The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
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International Atomic
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Conference and Euromech Colloquium #480

on

High Rayleigh Number Convection

4 - 8 Sept., 2006, ICTP, Trieste, Italy

Charge transport scaling in turbulent electroconvection

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Charge Transport Scaling in Turbulent Electroconvection

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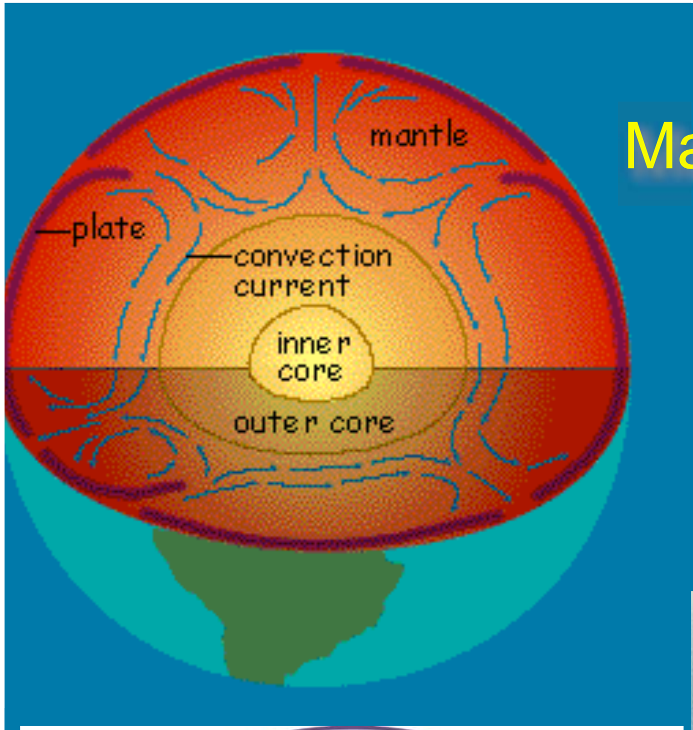
Dept. of Physics, University of Toronto

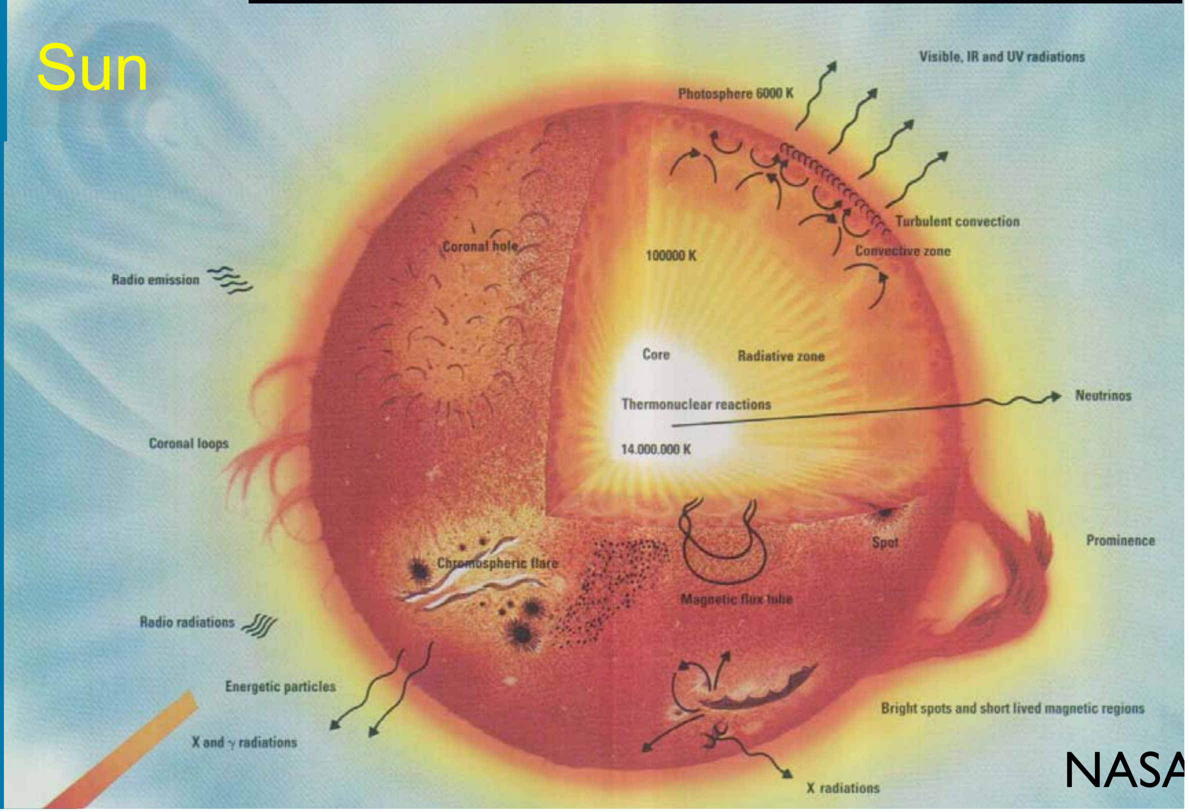
Experimental
Nonlinear
Physics
University of Toronto

ICTP, Trieste, Italy. Sept. 7, 2006



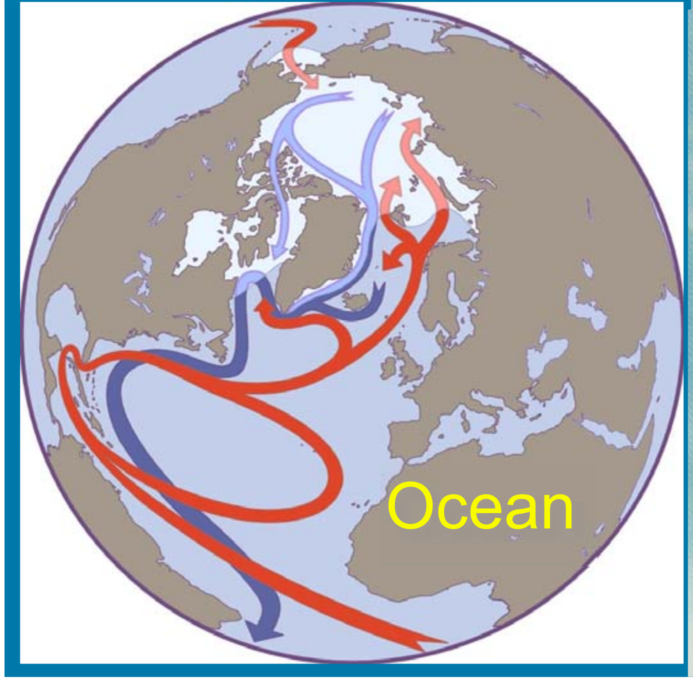
Mantle

Atmosphere



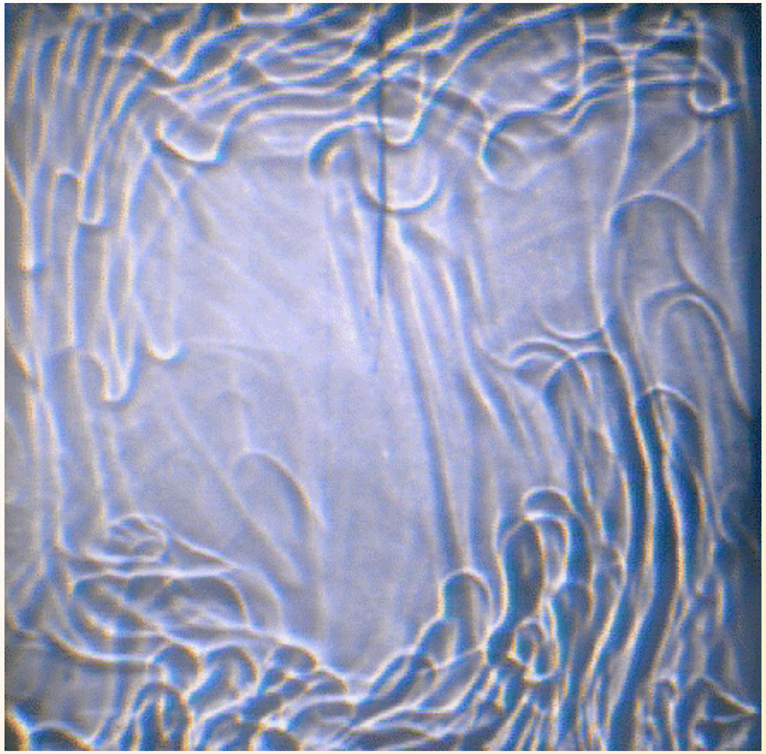
Sun

NASA

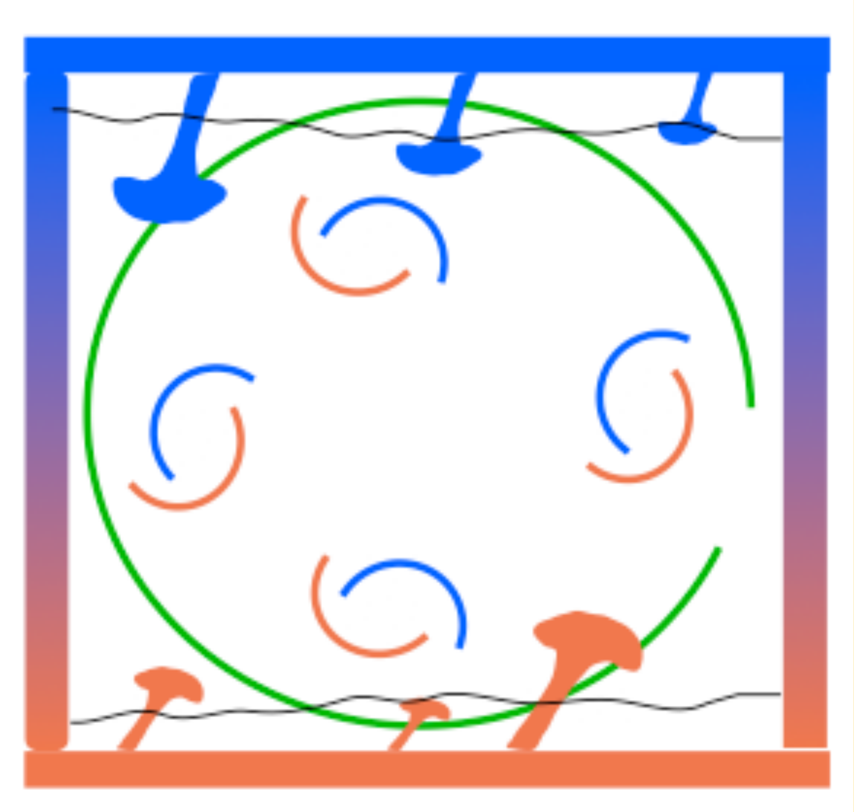


Ocean

High Rayleigh number thermal convection



By J. Zhang, S. Childress & A. Libchaber



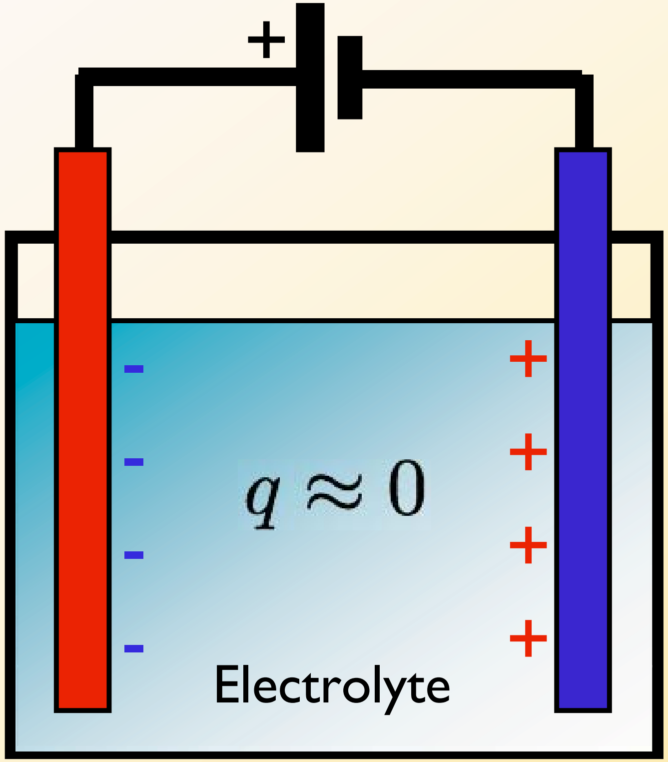
Flow organized into viscous and thermal boundary layers, plumes and large scale flow

Crucial question: How does heat transport (*Nusselt number*) depend on ΔT (*Rayleigh number*)?

Electroconvection?

Replace buoyancy forces with electrical forces

$$\alpha\rho\Delta T\vec{\mathbf{g}} \rightarrow q\vec{\mathbf{E}}$$



Not so easy!

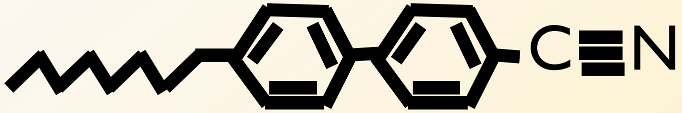
Bulk electrolyte forms Debye screening layers.

$$\lambda_D = \sqrt{\frac{\epsilon_o k_B T}{n e^2}} \sim 1\mu m$$

electroneutrality

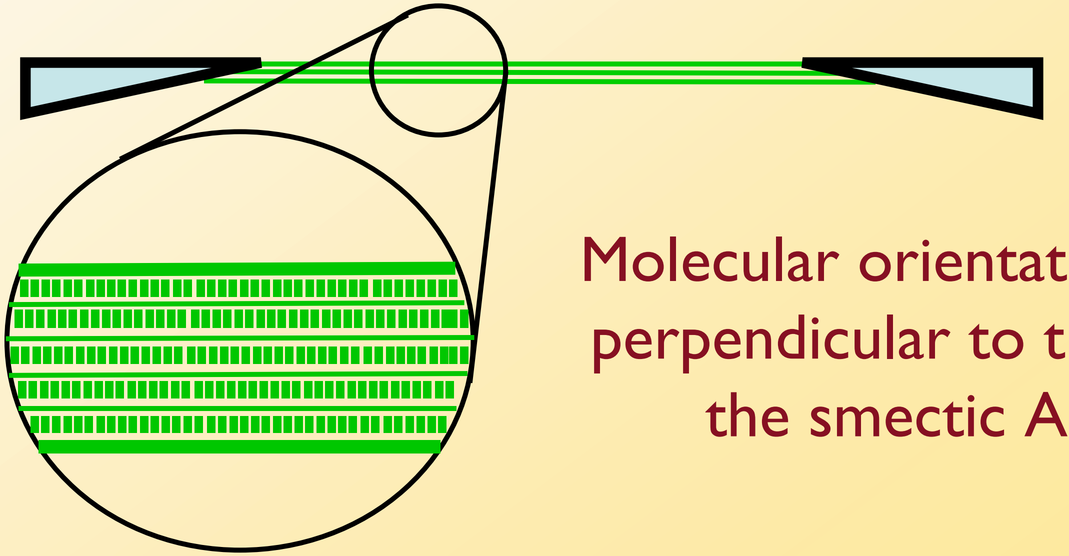
prevents body force from driving flow

Smectic liquid crystal films



8CB

Smectics form robust submicron thick suspended films which are an integer number of smectic layers thick. They are Newtonian for flows in the plane of the film and resist thickness change.



side view

Each layer is 3.16nm thick.

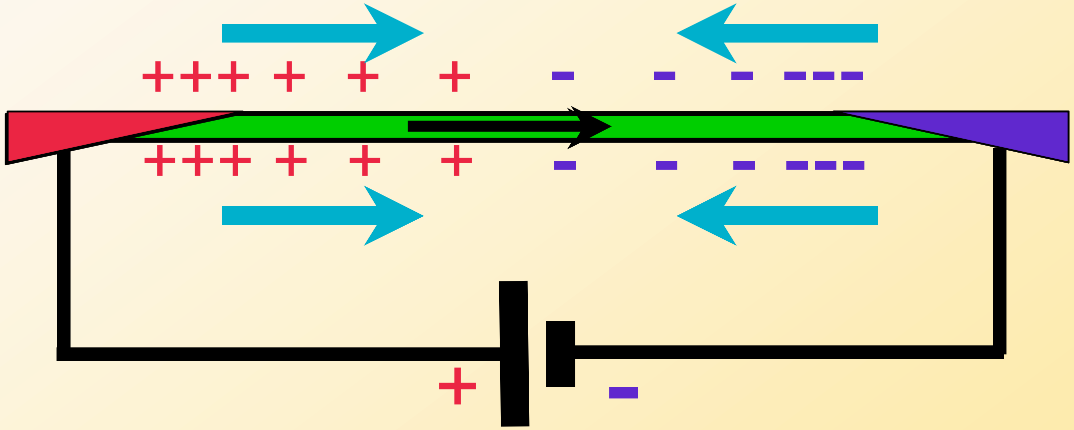
Films 2-100 layers thick

Molecular orientation remains perpendicular to the layers in the smectic A phase.

