The Abdus Salam
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Conference and Euromech Colloquium #480

on

High Rayleigh Number Convection

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Preliminary results for 2D simulation of convection in a rotating annulus

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2D simulation of convective heat transfer in a rotating annulus

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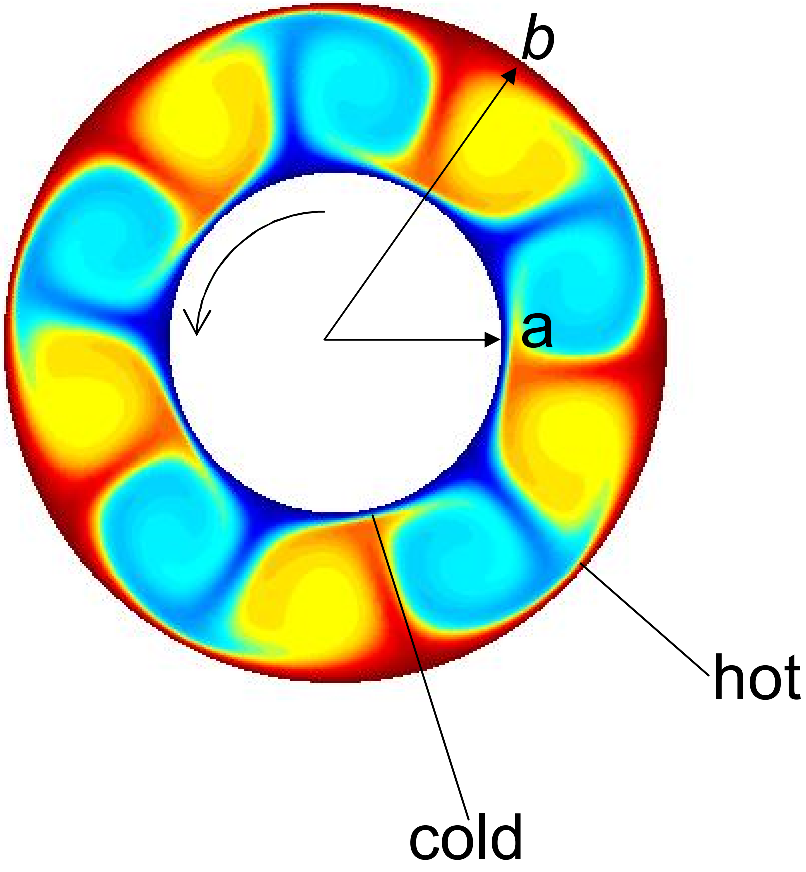
Euromech#480, 2006, High Ra number convection

