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3D simulations of double-diffusion convection

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Direct Numerical Simulations of Three-Dimensional Thermohaline Fingering

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Abstract

Three-dimensional Direct Numerical Simulations (DNS) of the salt fingering regime are described and shown to be applicable to the ocean. At high resolution ($512 \times 512 \times 128$) for a low density ratio of $R_\rho = 1.2$, fingers appear spontaneously from smooth initial conditions. With small scale seeding, similar results are found in smaller calculations ($64 \times 64 \times 256$), allowing several cases to be simulated and trends with R_ρ to be determined. These show a rapid decrease in the net salt diffusivity, K_S as R_ρ increases from 1.2 to 2.0. Fingering layers form on interfaces during an early stage if R_ρ is small and there is a large-scale perturbation. The statistical state at late times for lower R_ρ is a sea of vertically oriented fingers without staircases and is independent of the initial condition. These trends are consistent with oceanic observations and support a widely use ocean parametrization of double diffusion.

Why is double diffusion important

- It is estimated that 44% of the ocean is potentially unstable to double diffusion.
  - Mostly subtropical Atlantic and Indian Oceans.
  - Mediterranean outflow
  - Underneath ice shelves.
- Increased mixing could modify ocean currents, in particular deep water currents. **Conveyor Belt**
- Affects formation of deep water in the Arctic.
- Incorporated into the ocean part of NCAR's Climate System Model in a parameterization by Large, Doney and McWilliams.

Nature of instability

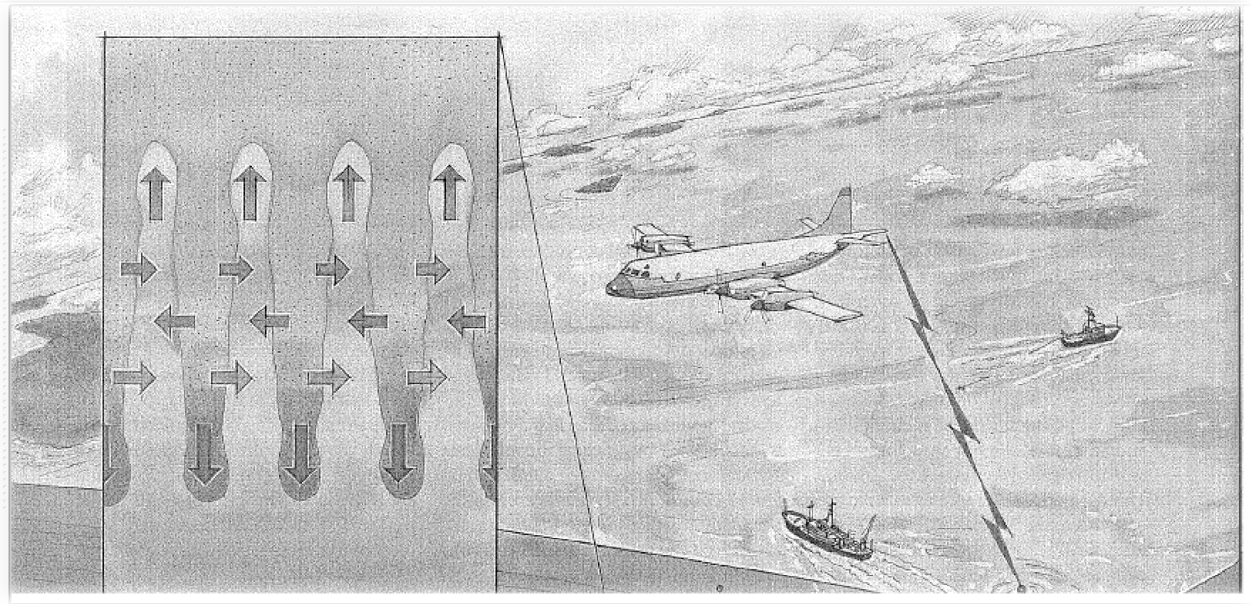
- Without diffusive terms, fluid is stably stratified.
  - Typically thermal stratification due to cold, deep water.
  - Overwhelms unstable salinity profile due to surface evaporation.
- But temperature diffuses more rapidly than salt.

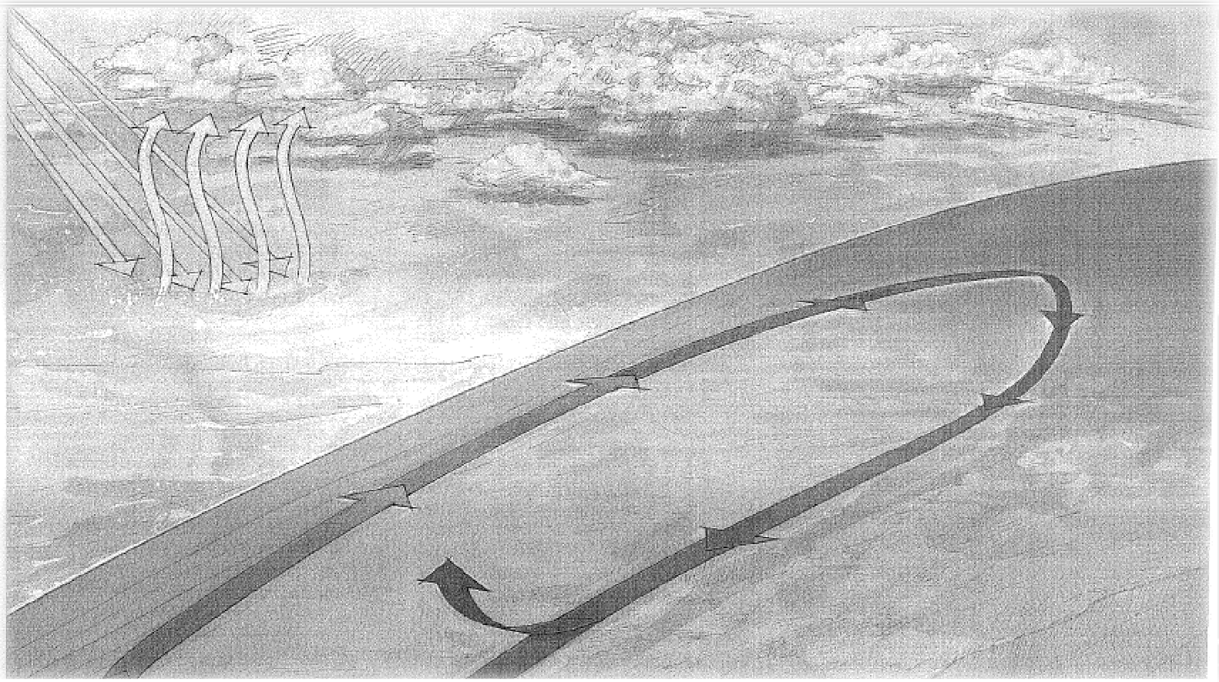
$$\tau = \frac{\kappa_T}{\kappa_S} \sim 100 >> 1 \qquad\qquad Pr_S = \frac{\nu}{\kappa_S} >> Pr_T = \frac{\nu}{\kappa_T} > 1$$

- Therefore local concentrations of salt can overcome the stability due to temperature.
- Define $R_\rho = \frac{\alpha\ \overline{T}_z}{\beta\ \overline{S}_z}$ where α and β are Boussinesq convective terms.
- Fluid can be double diffusively unstable only for $1 < R_\rho < \tau$.
- In practice, only for $1 < R_\rho < 1.6$, with some effects on waves up to $R_\rho = 3$.