Electroconvection in a smectic film

Thickness $s \ll \lambda_D$ all surface

Convection is driven by an unstable surface charge distribution on the two free surfaces



Apply a DC voltage.

Drive a current.

Produce surface charge

Surface forces appear that can drive convection

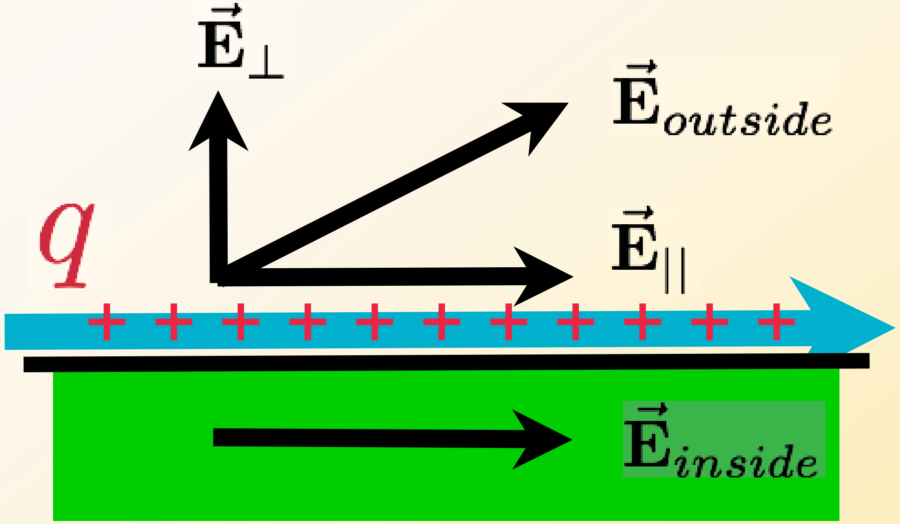
Mechanism of electroconvection

Current density $\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}_{inside}$

$$\vec{\mathbf{E}}_{outside} = -\nabla\psi$$

with $\nabla^2\psi = 0$

outside the film.



Boundary conditions require

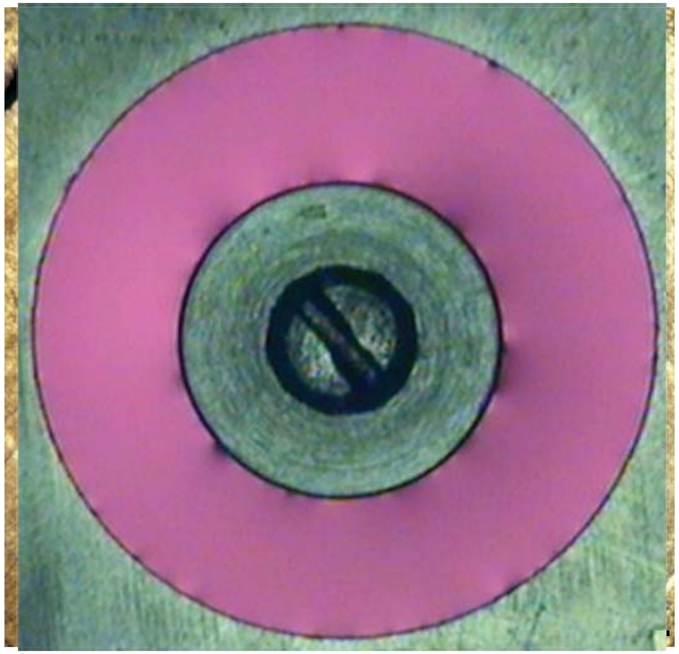
$$\vec{\mathbf{E}}_{||} = \vec{\mathbf{E}}_{inside}$$

Surface charge $q = \epsilon_o E_{\perp} = -\epsilon_o \partial_{\perp}\psi$

Surface force $q\vec{\mathbf{E}}_{||} = -\epsilon_o q \nabla_{||}\psi$

2D Annular Electroconvection

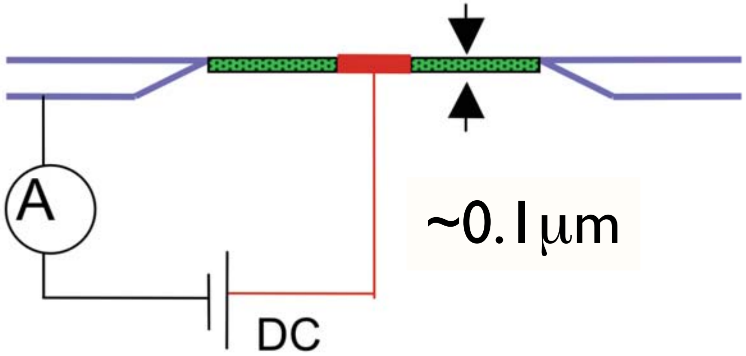
Electroconvection V > Vc



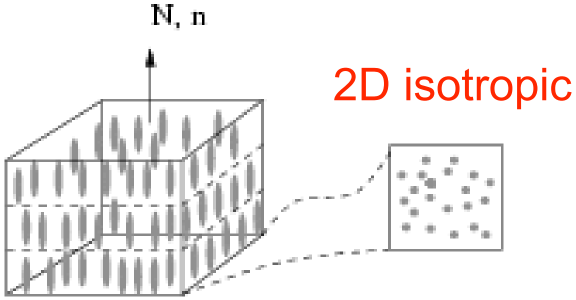
flow visualized by non-uniform thickness

Imposing DC Voltage, V
Measuring current, I

2D annular liquid crystal thin film

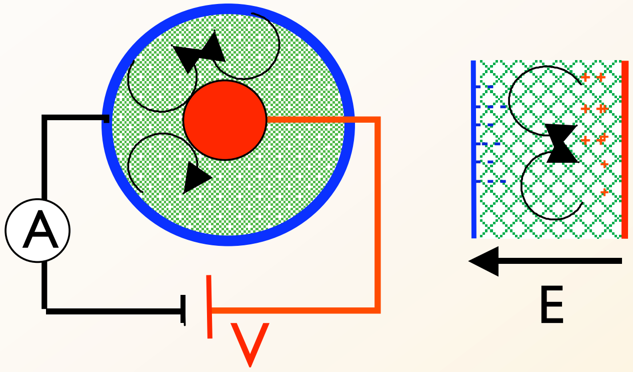


Smectic-A phase liquid crystal



Sample 8 CB (octylcyanobiphenyl)
Doping TCNQ (tetracyanoquinodimethane)

Analogy to Thermal Convection

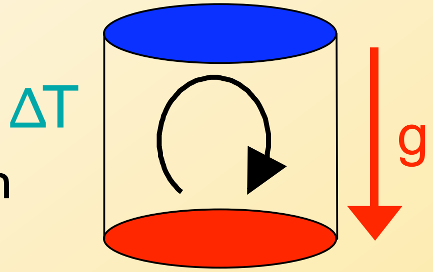


surface charge inversion

Rayleigh Bénard Convection (RBC)

Nu = heat flux

Ra ~ Δ T



fluid density inversion

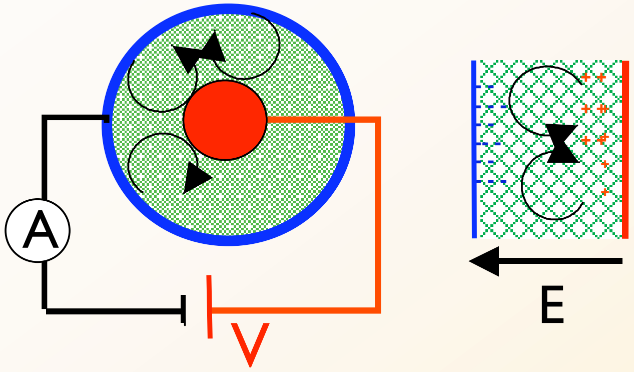
ΔT > Tc : convection

Dimensionless parameters

Rayleigh number	$Ra = \epsilon_0^2 V^2 / \eta \sigma s^2$	control parameter
Nusselt number	$Nu = I / I_{conduction}$	charge flux
Prandtl number	$Pr = \epsilon_0 \eta / \rho \sigma d s$	

Z. A. Daya et. al., Phys. Rev. E55, 2682 (1997)

Analogy to Thermal Convection



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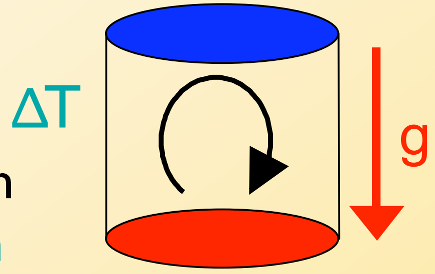


fluid density inversion

ΔT > Tc : convection

Dimensionless parameters

Rayleigh number $Ra = \epsilon_0^2 V^2 / \eta\sigma s^2$

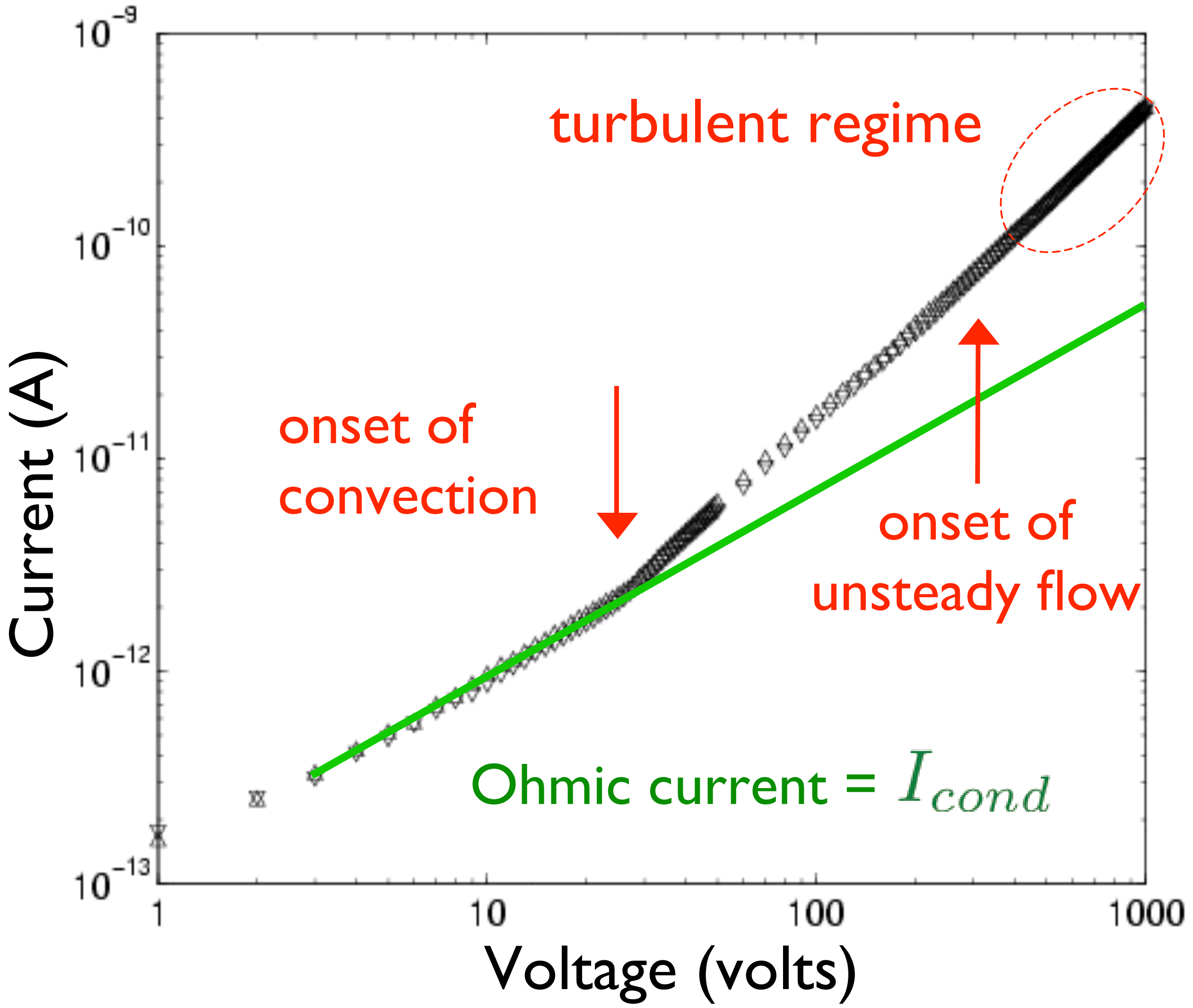
Nusselt number $Nu = I / I_{conduction}$

Prandtl number $Pr = \epsilon_0 \eta / \rho\sigma d s$

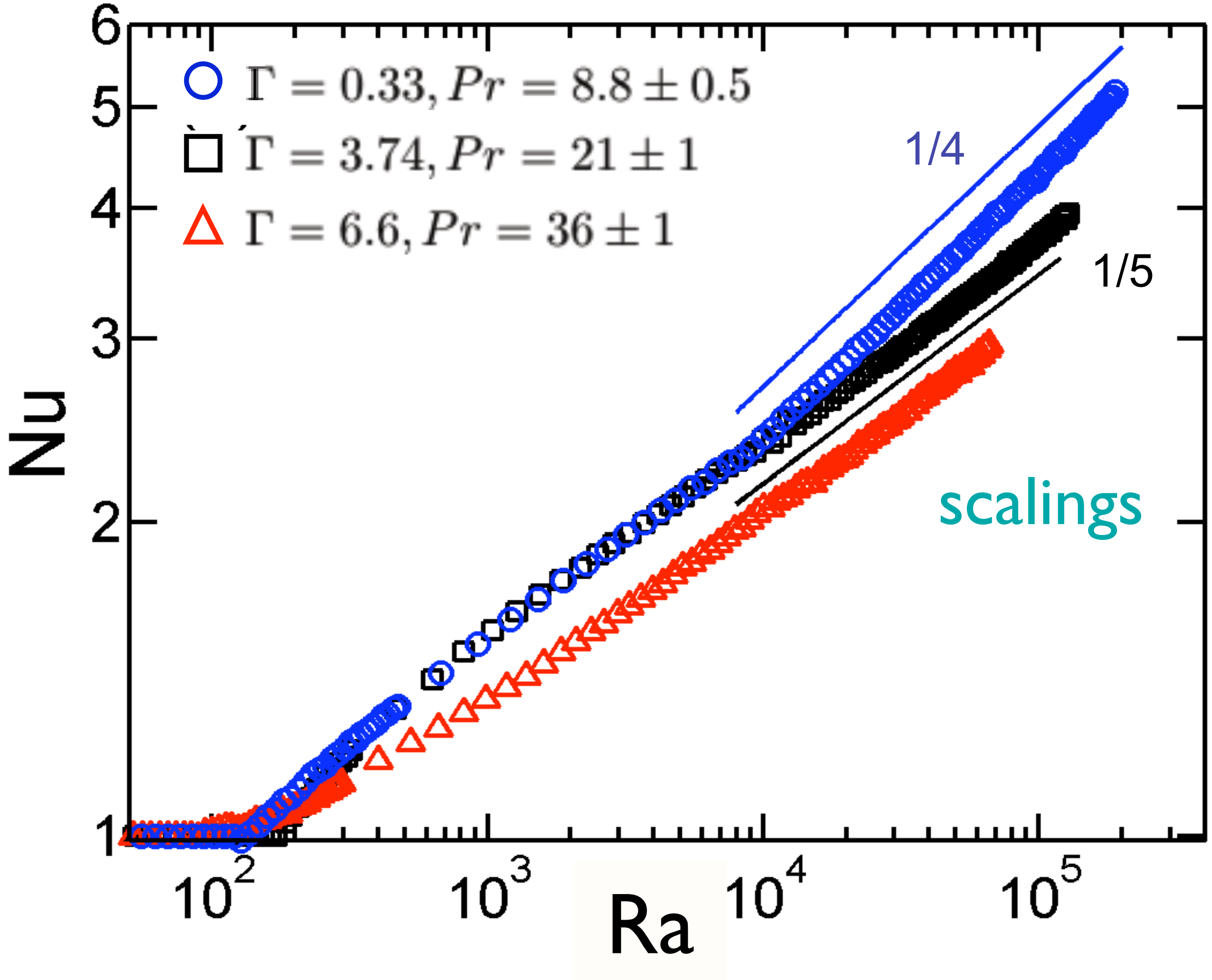
Reynolds number $Re = \dfrac{r_i \Omega}{Pr}$

