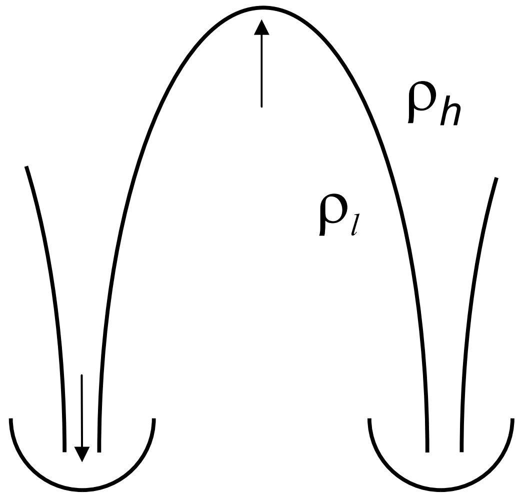


$P_0 = 10^5 Pa, \qquad P = \rho\, g\, h$

$\rho_h \sim 10^3\ kg/m^3, \quad g \sim 10\ m/s^2$

$h \sim 10$ m

grad P grad $\rho < 0$



Water flows out from an overturned cup

Lord Rayleigh, 1883,
Sir G.I. Taylor 1950

Non-Boussinesq convection and RT instability

non-Boussinesq convection

thermal plumes
thermal equilibrium
entropy grads/jumps

Rayleigh-Taylor instability

bubbles/spikes
mechanical equilibrium
density grads/jumps

This audience knows the non-Boussinesq convection better than anyone else.

Special care on the mining and interpretation of the experimental data is a distinct feature of the research presented at this meeting.

Here we suggest some ideas on how to connect the non-Boussinesq convection, the Rayleigh-Taylor instability, and a variety of turbulent processes in real unsteady multiphase flows.

The Rayleigh-Taylor turbulent mixing

Why is it important to study?

Rayleigh-Taylor turbulent mixing

is extensive interfacial mixing of the fluids

controls a wide variety of physical phenomena

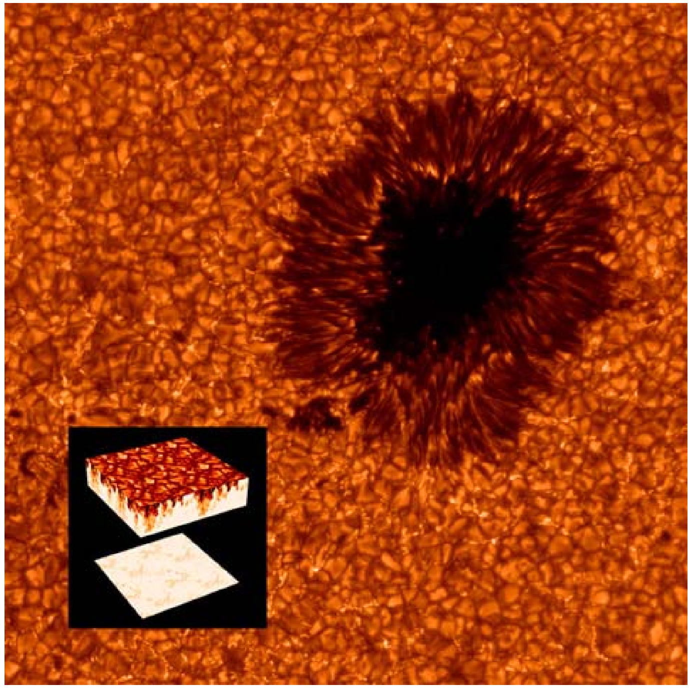
- ✓ inertial confinement and magnetic fusion
- ✓ plasmas, light-matter interaction
- ✓ supernovae explosions, thermonuclear flashes
- ✓ stellar and planetary convection
- ✓ premixed and non-premixed combustion (flames and fires)
- ✓ impact dynamics, properties of materials under high strain rate …

**RT turbulent flow is inhomogeneous, anisotropic and accelerated.
Its properties differ from those of the Kolmogorov turbulence.**

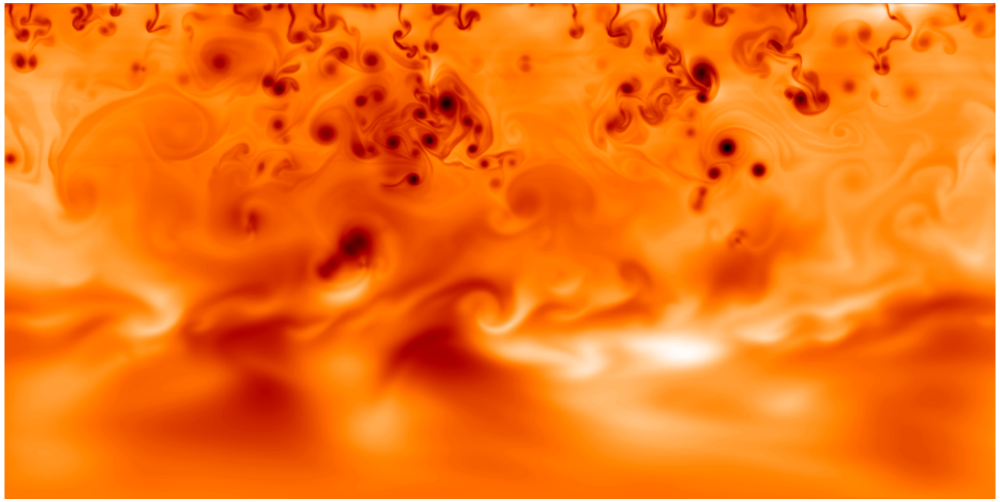
**Grasping essentials of the mixing process is
a fundamental problem in fluid dynamics.**