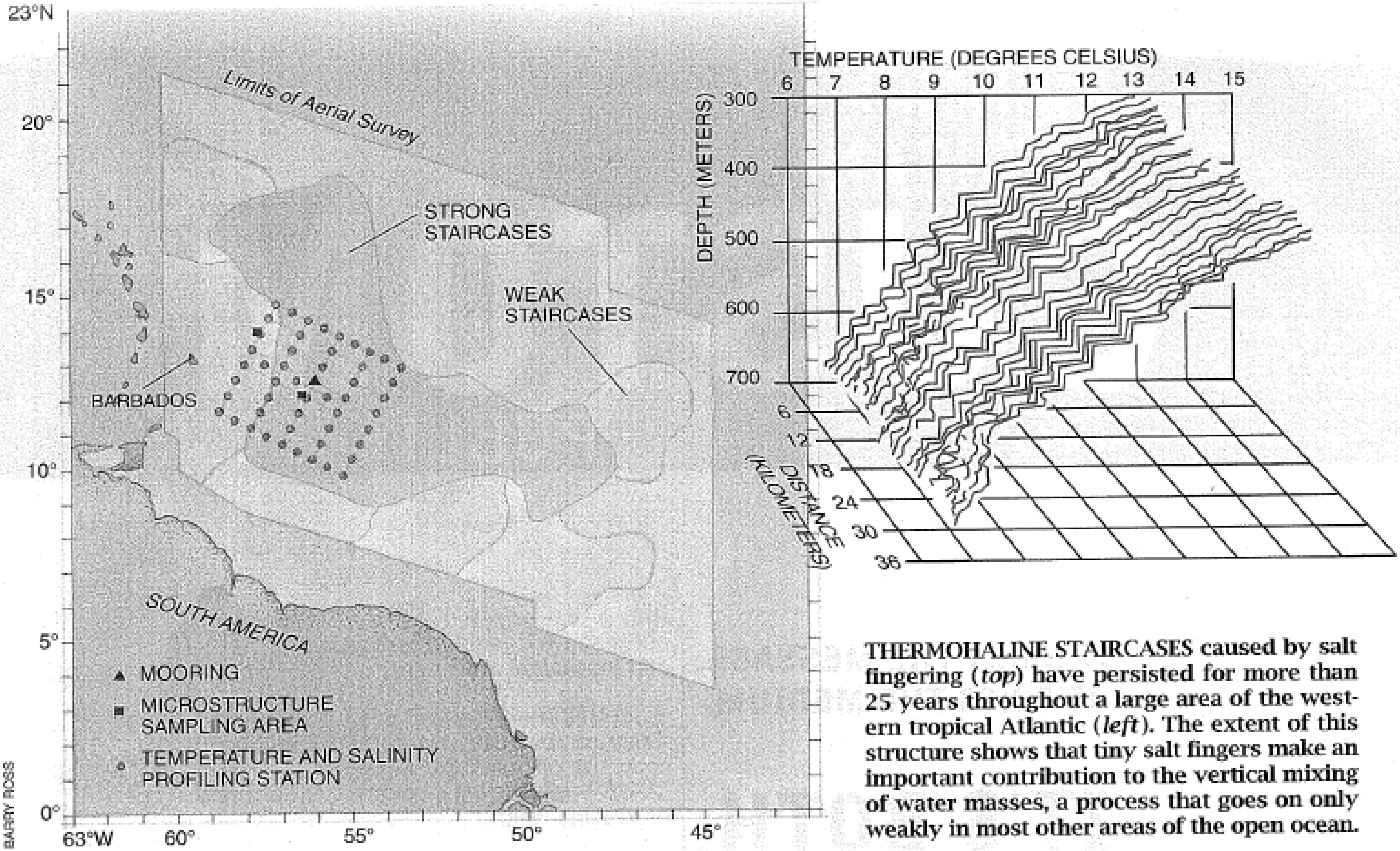
PURPLE FINGERS develop when warm, saline water containing potassium permanganate (which acts as dissolved salt and colorful tracer) floats on cold freshwater.

2 Observations



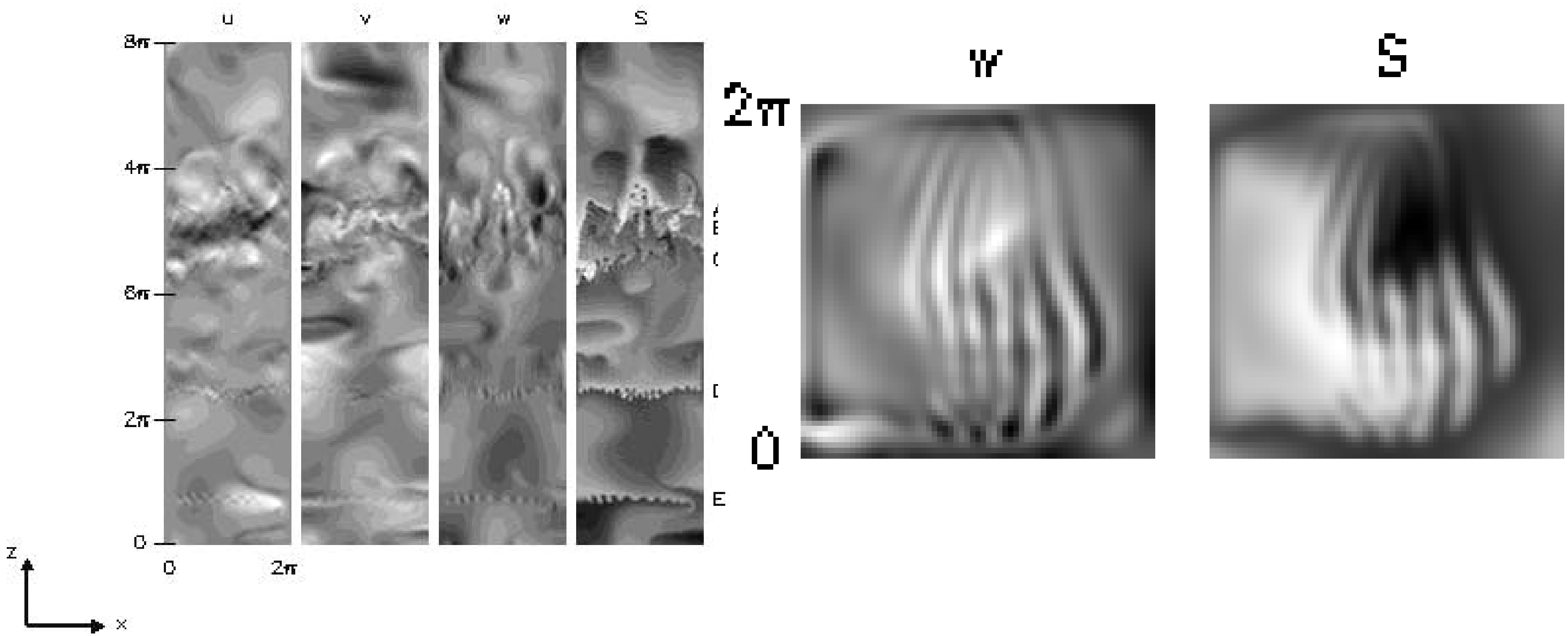


Caribbean staircases. The figures on the right represent the observation program C-Salt. What they believed was discovered were huge rolls separated by thin layers where salt and temperature jumped. The rolls might be 50 meters deep and the jumps 10s of centimeters. The inset in the upper figure shows fingers within one of those jumps. The light rays and arrows coming off the sea surface in the lower figure are supposed to represent evaporation and how it provides a source for a salt flux.



THERMOHALINE STAIRCASES caused by salt fingering (*top*) have persisted for more than 25 years throughout a large area of the western tropical Atlantic (*left*). The extent of this structure shows that tiny salt fingers make an important contribution to the vertical mixing of water masses, a process that goes on only weakly in most other areas of the open ocean.

• Sharp jumps in salinity and temperature in narrow layers.



Left: Vertical sections of perturbation velocities and salinity from: case 1.2L at $t = 10$, showing the interfaces A, B, C, D and E where salt fingers have formed. Fingers are characterized by upward (positive) vertical velocity and negative salt perturbations. Right: Horizontal sections of perturbation vertical velocity (left), and salinity (right) through interface A of case 1.2L at $t = 8$. **Layers require ribs.**

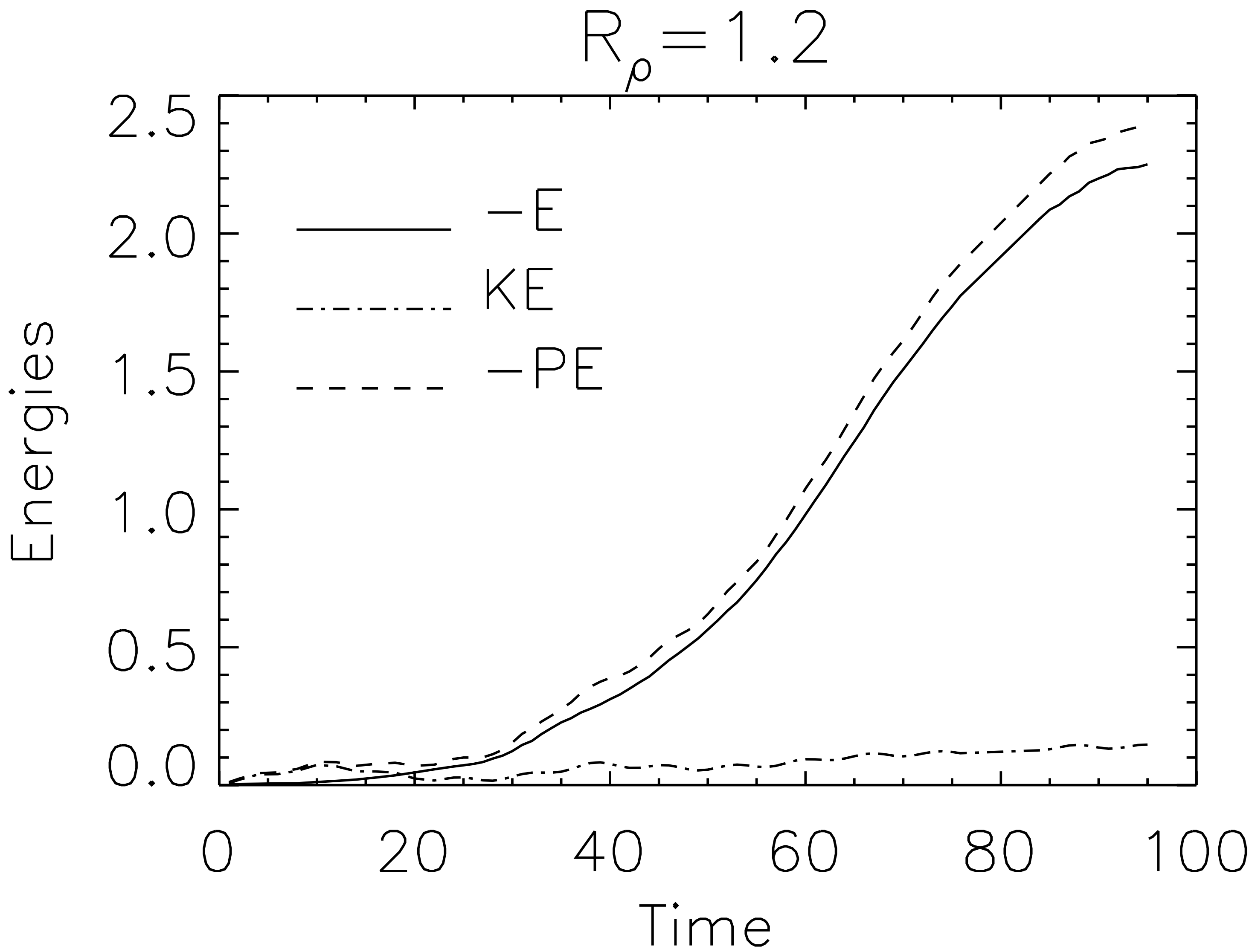
• But are these layers persistent?

