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on

High Rayleigh Number Convection

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Steady-state and turbulent convection: similarities and distinctions

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Steady-state and turbulent convection: Similarities and Distinctions

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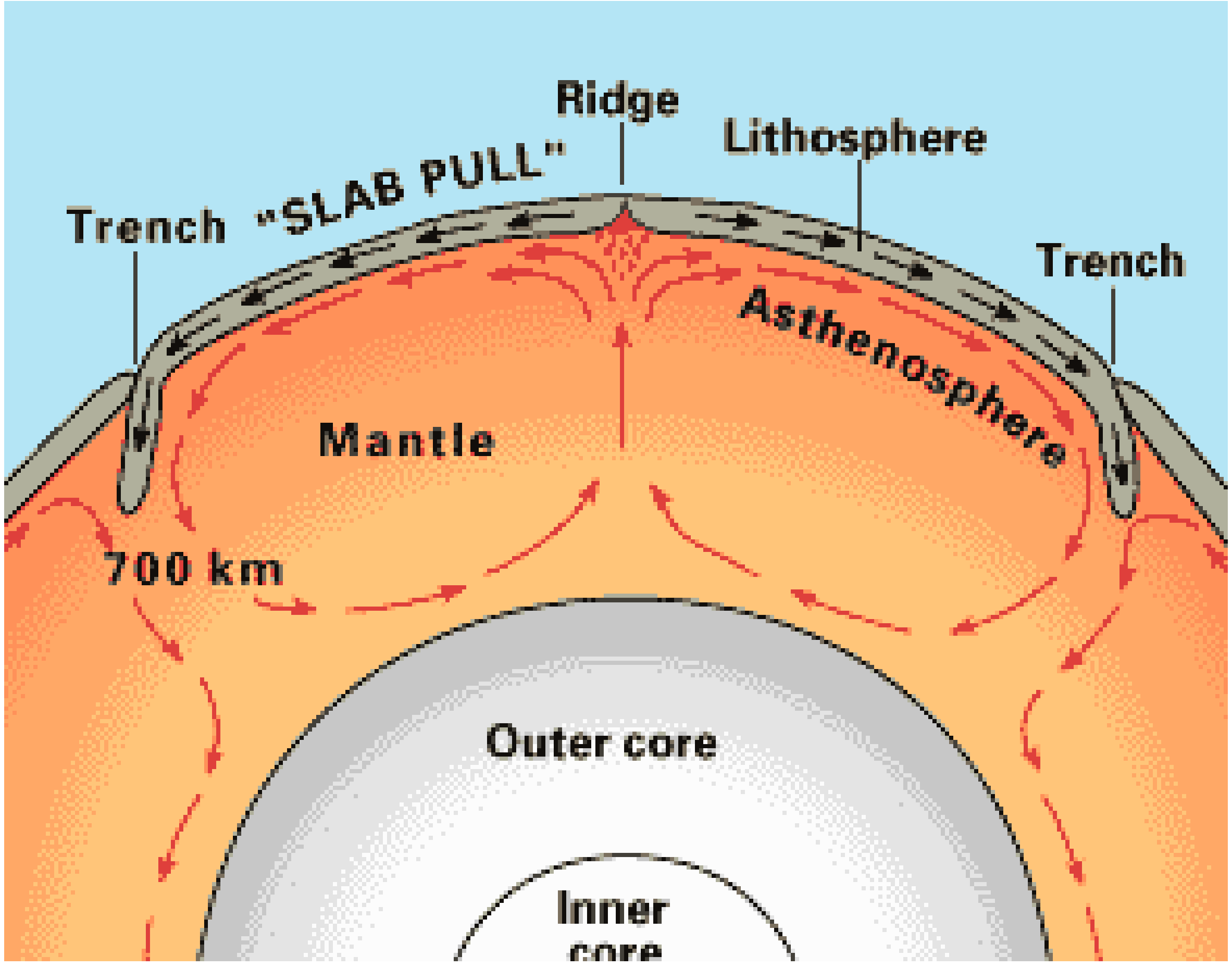
Stephan Stellmach, Tobias Höink, Martin Breuer,
Helmut Harder

Trieste, 7th September, 2006



Ridge
Lithosphere
"SLAB PULL"
Trench
Trench
Asthenosphere
Mantle
700 km
Outer core
Inner
core

Basics of Thermal Convection in the Earth‘s mantle

$$\frac{1}{\Pr}\left(\frac{\partial\vec{v}}{\partial t}+\vec{v}\nabla\overline{v}\right)=-\nabla P+\nu\nabla^2\vec{v}+RaT\vec{z}$$

$$\frac{\partial T}{\partial t}+\vec{v}\nabla T=\kappa\nabla^2 T+Q$$

$$Ra=\frac{g\alpha\Delta T d^3}{\kappa\nu}$$

Rayleighnumber

$$\Pr=\frac{\nu}{\kappa}$$

Prandtlnumber

α; thermal expansion coefficient, g gravitational sccelerationiom, ΔT temperature difference across the layer, d depth of the layer κ thermal diffusivity and ν viscosity.

Characteristics of mantle convection

Rayleighnumber $$Ra = \frac{\alpha g \Delta T d^3}{\kappa \nu} > O(10^7)$$

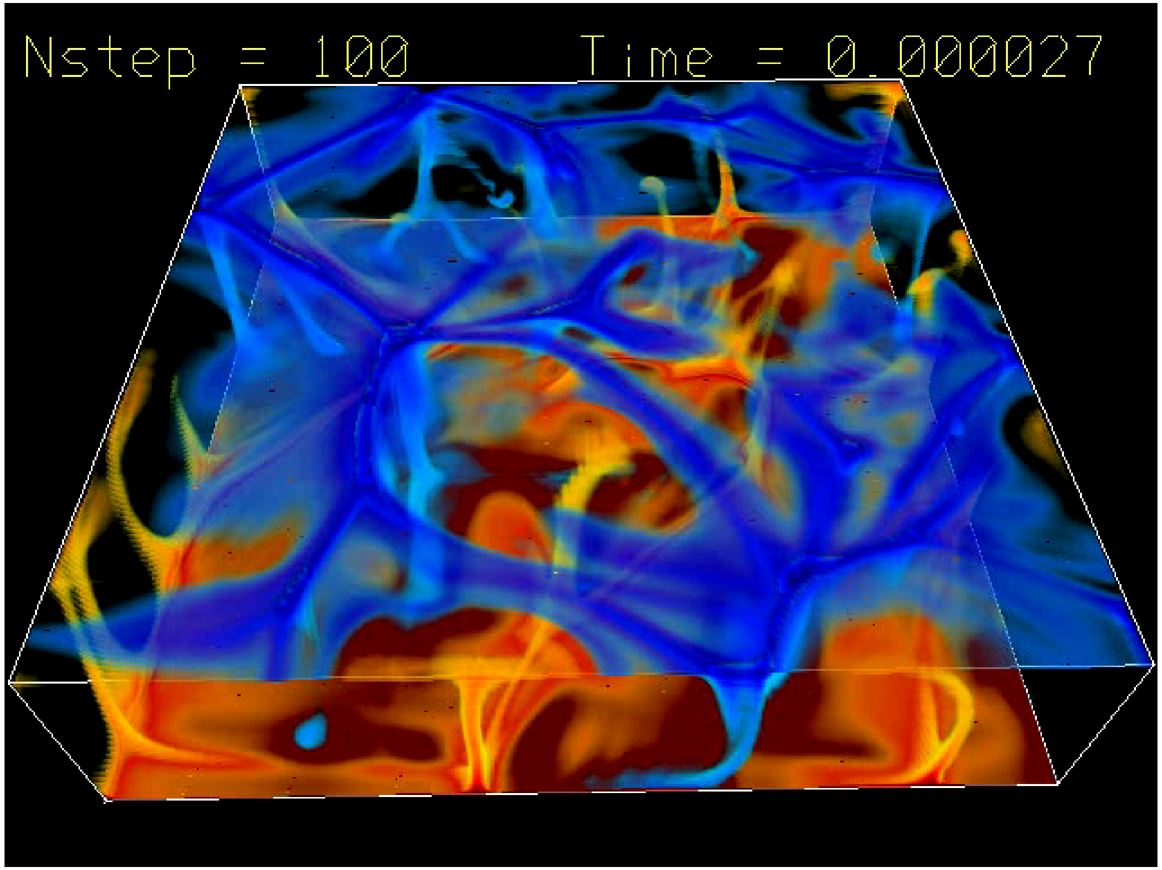
Prandtlnumber $$\Pr = \frac{\nu}{\kappa} = O(10^{23})$$

No Coriolis Forces, Viscosity depends strongly on temperature,
Flow velocities: cm/yr