Z. A. Daya et. al., Phys. Rev. E55, 2682 (1997)

Experimental Results



Electrical Nu vs *Ra*



P. Tsai, Z. A. Daya, & S.W. Morris, Phys. Rev. Lett. 92 084503 (2004)

The Pros and Cons of High *Ra* Smectic Electroconvection

Advantages over thermal convection:

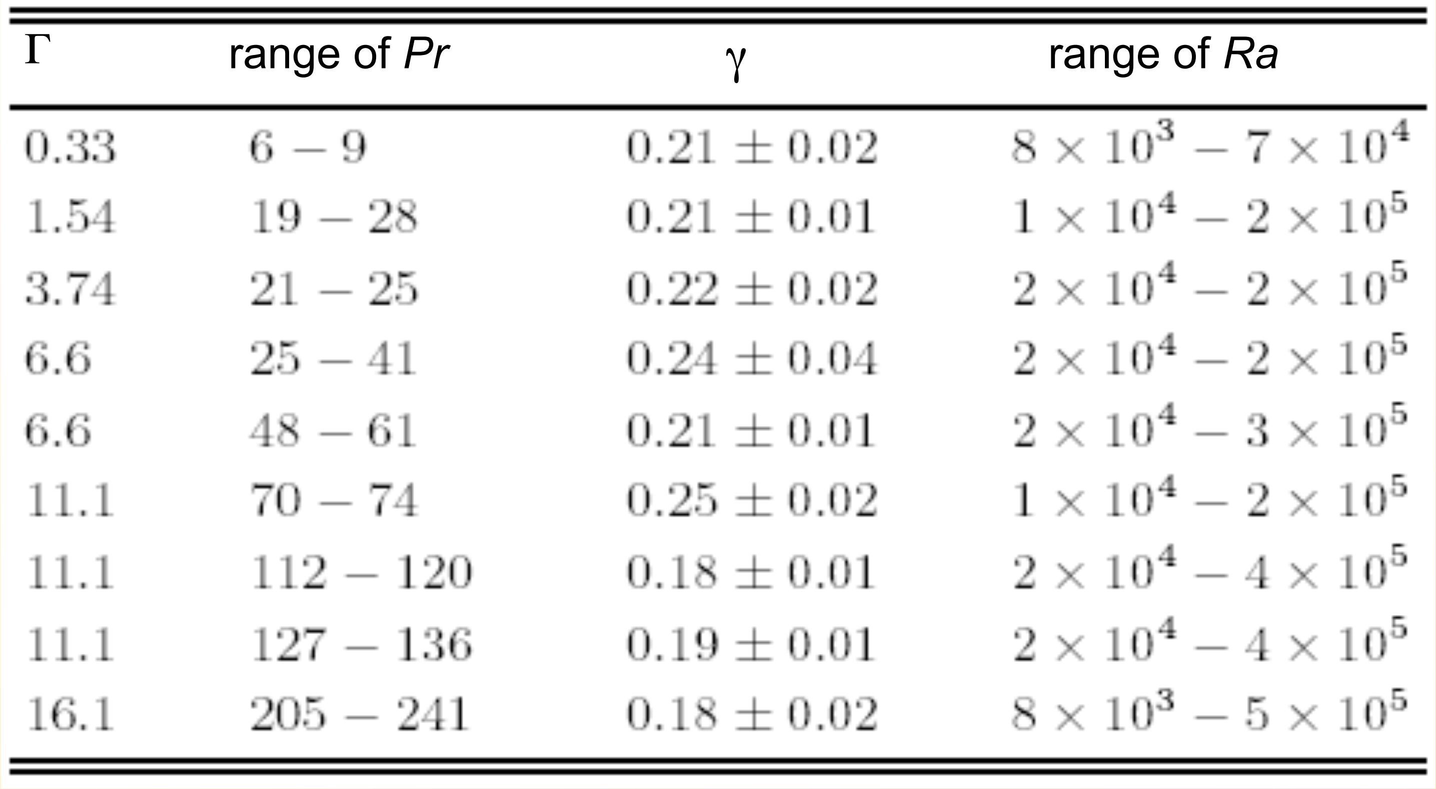
- *fast time scales*
- *annular geometry*
- *wide range of aspect ratio*
- *can impose external shear*

Disdvantages:

- *experimentally difficult to go above Ra* $\sim 10^6$
- *material degradation causes drifts*

$Nu = F(Ra, Pr, \Gamma)$?

$Nu \sim Ra^{\gamma}$



Γ	range of *Pr*	γ	range of *Ra*
0.33	$6-9$	0.21 ± 0.02	$8 \times 10^3 - 7 \times 10^4$
1.54	$19-28$	0.21 ± 0.01	$1 \times 10^4 - 2 \times 10^5$
3.74	$21-25$	0.22 ± 0.02	$2 \times 10^4 - 2 \times 10^5$
6.6	$25-41$	0.24 ± 0.04	$2 \times 10^4 - 2 \times 10^5$
6.6	$48-61$	0.21 ± 0.01	$2 \times 10^4 - 3 \times 10^5$
11.1	$70-74$	0.25 ± 0.02	$1 \times 10^4 - 2 \times 10^5$
11.1	$112-120$	0.18 ± 0.01	$2 \times 10^4 - 4 \times 10^5$
11.1	$127-136$	0.19 ± 0.01	$2 \times 10^4 - 4 \times 10^5$
16.1	$205-241$	0.18 ± 0.02	$8 \times 10^3 - 5 \times 10^5$

$\Gamma : 0.3 - 17$ $Pr : 5 - 250$

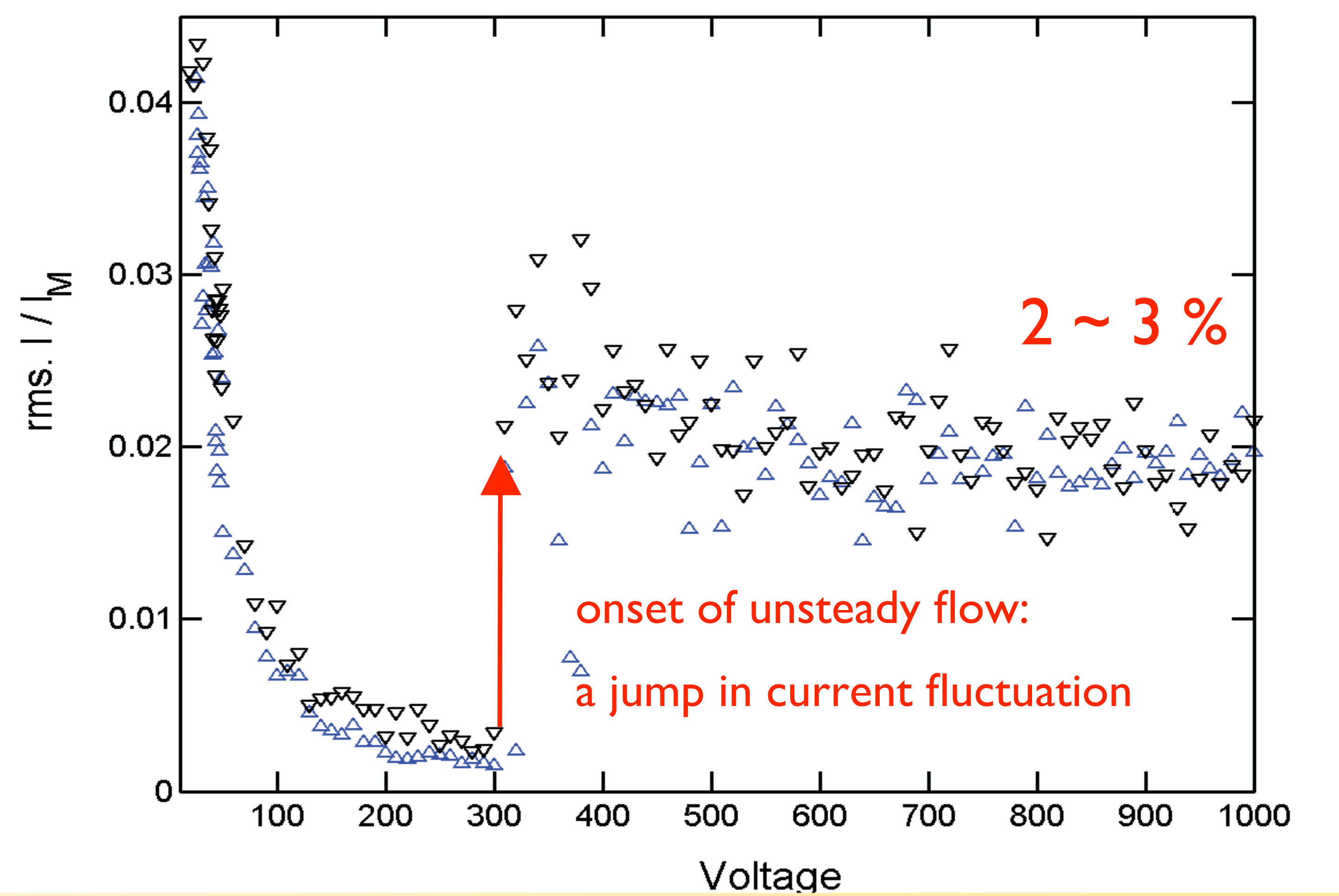
$< \gamma >= \mathbf{0.20 \pm 0.03}$ *or*

$< \gamma >= \mathbf{0.25 \pm 0.02}$

$Ra > 100\ Ra_c$

Charge Transport Fluctuations

normalized current fluctuation

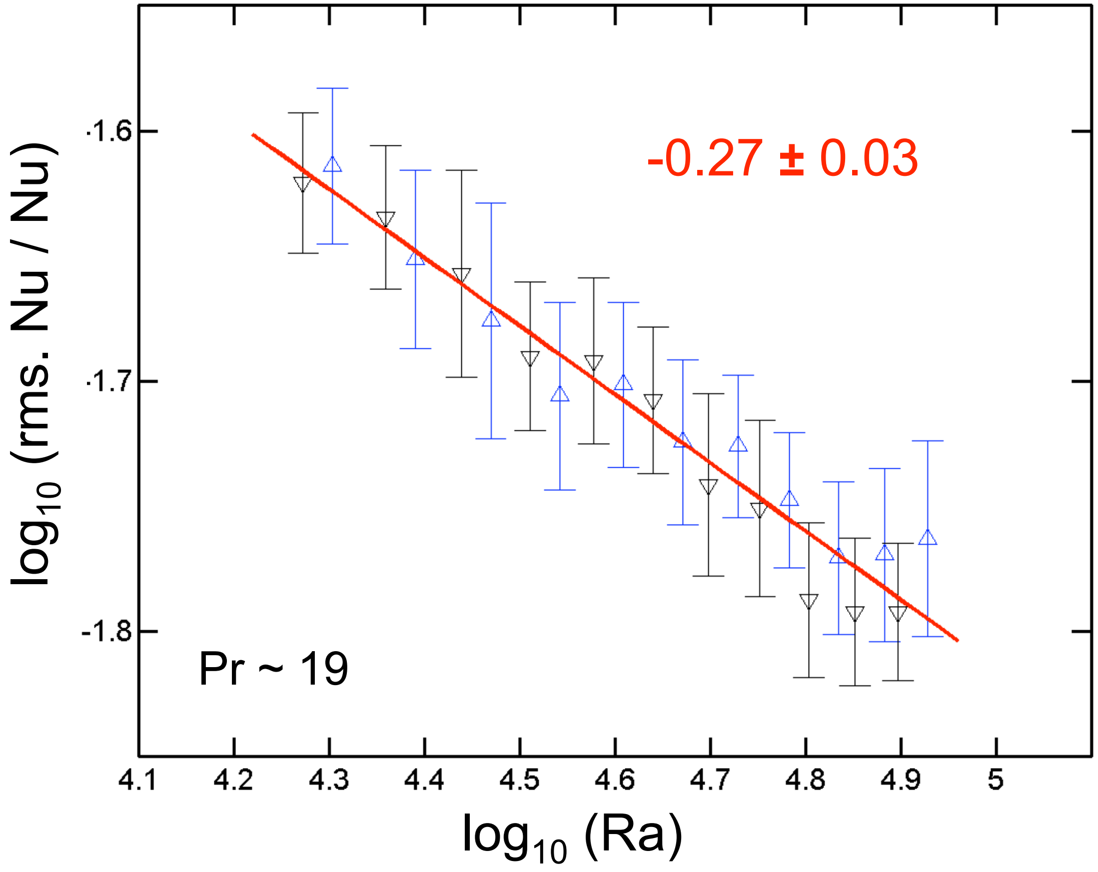


2 ~ 3 %

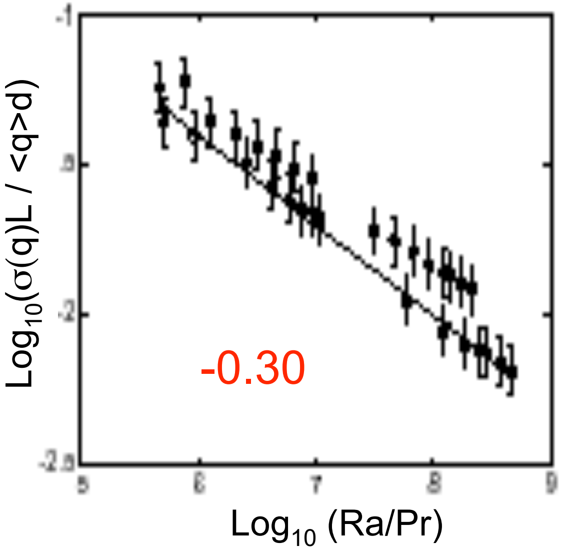
onset of unsteady flow:

a jump in current fluctuation

Scaling-law between Normalized Nu-fluctuation and Ra



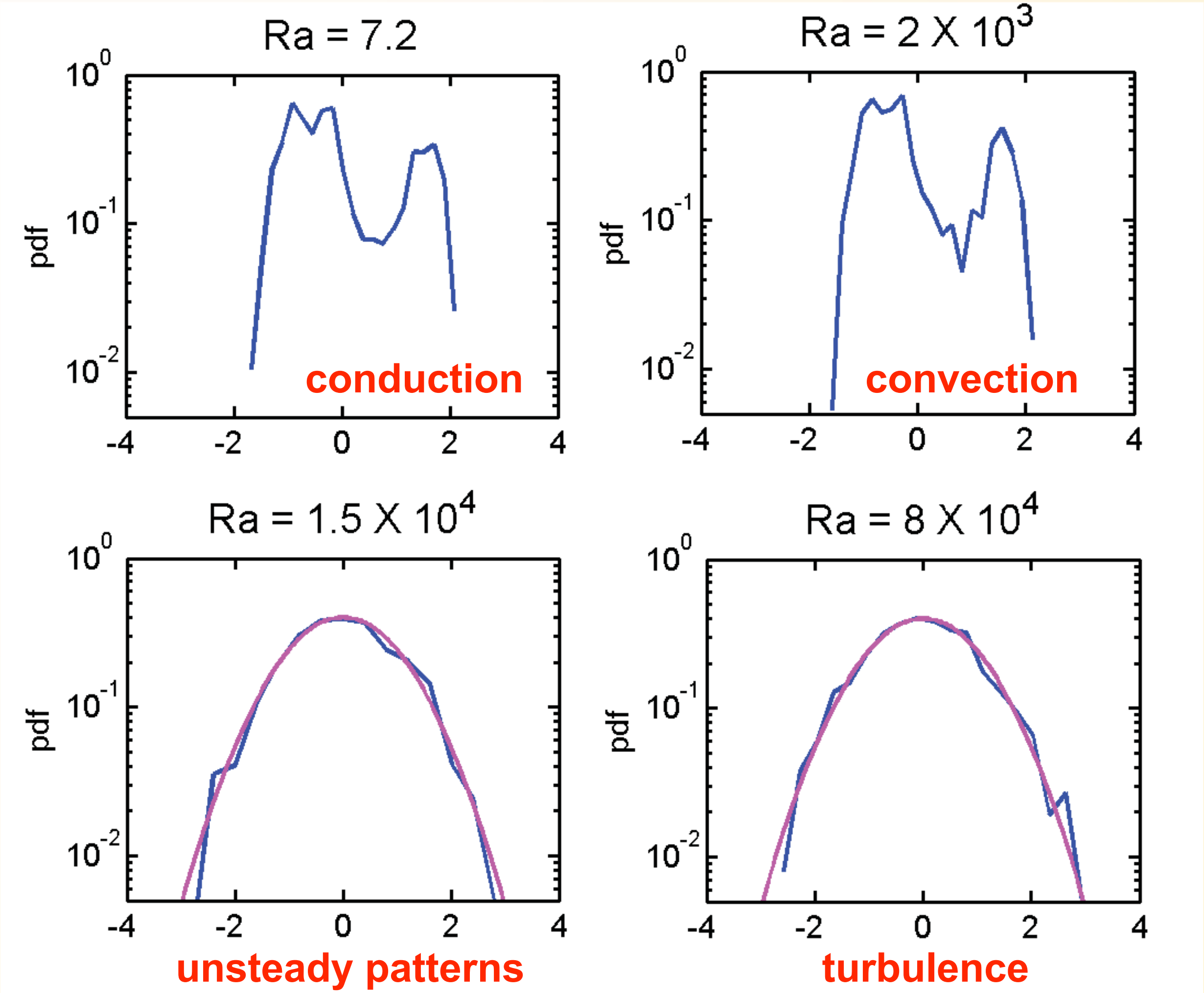
turbulent thermal convection



S. AUMAÎTRE and S. FAUVE

Europhys. Lett.,62 (6) p.822 (2003)