• Do they affect overall fluxes significantly?

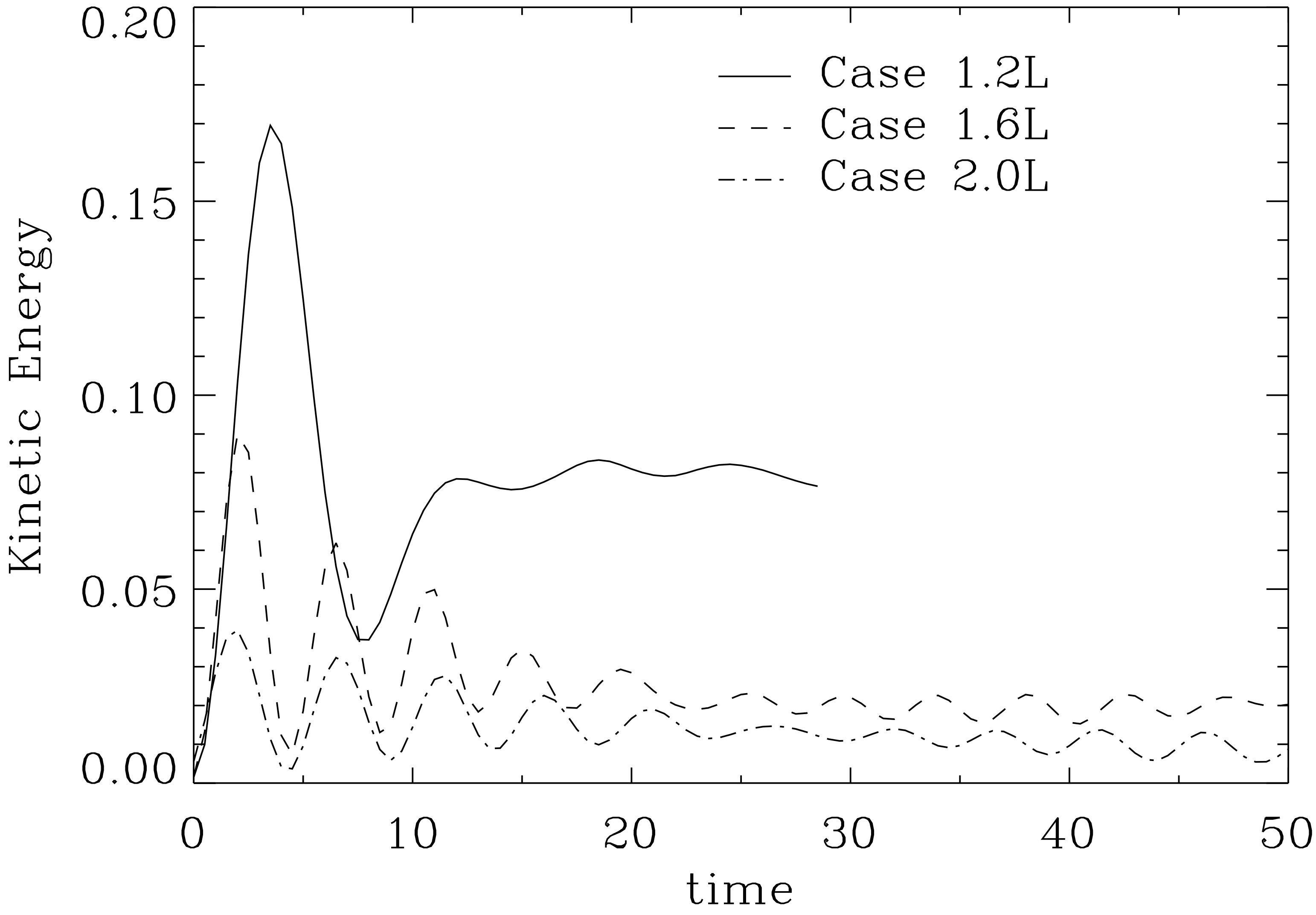
• Are they even due to double diffusion?

• **Linden** 3D instability.

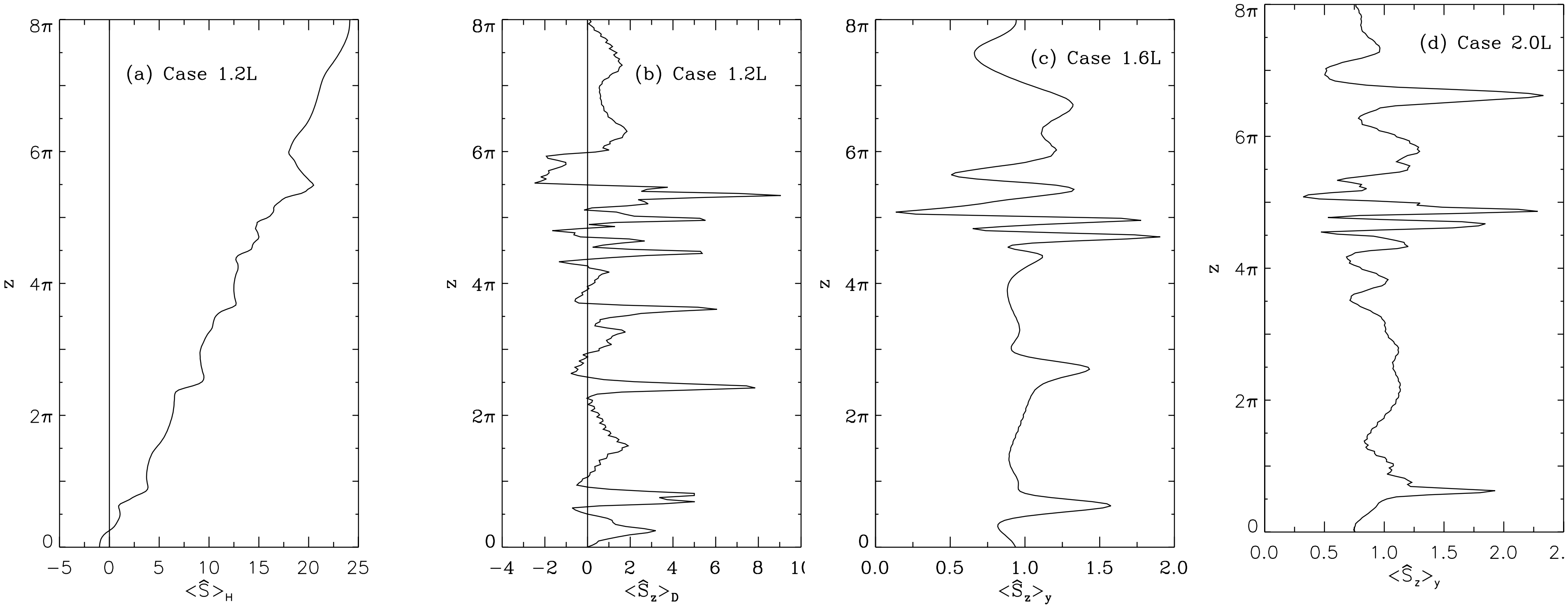
• Experiment.