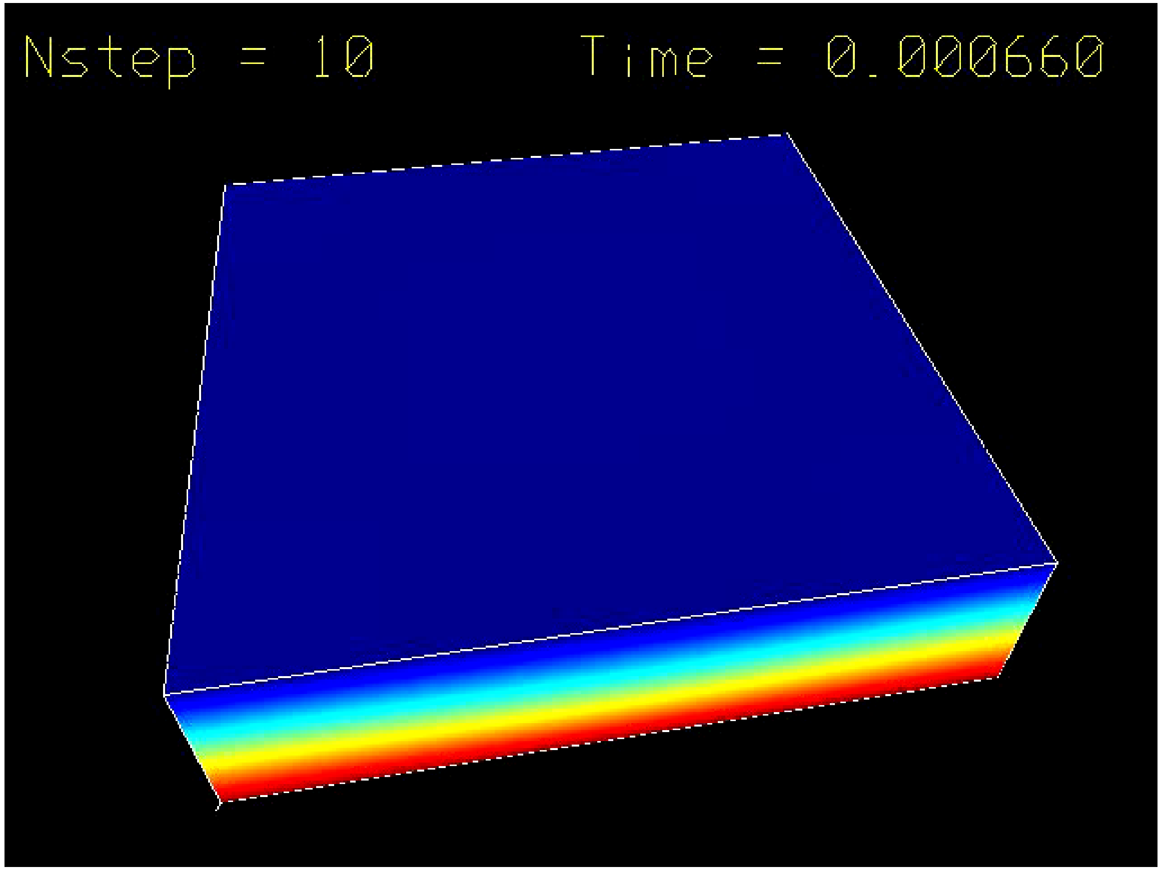
Characteristics of core convection

Ra > 10^{20}, , Pr < 1, Strongly influenced by Rotation
Flow velocity: mm/sec

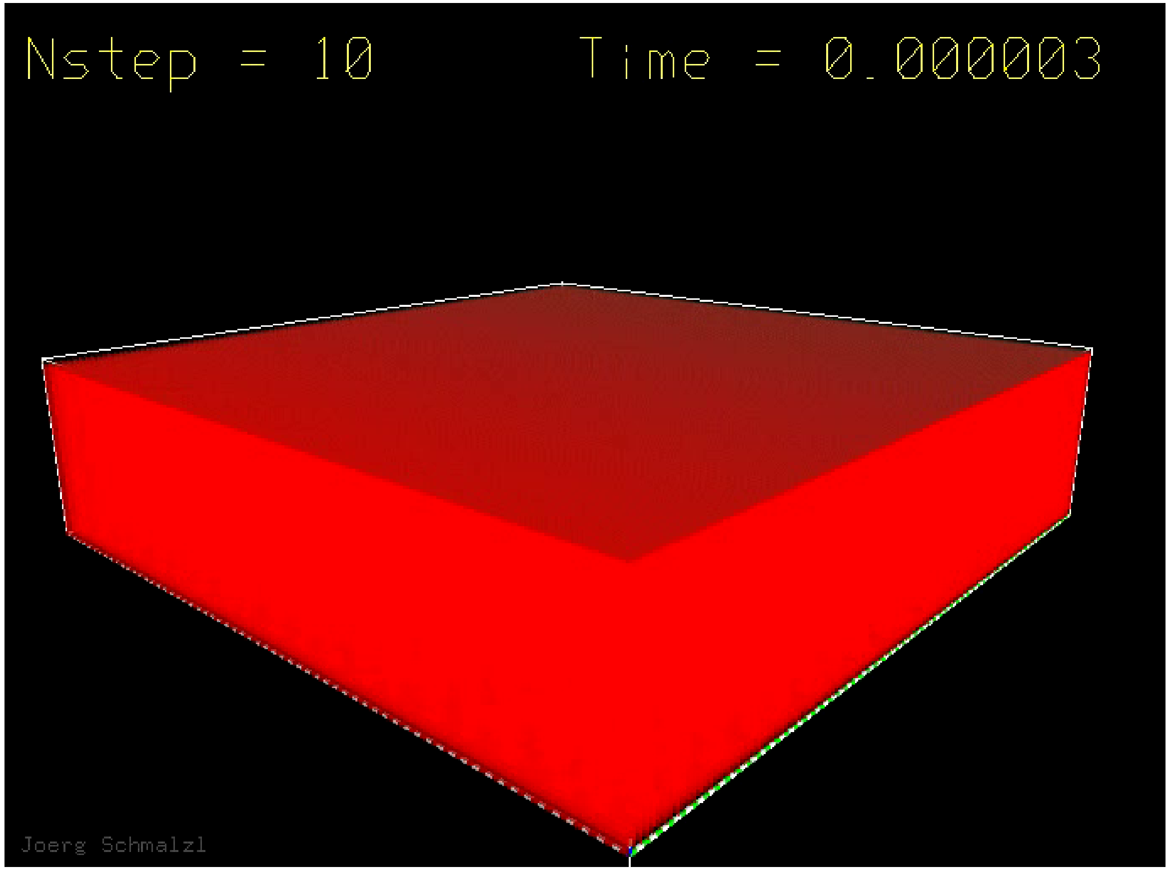
Convection at infinite Pr
Ra = 10**7



Toroidal motion at low Pr

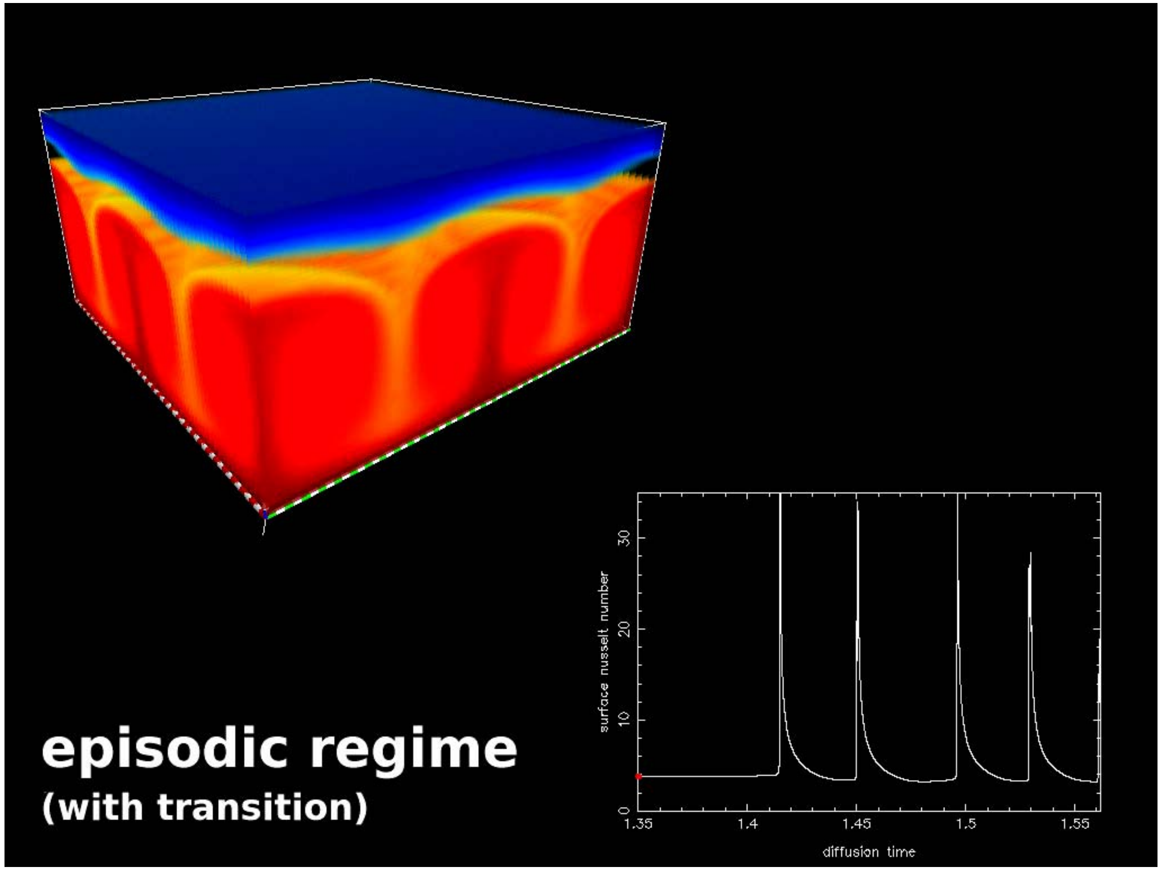


Strongly temperature-dependent viscosity – stagnant lid mode



Dynamics beneath the stagnant lid

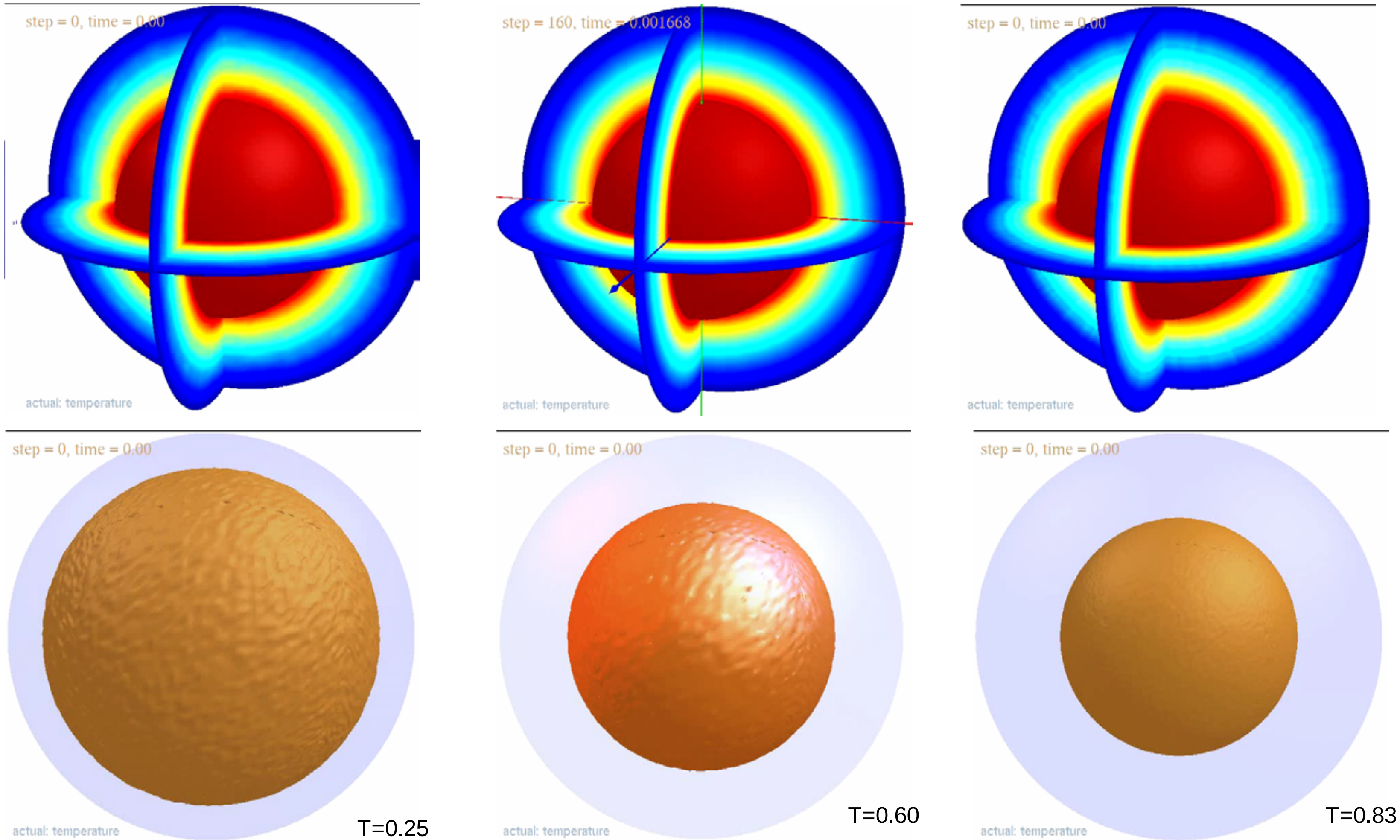
Episodicity and Resurfacing



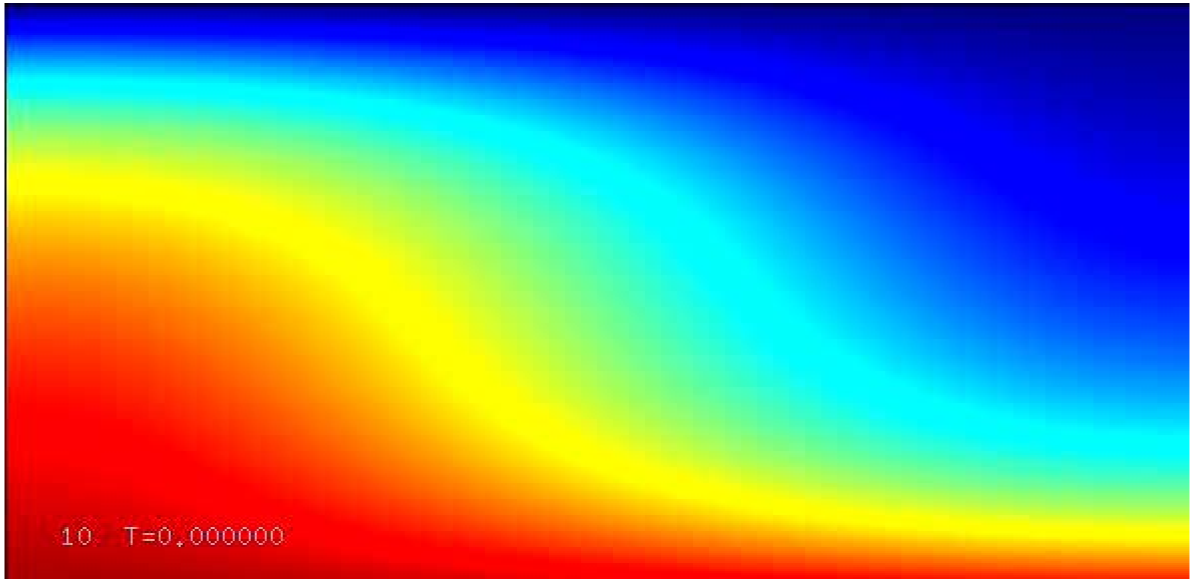
Thermal convection in a spherical shell

temperature-dependent viscosity, basal heating $Ra_{1/2} = 10^5$

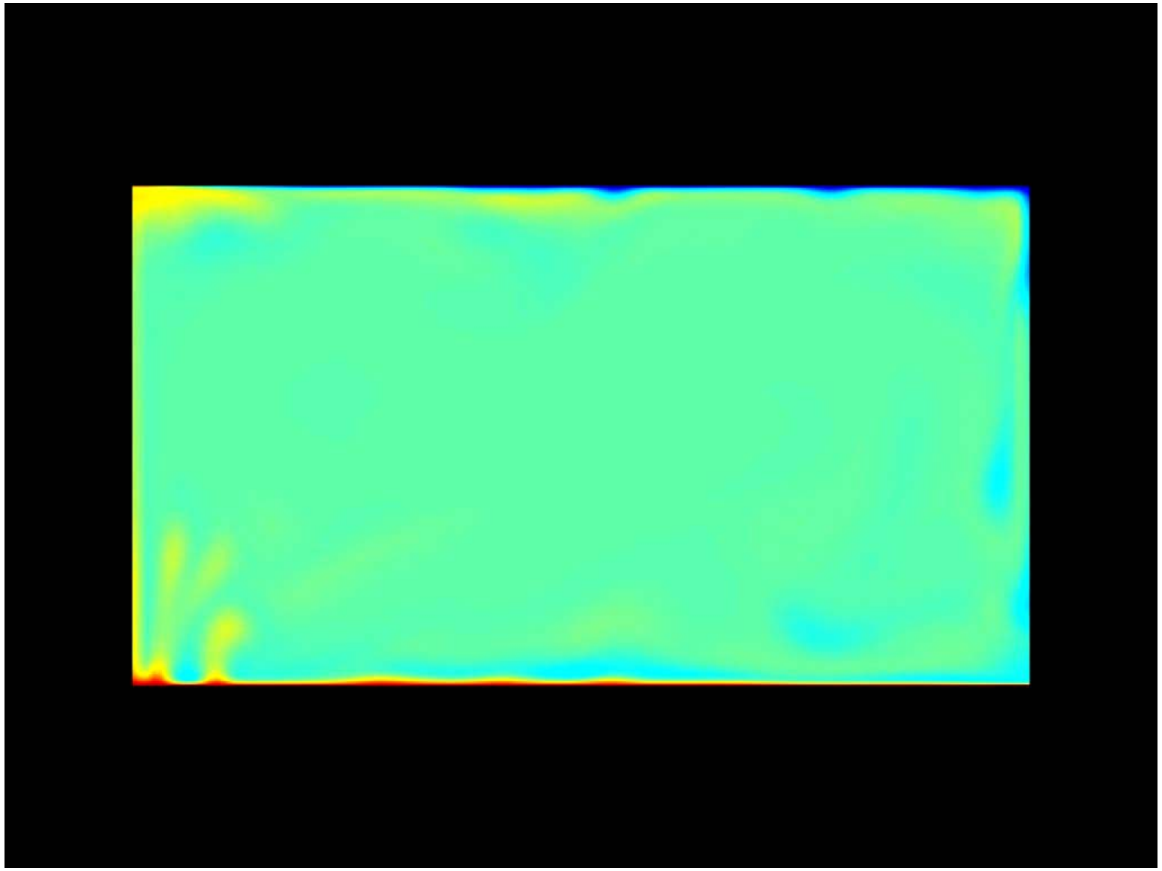