Probability Density Functions of $\frac{I - \langle I \rangle}{\sigma(I)}$



Pr ~ 19

α ~ 0.52

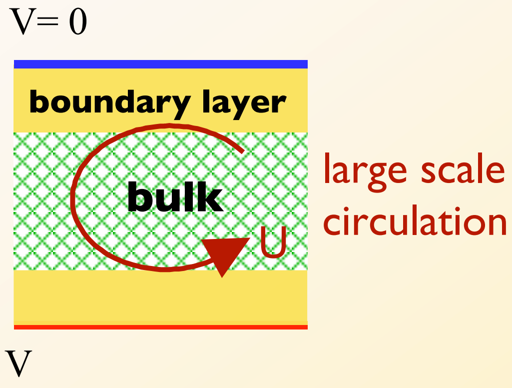
Gaussian distribution

“soft turbulence” at Rayleigh numbers ~ $2X10^4$

Scaling theory applied to smectic electroconvection

Global kinetic dissipation $$\epsilon_{\mathbf{u}} = \langle \nu (\nabla \mathbf{u})^2 \rangle = \frac{\nu^3 \mathcal{R}\mathcal{P}^{-2}(\mathrm{Nu}-1)}{\ln(1/\alpha)(r_o^2 - r_i^2)(r_o - r_i)^2}$$

Global electric potential dissipation $$\epsilon_{\psi} = \langle \sigma (\nabla \psi)^2 \rangle = \frac{2\sigma V^2 \mathrm{Nu}}{\ln(1/\alpha)(r_o^2 - r_i^2)}$$



$$\epsilon_{\mathbf{u}} = \epsilon_{\mathbf{u}}^{bulk} + \epsilon_{\mathbf{u}}^{BL}$$

$$\epsilon_{\psi} = \epsilon_{\psi}^{bulk} + \epsilon_{\psi}^{BL}$$

- $\epsilon_{\mathbf{u}} \sim \epsilon_{\mathbf{u}}^{BL}, \ \epsilon_{\psi} \sim \epsilon_{\psi}^{BL}$

  $$Nu \sim F(\Gamma)\ Ra^{1/4}\ Pr^{1/8}$$

- $\epsilon_{\mathbf{u}} \sim \epsilon_{\mathbf{u}}^{bulk}, \ \epsilon_{\psi} \sim \epsilon_{\psi}^{BL}$

  $$Nu \sim F(\Gamma)\ Ra^{1/5}\ Pr^{1/5}$$

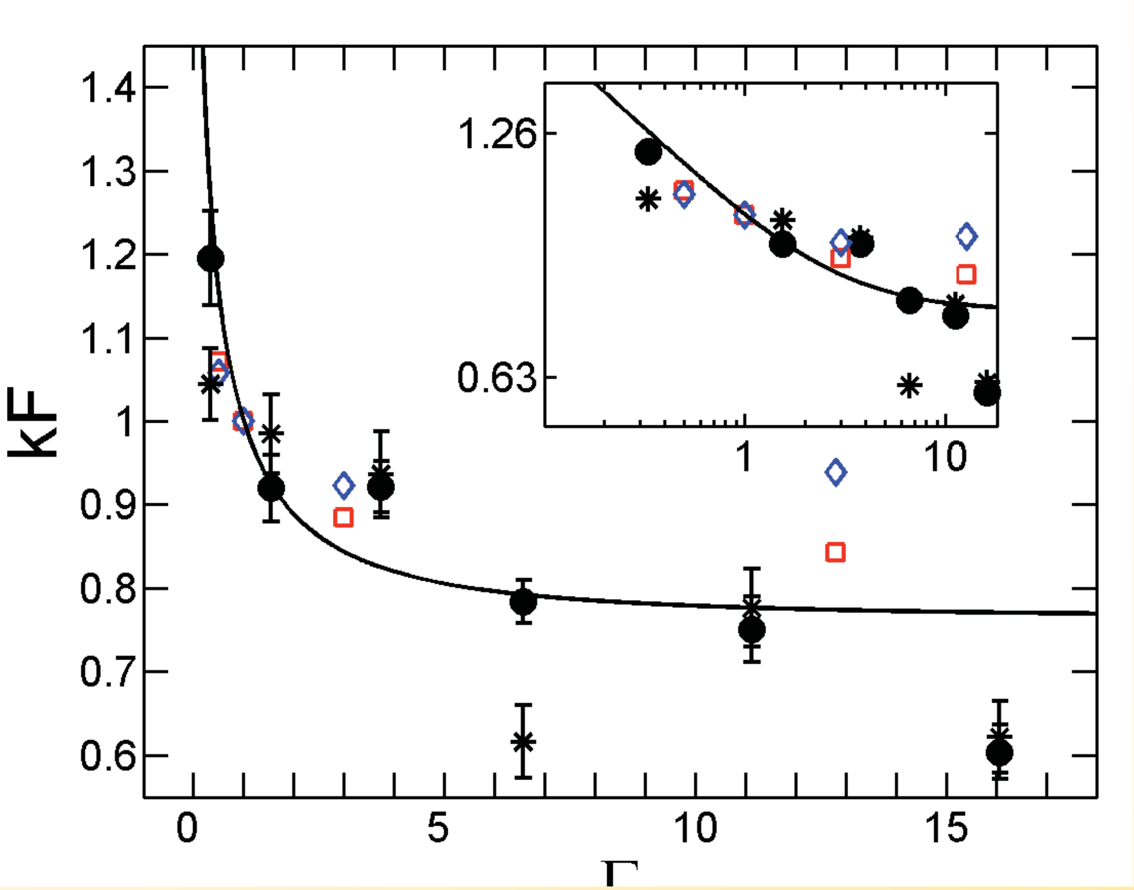
$$F(\Gamma) = \frac{\Gamma + \pi}{\pi} \log \left[\frac{\Gamma + 2\pi}{\Gamma} \right]$$

S. Grossmann & D. Lohse, *J. Fluid Mech.*, 407, 27 (2000)

Peichun Tsai, Zahir Daya, & Stephen Morris, *Phys. Rev. Lett.*, (2004)

Aspect Ratio Dependence of Charge Transport

the prefactor in power-law: $Nu \sim F(\Gamma)\ Ra^{\gamma}\ Pr^{\beta}$



Plot Nu / (Ra$^{\gamma}$ Pr $^{\beta}$)

γ, β : from Exp. results of the best fits ~ 1/5

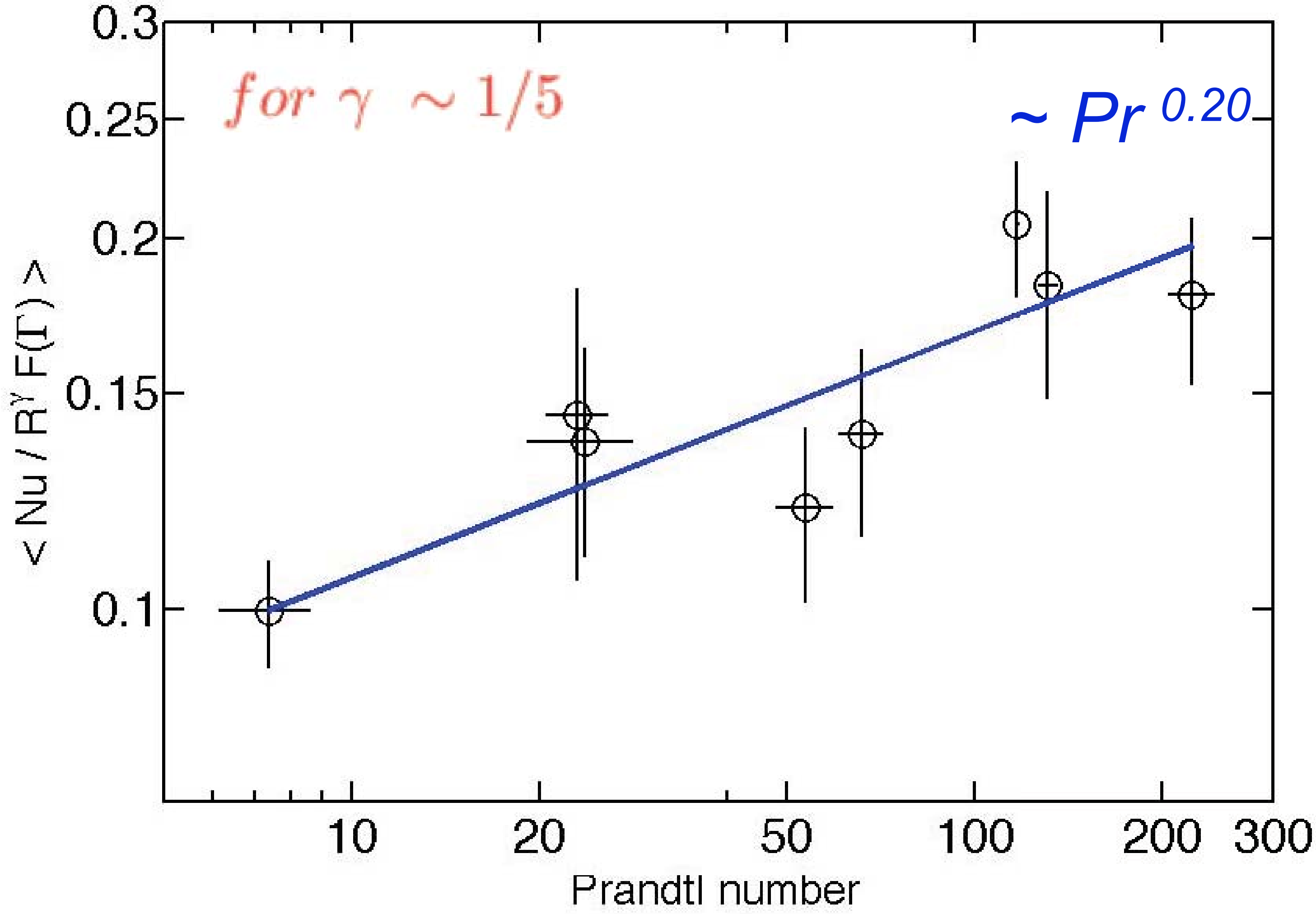
Theoretical prediction of *F*(Γ)

$$F(\Gamma) = \frac{\Gamma + \pi}{\pi} \ln\left(\frac{\Gamma + 2\pi}{\Gamma}\right)$$

● **Annular electroconvection** P.Tsai, Z. A. Daya, & S. W. Morris, Phys. Rev. E, **72**, 046311 (2005)

□ **Rayleigh-Bénard convection** X. Xu, K. Bajaj, & G. Ahlers, Phys. Rev. Lett. **84**, p.4357 (2000)

Electric Nu vs *Pr*



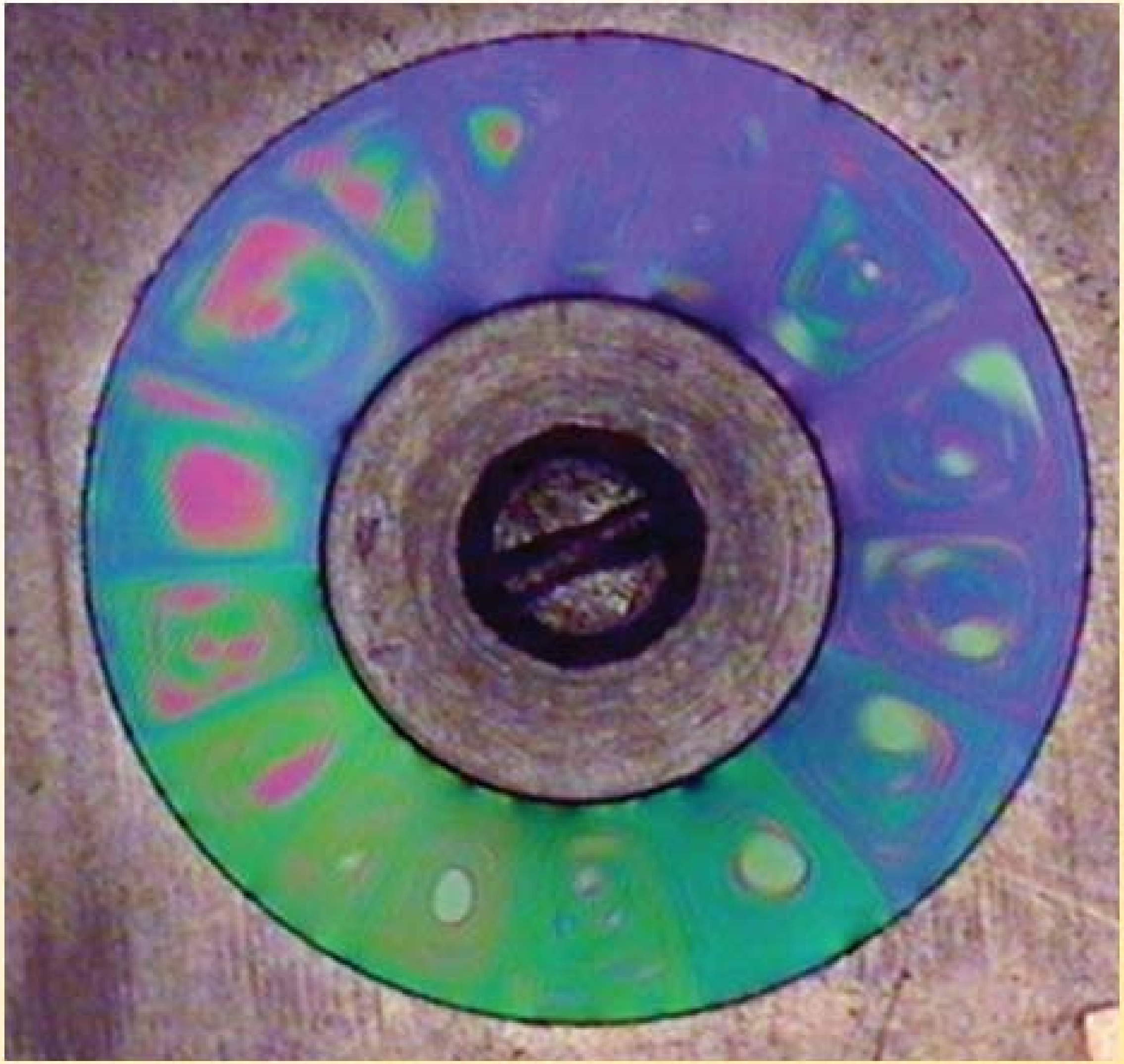
Flow Dynamics

@ 100 volts

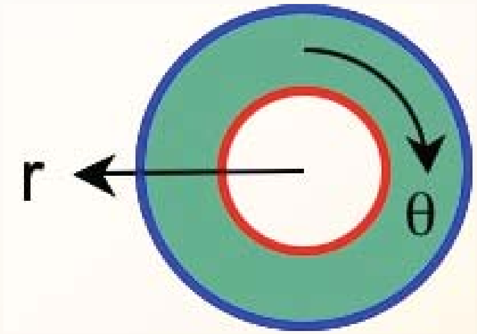
@ 250 volts

Vortex patterns at different applied voltages in a thin film, identified by non-uniform thickness

A Movie of Convective Patterns



Direct Numerical Simulation



2D Spectral method

θ : Fourier Galerkin

r : Chebyshev Collocation

Streamfunction-vorticity

Navier-Stokes $\rho(\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}) = -\nabla P + \eta\nabla^2\mathbf{u} + q\mathbf{E}$