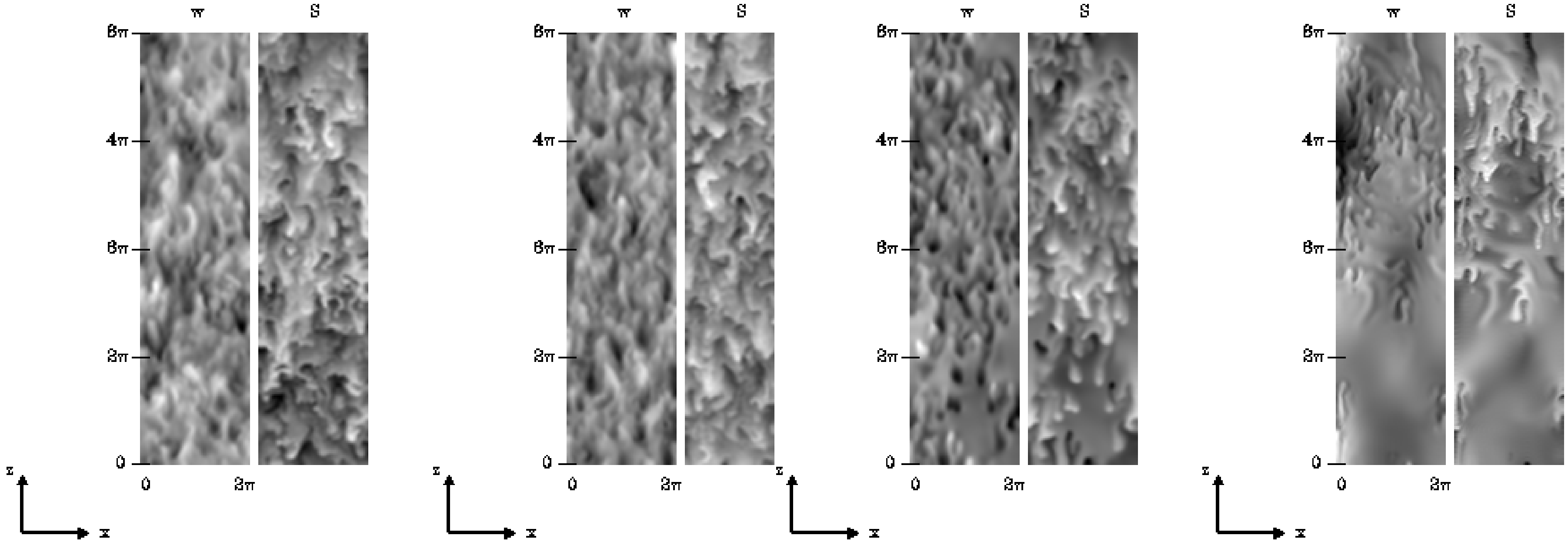
Global energy time series from case 1.2H, showing nondimensional kinetic energy, KE, minus potential energy, -PE, and minus total energy, -E.



Time series of non-dimensional kinetic energy, KE, from cases 1.2L, 1.6L, and 2.0L.



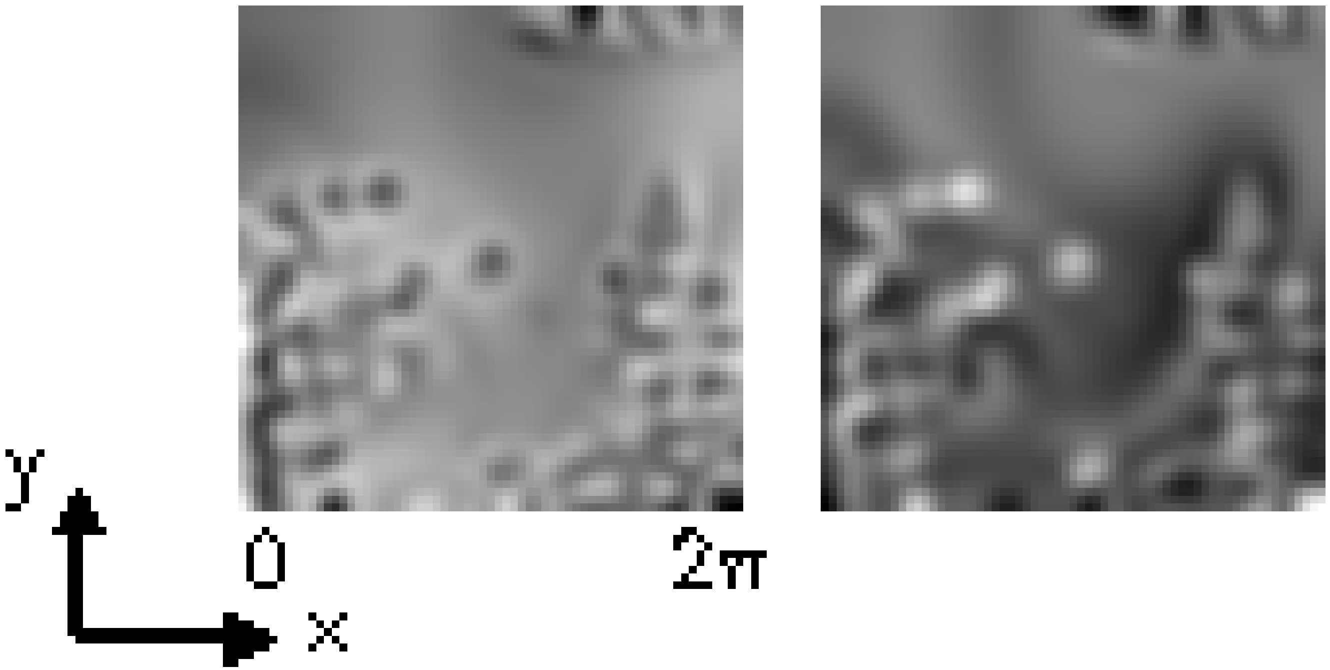
Vertical profiles of horizontally averaged total salinity and its vertical gradien; a) total salinity from case 1.2L and $t = 8$, b) salinity gradient from case 1.2L at $t = 8$, c) salinity gradient from case 1.6L at $t = 40$, d) salinity gradient from case 2.0L at $t = 43$.



Late time shaded contours from 4 cases. Two at left are from two $R_\rho = 1.2$ simulations, left with a large-scale perturbation and right with a small-scale perturbation. Two at right are for $R_\rho = 1.6$ and $R_\rho = 2$, both of which are wave dominated at large-scales, but 1.6 still has fingers.

The bottom line is that at late times when fingering is still important, a universal state seems to be reached.

Does this enhance diffusion?



Horizontal sections of perturbation vertical velocity through $z = 4.6\pi$ of case 2.0L at $t = 80$ (bottom panels). The case 2.0L values have been increased by a factor of 3 so that the same gray scales apply to both cases.

3 Governing equations and numerical method

The governing equations are:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mathbf{g}(\alpha T - \beta S) + \nu \nabla^2 \mathbf{u} \tag{1a}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T + w\overline{T}_z = \kappa_T \nabla^2 T \tag{1b}$$

$$\frac{\partial S}{\partial t} + \mathbf{u} \cdot \nabla S + w\overline{S}_z = \kappa_S \nabla^2 S \tag{1c}$$

$$\nabla \cdot \mathbf{u} = 0. \tag{1d}$$

- A pseudospectral numerical code periodic in the vertical with free-slip, insulating boundary conditions on the sides is used.
- Free-slip rather than periodic horizontal boundary conditions are used because the fluid must be confined.
- The major drawback of working in three-dimensions is that the entire physical range of scales cannot be spanned.

- A physical value of salt Prandtl number $Pr_S = 700$ implies that the viscous subrange must span a range of length scales of order $\sqrt{700} = 26$.

- With at most 512 grid points, there would only be at most a decade of length scales available to span the large-scale flow and the turbulent regime of salt.

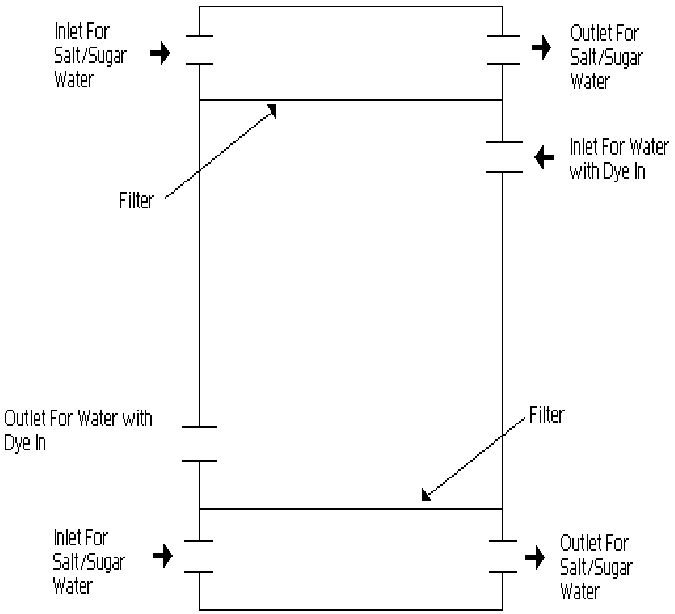
- Therefore give up on real turbulence and do simulations that are diffusively dominated.

- Furthermore, use smaller Prandtl numbers and compare to rescaled oceanic observations.

Case	R_ρ	$g\alpha$	$g\beta$	ν	N	P_T	P_S	$\overline{T}_z$	$\overline{S}_z$	τ	M	Λ
1.2H	1.2	1.2	1	.036	.45	7	48	1	1	6.9	.61	1.15
1.2L	1.2	1.2	1	.012	.45	2	6	1	1	3	.55	1.00
1.6L	1.6	1.6	1	.012	.78	2	6	1	1	3	.59	0.88
2.0L	2	1	.5	.006	.71	2	6	1	1	3	.56	0.73
1.2I	1.2	1.2	1	.012	.45	2	6	1	1	3	.55	1.00
1.2τ	1.2	1.2	1	.012	.45	6	6	1	1	1	–	––

Case	R_ρ	$g\alpha$	$g\beta$	ν	N	P_T	P_S	$\overline{T}_z$	$\overline{S}_z$	τ	M	Λ
1.2H	1.2	1.2	1	.036	.45	7	48	1	1	6.9	.61	1.15
1.2L	1.2	1.2	1	.012	.45	2	6	1	1	3	.55	1.00
1.6L	1.6	1.6	1	.012	.78	2	6	1	1	3	.59	0.88
2.0L	2	1	.5	.006	.71	2	6	1	1	3	.56	0.73
1.2I	1.2	1.2	1	.012	.45	2	6	1	1	3	.55	1.00
1.2τ	1.2	1.2	1	.012	.45	6	6	1	1	1	–	– –

Non-dimensional model parameters for each model case. Non-dimensional wavenumber M is computed as a function of R_ρ, P_T and τ from the expressions in Schmitt (1979). The wavelength, Λ, is given by (4), but with $\overline{T}_z$ substituted for $(\Delta T/\delta z)$.