$\Delta\eta_T = 10^0$ $\Delta\eta_T = 10^3$ $\Delta\eta_T = 10^6$

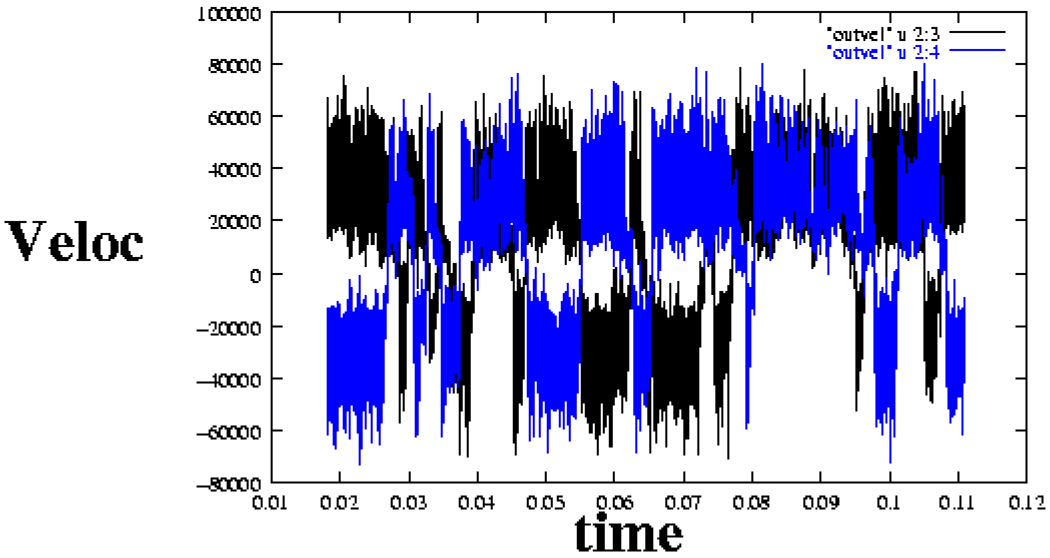


Generation of the Wind
Ra = 10**10
Stress-free boundaries

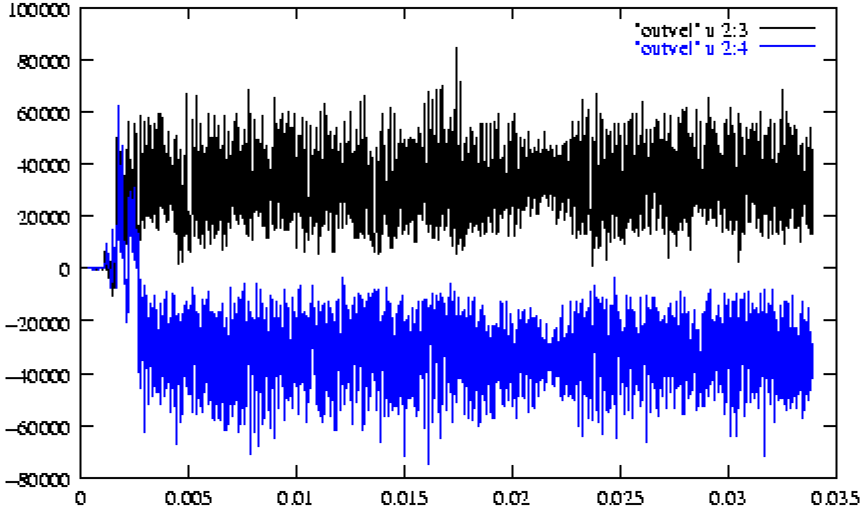


Flow Reversals – Ra=10**8
A = 1.8

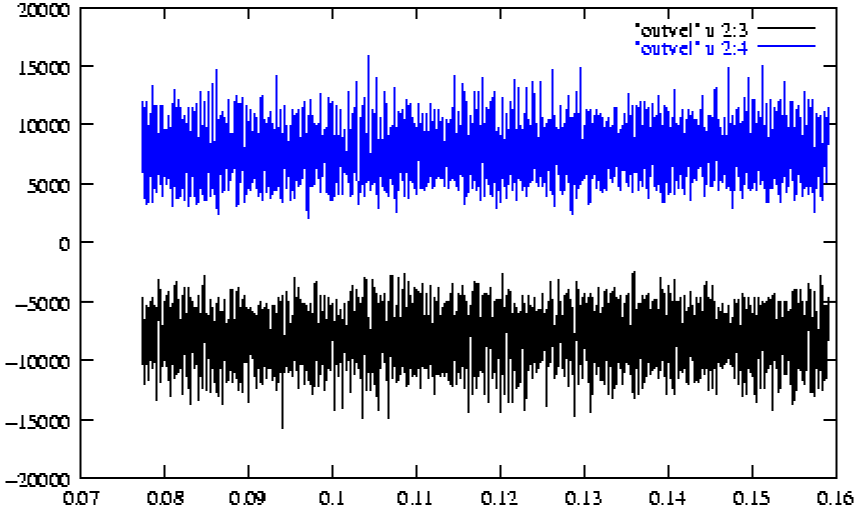




A = 1.8
Ra = 108**



A = 1.5, Ra = 108**



A = 1.8, Ra = 107**

Stationary solutions at infinite Prandtl number

$$0 = -\nabla P + \nu \nabla^2 \vec{v} + RaT\hat{z}$$

$$0 = \kappa \nabla^2 T - \vec{v} \nabla T + Q$$

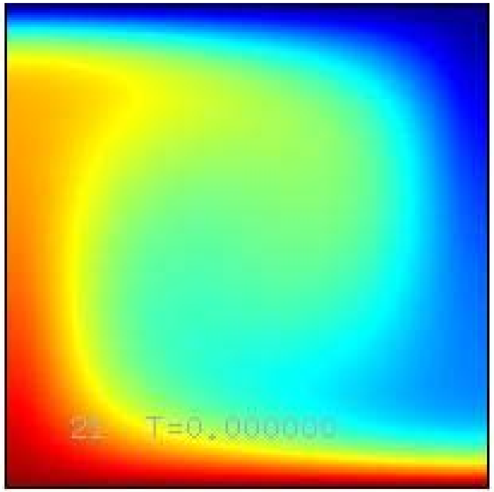
Solve the system iteratively

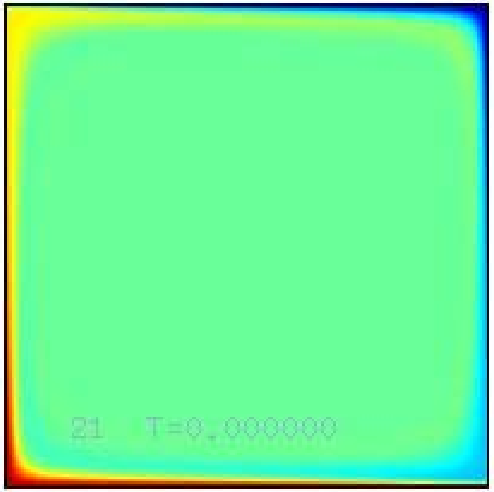
Can statinary states be used to derive the
Nu - Ra relation