Charge Continuity $\partial_t q + \nabla\cdot\mathbf{J} = 0, \quad \mathbf{J} = \sigma\mathbf{E} + q\mathbf{u}$

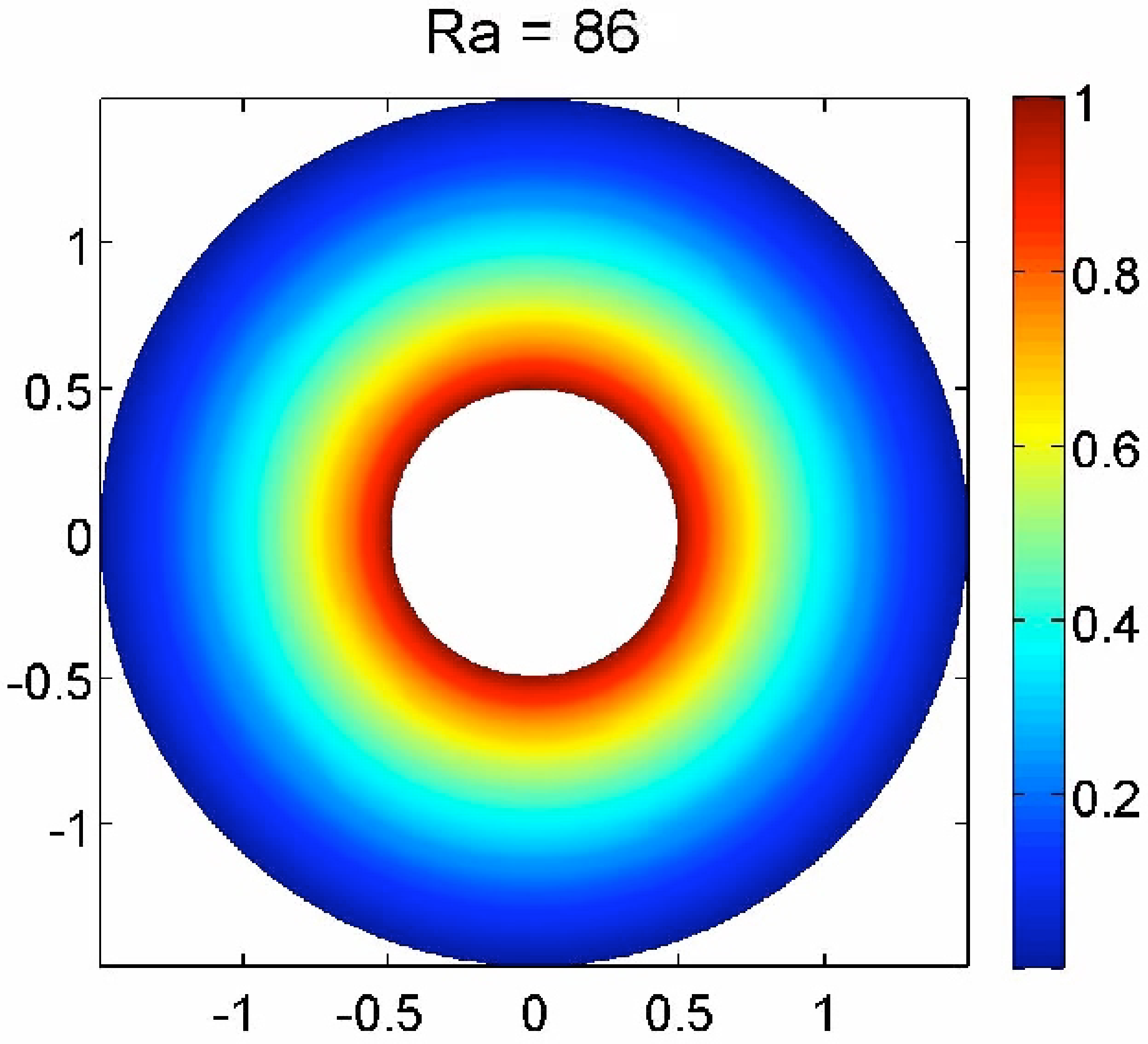
Mass Continuity $\nabla\cdot\mathbf{u} = 0$

Maxwell's Equation $\nabla^2\psi = 0, \quad q = -2\epsilon_0\partial_z\psi|_{z=0^+}$

Nonlocal, coupled relation

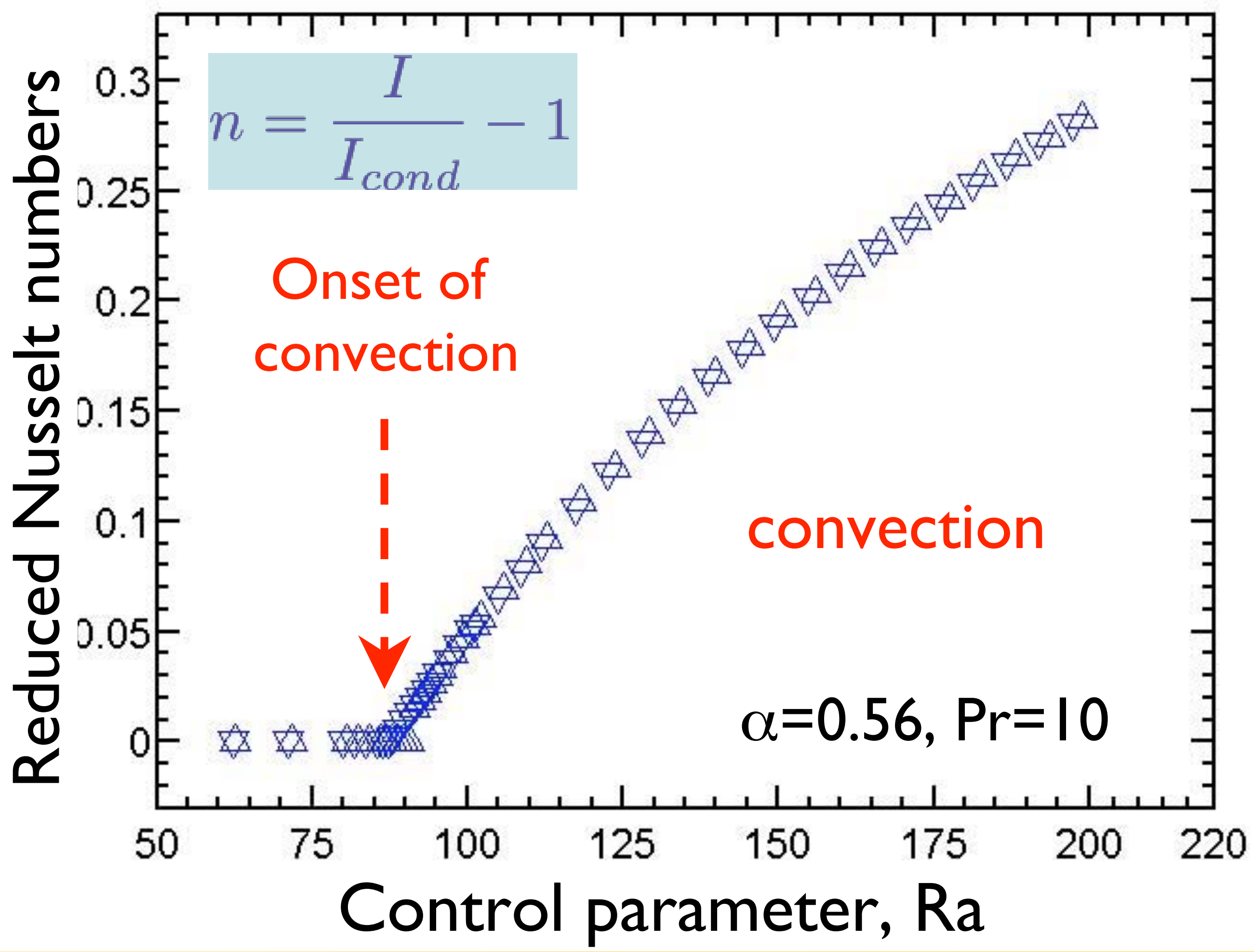
Initial condition: zero **u**, tiny random noise for ψ, for V < Vc

Electric Potential on a Film

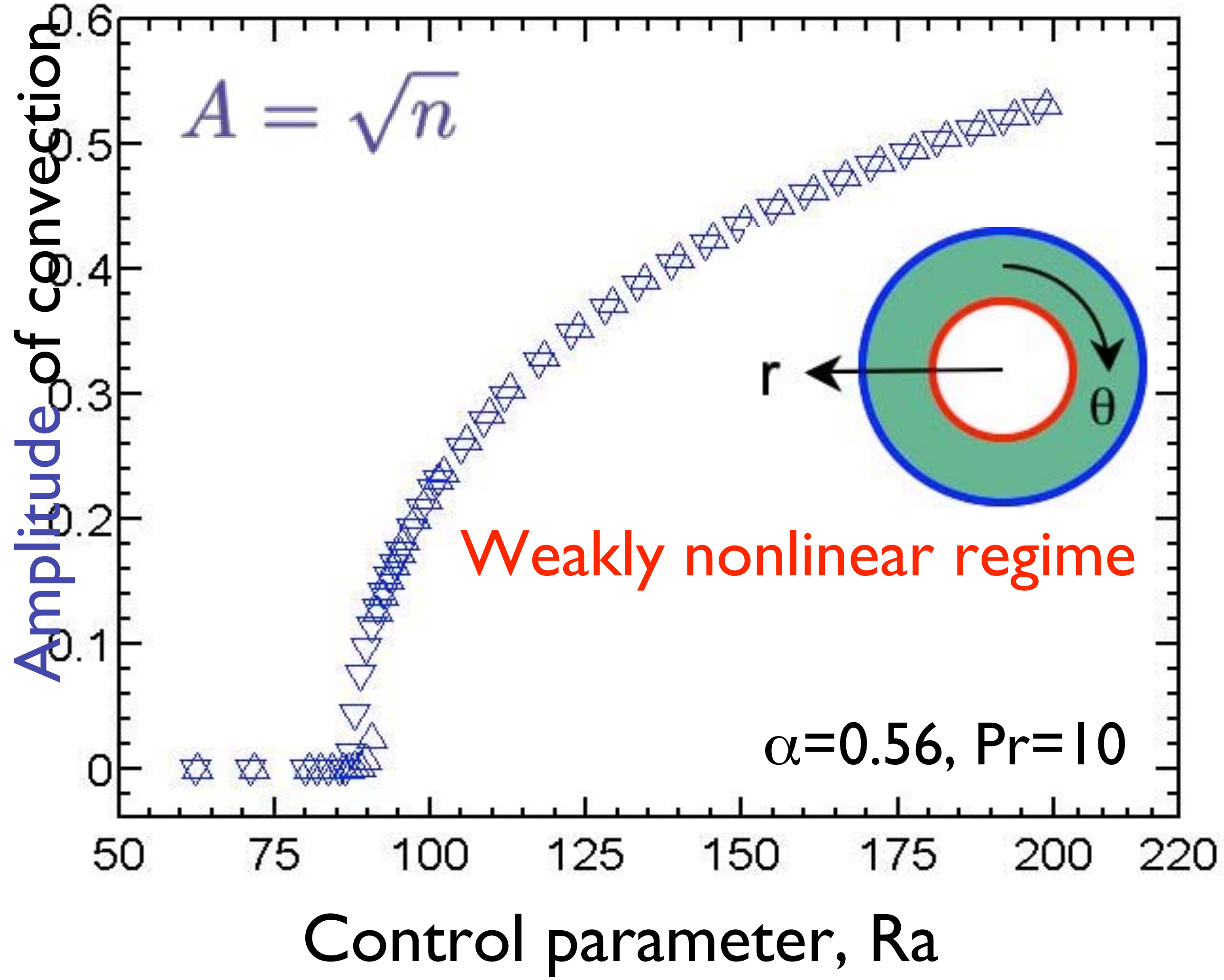


Numerical Data

in the Weakly Nonlinear Regime

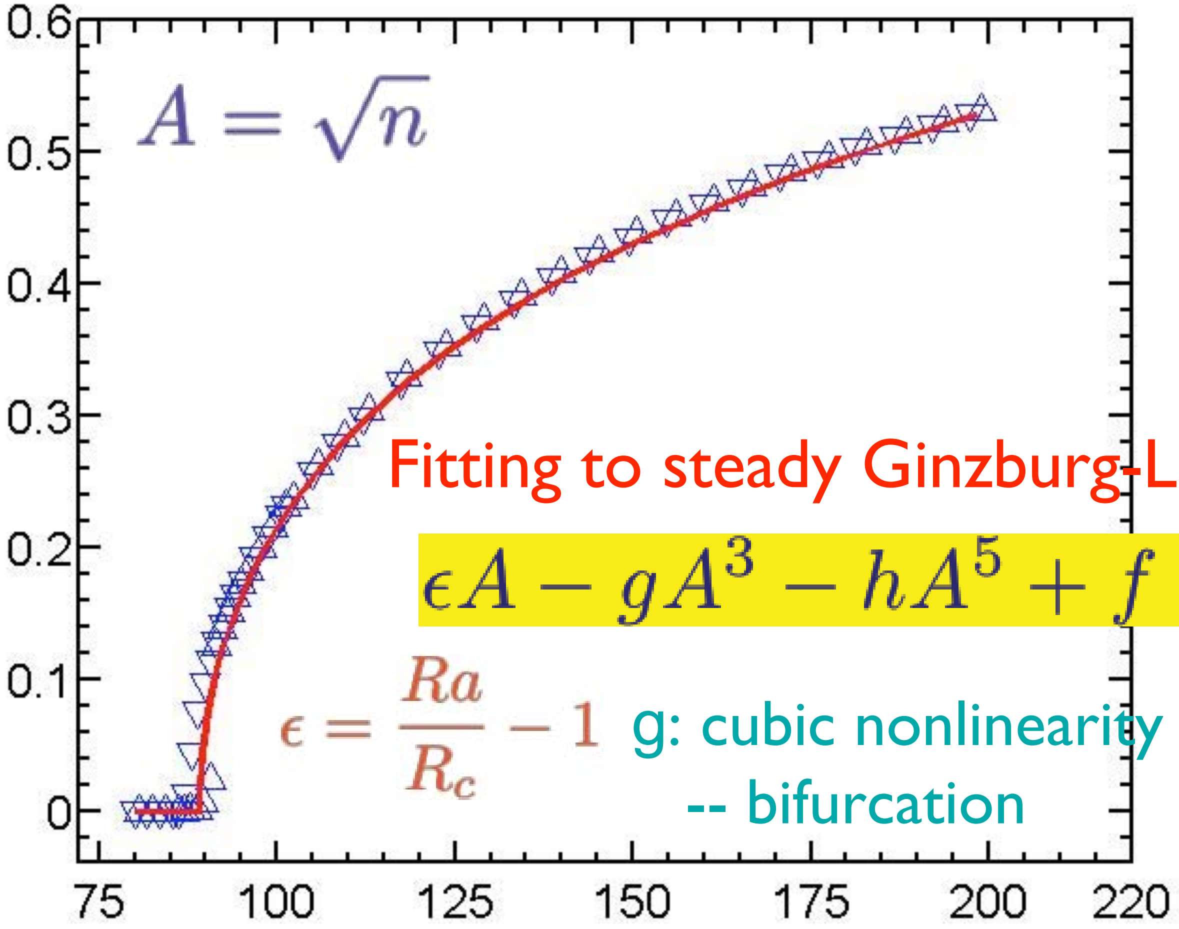


Numerical Data



Numerical Data

Amplitude of convection



$$A = \sqrt{n}$$

Fitting to steady Ginzburg-Landau Eqn.