Solar and Stellar Convection

Solar surface, LMSAL, 2002



Simulations of Solar convection
Cattaneo et al, U Chicago, 2002



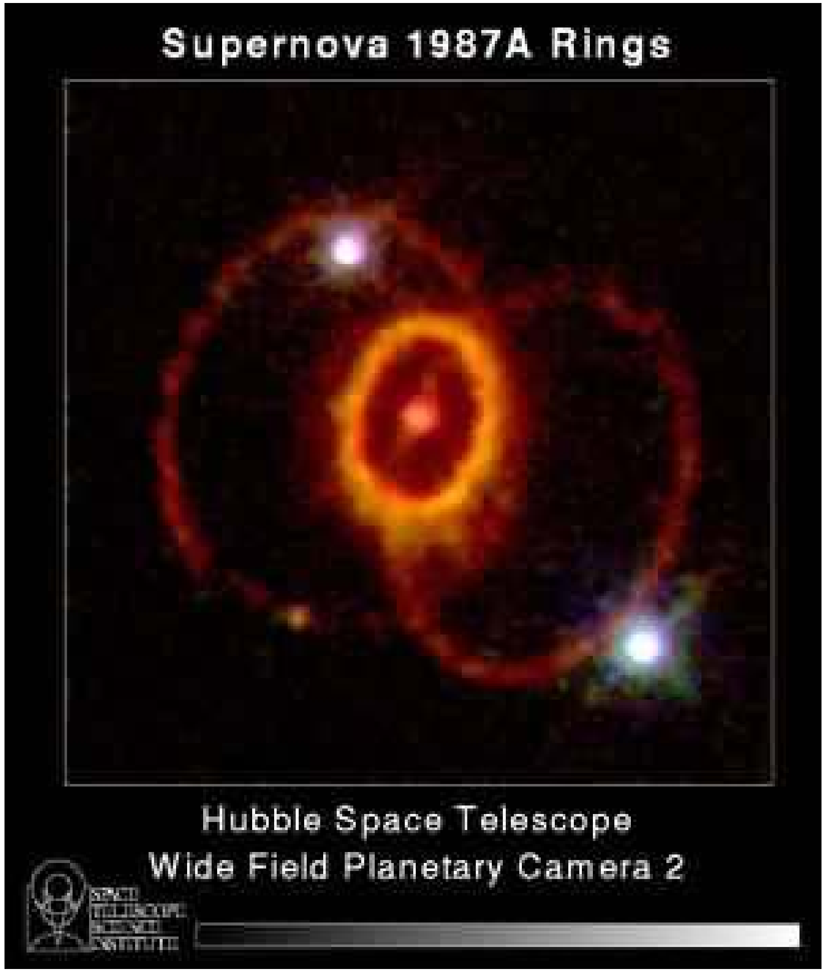
Observations indicate:

dynamics at Solar surface is governed by convection in the interior.

Simulations show:

Solar non-Boussinesq convection is dominated by downdrafts; which are either large-scale vortices (wind) or smaller-scale plumes (“RT-spikes”).

Supernovae



Burrows, ESA, NASA,1994

Supernovae and remnants

type II: RMI and RTI produce extensive mixing of the outer and inner layers of the progenitor star

type Ia: RTI turbulent mixing dominates the propagation of the flame front and may provide proper conditions for generation of heavy mass element

Pair of rings of glowing gas, caused perhaps by a high energy radiation beam of radiation, encircle the site of the stellar explosion.

Inertial confinement fusion

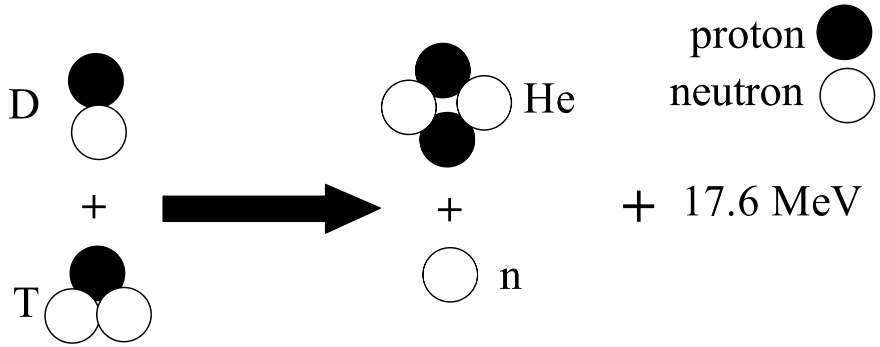


FIG. 5. Three-dimensional shape of the contact surface showing nonlinear bubble-spike structures for $(n,m)=(6,3)$.

Nishihara, ILE, Osaka, Japan, 1994

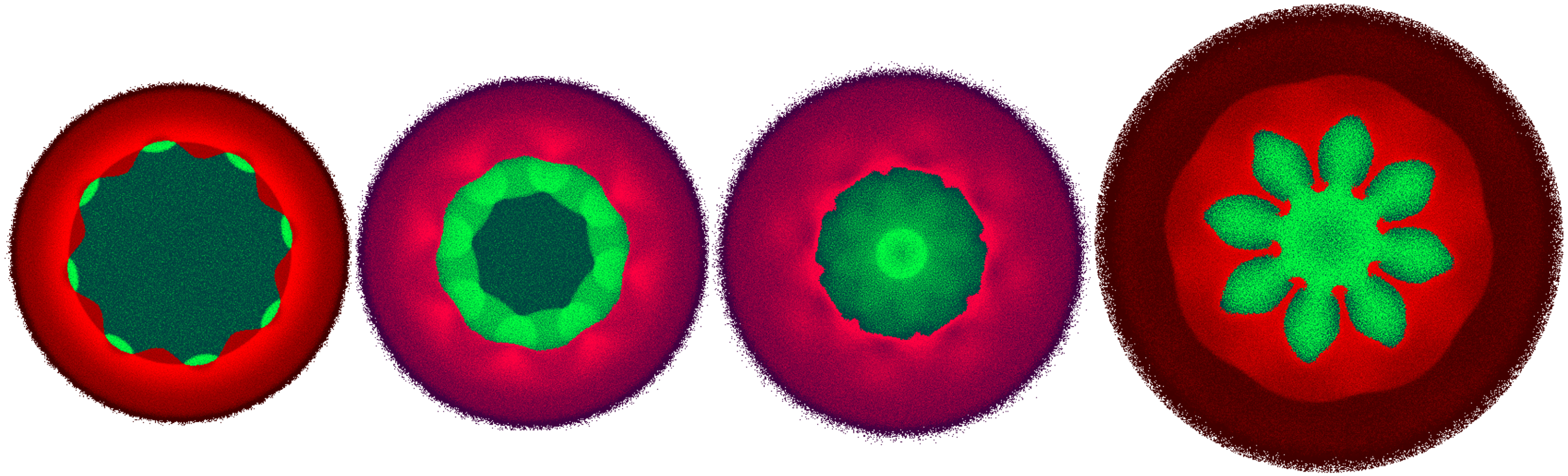
- For the nuclear fusion reaction, the DT fuel should be hot and dense plasma

- For the plasma compression in the laboratory it is used
  - ✓ magnetic implosion
  - ✓ laser implosion of DT targets

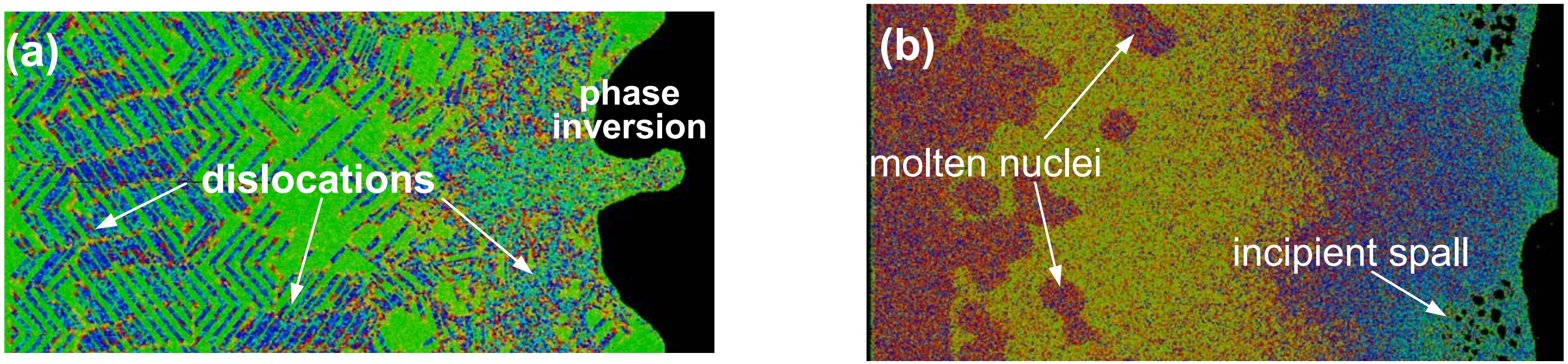
- RMI/RTI inherently occur during the implosion process