An annulus with corotating, hot outer, and cold inner cylindrical surfaces

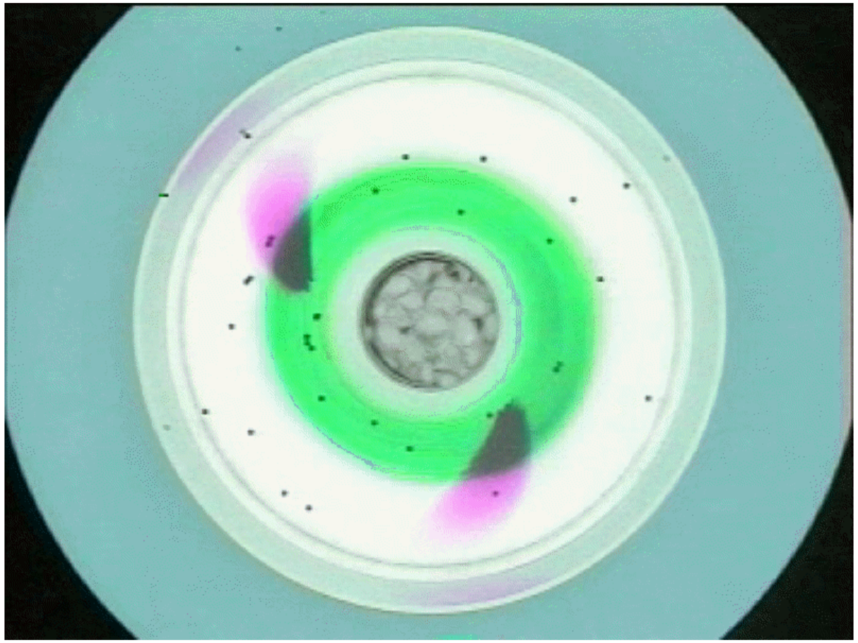


$$Ra = r_m \Omega^2 (b-a)^3 \beta \Delta T / \nu \kappa$$

$$Nu_a = a \ln(b/a) \partial_r [T] / \Delta T$$

$$Pr = 0.7$$

Laboratory modelling of geophysical flows-an example

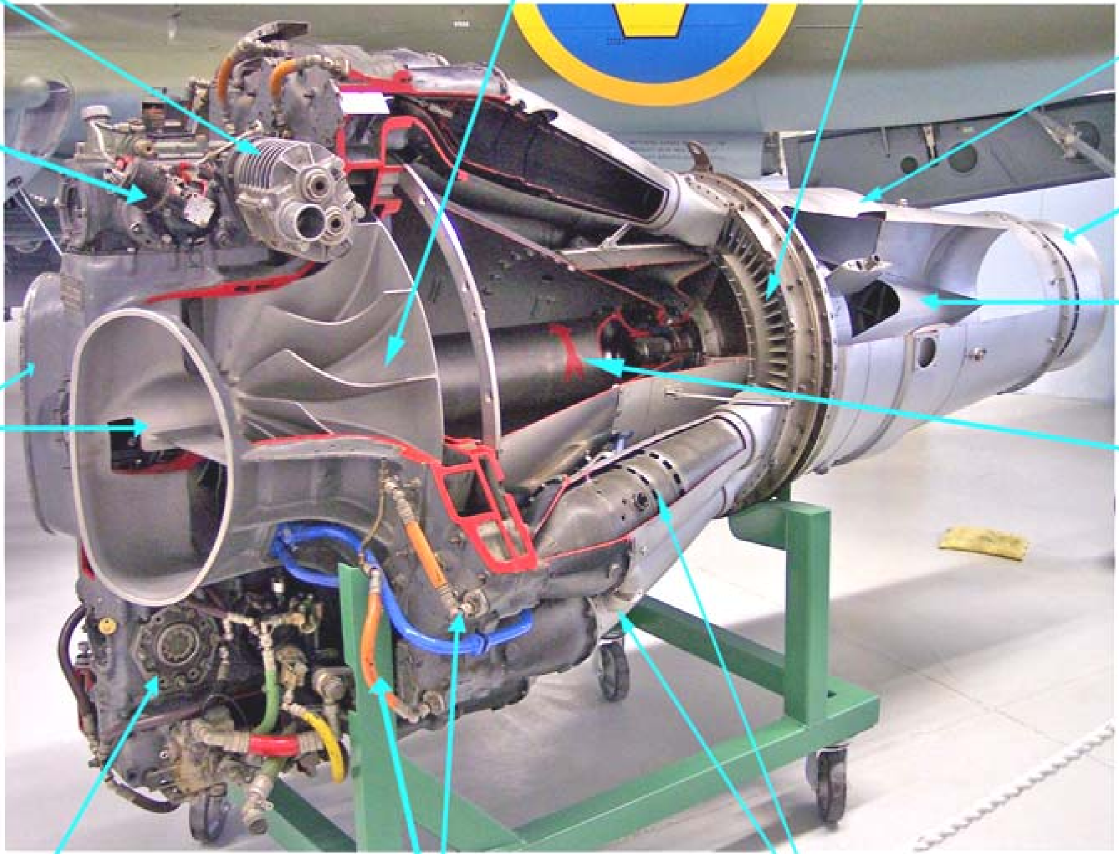


Westerly jet and Ekman spiral
0.8 rpm

Baroclinic instability
>1 rpm

Images from http://www-paoc.mit.edu/labweb, 12.003 Physics of atmosphere and oceans (an undergraduate course), John Marshall.

Rotating cavities in a jet engine



Pressure-cabin air compressor

Impeller (centrifugal compressor) attached to turbine via main shaft

Turbine stator blades

Muff around jet pipe supplying hot air for cabin heating

Pneumatic system air compressor (wheel brakes etc)

Jet pipe

Tail cone (turbine fairing)

Air inlets (supplied from intakes in aircraft wing root)