Inlet For Salt/Sugar Water
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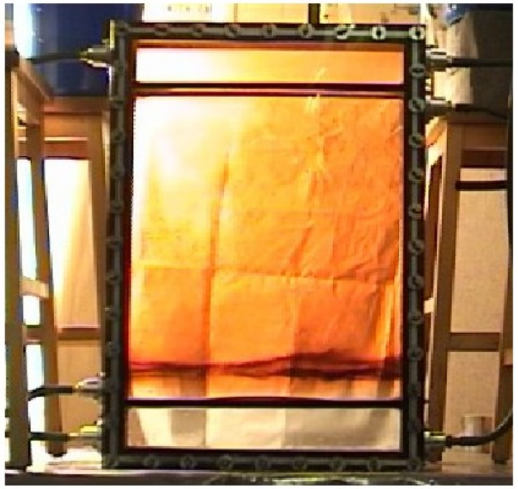
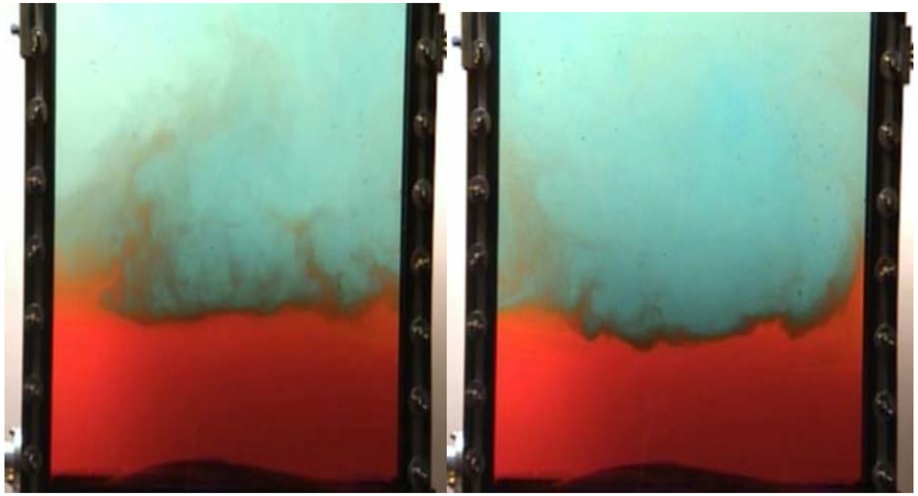
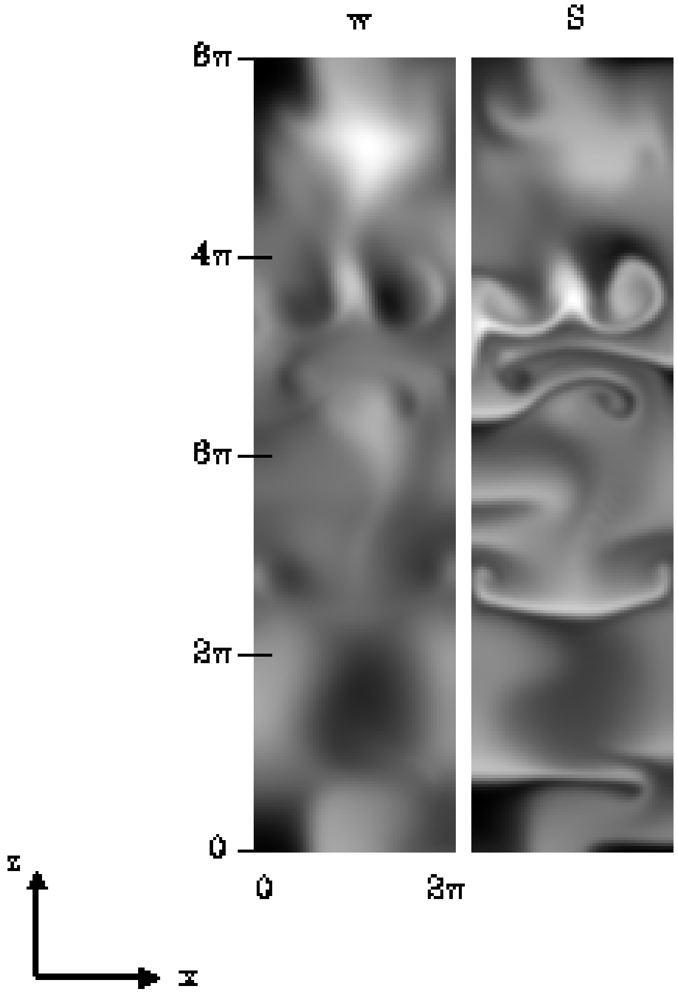
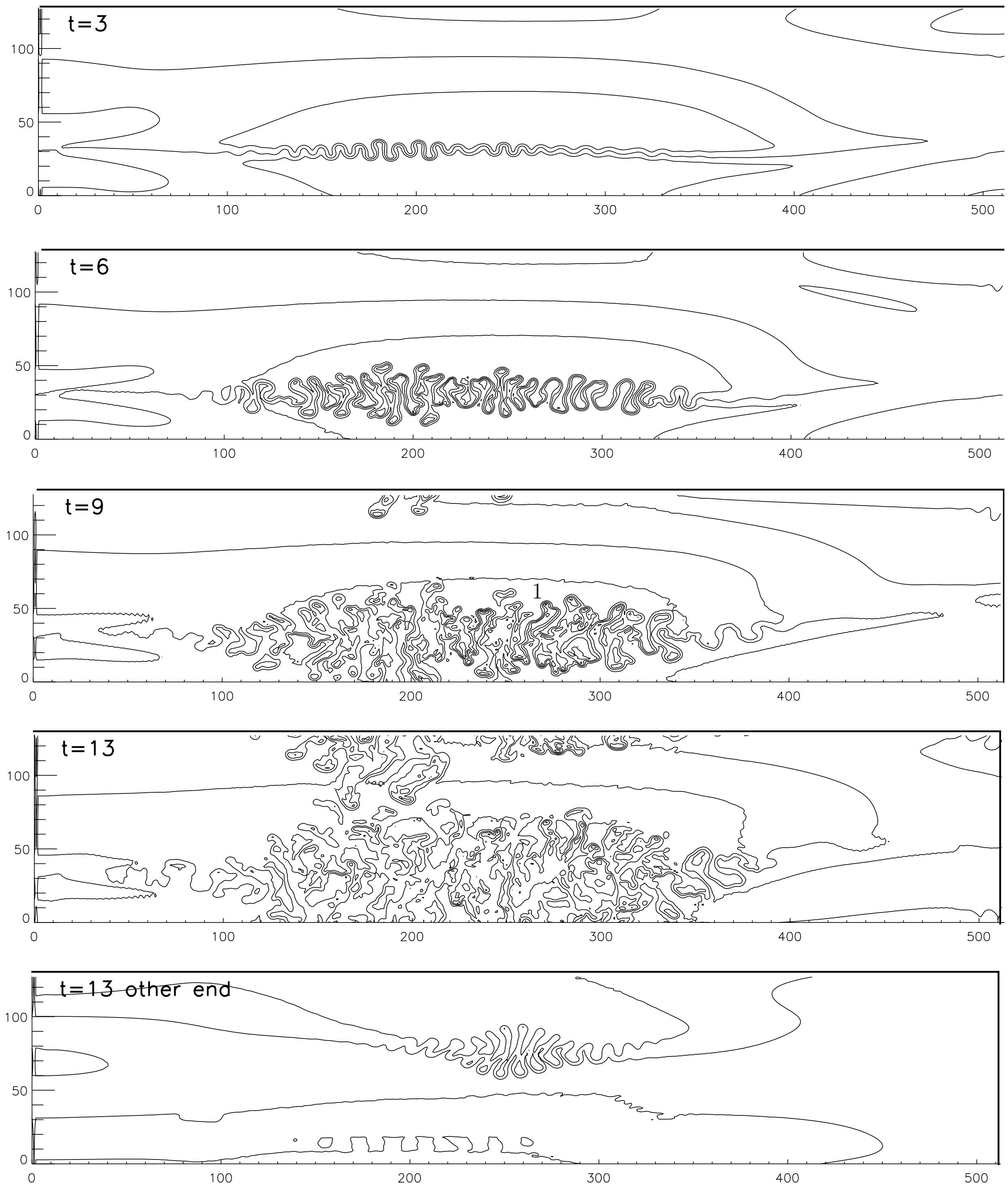