- RT turbulent mixing prevents the formation of hot spot

Impact dynamics in liquids and solids



MD simulations of the Richtmyer-Meshkov instability: a shock refracts though the liquid-liquid (up) and solid-solid (down) interfaces; nano-scales



Zhakhovskii, Zybin, Abarzhi et al DPP, DFD/APS 2005

Turbulent mixing induced by the Rayleigh-Taylor instability

What is known and unknown?

Rayleigh-Taylor evolution

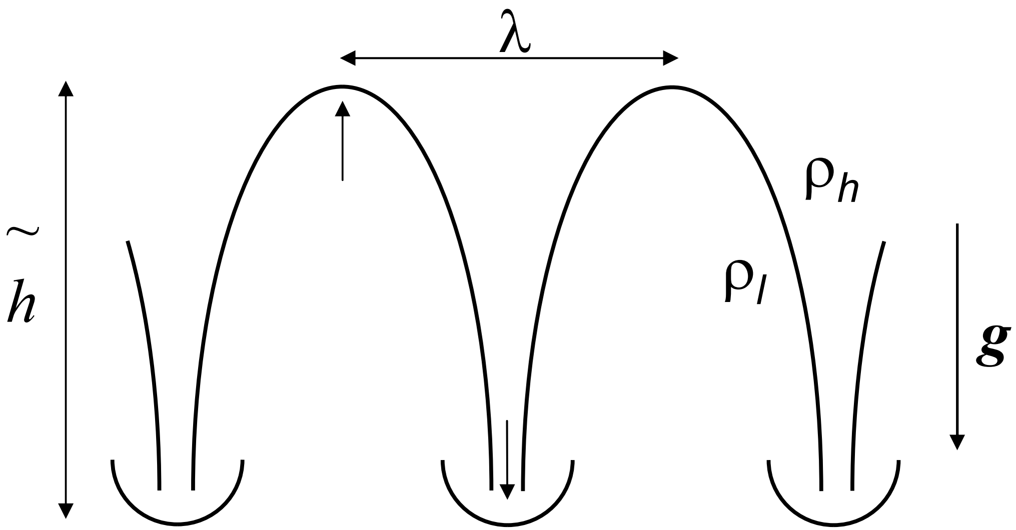
- linear regime $\tilde{h} \sim h_0 \exp(t/\tau)$

  $\tau \sim \sqrt{\lambda(\rho_h + \rho_l)/g(\rho_h - \rho_l)}, \quad \lambda \sim \lambda_{\max}$

- nonlinear regime $\tilde{h} \sim \lambda(t/\tau)$

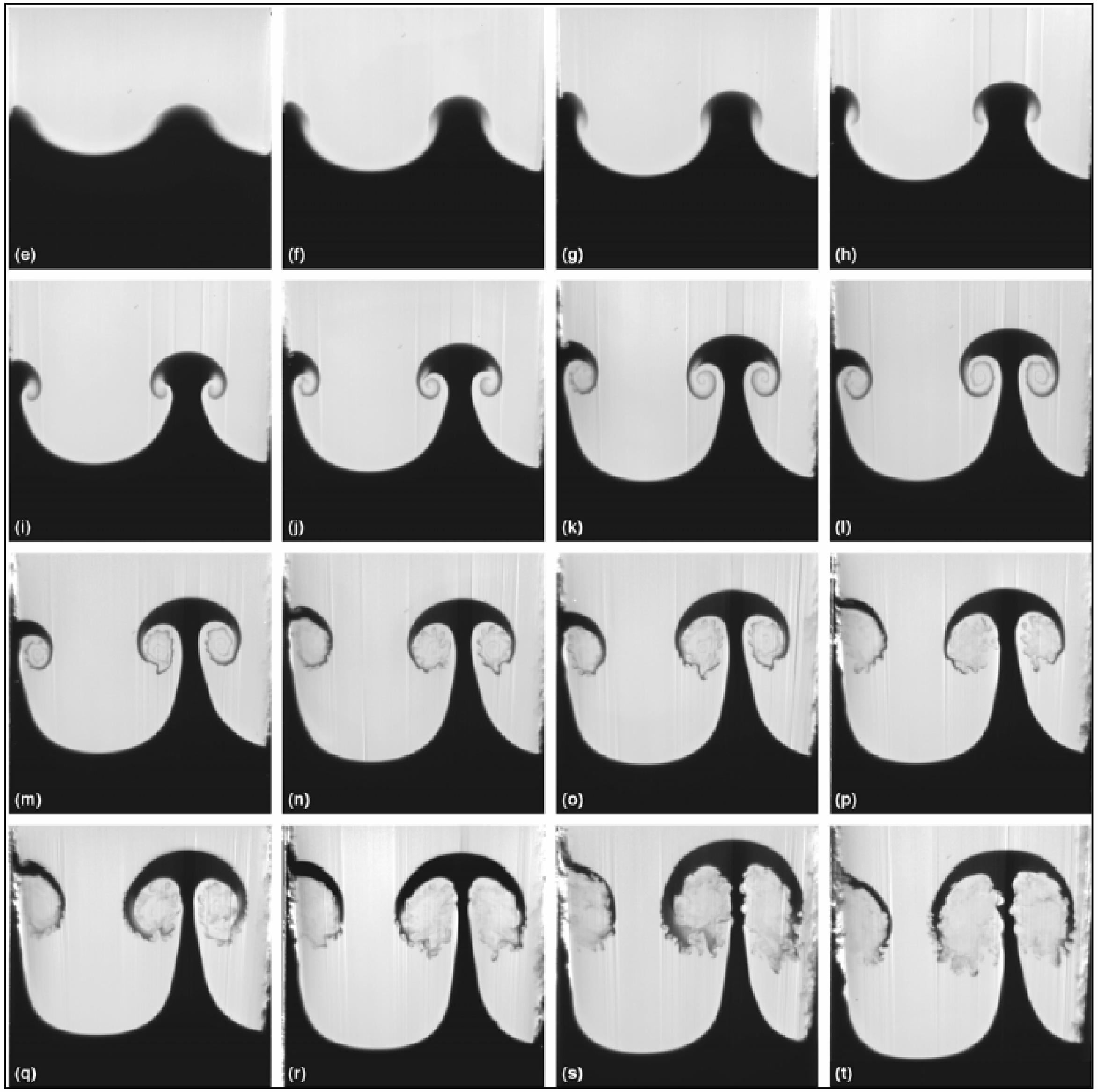
  light (heavy) fluid penetrates
  heavy (light) fluid in bubbles (spikes)