Main shaft connecting impeller and turbine

Starter motor (not fitted) mounting

Fuel lines to injectors inside combustion chambers

16 Combustion chambers (inside flame cans) arranged around engine

image from wikipedia

Numerical methods

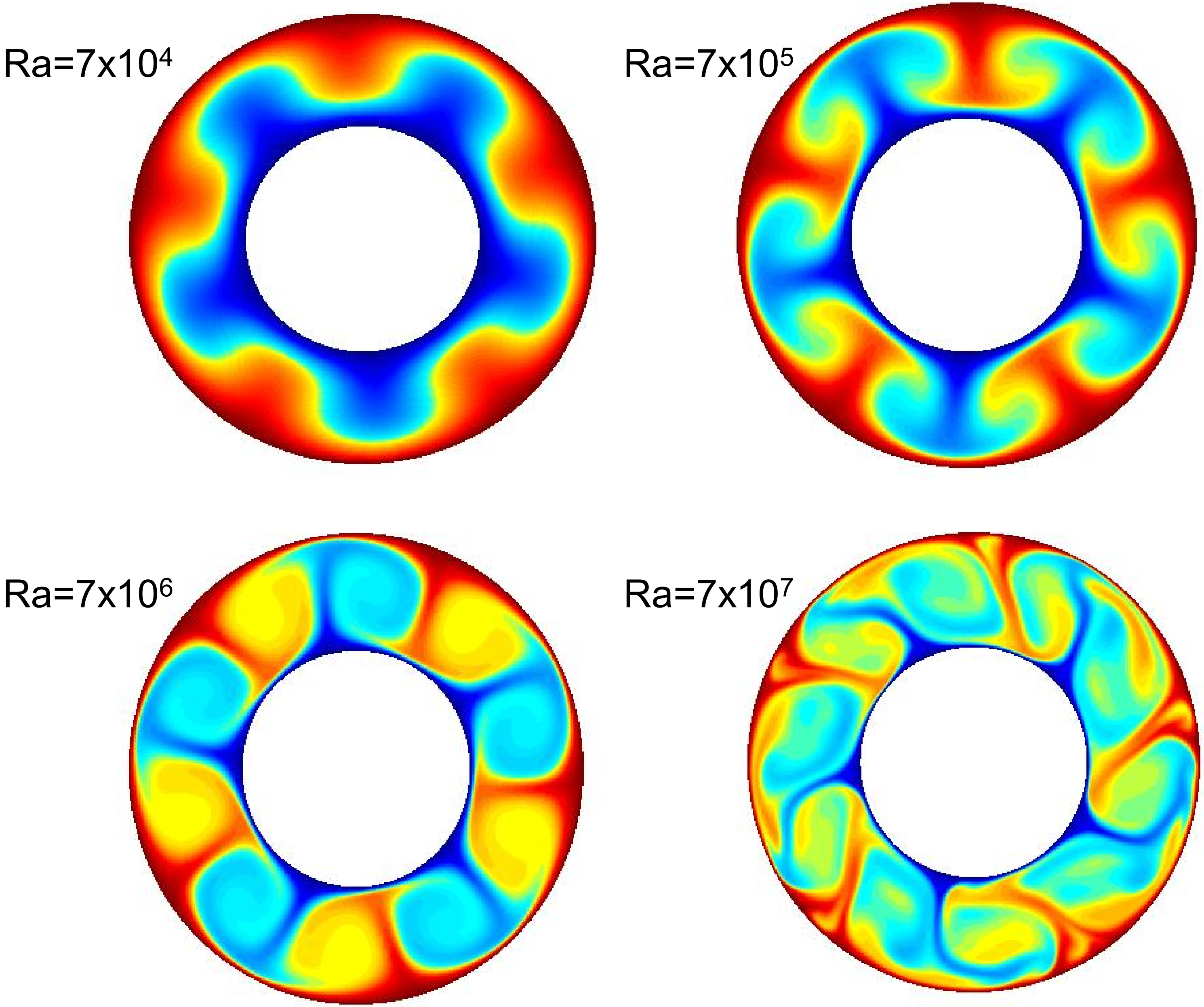
Governing equations in the vorticity-velocity formulation.

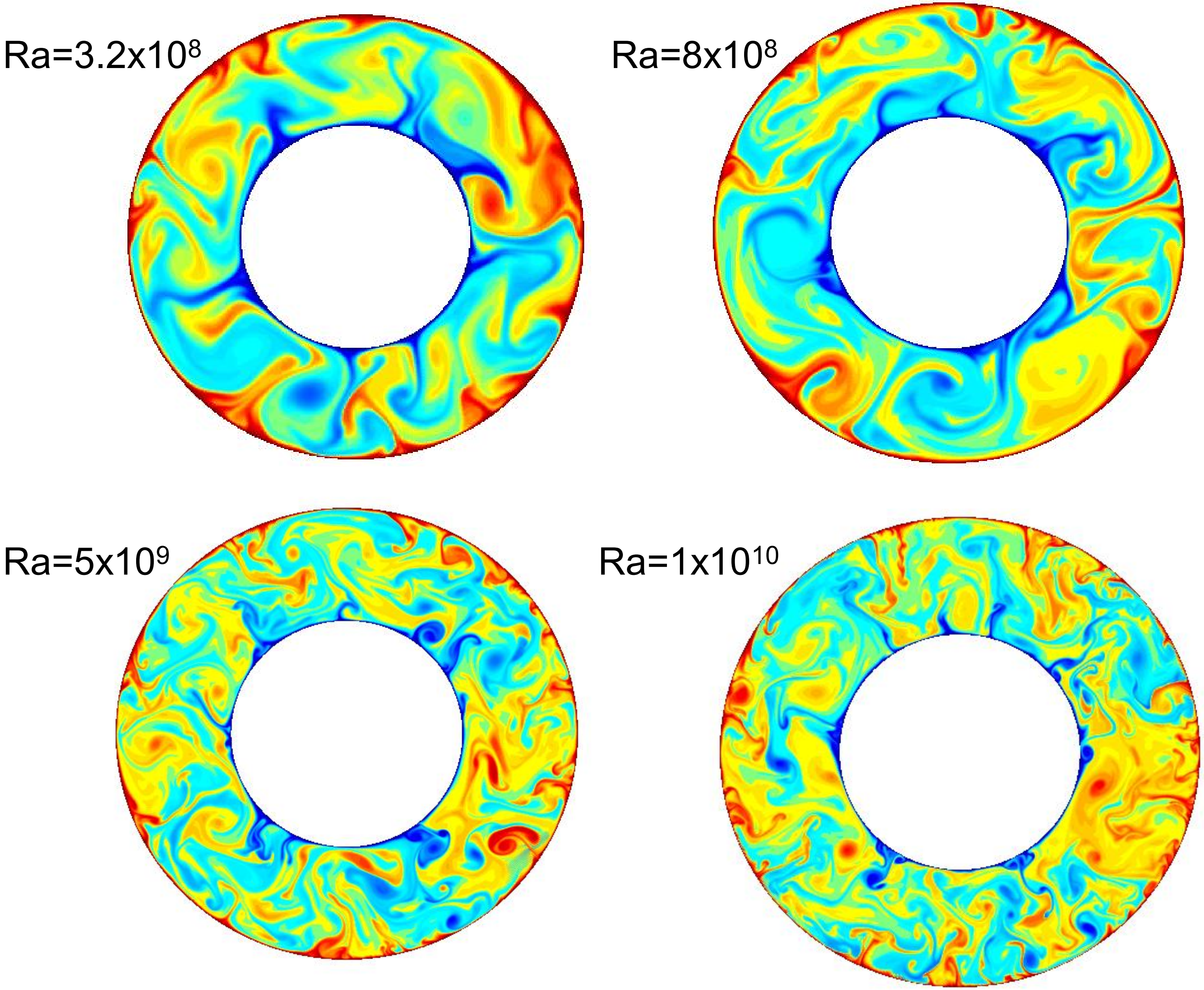
Fourth-order compact finite differencing for convective terms and conventional fourth-order finite differencing for the other terms.