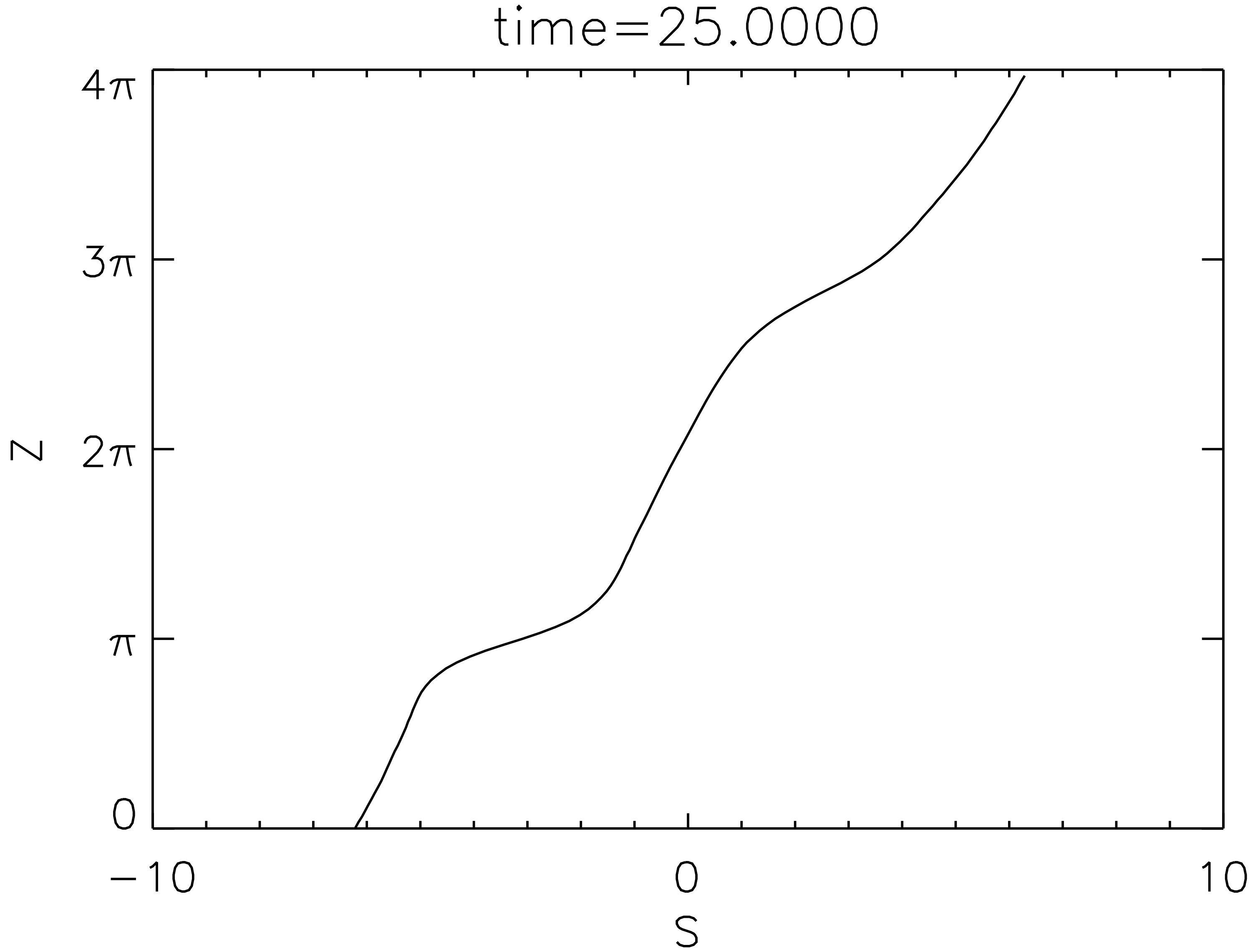




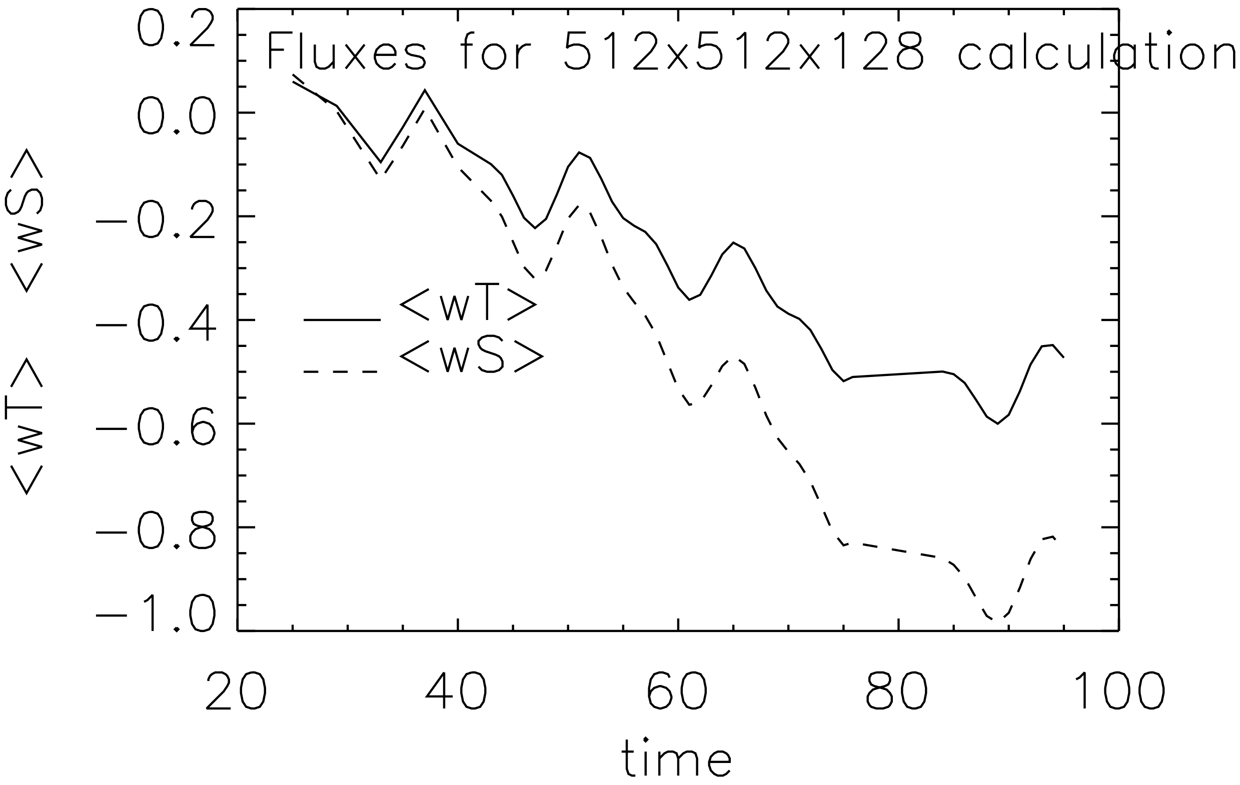
Vertical cross sections of the perturbation vertical velocity (left panel) and salinity (right panel) from case 1.2τ. The greyscale covers the ranges -0.8 to 1.2 in vertical velocity and from -5 to 6 in salinity, with negative values dark and positive values light.



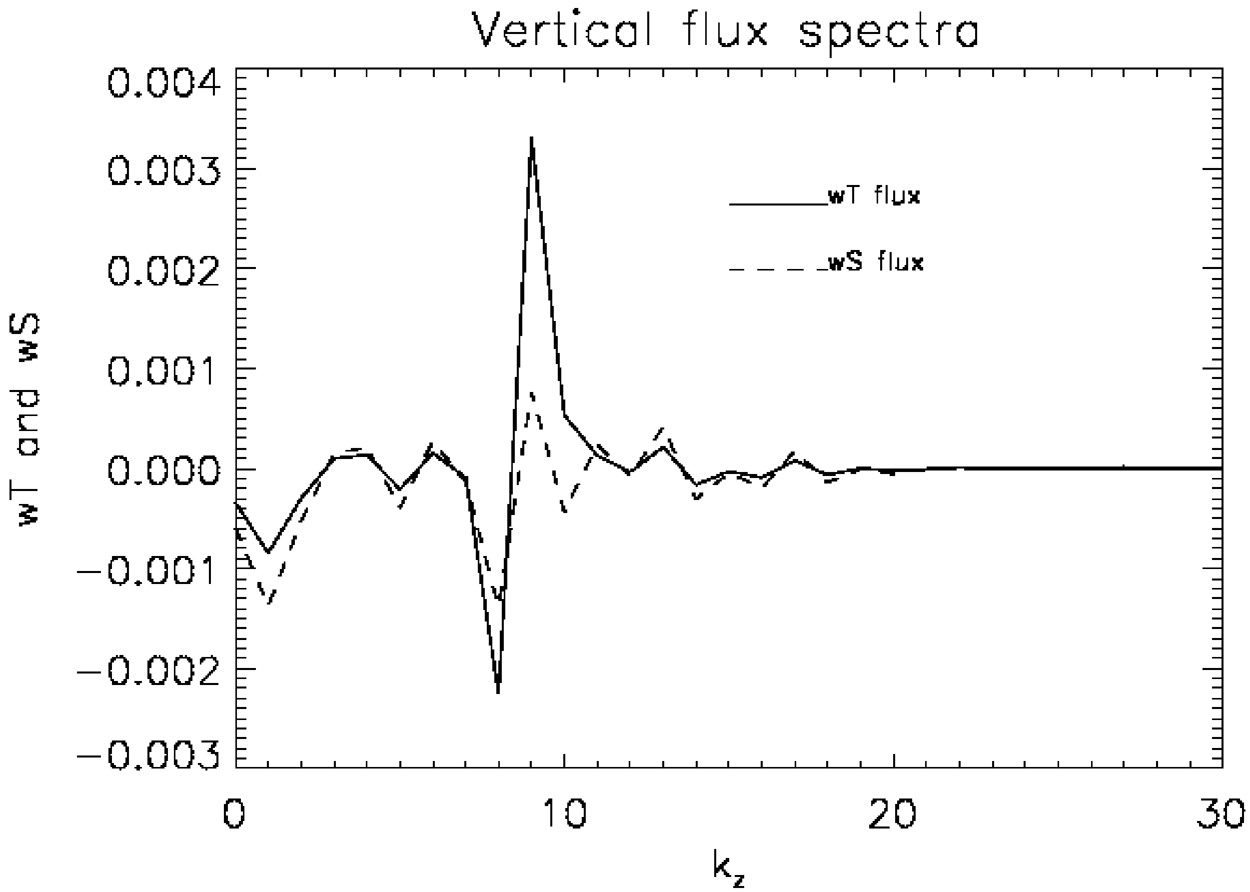
Vertical cross section at $y = 2\pi$ (one of the faces) of non-dimensional perturbation temperature from case 1.2H, at four early times, $t = 3$, $t = 6$, $t = 9$, and $t = 13$. The last slice is $t = 13$ is on the opposite face.