- turbulent mixing $\tilde{h} \sim gt^2$



RT flow is characterized by:
- ✓ large-scale structure
- ✓ small-scale structures
- ✓ energy transfers to large and small scales

Nonlinear Rayleigh-Taylor / Richtmyer-Meshkov



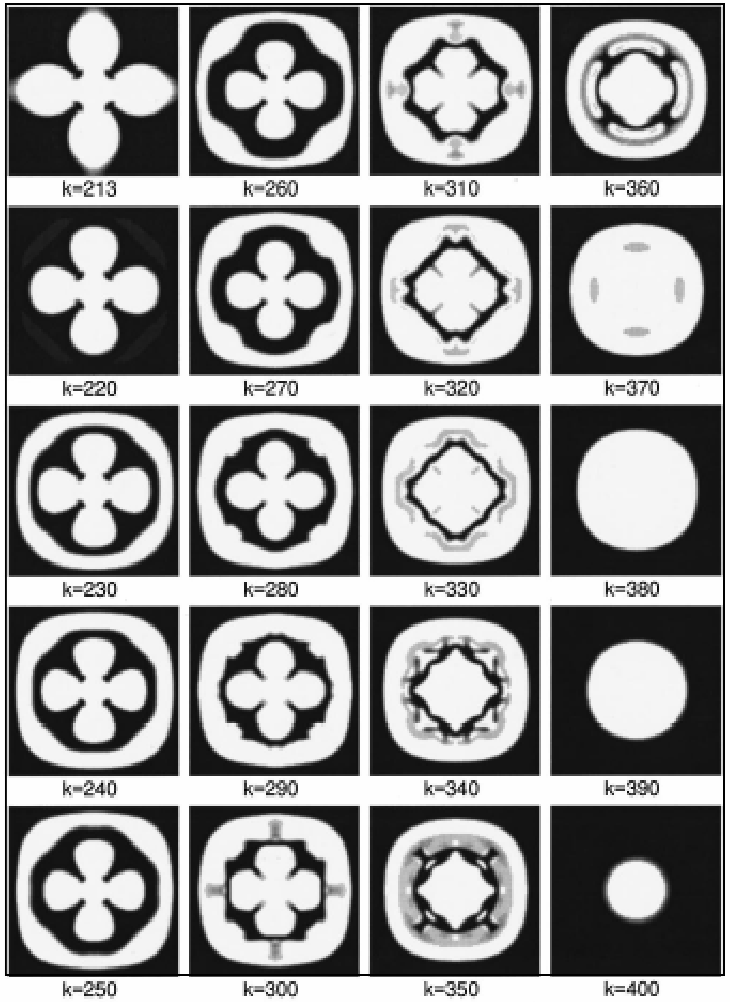
Krivets & Jacobs
Phys. Fluids, 2005

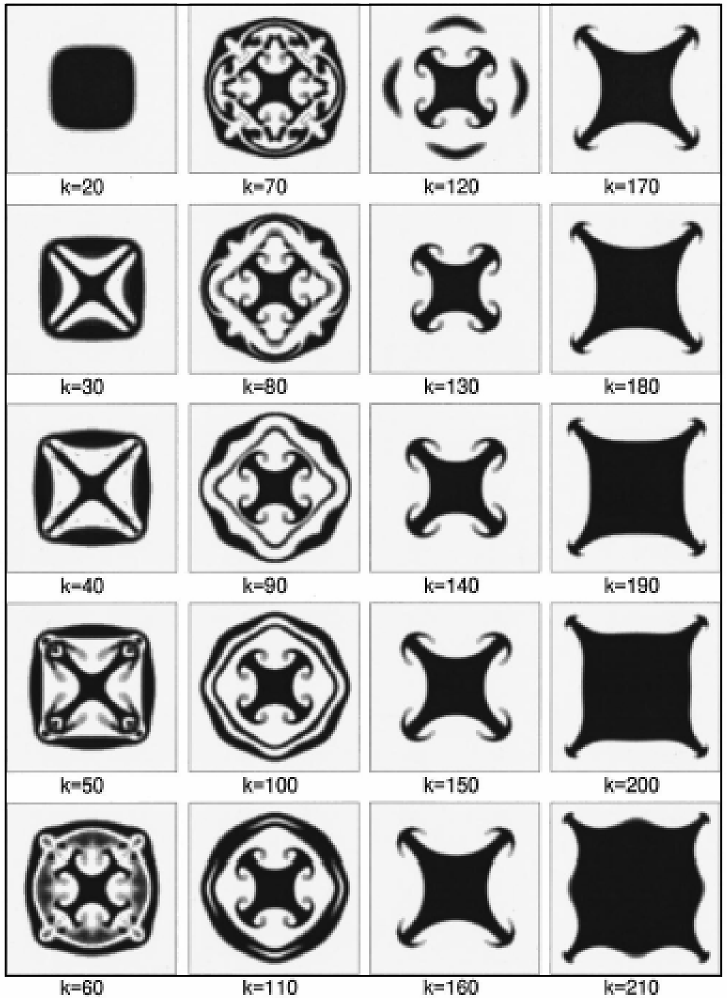
$$M = 1.27 \quad A = 0.5$$

$$\lambda \sim 6cm \quad t \sim ms$$

- large-scale dynamics is sensitive to the initial conditions
- small-scale dynamics is driven by shear