Leap-frog timestepping.

Using Full Multigrid (Briggs et al, 2000, A Multigrid Tutorial) to solve the Poisson equations for u and v at each timestep.

Temperature

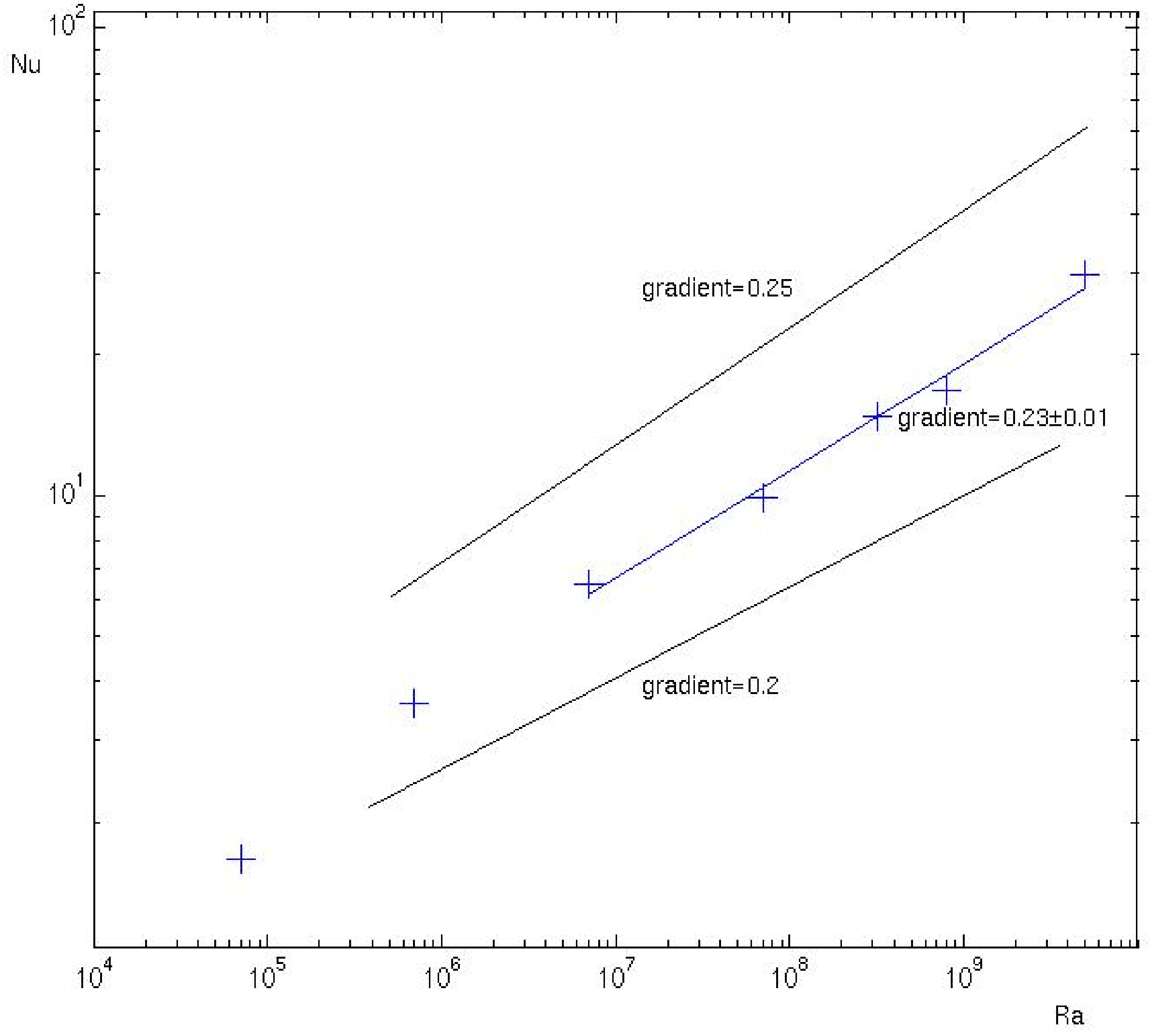



$Ra=3.2x10^8$
$Ra=8x10^8$
$Ra=5x10^9$
$Ra=1x10^{10}$

D. Bohn et al, 1995, Experimental and theoretical investigations of heat transfer in close gas-filled rotating annuli, *ASME Journal of Turbomachinery*, **117**, pp. 175—183.

Main result: $Nu \sim Ra^{\gamma}$, with $0.211 \leq \gamma \leq 0.228$ in the range $10^{7} < Ra < 10^{11}$.