$$Nu = a \bullet Ra^b$$

Stationary Flows – stress free boundaries

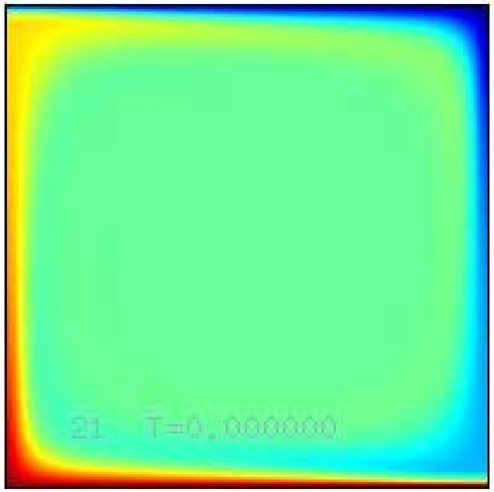
Ra = 10*4





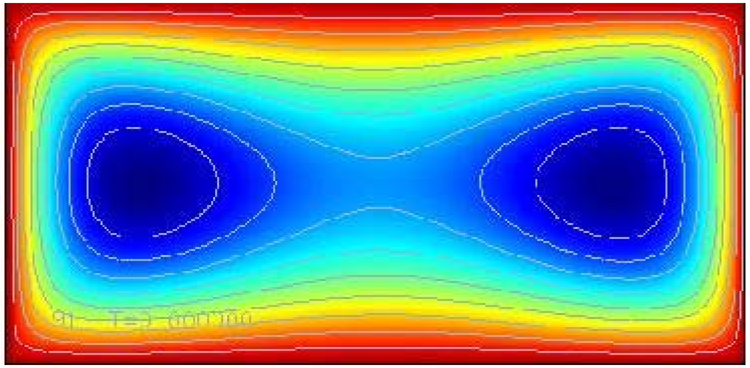
Ra = 107**

Ra = 106**



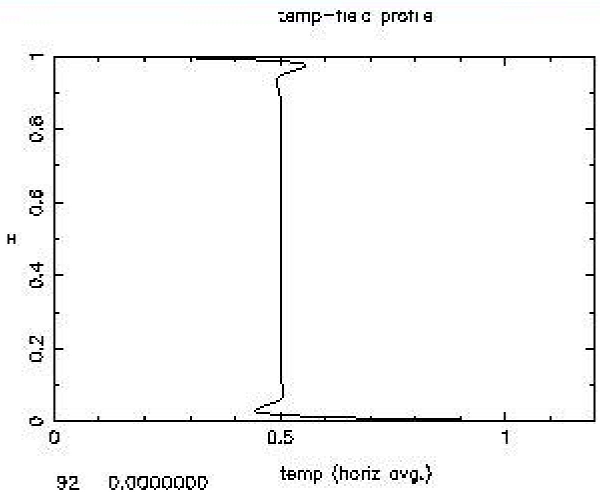
Ra = 108**

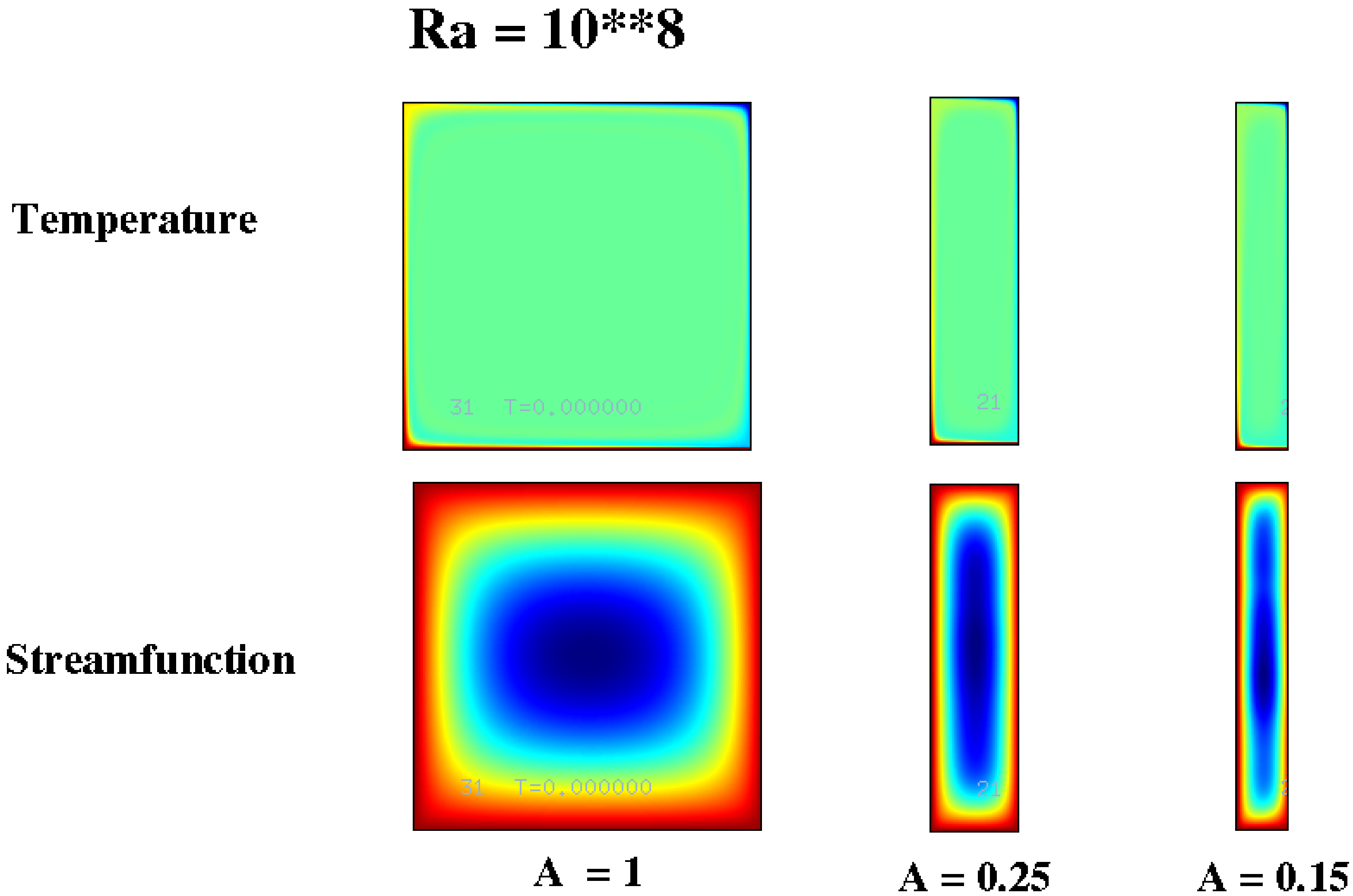
Streamfunction



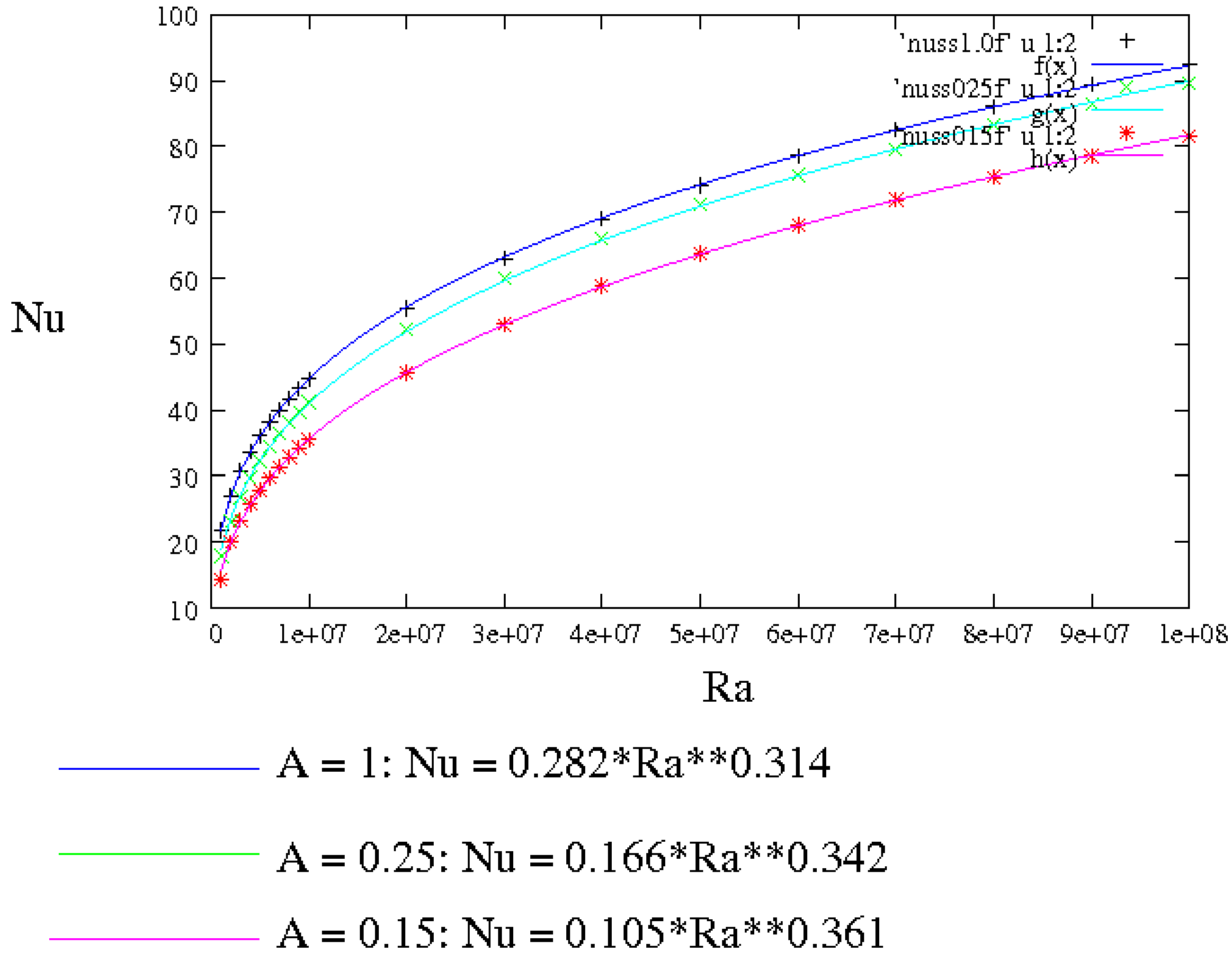
Temperature

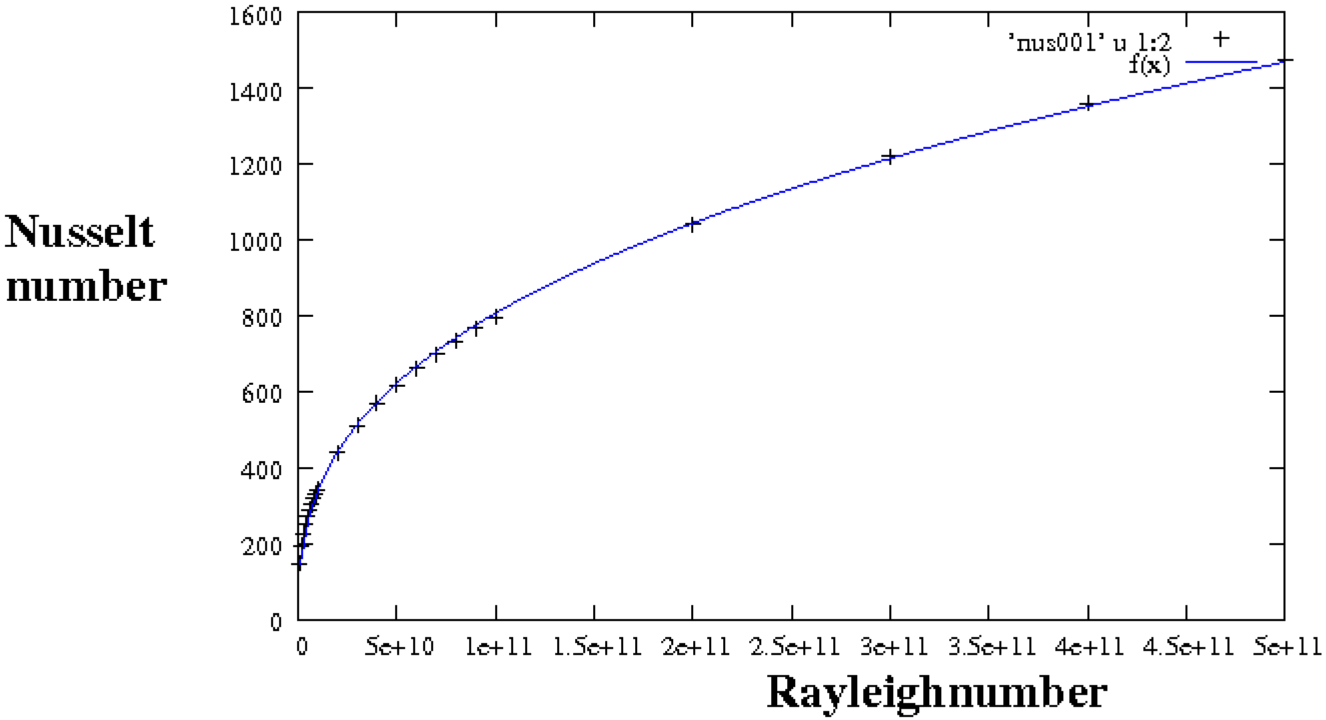
Ra = 108**



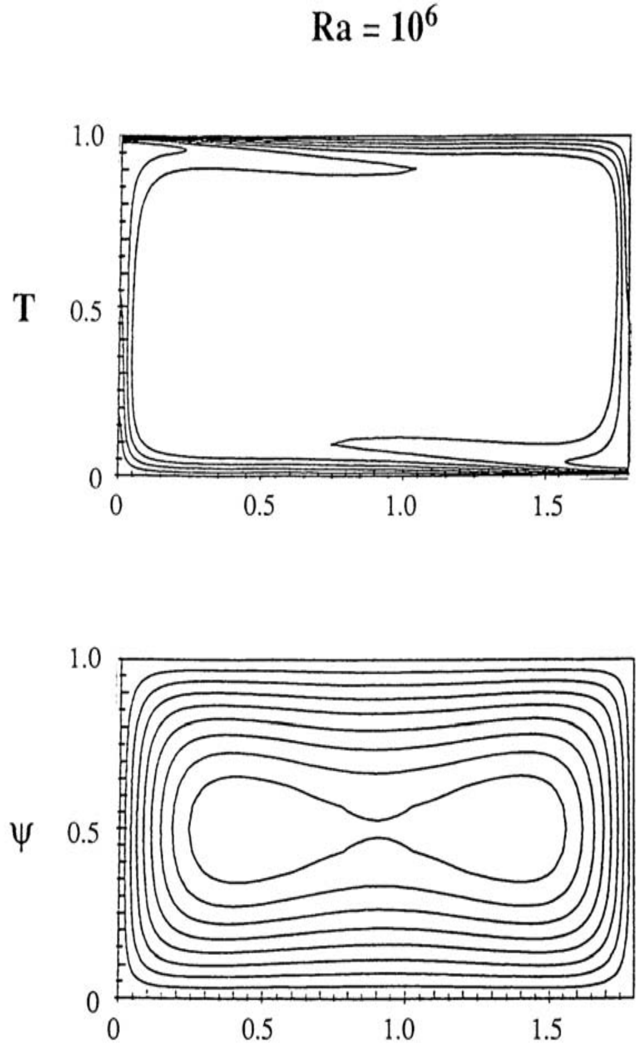

Ra = 10**8
Temperature
Streamfunction
31 T=0.000000
21
31 T=0.000000
21
A = 1
A = 0.25
A = 0.15

Effect of aspect ratio on Nu-Ra scaling, stress free b.c#s

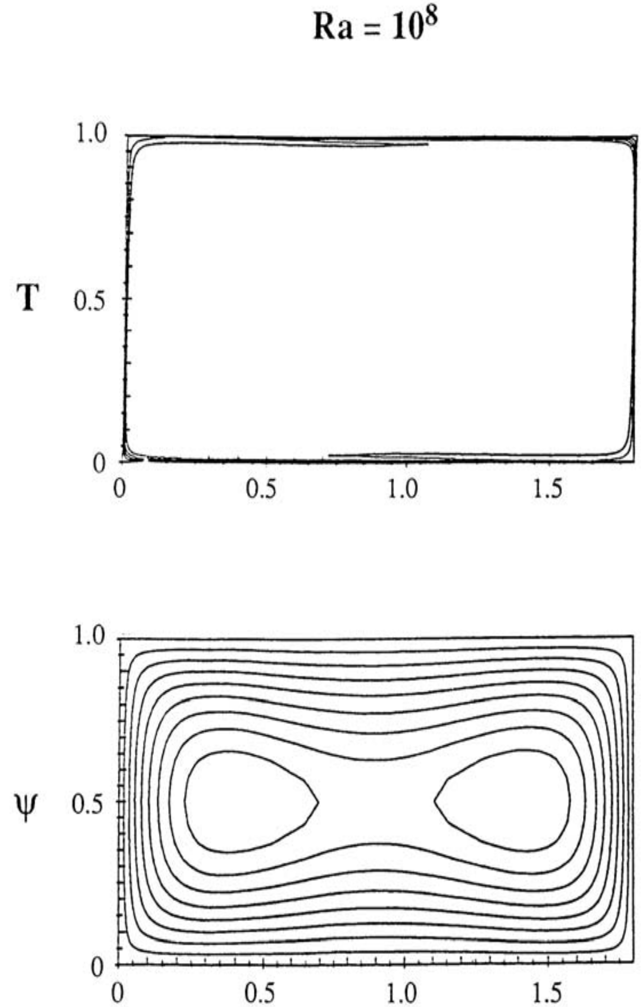




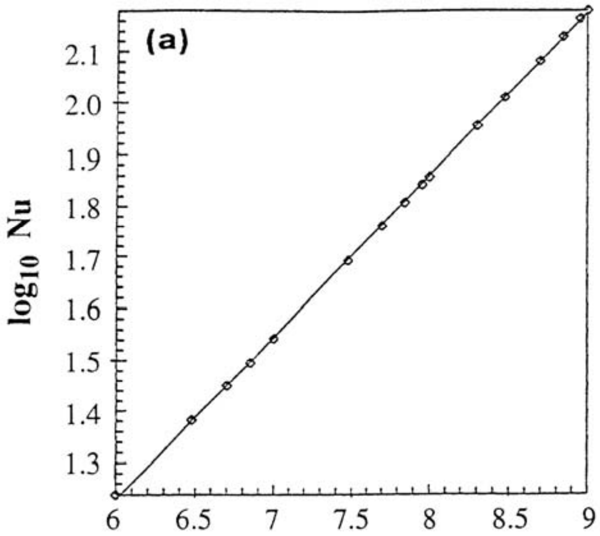
A = 0.001: Nu = 0.0635*Ra0.373**



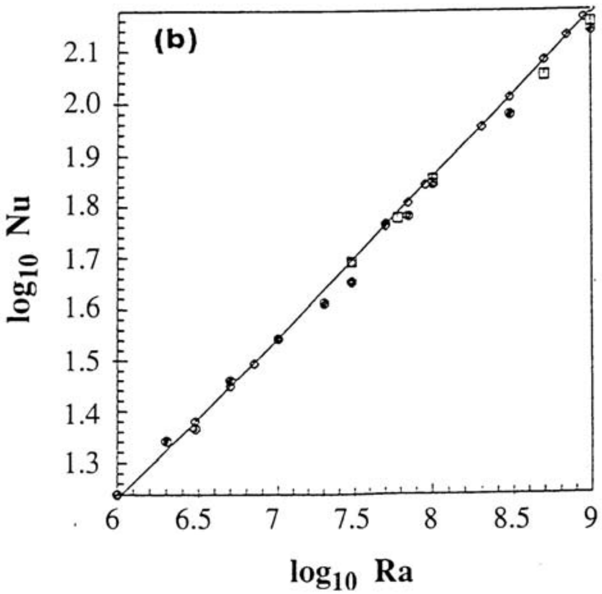
Streamfunction and temperature field of a stationary solution for Ra = 10^6.