Nonlinear Rayleigh-Taylor



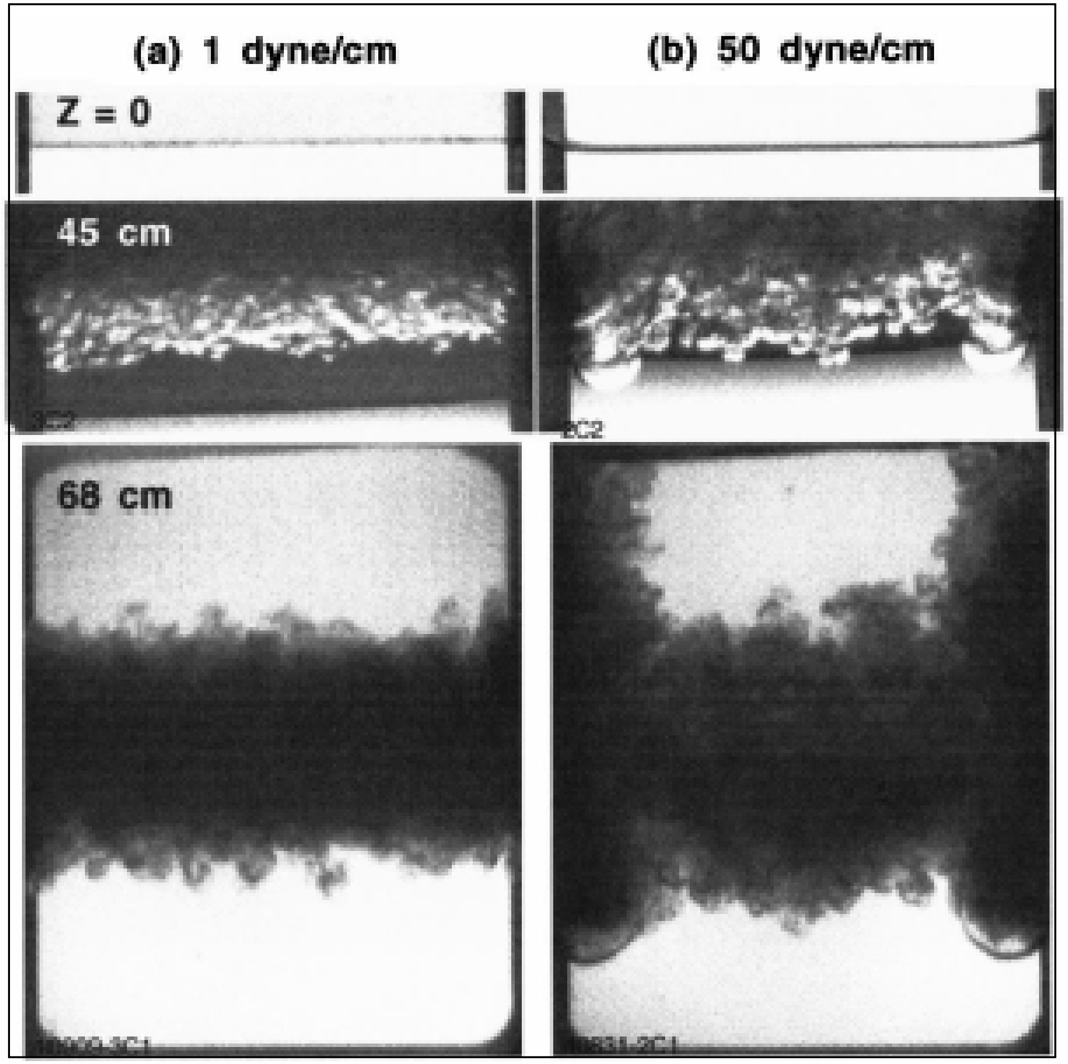


Density plots in horizontal planes
He, Chen, Doolen, 1999, Lattice Boltzman method

$A = 0.5 \qquad \mathrm{Re} = 1054$

$\sim \cos kx \cos ky$

Rayleigh-Taylor turbulent mixing



Dimonte, Remington, 1998

3D perspective view (top)
and
along the interface (bottom)

- internal structure of bubbles and spikes

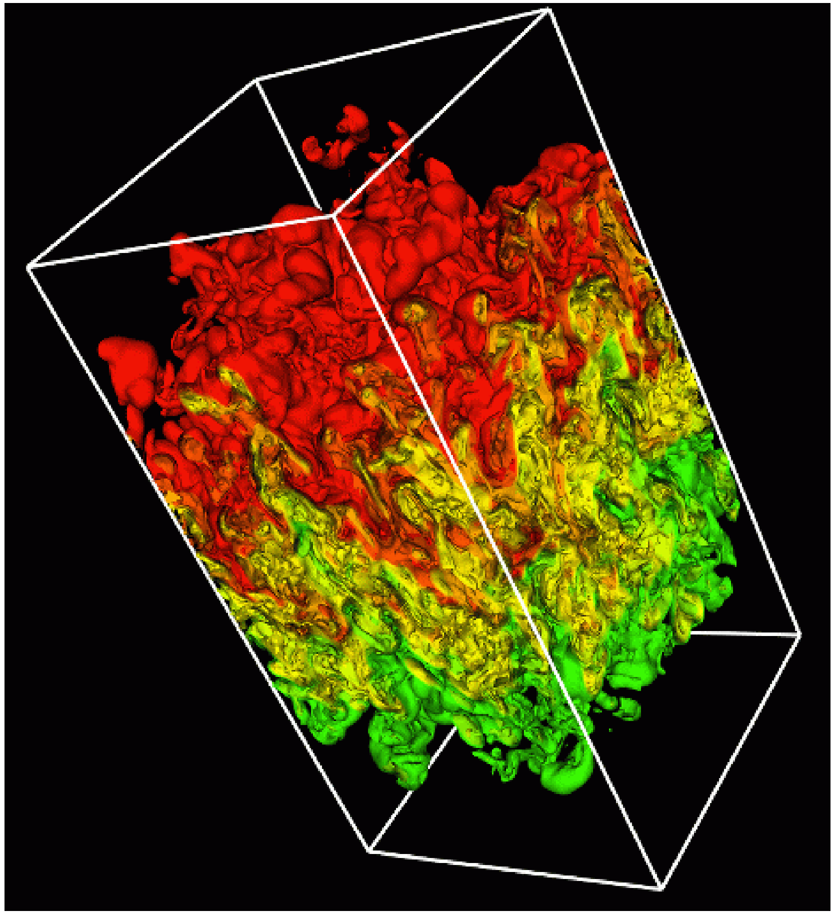
$$\lambda_{\max} \sim mm, \ \Lambda \sim O(\lambda_{\max})$$

$$A = 0.2, \ g = 73 g_0, \ L_{x(y)} = 7.3 cm, \ L_z = 8.8 cm \qquad We < 10^4, \quad \mathrm{Re} < 10^5$$

Rayleigh-Taylor turbulent mixing

FLASH 2004
3D flow
density plots

broad-band
initial
perturbation



small-amplitude
initial
perturbation

The flow is sensitive to the horizontal boundaries of the fluid tank,
is much less sensitive to the vertical boundaries,
and retains the memory of the initial conditions.

Rayleigh-Taylor turbulent mixing

Our phenomenological model

accounts for

- multi-scale and anisotropic character of the flow dynamics
- influence of turbulent diffusion and randomness of the dissipation process

identifies

- the new invariant, scaling and spectral properties of the accelerated mixing flow

discusses

- how to generalize the model and to apply it to physical phenomena, where the unsteady turbulent processes occur

Modeling of the turbulent mixing in unsteady multiphase flows

Any physical process is governed by a set of conservation laws:

conservation of mass, momentum (angular momentum), and energy

Kolmogorov turbulence isotropic, homogeneous …	transport of kinetic energy
Unsteady multiphase flow anisotropic, inhomogeneous…	transports of mass & momentum potential and kinetic energy

Turbulent mixing induced by the Rayleigh-Taylor
is driven by the momentum transport

Modeling of RT turbulent mixing

Dynamics: **balance per unit mass of the rate of momentum gain and the rate of momentum loss**

$$\frac{dh}{dt} = v, \qquad \frac{dv}{dt} = \tilde{\mu} - \mu$$

$$\tilde{\mu} = \frac{\delta\rho}{\rho}$$

These rates are the absolute values of vectors pointed in opposite directions and parallel to gravity.