$Nu \sim Ra^{0.23 \pm 0.01}$



A possible explanation of the exponent using the theory of Grossmann and Lohse.

Combination of

Regime I_l, dominance of $\epsilon_{u,BL}$ and $\epsilon_{\theta,BL}$:
$Nu \sim Ra^{0.25} Pr^{1/8}$, $Re \sim Ra^{0.5} Pr^{-3/4}$

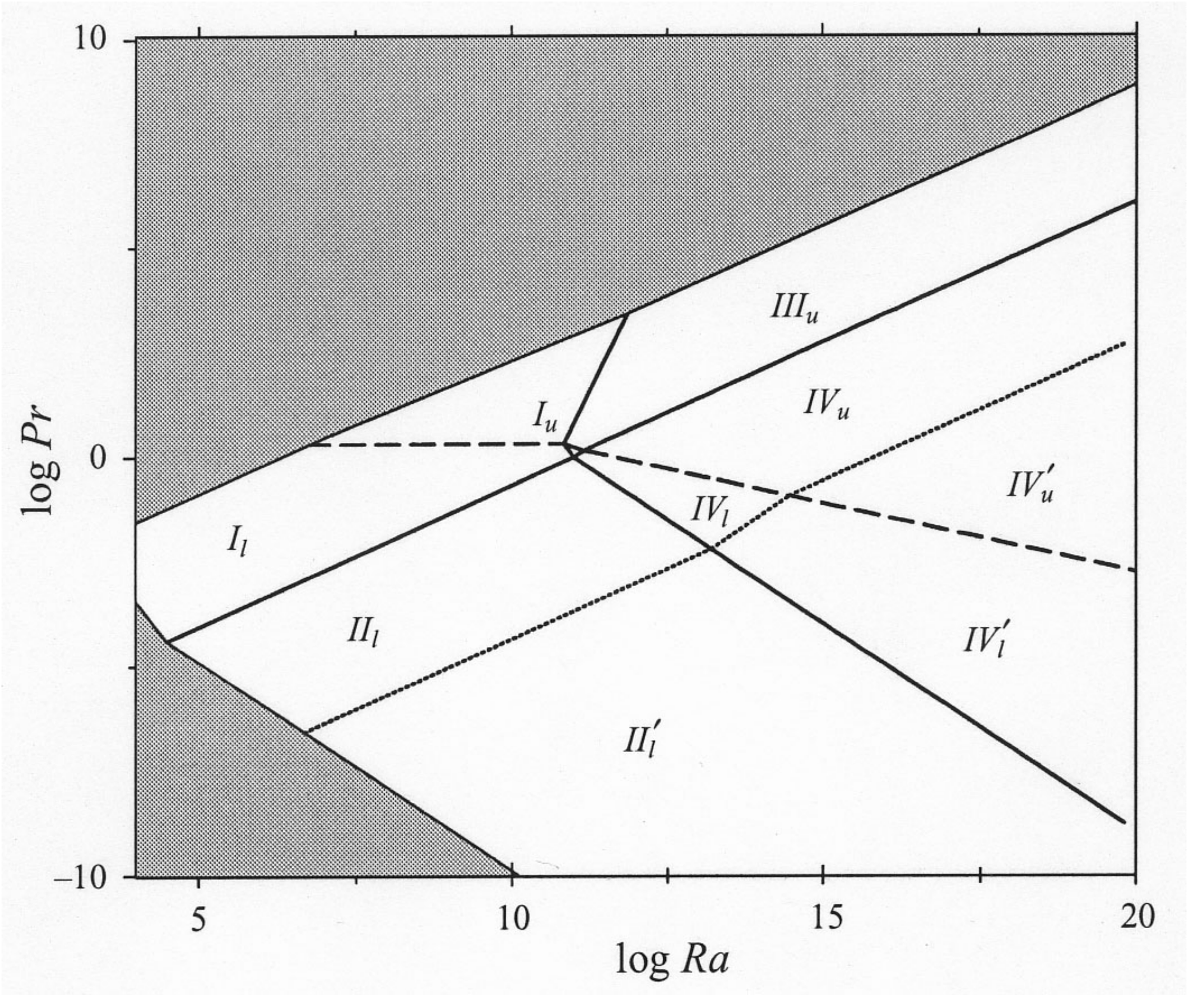
and

Regime II_l, dominance of $\epsilon_{u,bulk}$ and $\epsilon_{\theta,BL}$:
$Nu \sim Ra^{0.2} Pr^{1/5}$, $Re \sim Ra^{0.4} Pr^{-3/5}$