Streamfunction and temperature-field for a stationar solution at Ra = 10^8, under stress free conditions.

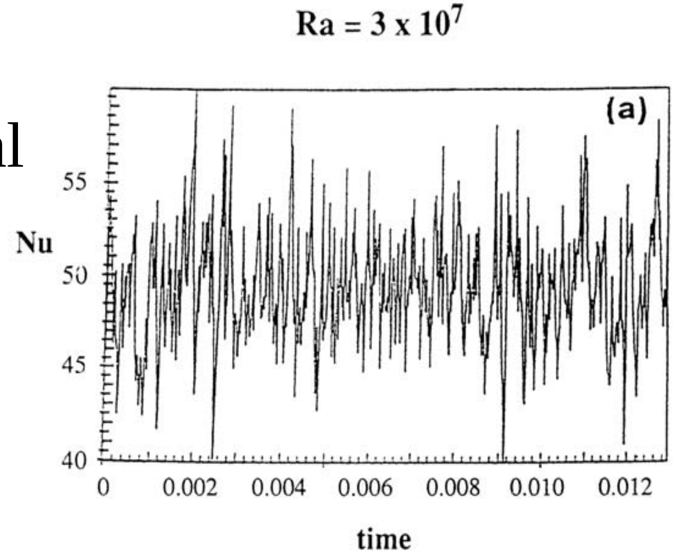


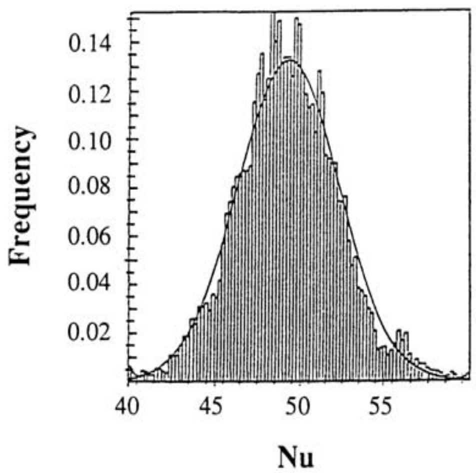
Functional dependence in this Ra-range is Nu = a Ra^{α} with a = 0.315.



Nusselt numbers as obtained from stationary solutions (a) and from corresponding time-dependent runs (b). Values, as obtained from stationary solutions resemble those from t.d. solutions in a statistical sense.

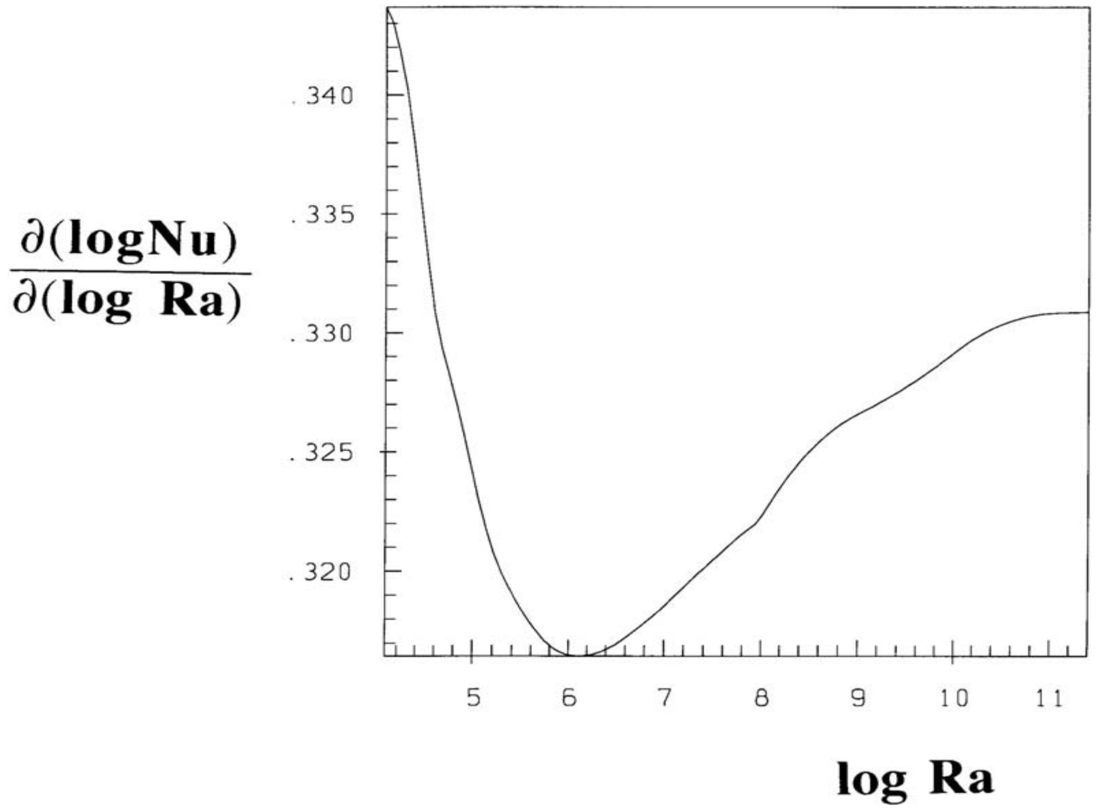
Do the stationary States have physical Meaning?





Time series of Nusselt Number Nu for a box of λ = 1.8 under stress free conditions with Ra = $3*10^7$.

Slope of the Nu-Ra curve $\partial(\log Nu)/\partial(\log Ra)$, as a function of Ra for stress free conditions.