rate of momentum gain	$\tilde{\mu} = \tilde{\varepsilon}/v$	**buoyant force**
rate of energy gain	$\tilde{\varepsilon} = v\,g\,\delta\rho/\rho$	
rate of momentum loss	$\mu = \varepsilon/v$	**dissipation force**
energy dissipation rate ε dimensional & Kolmogorov	$\varepsilon = C\,v^3/L$	

L is the flow characteristic length-scale, either horizontal λ or vertical h

Asymptotic dynamics

- characteristic length-scale is horizontal $L \sim \lambda$ nonlinear

$$v \sim \sqrt{g\lambda} \qquad h \sim t\sqrt{g\lambda}$$

$$\tilde{\mu} = g\,\delta\rho/\rho \qquad \mu = g\,\delta\rho/\rho$$

$$\tilde{\varepsilon} \sim (g\,\delta\rho/\rho)^{3/2}\lambda^{1/2} \qquad \varepsilon \sim (g\,\delta\rho/\rho)^{3/2}\lambda^{1/2}$$

- characteristic length-scale is vertical $L \sim h$ turbulent

$$v \sim g\,t \qquad h \sim g\,t^2/2$$

$$\tilde{\mu} = g\,\delta\rho/\rho \qquad \mu = (1-a)g\,\delta\rho/\rho \qquad a = 2h/g(\delta\rho/\rho)t^2$$

$$\tilde{\varepsilon} = a(g\,\delta\rho/\rho)^2 t \qquad \varepsilon = a(1-a)(g\,\delta\rho/\rho)^2 t \qquad a \sim 0.1$$

Accelerating turbulent mixing

The turbulent mixing develops:

- horizontal scale grow with time $\lambda \sim gt^2$
- vertical scale h dominates the flow and is regarded as the integral, **cumulative** scale for energy dissipation.
- the dissipation occurs in small-scale structures produced by shear at the interface.

In the turbulent mixing flow:

- length scale and velocity are time-dependent $v \sim g\,t, \quad L \sim gt^2$
- kinetic and potential energy both change
  changes in potential energy are due to buoyancy $\tilde{\varepsilon}$
  changes in kinetic energy are due to dissipation ε
- momentum gains and losses $\tilde{\mu}$
  μ

Accelerated mixing flow

- rates of momentum gain and momentum loss are scale and time invariant

$$\tilde{\mu} = g\,\delta\rho/\rho \qquad\qquad \mu \sim v^2/L$$

- rates of gain of potential energy gain and dissipation of kinetic energy are time-dependent

$$\tilde{\varepsilon} = vg\,\delta\rho/\rho \qquad\qquad \varepsilon \sim v^3/L$$

- ratio between the rates is the characteristic value of the flow

$$\Pi = \mu/\tilde{\mu} = \varepsilon/\tilde{\varepsilon}$$

✓ Π remains time- and scale-invariant value and the flow characteristic for time-dependent and spatially-varying acceleration, as long as potential energy is a similarity function on coordinate and time

Basic concept for the RT turbulent mixing

Kolmogorov turbulence is inertial (Galilean-invariant), isotropic, local and homogeneous.

- Energy dissipation rate is the basic invariant, determines the scaling properties of the turbulent flow.

$$\varepsilon \sim v^3 / L \sim v_l^3 / l$$

RT turbulent mixing is non-inertial (accelerated), anisotropic, non-local and inhomogeneous.

- The flow invariant is the rate of momentum loss

$$\mu \sim v^2 / L \sim v_l^2 / l$$

- We consider some consequences of time and scale invariance of the rate of momentum loss in the direction of gravity.

✓ The dynamics of momentum and energy depends on directions.
✓ There may be transports between the planar to vertical components.
✓ It may have a meaning to solve equations for 4D momentum-energy tensor and study their covariant and invariant properties in non-inertial frame of reference.

Invariant properties of the RT turbulent mixing

Kolmogorov turbulence		**RT turbulent mixing**
	• energy dissipation rate $$\varepsilon \sim v^3/L \sim v^2(v/L) \sim (vL)(v/L)^2$$	
time- and scale-invariant energy transport and inertial interval		time-dependent
	• rate of momentum loss $$\mu \sim v^2/L \sim v_l^2/l$$	
not a diagnostic parameter		time- and scale-invariant transport of momentum
	• enstrophy $$\omega^2 = (\nabla \times \mathbf{v})^2$$	
time- and scale-invariant		time-dependent
	• helicity $$\mathbf{v} \cdot (\nabla \times \mathbf{v})$$	
not-Galilean invariant		$\sim g$, time- and scale-inv

Scaling properties of the RT turbulent mixing

Kolmogorov turbulence

$$\varepsilon \sim v^3/L \sim v_l^3/l$$

transport of energy

RT turbulent mixing

$$\mu \sim v^2/L \sim v_l^2/l$$

transport of momentum

- **velocity scaling**

$$v_l/v \sim (l/L)^{1/3}$$

$$v_l/v \sim (l/L)^{1/2}$$

more ordered

- **scaling with Reynolds**

$$\mathrm{Re} \sim (vL/\nu) \sim const$$

$$\mathrm{Re}_l \sim \mathrm{Re}(l/L)^{4/3}$$

$$\mathrm{Re} \sim (vL/\nu) \sim g^2 t^3/\nu$$

$$\mathrm{Re}_l \sim \mathrm{Re}(l/L)^{3/2}$$

- **viscous scale**

$$l \sim \left(\nu^3/\varepsilon\right)^{1/4}$$

$$l \sim \left(\nu^2/\mu\right)^{1/3} \sim \left(\nu^2/g\right)^{1/3}$$

mode of fastest growth

- **similarly: structure function, dissipative scale, surface tension**

Spectral properties of RT mixing flow

Kolmogorov turbulence:

✓ spectrum of kinetic energy (velocity)

kinetic energy = $\int_k^\infty E(k)dk \sim \int_k^\infty \varepsilon^{2/3} k^{-5/3} dk \sim \frac{\varepsilon^{2/3}}{k^{2/3}} \sim (\varepsilon l)^{2/3} \sim v_l^2$

RT turbulent mixing:

- spectrum of kinetic energy

kinetic energy = $\int_k^\infty E(k)dk \sim \int_k^\infty \mu\, k^{-2} dk \sim \frac{\mu}{k} \sim \mu\, l \sim v_l^2$

- spectrum of momentum

momentum = $\mathbf{e}\int_k^\infty M(k)dk \sim \mathbf{e}\int_k^\infty \mu^{1/2} k^{-3/2} dk \sim \mathbf{e}\frac{\mu^{1/2}}{k^{1/2}} \sim \mathbf{e}(\mu l)^{1/2} \sim \mathbf{e} v_l$

These properties differ from Kolmogorov and/or Obukhov-Bolgiano

Time-dependent acceleration, turbulent diffusion

The transport of scalars (temperature or molecular diffusion) decreases the buoyant force and changes the mixing properties