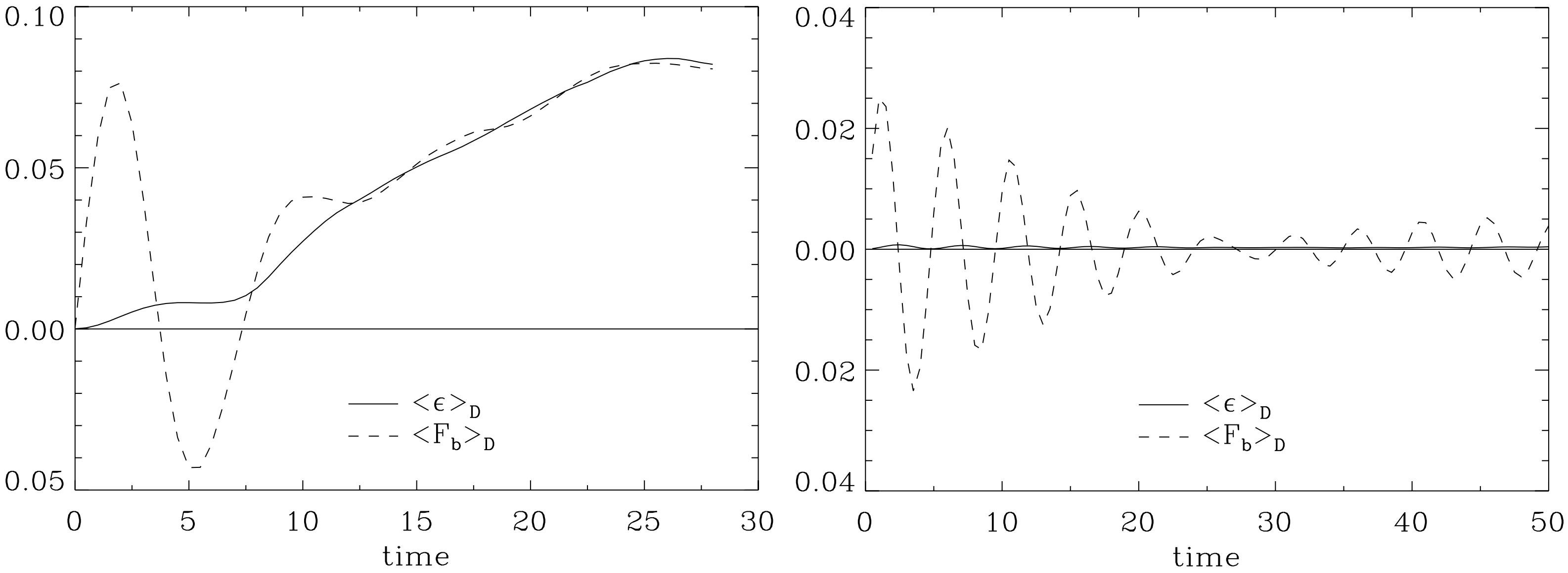
Vertical profile of horizontal average total salinity (non-dimensional) from case 1.2H at $t = 25$.



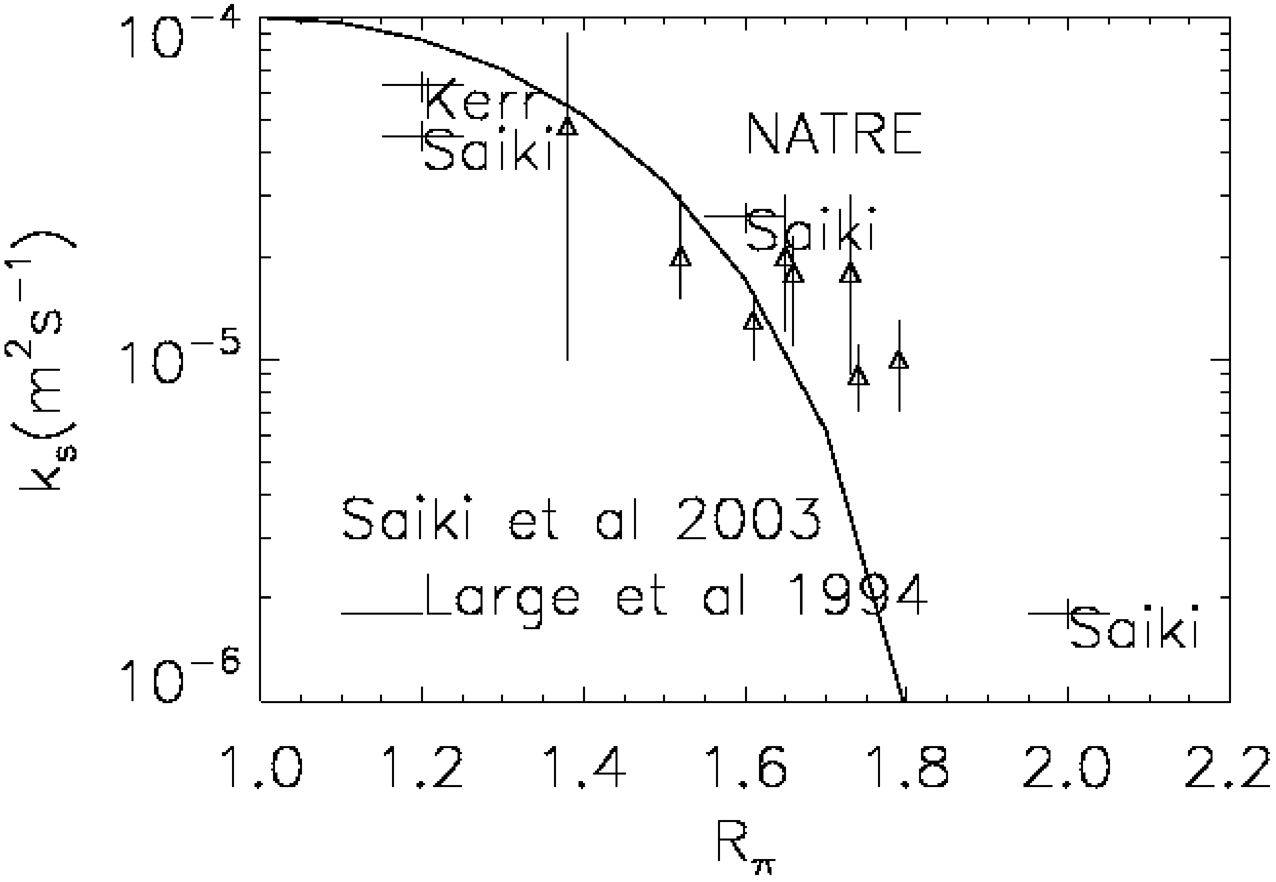
Time series of the nondimensional heat and salt fluxes from case 1.2H



Co-spectra of the vertical salt fluxes from a $256 \times 256 \times 64$ case at later time (66.5). The large peak represents the thickness of the fingers even though distinct layers do not exit at this time. I also need horizontal spectra of cases 1.2H, 1.2L, 1.6L, and 2.0L as a function of one of the non-dimensional horizontal wavenumbers and normalized by the domain average flux $\overline{ws}_D$.



Time series of domain average dissipation of turbulent kinetic energy ϵ and buoyancy flux $\overline{wb}$ from cases 1.2L (top) and 2.0L (bottom).