$$\epsilon A - gA^3 - hA^5 + f = 0$$

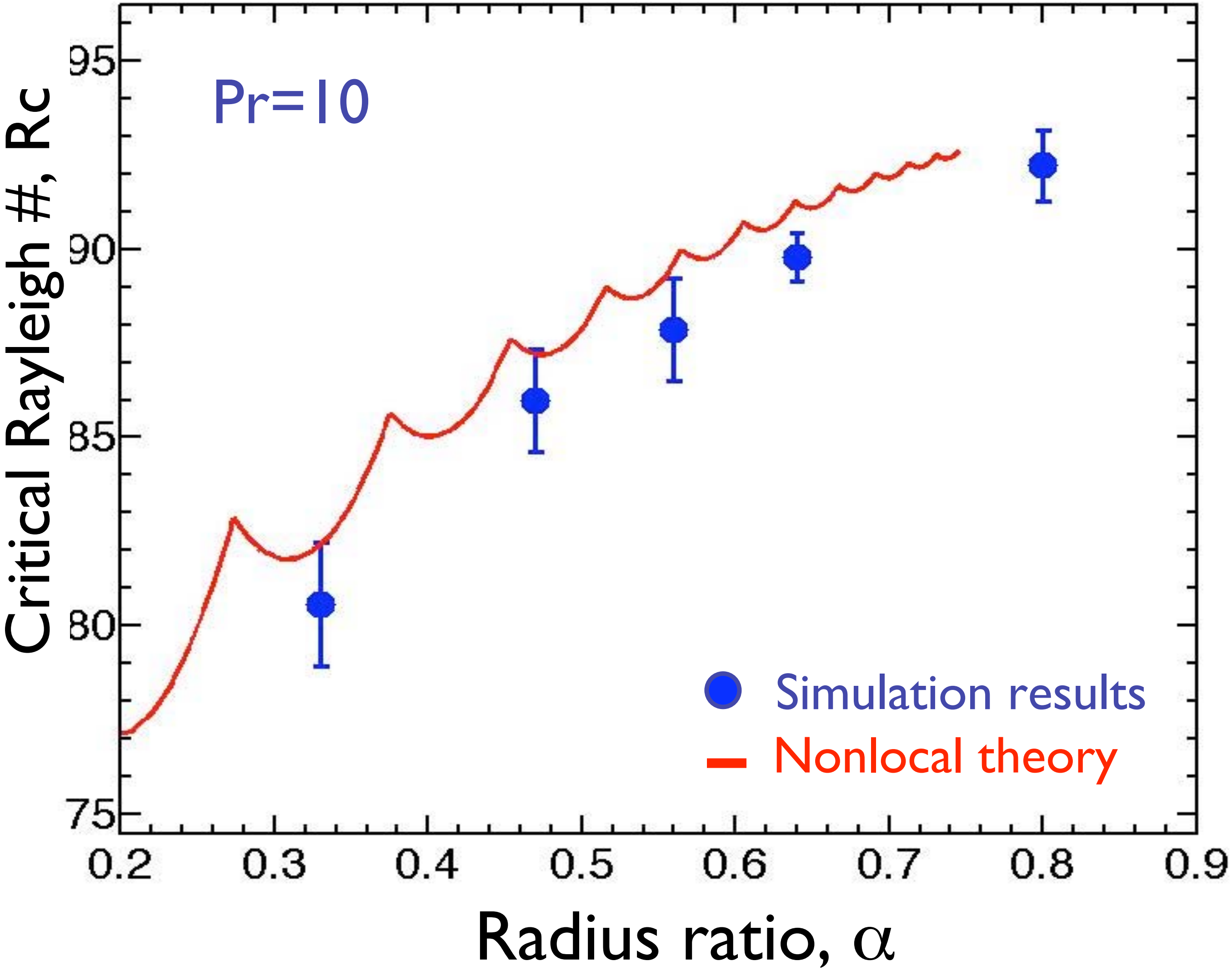
$$\epsilon = \frac{Ra}{R_c} - 1$$

g: cubic nonlinearity

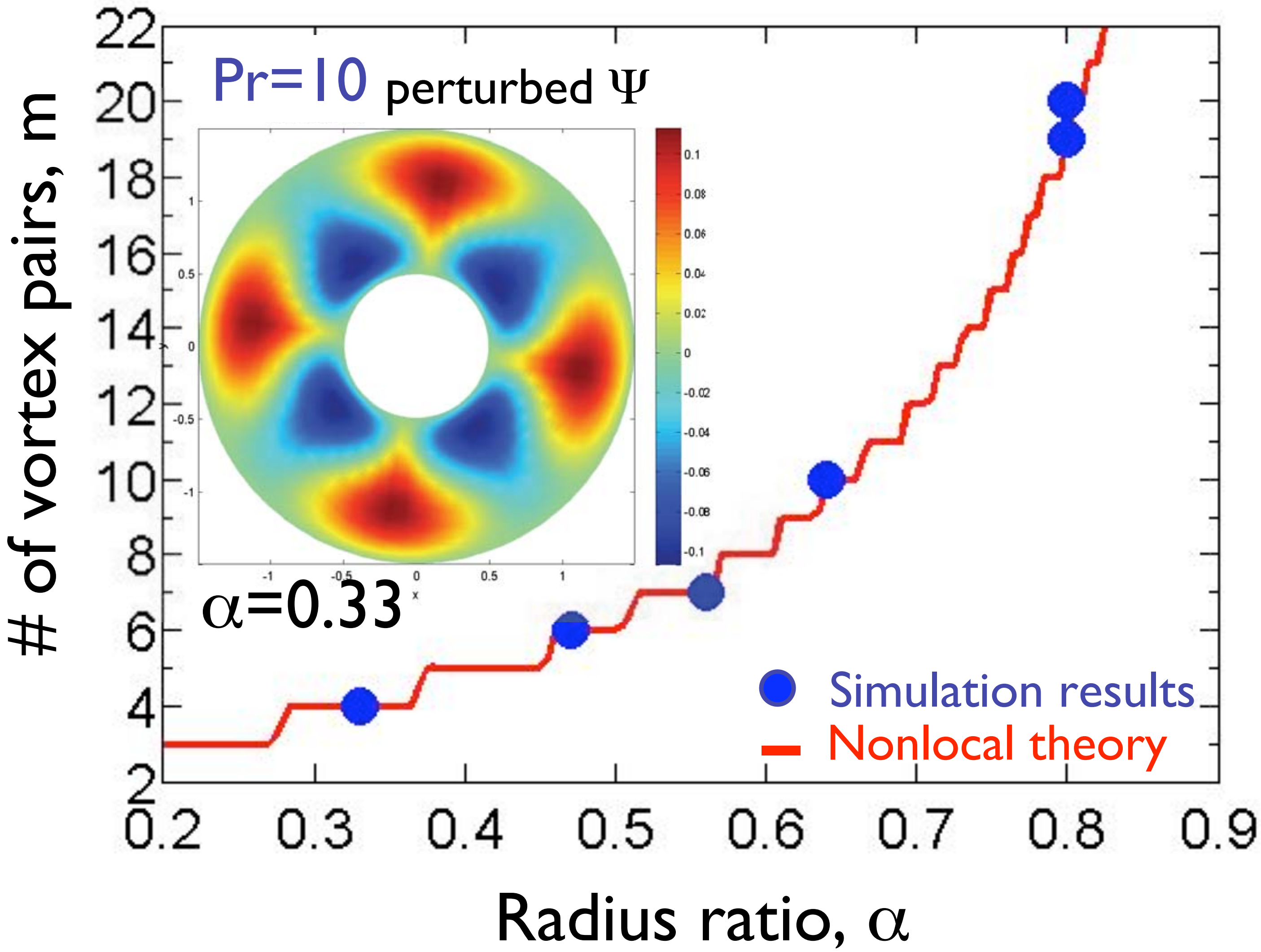
-- bifurcation

Control parameter, Ra

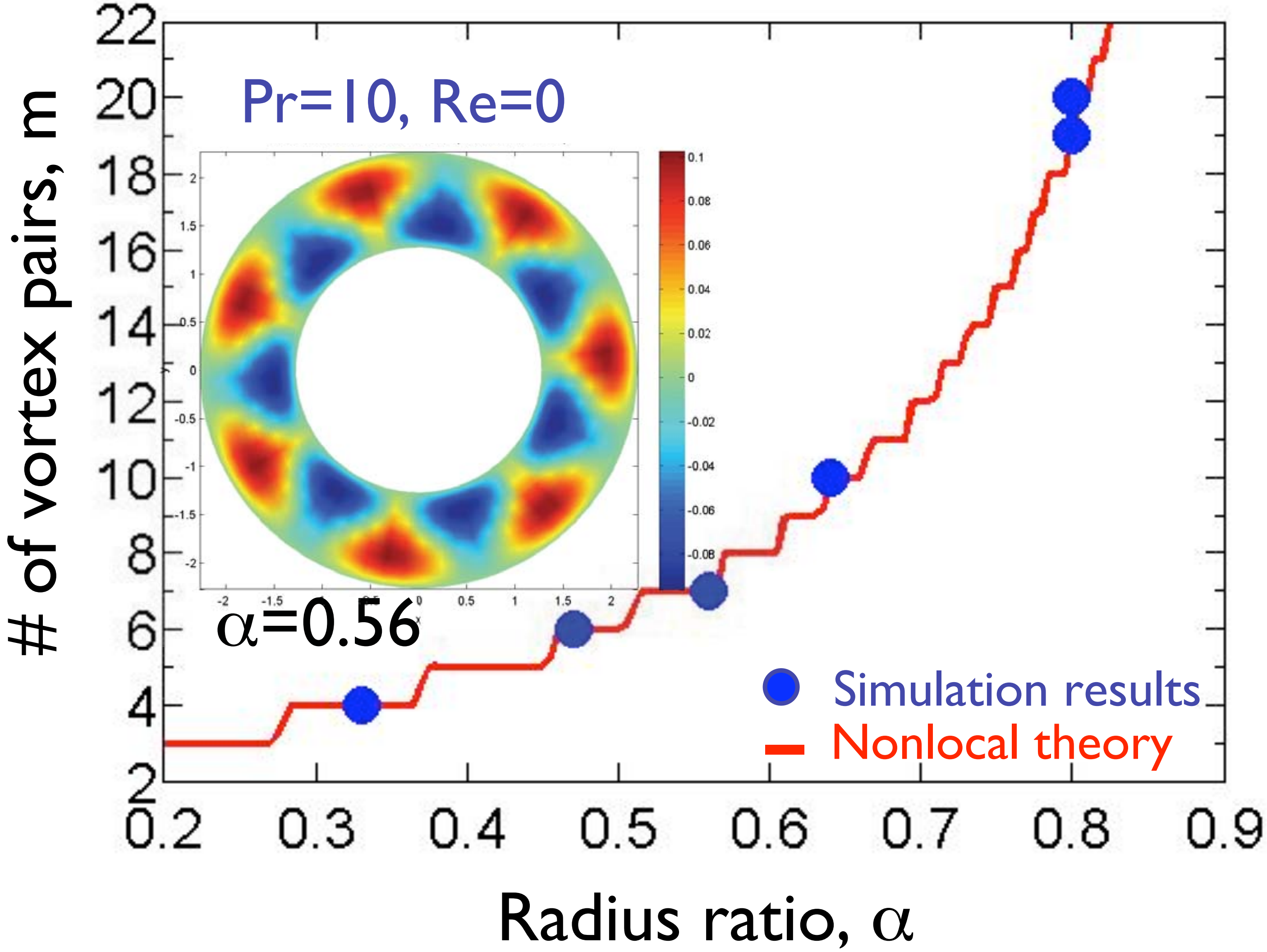
Aspect-ratio Dependence of Rc



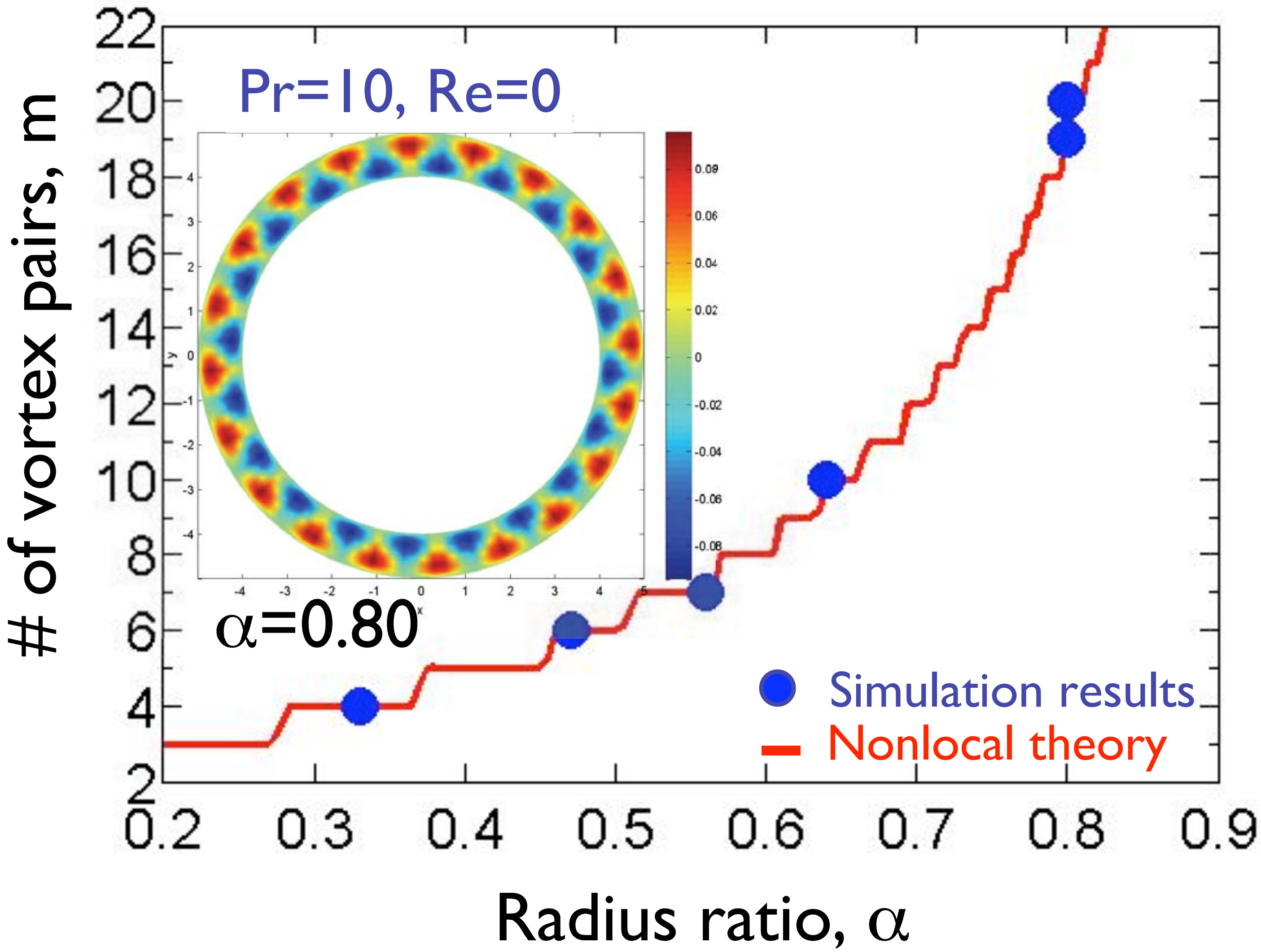
Aspect-ratio Dependence of m



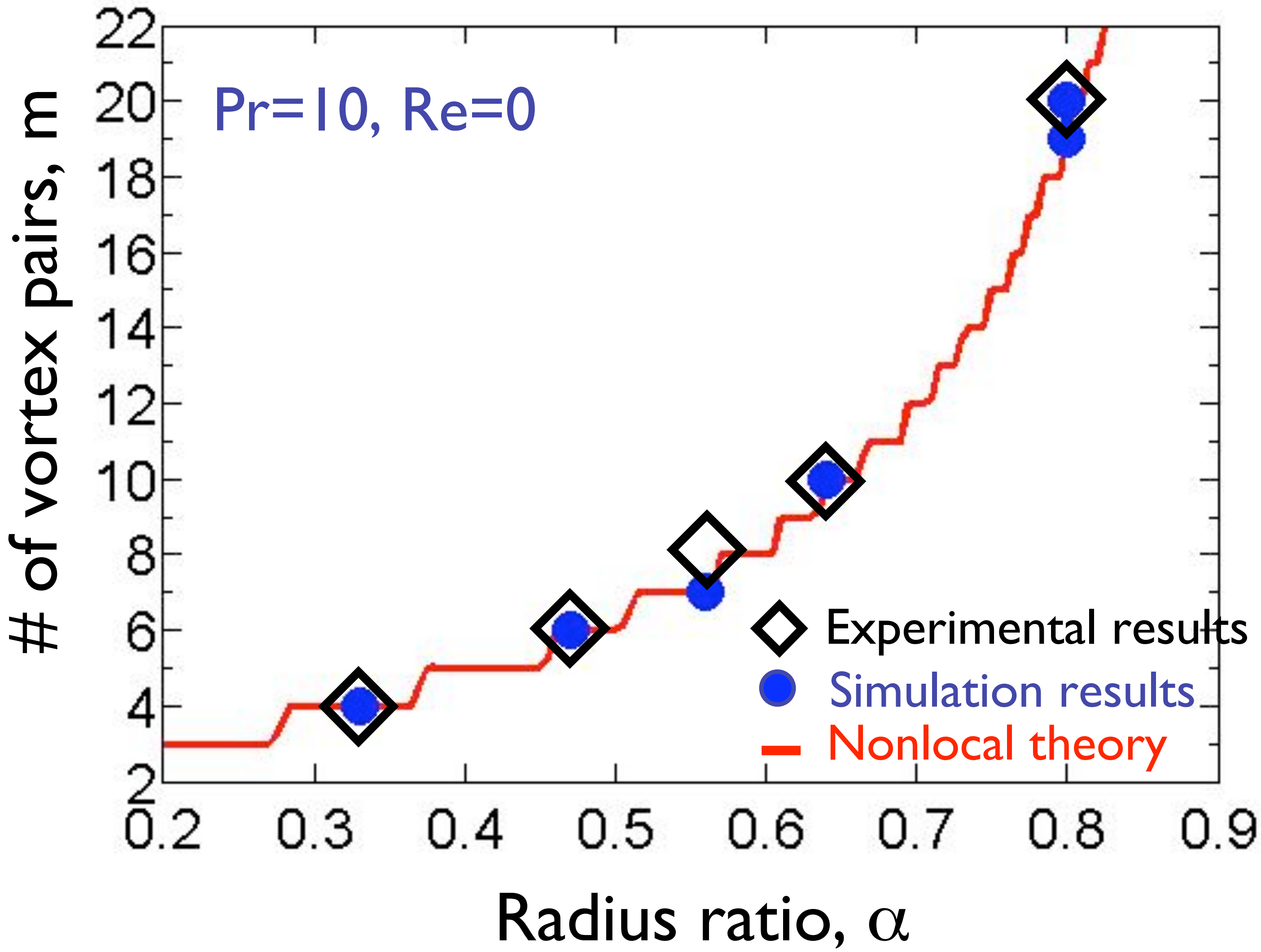
Aspect-ratio Dependence of m



Aspect-ratio Dependence of m

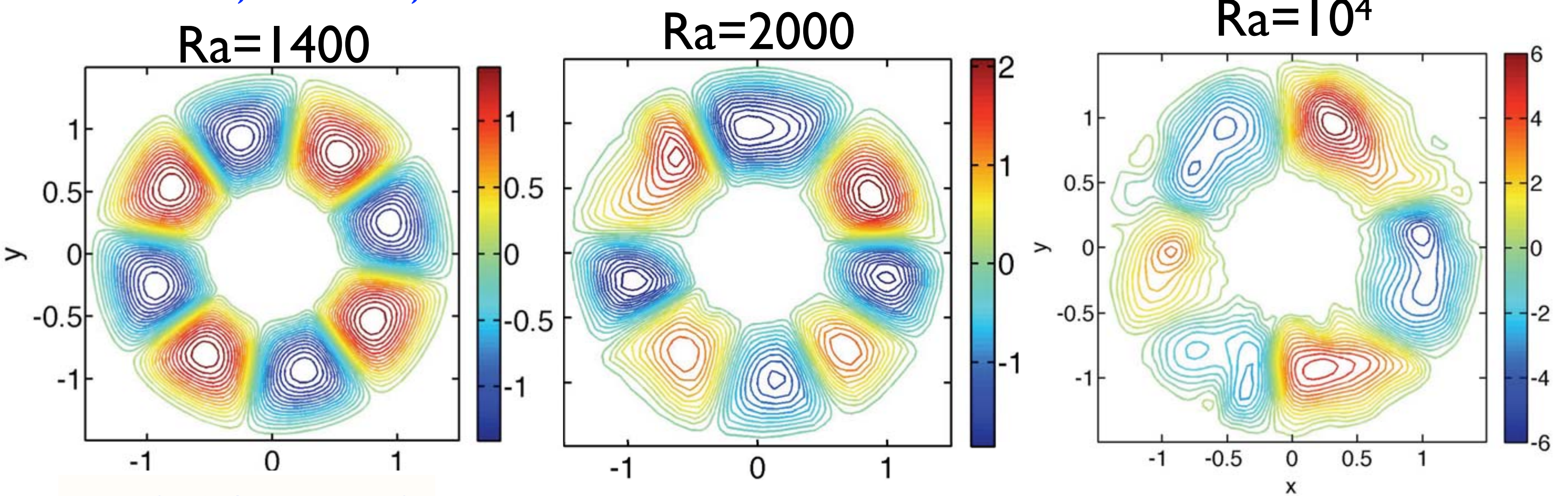


Aspect-ratio Dependence of m

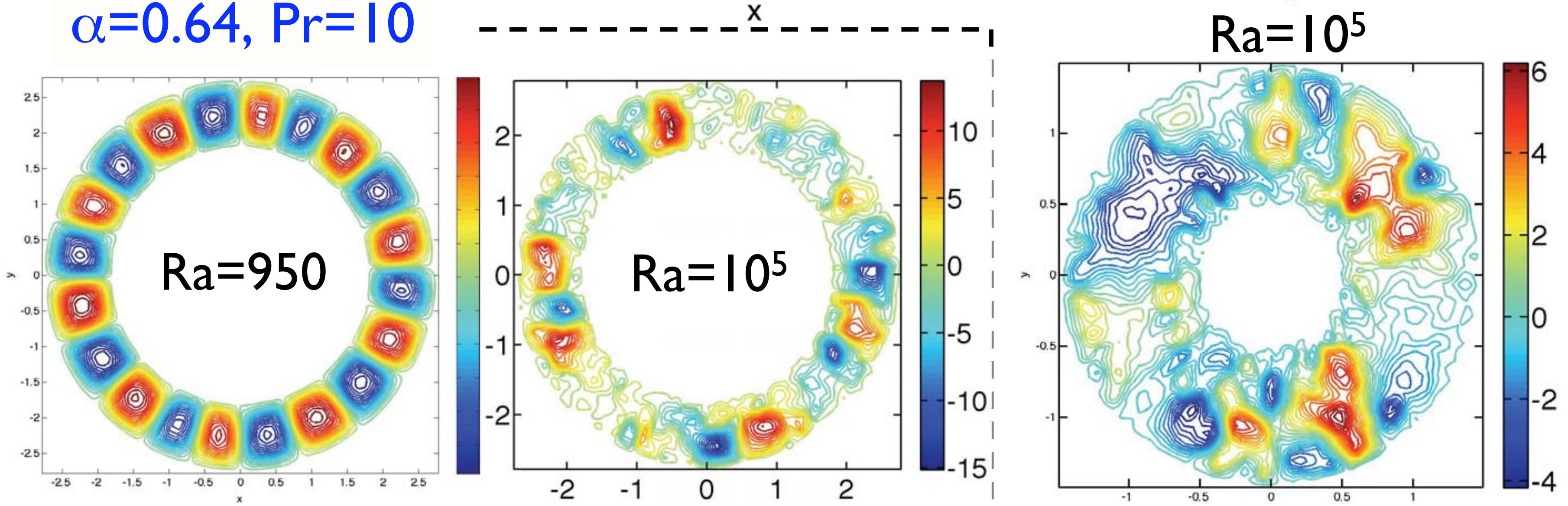


Route to Turbulence

α=0.33, Pr=10, Re=0: Iso-streamfunction



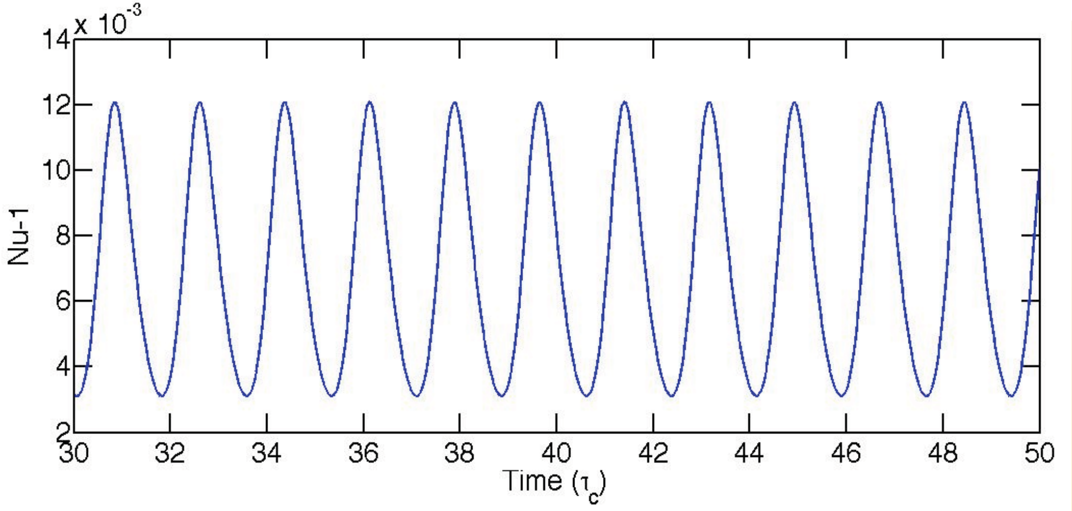
α=0.64, Pr=10



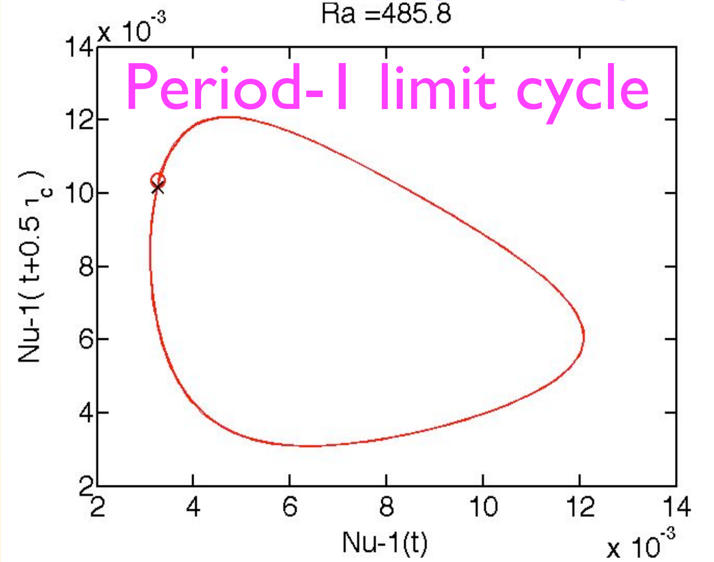
Route to Chaos

Shear Effects

Ra~486 α=0.47, Pr=16.3, Re=0.8



Re-construction of phase space