give $Nu \sim Ra^{0.22}$ and $Re \sim Ra^{0.43}$

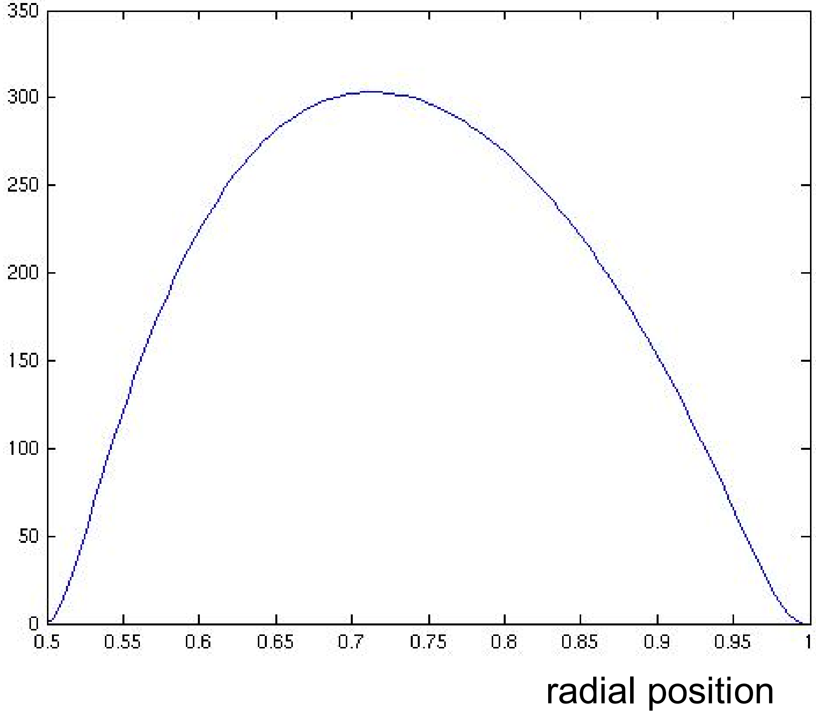
Major assumption and speculation: the theory is indeed ‘universal’, with its four regimes applicable to free convective heat transfer for rotating annulus convection, albeit the delineations may be different.

Regime diagram in Ra-Pr plane (Fig. 2 of Grossmann and Lohse, 2000, JFM)