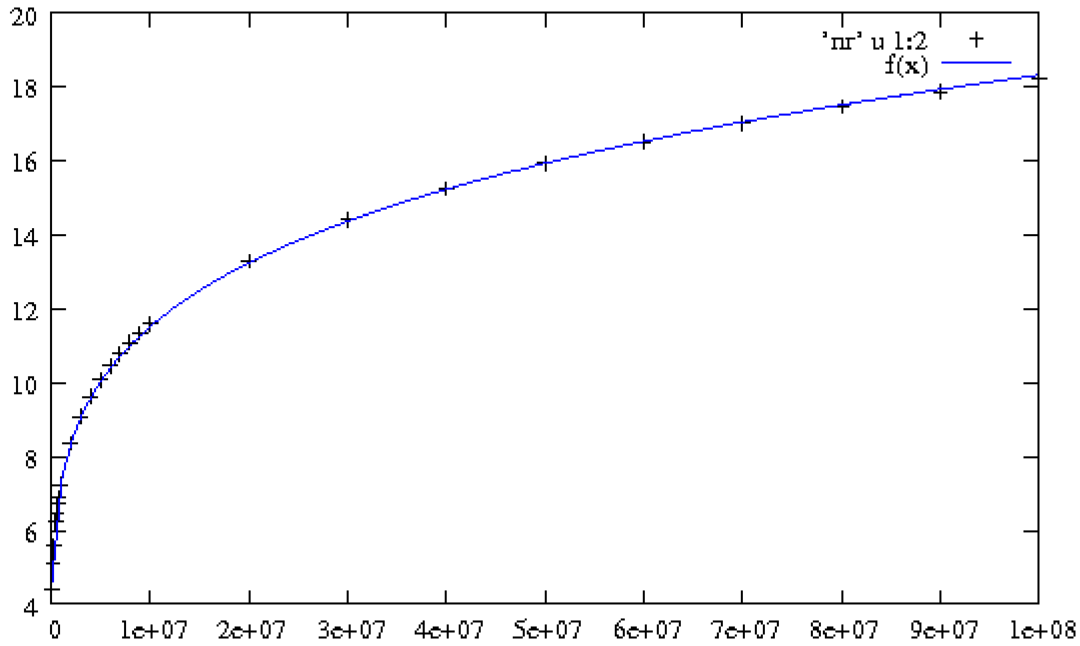


Aspect ratio 1 flow, top and bottom rigid – sides stress free

Nu = a*xb, with a = 0.447 and b= 0.201**

Theoretically obtained a = 1/5 (G.O. Roberts, 1977)

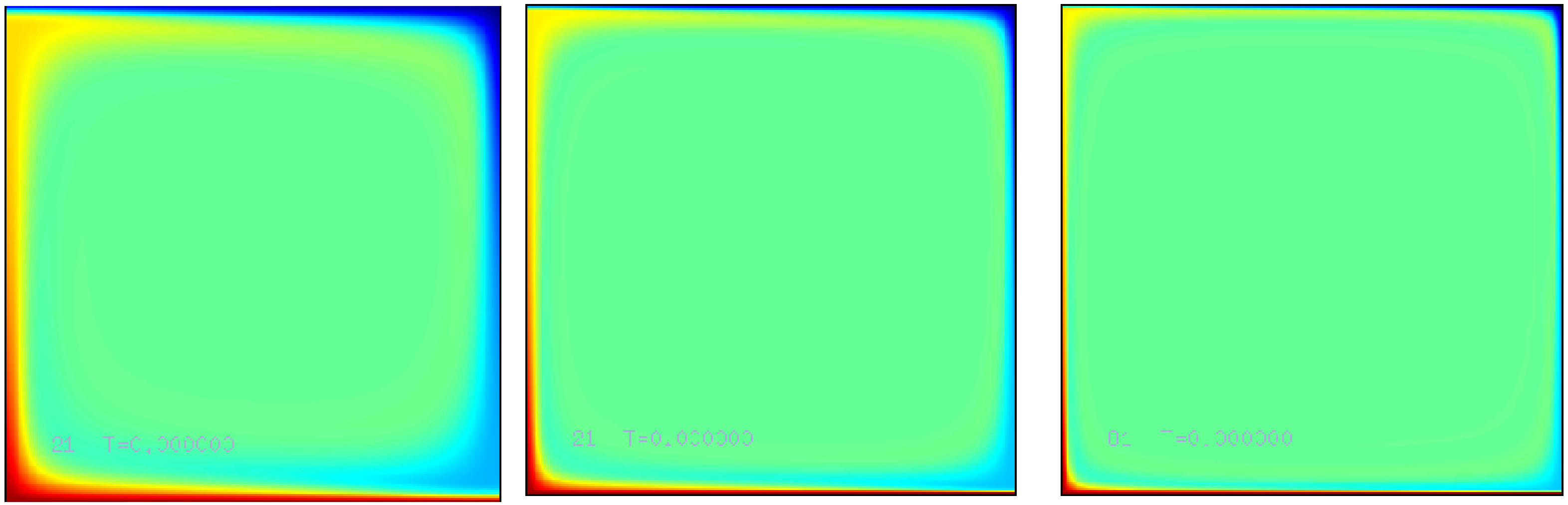
Nusselt number



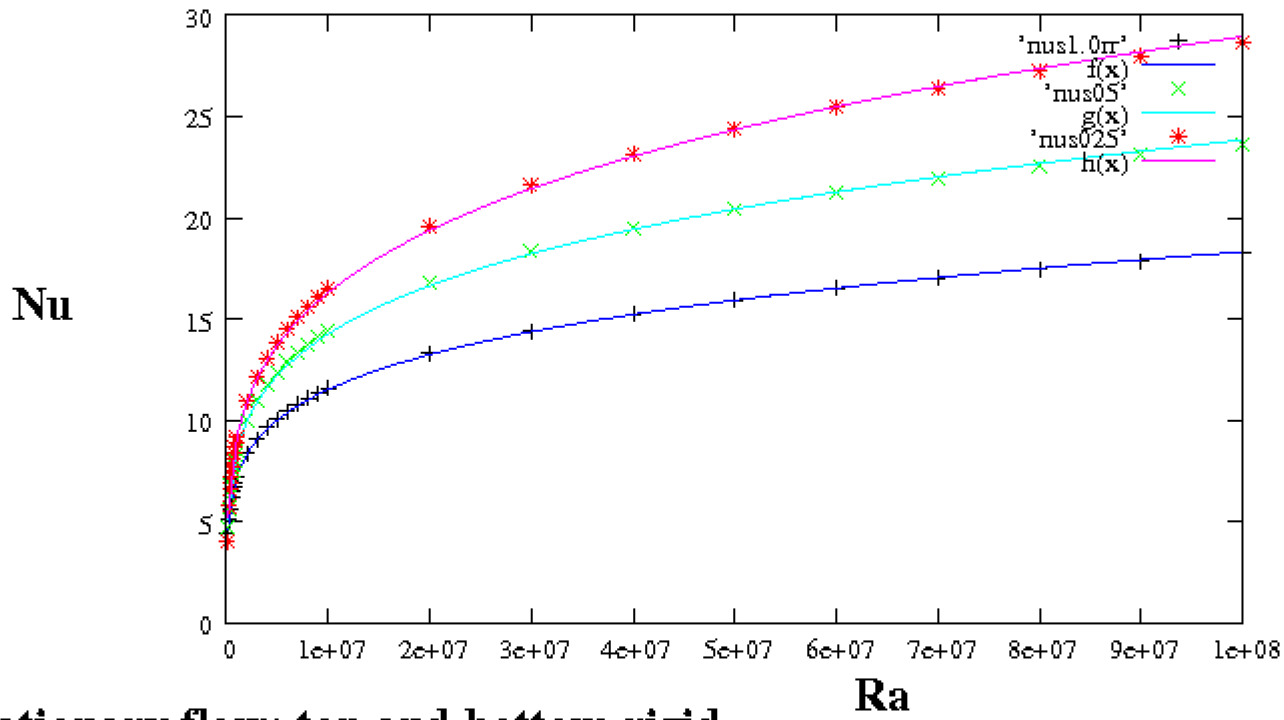
Rayleigh number

Nu = 0.447*Ra**0.201

Temperaturefields from stationary flows within rigid boundaries



Ra = 106** **Ra = 10**7** **Ra = 10**8**



Stationary flow: top and bottom rigid

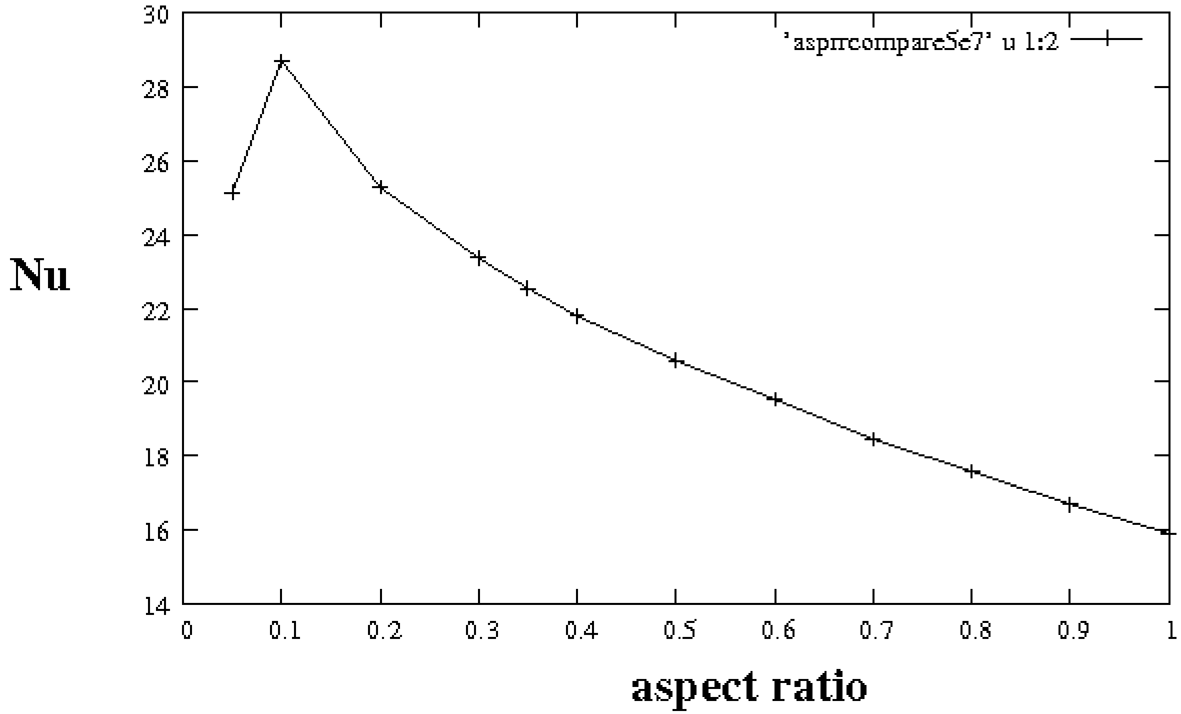
A=1: Nu = 0.448*Ra0.201**

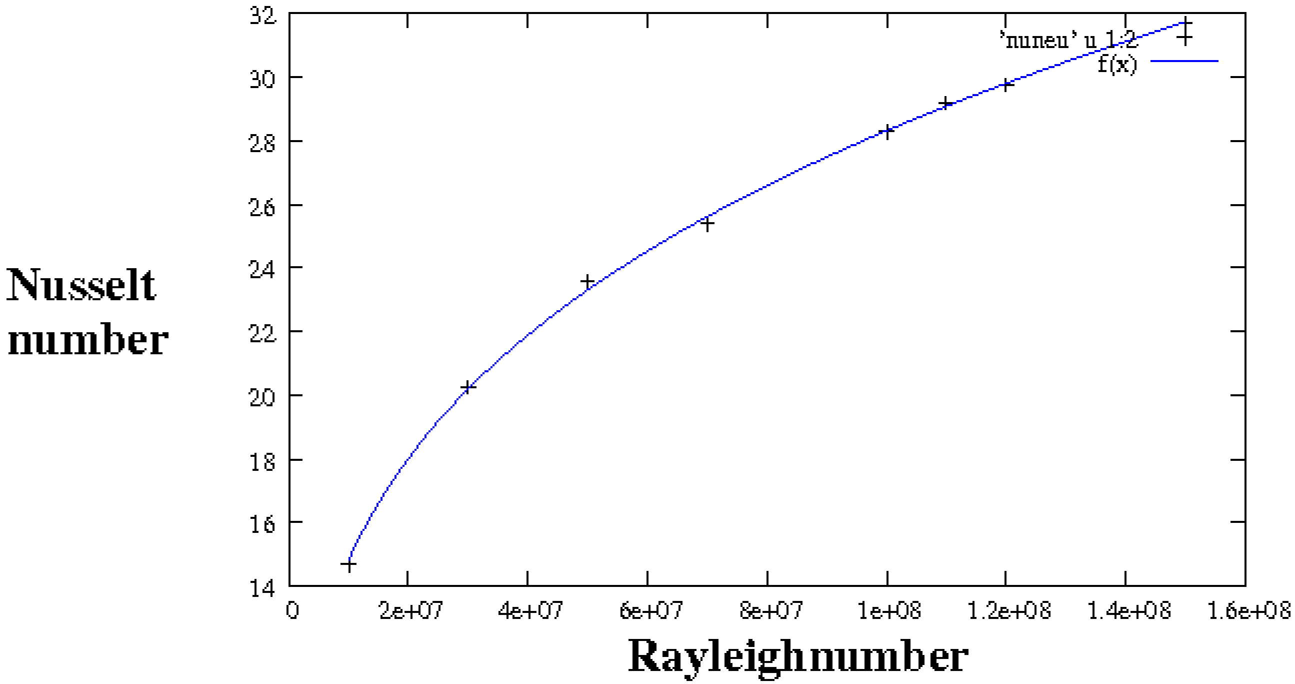
A = 0.5: Nu = 0.397*Ra0.223**

A = 0.25: Nu = 0.293*Ra0.249**

Effect of the aspect ratio on Nu, at a given Ra (5e7)