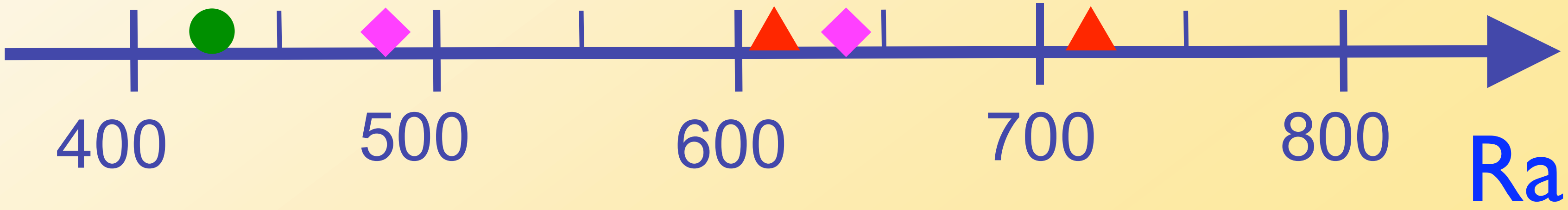


Period-1 limit cycle

Onset of convection

◆ Hopf Bifurcation

▲ Mode Change



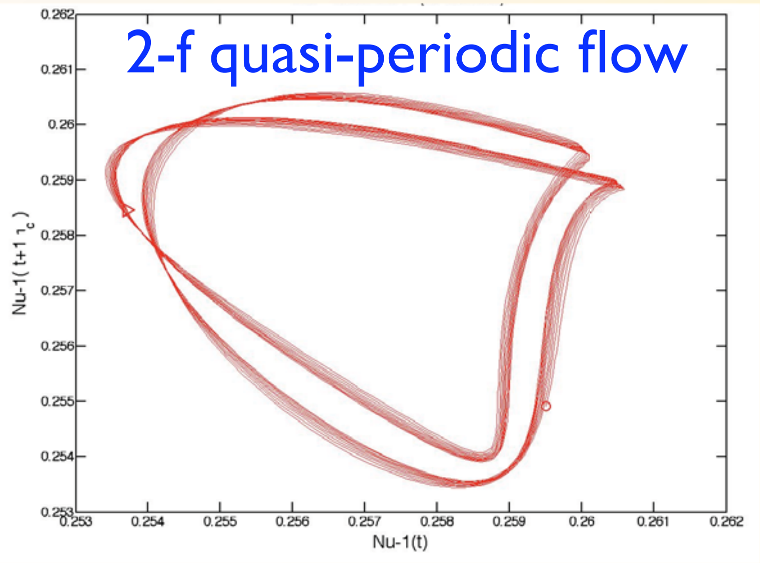
Route to Chaos

Shear Effects $\alpha=0.47$, Pr=16.3, Re=0.8

◆ Hopf Bifurcation

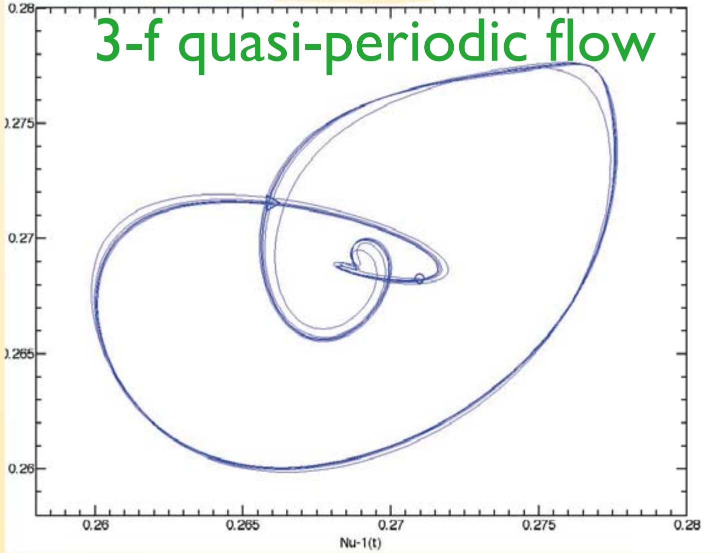
▲ Mode Change

Ra~639



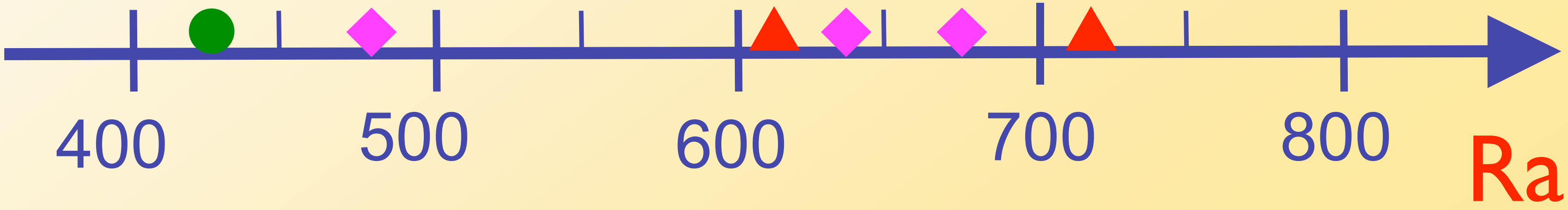
2-f quasi-periodic flow

Ra~673



3-f quasi-periodic flow

Onset of convection



The Ruelle-Takens-Newhouse scenario

Computational Resolution
in High Ra Number Regime

To resolve the Kolmogorov dissipation scale, η

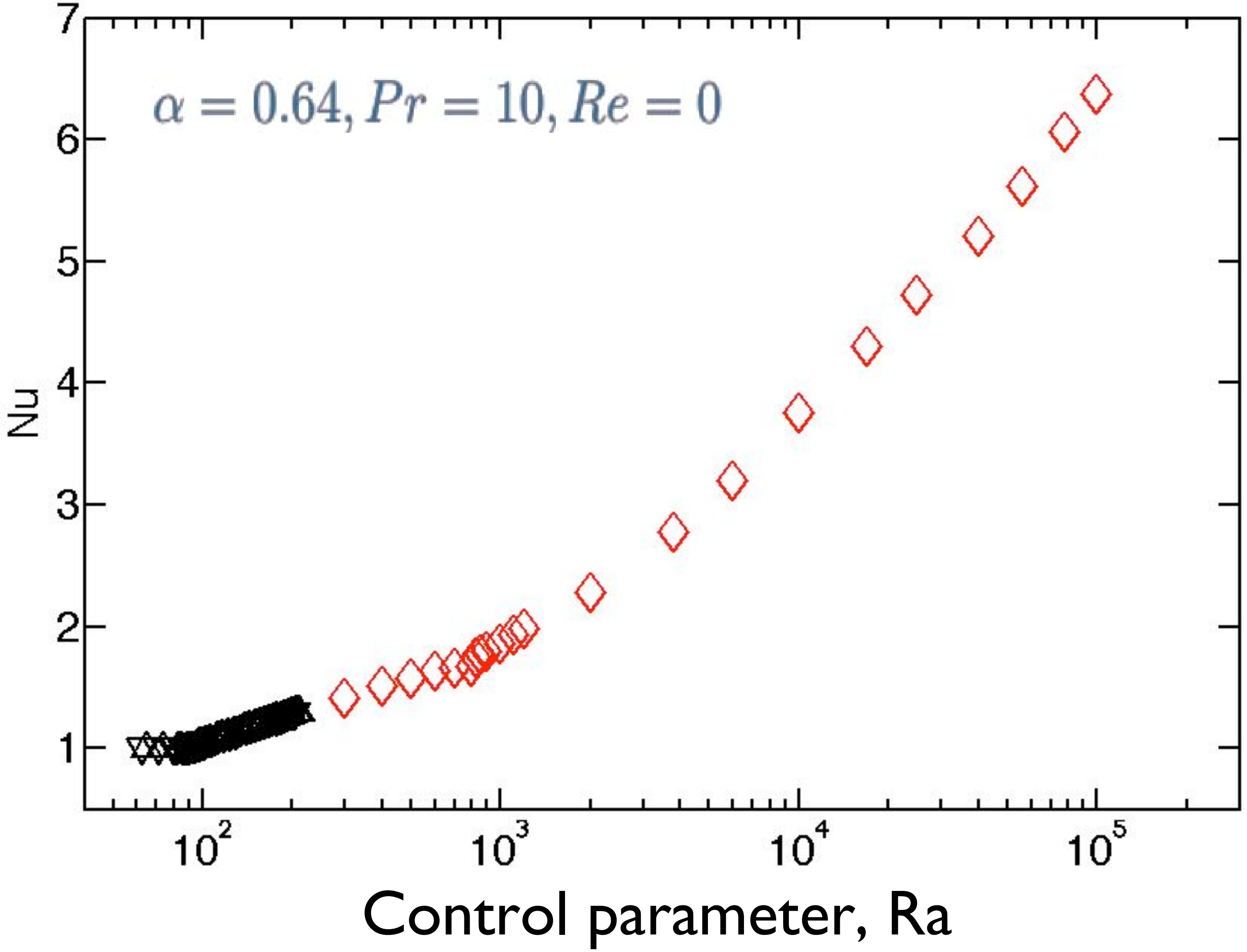
$$\frac{\eta}{d} = \frac{1}{d}\left(\frac{\nu^3}{\epsilon_u}\right)^{1/4} \sim \left(\frac{Pr^2}{RaNu}\right)^{1/4}, Pr > 1$$

Grid points: 129 azimuthally, 40 radially for Ra~10^6

Long computational time

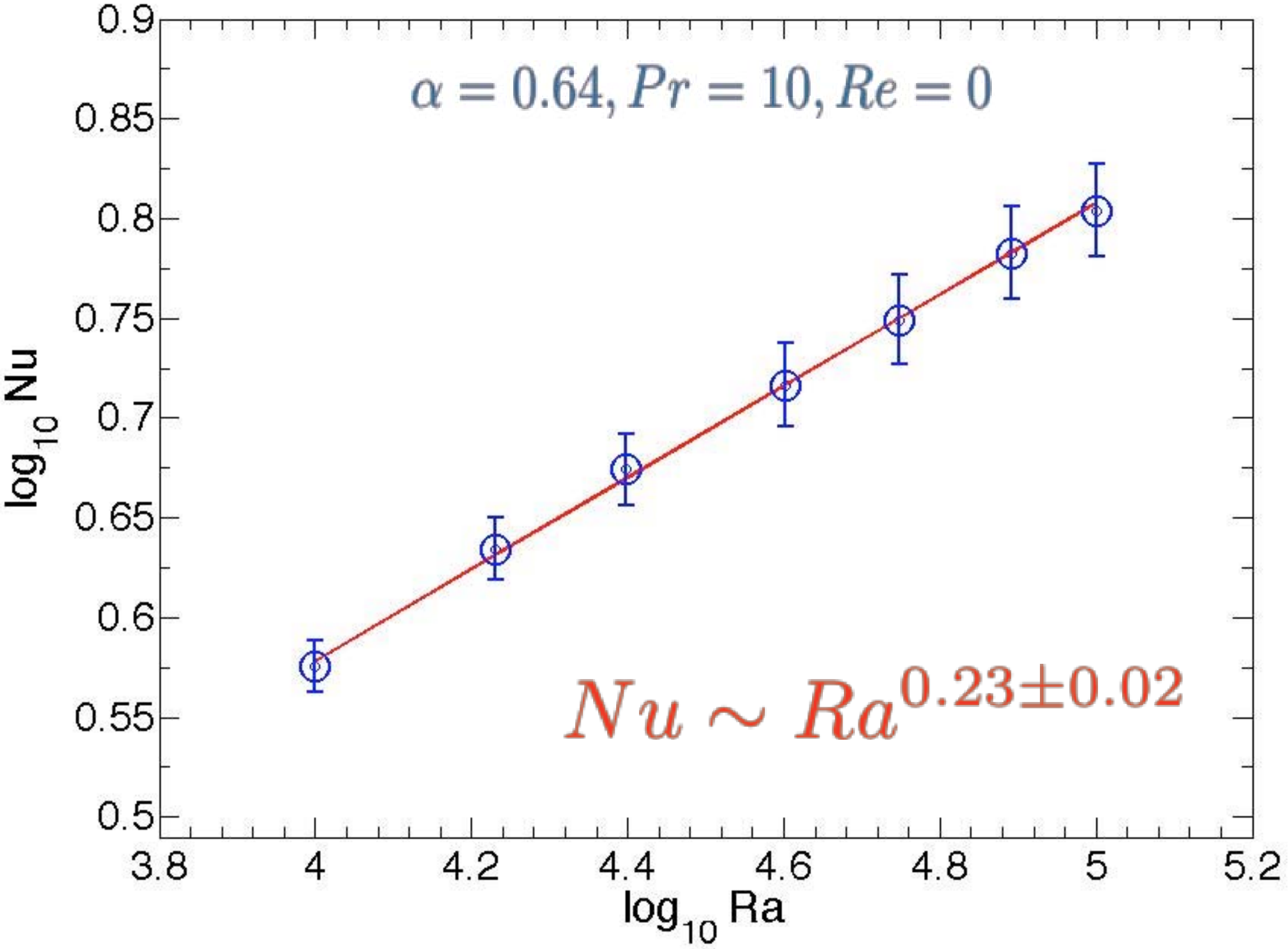