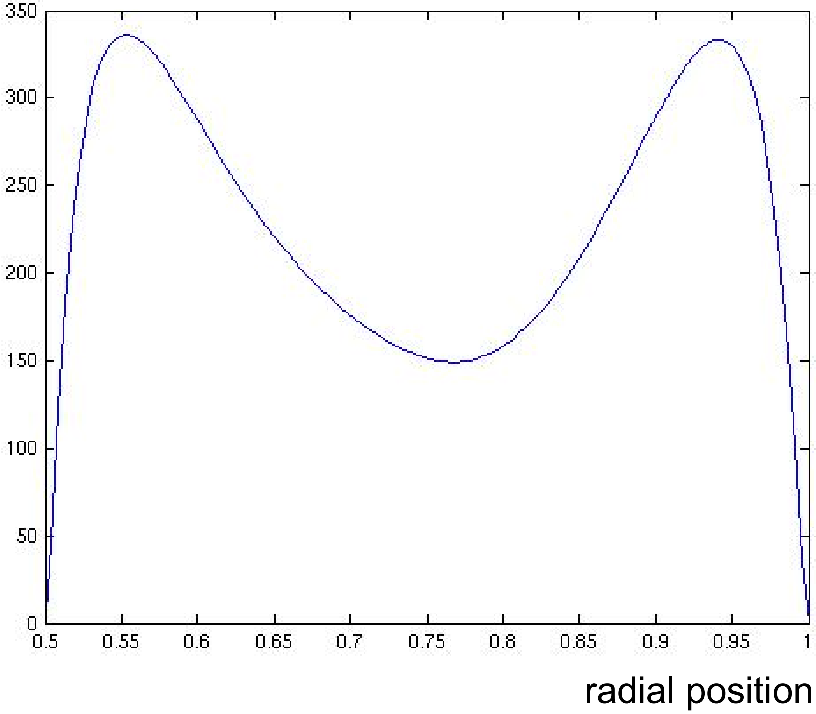


Ra=7x10^6

u_{rms} , u = radial velocity

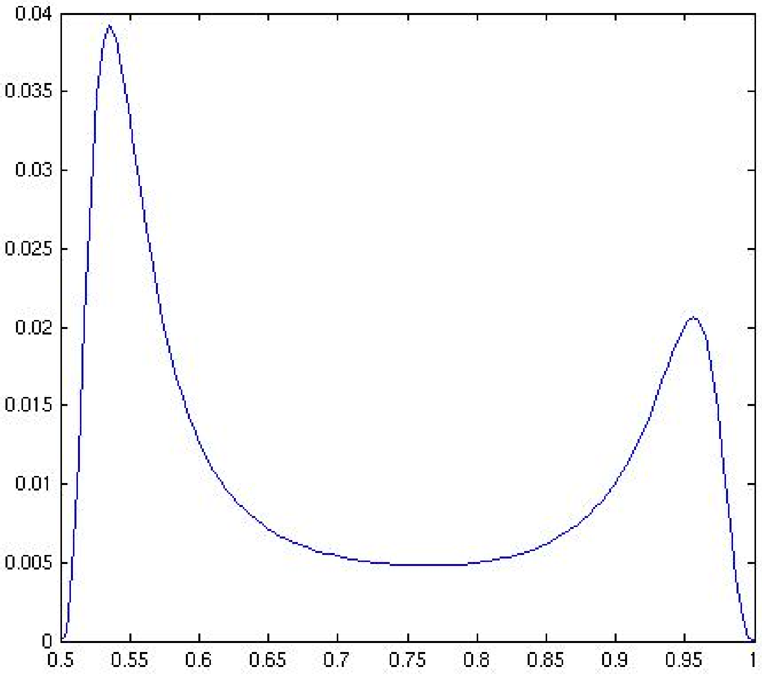


v_{rms} , v = tangential velocity



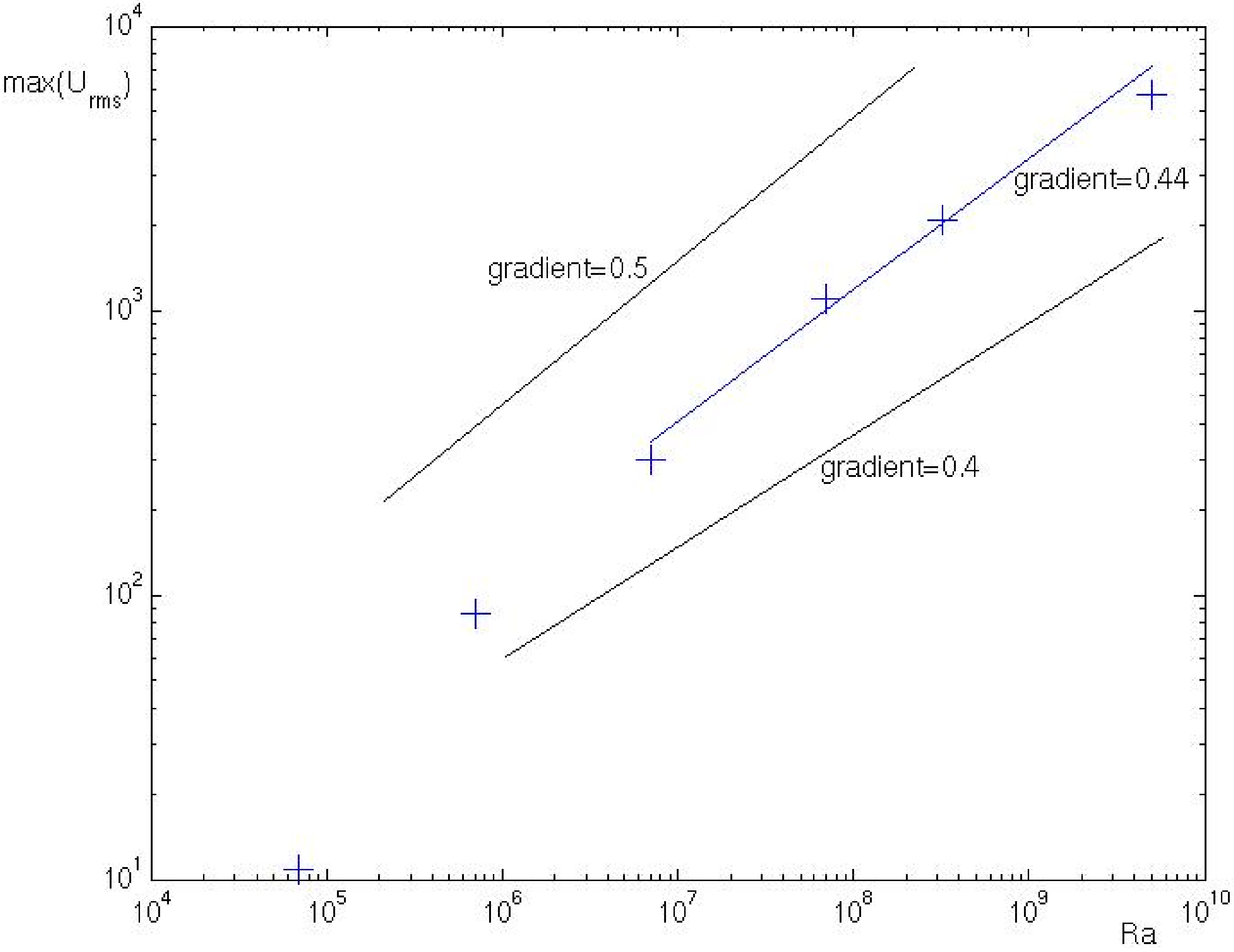
Ra=7x10^6

(T')^2, temperature variance

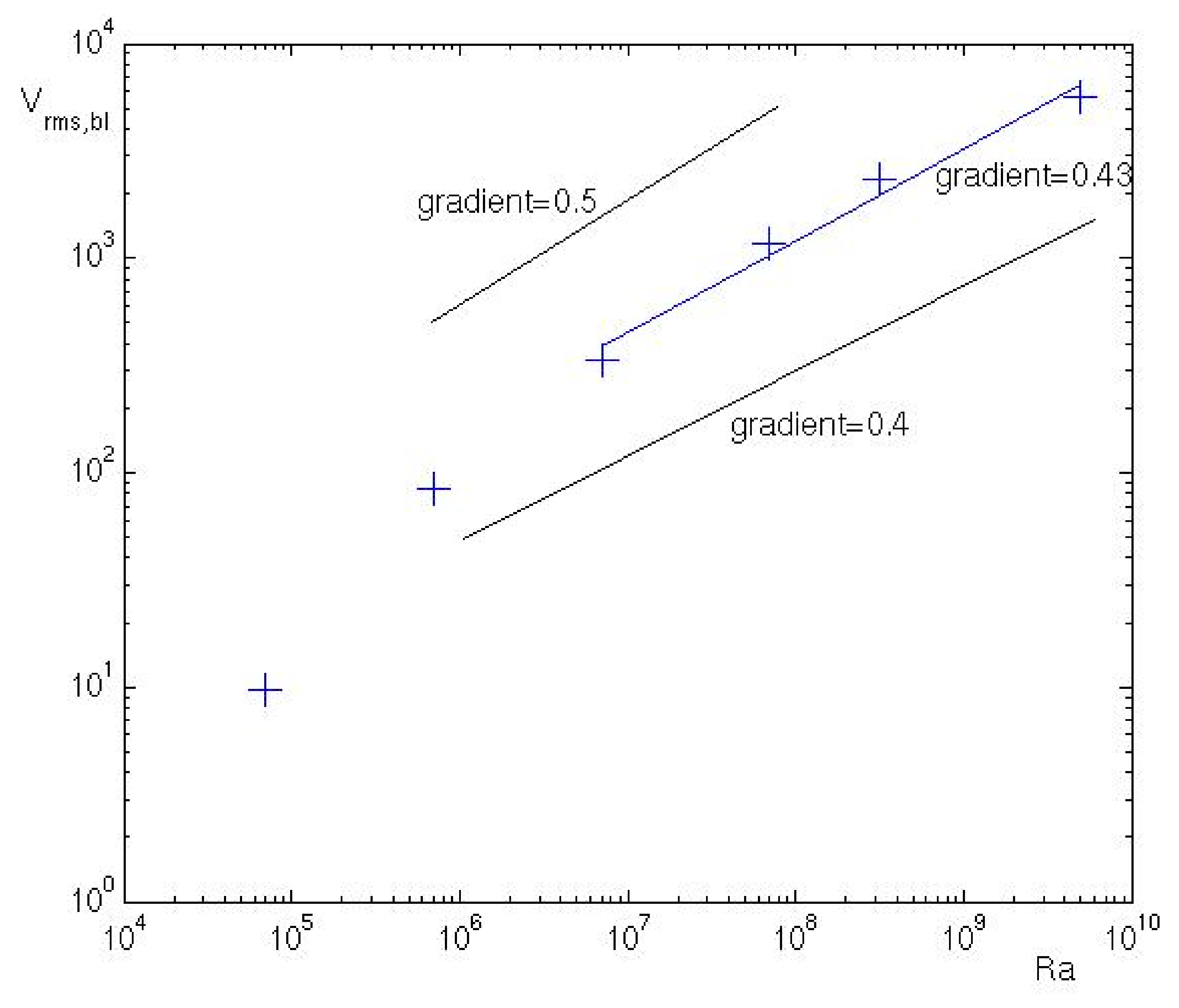


radial position

$\max(u_{rms}) \sim Ra^{0.44}$, GL theory: $Re \sim Ra^{0.43}$



$v_{rms,bl}$~$Ra^{0.43}$, GL Theory: Re~$Ra^{0.43}$



A few questions

1. What is the behaviour of the scaling exponents as higher Ra is reached?
2. What will flows in 3D simulation do? What will be the scalings produced? How are the dynamics of fluid and heat flows different from the 2D results?
3. Are there any additional ways to support or refute the suggestion that Grossmann and Lohse's theory can be used to explain the scalings for rotating annulus convection?
4. In what ways the rotational effects alter the regime diagram of Grossmann and Lohse? What dynamics of the flows are responsible?

Any comments and suggestions are welcome:
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