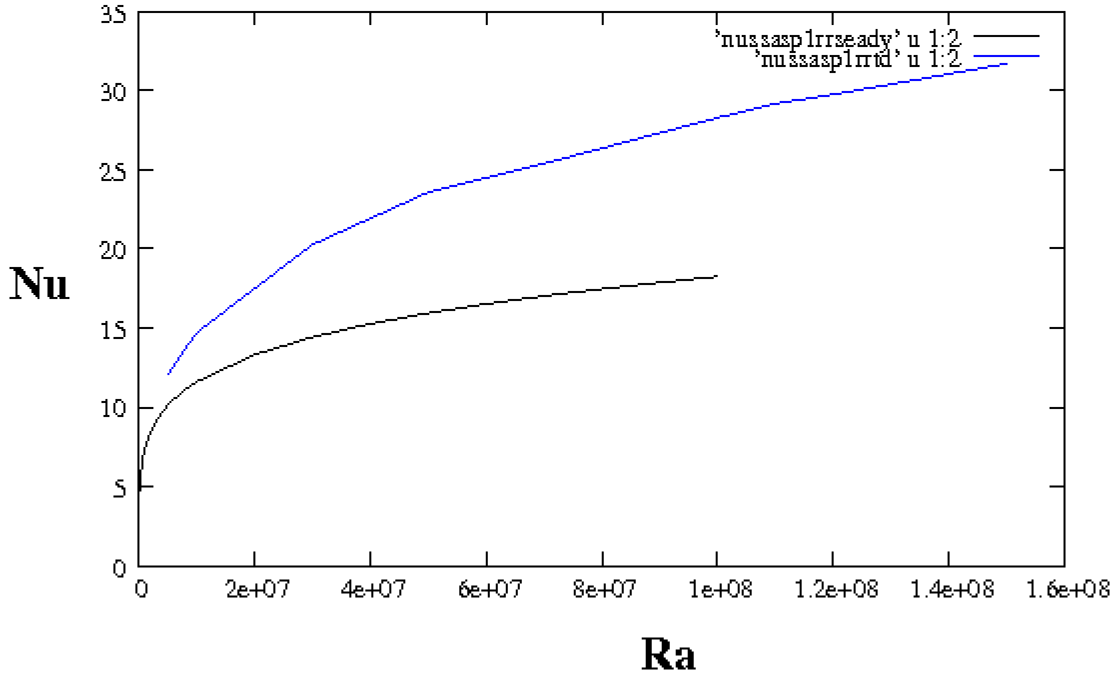
top – and bottom rigid





A=1: rigid bottom and top, time−dependent

Nu = 0.160*Ra0.281**

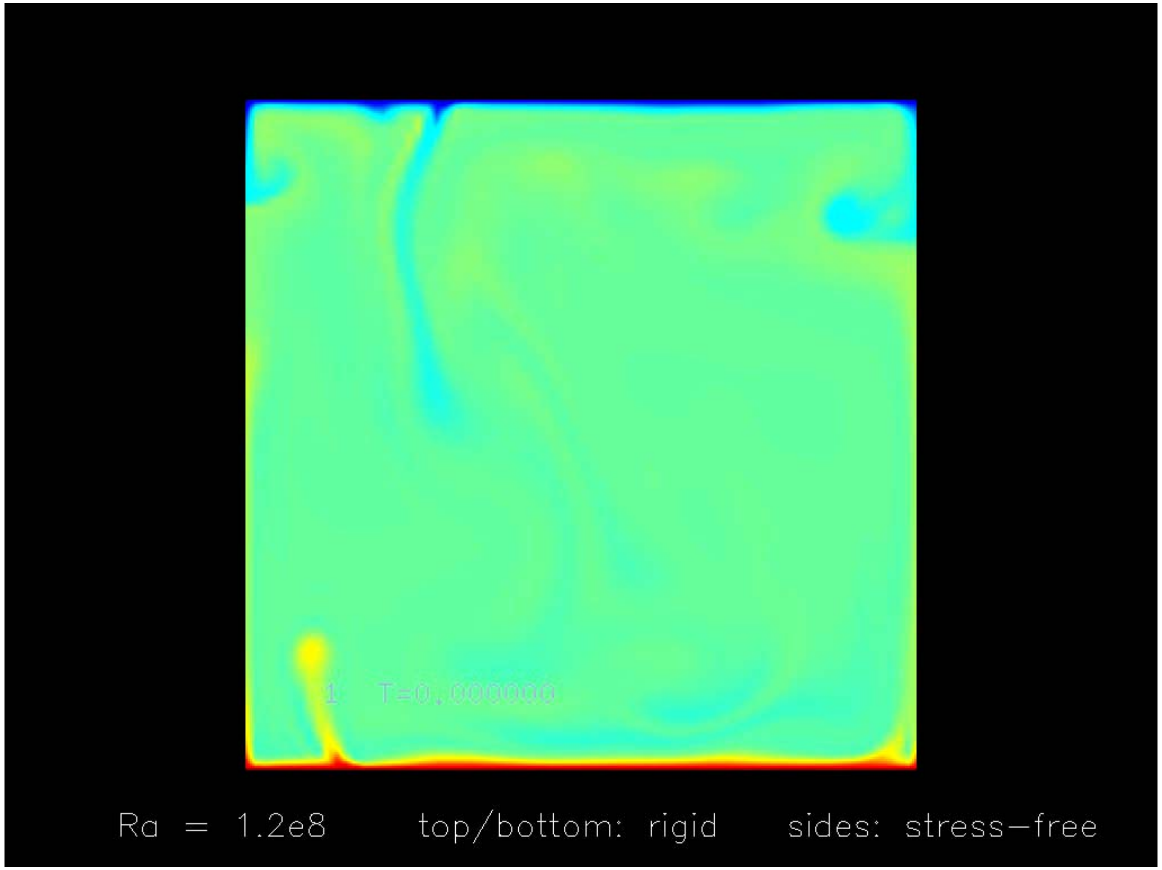


A=1: top and bottom rigid

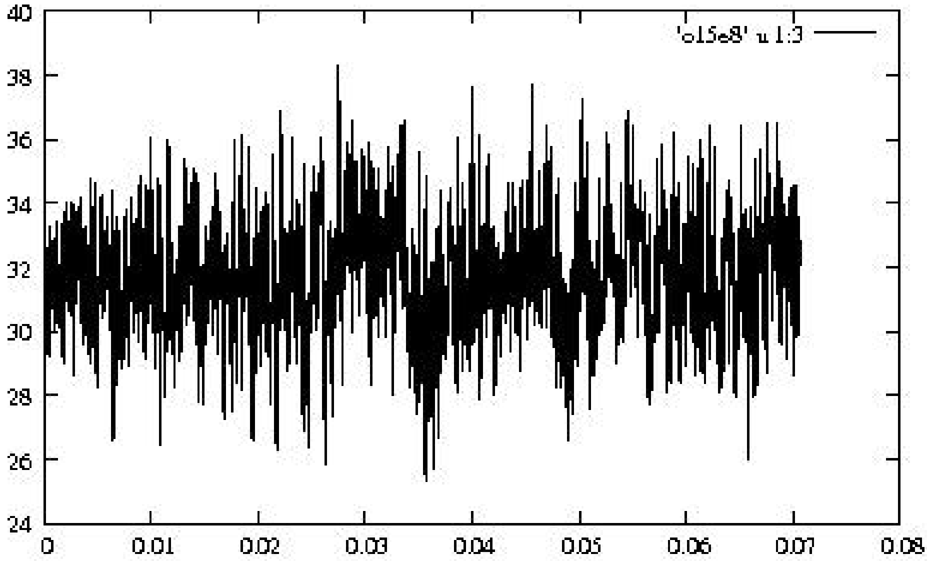
stationary flow

time−dependent flow

Rigid top- and bottom



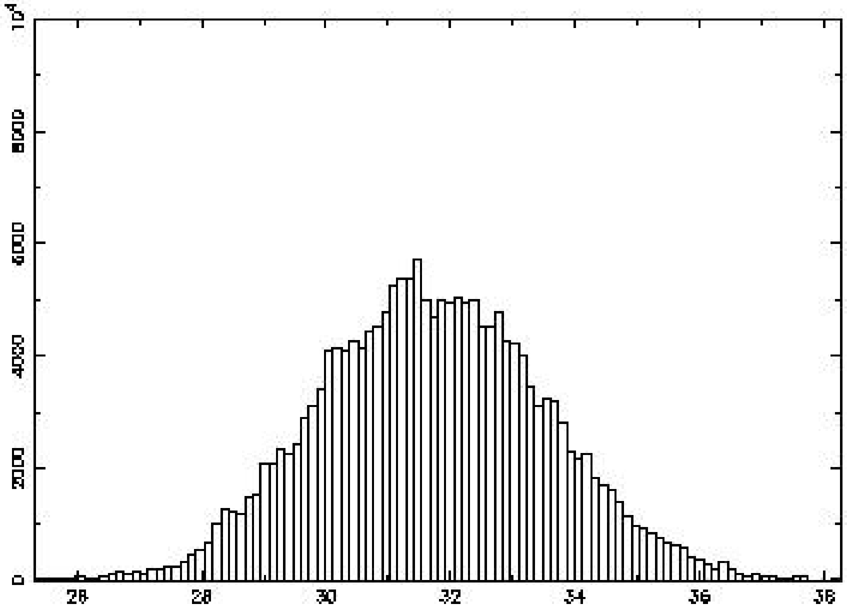
Nu



time

A = 1
Rigid
Ra= 1.5*10**8

Corresponds
To A = 0.25



Summary

Convection at high Rayleigh number and infinite Prandtl number exhibits features of turbulent convection, like ‘wind’, ‘plumes’ and ‘reversals’.

In a wide range of parameters stationary flows (with finite stability) can be found.

If a persistent ‘box-wide’ wind exists, the time evolution of Nu forms a Gaussian distribution with a mean value of the corresponding stationary state.