We assume $\rho \sim T$ $\qquad \delta\rho/\rho \sim \delta T/T$

Rate of temperature change is φ/T $\qquad \varphi = \chi(\nabla T)^2$

Landau & Lifshits $\qquad \varphi \sim (vL)(\delta T/L)^2 \sim (v/L)(\delta T)^2$

dynamical system

$$\frac{dh}{dt} = v \qquad \frac{dv}{dt} = \tilde{\mu} - \mu \qquad \frac{d\tilde{\mu}}{dt} = -C_t \frac{v}{gh}\tilde{\mu}^2 \qquad \tilde{\mu} = g\,\delta\rho/\rho$$

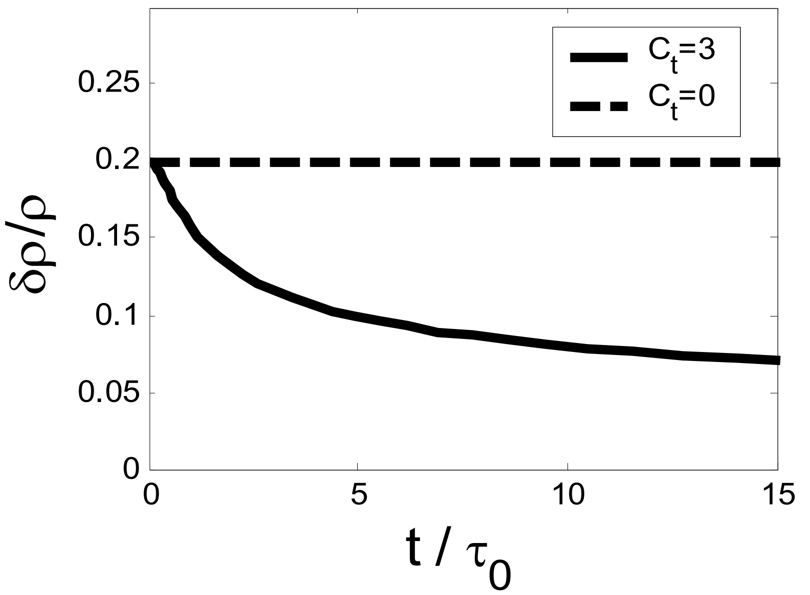
asymptotic solution

$$t \to \infty \qquad g\,\delta\rho/\rho \to 0 \qquad h/gt^2 \sim 1/\ln\left(gt^2\right) \to 0$$

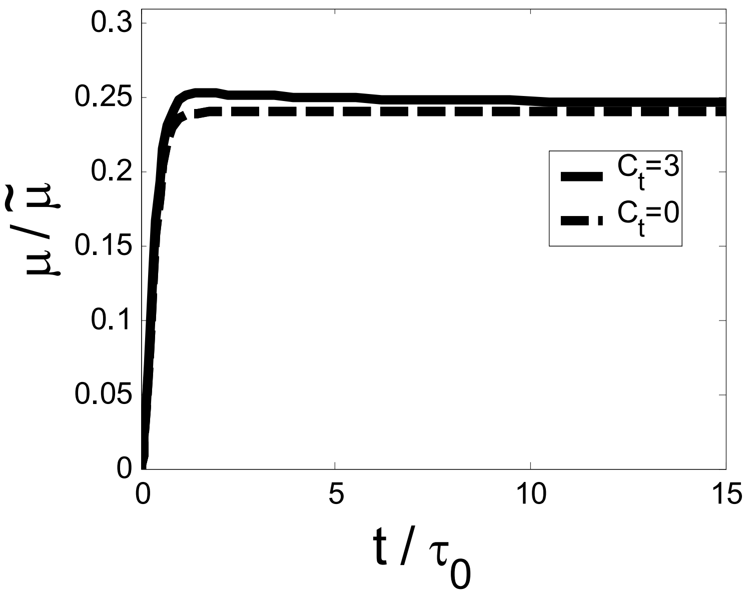
with A. Gorobets, K.R. Sreenivasan, Phys Fluids 2005

Asymptotic solutions and invariants

buoyancy $g\,\delta\rho/\rho$ vs time t



$\mu/\tilde{\mu}$ vs time t



dimensionless units

- ✓ **Buoyant force $g\,\delta\rho/\rho$ vanishes asymptotically with time.**
- ✓ **The ratio $\mu/\tilde{\mu}$ is time- and scale-invariant value, and the flow characteristics**

Randomness of the mixing process

The Rayleigh-Taylor turbulent mixing has a noisy character.

- The randomness is caused by
  the retained memory of the (unresolved) initial conditions
  the influence of shear-driven small-scale structures

- No detailed information is available from the observations.

- The rate of momentum loss is determined by
  random dissipative processes and fluctuates about its mean.

- We consider the influence of these fluctuations on the mixing dynamics
  and show that
  the mixing growth-rate is very sensitive to the stochastic effects,
  whereas the rate of momentum Π is a robust diagnostic parameter.

with M. Cadjun, S. Fedotov, L. Kadanoff, submitted

Stochastic model

The value of $\mu = C v^2 / h$ is statistically steady.

Its distribution is in general non-symmetric. $C > 0$

System of stochastic differential equations

$$dh = v dt, \quad dv = d\tilde{\mu} - d\mu, \quad d\tilde{\mu} = g dt, \quad d\mu = C\left(v^2 / gh\right) dt$$

$p(C)$ is log-normal

$$\sigma \sqrt{\frac{2}{\tau_C}} dW = \frac{dC}{C} + \frac{dt}{\tau_C}\left(\ln \frac{C}{\langle C \rangle} - \frac{\sigma^2}{2}\right)$$