Vertical salt diffusivity of salt fingering as a function of the density ratio R_ρ. The case names give the DNS results and the triangles with the standard error bars are replotted from [34]. Values based upon the application of laboratory flux laws are shown as LS [33] and S88 [3]. The prediction of [39] is dashed and the OGCM formulation Zhang *et al.* [37] is dotted. Equation 2 from [18] is solid.

$$K_S(R_\rho) = K_S^o\ (1 - (\frac{R_\rho - 1}{R_\rho^o - 1})\ )^3, \tag{2}$$

Equation 2 is based upon

$$K_S = \frac{(R_\rho - 1)}{(1-\gamma)}\ \frac{\epsilon}{N^2}. \tag{3}$$

where These fluxes are downgradient (negative) and have been parameterized in OGCMs as :

$$\langle wT \rangle = -K_T\ \overline{T_z} \tag{4a}$$

$$\langle wS \rangle = -K_S\ \overline{S_z}, \tag{4b}$$

where K_T and K_S are the double diffusive diffusivities that need to be determined, and the implied buoyancy diffusivity is $K_b = - < wb > N^{-2}$. These diffusivities are related through

$$K_S \quad = \quad \frac{R_\rho}{\gamma}\ \ K_T \quad = \quad \frac{(R_\rho - 1)}{\gamma - 1}\ \ K_b\ \ . \tag{5}$$

The total vertical buoyancy flux, $\langle wb \rangle$ is defined as,

$$\langle wb \rangle = g\ \alpha \langle wT \rangle - g\beta \langle wS \rangle = -g\ \beta \langle wS \rangle\ (1-\gamma), \tag{6}$$

Fingers tend to transport salt more efficiently than heat, because salinity variance is retained while temperature is molecularly diffused. K_T is modeled as a typical eddy diffusivity.

Conclusions

- Layers develop, but they are transient.
- Final state dependent only on R_ρ and independent of initial condition.
- Then, non-dimensional fluxes agree with NATRE experiment.
- And confirm model of Large et al (1994)

Acknowledgments

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References

[1] R. W. Schmitt, "Form of the temperature-salinity relationship in the Central Water: evidence for double diffusive mixing" J. Phys. Oceanogr. **11**, 1015 (1981).

[2] R. W. Schmitt,"Double Diffusion in Oceanography" Ann. Rev. Fluid Mech. **26**, 255 (1994).

[3] R. W. Schmitt, "Mixing in a thermohaline staircase". in *Small-scale turbulence and mixing in the ocean* (edited by J. C. J. Nihoul and B. M. Jamart), (Elsevier Science Publishers B. V., New York, 1988), pp. 435-452.

[4] R. W. Schmitt, "The Ocean's Salt Fingers" Scientific American **272**, 70 (1995).

[5] C. Y. Shen, "The evolution of the double-diffusive instability: Salt fingers" Phys. Fluids A **1**, 829 (1989).

[6] C. Y. Shen, "Heat-salt finger fluxes across a density interface" Phys. Fluids A **5**, 2633 (1993).

[7] C. Y. Shen, "Equilibrium salt-fingering convention" Phys. Fluids A **7**, 706 (1995).

[8] C. Y. Shen and G. Veronis, "Numerical simulation of two-dimensional salt fingers" J. Geophys. Res. **102**, 23131 (1997).

[9] P. F. Linden, "Salt fingers in a steady shear flow" Geophys. Fluid Dyn. **6**, 1-27 (1974).

[10] J. S. Turner, *Buoyancy Effects in Fluids.* (Cambridge University Press,Cambridge, 1973).

[11] R. M. Kerr, "Higher order derivative correlations and the alignment of small-scale structures in isotropic numerical turbulence" J. Fluid Mech. **153**, 31 (1985).

[12] R. M. Kerr, "Rayleigh number scaling in numerical convection" J. Fluid Mech. **310**, 139 (1996).

[13] J. R. Herring, R. M. Kerr, and R. Rotunno, "Ertel's potential vorticity in unstratified turbulence" J. Atmos. Sci. **51**, 35 (1994).

[14] J. R. Taylor and G. Veronis, "Experiments on double-diffusive sugar-salt fingers at high stability ratio" J. Fluid Mech. **321**, 315 (1996).

[15] C. Chan and C. F. Chen, " " **1**, (1997).

[16] T. G. L. Shirtcliffe and J. S. Turner, "Observations of the cell structure of salt fingers" J. Fluid Mech. **41**, 707 (1970).

[17] J. Werne E. E., DeLuca R., and Rosner "" 1991 **Phys. Rev. Letters**, 67 (,). Development of hard-turbulent convection in two dimensions: Numerical evidence

[18] W. G. Large, J. C. McWilliams, and S. C. Doney, "Oceanic vertical mixing: A review and a model with a nonlocal boundary layer parameterization" Rev. of Geophys. **32**, 363 (1994).

[19] R. W. Schmitt, "The growth rate of super-critical fingers" Deep-Sea Res. **26**, 23 (1979).

[20] W. W. Grabowski and T. L. Clark, "Cloud-environment interface instability: Rising thermal calculations in two spatial dimensions" J. Atmos. Sci. **48**, 527 (1991).

[21] A. E. Gill, *Atmosphere-Ocean Dynamics.* (Academic Press, New York, 1982).

[22] T. M. Özgökmen and O. E. Esenkov, "Asymmetric salt fingers induced by a nonlinear equation of state" Phys. Fluids **10**, 1882 (1998).

[23] T. M. Özgökmen, O. E. Esenkov, and D. B. Olson, "A numerical study of layer formation due to fingers in double-diffusive convection in a vertically-bounded domain" J. Marine Res. **56**, 463 (1998).

[24] J. Y. Holyer, "On the collective instability of salt fingers" J. Fluid Mech. **110**, 195 (1981).

[25] M. E. Stern, "The collective instability of salt fingers" J. Fluid Mech. **35**, 209 (1969).

[26] J. S. Turner, "Salt fingers across a density interface" Deep-Sea Res. **14**, 599 (1967).

[27] R. G. Lueck, "Microstructure measurements in a thermohaline staircase" Deep-Sea Res. **34**, 1677 (1988).

[28] M. C. Gregg and T. B. Sanford, "Shear and turbulence in thermohaline staircases" Deep-Sea Res. **34**, 1689 (1987).

[29] J. Y. Holyer, "The stability of long steady three-dimensional salt fingers" J. Fluid Mech. **156**, 495 (1985).

[30] M. E. Stern, "Maximum buoyancy flux across a salt interface" J. Marine Res. **34**, 95 (1976).

[31] T. J. McDougall,J. R.Taylor "Flux measurements across a finger interface at low values of the stability ratio" J. Marine Res. **42**, 1 (1984).

[32] R. B. Lambert,W.Sturges "A thermohaline staircase and vertical mixing in the thermocline" Deep-Sea Res. **24**, 211 (1977).

[33] R. B. Lambert,W.Sturges "C-Salt" Deep-Sea Res. , (1986).

[34] L St. Laurent and R. W. Schmitt, "The contribution of salt fingers to vertical mixing in the North Atlantic Tracer Release Experiment" J. Phys. Oceanogr. **29**, 1404-1424 (1999).

[35] A. Wong and G. Johnson, "South Pacific Eastern Subtropical Mode Water" J. Phys. Oceanogr. **33**, submitted (2003).

[36] Yeager,W.Large "other basins" , 1 (2003).

[37] J. Zhang, R.W. Schmitt, and R.X. Huang, "Sensitivity of the GFDL Modular Ocean Model to the parameterization of double-diffusive processes" J. Phys. Oceanogr. **28**, 589-605 (1998).

[38] E. Kunze, "Limits on growing finite-length salt fingers" J. Marine Res. **45**, 533-556 (1987).

[39] E. Kunze, "A proposed flux constraint for salt fingers in shear" J. Marine Res. **52**, 999-1016 (1994).

[40] M. E. Stern, T. Radko, and J. Simeonov, "Salt fingers in an unbounded thermocline" J. Marine Res. **59**, 355-390 (2001).

[41] W.J. Merryfield, G. Holloway, and A.E. Gargett, "A global ocean model with double-diffusive mixing" J. Phys. Oceanogr. **29**, 1124-1142 (1999).

[42] W. J. Merryfield, "Origin of thermohaline staircases" J. Phys. Oceanogr. , (2000).

[43] T. Radko and M.E. Stern, "Salt finger in three dimensions" J. Marine Res. **57**, 471-502 (1999).

[44] T. Radko and M.E. Stern, "Finite-amplitude salt fingers in a vertically bounded layer" J. Fluid Mech. **425**, 133-160 (2000).

[45] J.C Curry, J.R. Herring, J. Loncaric, and S.A. Orszag, "Order and disorder in two- and three-dimensional Bénard convection" J. Fluid Mech. **147**, 1-38 (1984).

[46] O. Métais and J.R. Herring, "Numerical experiments in forced stably stratified turbulence" J. Fluid Mech. **202**, 97-115 (1989).

[47] R. M. Kerr, "Rayleigh Number Scaling in Numerical Convection" J. Fluid Mech. **310**, 139-179 (1996).

[48] R. M. Kerr,J. R.Herring "Prandtl number dependence of Nusselt number in DNS" J. Fluid Mech. **419**, 325-345 (2000).

[49] G.O. Marmorino, W.K. Brown, and W.D. Morris, "Two-dimensional tempera-

ture structure in the C-SALT thermohaline staircase" Deep-Sea Res. **23**, 1675-1677 (1987).

[50] T.R. Osborn and C.S. Cox, "Oceanic fine structure" Geophys. Fluid Dyn. **3**, 321-345 (1972).

[51] Ledwell, "" , (1998).