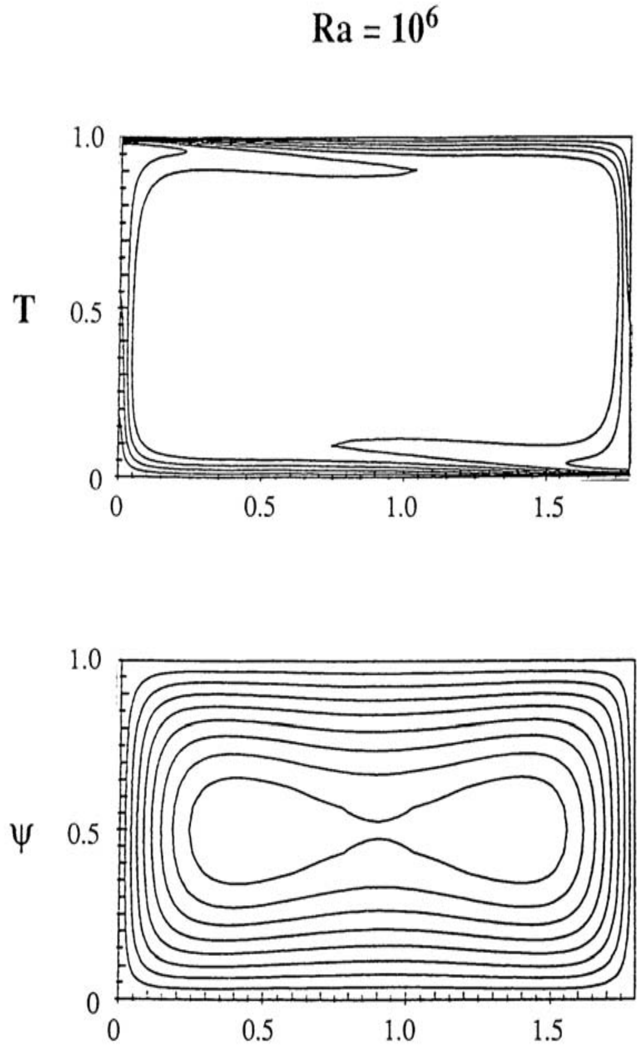
In a wide range of Rayleigh numbers the Nu-Ra scaling depends on the aspect ratio. The exponent in inversely proportional to the aspect ratio.

Under such conditions, in a wide parameter range, a layered flow pattern develops. The range includes temperature- and pressure dependence of the viscosity, internal heat generation and temporally decaying heat source

The results indicate that the formation of layers and thus of discontinuities are a typical feature of planetary formation

Low Prandtlnumber convection exhibits a significant toroidal flow component – 3D and 2D flows are similar at high Pr, but significantly different at low values of Pr.

IN



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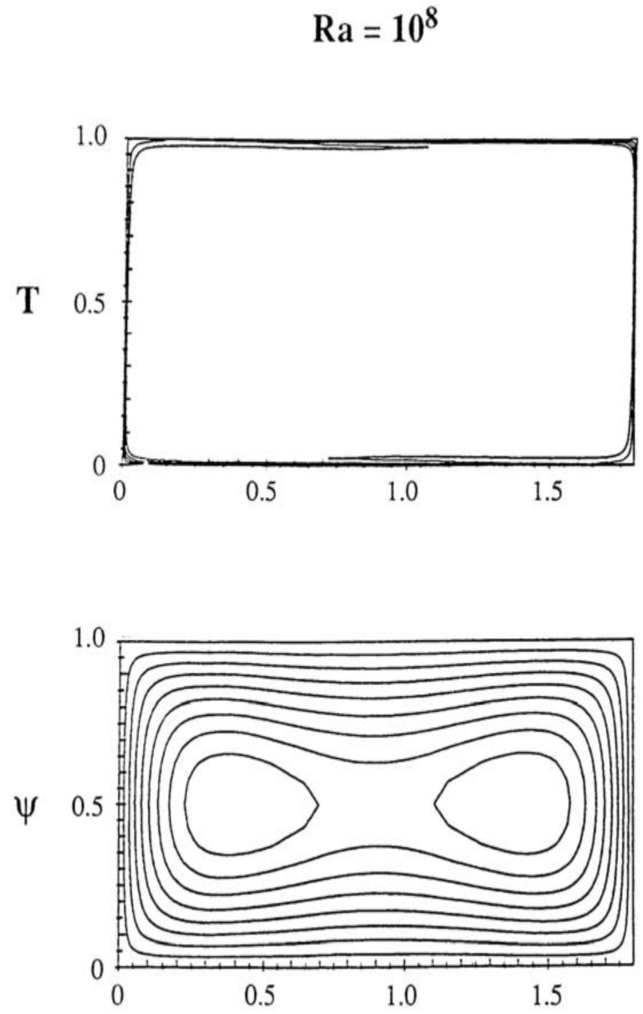
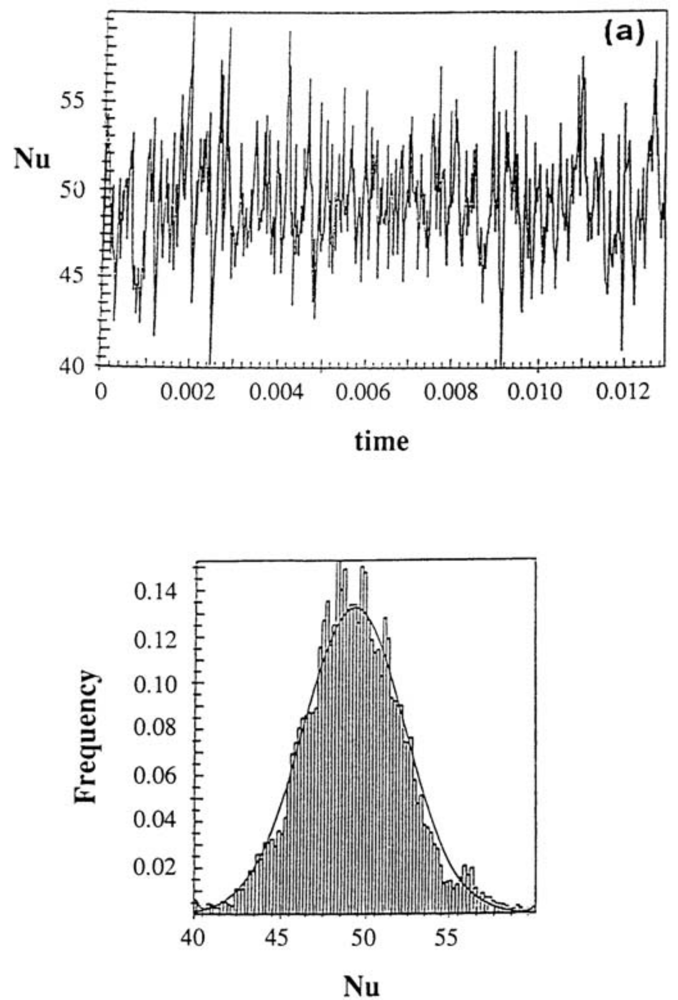


Streamfunction and temperature-field for a stationar solution at Ra = 10^8, under stress free conditions.

Ra = 3 x 10^7



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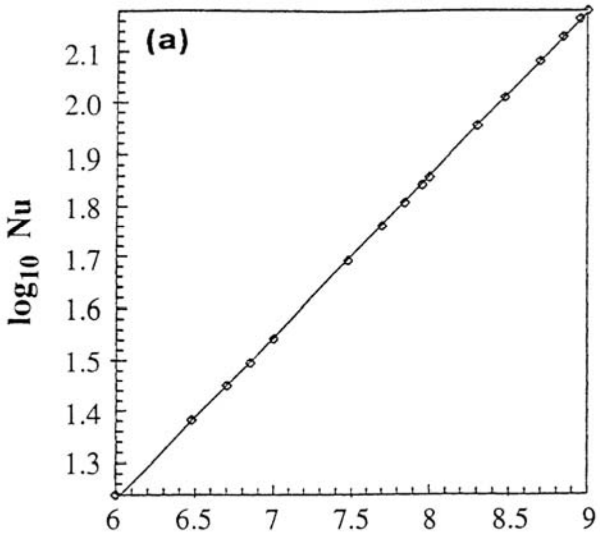

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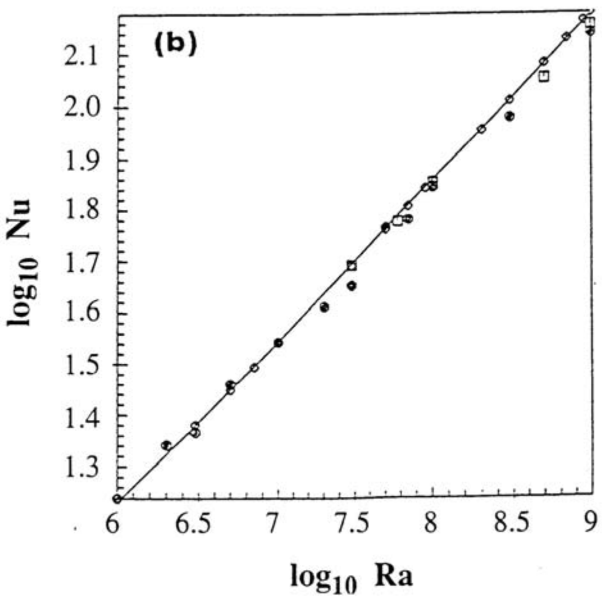

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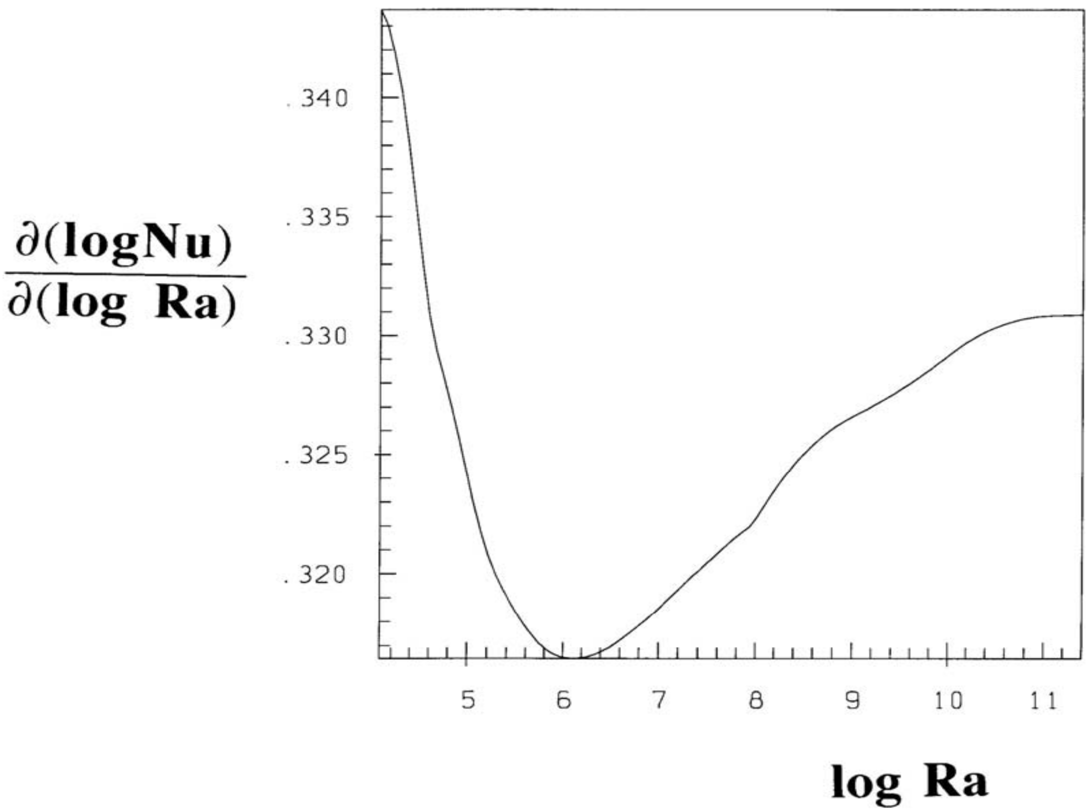

Two-dimensional conection in an incompressible Bousinessq-fluid at infinite Prandtl number

$$\nabla^4 \psi = Ra \bullet \frac{\partial T}{\partial x}$$

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial z} \bullet \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \bullet \frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}$$

With streamfunction $u = \frac{\partial \psi}{\partial z}, v = -\frac{\partial \psi}{\partial x}$

Consider only stationary flows

$$\nabla^4 \psi = Ra \bullet \frac{\partial T}{\partial x}$$

$$0 = -\frac{\partial \psi}{\partial z} \bullet \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \bullet \frac{\partial T}{\partial z} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}$$