Currently, Ra=10^5, dt ~ 10^{-7} τ_c (charge relaxation time)

Numerical Data of Nu vs. Ra in Electroconvection



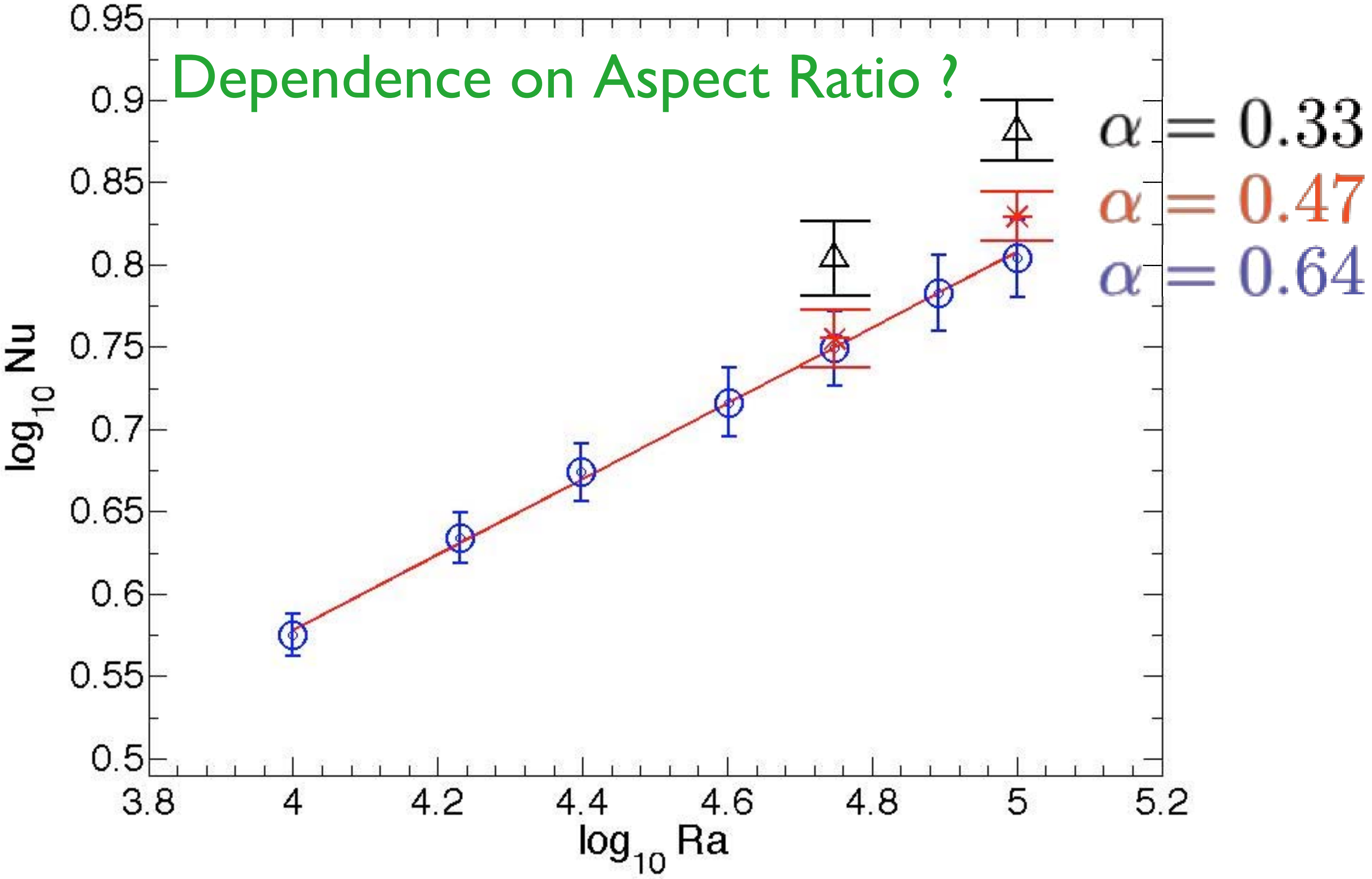
Turbulent Convection

Preliminary Numerical Data



Turbulent Convection

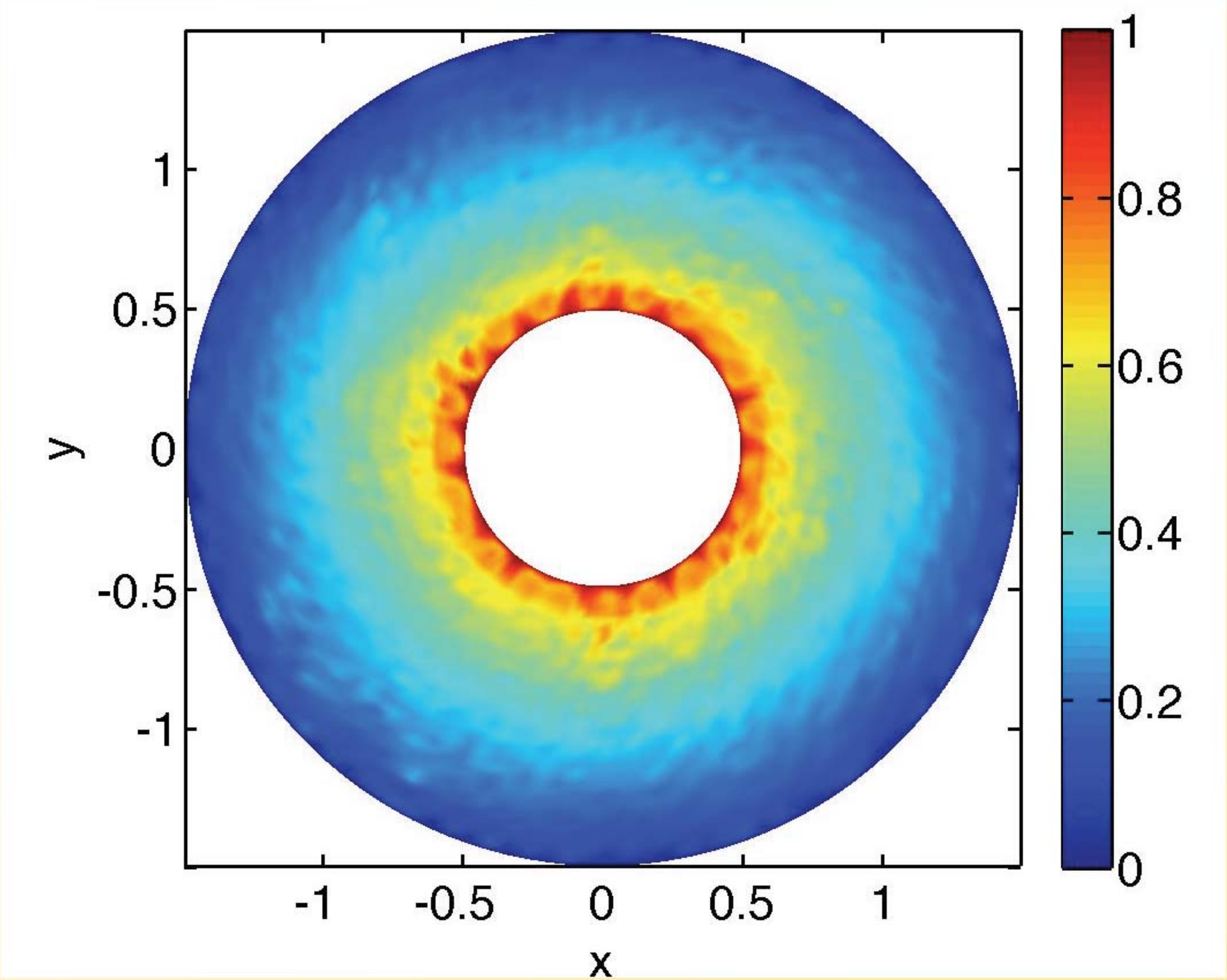
Preliminary Numerical Data



Dependence on Aspect Ratio ?

Electric Potential at $Ra=10^5$

$\alpha=0.33$, $Pr=10$, $Re=0$



Summary

Experiment

Onset of convection:

geometrically dependent.

Onset of unsteady flow:

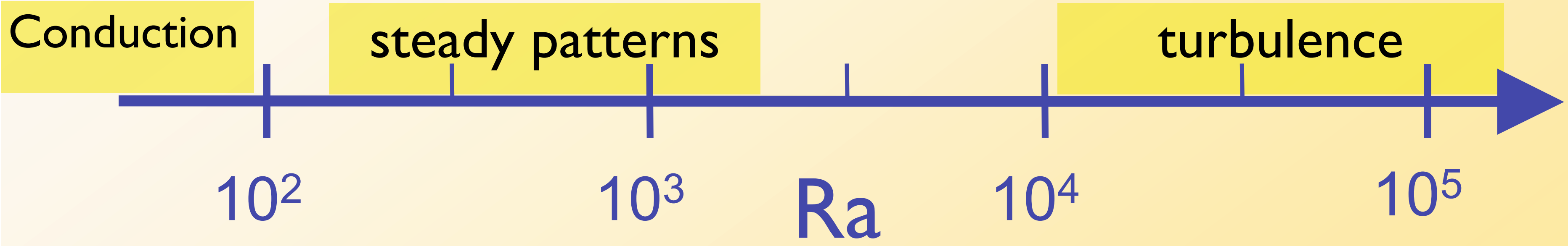
> 2% fluctuation/mean.

Soft turbulent regime:

- Gaussian PDF.
- Scaling-laws:

$Nu \sim Ra^{\gamma}$ local power laws

$\delta Nu / Nu \sim Ra^{-0.27 \pm 0.03}$



Simulation

Weakly Nonlinear Regime:

α, Pr, & Re affect bifurcations

Route to chaos:

α & Re effects

Convective turbulence

The End