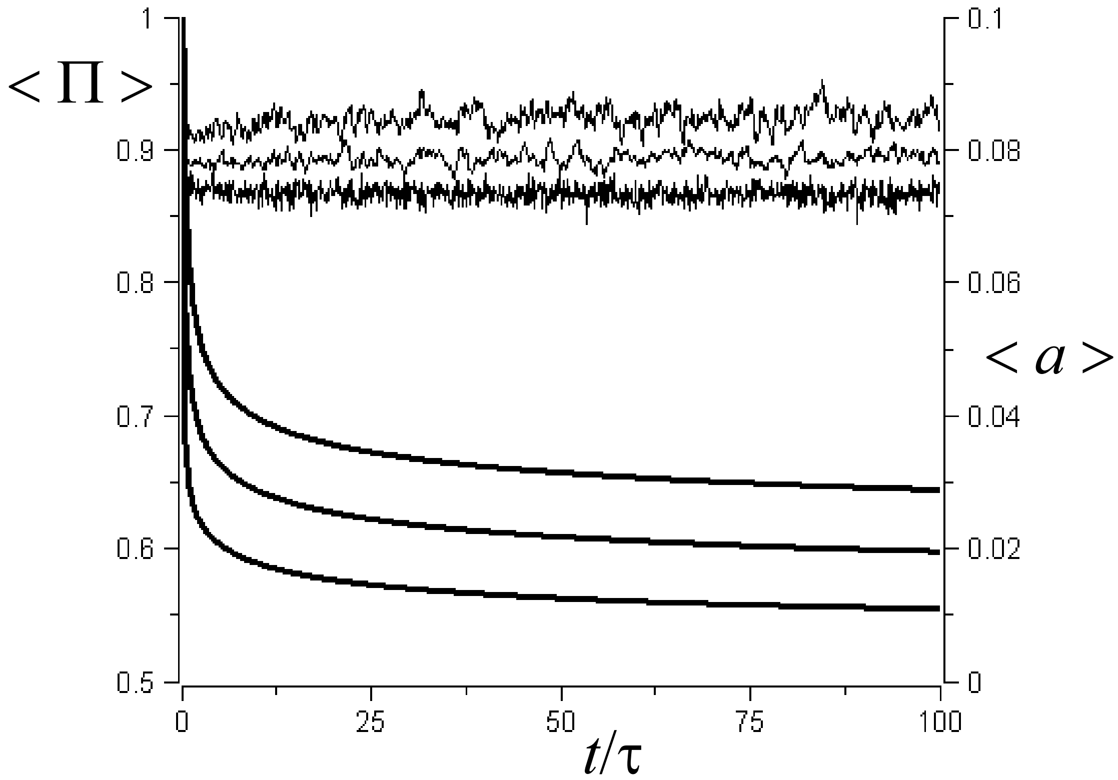
fluctuations intensity is σ

fluctuations time-scale is τ_c

Fluctuations

- ✓ do not change the time-dependence, $h \sim gt^2$
- ✓ influence the pre-factor $(h / gt\text{^}2)$
- ✓ long tails re-scale the mean significantly

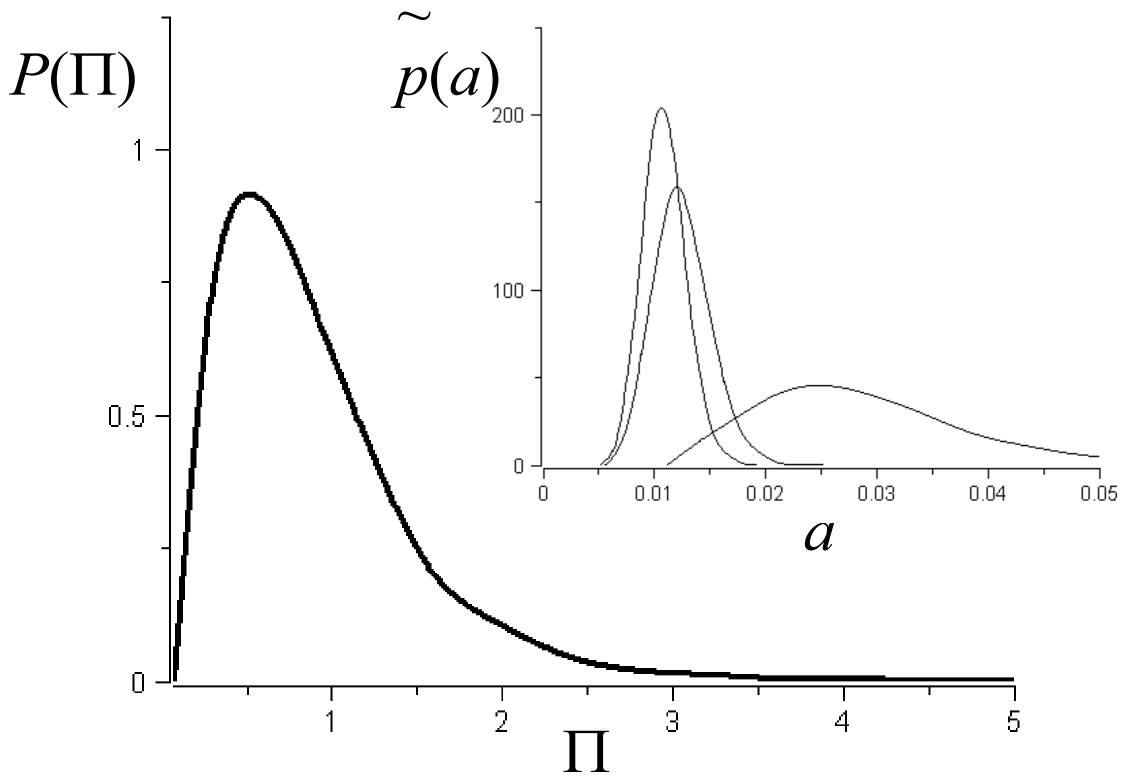
Statistical properties of accelerated mixing



- ✓ The value of $a = h / g(\delta\rho/\rho)t^2$ is very sensitive parameter.
- ✓ Asymptotically, its statistical properties are very sensitive to noise and retain a time-dependence.

Statistical properties of accelerated mixing

probability density function at distinct moments of time



The ratio between the momentum rates is

$$\Pi = \mu / \tilde{\mu}$$

- ✓ statistically steady for any type of acceleration
- ✓ a robust parameter to diagnose

Turbulent processes in unsteady multiphase flows

time and scale invariance of the parameter $\Pi = \mu/\tilde{\mu} = \varepsilon/\tilde{\varepsilon}$

- ✓ Non-Boussinesq convection
- ✓ Compressible turbulent flows
- ✓ Unsteady boundary layers
- ✓ Turbulent combustion in gravity field
- ✓ Large-eddy simulations and sub-grid-scale models for unsteady multiphase flows
- ✓ *Covariant, invariant and scaling dynamics of non-inertial, non-Galilean turbulent flows*
  - *4D for momentum-energy tensor transports*
  - *analogy with general relativity theory*

Diagnostics of unsteady turbulent processes

Basic invariant, scaling and spectral properties of accelerated mixing differ from those in the classical Kolmogorov turbulence.

- ✓ In Kolmogorov turbulence, energy dissipation rate is statistic invariant, rate of momentum loss is not a diagnostic parameter.
- ✓ In accelerating flow, the rate of momentum loss is the basic invariant, whereas energy dissipation rate is time-dependent.
- ✓ Energy is complimentary to time, momentum is complimentary to space.
- ✓ In classical turbulence, the signal is a (few) point measurement with detailed temporal statistics.
- ✓ Spatial distributions of the turbulent flow quantities are essential to diagnose for capturing the transports of momentum and energy in unsteady turbulent flows.

Conclusions

The model

- Suggested a phenomenological model to describe the accelerated turbulent mixing induced by Rayleigh-Taylor instability.
- Described the invariant, scaling and spectral properties of the flow.
- Considered the effects of randomness and turbulent diffusion

The results

- Accelerated turbulent flow is governed by the transport of momentum, whereas isotropic turbulence - by the transport of energy.
- The invariant, scaling, spectral properties and statistical properties of the accelerating mixing flow differ from those in Kolmogorov turbulence.
- The rate of momentum loss is the basic invariant of the accelerated flow, the energy dissipation rate is time-dependent.
- The ratio between the rates of momentum loss and gain is time and scale-invariant, for sustained and/or spatially varying and time-dependent acceleration